NBSIR 73-182

Analyses for Decision in the Office of Flammable Fabrics: The Level of the Standard

Technical Analysis Division
National Bureau of Standards
Washington, D. C. 20234

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Alexander R. Craw

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U. S. DEPARTMENT OF COMMERCE, Frederick B. Dent, Secretary

NATIONAL BUREAU OF STANDARDS, Richard W. Roberts, Director

ACKNOWLEDGEMENT

Particular thanks are due Dr. Joseph E. Clark, who as Chief of the Office of Flammable Fabrics and later on Acting Chief of the Fire Technology Division, gave strong support and encouragement throughout the effort represented here.

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EXECUTIVE SUMMARY

This paper serves as an expansion of the brief exposition in Myron Tribus' paper: Decision Analyses Approach to Satisfying the Requirement of the Flammable Fabrics Act. It starts with a decription of the rudiments of a single stage decision problem and how this problem is solved in principle by the formalism of decision analysis. For this one needs to identify explicity what alternatives form the basis of decision, what outcomes result from those alternatives, and what is the probability of each outcome if a particular alternative were in force. For each of the possible outcomes, one has to assign a value or utility to that outcome, and then for each alternative determine the expectation of the utility (disutility) of the outcome. The decision rule then is: Choose that alternative for which the expectation of the utility (disutility) is a maximum (minimum). All of the above is discussed in some detail in the light of a particular example, the level of the standard for children's sleepwear.

For the particular example, in place of having simply the outcomes burn (B) and no-burn (b or B_0), one allows the outcomes to include several degrees of burn injury B_1, B_2, B_3. Also included is whether or not the injured person survives (S) or dies (D), and if he survives, whether or not his body-image area (I_d) is affected or not.

Section 5 is a discourse on several of the possible measures of risk (disutility) that might be chosen. Some are relatively crude, others more sophisticated. The ultimate choice of outcomes may depend on the choice of the criteria to be used in the evaluation part of the decision process.

Note that if one uses the minimum expected number of burns as the measure of risk of the outcomes, this automatically implies that one should set the standard as stringently as possible, and we remark that <u>in this case one need not make any explicit calculation</u>. In addition to discussing what is to be included in the disutility measurement, a discussion is given of the essentials of benefit/cost analysis. If one is unable to complete a formal decision analysis, much of the information generated might be used to perform a conventional benefit/cost study, for example, or to adopt some other method.

Besides assigning a measure of disutility to the outcomes, one must assign the conditional probabilities of arriving at each outcome given that a particular alternative is in effect. It is usually extremely difficult to do this in a straightforward way. One then resorts to an insertion of intermediate events so that the sequence of conditional probabilities will combine to yield the desired probabilities of specifiec outcomes. This procedure is sometimes called "extending the conversation". One attempts to choose a set of intermediate events which make it possible to calculate each of the intermediate conditional probabilities. Section 6 concludes with a listing of the intermediate events suggested for the probability tree for the level of the standard for children's sleepwear. The details for the probability tree are:

1. Six probability trees with variables for each age (2) and income level (3) (3x2=6)
2. Alternatives (a)
3. Technology (T)
4. Nightwear (N)
5. Additional cost to the consumer (C)
6. Use (U) or non-use (u)
7. Existence of a hazard (H)
8. Exposure (E)
9. Ignition (I)
10. Burn (B) and burn-severity (B_i)
11. Survival (S) or death (D)
12. Body Image (I_d)

Section 7 provides discussion of the bases on which one might assign first estimates for each of the conditional probabilities needed. The probabilities that are known are stated. Suggestions are made as to the sources of information that would assist in assignment of probabilities for each item. Unsurprisingly, much of the needed information is not available, especially that dealing with the social and behavioral aspects of the problem. Section 7.8 contains a discussion of the concept of "exposure" to an ignition hazard for which an operational definition does not exist. One suggestion, in the nature of a trial-balloon, is to use a concept of <u>critical distance</u> to define an operationally measurable "exposure".

The paper concludes with a summary of current informational needs necessary to carry out a decision-making analysis. It is determined that the considerable information voids preclude an immediate application of the decision analysis technique to the problem of the level of the standard.

To make the report reasonably self-sufficient, we have summarized the work of M. Tribus and of K. MacDonald, as well as some of the recent work on the assessment of multi-attributed utility functions. These are included as appendices.

List of Symbols: (in order of appearance)

a	= alternative (here a particular level of standard)
0	= outcome
p	= probability
p_{ij}	= prob $(0_j \mid a_i)$=conditional probability of outcome 0_j given that alternative a_i is in effect
DM	= decision-maker
Σ	= summation symbol
u	= utility (also used to denote non-use)
u_{ij}	= $u(0_j \mid a_i) = u(a_i, 0_j)$
	= the utility of outcome 0_j conditional on the alternative a_i
	= expectation operator
OFF	= Office of Flammable Fabrics
FF	= Flammable Fabrics
FFACTS	= Flammable Fabrics Accident Case and Testing System
B	= Burn
b or B_o	= no-burn
B_1	= less than a "major" burn
B_2	= a major burn that is not extensive
B_3	= extensive burn
	Burn severity Levels (B_1, B_2, B_3)
S	= survival
D	= death
I	= ignition or ignite
i	= not ignite
I_d	= an individual's "body image" area is involved in burn

i_d	=	an individual's "body image" area is not involved in burn
n	=	number of burns per unit time
N	=	number in the population under discussion
d	=	disutility
p_{ij}	=	Prob $(0_j \mid a_i)$
HEW	=	Department of Health, Education, and Welfare
HEW-1	=	the first annual report of the Secretary of HEW under the FF Act to the President and The Congress
Shriner's	=	data from Shriner's Burn Institute
Rice	=	"Cost of Illness" by Mrs. Dorothy Rice
C	=	cost (additional) to consumer
U	=	use
u	=	non-use
E	=	exposed
e	=	not exposed
	=	perceived cost
Y	=	family invome
G	=	geographical area
W	=	washability
F	=	feel (or 'hand')
N	=	nightwear garment type
T	=	available technology
H	=	existence of a hazard
Q(c)	=	the demand function for quantity Q at price c.

1. Introduction

1.1 Purpose

This memorandum is intended to serve several purposes. The first is to introduce the rudiments of the formal discipline of Decision Analysis to personnel working in the area of flammable fabrics research and development. It extends in more elaborate detail the relatively brief exposition given by Myron Tribus in his paper: Decision Analysis Approach to Satisfying the Requirements of the Flammable Fabrics Act. An important contribution of that paper was the identification of certain gross information needs necessary for a rational approach (through the mechanism of Decision Analysis) to the problem of determining the level at which a particular standard for flammable fabrics should be set.

A second purpose of this note is to present this detail in the setting of a familiar, specific problem, and one for which some abundance of information is available. The problem of children's sleepwear is especially appropriate since, in addition to information availability, it was the subject of Tribus' original paper; the existing standard will likely be subject to review and/or revision in the future; and the problem is typical (in the sense of mathematical structure and needs) of standard setting for many other flammable fabrics.

The third purpose of this note is to start a formal, detailed decision analysis for the children's sleepwear problem. In this analysis we would hope to specifically indicate our present state of knowledge of the conditional probabilities necessary for the many branches of the probability tree.

This involves identifying what estimates of probabilities are known and possible sources of improvement for those probability estimates which are not available; indications are given of methods for establishing them and the likely sources of information.

Even if a complete decision analysis cannot be performed, much information will have been generated to serve usefully in other methods of decision making, e.g., benefit/cost analysis.

1.2 Caution

The paper is in the nature of a status report and reports on work that is ongoing but incomplete. Emphasis is placed on the logical structure of the analysis and attempts are made to identify those variables that are helpful or necessary to the analysis and for which information (data) need be obtained. While attempt will be made to indicate how the various pieces should be put together, some of the operational methods for doing this need to be tested.

We will not discuss other decision problems in the management area of flammable fabrics. In particular, we will not discourse on the decision problem of determining the priorities for standard setting nor the decision problem on the allocation of research and development in the flammable fabrics area.

2. The Single Stage Decision Problem

In this section we will develop the essentials of the solution of a single stage decision problem. The basic single-stage decision problem can be described generally as one in which the decision maker (DM) must choose a course of action from a set of k alternative actions, call them $a_1, a_2 \ldots, a_k$ which have associated a set of m possible outcomes, call them $0_1, 0_2, \ldots, 0_m$. This situation may be depicted graphically through a tree diagram, called a decision tree. Such a tree for three alternative actions and four outcomes is illustrated in Figure 1.*

If action a_1 is chosen (see Figure 1) the outcome may be either 0_1 or 0_2 or 0_3 or 0_4 since, by assumption, the outcomes are mutually exclusive and exhaustive. The decision maker (DM) usually has some information on the nature of the possible outcomes and part of the procedure of decision theory is to have the DM assign his (subjective) estimate of the probability of each possible outcome; this assignment is made on the basis of the information available to him at the time of assignment. This subjective probability is a measure of the DM's belief that if action a_1 is chosen that the particular outcome will result.

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*For the reader who may want to visualize a concrete rather than an abstract example, we suggest that the alternatives may be viewed as different levels of a standard for flammable fabrics (FF), and the outcomes are different severities of burns. As is done in sections 3 and 4, one may take $0_1 = B_o$ no burn, $0_2 = B_1$ = less than a major burn, $0_3 = B_2$ = a major burn that is not extensive, $0_4 = B_3$ = an extensive burn.

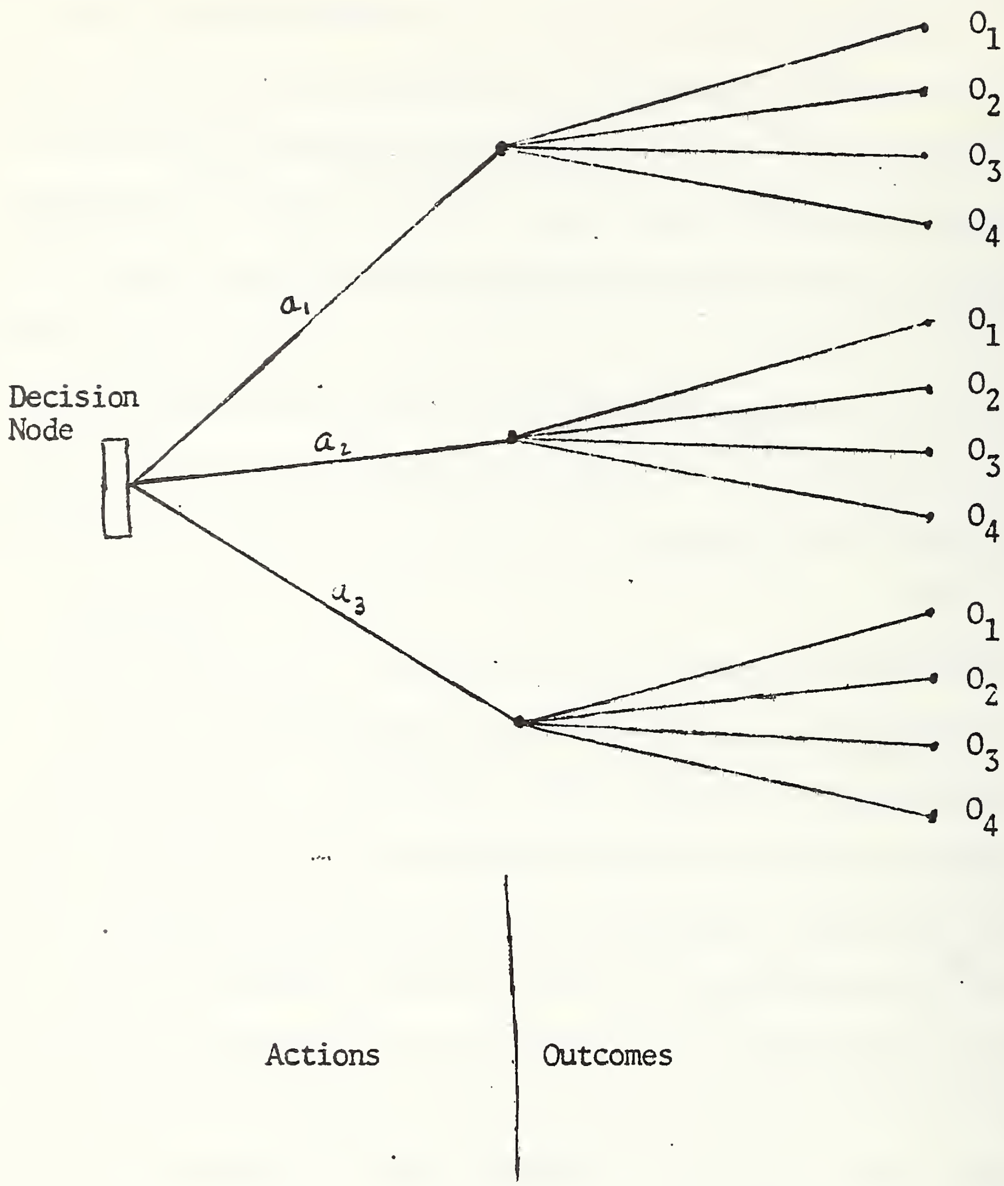


Figure 1. A Decision Tree for k=3, m=4

In our illustrative example we denote the subjective conditional probabilities that the outcomes will be 0_j (j=1,2,3,4) if action a_1 is chosen by $p_{1j} = \text{Prob}(0_j|a_1)$. Since the outcomes are assumed mutually exclusive and exhaustive the probabilities add to unity, i.e.,

$$\sum_{j=1}^{4} p_{ij} = p_{11} + p_{12} + p_{12} + p_{13} + p_{14} = 1.$$

It is convenient to indicate the probabilities on the outcome branches and to identify the outcomes at the tips of the branches (Figure 2).

It is the assignment of probabilities to outcomes that characterizes decision theory as decision making under uncertainty.

We remark that the information displayed in Figure 2 may also be displayed in matrix form (see Figure 3).

One of the difficult procedure problems in the application of decision theory is that of getting the DM's assignment of probabilities of outcomes. There are several operational ways of doing this, but for the time being we will defer discussion of these until we get to the concrete problem to be studied in detail.

In order to choose between the various actions or alternatives the DM must have some way of placing a value, worth or preference for each of the outcomes. If the decision problem is in a business context, monetary values of the outcomes are usually used to make the decision. If monetary value is used, there is a question of whether or not the DM

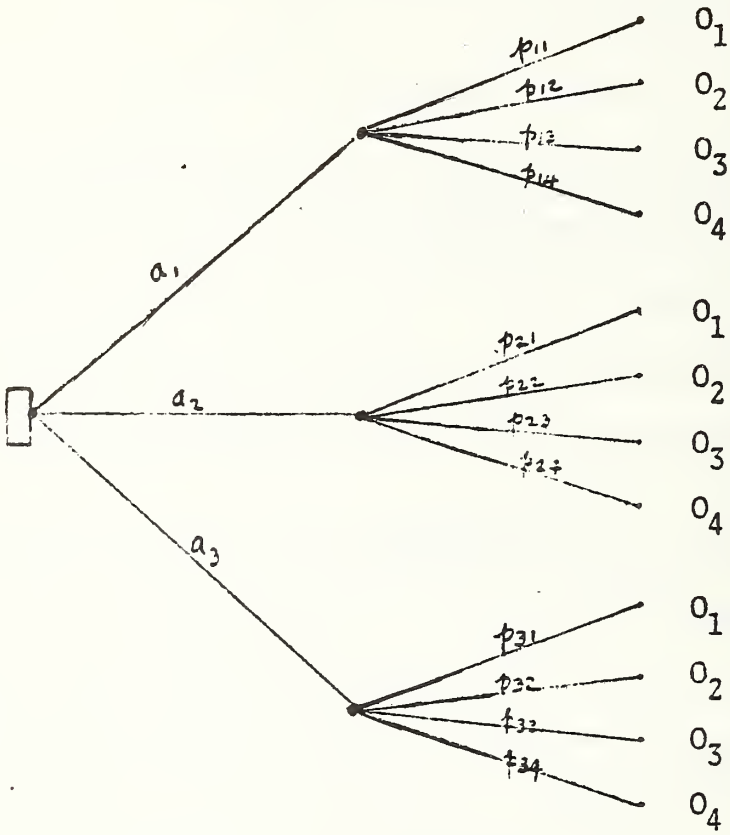


Figure 2. Probabilities on Decision Tree

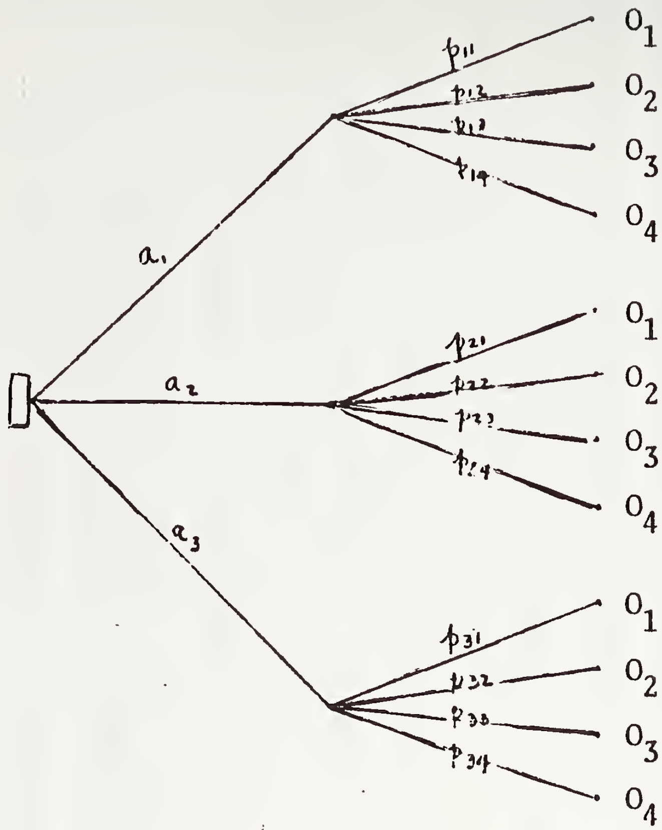
Action \ Outcome	o_1	o_2	o_3	o_4	Σ
a_1	p_{11}	p_{12}	p_{13}	p_{14}	1
a_2	p_{21}	p_{22}	p_{23}	p_{24}	1
a_3	p_{31}	p_{32}	p_{33}	p_{34}	1

Figure 3. The Probability Matrix for Three Actions With Four Outcomes

is a risk taker or a risk averter and this brings up the question of the utility of money to the DM rather than its face value. We will not pursue this point at this time.

In social problems, like that of consumer protection, the valuation will depend on a number of factors or attributes and recourse usually is made to the application of multi-attributed utility theory. This theory is based on a number of postulates that most rational persons are willing to accept, such as: if outcome A is preferred to outcome B, and outcome B is preferred to outcome C, then outcome A is preferred to outcome C. For the present discussion we will skip over the difficult problem of assessing a multi-attributed utility function and assume that the valuations have been made. We denote the utility of the outcome O_j conditional on the alternative a_i as $u(a_i,O_j)$ or simply as u_{ij}. In the graphical display of the decision problem it is convenient to write the valuations of the outcomes alongside the outcomes (see Figure 4). Correspondingly, we may display the utilities in a utility or "payoff" matrix (Figure 5).

Once the probabilities and values of outcomes have been assigned and determined, the DM is in a position to act. The usual decision rule is: Choose that alternative for which the expected utility of the outcomes is an optimum. The optimum is either a maximum or a minimum depending on whether the valuation is considered as a utility or disutility. The expected utility of the outcomes for a given act is obtained



$u(a_1,O_1) = u_{11}$
$u(a_1,O_2) = u_{12}$
$u(a_1,O_3) = u_{13}$
$u(a_1,O_4) = u_{14}$

$$\mathcal{E}\, u(O|a_1) = \sum_j p_{ij}\, u_{ij}$$

$u(a_2,O_1) = u_{21}$
$u(a_2,O_2) = u_{22}$
$u(a_2,O_3) = u_{23}$
$u(a_2,O_4) = u_{24}$

$$\mathcal{E}\, u(O|a_2) = \sum_j p_{2j}\, u_{2j}$$

$u(a_3,O_1) = u_{31}$
$u(a_3,O_2) = u_{32}$
$u(a_3,O_3) = u_{33}$
$u(a_3,O_4) = u_{34}$

$$\mathcal{E}\, u(O|a_j) = \sum_j p_{3j}\, u_{3j}$$

Actions | Probabilities of Outcome | Outcomes | Utilities | Expected Utilities

Figure 4. Tree Diagram for Analysis of Decision between Three Alternatives with Four Possible Outcomes

Action \ Outcomes	O_1	O_2	O_3	O_4
a_1	u_{11}	u_{12}	u_{13}	u_{14}
a_2	u_{21}	u_{22}	u_{23}	u_{24}
a_3	u_{31}	u_{32}	u_{33}	u_{34}

Figure 5. Utility or "Pay-Off" Matrix

by multiplying together the corresponding probabilities and utilities and adding over the set of outcomes.

$$\mathcal{E}\, u(0|a_i) = \sum_j p_{ij}\, u(a_i, 0_j) = \sum_j p_{ij}\, u_{ij}$$

In outline form the decision process may be summarized:

a) Determine the set of alternatives, $A = \{a_1, \ldots, a_k\}$

b) Determine the outcome set, $0 = \{0_1, 0_2, \ldots, 0_m\}$

c) Assign the DM's subjective probabilities for the outcomes, i.e., $p_{ij} = \text{Prob}(0_j | a_i)$

d) Evaluate the utility of the outcomes: $u(a_i, 0_j) = u_{ij}$

e) For each act, determine the expected utility of the outcomes, i.e., $\mathcal{E}\, u(0|a_i)$ for all i from 1 to k

f) Choose that act for which the expected utility of the outcomes is an optimum.

3. The Level of the Standard

In this section we recognize that the problem of the level at which to set the standard for a flammable fabric, in particular that for children's sleepwear, is a single stage decision problem, so that in principle, at least, we know how to go about the solution of the problem.

In principle the problem is solved once we have determined the valuations of the outcomes and the conditional probabilities of each possible outcome, i.e., the u_{ij}'s and the p_{ij}'s.

Our first problem is to delineate the set of possible alternative standards. In order to keep the problem within bounds some preliminary elimination of alternatives is essential. This process of elimination is sometimes colorfully referred to as "pruning the tree" to avoid a bushy mess.

As a start, the following three standards representing different levels of stringency, have been suggested:

a_1 = Present Commercial Standard CS191-53. In this test specimens are at an angle of 45° and the measured burning distance is 5 inches. A small flame impinges on the surface of the fabric for one (1) second. The fabric must not ignite and burn a 5-inch length in less than four (4) seconds.

a_2 = The so-called Blanket Flammability Test. This is a resistance to ignition test. The test specimen is 2½ inches in diameter. A small flame impinges on the center of the specimen for one second. The sample must not ignite, i.e., allow the establishment of a self-supporting flame.

a_3 = Vertical Test DOC FF3-71. Specimens are 3½ x 10 inches and are hung vertically in the test chamber. A flame source impinges on the bottom edge of the specimen for 3 seconds. Pass-fail criteria depend on char length of specimen and time of burning of drips or other fragments on the base of the cabinet.

The alternative a_1 is considered to be a minimal test, and is one which must be satisfied by all fabrics used for clothing in the United States. a_2 may be considered as a new baseline, while a_3 is very stringent. a_3 is the current standard for children's sleepwear.

4. Outcomes

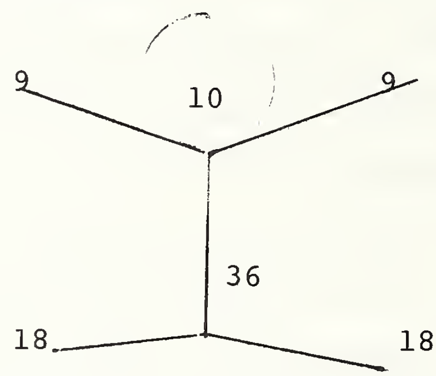
The stated objective of the OFF is to prevent death and reduce injuries due to FF. Here injury is interpreted as a burn injury. Tribus, in his expository paper, used burn (B) or no-burn (B_o), and suggested that in a more elaborate analysis one might want to take into account the level or severity of the burn. This seems to be a worthwhile suggestion.

The degree of burn injury is a continuous variable, but we will discretize this to a number of levels devoted by B_o, $B_1, \ldots, B_b$ in increasing level of severity. One might think of these as midpoints (or other representative points) of some class interval and that we are representing each entire class by a single member.

Several injury classification schemes were considered. After reviewing the burn-injury literature and discussions with physicians who treat burned individuals, and thinking ahead to the valuation of the outcomes, it seems advisable to adopt a classification scheme based on the depth of the burn and the total body area subjected to second and third degree burns. Sometimes this is referred to as Total Extent of Burn and is written (3°+2°). Most important, such data are available in the Flammable Fabrics Accident Case and Testing Systems (FFACTS), so there is hope of quantitative assessment.

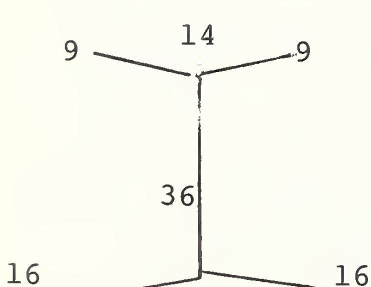
The depth of burns is divided into first, second, and third degrees. First degree burns have a redness; second degree, blisters; and third degree, full thickness injury. First second degree burns are partial thickness injury and will heal spontaneously, if infection does not supervene. Third degree is brown or gray, has considerable resiliency and is dry and anesthetic. The second degree burn, however, is red, wet soft and pliable: and sensation is usually present. (Larson, Nebraska Med. J., October 1969).

At the Shriner's Burn Institute at Galveston, Texas and at many other hospitals an estimate of the areal extent of burn injury is quickly evaluated by using the "rule of nines" (see diagram for age 15 years)

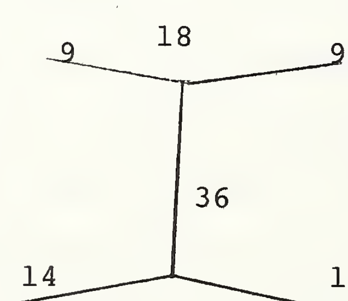


(Total is 100)

For younger children, the ratio of the sum of head and leg areas to the total body area is larger. For children of other ages the rule has been modified as indicated:



5 years



1 year

Note that 4 percent added to the head each time, whereas 2 percent is subtracted from the leg. (Larson)

The term "major" burn is used if more than 15 percent of body area has second and third degree injury, while the term "extensive" burn is used if 35 percent or more of body area is involved. (Ref: Quindlen and Abram, Southern Med. J., 1969). This suggests, as a start, that we adopt the following categories and symbols for burns:

B = burn

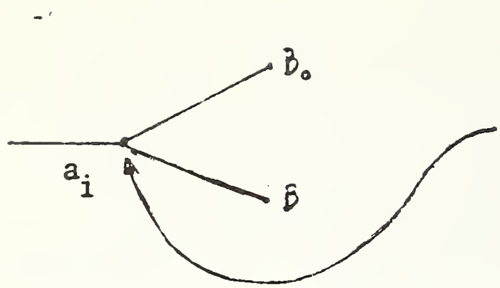
B_o (or b) = no-burn

B_1 = total extent of burn area less than or equal to 15 percent = less than a "major" burn

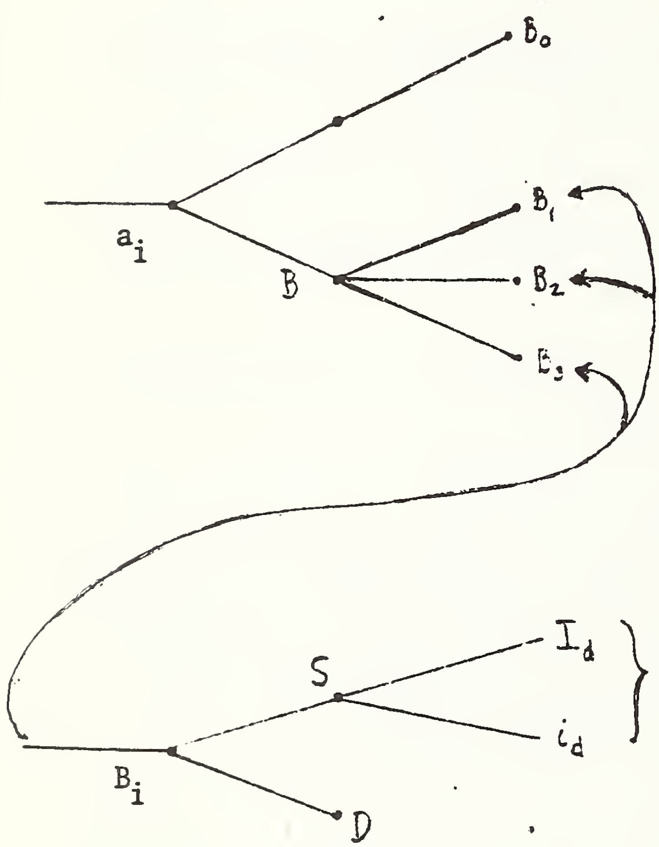
B_2 = total extent of burn area more than 15 percent but less than 35 percent = a major burn that is not extensive

B_3 = extensive burn = 35% or more of body area covered with second and third degree burns.

The economic "cost of illness" depends on whether or not the injured person survives or dies, and data is available on which to estimate the probability or survival (or death) by age, race, of persons who have suffered a given body area burn. We would like to allow for the outcomes the joint event of survival(S) for a person suffering a burn of severity level B_j. We will write this joint event as SB_j or B_jS, the order of symbols being unimportant here. Also let D denote the event of death.



Most likely will want to "Extend the conversation" to calculate $P(B|a_i)$ as Tribus did.



Note: these branches not included if psychic effects are ignored in evaluation.

Figure 6. "Extending the Conversation" for Outcomes

If we are to consider the psychic "cost of illness", psychiatrists advise that special psychic problems are encountered by those who suffer burns to the head, face and upper chest area, which effectively account for the individuals "body image"...this corresponds to the concept of "me, as a person," and that it appears to radiate outward; to a lesser extent the hands have importance. While the symbol I will be used to denote ignition (following Tribus' notation), it seems appropriate to use the symbol I_d if the person's body image area is involved in the burn injury, and to use the symbol i_d if it is not.

The ultimate choice of the outcomes to be considered will depend upon the choice of the criteria to be used in the evaluation part of the decision process. We take up this matter next.

5. Measurement of Worth (Risk) of Outcome

It is possible to provide a number of measures for the worth (risk) of outcomes, some very crude while others may possess a high degree of sophistication depending on the level of detail to be included.

5.1 Number of Burns

If we restrict the outcome level to that of burn (B) or no-burn (b), we can use the number (n) of burns per unit of time as a crude measure of worth (risk), and we could pick that level of standard, a_i for which the expected number of burns was a minimum. Our problem would then be resolved if if we could (1) determine the number N of the population at stake per unit of time and (2) determine the conditional probability of a burn if alternative a_i is chosen, since

$$\hat{\varepsilon}(n|a_i) = N\ p(B|a_i)$$

i.e., the expected number of burns conditional on the adoption of a_i equals the number in the population times the probability of a burn if alternative a_i is chosen. Tribus discusses a number of factors affecting the calculation of the probability of a burn, and we shall go into this matter in some detail in the Section 6.

We remark here that $(n|a_i)$ and $p(B|a_i)$ are on one-to-one correspondence above and we could take $p(B|a_i)$ as a measure of the "risk" involved (following the approach of Chauncey Starr*) in adopting the alternative a_i. The difficulty here is that we do not know whether the level of risk as measured by $p(B|a_i)$ is acceptable or not. We do, however, have the option of comparing these probabilities with those of other acts for which the public is at risk.

Note that if one uses the minimum expected number of burns as the measure of risk of the outcomes, this automatically says that one should set the standard as stringently as possible, and we remark that <u>in this case one need not make any explicit calculation</u>.

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*Starr, Chauncey (1969): "Social Benefit versus Technological Risk," Symposium on Public Safety, National Academy of Engineering, May 1, 1969. (See also, SCIENCE, 19 September 1969, Vol. 165, No. 3899, p. 1232-1238.

Another option mentioned by Tribus is to determine the curve of expected numbers of burns vs. expected increased cost to produce (this being the increased dollar cost to produce a garment) but indicates that this still begs the question of where is it reasonable to put the standard.

5.2 Disutility

If we proceed to consider outcomes that include severity levels of burn injury such as B_o, B_1, B_2, B_3 we could use the probability tree approach to calculate for each alternative each of the probabilities $p(B_o|a_i)$, $p(B_1|a_i)$, $p(B_2|a_i)$ and $p(B_3|a_i)$ and the expected number of burns of each severity level $n(B_o|a_i)$, $n(B_1|a_i)$, $n(B_2|a_i)$ and $n(B_3|a_i)$. Now, however, we are reluctant to add up these numbers and use the total number of burns as a measure of risk. Since we are considering classes of burn-injuries as outcomes it seems natural to consider these outcomes as undesirable events and to associate a degree of disutility or undesirableness with the outcomes. Since the symbol u is customarily used to denote utility, we choose here to designate the disutility of an outcome by $d(0)$ or $d(0|a_i)$ if the outcome is conditional. By the mere fact of introducing severity into the problem we recognize disutility varying with severity. We want to weight the severity levels in some way, i.e., we want to assess the disutility associated with each outcome.

If we consider the joint events B_jS and B_jD as outcomes, in effect we are recognizing that the events have different disutilities and want to take this into consideration. By further breakdown of the outcomes into compound events, such as B_jSI_d or B_jSi_d, we are taking into account that the disutilities of these events differ from that associated with B_jS.

By reverse token, if one cannot assess the difference in the disutilities between the events B_jS and B_jSI_d, there is no advantage to include the higher order outcome in the decision tree.

No matter what level of detail is considered for the outcome it is possible to use the probability tree approach to determine each $p(0_j|a_i) = p_{ij}$, i.e., the probability of the j^{th} outcome given that alternative a_i is in effect and also the corresponding expected number $n(0_j|a_i)$, items of interest in their own right.

The above probability tree might be referred to as an "Outcome Tree" or as a "Result Tree". A separate but similar "Cost Tree" might be constructed to examine the costs (in whatever coin) that would have to be paid to gain the increased protection. Here as in many public decisions the benefit and costs accrue to different people or classes of people.

5.3 Assessment of Disutility (loss) - The Cost Tree

One must decide on what factors (attributes) are to be considered in the disutility (or loss) function and to whom the disutility accrues. For example, McDonald's loss function represents the average financial loss to a single individual as a consequence of O_j, given action a_i. The value of the function is the sum of (1) the individual's loss of earnings (mortality or morbidity), (2) hospital costs (3) costs of physicians services, and (4) the cost to implement the action a_i. For mortality losses McDonald used the present value of lifetime earnings, discounted at a 6% rate.

We remark here that these are economic losses and losses to a single individual. They are quantifiable. But they are not necessarily exhaustive. Practioners of benefit/cost analysis in the health and safety area theorize on what should be included but often do not give any practical methods for quantifying these. They also refer to losses due to intangibles, primarily pain and suffering associated with injury, but usually do not evaluate these in the analysis. (Mishan, Wiederkehr).

We choose to consider the disutility to an individual in the population who might be subject to burns from use or non-use of children's sleepwear, in some specified age group, say 5 years or younger. Other age groupings can be cared for in the analysis as well. Actually, while we talk of an individual, we will have in mind the members of the individuals immediate family, particularly the parents or guardians (who are the decision or rule makers for the young children). We will refrain from considering a personal individual, such as your child, but will consider a statistical or representative individual from the population. Although, for any particular property such as psychological effect, there is a distribution of individuals within the particular age class, when one views these from the point of view of the population of all people, the individuals in one age class are viewed as fairly homogeneous, especially if we consider such factors as the loss of potential earnings, treatment costs, etc.

With regard to intangibles, physical pain in an individual is an extremely subjective phenomenon, but is usually restricted to some relatively short period of time, the treatment period, in comparison to the time of psychological effects. We are currently examining the literature on psychological effects associated with burn injuries in the hope of at least identifying qualitatively the major effects. We have found that in addition to the individual's immediate family; there are considerable psychological problems encountered by those who are involved in the medical care of the burned children, in particular nurses, doctors, and physical therapists.

These are the other intangibles that have to be presented to the decision maker in the prose which accompanies the quantification, and we would like to include some allowance for the disutilities of the intangibles, even if only crudely. We have looked at several possible approaches for doing some quantification of these factors which others have classed as intangibles. Discussion of these is contained in Appendix B. In particular we give there a discussion of some of the recent results in the methods of the assessment of multi-attribute utility functions.

We will not include in the discourse other losses to society as a result of outcomes.

5.4 The Disutility to the Manufacturer

To this point we have included the consumer and the public in the measurement of disutility of the outcomes. In some way or other we would like to take into account the effect on the manufacturing industry (manufacturers and their employees) of the act of setting the standard at a particular level.

Any change in the standard level will undoubtedly cause a change in the cost of manufacture of fabrics, which eventually will result in a change in cost to the consumer who purchases the fabric (or garment).

The law requires that the level of the standard set should be technically feasible and reasonable. We assume here that in order that an alternative standard (a_i) be under consideration that (a) there exist a technological means for the manufacturing industry to meet the standard, (b) that there is a way of estimating the cost to the manufacturer to comply with the standards and (c) that there is a way of transforming this cost into a cost to the consumer for the unit of time under consideration in the valuation of outcomes.

There are then at least two possible ways to proceed in selecting the level of the standard:

Introduce a cost variable in the probability tree for the calculation of the probability of each possible outcome and (1) use the expected disutility of the outcomes to the consumer public as the decision rule for the selection of the level of the standards, or (2) use the probability tree to calculate the expected numbers of lives saved and injuries (of different severities) avoided, treat them as benefits, and then calculate a benefit to cost ratio for each alternative standard and perform a conventional benefit/cost analysis.

The first alternative has been the topic of discussion of this note to the present. The theoretical discussion of this methodology will be complete when we discuss the detailed make-up of the structure of the probability tree in Section 6.

5.5 Benefit/Cost Analysis

The benefit/cost ratio is simply the present value of all benefits expected to be realized over a selected period of time attributable to a given standard divided by the present value of all costs expected to be incurred over the same time period and attributable to the same standard. It is traditionally assumed that money (dollars) can serve as a common unit of measurement of all benefit variables as well as of all costs. The costs reflect actual losses to society resulting from the accidental events.

Costs are measured by (1) expenditures on the part of government to develop, promulgate and enforce standards programs and (2) expenditures on the part of consumers with respect to the purchase and use of flammable fabrics and garments. In this problem we may treat governmental expenditure as fixed, i.e., federal expenditures do not vary significantly with the output of rule making actions.

The consumer cost component includes not only the manufacturer and/or suppliers cost but also reflects any additional expenditures on the part of the user and any additional costs that are borne by society as a whole that are caused by and attributable to the rule making action, e.g., the need to take special precautions in the laundering or cleaning of the fabrics, or the loss of sectors of employment due to displacement of workers due to change in technologies.

The benefit/cost ratio may be used to rank the alternatives, i.e., the greater the ratio the higher the ranking. Caution should be exercised in the use of the ratio alone as a measure of program worth for in certain instances it may lead to a choice that does not maximize the net benefit (i.e., benefits minus costs) to society.

5.6 Factors to be Included in "Cost" (Disutility) Assessment

We list the factors that we will consider in the assessment of disutility, and indicate possible sources of information for data as well as some comments on each of the items. HEW-1 will be used to designate the first annual report of the Secretary of Health, Education, and Welfare under the Flammable Fabrics Act to the President and the Congress: Studies of Death, Injuries, and Economic Losses

Note: This discussion of benefit/cost analysis reflects the summary of this topic by Dr. Harris Hordon in an unpublished working paper for the National Highway Traffic Safety Administration (NHTSA).

Resulting from Accidental Burning of Products, Fabrics or Related Materials (Sept. 1965-Feb. 1969); (undated).

Shriners will designate data available from the Shriners Burn Institutes. Rice, is used to designate "the HEW Report", "The Cost of Illness" by Mrs. Dorothy Rice.

1. Hospital Costs (HEW-1, pp. 48-49 and data from Shriners Burn Institute.

2. Costs of physicians services (HEW-1, Shriners).

3. Individual's loss of earnings - mortality (Rice, p. 93).

4. Individual's loss of earnings - morbidity (HEW-1,p. 56).

   Note: This item will most likely be omitted for children.

5. Transportation of victims or family or both to and from hospital (Shriners).

   Note: Data may be difficult to obtain; not in HEW-1.

6. Costs of subsequent hospital visits. (Shriners)

   Note: This will vary with severity levels. Primarily elective surgery, skin grafting, prosthetic devices, psychiatric treatment.

7. Cost to implement action (government agencies).

   Note: This may be considered as fixed across alternatives. Some estimates should be made to ascertain magnitude.

8. Costs to manufacturer.

   Note: This will probably require an independent study. Needed to Determine "Reasonablenes " and also additional costs to individual.

9. Purchase cost to individual.

   Note: Also needed for calculation of probabilities of outcomes.

10. Intangibles:

   Note: Here we feel the primary intangible is the psychic effect.

We feel that physical suffering ("pain") has been measured to a large extent in part of the economic valuation, particularly in hospital costs and costs of physician's services. We will try to at least discuss the psychic effects qualitatively. We will investigate the possibility of use of newly developed techniques of assessment of multi-attributed utility functions (see Appendix B).

6. Probabilities of Outcomes

6.1 Extending the Conversation

In Section 2 we denoted the probability of the outcome 0_j conditional on the selection of alternative a_i by p_{ij} = Prob $(0_j|a_i)$. A major portion of the work in decision analysis is over once we have arrived at estimates for the national values of the p_{ij}'s that are acceptable to the decision maker. But arriving at these estimates directly is almost an impossible task, and we look around for means of accomplishing our goal.

This is done by introducing into the problem auxiliary events for which the estimation of the probabilities of the auxiliary events is more easily accomplished, and from which, by using the laws for the combinations of probabilities, it is possible to calculate the p_{ij}'s. This process is sometimes referred to as "Extending the Conversation".

In his previous work on this problem Tribus introduced the auxiliary variables of additional costs (C_j), use (U) or non-use (u), exposure (E) and ignition (I) to calculate the probability of a burn (B) conditional upon the adoption of the level of standard a_i by

$$p(B|a_i) = \sum_{\substack{\text{sum over} \\ \text{all } j}} p(B|UEIC_ja_i)\ p(I|UEC_ja_i)\ p(E|UC_ja_i)\ p(U|C_ja_i)\ p(C_j|a_i)$$

$$+ \sum_{\substack{\text{sum over} \\ \text{all } j}} p(B|uEIC_ja_i)\ p(I|uEC_ja_i)\ p(E|uC_ja_i)\ p(uC_ja_i)\ pC_j|a_i)$$

This somewhat formidable collecting of symbols can be depicted graphically by a"probability tree" (see Figure 6). The terms following the summation signs represent the probability of a burn following a particular path through the tree from a_i to B; the total probability is the sum of the probabilities over all the paths from a_i to B. Tribus also gave an indication of the interpretation of each term and how it might be determined. For example, "the term $p(I|UEa_iC_j)$ represents the probability that a fabric will ignite when exposed to a flame. This probability is reckoned conditional on the truth of UEa_i (C_j is immaterial here), and a study of the various conditions of exposure as gleaned from field studies correlated with laboratory tests will enable an assignment of this term".

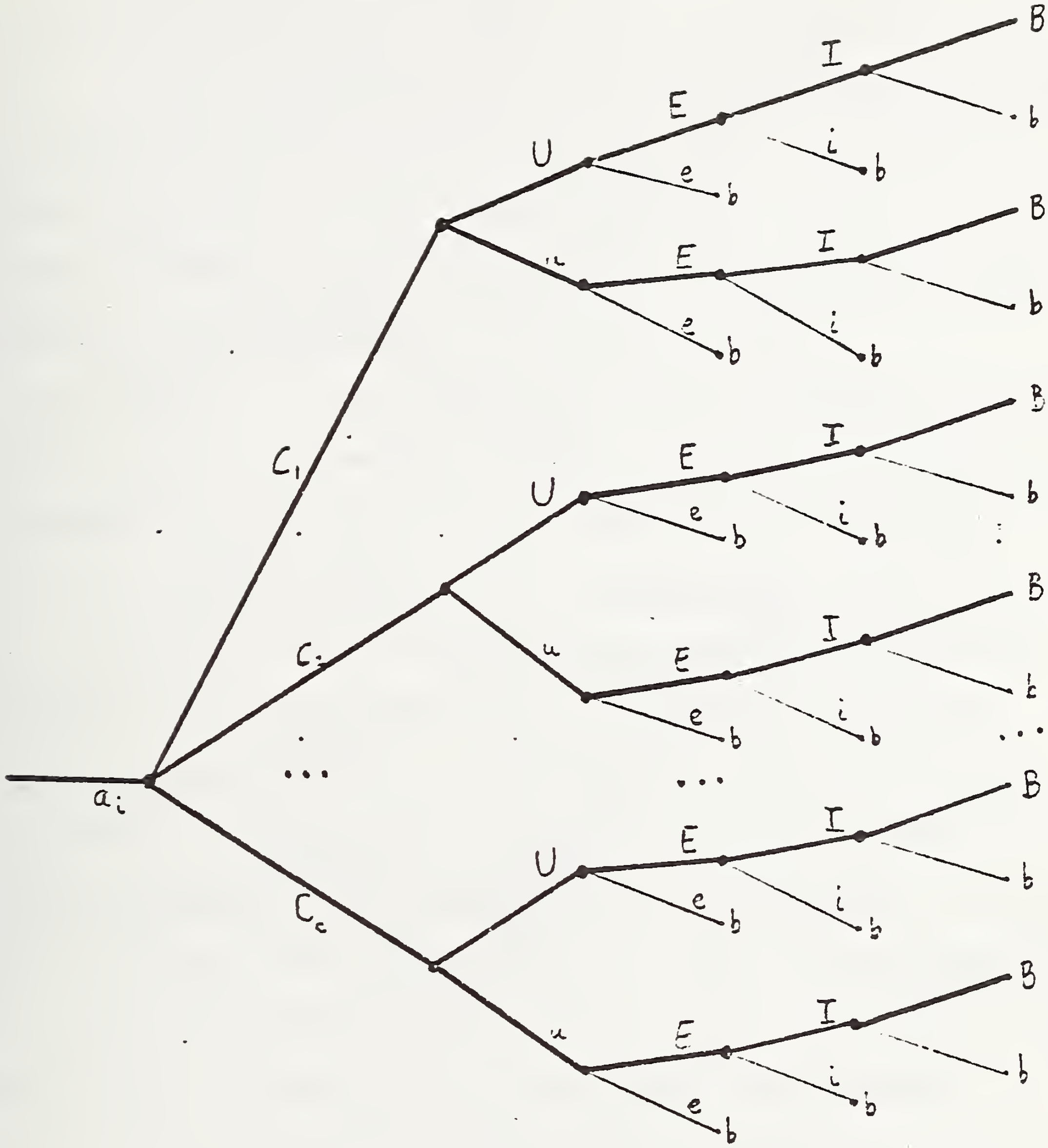


Figure 7. Probability Tree for Prob($B|a_i$)

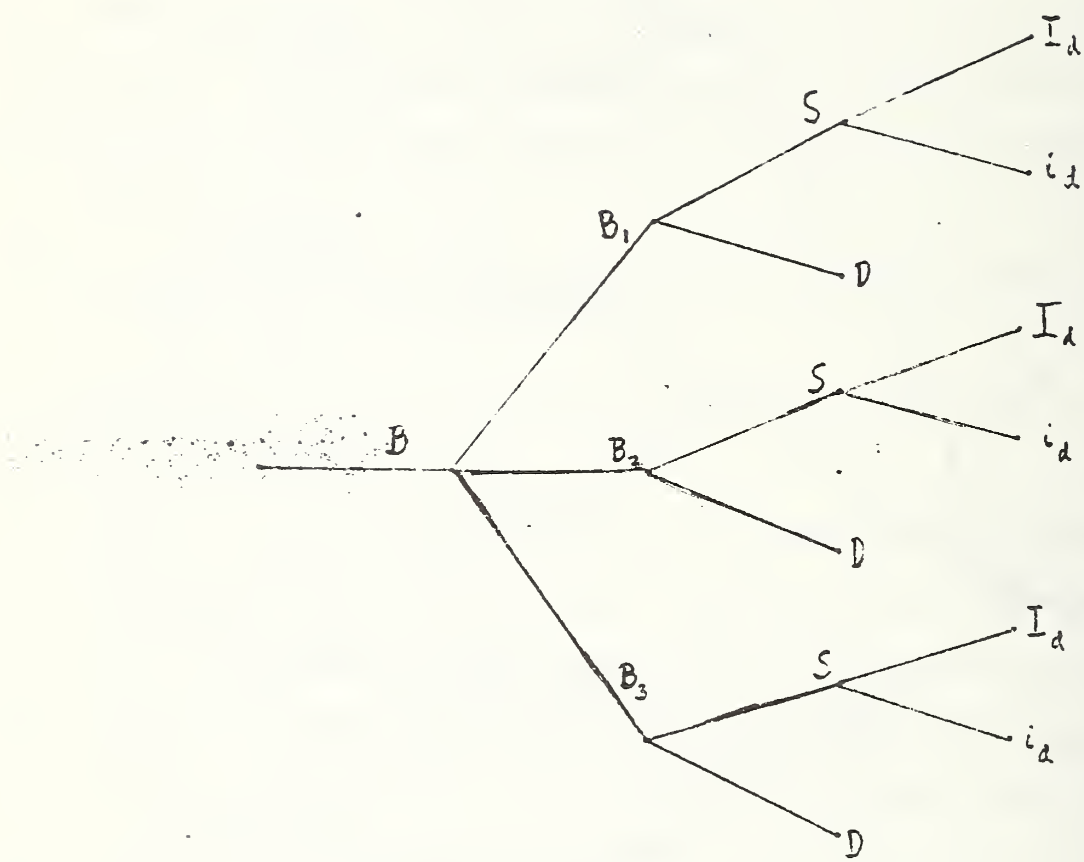


Figure 8. Extending the Tree for Outcomes

In Section 4 we chose to include several categories of outcomes by severity burn levels, which we denoted by B_1, B_2, and B_3. We can determine each of the probabilities $\mathrm{Prob}(S|a_iB_j)$ by extending the probability tree from each B to B_j (j=1, 2, 3) and then extending to S or D from each B_j.

Other variables can be introduced to facilitate the calculation of the probability, $\mathrm{Prob}(0_k|a_i) = p_{ik}$.

For example, in the case of children's sleepwear, we might ask the question of how the manufacturers might possibly meet a standard by using different materials and/or techniques. For convenience let us call these <u>technologies</u>, and let T_1, $T_2,\ldots,T_t$ denote the totality of technologies available to the manufacturers to comply with the candidate standards a_1,a_2,a_3. As an example, T_1 might denote the use of cotton fabric alone; T_2 might denote the use of cotton treated in some particular way; etc. We then ask the question: If standard a_1 is chosen, what fraction of (or what is the probability that) the children's sleepwear will be made using each of the technologies? It may well turn out that, for example, if standard a_i is chosen the probability of compliance with T_1 is zero, the fraction of garments using T_2 is .30, etc. The fractions T_1, $T_2,\ldots,T_t$ may differ for different levels of standards. For convenience, let

$$\mathrm{Prob}(T_k|a_i) = t_{ki}$$

denote the probability of a garment manufactured using T_k given that standard a_i is adopted. If we know what particular

T_k is being considered, we may be in a better position to estimate the probability of outcome 0_i.

It is suggested that the T variable be inserted between a_i and C in the tree. We must remember that our aim is to facilitate the calculation of the p_{ij}'s and that we are at liberty to do this in many ways. The more intermediate variables we introduce into the discussion the greater is the number of intermediate conditional probabilities that have to be estimated. So it behooves us to introduce as few intermediate variables as possible and to choose those variables which lead to easy estimation of the intermediate conditional probatilities.

6.2 Example: $\mathrm{Prob}(U|a_i)$

In attempting to estimate the probability of use (by a consumer and nationally) of garments of children's sleepwear manufactured to conform to a given level of standard a_i [call this $\mathrm{Prob}(U|a_i)$], if we can do this well without too much attention to detail - all the better. However, use will depend on a variety of things: what the garment is made of and behavioral characteristics of the user. The latter may depend on perceived cost (actual cost plus "cost" of discomfort, etc.) (π), and these in turn may depend on family income (Y) and local geographical area (G), washability (W), and feel (F) or hand of the garment.

We would like to throw in as many variables as needed to make the estimation of $\mathrm{Prob}(U|a_i)$ easier, since it is easy to combine probabilities of lower order events. On the other hand, if a variable does not enter the probabilities significantly, one shouldn't bring it in. If the decision-maker (DM) knows, or feels that, say, geographical location doesn't effect use vary much, then we would not want to include this variable in the chain of calculation. We should have some basis (like data) for including or excluding some variable from discussion.

Use might depend on garment type (N) [for nightwear] and this might include pajamas, nightgowns, robes, single or multiple layers and combinations of them, such as robes (single layer) robes (multiple layers), etc.

Information on family income by geographical area is available from the latest census. Data on sales of garmant type by geographical area should be available from the manufacturers and distributors. Behavioral scientists might be called upon to furnish information on the behavioral characteristics relating to use of non-use of garment type by various sections of the population of potential users.

It is conjectured that information on perceived cost (ζ) may be difficult to come by but here again one might appeal the question to behavioral scientists.

6.3 Suggested Detail for Initial Probability Tree for Outcomes

For separate age class 0-5 years and 6-12 years, by income class:

alternative (a)

available technologies (T)

additional cost to consumer (C)

use or non-use (U) or (u)

existence of a hazard (H)

exposure (E)

ignition (I)

burn (B)

burn-severity (B_i)

survival or death (S) or (D)

body image (I_d)

7. Example of Detail for a Decision Tree for Level of Standard

7.1 Several Probability Trees with Variables for Each

For the present, and guided by members of OFF, we suggest the following level of detail for the decision trees. For the time being we will ignore geographical area (G), washability (W), feel or hand (F). In order to bring in socio-economic conditions and their effect on the behavioral characteristics of individuals with respect to "use" and "exposure" and noting that the morbidity-mortality effects may vary by age* and to differentiate the effects by age, we suggest that six (6) identically structured probability trees be developed: age (2), income level (3); 2 x 3 = 6. The age groups are: 0-5 years, 6-12 years.

The income level classes for families are:

Y_1 = less than or equal to the poverty level,

y_2 = above the poverty level and below the median level,

Y_3 = above median level.

Data on the numbers of children in the various age classes, by race and income level can be obtained from the Tables 491 and 499 of the Statistical Abstract of the United States and from the Bureau of Census Current Population Reports. These data on the number of children in each group are needed to calculate the expected numbers of burns by severity and the expected numbers of burn deaths for each of the trees. From these probability trees one can obtain first estimates of the expected numbers of each of the outcomes. These could serve as crude measures of disutility as discussed in sections 5.1 and 5.2. Further, by choosing some alternative, say a_1, as a reference level, one could then have estimates of the number of lives saved and injuries avoided, items which furnish the

*Rittenbury et al present data to show that there is a racial difference in mortality for children in the 0 to 14 age group, but further indicate that there is insufficient data to determine why the racial difference exists. However, the lower socioeconomic status of the Negro in the population from which the patients studied were taken may explain this difference.

basis for a conventional benefit/cost analysis. In the latter the net benefits (benefits minus costs) could serve as a measure of the expected utility for purposes of decision making (see section 5).

Purposefully, we will try to steer some sort of middle-ground in the detail to be incorporated in the decision trees, in order to have some means of possibly collasping (at some later time) the details of the tree, or on the other hand, of pointing to the necessity for greater detail should this be borne out in the analysis.

Each tree will have the following sequence of branches: Alternatives (a), Technology (T), Nightwear (N), Cost (C), Use (U), Hazard (H), Exposure (E), Ignition (I), Burn (B), Survival (S), Body Image (I_d). Hazard is used here as a short term for "existence of a hazardous condition, in particular, the presence (in the home) of ignition sources". We will elaborate on the definition and operational evaluation of "exposure" when we take up the detail of this variable.

The suggested detail for each of the variables is given below along with some elaborating discussion.

7.2 Alternatives (a)

a_1 = Present Commercial Standards CS 191-53

a_2 = Blanket Flammability Test

a_3 = Vertical Test DOC-FF-3-71

The detail of these alternatives have been discussed in section 3. In the real life decision process these will have to be expanded in number, probably with several levels between a_1 and a_3.

7.3 Technology (T)

T_1 = Cottons

T_2 = Man-mades (synthetics)

L. J. Sharman has suggested the following probability (fraction of market) matrix:

	a_1	a_2	a_3
T_1	0.90	0.60	0.30
T_2	0.10	0.40	0.70
Σ	1.00	1.00	1.00

7.4 Nightwear (N)

N_1 = Loose-fitting (nightgowns)

N_2 = Semi-loose (pajamas)

N_3 = Snug-fitting (knit sleepwear)

N_4 = Robe

*Many classifications of technologies are possible. For example, Lyman Fourt of Gillette Research Institute has used four principal groups: (a) cotton, (b) polyester/cellulose blends from 35 to 50% cellulose, (c) melt-shrink thermoplastics, and (d) wool.

For each of the alternatives a_i (i = 1, 2, 3) we need to develop a matrix of the market split:

a_i	N_1	N_2	N_3	N_4	Σ
T_1	t_{11}	t_{12}	t_{13}	t_{14}	1.00
T_2	t_{21}	t_{22}	t_{23}	t_{24}	1.00

Here, $t_{11} = \text{Prob}(N_1 | a_i T_1)$ = Fraction of <u>cotton</u> made nightwear that are devoted to <u>nightgowns</u> under condition that a_i is in effect.

$t_{12} = \text{Prob}(N_2\ a_i T_1)$ = Fraction of <u>cotton</u> nightwear that are devoted to <u>pajamas</u> under condithat a_i is in effect.

7.5 <u>Additional Costs to Consumer (C)</u>

Here we mean the relative additional <u>dollar</u> costs to the purchaser, with the reference cost taken as that to comply with the weakest alternative (a_1); i.e. $\Delta_k/c(a_1)$ where $\Delta_k = c(a_k) - c(a_1)$ and $c(a_k)$ is the cost under alternative a_k. Tentative levels of fractional additional cost are:

C_0 = Essentially no additional cost,

C_1 = Up to 10 per cent additional cost,

C_2 = Between 10 and 20 per cent additional cost,

C_3 = Between 20 and 30 per cent additional cost,

C_4 = Above 30 per cent additional.

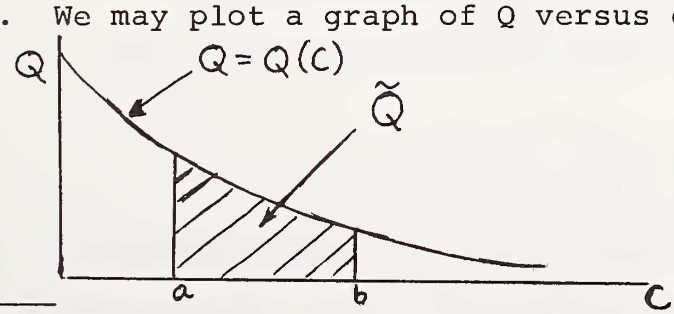
Under the assumptions that:

- The typical garment contains 1.5 yards of fabic
- The increased cost per yard of the fabric is 30 cents
- The demand for the product is inelastic
- The manufacturing process will not change with a change in fabric
- The fire retardant chemical increases the fabric weight by 22 per cent

the Ernst and Ernst* study for the Office of Textiles, Department of Commerce estimates the (average) increased retail price is \$0.89 to \$0.92 per garment. This applied under alternative a_3.

We will now proceed to demonstrate how to get some estimates for the probabilities like Prob $(C_1|a_3T,N_4)$. For this particular probability we will show that Prob$(C_1|a_3T_1N_4) \approx$ Fraction of <u>Robes</u> made of treated cotton orginally (under a_1) retailing for more than \$9.00.

Proof: Let Q = Q(c) denote the demand function for quantity Q at Price C. We may plot a graph of Q versus c.



*Ernst and Ernst (April 1970): A Study to Estimate the Additional Cost of Children's sleepwear under Flammability Standards, COM-71-00847.

The total demand for a quantity with cost between a and b is given by $\tilde{Q}(a,b) = \int_a^b Q(c)dc$.

We now assume that the demand curves for treated and untreated materials are the same, i.e., they depend on cost alone and not on any other factors such as wear, feel, and washability.

To determine limits of integration for assessing $\tilde{Q}(a,b)$ we assume that $\Delta_3(c) = \$0.90$ under alternative a_3.

C_1 is in effect if $\Delta_3(c)/c(a_1) \leq 0.10$; or $c(a_1) \geq \Delta_3/.10 = \$9.00$

C_2 is in effect if $0.10 < \Delta_3/c(a_1) \leq 0.20$; or $\$4.50 \leq c(a_1) < \$9.00$

C_3 is in effect if $0.20 < \Delta_3/c(a_1) \leq 0.30$; or $\$3.00 \leq c(a_1) < \$4.50$

C_4 is in effect if $0.30 < \Delta_3/c(a_1)$ or $c(a_1) < \$3.00$

Let $\tilde{Q}_{1i}$ denote the total quantity of sales of untreated cotton robes corresponding to condition C_i. In particular,

$$\tilde{Q}_{11} = \int_{9.00}^{\infty} Q(c)dc.$$

Let $\tilde{Q}_{3i}$ denote the quantity of sales of treated cotton robes corresponding to C_i. In particular,

$$\tilde{Q}_{31} = \int_{9.90}^{\infty} Q(c)dc,$$

and the change in quantity due to the increased cost of treatment is

$$\tilde{Q}_{31} - \tilde{Q}_{11} = \int_{9.00}^{\infty} Q(c)dc$$

or

$$\tilde{Q}_{31} = \tilde{Q}_{11} - \int_{9.00}^{9.90} Q(c)dc$$

This states that the total quantity of sales of treated cotton robes ($a_3T_1N_4$) corresponding to C_1 equals the total quantity of sales of untreated cotton robes ($a_1T_1N_4$) originally selling for \$9.00 or more diminished by the amount of sales of untreated robes originally selling between \$9.00 and \$9.90.

Now Prob ($C_1|a_3T_1N_4$) = Number of sales of treated cotton robes corresponding to condition C_1 divided by the total number of sales of treated cotton robes.

$$= \frac{\tilde{Q}_{31}}{\tilde{Q}_3} = \frac{\tilde{Q}_{11}}{\tilde{Q}_3} - \frac{\int_{9.00}^{9.90} Q(c)dc}{\tilde{Q}_3}$$

where $\tilde{Q}_3 = \int_{0+.90}^{\infty} Q(c)dc$

$$\simeq \frac{\tilde{Q}_{11}}{\tilde{Q}_3}$$, since the second term is small

$$\simeq \frac{\tilde{Q}_{11}}{\tilde{Q}_1} \text{ since } \tilde{Q}_3 \simeq \tilde{Q}_1 = \int_0^{\infty} Q(c)dc$$

Thus,

Prob ($C_1|a_3T_1N_4$) $\simeq$ Fraction or Robes made of treated cottons originally (under a_1) retailing for more than \$9.00.

This completes the proof for the estimation of the approximate value for Prob($C_1|a_3T_1N_4$).

This first approximation for the probability may be refined by taking into account the term $-\int_{9.00}^{9.90} Q(c)\, dc/\tilde{Q}_3$ to obtain the approximation. These estimates may be adjusted further by taking into account factors other than price in determining the demand function for sales.

In like manner,

$$\tilde{Q}_{32} = \int_{5.40}^{9.90} Q(c)dc = \left[\int_{4.50}^{9.00} + \int_{9.00}^{9.90} - \int_{4.50}^{5.40}\right] Q(c)dc$$

$$= \tilde{Q}_{12} - \left[\int_{4.50}^{5.40} - \int_{9.00}^{9.90}\right] Q(c)dc$$

$$\frac{\tilde{Q}_{32}}{\tilde{Q}_3} \simeq \frac{\tilde{Q}_{12}}{\tilde{Q}_3} \simeq \frac{\tilde{Q}_{12}}{\tilde{Q}_1}$$

Thus, $\text{Prob}(C_2|a_3T_1N_4) \simeq$ Fraction of Robes made of treated cotton originally (under a_1) retailing between \$4.50 and \$9.00.

Similarly, $\text{Prob}(C_3|a_3T_1N_4) \simeq$ Fraction of robes made of treated cottons originally retailing between \$3.00 and \$4.00

and

$\text{Prob}(C_4|a_3T_1N_4) \simeq$ Fraction of robes of treated cottons originally (under a_1) retailing for less than \$3.00

while

$\text{Prob}(C_o|a_3T_1N_4) = 0$

Note that if we are satisfied to use an average $\Delta_3 = \$0.90$, we would need information on retail pricing of nightwear and the original market splits to estimate the various probabilities above.

We remark that we automatically have

$Prob(C_o | a_1 T_1 N_4) = 1.00$

$Prob(C_j | a_1 T_1 N_4) = 0$ for $j = 1,2,3,4$.

Using a different demand curve for pajamas as a function of cost and making similar assumptions about the invariance of the demand curve with respect to factors other than cost, one can approximate $Prob(C_1 | a_3 T_2 N_2)$ by the fraction or pajamas made of treated synthetics retailing (under a_1) for more than $9.00, etcetera. Also

$Prob(C_o | a_1 T_2 N_2) = 1.00$

$Prob(C_j | a_1 T_2 N_2) = 0$ for $j = 1,2,3,4$.

This takes care of the estimation of the conditional probabilities of additional costs for alternatives a_1 and a_3.

To estimate the conditional probabilities $Prob(C_j | a_2 T_j N_k)$ under alternative a_2, one would need to determine the value of $\Delta_2 = c(a_2) - c(a_1)$ along the lines in the Ernst and Ernst report for Δ_3. Then one could apply the same type reasoning as done in the case for a_3 to estimate $Prob(C_i | a_2 T_j N_k)$.

To complete the numerical evaluation of the conditional cost probabilities one needs two things:

(a) a market study to determine the demand curves for each type nightwear under the assumption a_1 was in effect. This was the state of affairs before a_3 was promulgated.

(b) A costing study to determine the additional cost per treated garment to meed the standard a_2, i.e., determine Δ_2.

7.6 Use (U) or Non-Use (u)

To determine quantities like $\text{Prob}(U|a_i T_j N_k C_\ell)$ will to require some sort of behavioral and/or market study.

7.7 Hazard (H)

Examination of FFACTS Quick Query for August 7, 1972 reveals that the following were the principal hazardous conditions that led to ignition (scorch) and burn while children (0-12 years) were wearing sleepwear:

H_1 = Matches/lighters

H_2 = Open flame/hearth/fireplace/candle

H_3 = Space heater/gas heat/stove

H_4 = Ranges, gas and electric

H_5 = Miscellaneious hazards other than H_1, H_2, H_3, H_4.

Let H_o = No-Hazard.

The information on the existence of hazardous conditions can be obtained from a survey, but undoubtedly some excellent estimates should be available from home economists.

The probability of the existence of a particular hazardous condition would appear to be independent of the level of the standard.

7.8 Exposure (E)

As yet, the meaning of "exposure" has not been defined. We would like to let E_j denote the event of "exposure to hazardous condition H_j" and let E_0 (or e) denote the event of not being "exposed". Intuitively, one would not like to say that he is "exposed" to a hazard such as a fireplace, just by being in the room containing the fireplace. Some idea of closeness in space and length of time is involved. We need a definition of "exposure" that is operationally measureable, so that (a) one can determine the probability of "exposure" to hazard H:

$$\text{Prob}(E) = \text{Prob}(H)\text{Prob}(E|H)$$

and equally important that (b) one can calculate the probability of ignition given exposure to hazard H.

$$\text{Prob}(I) = \text{Prob}(I|E)\ \text{Prob}(E)$$

One must look ahead to see what information is available to calculate the Prob(I|E). The research program recommended by the Government, Industry Research Committee on Flammable Fabrics (GIRCFF) has been aimed at calculating Prob(I|E) when "exposure" is given by some set of factors.

Before proceeding to an operational definition of "exposure" one can indicate what one would call the probability of "exposure" to hazard H_j. Time enters in several ways. One important time is the time the individual spends in nightwear. One might choose to call this the <u>wearing time</u> (t_w). This wearing time will be dependent on age and socio-economic conditions, family habits and doctrine. A behavioral study could determine representative wearing times. One would like to exclude the <u>sleeping time</u> (t_s) as a time under which the child is "exposed" to hazards so that the total <u>time available</u> (t_A) for "exposure" to hazards while in nightwear is the difference between wearing time and sleeping time. Thus, one could define the probability of exposure to hazard H_j as the fraction (f_j) of the wearing time less sleeping time that the individual is "exposed" to H_j. In symbols,

$$\text{Prob}(E_j|H_j111) = f_j t_A/t_A = f_j$$

where $t_A = t_W - t_S$

So far, exposed (or exposure) to hazard H_j is still undefined. Experience and experiments tell us that for a specific H_j and material T_i some combination of critical values of factors that represent the properties and environment of the ignition source, of the fabric and of distance and time, like distance, intensity of heat, time, RH, etc., is needed before "ignition" is possible. Call this multi-dimensional space of ignition factors the factor space for the given specific situation and materials. The factor space can be separated into two complementary subsets $F_1(H_j)$ and $F_2(H_j)$. The first subset is one in which ignition is impossible (one is just too far away or the heat intensity is not enough,...). The complementary set $F_2(H_j)$, which we shall call the critical factor set for H_j, is that collection of factor values for which "ignition" is possible. The critical factor set for H_j implicity defines the exposure to H_j, which we have previously labeled E_j. When one is outside the critical factor set for H_j, i.e., in $F_1(H_j)$, we would like to say the person is not exposed to the hazard H_j. The above is merely a refined way of saying that a person is exposed to a hazard if something detrimental (ignition) can happen. If a detrimental effect cannot occur, a person would not be "exposed" to a hazard.

Unfortunately, the critical factor set thus defined is not reasonably good for operational evaluation of exposure to a hazard, for each candidate fabric would have a separate critical factor set and we would have to know all the possible combinations of factors for which ignition were possible for the fabric in order to define the set. We have to inject some degree of arbitrariness into the definition and slack off a little.

If we concentrate on a particular hazard, say an open flame, hearth or fireplace (H_2) one has some inherent representative characterization of the flame (which will eventually have to be determined). Then the remaining factors are essentially distance, time, and properties of the materials. For hazard H_2 there is some critical distance, which when exceeded would lead to no ignition, independent of the length of time any kind of candidate fabric (T) is used in the vicinity of the particular hazard. This critical distance for a particular hazard depends on the alternative (to define the class of candidate fabrics) but not on a particular fabric nor on time.

In place of using the critical factor set to define the exposure to a hazard we suggest the use of critical distance. The ratio of the sum of the times spent within the hazard's critical distance to the time available (t_A) for exposure to hazards will be taken as the <u>definition</u> of the <u>probability of exposure to the hazard H_j</u>, given the existence of the hazard H_j, i.e., Prob $(E_j|H_j\ldots)$. Note that this definition of probability of exposure allows for a frequency of residence occasions within the critical distance associated with the particular hazard and is independent of the fabric (T).

Laboratory experiments should lead one to a determination of the <u>critical distance</u> for each type of hazard, H_j. The evaluation of the Prob$(E_j|H_j\ldots)$ can only be evaluated through a knowledge of the behavioral patterns of individuals in the presence of a hazard.

To this point we have used the properties of the hazard (ignition source) the properties of the materials (T) to define a critical distance, and the total residence time and wearing and sleeping times to define the Prob$(E_j|H_j)$.

To relate exposure to the next variable (I) in the probability tree we need a concept of exposure time. By <u>exposure time</u> on a single occasion (t_{E_j}) to the hazard H_j we shall mean the time (for that occasion) spent within the critical distance set for H_j. The determination of representative exposure times is again dependent on the behavioral characteristics of individuals.

7.9 <u>Ignition (I)</u>

Information of Prob$(I|E_j\ldots)$ will have to come from the experimental program, both in-house and that done under contract at the recommendation of GIRCFF.

Ignition time (τ_i) appears to be a central element in the determination of the probability of ignition under fixed conditions of exposure. A major question is whether (under fixed conditions of exposure, labelled X here) ignition time is a random of a deterministic variable. The ignition time is deterministic if under each specified set of conditions, there is a unique value for the ignition time. If the ignition time, under repeated application of the same conditions, exhibits a range of times with an associated probability (frequency) distribution, then the ignition time is a random variable.

In the deterministic case, if the time of exposure (τ_e) is less than the ignition time, i.e., $\tau_e < \tau_i$, then the $\mathrm{Prob}(I | \tau_e < \tau_i ; X) = 0$ and ignition is not possible; however, if the exposure time is greater than or equal to the ignition time, i.e., $\tau_e \geq \tau_i$, then $\mathrm{Prob}(I | \tau_e \geq \tau_i ; X) = 1$, i.e., ignition is certain.

In the random variable case, the best that can be said is something like this: if for given fixed conditions X, the exposure time is below the range of ignition times, then the probability of ignition is zero; if the exposure time is within the range of ignition time there is a non-zero probability of ignition. This non-zero probability can be determined provided the frequency distribution for ignition times is known.

The work done at Georgia Institute of Technology (GT) for the case of radiative transfer indicates that the "ignition time depends strongly and deterministically on fabric properties". (pages 11, 61 Final Report 1971). However, a fourth phase of their work for radiative heating is to determine whether or not the measured ignition time <u>is</u> a unique function of heating intensity for fixed fabric properties and conditions. Thus, at the present this question is not conclusively settled, but fortunately, the radiative case applies only to one part (electric ranges) of hazard H_4.

The predominant hazards leading to burn injuries in children using nightwear are <u>convective</u> in nature (flaming heat sources) and here the evidence indicates strongly that the ignition time is a random variable.

Work at Gillette Research Institute* on materials held at fixed distances above a gas stove and in the flame provide a considerable body of data that can be used to determine the probability of ignition for different exposures, certainly for the gas range part of hazard H_4, if not for the other flame heat sources. Results are presented for GIRCFF Fabrics using the classification of (a) Cotton, (b) Polyester/Cellulose, (c) Melt-Shrink, (d) Wool.

*Lyman Fourth (Dec. 31, 1971): Study of Ignition and Exposure, Gillette Research Institute, Harris Research Laboratories.

Some of the above data may help define the critical distances defined in the previous section. Before these data can be applied one must have some behavioral information on the distances and exposure times actually involved with children in the proximity of flame sources.

7.10 Burn (B) and Burn Serverity (B_i)

At this level we find it convenient to extend the conversation by expanding the outcomes from burn (B) and no burn (B_0 or b) to include severity levels of burns (B_i), survival (S) or death (D), and body image area (I_d). This is indicated in Figure 8 (page 36).

As described in Chapter 4, we suggest adopting for severity levels of burn inquiry:

$B_0 \equiv$ no-burn

$B_1 \equiv$ second and third degree burn area less than or equal to 15 percent $\equiv$ than a "major" burn.

$B_2 \equiv$ second and third degree burn area more than 15 percent, but less than 35 percent $\equiv$ a "major" burn that is not "extensive".

$B_3 \equiv$ second and third degree burn area greater than 35 percent $\equiv$ an "extensive" burn.

One needs to determine the probability of a burn given that ignition has occurred while the child was wearing nightwear of a particular kind satisfying the requirements of a designated alternative, i.e., Prob $(B|a_i T_j U E_e I)$.

As pointed out by the Factory Mutual Research Corporation*:

> Clearly, the behavior of people enters into the problem. If a person is able to sense the heat source (by means of pain mechanism in skin tissues) before ignition occurs, he may draw away from the source and not be injured. However, if the fabric worn by the person is heavy and opaque, (or if the person is inebriated or is a very small child) ignition may precede detection by pain and the chances os a severe burn injury are greatly increased.

We remark also that the behavior of children subsequent to recognition of ignition of their nightwear will effect the severity of the burn injury.

However, if we assume that a burn has occurred, we may turn to available accident data on burns such as that contained in the FFACTS system to get some preliminary estimates of Prob $(B_1|B...)$. We have prepared such a preliminary table from the FFACTS Quick Query Output dated August 7, 1972 to illustrate the procedure. (See Table 7.1)

For Children from 0-5 years old, ignoring the technology, fabric and income groups, but for alternative a_1 and wearing Robes (N_4):

$$\text{Prob } (B_1|BIa_1N_4UE_1) = 2/3$$

$$\text{Prob } (B_2|BIa_1N_4UE_1) = 0$$

$$\text{Prob } (B_3|BIa_1N_4UE_1) = 1/3$$

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*A. S. Kaleekar and H. C. Kung (October 1, 1971): A Study of Preignition Heat Transfer Through a Fabric-Skin System Subjected to a Heat Source, Factory Mutual Research Corporation.

TABLE 7.1: SEVERITY OF BURNS AFTER IGNITION BY EXPOSURE TO HAZARD CLASS BY NUMBER

Age

Age 0-5 Race (All) Income (All) No Restriction on Technology

	ROBES					PAJAMAS					NIGHTGOWN					SLEEPER				
	B_1	B_2	B_3	Unk	Totals	B_1	B_2	B_3	Unk	Total	B_1	B_2	B_3	Unk	Total	B_1	B_2	B_3	Unk	Total
E_1	2	0	1	0	3	26	4	5	7	42	6	1	3	2	12					
E_2	1	0	0	0	1	3	0	0	0	3	0	1	1	1	3					
E_3	0	0	0	0	0	1	0	0	0	1	6	0	2	0	8					
E_4	0	2	0	0	2	10	1	0	2	13	2	1	0	1	4					
E_5	0	1	0	1	2	2	3	2	2	10	1	2	0	0	3					
	3	3	1	1	8	42	8	8	11	69	15	5	6	5	30					
Age 6-12																				
E_1	1	0	1	0	2	2	0	0	4	6	3	1	1	2	7					
E_2	2	0	0	1	3	4	1	0	1	6	3	0	1	2	7					
E_3	0	0	0	0	0	5	1	0	0	6	10	1	1	2	12					
E_4	2	1	0	0	3	20	1	0	3	24	3	1	2	1	7					
E_5	0	0	0	0	0	2	0	0	0	2	1	0	0	0	1					
	5	1	1	1	8	33	3	0	8	44	20	3	5	5	33					

while for children aged 0-5, wearing pajamas (N_2):

Prob $(B_1 | BIa_1N_2UE_1) \geq 26/42$

Prob $(B_2 | BIa_1N_2UE_1) \geq 4/42$

Prob $(B_3 | BIa_1N_2UE_1) > 7/42$, etc.

In the table Unk denotes that the severity of burn level is unknown. If the case of Pajamas (N_2) for ignition (I) due to exposure to matches/lighters (E_1) there are 7 such entries. These 7 cases could be allocated to the severity levels B1, B_2 and B_3 in a variety of ways. It is for the reason that we have indicated $\geq$ in the second example.

Similar tables could (in principle) be constructed for each of the various technologies and income groups. However, one should expect even smaller values for the entries in the tables.

7.11 Survival (S) or Death (D)

For preliminary estimates we suggest the use of the clinical burn data for the Medical College of Virginia (MCV) published in Rittenbury et al* (Table 1, p. 124). This is based on percent of total area burned. We will use as preliminary estimates the age grouping 0-4 and 5-14 for the 0-5 and 6-12 respectively. From Table 1 of that article we

*Rittenbury M.S. et al (July 1966): Probit Analysis of Burn Mortality in 1,831 Patients, Annuals of Surgery, Vol. 144, 123-138.

aggregate cases and otbain the following table:

MCV Clinical Burn Data 1949-1962

Burn Severity Class	% Total Area Burned	0-4 Yr.			5-14 Yr.		
		No. Pts.	No. Deaths	% Mort.	No. Pts.	No. Deaths	% Mort.
B_1	< 15	339	3	0.8	145	0	0
B_2	15-34	110	15	13.6	80	14	17.5
B_3	$\geq$ 35	44	37	84.1	50	35	71.4
	Totals	493	55	11.1	275	49	17.7

This will provide $\text{Prob}(D|B_i....)$.
$\text{Prob}(S|B_i...) = 1-\text{Prob}(D|B_i...)$.

7.12 Body Image (I_d)

We suggest that preliminary estimates of $\text{Prob}(I_d|SB_i...)$ be obtained from an examination of the details of the case histories of accidents in FFACTS.

8. Summary of Current Needs

8.1 Probability Trees for Outcomes

In the previous sections (7.1 through 7.12) we have discussed in some detail what needs to be done in order to calculate the probabilities of the outcomes associated with the alternative levels of standard for flammable fabrics for children's nightwear. The solution of this problem depends on the availability of information on social and economic factors, on the behavioral patterns of individuals as well as on engineering type data. We will indicate a "shopping list" of current informational needs, and will try to place them in some semblance of priority.

a) In order to have all of the branches of the tree fit together there is need to have some agreed upon operational definition of "exposure" to a hazard. We have discussed this at some length in section 7.8 and have suggested the use of a critical distance to define this exposure. If this is an acceptable concept there is need to determine a critical distance for each of the hazards under consideration. This will probably entail a professional engineering judgment based on existing engineering data. Once this distance has been defined for each hazard, the probability of exposure to a particular hazard can be determined from a study of the behavior of individuals in the presence of the hazard. The behavioral study should, at the same time, determine the distribution of representative exposure times; these are needed in order to estimate the probability of ignition given exposure.

b) There is need to summarize all of the experimental work done at the recommendation of the GIRCFF to determine Prob(I|E...) and the availability of other data on this topic for the hazards under consideration. From these data, professional engineering judgment based on an "up and down" procedure* probably could finish preliminary estimates of probabilities of ignition for the remaining cases.

*By an "up and down" procedure we mean an adjustment upward or downward from some known reference level, with the amount of adjustment estimated by professional judgement.

c) There is need to determine the probability of a burn given that ignition has occurred. This can be addressed best through a behavioral study. (See Section 7.10)

d) The FFACTS can generate reasonable information on burn severity levels only for fabrics satisfying alternative a_1. The extent of what detailed information, on burn severity can be extracted from this system should be determined. Then one probably could utilize the "up and down" procedure to provide the remaining preliminary estimates of probabilities. (See Section 7.10)

e) Social and behavioral studies are needed to determine probabilities of use and the existence of hazards.

f) Economic studies are needed to determine the market splits for the various technologies used to satisfy the different alternate levels of standards. The present study by Ernst and Ernst is applicable only to the case of treated cottons for alternative a_3. (See also Section 8.3 below)

8.2 Measures of Worth or Risk

From this probability tree one could then obtain first estimates of the expected numbers of each of the outcomes. These could serve as crude measures of disutility. Further, by choosing some alternative as a reference level, one could then have estimates of the number of lives saved and injuries avoided, items which furnish the basis for a conventional benefit/cost analysis. In the latter the net benefits (benefits minus costs) could serve as a measure of the expected utility for purposes of decision making.

8.3 Manufacturers Cost Analysis

In order to carry out a benefit/cost analyses or to calculate an expected net disutility, it will be necessary to have an analysis performed on the costs to the manufacturing industry to comply with the requirements of each alternative standard. This cost analysis is also essential to the determination of the probabilities of the levels of increased cost to the consumer in the probability tree for outcomes. Part of the cost analysis will involve identification of the technologies (fabrics and processes) (T_k) needed to comply with each of the prospective standards, and will be most helpful in determining the $Prob(T_k|a_i)$. The analysis should also provide evidence of the "reasonableness" of the requirements imposed on the industry. This cost analysis appears to play a central role in the decision making problem at hand.

8.4 Intangibles

We consider the primary intangible in this problem to be the psychic effects, which accrue not only to the individual and his immediate family but also to those who are involved in the burn victims therapy and rehabilitation. We have been reviewing the literature on the psychic effects of burn victims and have had discussions with physicians, nursing staff, and psychiatrists who treat these personnel. It is our intention to write a separate paper which will review and summarize what is published in this area. This will permit the decision makers in this problem area to have at least qualitative information on this factor. It is recommended that we try to do more, particularly in attempting to evaluate multi-attributed disutility functions which contain psychic effects as a component of a vector factor, following the procedures recommended by Keeney and others.

9. Conclusion

We have looked at the details of the problem of applying decision analysis to determine the level of the standard for a flammable fabric. We find that there are considerable information voids that preclude immediate application of this technique and have listed these in detail in Section 8. The solution of this problem depends on the availability of information on social and economic factors, on the behavioral patterns of individuals as well as on engineering type data. Much of the behavioral and social information needs may be very difficult to obtain. Survey techniques, a tool used often by social scientists, will undoubtedly be needed. The results of surveys will have to be supplemented by the judgements of social and behavioral specialists in exactly the same way that the engineering data on ignition will have to abetted by the judgments of physical scientists and engineers.

The most important need at this time is a conceptual one, that of defining what is meant by "exposure" to a hazard. In addition, there is need for an operational way of assessing the probability of exposure, once "exposure" has been defined. Suggestions for these have been given in Section 7.8.

Most of the information needed for a decision analysis is equally necessary to conduct other methods of decision making, whatever they may be. We feel that the examination of this problem from the point of view of decision analysis has been benefical in identifying those areas where additional information is needed.

Appendix A

Summaries of Background Papers

A.1 M. Tribus: Decision Analysis Approach to Satisfying the Requirements of the Flammable Fabrics Act. (ASTM Standardization News, pp. 22-27, February 1973)

This paper was presented by the then Assistant Secretary of Commerce for Science ant Technology at the meeting of the Textiles and Needle Trades Division of the American Society for Quality Control at Greensboro, North Carolina, February 12, 1970. The problem addressed was that of determining the level at which a standard for flammable fabrics should be set.

After discussing the requirements placed on the Secretary of Commerce by the Flammable Fabrics Act and reviewing some of the beginning work of Dr. Chauncey Starr of the University of Califormia at Los Angeles relative to the appropriate risk levels which might be used by public officials in setting standards for the protection of consumers, the author lays down the elements of a probability tree to calculate the probability of a burn, if a given level of flammable fabrics standard for children's sleepwear is adopted. The raw data required were not immediately available. However, he did feel that the analysis provided a conceptual framework required to guide both the data gathering and laboratory experimentation.

Tribus depicted a probability tree concerned with five (5) variables, given that the decision maker had selected a particular standard (S_i). These variables were: (1) additional cost (C) to the user to comply with the standard; (2) use (U) or non-use (u) by the consumer of materials satisfying the standard; (3) exposure (E) or non-exposure (e) of the wearer of the flammable fabrics to ignition sources; (4) ignition (I) or non-ignition (i) of the exposed fabric and finally the outcome variables burn (B) or non-burn (b). This probability tree may be depicted as in Figure A1.

The total probability of a burn, contingent on a choice of S_i

$$p(B|S_i) = \sum_{\text{sum over all } j} p(B|UEIC_jS_i)p(I|UEC_jS_i)p(E|UC_jS_i)p(U|C_jS_i)p(C_j|S_i)$$

$$+ \sum_{\text{sum over all } j} p(B|uEIC_jS_i)p(I|uEC_jS_i)p(E|uC_jS_i)p(u|C_jS_i)p(C_j|S_i)$$

Each product of terms following the summation signs represents the probability of a burn by proceeding along a particular path through the branches of the tree starting at S_i and ending at B. The summation over all j merely adds up the probabilities of all ways of going from S_i to the outcome B.

The author gives an interpretation of each term and how it might be determined in practice. For example: "$p(B|UEIC_jS_i)$ is the probability of a burn, given use, exposure to an ignition, cost and a certain standard....(It) can be determined from laboratory tests on various flame retardant materials. (It turns out that given $UEIS_i$ true, C_j is immaterial)".

The analyses provide a means for computing the expected number of burn cases at various levels of expenditure for protection.

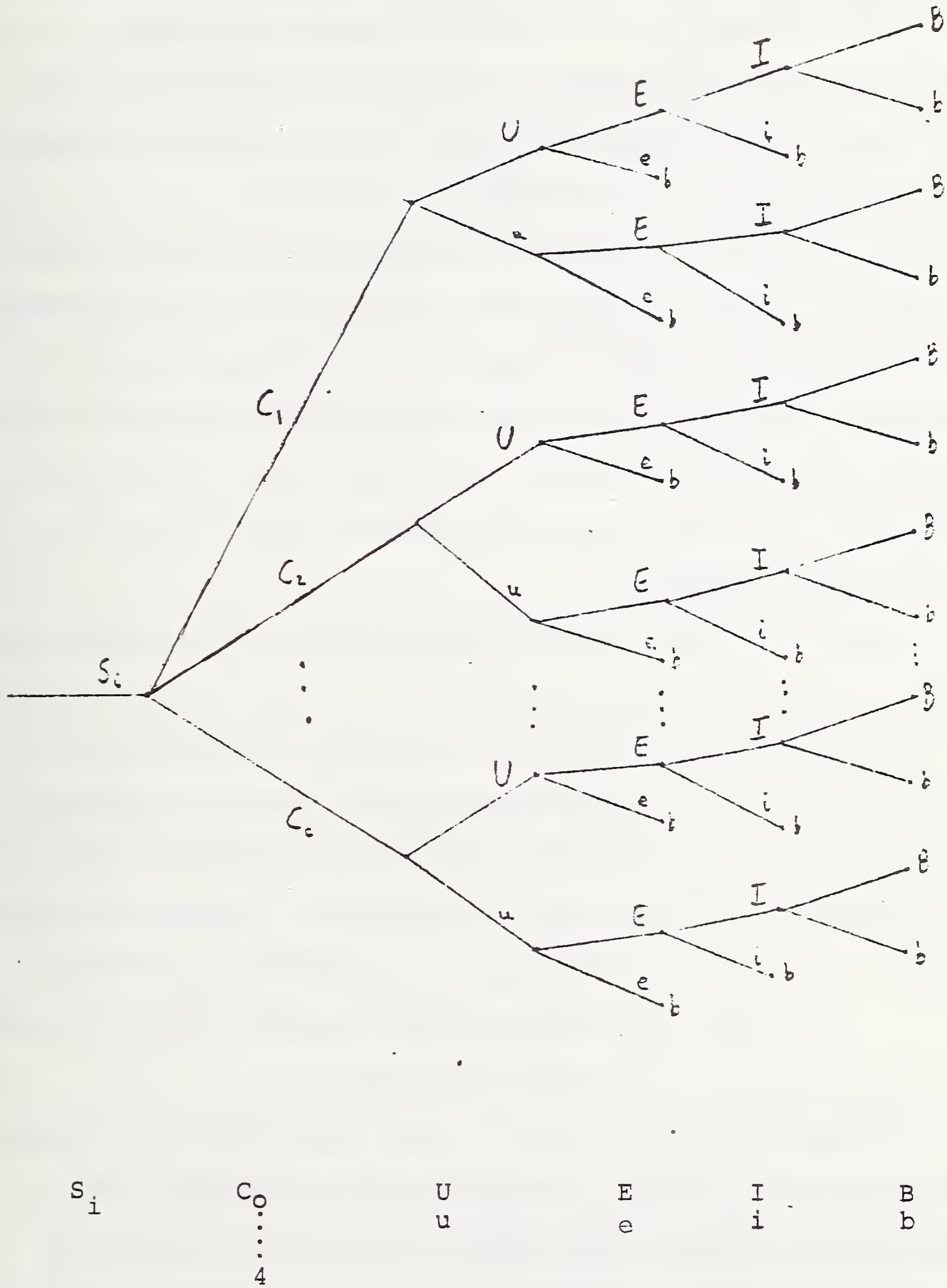


Figure A1. The Probability Tree for Prob ($B|S_i$)

The author then discusses a possible way at arriving at the level of the standard by considering possible trade-offs through non-use of the garments meeting the standard by consumers of the disutility associated with increasing costs accompanying the increase in stringency of standards.

Tribus points out that the above trade-off considerations also provide a basis for considering the effects on the number of burns by programs complementary to standard setting such as publicity and education. [This latter type of analysis was the subject of Kathryn McDonald's masters thesis at Cornell University in 1970; the thesis is summarized in the next section of this appendix.]

No attempt at actual analysis was made in this concept paper.

A.2 Kathryn McDonald: Consumer Protection: The Case of Flammable Fabrics, Master's Thesis, Cornell University, September 1970.

and K. McDonald, B.F. Smith, and R. Dardis: Flammable Fabrics and The Cost of Consumer Protection, Textile Research Journal, Vol. 41, No.6 492-500, June 1971.

The McDonald thesis is in two parts, the first on an investigation of burn injuries in general with a review of the literature and a sample survey and analysis of burn injuries in the city of Syracuse, New York for the four year period 1966 through 1969. The second part of the thesis is devoted to the decision analysis and it is this part of the thesis that appears in published form.

For the Children's Sleepwear problem the following points from the first part appear important. There are cultural and sociological influences on the incidence of accidental burns: age, sex, culture, geographical location and whether are factors. Wilkinson is quoted that a prominent factor was not bad living accommodations but rather "domestic habits of the family." Another study points out that most burn accidents occurred in the three lowest (out of five) social groups (based on occupation of head of household). An early study (1950's) by Mayor indicated that death rates from burn injuries were highest in the East South Central States and lowest in Mid-Atlantic States, that non-whites were more succeptible to burn fatalities than whites and that (at that time at least) in the South and Southwest there was a predominant use of movable, open-flame heaters. For Children under 5 years in Birmingham, England one study showed a male: female ratio of 3:2 in home accidents. Source of ignition, such as open solid-fuel fires, electric and gas heaters, floor heaters are important. A probit analysis of burn cases indicates that mortality was related to age, depth of burn and extent (size of burn).

We now comment on the second part of the thesis dealing with the decision analysis. We also draw out a decision-tree picturization rather than present the matrix representation of the problem. In what follows page numbers are those in the published paper.

K. McDonald's Decision Analysis Problem: choose between four actions a_1, a_2, a_3, a_4 each with three possible outcomes, θ_1, θ_2, θ_3. Definition of symbols:

a_1 = do nothing (status quo) θ_1 = ...potentially fatal

p493 a_2 = strengthen standard alone θ_2 = ...potentially injurious

a_3 = educational program alone θ_3 = ...neither θ_1 nor θ_2

a_4 = do both

p.493

$v(\theta_j, a_i)$ represents average financial loss to a single individual as a consequence of θ_j, given action a_i, and is the sum of

(1) individual's loss of earnings (mortality or morbidity)

(2) hospital costs

(3) cost of physician services

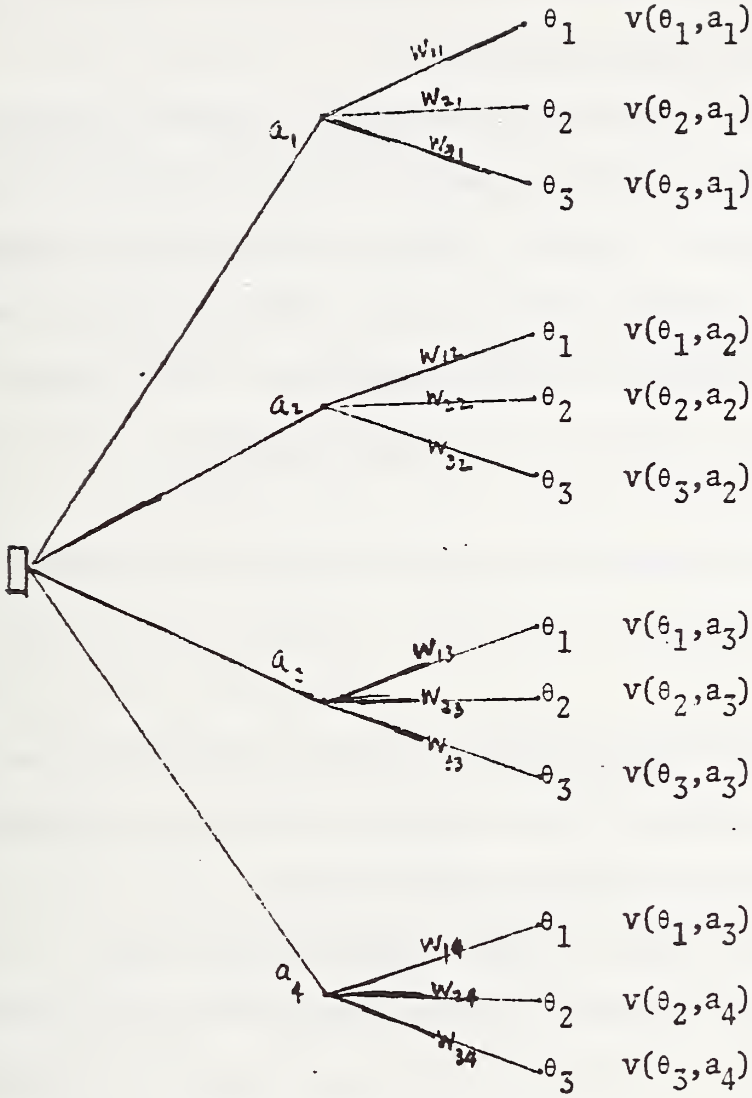
(4) cost to implement action a_i.

Loss caused by property (structural) damage was not included. No allowance was made for the pain and grief suffered by the injured person or his family and the author notes (and quotes D. Rice) that the loss is intangible and difficult to measure.

Two different decision criteria are employed: (a) minimize expected "loss" (disutility) and (b) minimize the maximum expected disutility.

As pointed out previously, this problem is different from that addressed by M. Tribus, who was concerned about the level at which action a_2 = (strengthen standard alone) should be set.

However, the McDonald paper does address the level at which a standard could be cost-effective. This is done through the



$w_{ij} = \text{prob}(\theta_i | a_j)$

For matrix form see Table I, p. 495

Action Outcome Value of outcome "a loss" (Evaluated as a dollar cost)

Figure A2: Decision Analysis Diagram for McDonald's Problem introduction of a parameter β (called b in the original thesis) - (see p. 493), which is meant to be a measure of the effectiveness of the fire-retardant material.

The assumption is made that protective clothing would never cause more loss than was anticipated when no protective clothing was used. Thus, $0 \leq \beta \leq 1$. When $\beta=1$ the treated apparel provided no additional protection to the individuals and when $\beta=o$, 100% burn protection was provided [Note: The writer would think of β as a measure of <u>ineffectiveness</u> of the fire-retardant material].

This β factor is used in a study of cost-effectiveness with the cost to implement action a_2, labelled C_a, in the following way.

Let $E(a_i)$ denote the expected loss from action a_i, and let $E(a_1) = S$; then $E(a_2) = \beta S = C_a$ (p. 496). The expected losses for actions a_1 and a_2 are equal when

$D(a_1) = E(a_2)$ or when $S = \beta S + C_a$ or $S(1-\beta) = C_a$.

By studying this relationship one can determine the relation between program cost and effectiveness (protective clothing). This is done in the published paper on pp. 496-498.

There are several weaknesses in the paper and the authors seem to be aware of them. These weaknesses have to do with the value of loss functions $v(\theta_i, a_j)$. These are treated as cost functions and are expressed in dollars and all costs are additive. "No allowance was made for "psychic" losses, that is, pain and grief suffered by the injured person and his family.... In addition, it was not possible to take into account the loss due to non-hospitalized injuries. Thus, the cost of burn injuries is likely to have been underestimated in the present model" (p.500). This latter point could affect the detail of much of the cost-effectiveness arguments. For example, in Figure 1, p. 496, the a_3 line may be displaced upward and intersect the a_4 line, and Figure 2, p. 497 may well turn out to be the <u>actual</u> rather than the <u>hypothetical</u> case.

Also, the arguments may well turn around, if the loss to the individual is "expressed in utility rather than monetary terms" (p.499).

The authors' conclude:

> "Because the present cost to treat apparel for fire retardance exceeds the expected loss from apparel burn injuries, the expected-loss criterion directed the decision maker to purchase treated apparel. The minimax criterion, on the other hand, led to a decision in favor of protective clothing."
>
> "The authors believe that therein may lie a solution to the problem of flammable apparel - let the consumer decide, on an individual basis, which criterion he wishes to employ and then select the appropriate course of action. In such instances, education is essential."

Appendix B

The Assessment of Multi-Attributed Utility Functions

In the evaluation of the outcomes one leans heavily to the use of modern utility theory as developed by von Neumann and Morgenstern, Luce, Raiffa, Peter Fishburn and others. A utility function, u, is a function which assigns a real value to every outcome (consequence) in a manner such that $u(b)>u(c)$ if and only if consequence b is preferred to consequence c. The importance of utility functions is that they may be used as a guide to rational behavior. However, a major operational problem is that of assessing the DM's utility function for the outcomes. The technique of assessment of utility functions is primarily that of asking appropriate questions relative to trade-offs. Here the concept of a "lottery" (or its equivalent) and of a "certainty equivalent" is useful. The reader is referred to Chapter 4 of the delightful paperback by Raiffa for a convenient discussion, with example, of the assessment of a single attribute utility function, and to pages 246-255 of the same book for an excellent introduction to the multi-attribute problem. The latter terminates with a discussion of a medical treatment problem, in which the weakest link in the chain turned out to be the treatment of the utility structure. The task in that problem was to assess a utility function involving 7 attributes.

For the case of a multi-attributed utility function much of the operationally useful techniques for its assessment have been based on theoretical developments due to Raiffa and his colleagues and students, notably R. L. Keeney. Central in this work is the concept of utility independence of one attribute Y on another, Z. It is possible for Y to be utility independent of Z, but for Z not to be utility independent of Y. An example of this phenomenon is given at the end of this analysis. However, in the case the attributes Y and Z are mutually utility independent, it can be shown that the utility function u(y,z) can be evaluated from the quasi-additive functional form

$$u(y, z) = u(y_o, z) + u(y, z_o) + ku(y_o,z)\, u(y,z_o)$$

where k is an empirically evaluated constant. The notation here indicates that y and z denote amounts of the attributes Y and Z and y_o and z_o are fixed values of y and z. It should be noted that $u(y_o, z)$ and $u(y, z_o)$ are one-attribute conditional utility functions.

In some problems it may be more convenient to assess an iso-preference curve than a conditional utility function. An iso-preference curve (also called an indifference curve) is a curve along which the DM has a fixed value of utility, i.e., he is indifferent to combinations of attributes that make up the curve. Thus, if (y_1, z_1) and (y_2, z_2) are two arbitrary points on an iso-preference curve, the utility of each combination of attributes is the same, i.e., $u(y_1, z_1) = u(y_2, z_2) =$ a constant.

Under the same consitions of mutual utility independence of the attributes Y and Z, an iso-preference curve may be substituted for either a conditional utility function for Y or Z provided it covers the same range; details are not indicated in this note. We mention, in passing, that there are operational ways of determining when Z is utility independent of Y, and there are ways of assessing $u(y, z)$ if Z is utility independent of Y, but Y is not utility independent of Z.

In his dissertation Keeney outlines a procedures for assessing multi-dimensional utility functions in five steps:

1. Introducing the terminology and ideas of DA and utility theory to the DM.

   After these discussions about the definitions and the DM's understanding of them one should make clear to the DM that the preferences one is interested in are his, that there are no objectively correct preferences, that the preferences of importance represent the subjective feelings of the DM and that he is free at any time to indicate a change in his feelings about any of the preferences stated. One of the purposes of a DA is to require DM to reflect on his preferences and hopefully straighten them out in his own mind.

2. Identifying the applicable utility independence assumptions.

   If, for example, there are two attributes Y and Z under discussion one tries to determine if Y is utility independent of Z and whether Z is utility independent of Y. The independence properties will determine the mathematical functional form of the multi-attributed utility function, and how to proceed further in the assessment of that utility function.

3. Assessing conditional utility functions.

   One assesses each of the required conditional utility functions on an arbitrary scale.

   In this part of the procedure, for example, one assesses say the utility of the attribute Z for a fixed amount of attribute Y, say y_o, and then one assesses the utility of the attribute Y keeping the amount z_o of the attribute Z fixed in the process.

   Recall that in certain cases it may be easier to assess an iso-preference curve than a conditional utility function.

4. Scaling the conditional utility functions.

   In this step one scales the conditional utility functions so that they have a common origin and unit of measure.

   After this stage of the procedure one puts the scaled conditional utility functions in the appropriate mathematically functional form and one then has a utility function for the DM.

5. Checking the consistency of the utility function.

   This is accomplished by further questioning of the DM about his preferences. If any inconsistencies occur one repeats some or all of the steps 1 through 5 until one becomes satisfied that the utility function represents the DM's true preferences.

The systematic assessment of multi-dimensional utility functions is probably the most difficult part of the procedure of applying DA and may be a limitation on the DA approach.

The following discussion, due to Keeney, indicates the possibility of a one-way utility independence between two variables without the existence of a mutual utility independence:

> Suppose one has a serious disorder and must undergo a serious operation. Also more than one type of operation exists for controlling this particular disorder. A choice must be made as to what type of operation will be proformed. The consequences are to be evaluated in terms of the probability of death and patient expense.
>
> One might assume the probability of death is utility independent of patient expense by arguing that the relative preferences for death should be the same regardless of cost. <u>However, the patient expense might not be utility independent of the probability of death</u>. One might have a completely different preference structure for expense, given that he is almost sure to live, than he would given the operation is likely to be fatal.

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16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

The fundamentals of a single-stage decision problem are discussed and illustrated in the problem: The Level of the Standard for Children's Sleepwear, originally discussed by M. Tribus. Outcomes are identified, and various potential measures of disutility are discussed.

Given a particular alternative is in effect one must assign the conditional probabilities of arriving at each outcome. This process is aided by introducing intermediate events (extending the conversation). For the childrens' sleepwear problem this is done by considering for each of two age groups and three income levels the probability tree with branches: alternatives (a), technology (T), nightwear (N), additional cost to the consumer (C), Use (U) or non-use (u); existence of a hazard (H), exposure (E), ignition (I), burn (B) and burn-severity (B_i), survival (S) or death (D) and body image (I_d).

Attention is given to the preliminary assignment of each of the conditional probabilities needed. Suggestions are made as to sources of information. Much of the needed information is not available, especially that dealing with the social and behavioral aspects of the problem. The concept of exposure to an ignition hazard, for which an operational definition does not exist, is discussed.

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