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A Summary of the Relationships Between Accident Indices and Rates Following a Redefinition of "Failure"

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Final Report

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Note: This report to the National Highway Traffic Safety Administration (NHTSA) has been prepared to serve as a mathematical appendix to a report to be published by the NHTSA.

SUMMARY

Using Cerrelli's association of "failure" with active involvements in the case of two vehicles accidents, one can define failure indices, accident involvement indices, failure rates, accident involvement rates for 1-vehicle, 2-vehicle and for the union class of 1- and 2-vehicle accidents. This note presents in a condensed form all of the definitions of these measures, and a number of useful relationships and interrelationships that exist between these different measures. These formulae should prove useful in the calculation of the resulting indices and rates and of converting from one set of measures to another.

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1.0 Introduction

In his recent paper: "Interpretation of the Accident Involvement Index,"[1] Ezio Cerrelli points out the pitfalls in interpretation (primarily by administrators) commonly associated with the currently used Accident Involvement Index (AII). A most common misinterpretation is that of considering it as an absolute measure of misperformance of a category of driver-vehicle combination, when in fact it is a relative measure. A second difficulty is that of interpreting each involvement as a "failure" in some sense and using involvements to measure the degree of "failure" for each category of driver-vehicle combination.

Cerrelli then proceeds with a rationale for associating one "failure" with each accident and then defines a Failure Index (FI) to be associated with each category of driver-vehicle. In the case of a one-vehicle accident, "failure" is identified with "involvement," but in the case of a two-vehicle accident, "failure" is associated only with an "active" involvement while passive involvement is considered as "non-failure." The basic relation is that "failure" implies involvement, but involvement does not necessarily imply "failure." This may be diagrammed for

1-vehicle accidents: 2-vehicle accidents:

Failure ⟷ Involvement   Failure ⟶ Involvement

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[1]To be published as a National Highway Traffic Safety Administration (NHTSA) Report.

As currently used the Accident Involvement Index is defined for the composite set (union) of the sets of 1- and 2-vehicle accidents. Originally Cerrelli defined separate failure indices for one-vehicle and two-vehicle accidents. It was pointed out by this author that failure and involvement indices could be obtained for each set of accident types (1-vehicle, 2-vehicle, composite), and that corresponding rates (ratios) could be defined as well. The latter turn out to be absolute rather than relative measures.

The purpose of this note is to present in a condensed form, for possible rapid reference and use, the definitions, relationships and interrelationships between traffic accident related indices and rates that result from Cerrelli's explicit identification of "failure" in the case of two vehicle accidents. These formulas should prove useful in the calculation of the indices and rates that result and of converting from one set to another. The notation used is that suggested to and employed by Cerrelli in his paper.

The reader should anticipate some general statements, such as:
a. the ratio of fractional deviations from the norms for failure $\left\{ \begin{matrix} \text{indices} \\ \text{rates} \end{matrix} \right\}$ is the ratio of total involvements to total failures time the fractional deviations from the norms for involvement $\left\{ \begin{matrix} \text{indices} \\ \text{rates} \end{matrix} \right\}$.

This holds true for each type of accident: 1-vehicle, 2-vehicle and the union of 1- and 2- vehicle accidents.

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b. For the union of 1- and 2-vehicle accidents the

$$\left\{\begin{matrix} \text{accident involvement} \\ \text{failures} \end{matrix}\right\} \quad \text{index}$$

$$\left\{\begin{matrix} \text{fractional deviations} \\ \text{from norm for} \end{matrix}\right\} \quad \begin{matrix} \text{indices} \\ \text{rates} \end{matrix}$$

is equal to a weighted sum of the

$$\left\{\begin{matrix} \text{accident involvement} \\ \text{failure} \end{matrix}\right\} \quad \text{indices}$$

$$\left\{\begin{matrix} \text{fractional deviations} \\ \text{from norm for} \end{matrix}\right\} \quad \begin{matrix} \text{indices} \\ \text{rates} \end{matrix}$$

for 1- and 2-vehicle accidents.

2.0 Notational Conventions

Script letters (e.g., $\mathscr{C}$) will denote categories (classes) or sets.

Script letters with subscripts denote subsets: $\mathscr{C}_k$.

Upper case letters, with or without subscripts, denote integers: E_k.

Lower case letters, with or without subscripts, denote proportions: $e_k = E_k/E$.

An unscripted single upper case letter denotes a summation over the range of the subscript:

$$E = E_1 + E_2 + \ldots + E_n = \sum_{k=1}^{n} E_k.$$

1 refers to the set of 1-vehicle accidents;

2 refers to the set of 2-vehicle accidents;

* (or u) refers to the union of the sets of 1- and 2-vehicle accidents.

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When double subscripts are used, the first subscript refers to the number of vehicles in the class of accidents under consideration, while the second subscript refers to a particular category.

AII denotes an <u>A</u>ccident <u>I</u>nvolvement <u>I</u>ndex;

AIR denotes an <u>A</u>ccident <u>I</u>nvolvement <u>R</u>ate;

FI denotes a <u>F</u>ailure <u>R</u>ate.

3.0 <u>Definitions</u>

E_k = the <u>number</u> of exposure units associated with category ℓ_k

e_k = the <u>proportion</u> of total exposure units associated with category $\ell_k = E_k/E$

3.1 1-Vehicle Accidents

S_k = the number of <u>single</u> vehicle accident <u>involvements</u>

= the number of <u>single</u> vehicle accident <u>failures</u> associated with ℓ_k.

$$S = S_1 + S_2 + \ldots + S_n = \sum_{k=1}^{n} S_k$$

$s_k = S_k/S$

3.2 2-Vehicle Accidents

I_k = the number of involvements in two vehicle accidents associated with ℓ_k.

$$I = \sum_{k=1}^{n} I_k$$

$i_k = I_k/I$

P = $I/2$ = the total number of <u>passive</u> involvements.

$p_k = P_k/P$ (assumption iii, Cerrelli $\rightarrow$ $p_k = e_k$ for 2-vehicle accidents)

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A_k = the number of active involvements in $\mathcal{L}_k$

$\quad = I_k - P_k$

$A = I - P = I/2$

$$a_k = \frac{A_k}{A} = \frac{I_k - P_k}{I/2} \left[= \frac{I_k - e_k P}{I/2} = \frac{I_k - e_k I/2}{I/2} \right. \quad \text{by iii}$$

$a_k + p_k = 2i_k$

3.3 Union of 1- and 2-Vehicle Accidents

I_{*k} (or I_k^*) $= S_k + I_k$ = total <u>involvements</u> associated with $\mathcal{L}_k$.

I_* (or I^*) $= S + I$

I_{*k} (or i_k^*) $= I_{*k}/I_*$

A_{*k} (or A_k^*) $= S_k + A_k$ = total failures associated with $\mathcal{L}_k$

A_* (or A^*) $= S + A$

a_{*k} (or a_k^*) $= A_{*k}/A_*$

4.0 <u>Accident Involvement Indices</u>

4.1 Definitions

$$(\text{AII})_{1k} = \frac{S_k/S}{e_k} = \frac{S_k}{e_k S} = \frac{s_k}{e_k} \quad ; \qquad (\text{AII})_1 = \frac{S/S}{e} = 1$$

$$(\text{AII})_{2k} = \frac{I_k/I}{e_k} = \frac{I_k}{e_k I} = \frac{i_k}{e_k} \quad ; \qquad (\text{AII})_2 = \frac{I/I}{e} = 1$$

$$(\text{AII})_{*k} = \frac{I_{*k}/I_*}{e_k} = \frac{i_{*k}}{e_k} \quad ; \qquad (\text{AII})_* = \frac{I_*/I_*}{e} = 1$$

4.2 Relationships

$$(AII)_1 + (AII)_2 = 2(AII)_*$$

$$(AII)_{*k} = \frac{S}{I_*} (AII)_{1k} + \frac{I}{I_*} (AII)_{2k}$$

$$= \text{weighted average of } (AII)_{1k} \text{ and } (AII)_{2k}$$

5.0 Failure Indices

5.1 Definitions

$$(FI)_{1k} = \frac{S_k/S}{e_k} = \frac{s_k}{e_k} \quad ; \quad (FI)_1 = \frac{S/S}{e} = 1$$

$$(FI)_{2k} = \frac{A_k/A}{e_k} = \frac{a_k}{e_k} \quad ; \quad (FI)_2 = \frac{A/A}{e} = 1$$

$$(FI)_{*k} = \frac{A_{*k}/A_*}{e_k} \equiv \frac{a_{*k}}{e_k} \quad ; \quad (FI)_* = \frac{A_*/A_*}{e} = 1$$

5.2 Relationships

$$(FI)_{*k} = \frac{S}{A} (FI)_{1k} + \frac{A}{A_*} (FI)_{2k} \; ; \; (FI)_1 + (FI)_2 = 2(FI)_*$$

6.0 Failure Rates

6.1 Definitions:

$$(FR)_{1k} = S_k/E_k \quad ; \quad (FR)_1 = S/E$$

$$(FR)_{2k} = A_k/E_k \quad ; \quad (FR)_2 = A/E$$

$$(FR)_{*k} = A_{*k}/E_k \quad ; \quad (FR)_* = A_*/E$$

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6.2 Relationships

$$(FR)_{*k} = (FR)_{1k} + (FR)_{2k} \quad ; \quad (FR)_{*} = (FR)_{1} + (FR)_{2}$$

$$(FR)_{1} = \sum_{k} e_{k}(FR)_{1k} = \text{weighted average over } (FR)_{1k}$$

$$(FR)_{2} = \sum_{k} e_{k}(FR)_{2k} = \text{weighted average over } (FR)_{2k}$$

$$(FR)_{*} = \sum_{k} e_{k}(FR)_{*k} = \text{weighted average over } (FR)_{*k}$$

or $$(FR)_{j} = \sum_{k} e_{k}(FR)_{jk}, \ j = 1, 2, *$$

7.0 Accident Involvement Rates

7.1 Definitions

$$(AIR)_{1k} = S_{k}/E_{k} \quad ; \quad (AIR)_{1} = S/E$$

$$(AIR)_{2k} = I_{k}/E_{k} \quad ; \quad (AIR)_{2} = I/E$$

$$(AIR)_{*k} = I_{*}/E_{k} \quad ; \quad (AIR)_{*} = I_{*}/E$$

7.2 Relationships

$$(AIR)_{1} = \sum_{k} e_{k}(AIR)_{1k} = \text{weighted average of } (AIR)_{1k}$$

$$(AIR)_{2} = \sum_{k} e_{k}(AIR)_{2k} = \text{weighted average of } (AIR)_{2k}$$

$$(AIR)_{*} = \sum_{k} e_{k}(AIR)_{*k} = \text{weighted average of } (AIR)_{*k}$$

or $$(AIR)_{j} = \sum_{k} e_{k}(AIR)_{jk}, \ j = 1, 2, *$$

$$(AIR)_{*k} = (AIR)_{1k} + (AIR)_{2k} \quad ; \quad (AIR)_{1} + (AIR)_{2} = (AIR)_{*}$$

8.0 Interrelationships between Rates

$$(AIR)_{1k} = (FR)_{1k} \qquad\qquad \frac{(FR)_1}{(AIR)_1} = \frac{S}{S}$$

$$(AIR)_{2k} = (FR)_{2k} + P_k/E_k \qquad\qquad \frac{(FR)_2}{(AIR)_2} = \frac{A}{I}$$

$$(AIR)_{*k} = (FR)_{*k} + P_k/E_k \qquad\qquad \frac{(FR)_*}{(AIR)_*} = \frac{A_*}{I_*}$$

If we define <u>passive rates</u>:

$$(PR)_{1k} \equiv 0$$

$$(PR)_{2k} = P_k/E_k$$

$$(PR)_{*k} = P_k/E_k$$

then $\boxed{(AIR)_{jk} = (FR)_{jk} + (PR)_{jk}, \ j = 1, 2, *}$

For $j = 1, 2, *$ $\dfrac{(FR)_j}{(AIR)_j}$ equals the ratio of total failures to total involvements

9.0 Interrelationships between Indices

$$(AII)_{1k} = (FI)_{1k} \qquad\qquad (FI)_{1k} = (AII)_{1k}$$

$$(AII)_{2k} = \tfrac{1}{2}[1 + (FI)_{2k}] \qquad\qquad (FI)_{2k} = 2(AII)_{2k} - 1$$

$$(AII)_{*k} = \frac{A_*}{I_*}(FI)_{*k} + \frac{A}{I_*} \qquad\qquad (FI)_{*k} = \frac{I_*}{A_*}(AII)_{*k} - \frac{A}{A_*}$$

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10.0 Interrelationships between Rates and Indices

$$(FI)_{1k} = \frac{(FR)_{1k}}{(FR)_1} = (AII)_{1k} \qquad\qquad (AII)_{1k} = \frac{(AIR)_{1k}}{(AIR)_1}$$

$$(FI)_{2k} = \frac{(FR)_{2k}}{(FR)_2} \qquad\qquad\qquad (AII)_{2k} = \frac{(AIR)_{2k}}{(AIR)_2}$$

$$(FI)_{*k} = \frac{(FR)_{jk}}{(FR)_*} \qquad\qquad\qquad (AII)_{*k} = \frac{(AIR)_{*k}}{(AIR)_*}$$

$$\boxed{(FI)_{jk} = \frac{(FR)_{jk}}{(FR)_j} \;,\; j = 1,\,2,\,* \qquad (AII)_{jk} = \frac{(AIR)_{jk}}{(AIR)_j} = j = 1,\,2,\,*}$$

11.0 Interrelationships between Fractional Deviations from Norm of Indices and Rates

Since $(FI)_j = (AII)_j = 1$ for $j = 1,\,2,\,*$

and $(FI)_{jk} = \dfrac{(FR)_{jk}}{(FR)_j}$ and $(AII)_{jk} = \dfrac{(AIR)_{jk}}{(AIR)_j}$ for $j = 1,\,2,\,*$,

$$(FI)_{jk} - 1 = \frac{(FI)_{jk} - 1}{1} = \frac{(FI)_{jk} - (FI)_j}{(FI)_j}$$

$$= \frac{(FR)_{jk}}{(FR)_j} - 1 = \frac{(FR)_{jk} - (FR)_j}{(FR)_j}$$

so $\boxed{(FI)_{jk} - 1 = \dfrac{(FI)_{jk} - (FI)_j}{(FI)_j} = \dfrac{(FR)_{jk} - (FR)_j}{(FR)_j} \;,\; j = 1,\,2,\,*}$

and in the same way

$$\boxed{(AII)_{jk} - 1 = \frac{(AII)_{jk} - (AII)_j}{(AII)_j} = \frac{(AIR)_{jk} - (AIR)_j}{(AIR)_j} \;,\; j = 1,\,2,\,*}$$

For j = 1,

$$\frac{(FI)_{1k} - (FI)_1}{(FI)_1} = \frac{(FR)_{1k} - (FR)_1}{(FR)_1} = \frac{(AII)_{1k} - (AII)_1}{(AIR)_{1k}} = \frac{(AIR)_{1k} - (AIR)_1}{(AIR)_1}$$

i.e., all fractional deviations are the same.

For j = 2,

$$\frac{(FI)_{2k} - (FI)_2}{(FI)_2} = \frac{(FR)_{2k} - (FR)_2}{(FR)_2} = \frac{2(AII)_{2k} - (AII)_2}{(AII)_2} = \frac{2(AIR)_{2k} - (AIR)_2}{(AIR)_2}$$

i.e., fractional deviations for failures are twice the fractional deviations for involvements.

For j = *, $(FI)_{*k} - 1 = \dfrac{I_*}{A_*} [(AII)_{*k} - 1]$

i.e., fractional deviations for failures is I_*/A_* times the fractional deviations for involvements. Note: $I_*/A_* > 1$.

Note that I_*/A_* is the ratio of total involvements to total failures (active involvements).

For j = 1, this ratio is 1

j = 2, this ratio is 2

j = *, this ratio is I_*/A_*,

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so we can generalize:

> The ratio of fractional deviations from the norms for failure is the ratio of total involvements to total failures times the fractional deviations from the norm of involvements. This holds for j = 1, 2, *.

> For j = *,
>
> $$(FI)_{*k} - 1 = \frac{S}{A_*} [(FI)_{1k} - 1] + \frac{A}{A_*} [(FI)_{2k} - 1], \quad A_* = S + A$$
>
> i.e., the fractional deviation of $\begin{Bmatrix} \text{failures} \\ \text{involvement} \end{Bmatrix}$ from the norm for the union of the classes of 1- and 2-vehicle accidents is equal to the weighted sum of the fractional deviations of $\begin{Bmatrix} \text{failures} \\ \text{involvements} \end{Bmatrix}$ from the norm for 1- and 2-vehicle accidents.
>
> $$(AII)_{*k} - 1 = \frac{S}{I_*} [(AII)_{1k} - 1] + \frac{I}{I_*} [(AII)_{2k} - 1], \quad I_* = S + I$$

One can combine the above results with those from sections 4.2 and 5.2 to state more generally:

> For the union of 1- and 2-vehicle accidents the
>
> $\begin{Bmatrix} \text{accident involvement} \\ \text{failure} \end{Bmatrix}$ index
>
> is equal to a weighted sum of the
>
> $\begin{Bmatrix} \text{fractional deviations} \\ \text{from norm for} \end{Bmatrix}$ indices rates
>
> $\begin{Bmatrix} \text{accident involvement} \\ \text{failures} \end{Bmatrix}$ indices
>
> for 1- and 2-vehicle accidents.
>
> $\begin{Bmatrix} \text{fractional deviations} \\ \text{from norm for} \end{Bmatrix}$ indices rates

12.0 Derivation of Interrelationships between Indices

Starting from

$$(AII)_{*k} = \frac{S}{S+I} (AII)_{1k} + \frac{I}{S+I} (AII)_{2k}, \quad I = 2A \qquad \text{(Sec. 4.2)}$$

$$= \frac{S+A}{S+I} \left[\frac{S}{S+A} (AII)_{1k} + \frac{2A}{S+A} (AII)_{2k} \right]$$

$$= \frac{S+A}{S+I} \left[\frac{S}{S+A} (FI)_{1k} + \frac{2A}{S+A} \frac{1}{2} (1 + (FI)_{2k}) \right] \qquad \text{(Sec. 9)}$$

$$= \frac{S+A}{S+I} \left[\underbrace{\frac{S}{S+A} (FI)_{1k} + \frac{A}{S+A} (FI)_{2k}}_{(FI)_{*k}} + \frac{A}{S+A} \right] \qquad \text{(Sec. 5.2)}$$

$$= \frac{S+A}{S+I} (FI)_{*k} + \frac{A}{S+I} = \frac{A_*}{I_*} (FI)_{*k} + \frac{A}{I_*}, \quad \begin{aligned} A_* &= S + A \\ I_* &= S + I \end{aligned}$$

or

$$(AII)_{*k} = \frac{A_*}{I_*} (FI)_{*k} + \frac{A}{I_*}$$

$$= \frac{A_*}{I_*} (FI)_{*k} - \frac{A_*}{I_*} + \frac{A_*}{I_*} + \frac{A}{I_*}$$

$$= \frac{A_*}{I_*} [(FI)_{*k} - 1] + \frac{A_* + A}{I_*}$$

and

$$(AII)_{*k} - 1 = \frac{A_*}{I_*} [(FI)_{*k} - 1] + \frac{A_* + A - I_*}{I_*}$$

Since $A_* + A - I_* = S + 2A - S - I = 2A - I = 0$, the last term

falls out, and

$$(AII)_{*k} - 1 = \frac{A_*}{I_*} [(FI)_{*k} - 1] \qquad \text{(Sec. 10)}$$

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Using Cerrelli's association of 'failure' with active involvements in the case of two vehicles accidents, one can define failure indices, accident involvement indices, failure rates, accident involvement rates for 1-vehicle, 2-vehicle and for the union class of 1- and 2-vehicle accidents. This note presents in a condensed form all of the definitions of these measures, and a number of useful relationships and interrelationships that exist between these different measures. These formulae should prove useful in the calculation of the resulting indices and rates and of converting from one set of measures to another.

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