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VIBRATION ISOLATION:
Use and Characterization

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VIBRATION ISOLATION:
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Preface

Unwanted vibrations at even very low levels of acceleration can lead to serious problems in monitoring and control of industrial processes as well as in the conduct of accurate measurements of many kinds. At higher levels of acceleration, vibrations can cause structural damage, degradation of product quality, can directly result in human discomfort, and produce airborne acoustical noise. When possible, it is often preferable to control vibration at the source; however, this may not be practical if to do so would require costly redesign or modification of equipment or structures. Modification of the transmission path between a source of vibration energy and the equipment that must be protected from excessive vibration is frequently the most cost-effective means of vibration control.

In this handbook, Dr. Snowdon has carefully reviewed, evaluated and synthesized a large body of literature concerned with the use and the characterization of the performance of vibration isolators and has summarized analytical and experimental procedures for characterizing the effectiveness of antivibration mountings. This state-of-the-art review also provides a basis for further research in the development of improved techniques for evaluation of vibration isolation.

We are pleased to make this report available as a resource for designers and users of vibration isolation systems and for scientists and engineers who are carrying out research on this important topic.

John A. Simpson, Director
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Abstract

The results of a search and critical evaluation of the literature pertinent to both the use and the characterization of the performance of antivibration mountings for the control of noise and vibration are described. First to be discussed are the static and dynamic properties of rubberlike materials that are suited for use in antivibration mountings. This is followed by analyses of the simple (one-stage) mounting system and its subsequent, impaired performance when second-order resonances occur either in the isolator (wave effects) or in the structure of the mounted item itself (nonrigid supporting feet). A discussion is then given to the performance of the compound or two-stage mounting system which possesses superior isolation properties for high frequencies. Next, the four-pole parameter technique of analysis is described and applied, in general terms, to the characterization of the performance of an antivibration mounting with wave effects for both the cases where either the supporting foundation or mounted item are nonrigid. The adopted methods for the direct measurement of antivibration-mounting performance are described, followed by an explanation of how this same experimental determination of transmissibility can also be made using an indirect measurement technique based upon four-pole parameter analysis considerations. Finally, recommendations for future work in various areas of research on antivibration mountings are given.

Key Words: Antivibration mounting; damping; dynamic properties; industrial engineering; isolation; machinery and equipment; mechanical impedance; mechanical vibrations; noise control; transmissibility; vibrations; vibration isolation.

Table of Contents

List of Figures

Introduction

This report is concerned with vibration isolation, with antivibration mountings (resilient isolators), and with the static and dynamic properties of rubberlike materials that are suited for use in antivibration mountings. The design of practical antivibration mounts incorporating rubber or coiled-steel springs is described in Refs. 1-27; pneumatic isolators (air mounts, etc.) are described in Refs. 5, 28-35.* Throughout the literature, as here, attention is focussed predominantly on the translational (vertical) effectiveness of antivibration mountings. However, the two- and three-dimensional vibration of one- or two-stage mounting systems is addressed in Refs. 4, 10, 12, 36-56.

Following a description of the static and dynamic properties of rubberlike materials, the performance of the simple or one-stage mounting system is analyzed, account being taken of the occurrence of second-order resonances in the isolator and in the mounted item. In the latter case, as likely in practice, the bulk of the mounted item is assumed to remain masslike whereas the feet of the item are assumed to be nonrigid (multi-resonant). Discussion is then given to the two-stage or compound mounting system, which affords superior vibration isolation at high frequencies. Subsequently, the powerful four-pole parameter technique (Ref. 57) is employed to analyze, in general terms, the performance of an antivibration mounting with second-order resonances (wave effects) when both the foundation that supports the mounting system and the machine are nonrigid.

The universally adopted method of measuring mount transmissibility is then described, followed by an explanation of how transmissibility can also

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* Occasionally in this report, trade names are given in order to provide adequate identification of materials or products. Such identification does not constitute endorsement by the National Bureau of Standards.

be determined by four-pole parameter techniques based on an apparatus used by F. Schloss (Ref. 58). The four-pole measurement approach has not been exploited hitherto, but it is apparently feasible and valuable because it enables mounts to be tested under compressive loads equal to those routinely encountered in service.

1. Static Properties of Rubberlike Materials

The strain induced in a purely elastic linear material is proportional to the stress that produces the deformation. As explained in Ref. 59, two fundamental types of deformation that a rubberlike material may experience are described by two independent elastic moduli. Thus, the shear modulus G describes a shear deformation for which the material does not change in volume [Fig. 1(a)], and the bulk modulus B describes a volume deformation for which the material does not change in shape [Fig. 1(b)]. Rubbers that do not contain fine particles of carbon black reinforcement (filler) have shear and bulk moduli of approximately 0.7 and 10^3 MPa (7×10^6 and 1×10^{10} dyn/cm^2 or 100 and 10^5 psi).

A sample of material sandwiched between plane, parallel, rigid surfaces in the configuration of Fig. 1(c) is frequently said to be in compression, but it is not homogeneous compression governed by the bulk modulus B. In fact, the mechanical behavior is governed primarily by B only when the lateral dimensions of the sample are very large in comparison with the sample thickness [Fig. 1(d)]. In this event the material changes in both shape and volume, and the ratio of stress to strain in the material is described by a modulus M given by

$$M = B + (4G/3) \approx B \quad . \tag{1}$$

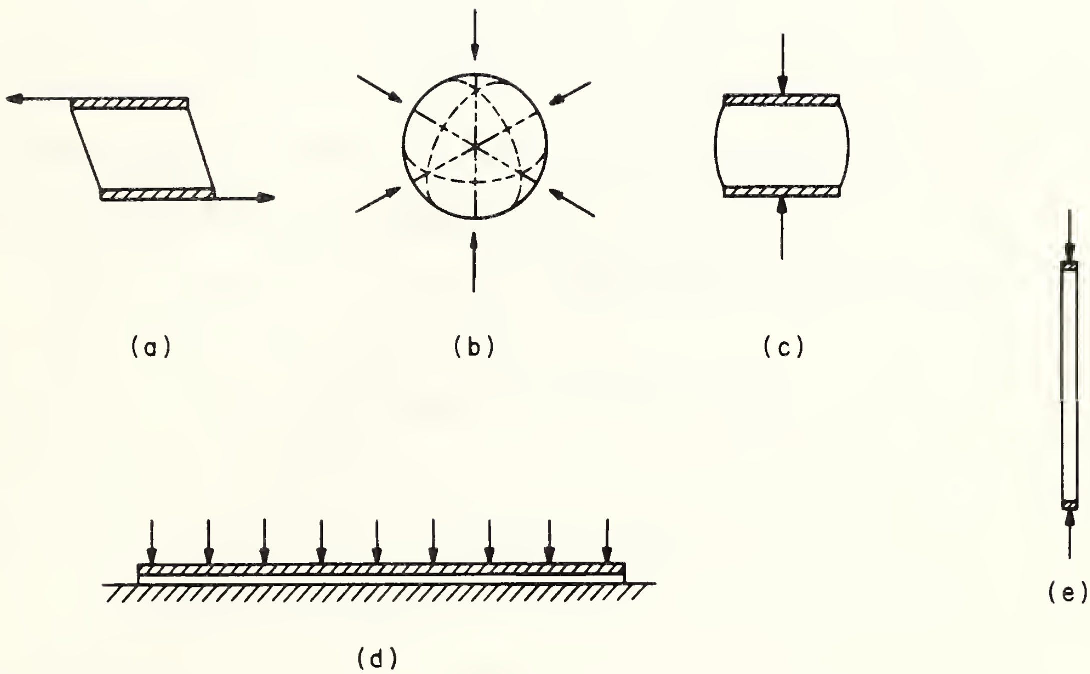


Fig. 1 Simple deformations of a rubberlike material. (Ref. 59.)

This is to say the resilience that is normally associated with the rubber-like material is not apparent because B >> G. If resilience is required in this situation, it is necessary to use spaced strips of material or a perforated sheet (Ref. 3), thereby leaving the material free to expand laterally when it is compressed vertically.

Also considered must be the other geometric extreme, in which the lateral dimensions of the sample are small in comparison with the sample thickness; namely, the sample is a rod or bar and the stress is applied along its axis as in Fig. 1(e). In this event the ratio of stress to strain in the material is governed by the Young's modulus E (approximately 2 MPa, 2×10^7 dyn/cm^2, or 300 psi for unfilled rubbers), and the ratio of the resulting lateral to axial strain is described by Poisson's ratio ν. For rubbers, it is well known that

$$E = 9\,BG/(3B + G) \approx 3G \tag{2}$$

and

$$\nu = [(E/2G) - 1] \approx 0.5 \quad . \tag{3}$$

An element of rubberlike material in the configuration of Fig. 1(c) possesses an apparent modulus of elasticity E_a that is intermediate in value to the moduli E and M [Figs. 1(e) and (d)]. The rubberlike material is usually bonded to the rigid surfaces between which it is compressed, in which case (Refs. 21, 23, 60, 61) it is possible to state that

$$E_a = \frac{E(1 + \beta S^2)}{[1 + (E/B)(1 + \beta S^2)]} \quad , \tag{4}$$

where the so-called shape factor S (Refs. 3, 5, 11, 13, 15, 21-23, 26, 39, 59-75) is equal to the ratio of the area of one loaded surface to the total force-free area, and β is a numerical constant. The shape factor of a rubber cylinder of diameter D and height ℓ is equal to $D/4\ell$; the shape factor of a rectangular rubber block of sides a and b and height ℓ is equal to $ab/2\ell(a + b)$. For all samples except those with large lateral dimensions (large shape factors), Eq. 4 can be written as

$$E_a = E(1 + \beta S^2) \quad . \tag{5}$$

Note that because $E \approx 3G$, the apparent modulus of elasticity E_a is some simple numerical multiple of the shear modulus G.

The dependence of the apparent modulus E_a on shape factor is plotted in Fig. 2 for rubbers of various hardness (Refs. 21, 23). The curves of this figure have the form predicted by Eq. 4. Measured values of E, G, and B for natural rubbers of increasing hardness (increasing volume of carbon black filler) are listed in Table I (Ref. 23); the related values of β are also listed. For rubbers unfilled by carbon black, $\beta = 2$. Equation 5 is valid for samples that are circular, square, or moderately rectangular in cross section. However, for a pronounced rectangular rail-type sample--a so-called compression strip for which $b \gg a$--a companion equation pertains (Refs. 21, 23); that is,

$$E_a = (2/3)\, E(2 + \beta S^2) \quad , \tag{6}$$

where $S = a/2\ell$.

Hardness measurements can provide an estimate of E and G (Refs. 23 and 3, 5, 11, 13, 17, 39, 76, 77). Hardness is readily measured but is

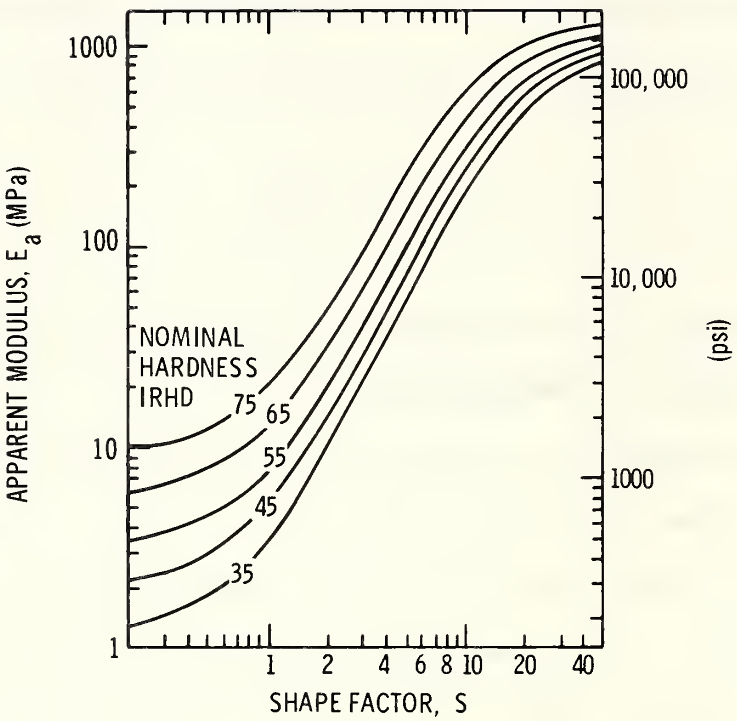


Fig. 2 Dependence of the apparent modulus E_a on shape factor S for natural rubbers of various hardnesses. (Refs. 21, 23.)

Table 1. Hardness and elastic moduli of natural rubber spring vulcanizates containing (above 48 IRHD) SRF carbon black as filler. (Ref. 23.)

Hardness IRHD $\pm$ 2	β (Eqs. 4-6)	Youngs modulus (MPa)	Shear modulus (MPa)	Bulk modulus (MPa)
30	1.86	0.92	0.30	1000
35	1.78	1.18	0.37	1000
40	1.70	1.50	0.45	1000
45	1.60	1.80	0.54	1000
50	1.46	2.20	0.64	1030
55	1.28	3.25	0.81	1090
60	1.14	4.45	1.06	1150
65	1.08	5.85	1.37	1210
70	1.06	7.35	1.73	1270
75	1.04	9.40	2.22	1330

subject to some uncertainty, hence the tolerance in the hardness values quoted in Table I. Rubber hardness is essentially a measure of the reversible, elastic deformation produced by a specially shaped indentor under a specified load and is therefore related to E. Readings in International Rubber Hardness degrees (IRHD) and on the Shore Durometer A scale are approximately the same. An objection to such hardness measurements is said to be that both stress and strain vary throughout the test. Thus, as the indentation proceeds, the load is distributed by an increasing area of contact between the indentor and the sample, so causing the average contact pressure to diminish.

Creep is the continued deformation (drift) of a rubber under static load (Refs. 23 and 11, 12, 15, 39, 62, 78-81). When the load is removed, all but a few percent of the original deformation is recovered immediately; further recovery takes much longer and may never be achieved. The incompletely recovered deformation is termed permanent set. Creep varies linearly with the logarithm of time; for example, the amount of creep occurring in the decade of time from 1 to 10 minutes after loading is the same as the amount in the decade from 1 to 10 weeks after loading. Creep under load should not exceed 20% (for 70 IRHD) of the initial deflection during the first several weeks; only a further 5 - 10% increase in deflection should then occur over a period of many years.

Load-deflection and stress-strain curves for statically compressed rubber are referred to throughout the literature (Refs. 3, 5, 11, 13, 17, 21, 23, 27, 36, 39, 60, 61, 63, 64, 67, 69, 70, 73, 74, 76, 77, 82-91). A series of stress-strain curves from Ref. 11 is shown in Fig. 3, which refers to various shape factors and deflections up to 50% (a value seldom reached in practice) for a rubber hardness of 40 Shore Durometer. These data are said not to be limited to one type of rubber but they do relate to room temperature and to rubber samples bonded to rigid surfaces as in the manner of an antivibration mount [Fig. 1(c)].

To conclude, it is appropriate to mention that the natural frequency f_o of a resiliently mounted item (Sec. 3) can be expressed in terms of the static deflection d of the resilient element as follows:

$$f_o = 0.4984/\sqrt{d} \text{ Hz} \qquad \text{(d meters)} \qquad (7)$$

or

$$f_o = 3.1273/\sqrt{d} \text{ Hz} \qquad \text{(d inches)} \quad . \qquad (8)$$

Values of f_o can be read from the straight-line plot of Fig. 4.

2. Dynamic Properties of Rubberlike Materials

The dynamic properties of rubberlike materials that experience sinusoidal vibration are readily accounted for by writing the elastic moduli

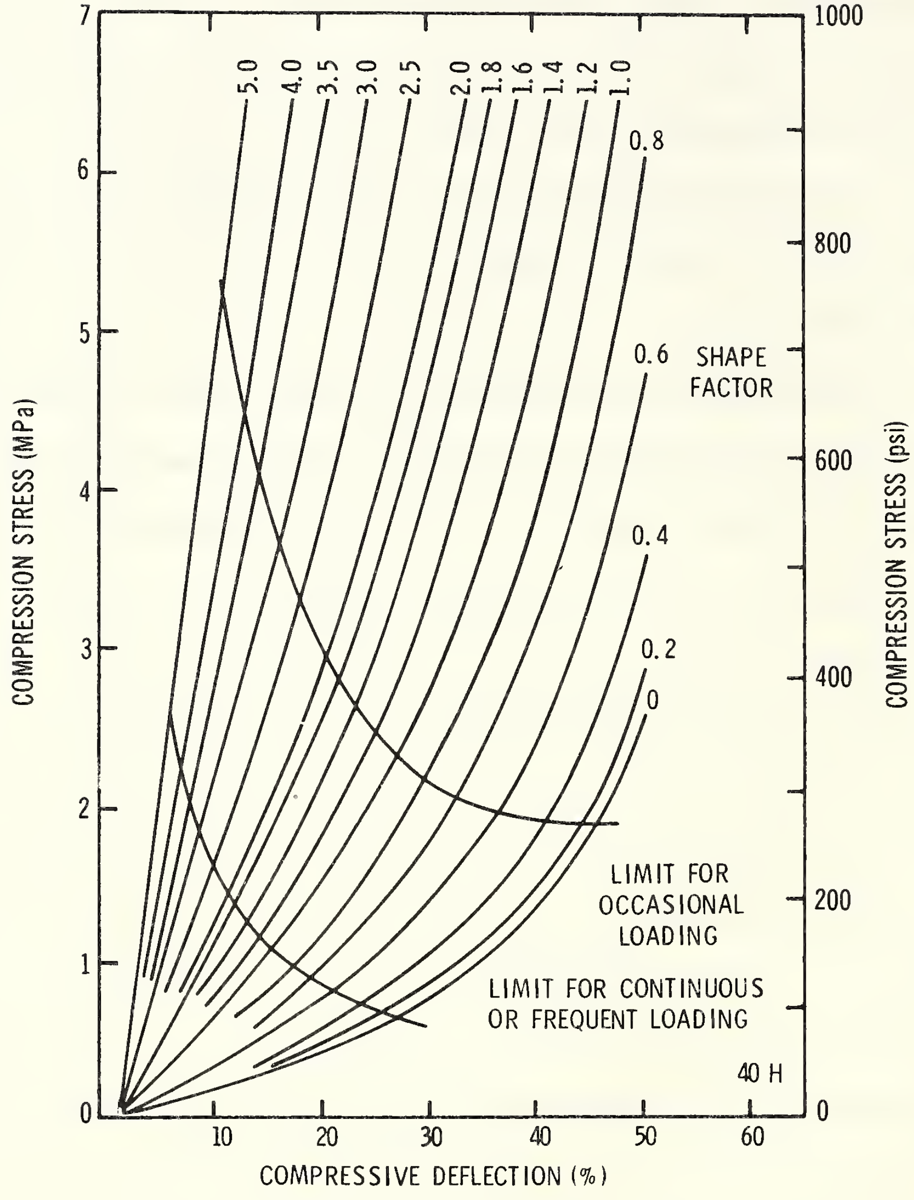


Fig. 3 Stress-strain curves of 40 durometer rubber in compression with various shape factors. (Ref. 11.)

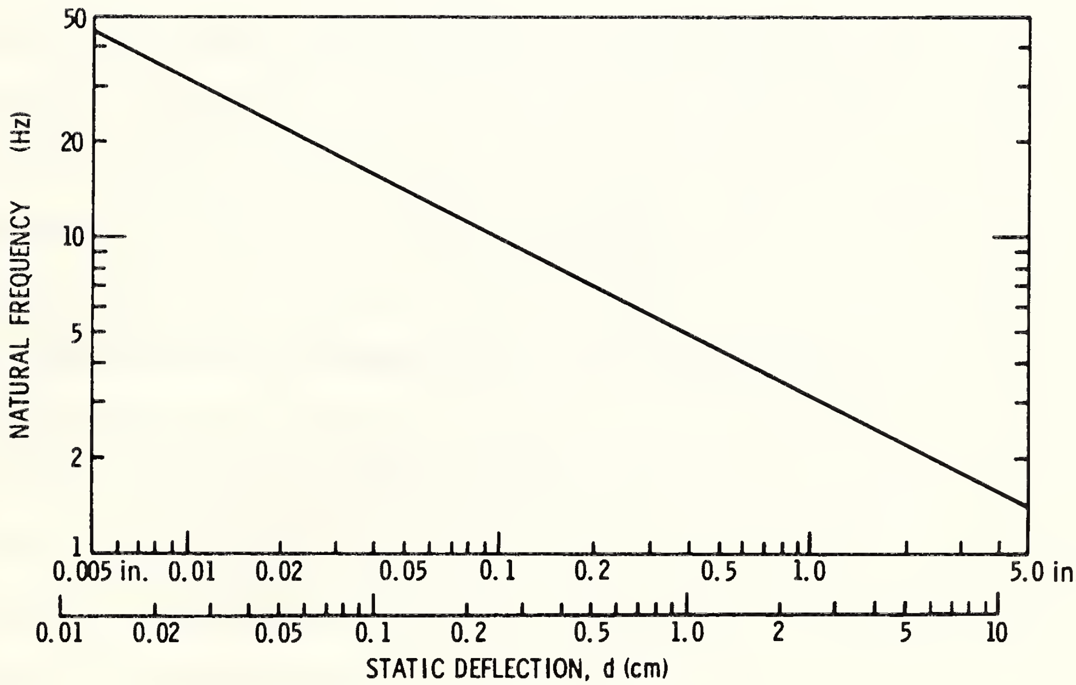


Fig. 4 Natural frequency of a simple mounting system versus static deflection of the antivibration mount.

that govern the vibration as complex quantities (Ref. 59). For example, the Young's modulus and the shear modulus are most generally written

$$E^*_{\omega,\theta} = E_{\omega,\theta} (1 + j\delta_{E\omega,\theta}) \quad (9)$$

and

$$G^*_{\omega,\theta} = G_{\omega,\theta} (1 + j\delta_{G\omega,\theta}) \quad . \quad (10)$$

Here, the star superscripts denote complex quantities and $j = \sqrt{-1}$; the so-called dynamic moduli $E_{\omega,\theta}$ and $G_{\omega,\theta}$ are the real parts of the complex moduli $E^*_{\omega,\theta}$ and $G^*_{\omega,\theta}$, and $\delta_{E\omega,\theta}$ and $\delta_{G\omega,\theta}$ are the so-called damping or loss factors associated with the Young's modulus and shear deformations of the material. The subscripts ω and θ indicate that the dynamic moduli and damping factors are, in general, functions of both angular frequency ω and temperature θ. The damping factors are equal to the ratios of the imaginary to the real parts of the complex moduli, and are directly equivalent to the reciprocal of the quality factor Q that is employed in electrical circuit theory to describe the ratio of an inductive reactance to a resistance. The damping factors are also equivalent to other commonly employed measures of damping such as those listed in Fig. 5.

There is nothing magical in the concept of a complex modulus--it means only that strain lags in phase behind stress in the rubberlike material by an angle the tangent of which is the damping factor $\delta_{E\omega,\theta}$ or $\delta_{G\omega,\theta}$. The damping factors $\approx$ 1.0 for "high-damping" rubbers, and $\approx$ 0.1 or less for "low-damping" rubbers--in which case the dynamic moduli and damping factors vary only slowly with frequency through the audio-frequency range at room temperature, as will be illustrated subsequently.

$$
\begin{aligned}
\text{DAMPING FACTOR } \delta &= \text{LOSS FACTOR } \eta \text{ or } \beta \\
&= \text{TAN } \delta \\
&= 2\,(\text{DAMPING RATIO } C/C_c) \\
&= (1/\pi)\,(\text{LOGARITHMIC DECREMENT}) \\
&= (1/2\pi)\,(\text{SPECIFIC DAMPING CAPACITY}) \\
&= 1/(\text{QUALITY FACTOR } Q) \\
&= (\text{RESONANT BANDWIDTH})/\omega_0
\end{aligned}
$$

(PROVIDED THAT THE DAMPING FACTOR IS LESS THAN APPROXIMATELY 0.3)

Fig. 5 Equivalence between the damping factor δ employed in this report and other commonly employed measures of damping.

For rubberlike materials, the complex shear and Young's moduli exhibit the same frequency dependence (Ref. 59); that is to say,

$$E_{\omega,\theta} = 3\ G_{\omega,\theta} \tag{11}$$

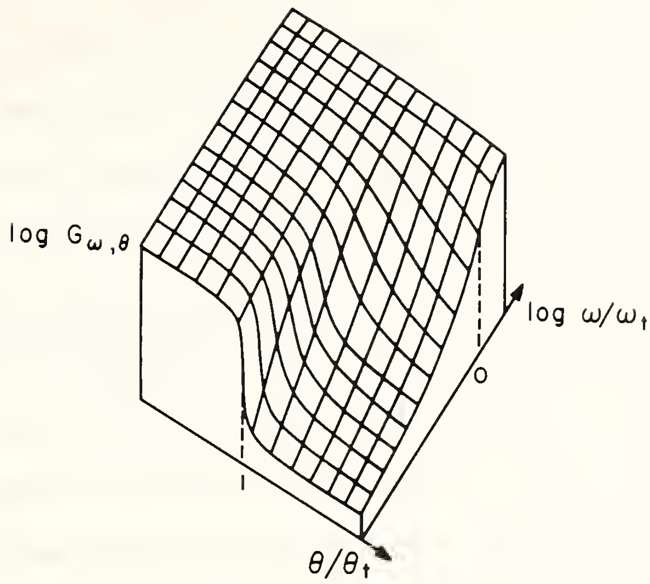
and

$$\delta_{E\omega,\theta} = \delta_{G\omega,\theta} \quad . \tag{12}$$

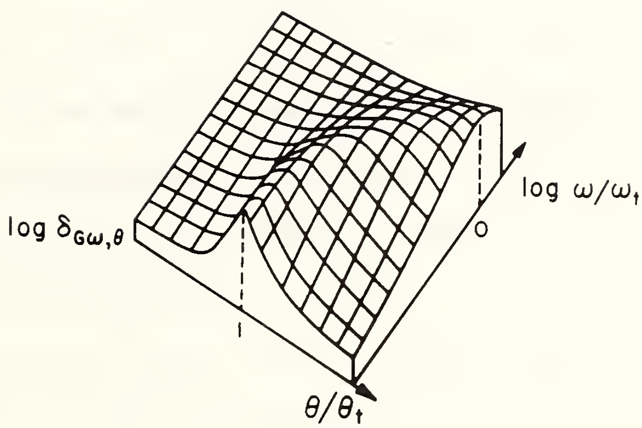
The dynamic moduli of Eq. 11 are found experimentally to increase in value when frequency increases or when temperature decreases. This is best visualized by reference to Fig. 6, where, for example, the dynamic modulus $G_{\omega,\theta}$ and damping factor $\delta_{G\omega,\theta}$ are shown diagrammatically as a function of angular frequency ω (hereafter referred to simply as frequency) and temperature θ. The transition frequency ω_t and temperature θ_t refer to the transition of rubberlike materials at sufficiently high frequencies or sufficiently low temperatures to an "inextensible" or glasslike state, $G_{\omega,\theta}$ becoming so large that the characteristic resilience of the material is no longer apparent. At the so-called rubber-to-glass transition, the damping factor passes through a maximum value that lies approximately in the frequency or temperature range through which $G_{\omega,\theta}$ is increasing most rapidly.

Much effort has been expended over the years to develop test apparatus to yield values of the dynamic moduli and their associated damping factors (Refs. 13, 17, 21, 66, 75, 92-119). One apparatus has been particularly criticized (Refs. 110, 120-127), but for soft rubberlike materials and data recorded away from regions of fluctuating response, the results obtained are thought to be reliable.

Any single piece of apparatus is limited in that dynamic measurements cannot be made through an extensive frequency range; however, it is generally



(a)



(b)

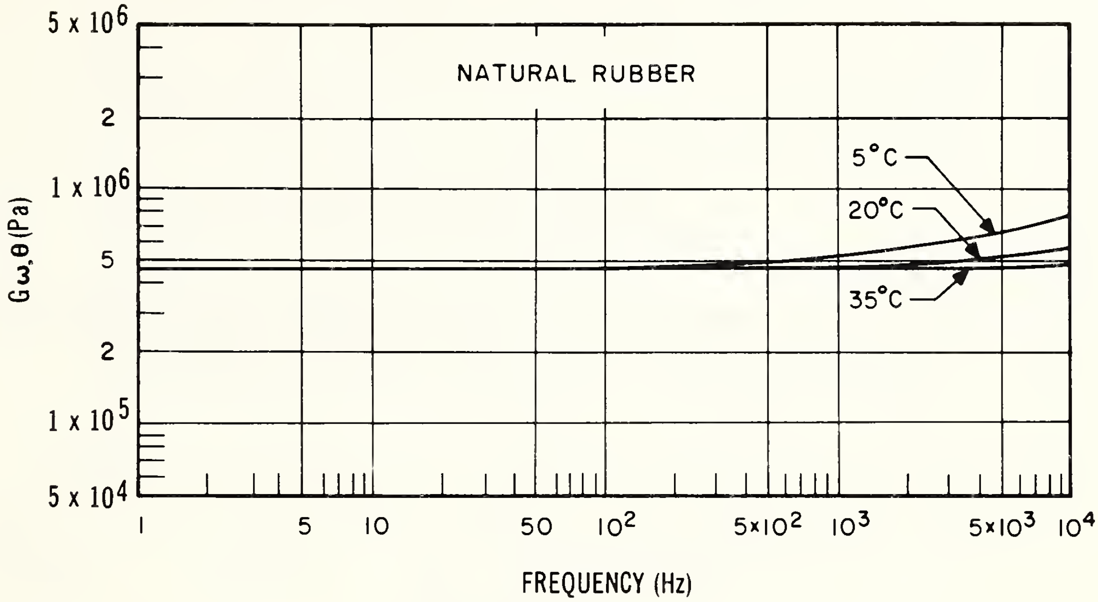
Fig. 6 Dependence of (a) the dynamic shear modulus $G_{\omega,\theta}$, and (b) the shear damping factor $\delta_{G\omega,\theta}$ of a rubberlike material on angular frequency ω and temperature θ. (Ref. 59.)

straightforward to make measurements through a wide temperature range. Then, with a well-established technique known as the method of reduced variables, the dynamic moduli and damping factors can be predicted through a very broad frequency range at a single temperature of interest such as room temperature. The method of reduced variables (Ref. 59), although originally semiempirical, is now well validated both by theory and by successful usage.

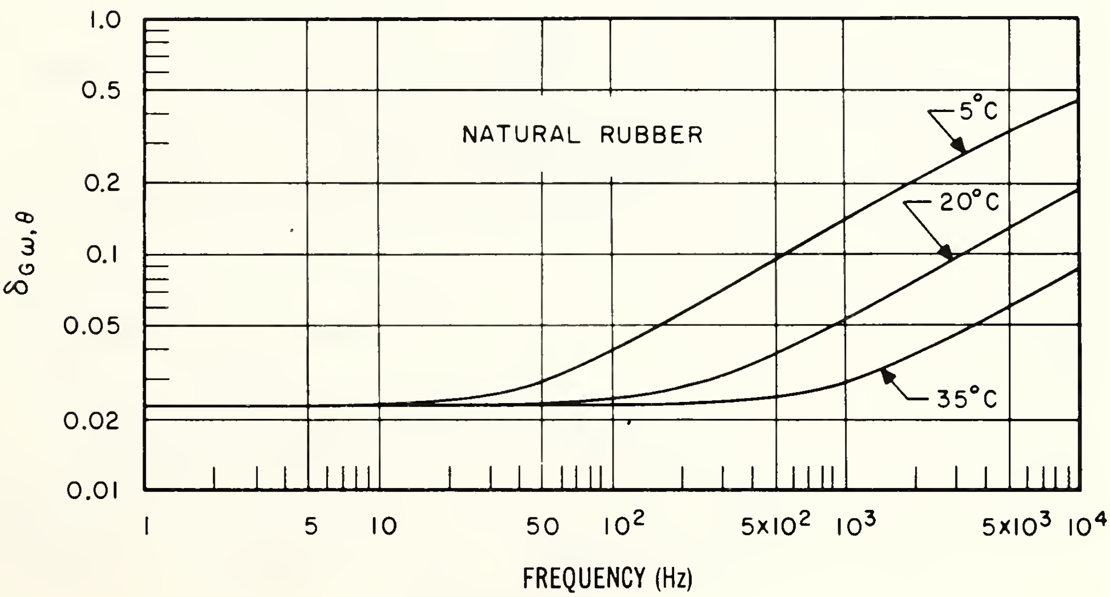
Examples of data established in the foregoing way (Ref. 59) are reproduced in Figs. 7-9, where the dynamic shear moduli and damping factors of unfilled natural rubber, natural rubber filled with 50 parts by weight of high-abrasion furnace (HAF) black, and Thiokol RD rubber are plotted versus frequency in the audio frequency range 1 Hz - 10 kHz at $5^{o}C$, $20^{o}C$, and $35^{o}C$. Other measurements of the dynamic shear and Young's moduli and their associated damping factors are reported in Refs. 21, 59, 62, 65, 73, 79, 86, 92, 94-96, 99, 100, 102, 104, 105, 108, 110, 111, 116-118, 128-142.

Rubbers are reinforced with carbon black to increase their stiffness, tear resistance, and abrasion resistance--to an extent that depends on the type of black utilized. Furnace, channel, lamp, and thermal blacks cover a wide range of particle sizes; furnace and channel blacks are the most finely divided. Note that the presence of carbon black (1) has increased the dynamic shear modulus of the natural rubber of Fig. 8 by a factor of approximately 10 above that of the unfilled rubber of Fig. 7, and (2) has increased the value of the damping factor, particularly at low frequencies. It should be recognized, however, that the addition of carbon black may reduce the damping factor significantly at frequencies above the range considered here.

Although $G_{\omega,\theta}$ and $\delta_{G\omega,\theta}$ increase only by a factor of two or three at room temperature through the four decades in frequency considered in Fig. 8, it is well to remember this fact if satisfactory engineering design is to

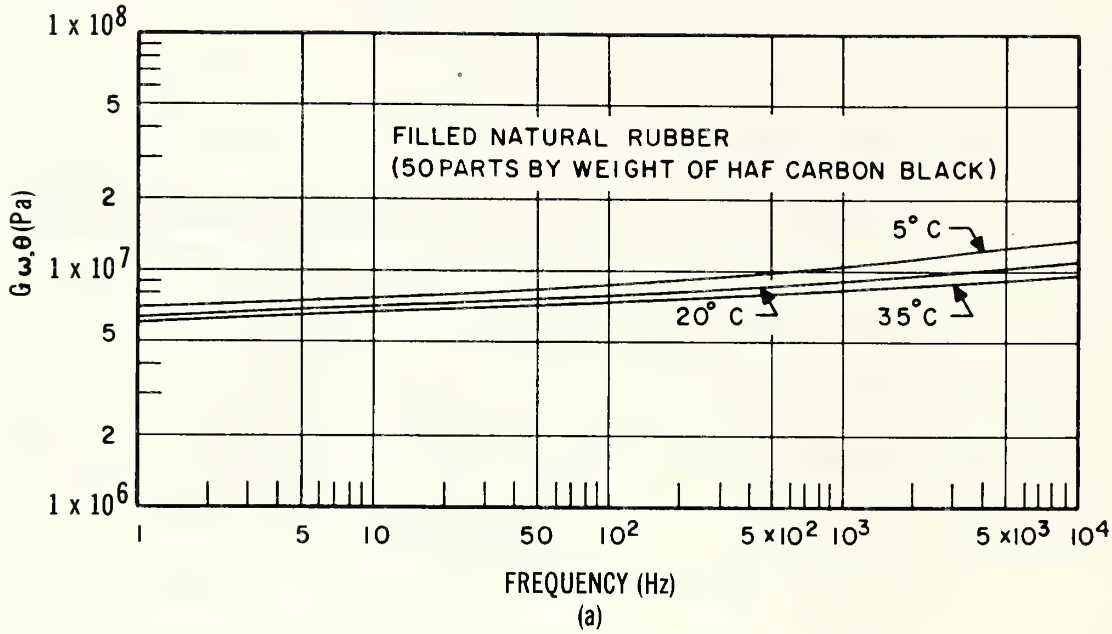


(a)



(b)

Fig. 7 Frequency dependence of (a) the dynamic shear modulus, and (b) the damping factor of unfilled natural rubber at 5, 20, and 35°C (41, 68, and 95°F). (Ref. 59.)



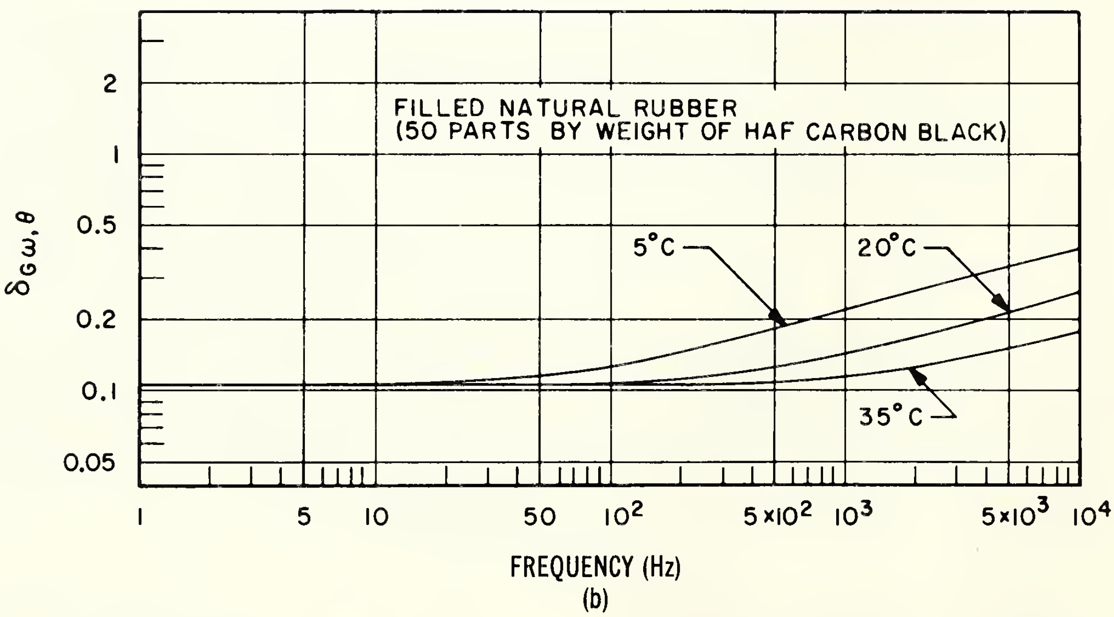


Fig. 8 Frequency dependence of (a) the dynamic shear modulus, and (b) the damping factor of natural rubber filled with 50 parts by weight of HAF carbon black per 100 parts rubber. (Ref. 59.)

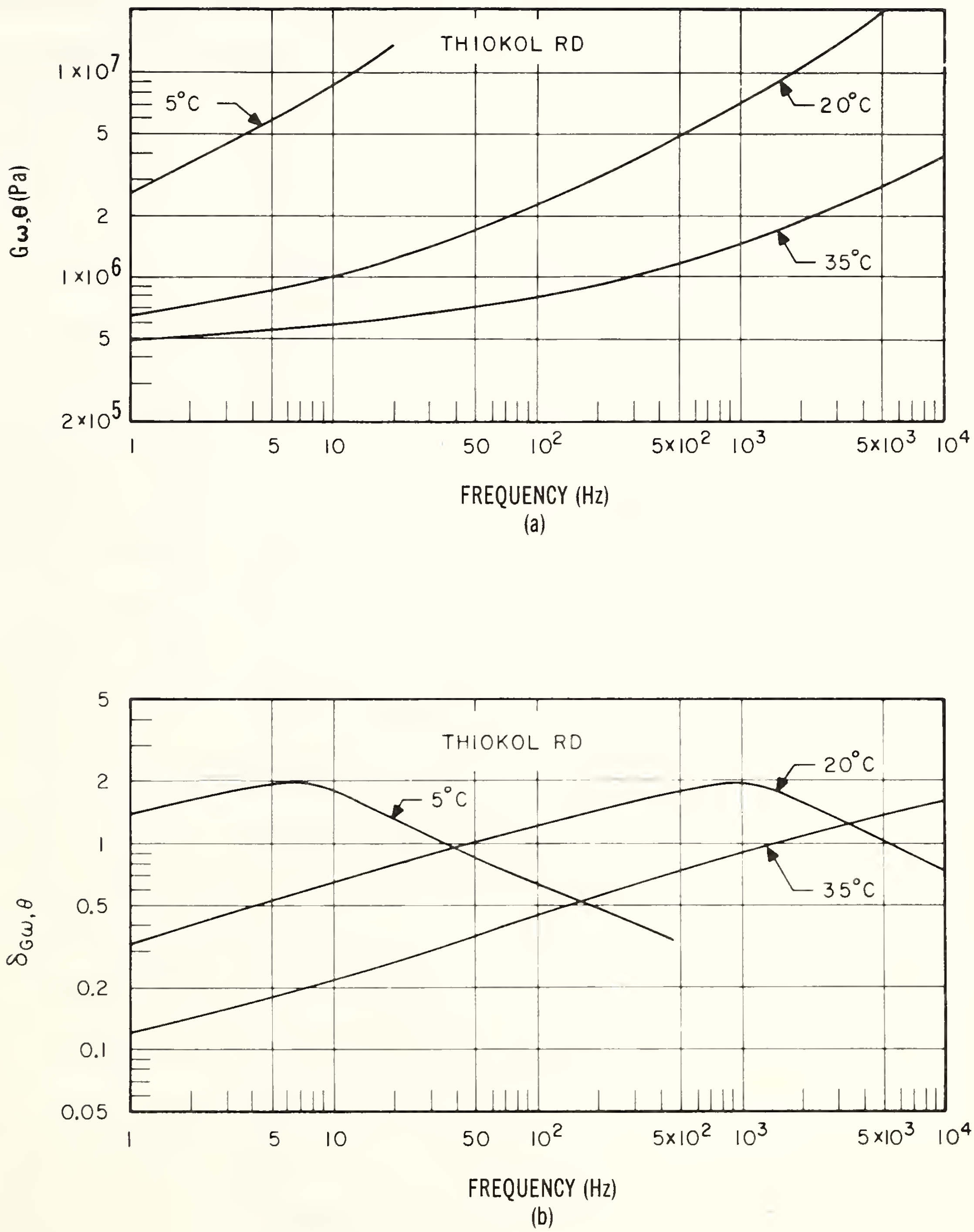
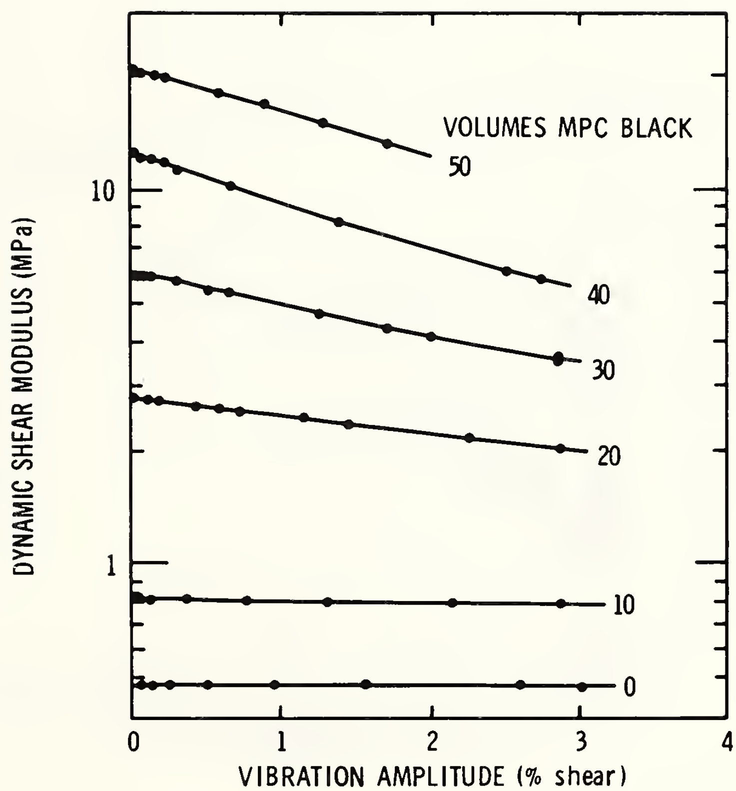


Fig. 9 Frequency dependence of (a) the dynamic shear modulus, and (b) the damping factor of an unfilled rubber Thiokol RD. (Ref. 59.)

be achieved when high-frequency vibration is of concern. By contrast, the dynamic modulus $G_{\omega,\theta}$ of high-damping rubberlike materials increases greatly with frequency, and $G_{\omega,\theta}$ and $\delta_{G\omega,\theta}$ are strongly dependent on temperature, as the curves of Fig. 9 for Thiokol RD rubber illustrate. Thiokol RD was produced some years ago as an experimental copolymer of butadiene-acrylonitrile and, most probably, chloroprene. It is referred to here because, in analytical studies of antivibration mountings that are described subsequently, its properties have been considered to typify those of high-damping rubbers.

The data of Figs. 7-9 relate to small amplitudes of vibration for which the rubberlike materials exhibit linear behavior. Whereas unfilled and lightly filled rubbers remain linear for increasing strain, up to relatively large strains, the dynamic moduli and damping factors of moderately and heavily filled rubbers show a strong amplitude dependence (Refs. 15, 73, 81, 86, 98, 103, 106, 107, 135, 136, 143-154). This fact is exemplified by the curves of Figs. 10 and 11, which are drawn from Refs. 144 and 135, respectively. The data of Fig. 10, for example, which refer to exciting frequencies in the range 20 - 120 Hz, show that the dynamic shear modulus of natural rubber containing 40 parts by volume of medium processing channel (MPC) black is more than halved when an alternating shear strain of 3% breaks down the three-dimensional aggregates or so-called matrix of carbon particles within the rubber. The greatly increased damping factor that accompanies this strain amplitude is primarily a reflection of the reduction in value of the dynamic modulus--rather than an increase per se in the imaginary part of the complex modulus, which remains essentially constant.

In conclusion, it is appropriate to note that the static load experienced by a rubber sample can also influence its dynamic properties. This fact is addressed in Refs. 62, 82, 88, 95, 96, 109, 112, 115, 119, 142, 155, 156. For example, data from Ref. 96 are reproduced in Fig. 12, where the dynamic



(a)

Fig. 10 Dependence on vibration amplitude (% shear) of (a) the dynamic shear modulus, and (b) the damping factor of natural rubber filled with various parts by volume of MPC carbon black per 100 volumes rubber. (Ref. 144.)

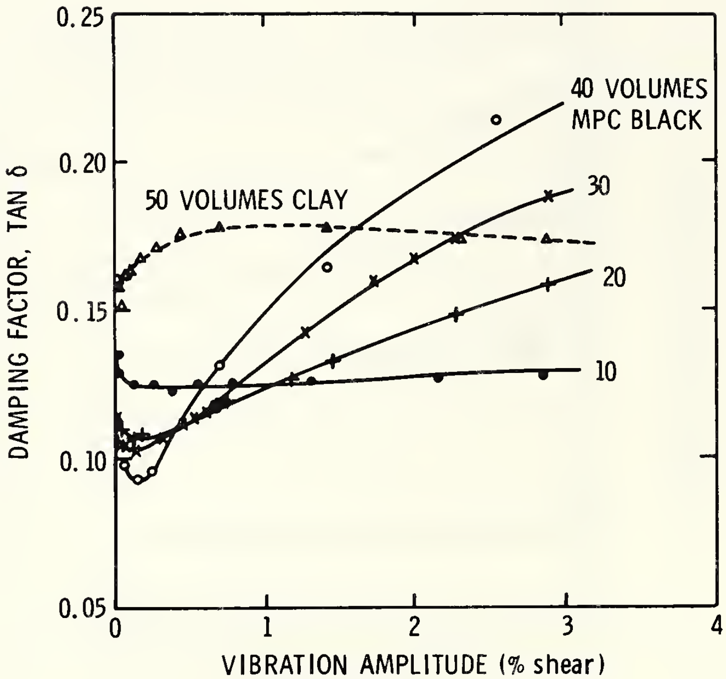


Fig. 10(b) Dependence on vibration amplitude (% shear) of the damping factor of natural rubber filled with various parts by volume of MPC carbon black per 100 volumes rubber. (Ref. 144.)

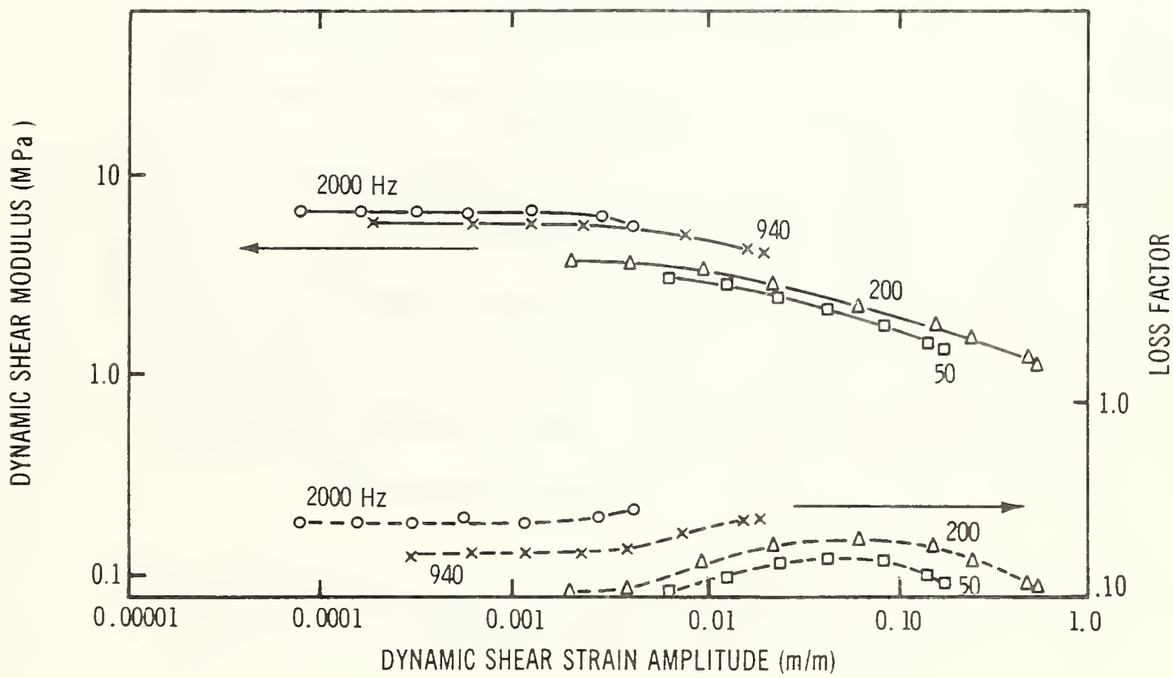


Fig. 11 Dependence on dynamic shear strain amplitude of the dynamic shear modulus and damping factor of natural rubber filled with 50 parts by weight of carbon black per 100 parts rubber. (Ref. 135.)

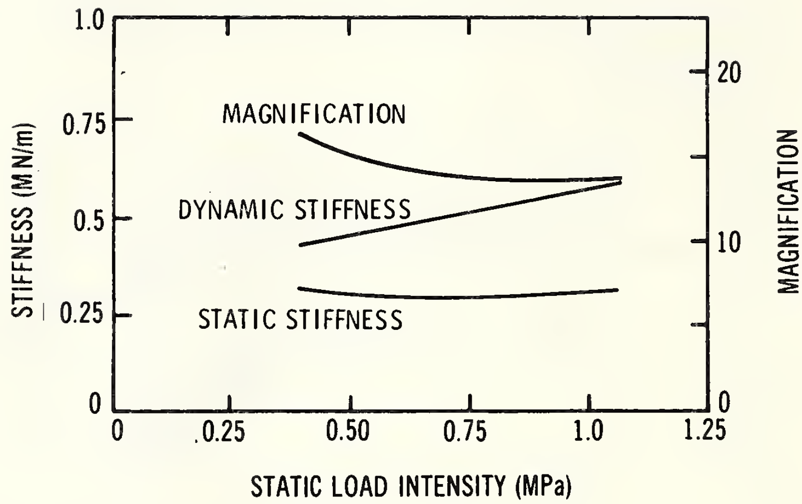


Fig. 12 Dependence on static stress of the dynamic stiffness and magnification Q = (damping factor)$^{-1}$ of natural rubber. Excitation frequency 60 Hz; rubber hardness approximately 50 Shore Durometer. (Ref. 96.)

stiffness and magnification $Q = (\text{damping factor})^{-1}$ of a natural rubber sample having a hardness of approximately 50 Shore Durometer are plotted versus static stress in the range 0.35 - 1.05 MPa ($0.35 - 1.05 \times 10^7$ dyn/cm^2, or 50 - 150 psi). Superimposed on the static displacement produced by this stress is a small displacement of 60 Hz frequency and rms amplitude of $0.5 - 1.8 \times 10^{-5}$m ($0.2 - 0.7 \times 10^{-3}$in.).

3. Simple Mounting System

The simple mounting system (Refs. 4, 9, 11, 13, 20, 24, 59, 68, 81, 133, 157-178) is shown in Fig. 13, where an element of mass M is supported by a linear rubberlike material utilized so that its behavior is governed by the complex shear modulus $G^*_{\omega,\theta}$ (Sec. 2). Here, and subsequently, it is assumed that the temperature remains constant, so that $G^*_{\omega,\theta}$ may be written as

$$G^*_\omega = G_\omega\,(1 + j\delta_{G\omega}) \quad . \tag{13}$$

The mounted item M is assumed to be supported at its center of gravity, and to vibrate only in the vertical direction; it is excited either by a sinusoidally varying ground displacement $\tilde{x}_1$, as in Fig. 13(a), or by a sinusoidally varying force $\tilde{F}_1$, as in Fig. 13(b). If the transmissibility T across system (a) is defined as the magnitude of the displacement ratio $|\tilde{x}_2/\tilde{x}_1|$, and if the transmissibility across system (b) is defined as the magnitude of the force ratio $|\tilde{F}_2/\tilde{F}_1|$--then, at any one frequency,

$$T = |\tilde{x}_2/\tilde{x}_1| = |\tilde{F}_2/\tilde{F}_1| \quad , \tag{14}$$

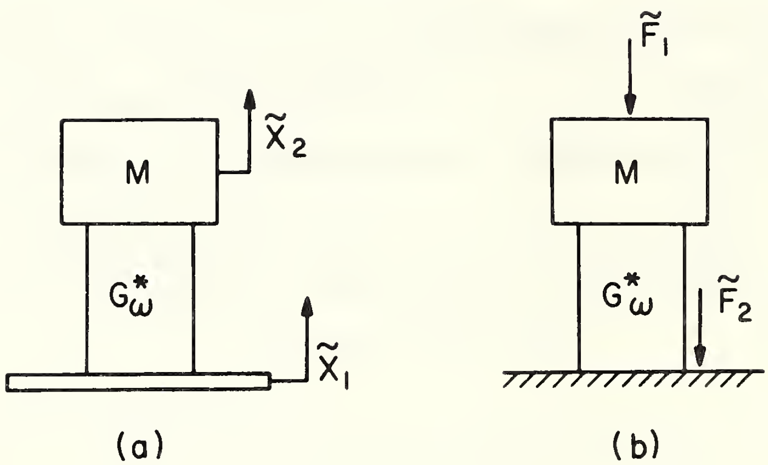


Fig. 13 Simple mounting system with rubberlike material.

where $\tilde{x}_2$ is the displacement of M in Fig. 13(a) and $\tilde{F}_2$ is the force transmitted to the ideally rigid foundation in Fig. 13(b). Thus, the results of a single calculation or measurement of transmissibility have dual significance. Further, because sinusoidal motion is of concern, T can equally well be expressed as the acceleration ratio $|\tilde{A}_2/\tilde{A}_1|$, where $\tilde{A}_i = (j\omega)^2\tilde{x}_i$, i = 1, 2.

The transmissibility across the simple mounting system is given (Ref. 59) by

$$T = \frac{(1 + \delta_{G\omega}^2)^{1/2}}{\{[1 - (\omega/\omega_o)^2(G_o/G_\omega)]^2 + \delta_{G\omega}^2\}^{1/2}} \quad . \tag{15}$$

From this general equation, the transmissibility of any linear rubberlike material can be calculated, provided that the dependence of G_ω and $\delta_{G\omega}$ upon frequency is known. The quantity G_o is the value of G_ω at the natural frequency ω_o of the system, which is defined as the frequency for which, in the absence of damping, T becomes infinitely large; that is

$$\omega_o^2 = kG_o/M \quad , \tag{16}$$

where the constant k has the dimensions of length. For a rubber mount of cross-sectional area A and length (height) ℓ, reference to Eq. 5 shows that

$$k = 3(A/\ell)(1 + \beta S^2) \quad ; \tag{17}$$

more simply, if the rubber element is used directly in shear, rather than as drawn in Fig. 13, then $k = (A/\ell)$.

The parallel spring and viscous dashpot combination of Fig. 14(a) is frequently discussed in the literature concerned with antivibration mountings (Refs. 6, 27, 36, 39, 55, 59, 77, 78, 88, 175, 176, 179-190); however, it is important to recognize (Ref. 59) that the dynamic properties of the combination poorly represent those of rubberlike materials. In fact, the damping factor of the combination is directly proportional to frequency--in contrast to the damping factors of either low or high-damping rubbers (Sec. 2). Again, it is important to recognize that the three-element combination of two springs and one dashpot shown in Fig. 14(b) (Refs. 39, 59, 98, 159, 170, 175, 176, 180, 182, 184, 187, 188, 191-193) also fails to provide a satisfactory representation of a rubberlike material, even though (1) the combination does exhibit creep under constant stress (Sec. 1), and stress relaxation under constant strain, and (2) the combination does stiffen by a small constant amount as frequency increases and, by association, is said to possess a transition frequency. However, the overall stiffness increases by a factor that is of the order of units rather than hundreds or thousands as observed for rubberlike materials in practice (Sec. 2 and Ref. 59). Consequently, analyses based on the spring and damper combinations of Fig. 14 can provide misleading conclusions unless the springs and dampers are viewed solely as mechanical devices.

The transmissibility of natural rubber, natural rubber filled with carbon black, and the high-damping rubber Thiokol RD is shown in Fig. 15. Data have been taken from Figs. 7-9 for these rubbers and inserted numerically into the expression for transmissibility given by Eq. 15. The natural mounting frequency has been chosen as 5 Hz and the ambient temperature is $20^{o}C$. Transmissibility is plotted on a decibel scale; thus, a displacement ratio or a force ratio T appears on a decibel scale as $20 \log_{10} T$ decibels (dB). Negative values of T(dB) mean that the input displacement or force

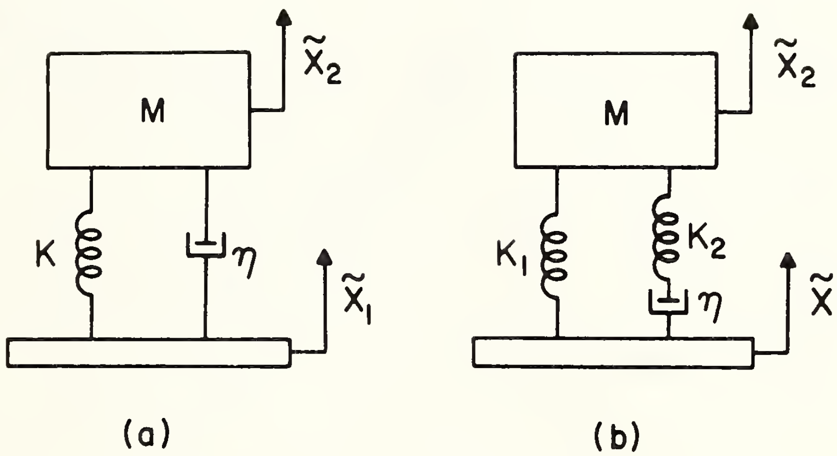


Fig. 14 Simple mounting system with two- and three-element spring- and viscous-dashpot combinations.

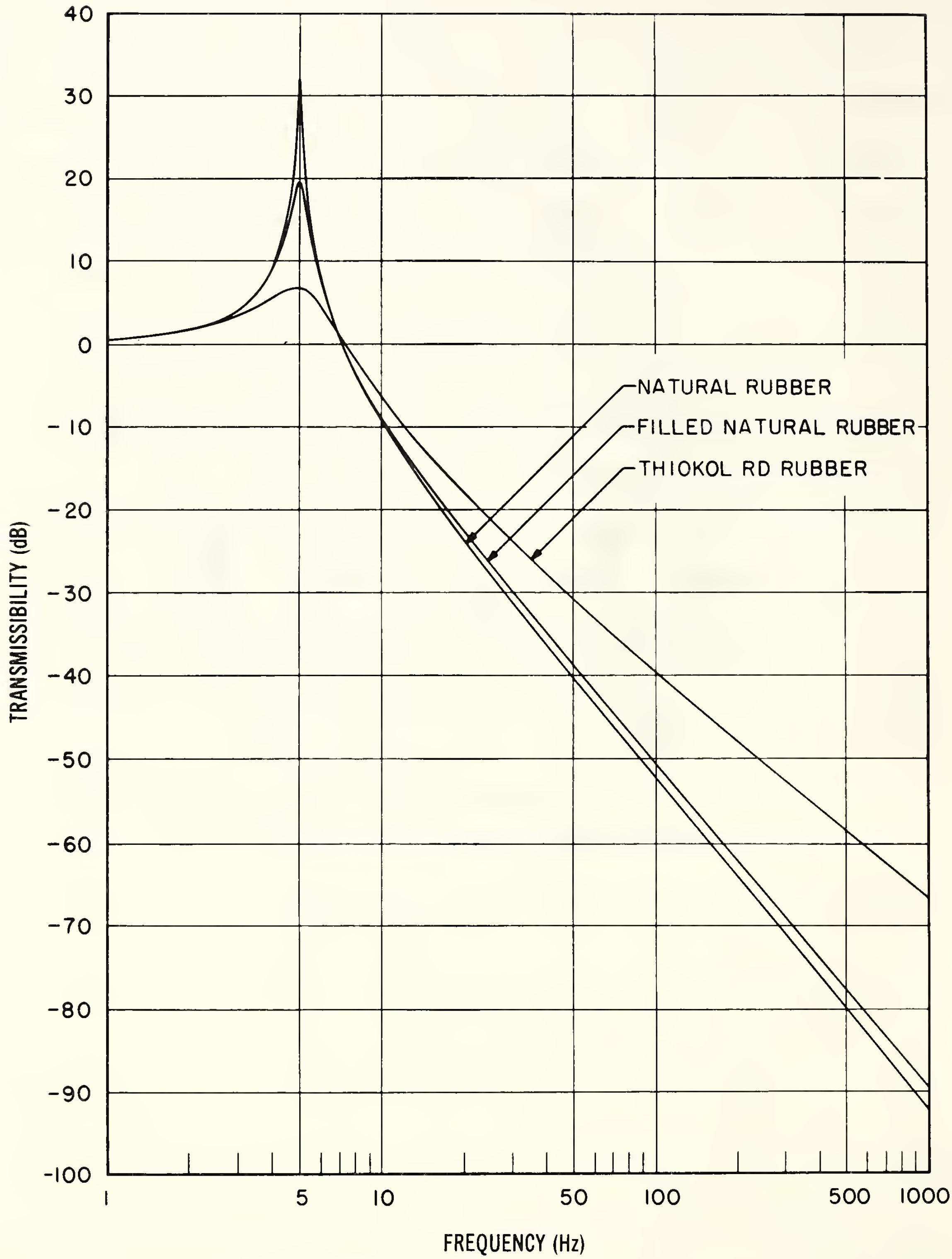


Fig. 15 Transmissibility of simple mounts on natural rubber, natural rubber filled with carbon black, and Thiokol RD rubber at 20°C; natural mounting frequency = 5 Hz. (Ref. 171.)

has been attenuated by the introduction of the rubber mounting; positive values of T(dB) mean that undesired magnification has occurred.

An antivibration mounting is required to provide small values of transmissibility at all frequencies that are contained in the Fourier spectrum of the displacement applied to its foundation, as in Fig. 13(a), or in the spectrum of the force applied to the item of equipment or machinery that it supports, as in Fig. 13(b). Thus, an effective antivibration mount should afford

(1) a low natural mounting frequency ω_o,

(2) a low transmissibility at resonance,

and

(3) a transmissibility that decreases rapidly with frequency at frequencies greater than ω_o.

A low natural frequency can be obtained by employing a mount of suitably low stiffness (or by increasing the mass of the mounted item). Because the lateral stability of the mounting system must be maintained, the extent to which the stiffness of the mounting can be reduced is limited. In practice, the natural frequencies of mounting systems are generally selected to be equal to or greater than 5 Hz. The use of a high-damping rubber can ensure that the resonant transmissibility will take small values.

The rate at which transmissibility decreases with frequency above ω_o varies considerably with the type of rubberlike material utilized in the mounting. Transmissibility decreases most rapidly with frequency for natural and other low-damping rubbers--essentially in proportion to $1/\omega^2$ (12 dB/octave). The transmissibility of Thiokol RD and other high-damping rubbers decreases at a much slower rate. This is one of the major drawbacks to the use of high-damping rubbers in antivibration mountings; the low resistance to creep (Sec. 1) of some high-damping rubbers is another drawback. The poor performance of these rubbers at high frequencies is predominantly caused by the significant increase in value of their dynamic moduli G_ω with frequency (as in Fig. 9). Contrary to the supposition often made, the inherent high damping of the rubbers has relatively small influence upon the values of transmissibility above resonance. To explain these facts, it is helpful to refer again to the general transmissibility equation (Eq. 15). Thus, at frequencies well above ω_o, this equation may be approximated as

$$T \approx \left[\frac{G_\omega}{G_o}\right] \frac{(1 + \delta_{G\omega}^2)^{\frac{1}{2}}}{(\omega/\omega_o)^2} \quad . \tag{18}$$

The values of G_ω possessed by natural and other low-damping rubbers increase only slowly with frequency, and $\delta_{G\omega}$ remains small; consequently, T decreases almost in proportion to the square of the exciting frequency. By contrast, the values of G_ω possessed by high-damping rubbers increase rapidly with frequency--a fact that, as mentioned, is primarily responsible for the large values of transmissibility observed for these rubbers at high frequencies (Ref. 59).

Although natural rubber and other low-damping rubbers such as neoprene are the rubbers normally utilized in antivibration mountings, high-damping rubbers would have greater application if they could be produced such that their dynamic moduli G_ω remained constant or increased only slowly with frequency. To date, it has proved impossible to satisfy this requirement; however, the suggestion has been made (Ref. 59) that natural rubber be used mechanically in parallel with a high-damping rubber of suitably smaller cross-sectional area. In this way, the dynamic modulus of the combination of rubbers can be adjusted to increase relatively slowly with frequency, while the associated damping factor takes values of significant magnitude intermediate to those of the constituent rubbers.

4. Simple Mounting System—Impairment of Performance

4.1 General Discussion

It is appropriate now to mention some reasons why larger values of transmissibility (reduced isolation) may occur at frequencies above resonance than the curves of Fig. 15 predict. These reasons (a) may simply be mechanical or (b) they may be basic. Thus,

(a) Vibration isolation may be impaired by mechanical links that have significant stiffness and hence that bypass, to some extent, the anti-vibration mounts. For example, vibration from a resiliently mounted diesel engine may reach its foundation via an exhaust pipe that is still rigidly connected to a surrounding enclosure, or it may reach the foundation via a bearing pedestal that supports a rotating shaft extending from the engine.

(b) Vibration isolation may be inadequately predicted at higher frequencies for the basic reason that the mounting system of Fig. 13 is too simplified a model of the practical situation. The mounting system can be criticized for three primary reasons, which are outlined in what follows:

First, values of transmissibility have been derived theoretically from knowledge of the mechanical properties of rubbers measured at small dynamic strains. It may be thought, therefore--particularly in the case of rubbers filled with substantial proportions of carbon black (Sec. 2)--that the performance of the rubbers under greater strains would differ from the performance predicted by curves such as those of Fig. 15. However, two comments may be made. First, although it is possible that the character of the transmissibility curves of filled rubbers will be strain dependent near resonance ($\omega \approx \omega_o$), well designed mounting systems normally possess natural frequencies that fall significantly below the spectrum of frequencies that the mountings are required to isolate. In consequence, the exciting frequencies should fall where $\omega >> \omega_o$, and where the strain is relatively small and is decreasing rapidly as ω increases. Second, even should a filled rubber exhibit nonlinear properties at frequencies above resonance, the dynamic stiffness of the rubber would decrease in magnitude (Figs. 10 and 11), so that the transmissibility of the mounting system would also decrease; that is, the effectiveness of the mounting would become greater.

Second, "wave effects" may be observed at high frequencies when the mount dimensions become comparable with multiples of the half-wavelengths of the elastic waves traveling through the mounting. Alternatively, wave effects may be thought of as occuring when the elasticity and the distributed mass of the rubber mounting interact at high frequencies. Wave effects are discussed in Refs. 4, 6, 20, 39, 59, 68, 81, 157, 158, 161, 162, 164, 165, 170, 171, 182, 189, 194-203 (waves in individual springs and rubber mounts are discussed in Refs. 156, 204-206). Wave effects are evident, for example, in the measured transmissibility curves of Fig. 16, which relates to a small natural-rubber mount containing 40 parts by weight of EPC carbon black (Ref. 59), and Fig. 17, which relates to a helical-spring and two natural-rubber mounts (Ref. 164). Other measurements of the transmissibility of rubber mounts in the simple system are described in Refs. 13, 21, 59, 68, 145, 152, 157, 158, 161, 165-171, 194, 195, 201, 207-209. Although many pronounced wave resonances occur in the transmissibility curves of springs, as in Fig. 17, the resonances in the transmissibility curves of practical rubber mounts are not always of primary concern. In fact, (a) the resonances are suppressed reasonably

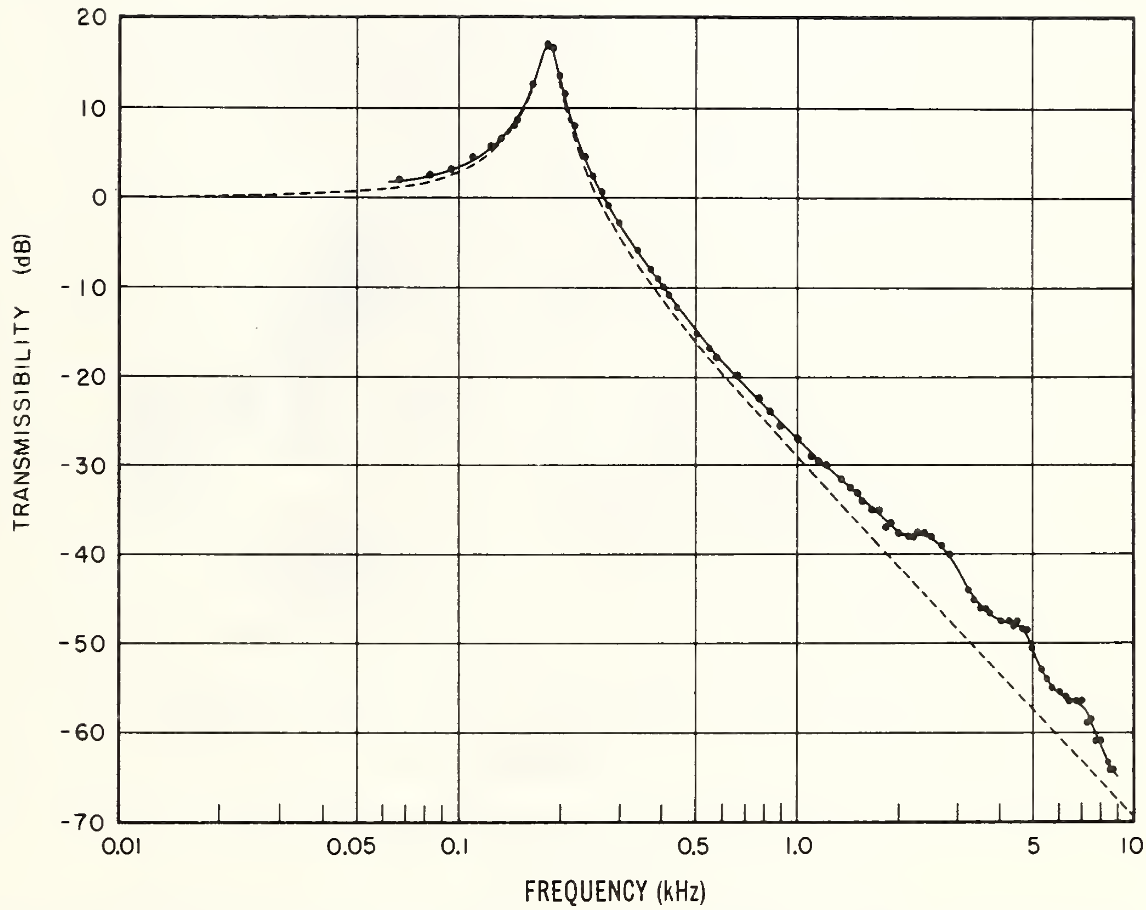


Fig. 16 Transmissibility of a simple mount of natural rubber filled with 40 parts by weight of EPC carbon black at 19°C. The dashed curve has been calculated from Eq. 15 assuming that G_{ω} and $\delta_{G\omega}$ are frequency independent. (Ref. 59.)

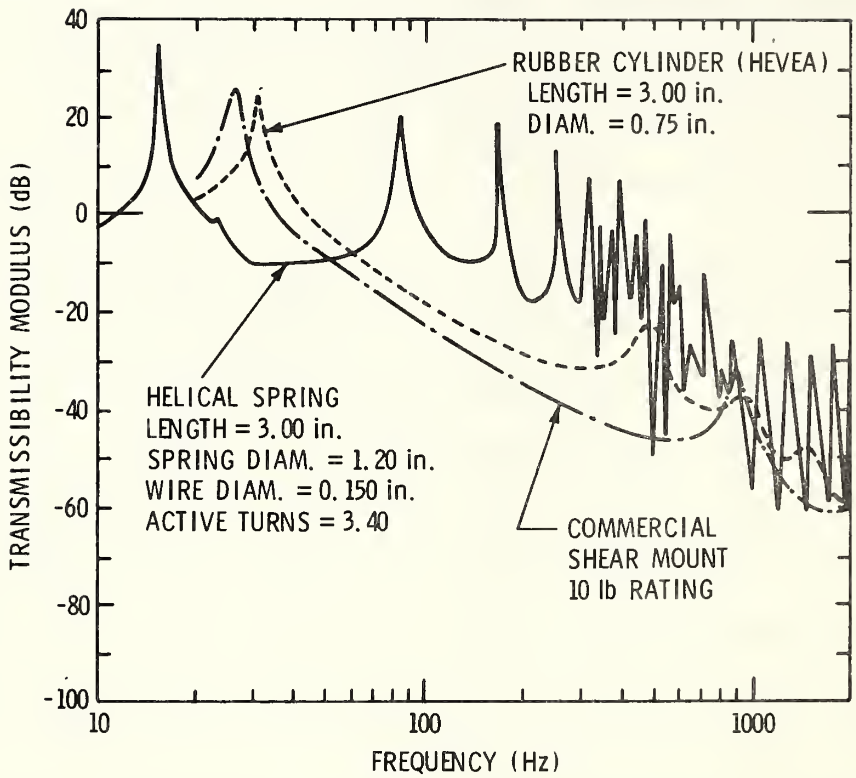


Fig. 17 Transmissibility of simple mounts that incorporate (a) a helical spring (solid curve), (b) a natural rubber cylinder (chain curve), and (c) a commercial shear mount (dashed curve). (Ref. 164.)

well by the internal damping of the rubber mounts, and (b), as will be demonstrated, even the first of the wave resonances invariably occurs at frequencies in excess of 20 ω_o, where significant isolation has already been achieved.

Third, the mounted item M may not behave as an ideally rigid mass. For example, the flanges or feet on which M is mounted may fail to remain ideally rigid and may resonate because of their poor design, so giving rise to other peaks in the transmissibility curve at high frequencies--even though the bulk of the mounted item may continue to behave as a lumped mass well into the high-frequency region. The peaks in the transmissibility curve may be troublesome because the internal damping of the metal feet will be at least 5 or 10 times smaller than the damping of the rubber mounts in which the previously discussed wave effects occurred. The feet may protrude from the bottom of the mounted item, or from its sides. This will be the case if the usually beneficial step is taken to locate the mounts in a plane that passes through the center of gravity of the mounted item (so minimizing the rocking motion it experiences if subjected to horizontally directed vibratory forces). Analyses of a mounted item with self resonances will be described subsequently; other discussions of the problem appear in Refs. 30, 160, 162, 164, 165, 173, 174, 188, 210.

4.2 Wave Effects

The geometry of the rubber components of antivibration mountings is frequently complex, which makes precise theoretical calculations of transmissibility difficult at high frequencies. A guide to the character of wave effects in antivibration mountings has been obtained, however, by considering the transmissibility of "mountings" that obey the simple wave equation for the longitudinal vibration of a rod of uniform cross section.

This approach has been taken by all earlier workers although, on two occasions (Refs. 199 and 200), "mountings" have also been considered to obey the wave equation for a transversely vibrating uniform beam. A disadvantage of these analyses is that they relate to "long" rods and beams with lateral dimensions that remain small in comparison with the wavelength. However, wave effects in a cylindrical rodlike mount of significant lateral dimensions have been analyzed (Ref. 59) using a "corrected" wave equation given by A. E. H. Love. In the Love theory, the radial motion of the plane cross section of the mount caused by axial compression and extension is, in some measure, accounted for.

The transmissibility T derived from the simple wave equation for a "long" rod with internal damping can be written

$$T = \left|[\cos n^*\ell - \gamma(n^*\ell)\sin n^*\ell]\right|^{-1} \quad , \tag{19}$$

where n^* is the complex wavenumber of the rodlike mount. The mass ratio

$$\gamma = \frac{M}{M_R} = \frac{M}{\rho A \ell} \quad , \tag{20}$$

where ρ and A are the density and cross-sectional area of the mount and ℓ is its length. The dimensionless product $(n^*\ell)$ is conveniently written

$$(n^*\ell) = (p + jq) \quad , \tag{21}$$

where

$$p = \frac{n\ell}{D_{E\omega}} \left(\frac{E_o}{E_\omega}\right)^{1/2} \left(\frac{D_{E\omega} + 1}{2}\right)^{1/2} \quad , \tag{22}$$

and

$$q = -\frac{n\ell}{D_{E\omega}}\left(\frac{E_o}{E_\omega}\right)^{\frac{1}{2}}\left(\frac{D_{E\omega}-1}{2}\right)^{\frac{1}{2}} \quad . \tag{23}$$

In these equations, the dynamic Young's modulus $E_\omega = E_o$ at the natural mounting frequency ω_o,

$$n = \omega(\rho/E_o)^{\frac{1}{2}} \quad , \tag{24}$$

and

$$D_{E\omega} = (1 + \delta^2_{E\omega})^{\frac{1}{2}} \tag{25}$$

It can be demonstrated that, if the dimensionless quantity $n\ell$ takes the value N_R when $\omega = \omega_o$,

$$n\ell = (\omega/\omega_o)\ N_R \rightarrow (\omega/\omega_o)(\gamma)^{-\frac{1}{2}} \quad . \tag{26}$$

Consequently, as $n\ell$ is varied, corresponding values of ω will be specified because N_R and ω_o will have been designated. In turn, values of E_ω and $\delta_{E\omega}$ will be known for each value of ω (e.g., Figs. 7-9), so that the expressions for p and q can be determined. In practice, it appears that the mass ratios for the majority of mounting systems take values in the range $50 < \gamma < 350$. The smallest value of γ yields the <u>least</u> favorable transmissibility curve. Thus, the wave resonances correspond closely with the natural frequencies

$$\omega_i = i\,\pi\,\omega_o\sqrt{\gamma} \quad , \qquad i = 1, 2, 3, \ldots \tag{27}$$

of the mount when clamped rigidly at each end; consequently, the smaller the values of γ, the lower the frequency at which the first wave resonance occurs (i = 1) and the more apparent the departure of the transmissibility curve from the predictions of the simple one-degree-of-freedom theory (e.g., Fig. 15). Note that, if $\gamma > 50$, then $\omega_1 > \pi \omega_o \sqrt{50} > 20 \omega_o$.

The transmissibility T determined from the Love theory is identical in form to Eq. 19:

$$T = |[\cos N^*\ell - \gamma(N^*\ell) \sin N^*\ell]|^{-1} \quad . \tag{28}$$

In this equation, the parameter $N^*\ell$ represents the complex number (P + jQ), where P and Q are functions of the foregoing quantities p and q. In the case of rubberlike materials for which $E_\omega = 3G_\omega$ and $\delta_{E\omega} = \delta_{G\omega}$ (Sec. 2), the expressions for P and Q may be written

$$P = \tfrac{1}{4} [\mu + (\mu^2 + \chi^2)^{\frac{1}{2}}]^{\frac{1}{2}} \tag{29}$$

and

$$Q = \tfrac{1}{4} [-\mu + (\mu^2 + \chi^2)^{\frac{1}{2}}]^{\frac{1}{2}} \quad , \tag{30}$$

where

$$\mu = [(p^2 - q^2) - \phi^2(p^2 + q^2)^2]/\xi \quad , \tag{31}$$

$$\chi = 2pq/\xi \quad , \tag{32}$$

and

$$\xi = [1 - 2\phi^2(p^2 - q^2) + \phi^4(p^2 + q^2)^2] \quad , \tag{33}$$

$$\phi = (r/2\ell) \quad . \tag{34}$$

The quantity r is the radius of gyration of an elementary section of the mount about its longitudinal axis; for example, if the mount is circular in cross section and has a diameter D, then $r = D/2\sqrt{2}$.

Wave effect calculations based on the "long-rod" theory (Eq. 19) are plotted in Fig. 18 for values of the mass ratio $\gamma = 50$, 100, and 250. It has been assumed that the dynamic Young's modulus and associated damping factor are frequency independent, that $\delta_E = 0.1$, and that the first natural frequency of the mounting system--for which $n\ell = N_R = 0.141$ when $\gamma = 50$--is again $f_o = \omega_o/2\pi = 5$ Hz. The curves of Fig. 18, which may be thought of as describing the transmissibility of natural rubber mounts heavily reinforced with carbon black, show how the level to which T is increased by the wave resonances depends upon the value of γ. As mentioned previously, the occurrence of wave resonances becomes of less concern as γ becomes larger; from this point of view, therefore, it is desirable to utilize antivibration mounts as near their maximum rated load as possible, thereby making γ a relatively large quantity.

Wave effect calculations based on the Love theory (Eq. 28) are plotted in Fig. 19 for a representative value of $\gamma = 200$ and for cylindrical mounts having a length-to-diameter ratio $\ell/D = 5$. Values of $E_\omega = 3G_\omega$ and $\delta_{E\omega} = \delta_{G\omega}$ drawn from Figs. 7 - 9 for unfilled natural rubber, natural rubber filled with 50 parts by weight of carbon black, and the high-damping rubber Thiokol RD, have been inserted numerically into Eqs. 22, 23, for p, q, and hence into Eqs. 29 and 30 for P and Q. Transmissibility curves calculated from the simple one-degree-of-freedom theory (Eq. 15) for the same three rubberlike materials are redrawn in Fig. 19 for comparison. Although the transmissibility of the natural rubber mountings is increased appreciably by the occurrence of wave resonances at high frequencies, the peak values of transmissibility occur at significantly lower levels than would be observed if $\gamma = 50$. Wave effects increase the

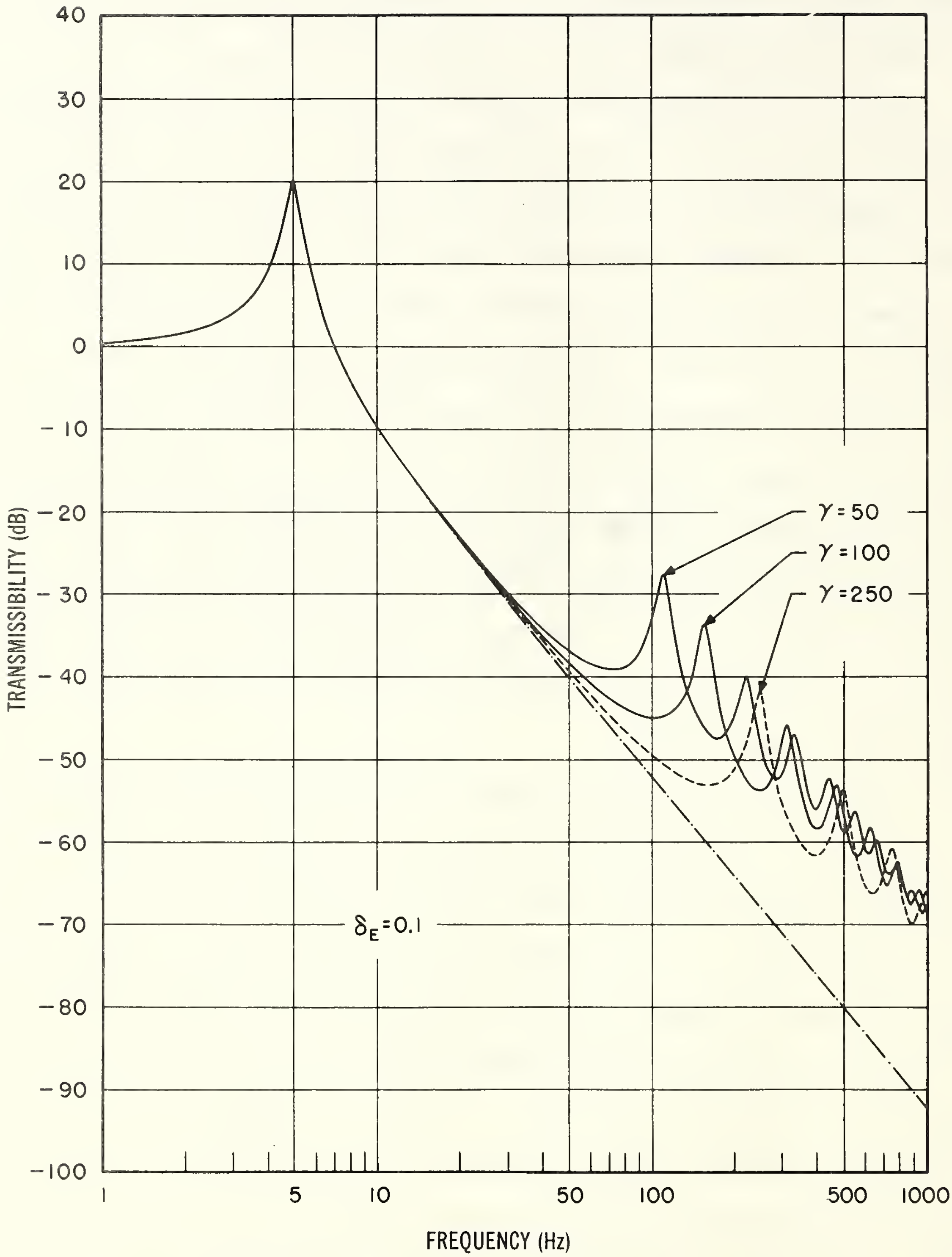


Fig. 18 Transmissibility of a simple mount with wave effects calculated from the "long"-rod theory. Damping factor δ_E = 0.1; mass ratio γ = 50, 100, and 250; natural mounting frequency = 5 Hz. (Ref. 59.)

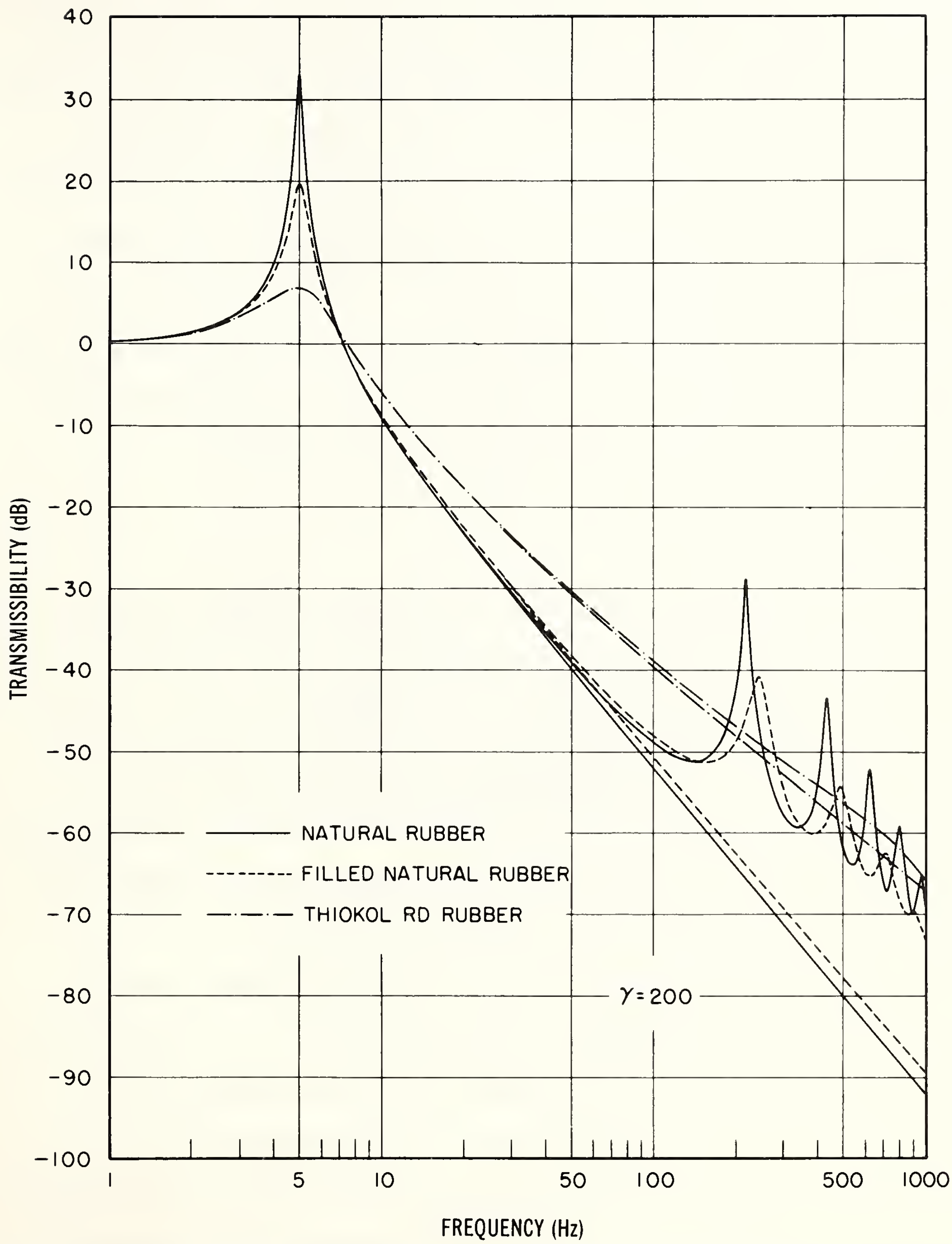


Fig. 19 Transmissibility calculated from the Love theory of rod vibration for simple mounts of unfilled natural rubber, natural rubber filled with 50 parts by weight of HAF carbon black, and Thiokol RD rubber. Cylindrical mounts have a length-to-diameter ratio of 5; mass ratio $\gamma = 200$; natural mounting frequency = 5 Hz. (Ref. 59.)

transmissibility of Thiokol RD rubber by a relatively small amount; in fact, the simple theory provides a remarkably accurate prediction of the transmissibility of this and other high-damping rubbers. On the other hand, the transmissibility of the heavily filled natural rubber is increased by approximately 20 dB at high frequencies as compared with the predictions of the simple theory. However, if γ were larger and the ratio ℓ/D were smaller than considered here, as could well be the case in practice, the wave resonances would shift to higher frequencies and lower levels, and the transmissibility curve would roll off at frequencies following the first wave resonance ($> \omega_1$) more rapidly than observed at present.

4.3 Nonrigid Flanges

An item supported by nonrigid (multiresonant) flanges or feet is shown in Fig. 20(a). This is not a contrived problem; in fact, one does not have to look far to find examples of such situations. For instance, a marine engine attached to a subframe having significant unsupported length is shown in Fig. 20(b); here, the subframe is fashioned so that the mounting points lie on the same horizontal as the center of gravity of the engine (Ref. 19).

A guide to the transmissibility T across the simple system of Fig. 20(a) has been obtained by visualizing the feet of the mounted item as short shear beams; that is, as beams with length-to-depth ratios of approximately three or less for which it can realistically be assumed that the beam deflection due to bending is much less than the deflection due to shear (Ref. 177). The mounts are assumed here, and subsequently, to have the complex stiffness

$$K^* = K\,(1 + j\delta_K) \quad , \qquad (35)$$

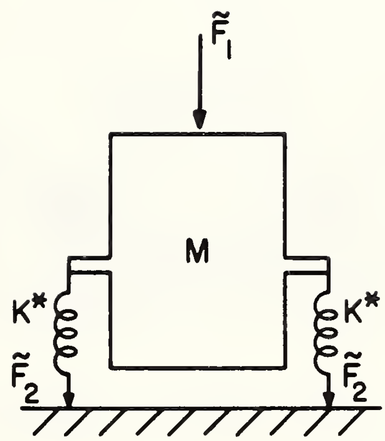


Fig. 20 (a) Idealized simple mounting system with a rigid mounted item supported via nonrigid (multiresonant) flanges or feet. (Ref. 177.)

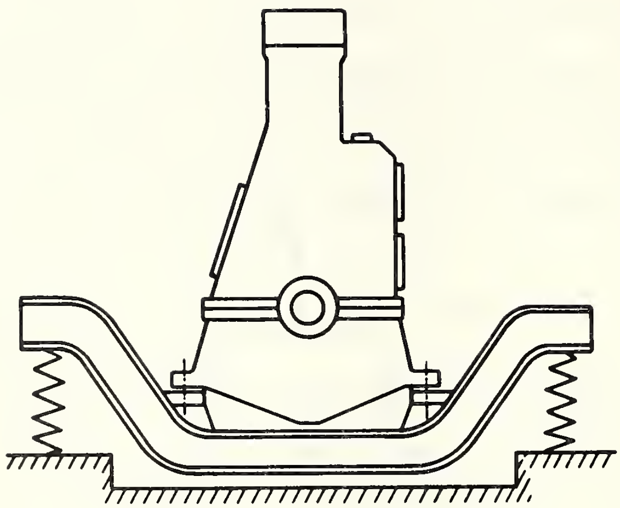


Fig. 20 (b) A practical example of (essentially) the case of a simple mounting system with an ideally rigid mounted item supported via nonrigid (multiresonant) flanges or feet. (Ref. 19.)

where the stiffness K and the damping factor δ_K are directly analogous to the previously utilized quantities kG_ω (k is specified by Eq. 17) and $\delta_{E\omega} = \delta_{G\omega}$. Because future discussions will be restricted to small values of $\delta_K \approx 0.05$, the quantities K and δ_K are taken to be frequency independent, a justifiable assumption for natural, neoprene, SBR, and other low-damping rubbers.

With the foregoing premises, the transmissibility across the system of Fig. 20(a) can be expressed as follows:

$$T = |2\tilde{F}_2/\tilde{F}_1| = |[\psi^* - (n^*\ell)\ \Gamma^*\eta^*]|^{-1} \quad , \tag{36}$$

where

$$\psi^* = [\cos n^*\ell - \gamma_F(n^*\ell) \sin n^*\ell] \quad , \tag{37}$$

$$\eta^* = [\sin n^*\ell + \gamma_F(n^*\ell) \cos n^*\ell] \quad , \tag{38}$$

$$n^*\ell = (p + jq) \quad , \tag{39}$$

and

$$\Gamma^* = \Gamma\ (1 + j\delta_F)/(1 + j\delta_K) \quad . \tag{40}$$

In these equations, n^* is the complex wavenumber of the shear-beam feet, ℓ is their length,

$$\gamma_F = M/2M_F \quad , \tag{41}$$

and

$$\Gamma = K_F/K \quad , \tag{42}$$

where M_F, K_F, and δ_F, are the mass, static stiffness, and damping factor of each shear-beam foot. In addition,

$$p = \frac{n\ell}{D_F}\left(\frac{D_F + 1}{2}\right)^{\frac{1}{2}} \tag{43}$$

and

$$q = -\frac{n\ell}{D_F}\left(\frac{D_F - 1}{2}\right)^{\frac{1}{2}} \quad , \tag{44}$$

where

$$D_F = (1 + \delta_F^2)^{\frac{1}{2}} \tag{45}$$

and

$$n\ell = (\omega/\omega_o)\, N_F = \Omega N_F \quad . \tag{46}$$

Here, the natural frequency of the mounting system is given by the equation

$$\omega_o^2 = \frac{2K\, K_F}{M(K + K_F)} = \frac{2K}{M}\left[\frac{\Gamma}{1 + \Gamma}\right] \quad , \tag{47}$$

and N_F is the value of $n\ell$ for which the first peak value of T would be observed (when $\omega = \omega_o$) if $\delta_F = \delta_K = 0$. A close guide to this value of N_F can be obtained from the relation

$$N_F \approx [(\gamma_F + 1)\, \Gamma + \gamma_F]^{-\frac{1}{2}} \tag{48}$$

or, if both γ_F and Γ are large, from

$$N_F \approx (\gamma_F \Gamma)^{-\frac{1}{2}} \quad . \tag{49}$$

Therefore, as the angular frequency ω of the impressed force is varied, corresponding values of $n\ell$ are specified by Eq. 46, and the expressions for p and q, and hence T, may be evaluated.

Representative calculations of T are plotted as a function of the frequency ratio $\Omega = \omega/\omega_o$ in Fig. 21, where the shear-beam feet have 1/40 of the mass of the mounted item ($\gamma_F = M/2M_F = 40$) and stiffnesses 5, 25, and 100 times greater than that of the mounts supporting them from below; the damping factors $\delta_K = 0.005$ and $\delta_F = 0.01$. The resonances of the shear-beam feet, which are responsible for the pronounced peak values of T at high frequencies, are seen to be of the least consequence when the stiffness ratio Γ is largest. In fact, the resonances will advantageously occur at the highest possible frequency when the ratio of the static stiffness to mass of the feet is made as large as possible; that is, in this simple example, when the shear-beam feet are made as short as possible. Their first resonance occurs at the approximate frequency

$$\omega_1 \approx (\pi\omega_o/2)\ \sqrt{(\gamma_F \Gamma)} = (\pi/2)\ \sqrt{(K_F/M_F)} \quad , \tag{50}$$

provided that γ_F and Γ are relatively large ($\Gamma > 5$).

5. Compound Mounting System

It is natural to question how it is possible to obtain greater vibration isolation than that afforded by the simple mounting system. If added mass can be tolerated, the two-stage or compound mounting shown in Fig. 22(a) can provide especially low values of transmissibility at high frequencies. Antivibration mounts of complex stiffness $2K_1^*$ and $2K_2^*$ in the upper and lower stages of

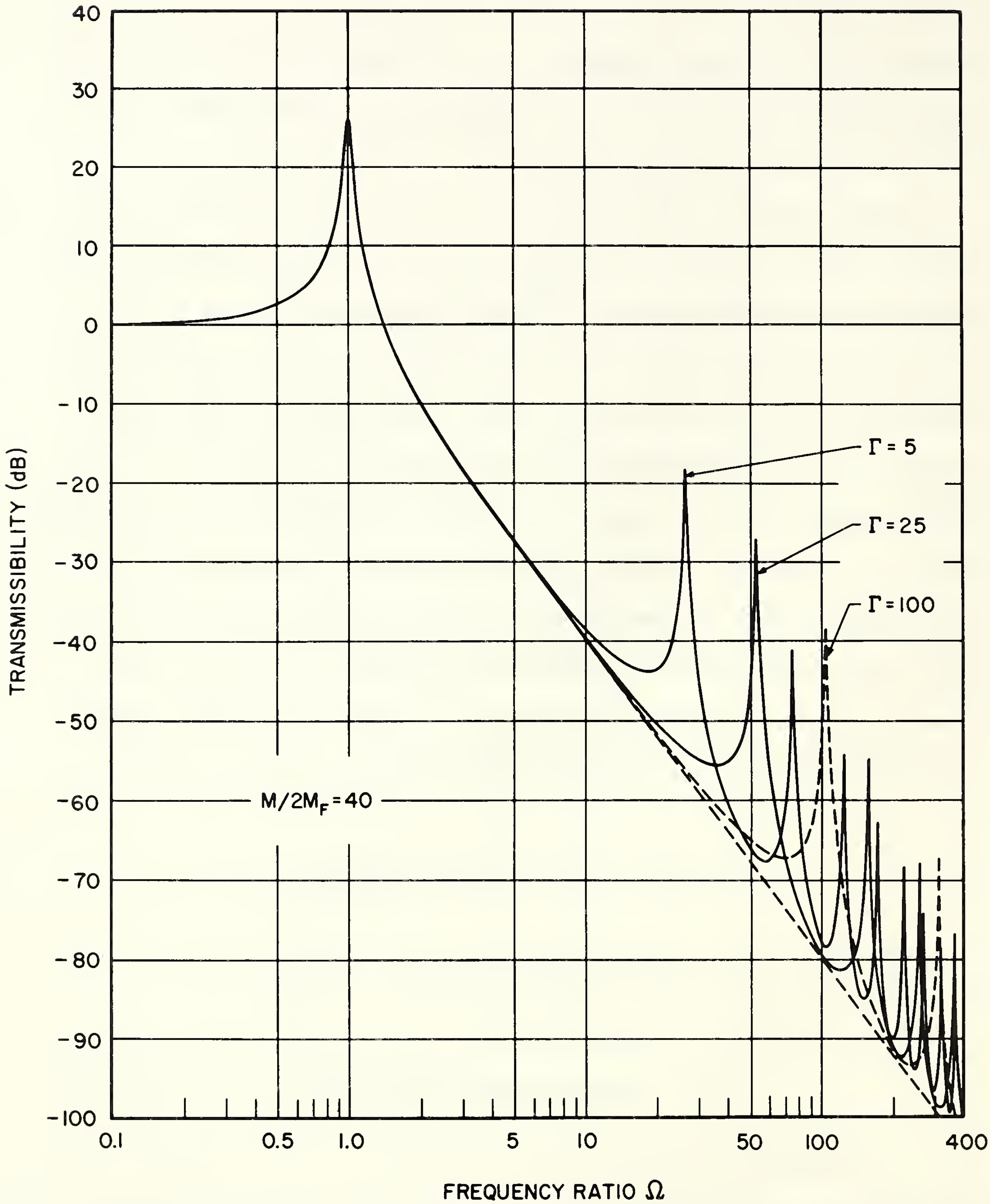


Fig. 21 Transmissibility of the simple mounting system of Fig. 20(a) with shear-beam resonances in feet of mounted item. Mass ratio $\gamma_F = 40$; stiffness ratio $\Gamma = K_F/K = 5$, 25, and 100; damping factors $\delta_K = 0.05$ and $\delta_F = 0.01$. (Ref. 177.)

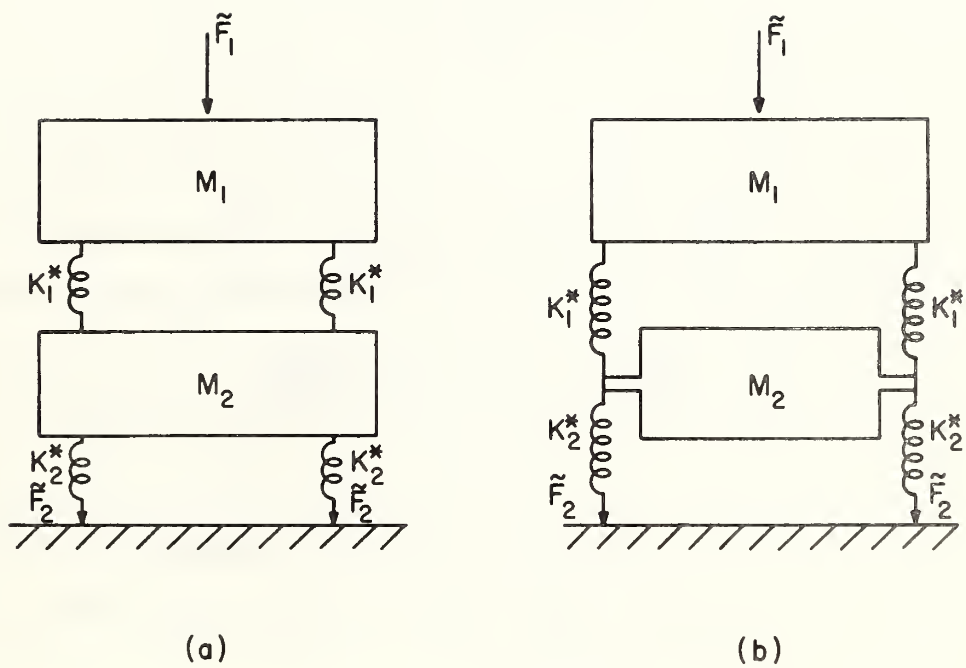


Fig. 22 Compound mounting system with a mounted item of mass M_1, and an intermediate mass M_2 that is supported (a) directly, and (b) via nonrigid (multiresonant) flanges or feet.

the system are separated by an additional or intermediate mass M_2. The system possesses a secondary as well as a primary resonance, which is a disadvantage, but above the secondary resonant frequency ω_2, transmissibility falls off in proportion to $1/\omega^4$ (24 dB/octave) provided that the stiffness and damping of the mounts remain independent of frequency, as assumed here. This is twice the rate of 12 dB/octave at which the transmissibility across the simple system dimmishes at high frequencies under the same circumstances.

The compound system is discussed in Refs. 13, 44, 45, 50, 54, 56, 59, 78, 88, 157, 159, 161, 167, 169, 170, 173, 174, 176-179, 186, 202, 211-218. A large-scale application of the system is considered in Ref. 186, which describes the compound mounting of 7,700-kg and 36,000-kg diesel generators on one extensive intermediate mass. An adaption of the arrangement employed is shown in Fig. 23(a). Two, much smaller, applications of the compound system are described in Refs. 173 and 217, in both of which the system has effectively been compacted into an "off-the-shelf" antivibration mount. The design of one mount (Ref. 217) is shown in Fig. 23(b), where the secondary mass comprises two cylindrical lumped masses 10 and a spacer yoke 12, and the resilient elements comprise 16.

For the compound system to have the greatest effectiveness as an antivibration mounting at high frequencies, it is desirable that the secondary resonance ω_2 occur, for any given value of the primary resonance ω_1, at the lowest possible frequency. This situation can be realized (Ref. 59) when the mount stiffness ratio takes the optimum value

$$K_2/K_1 = [1 + (M_2/M_1)] = (1 + \beta) \quad , \tag{51}$$

where M_1 is the mass of the mounted item. This is otherwise an appropriate result because the lower mounts support a static load that is greater, by

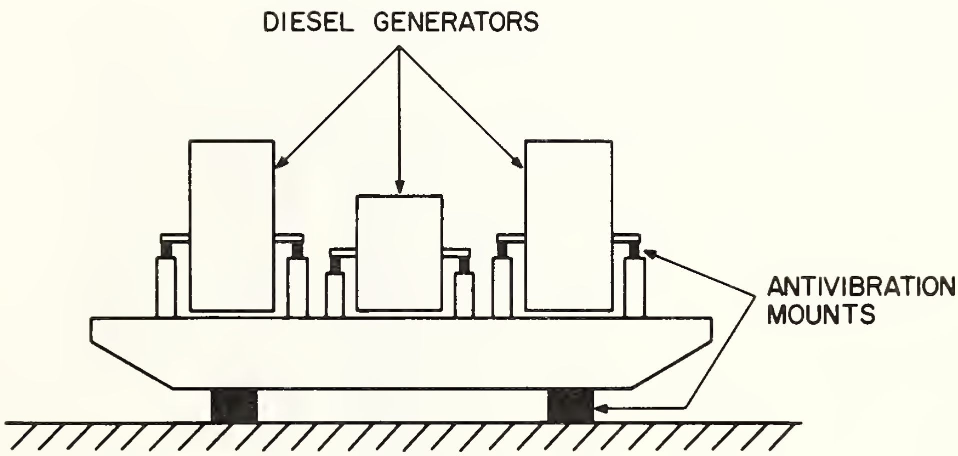


Fig. 23 (a) Compound mounting of 7,700-kg and 36,000-kg diesel generators on one extensive intermediate mass. (Ref. 186)

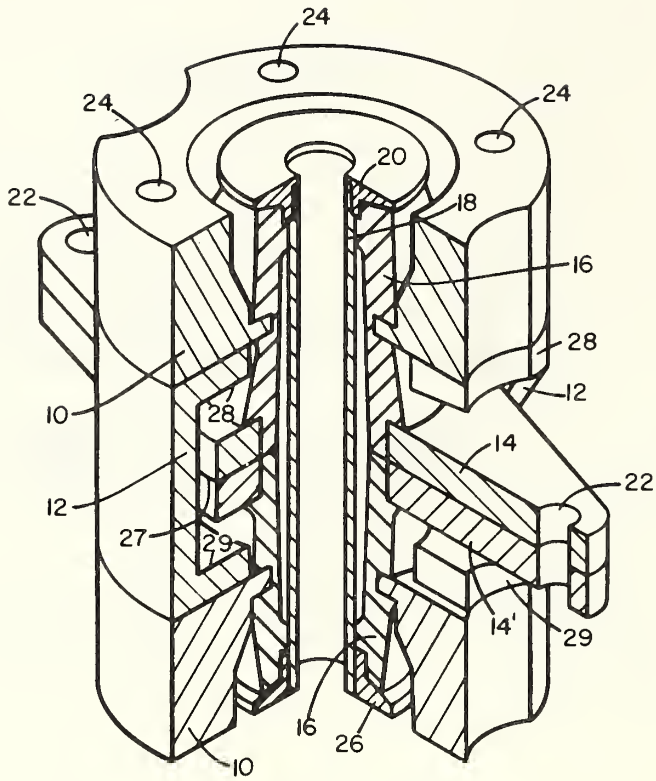


Fig. 23(b) Small-scale compound mounting with an intermediate mass M_2 comprising two cylindrical masses 10 and a spacer yoke 12 (resilient elements comprise 16). (Ref. 217.)

the same factor $(1 + \beta)$, than the load supported by the upper mounts. For this optimum stiffness ratio, the frequency ratio ω_2/ω_1 and the transmissibility T across the compound system can be expressed as

$$\frac{\omega_2}{\omega_1} = \frac{[1 + \sqrt{(1 + \beta)}]}{\sqrt{\beta}} \qquad (52)$$

and

$$T = \frac{(1 + \delta_K^2)}{\{[\beta\lambda(1 - \lambda)\Omega^4 - 2\lambda\Omega^2 + 1 - \delta_K^2]^2 + (2\delta_K)^2\ (1 - \lambda\Omega^2)^2\}^{\frac{1}{2}}} , \qquad (53)$$

where

$$\lambda = (1 + \beta)/(2 + \beta) \qquad (54)$$

and $\Omega = \omega/\omega_o$ is again a frequency ratio against which T is conveniently plotted. The reference frequency $\omega_o \approx \omega_1$ is actually the natural frequency of the one-degree-of-freedom system obtained when $M_2 = 0$; thus,

$$\omega_o^2 = \frac{2K_1K_2}{(K_1 + K_2)\ M_1} . \qquad (55)$$

It is noteworthy that, at high frequencies, Eq. 53 can be written

$$T_{HF} = \frac{(1 + \delta_K^2)(2 + \beta)^2}{\beta\Omega^4(1 + \beta)} \approx \frac{4(1 + \delta_K^2)}{\beta\Omega^4} , \qquad (\beta \leqq 0.5) \qquad (56)$$

so that is normally advantageous to employ the largest acceptable value of β; that is, the largest possible intermediate mass M_2.

Although the foregoing discussion relates to the force-driven system of Fig. 22(a), it is important to recognize that Eq. 53 for T pertains equally well to the dual situation where the mounted item M_1 experiences a displacement $\tilde{x}_2$ as the result of an applied vibratory ground displacement $\tilde{x}_1$; thus,

$$T = |2\tilde{F}_2/\tilde{F}_1| = |\tilde{x}_2/\tilde{x}_1| \quad . \tag{57}$$

An equivalent result for the simple mounting system appears in Sec. 3 (Eq. 14).

Transmissibility calculations made from Eq. 53 for three compound systems are plotted in Fig. 24. The mass ratio $\beta = M_2/M_1 = 0.1$, 0.2, and 1.0, and $\delta_K = 0.05$. Note how the position of the secondary resonance depends markedly on the value chosen for β. The potential value of the compound system as an especially effective antivibration mounting at high frequencies is immediately apparent when comparison is made with the dashed curve, which shows the transmissibility across the simple mounting system. Clearly evident are the benefits that result from the use of large intermediate masses (large β). It will be recognized that the compound system can be of particular value in mitigating the increase in transmissibility that occurs, for example, when it is necessary to mount machinery on a nonrigid foundation such as a system of metal girders in shipboard and aerospace applications. The foundation resonances are then superimposed, approximately speaking, on one of the lower solid curves rather than on the dashed curve. However, if the compound system is to provide the small values of transmissibility predicted at high frequencies, it is vital that the intermediate mass M_2 remains masslike in character. If it does not, then the performance of the compound system will be seriously impaired.

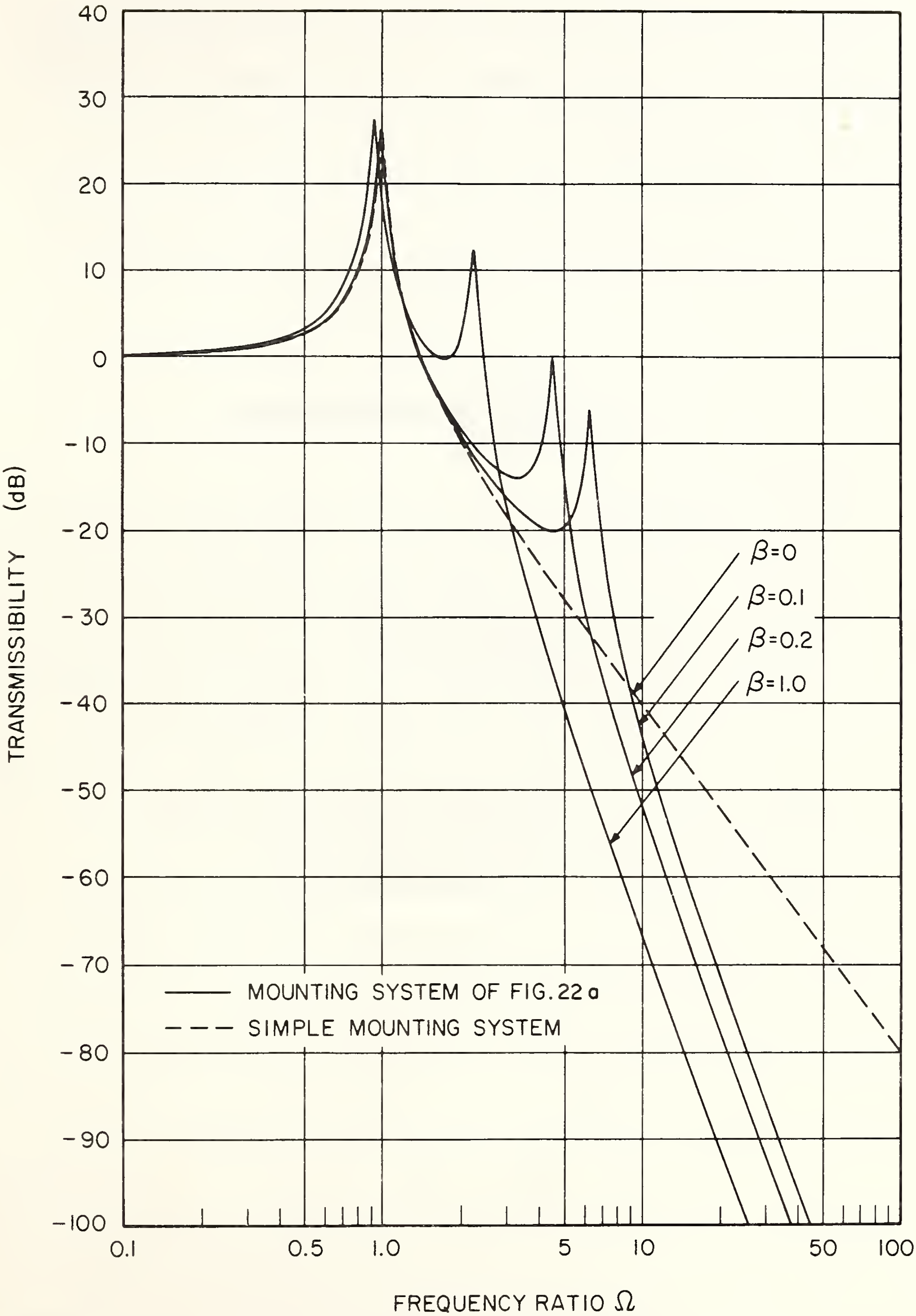


Fig. 24 Transmissibility of the compound mounting system of Fig. 22(a). Mass ratio $\beta = M_2/M_1 = 0.1$, 0.2, and 1.0; damping factor $\delta_K = 0.05$. (Ref. 218.)

This situation has been analyzed in Ref. 177, where flanges from which M_2 is supported, as in Fig. 22(b), behave as springs at some high frequencies because of their poor design. The inertia of M_2 is large at high frequencies and, therefore, to a first approximation, the points of juncture of the springs K_1 and K_2 [Fig. 22(b)] are restrained only by a resilient element the other end of which is attached to the "stationary" mass M_2.

6. Four-Pole Parameter Analyses

6.1 Introduction

Four-pole parameters will be referred to widely in the remainder of this report and, consequently, it is appropriate to review briefly some of their relevant properties. Detailed discussions of four-pole parameters can be found, for example, in Refs. 57 and 202, where many other pertinent articles are listed. Application of four-pole parameter techniques enables a general account to be taken of wave effects in antivibration mounts and of lack of rigidity in the foundation and the mounted item.

A linear mechanical system is shown schematically in Fig. 25(a). The system may be comprised of one or more lumped or distributed elements, or be constructed from any combination of such elements. The input side of the system vibrates sinusoidally with a velocity $\tilde{V}_1$ in response to an applied force $\tilde{F}_1$. In turn, the output side of the system exerts a force $\tilde{F}_2$ on the input side of some further system, sharing with it a common velocity $\tilde{V}_2$. Thus, the system is said to have input and output terminal pairs, a force $\tilde{F}_1$ and velocity $\tilde{V}_1$ at the input terminal pair giving rise to a force $\tilde{F}_2$ and velocity $\tilde{V}_2$ at the output terminal pair, the reaction of any subsequent mechanical system being accounted for. Forces are considered positive when directed to the right.

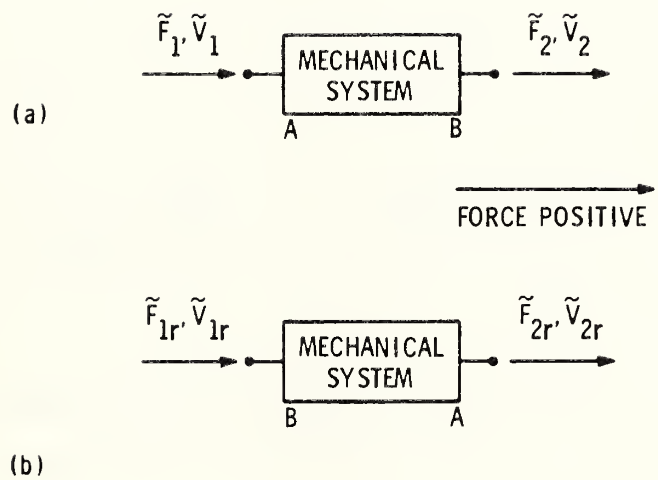


Fig. 25 (a) General four-terminal mechanical system, and (b) system reversed so that input and output terminal pairs are interchanged.

Consider now the mechanical impedance Z of a mass M and a spring of stiffness K (Fig. 26) in the context of the foregoing discussion; thus,

$$Z_M = j\omega M = (\tilde{F}_1 - \tilde{F}_2)/\tilde{V}_1 = (\tilde{F}_1 - \tilde{F}_2)/\tilde{V}_2 \tag{58}$$

or

$$\tilde{F}_1 = \tilde{F}_2 + j\omega M\tilde{V}_2 \quad , \tag{59}$$

$$\tilde{V}_1 = \tilde{V}_2 \quad , \tag{60}$$

and

$$Z_K = (K/j\omega) = \tilde{F}_1/(\tilde{V}_1 - \tilde{V}_2) = \tilde{F}_2/(\tilde{V}_1 - \tilde{V}_2) \tag{61}$$

or

$$\tilde{F}_1 = \tilde{F}_2 \quad , \tag{62}$$

$$\tilde{V}_1 = \tilde{F}_2\,(j\omega/K) + \tilde{V}_2 \quad . \tag{63}$$

Inspection of these equations makes it possible to understand that the vibration response of the general four-terminal system of Fig. 25(a) can be represented by the following equations:

$$\tilde{F}_1 = \alpha_{11}\tilde{F}_2 + \alpha_{12}\tilde{V}_2 \tag{64}$$

$$\tilde{V}_1 = \alpha_{21}\tilde{F}_2 + \alpha_{22}\tilde{V}_2 \quad , \tag{65}$$

where α_{11}, α_{12}, α_{21}, and α_{22} are known as four-pole parameters. It is directly apparent that

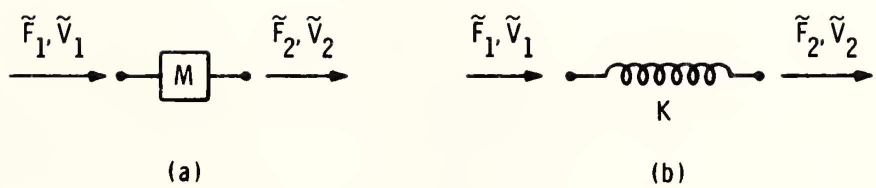


Fig. 26 (a) Lumped mass obeying Newton's second law, and (b) a massless spring obeying Hooke's Law.

$$\alpha_{11} = \left.\frac{\tilde{F}_1}{\tilde{F}_2}\right|_{\tilde{V}_2 = 0} \tag{66}$$

$$\alpha_{12} = \left.\frac{\tilde{F}_1}{\tilde{V}_2}\right|_{\tilde{F}_2 = 0} , \tag{67}$$

$$\alpha_{21} = \left.\frac{\tilde{V}_1}{\tilde{F}_2}\right|_{\tilde{V}_2 = 0} , \tag{68}$$

and

$$\alpha_{22} = \left.\frac{\tilde{V}_1}{\tilde{V}_2}\right|_{\tilde{F}_2 = 0} , \tag{69}$$

where the subscript $\tilde{V}_2 = 0$ indicates that the output terminal pair is blocked and the subscript $\tilde{F}_2 = 0$ indicates that the output terminal pair is free (unrestrained). The parameters α_{11} and α_{22} are dimensionless; α_{12} has the dimensions of impedance and α_{21} the dimensions of $(\text{impedance})^{-1}$.

In general, the four-pole parameters are frequency-dependent complex quantities. Of considerable advantage is the fact that the parameters characterize only the system for which they are determined; their value is not influenced by the preceding and subsequent mechanical systems. Equations 64 and 65 enable expressions for the driving-point and transfer impedances and for the force and displacement transmissibilities across the system to be written down concisely; thus, driving-point impedance,

$$Z_1 = \frac{\tilde{F}_1}{\tilde{V}_1} = \left(\frac{\alpha_{11}\tilde{F}_2 + \alpha_{12}\tilde{V}_2}{\alpha_{21}\tilde{F}_2 + \alpha_{22}\tilde{V}_2}\right) = \left(\frac{\alpha_{11}Z_T + \alpha_{12}}{\alpha_{21}Z_T + \alpha_{22}}\right) \quad ; \tag{70}$$

transfer impedance,

$$TZ_{12} = \frac{\tilde{F}_1}{\tilde{V}_2} = (\alpha_{11}Z_T + \alpha_{12}) \quad ; \tag{71}$$

force transmissibility

$$T_{F12} = \left|\frac{\tilde{F}_2}{\tilde{F}_1}\right| = \left|\frac{Z_T}{(\alpha_{12} + \alpha_{11}Z_T)}\right| \quad ; \tag{72}$$

and displacement transmissibility

$$T_{D12} = \left|\frac{\tilde{V}_2}{\tilde{V}_1}\right| = \left|\frac{1}{(\alpha_{22} + \alpha_{21}Z_T)}\right| \quad . \tag{73}$$

In these equations, $Z_T = \tilde{F}_2/\tilde{V}_2$ is the driving-point impedance of the mechanical system subsequent to the one under consideration.

It can be demonstrated that, without exception,

$$(\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}) = 1 \quad ; \tag{74}$$

consequently, knowledge of only three of the four-pole parameters is sufficient to specify the performance of the system completely (Refs. 57 and 202). Further, in the special case of a symmetrical system (when it does not matter which terminal pair is input or output),

$$\alpha_{11} = \alpha_{22} \tag{75}$$

and knowledge of only two independent four-pole parameters is sufficient to determine the system performance completely.

Should the mechanical system be reversed, so that the original output and input terminal pairs are interchanged, as in Fig. 25(b), then the relevant four-pole equations become

$$\tilde{F}_{1r} = \alpha_{22}\tilde{F}_{2r} + \alpha_{12}\tilde{V}_{2r} \quad , \tag{76}$$

$$\tilde{V}_{1r} = \alpha_{21}\tilde{F}_{2r} + \alpha_{11}\tilde{V}_{2r} \quad , \tag{77}$$

where the input and output forces and velocities are now $\tilde{F}_{1r}$, $\tilde{V}_{1r}$ and $\tilde{F}_{2r}$, $\tilde{V}_{2r}$, respectively, and

$$\alpha_{11} = \left.\frac{\tilde{V}_{1r}}{\tilde{V}_{2r}}\right|_{\tilde{F}_{2r} = 0} \quad , \tag{78}$$

$$\alpha_{12} = \left.\frac{\tilde{F}_{1r}}{\tilde{V}_{2r}}\right|_{\tilde{F}_{2r} = 0} \quad , \tag{79}$$

$$\alpha_{21} = \left.\frac{\tilde{V}_{1r}}{\tilde{F}_{2r}}\right|_{\tilde{V}_{2r} = 0} \quad , \tag{80}$$

and

$$\alpha_{22} = \left.\frac{\tilde{F}_{1r}}{\tilde{F}_{2r}}\right|_{\tilde{V}_{2r} = 0} \quad . \tag{81}$$

Although the values of the four-pole parameters α_{11} and α_{22} remain unchanged, their definitions differ here from those of Eqs. 66 and 69; in fact, the parameters have dual significance and can be determined in alternate ways--an advantage that will become apparent subsequently. By comparing the companion definitions of α_{11} and α_{22}, it follows that

$$\frac{1}{\alpha_{11}} = \left.\frac{\tilde{F}_2}{\tilde{F}_1}\right|_{\tilde{V}_2 = 0} = \left.\frac{\tilde{V}_{2r}}{\tilde{V}_{1r}}\right|_{\tilde{F}_{2r} = 0} \tag{82}$$

and

$$\frac{1}{\alpha_{22}} = \left.\frac{\tilde{V}_2}{\tilde{V}_1}\right|_{\tilde{F}_2 = 0} = \left.\frac{\tilde{F}_{2r}}{\tilde{F}_{1r}}\right|_{\tilde{V}_{2r} = 0} . \tag{83}$$

These equations show, as noted previously (Eqs. 14 and 57) that the force and displacement (and, therefore, the velocity and acceleration) transmissibilities in opposite directions between the two terminal pairs of a mechanical system are identical.

If the output pair of one mechanical system is rigidly connected to the input terminal pair of another system (Fig. 27), so that the output from the first is exactly the input to the second, the two systems are connected in series. Moreover, for n systems in series, the output force and velocity $\tilde{F}_{(n+1)}$, $\tilde{V}_{(n+1)}$ can be related to the input force and velocity $\tilde{F}_1$, $\tilde{V}_1$ by the continued product of n 2 x 2 matrices (Refs. 57 and 202). Simply, for a two-stage system,

$$\begin{bmatrix} \tilde{F}_1 \\ \tilde{V}_1 \end{bmatrix} = \begin{bmatrix} \alpha'_{11} & \alpha'_{12} \\ \alpha'_{21} & \alpha'_{22} \end{bmatrix} \begin{bmatrix} \alpha''_{11} & \alpha''_{12} \\ \alpha''_{21} & \alpha''_{22} \end{bmatrix} \begin{bmatrix} \tilde{F}_3 \\ \tilde{V}_3 \end{bmatrix} \tag{84}$$

or

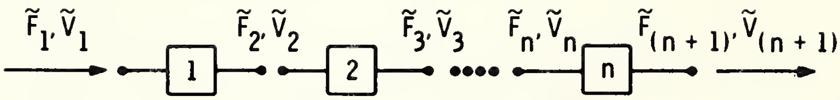


Fig. 27 Series connection of n four-terminal systems.

$$\begin{bmatrix} \tilde{F}_1 \\ \tilde{V}_1 \end{bmatrix} = \begin{bmatrix} \upsilon_{11} & \upsilon_{12} \\ \upsilon_{21} & \upsilon_{22} \end{bmatrix} \begin{bmatrix} \tilde{F}_3 \\ \tilde{V}_3 \end{bmatrix} , \tag{85}$$

where

$$\begin{bmatrix} \upsilon_{11} & \upsilon_{12} \\ \upsilon_{21} & \upsilon_{22} \end{bmatrix} = \begin{bmatrix} \alpha'_{11}\alpha''_{11} + \alpha'_{12}\alpha''_{21} & \alpha'_{11}\alpha''_{12} + \alpha'_{12}\alpha''_{22} \\ \alpha'_{21}\alpha''_{11} + \alpha'_{22}\alpha''_{21} & \alpha'_{21}\alpha''_{12} + \alpha'_{22}\alpha''_{22} \end{bmatrix} . \tag{86}$$

To conclude, it is appropriate to list the four-pole parameters for a slender rod of uniform cross-sectional area A, length ℓ, and density ρ, when the rod is driven axially by a sinusoidally varying force. For this symmetrical system,

$$\alpha_{11} = \alpha_{22} = \cos n^* \ell \tag{87}$$

$$\alpha_{12} = \mu_R^* \sin n^* \ell \tag{88}$$

and

$$\alpha_{21} = - \sin n^* \ell / \mu_R^* , \tag{89}$$

where

$$\mu_R^* = (j\omega M_R / n^* \ell) . \tag{90}$$

In these equations, M_R and n^* are the mass and the complex wavenumber of the rod; that is, $M_R = \rho A \ell$ and

$$n^* = (\omega^2 \rho / E^*)^{\frac{1}{2}} , \tag{91}$$

where E^* is the complex Young's modulus. As in Sec. 4.2, it is convenient to visualize the dimensionless product $n^*\ell$ as the complex number $(p + jq)$, where p and q are given by Eqs. 22 and 23 in which the frequency dependence of E_ω and $\delta_{E\omega}$ is now assumed to be negligible.

6.2 Characterization of an Antivibration Mounting

Consider now the antivibration mounting of Fig. 28 that is comprised of a uniform rodlike sample of rubberlike material bonded to metal end plates of masses M_1 and M_2. The mechanical behavior of the rubberlike material is assumed to be governed by Eqs. 87-90. An input force and velocity $\tilde{F}_1$, $\tilde{V}_1$ produce an output force and velocity $\tilde{F}_2$, $\tilde{V}_2$ at some termination subsequent to the end plate of mass M_2. An equation of the form of Eq. 85 remains relevant to this three-stage system, but now

$$\begin{bmatrix} \upsilon_{11} & \upsilon_{12} \\ \upsilon_{21} & \upsilon_{22} \end{bmatrix} = \begin{bmatrix} 1 & j\omega M_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c. & \mu_R^* s. \\ (\mu_R^*)^{-1} s. & c. \end{bmatrix} \begin{bmatrix} 1 & j\omega M_2 \\ 0 & 1 \end{bmatrix} \quad , \tag{92}$$

where the abbreviations s. and c. represent the complex circular functions $\sin n^*\ell$ and $\cos n^*\ell$, and the matrices for the masses M_1 and M_2 follow directly from inspection of Eqs. 59 and 60. It is readily shown that

$$\upsilon_{11} = [c. - \gamma_1(n^*\ell)s.] \quad , \tag{93}$$

$$\upsilon_{12} = \mu_R^* \{[s. + \gamma_1(n^*\ell)c.] + \gamma_2(n^*\ell)\ [c. - \gamma_1(n^*\ell)s.]\} \quad , \tag{94}$$

$$\upsilon_{21} = -\ (\mu_R^*)^{-1}(s.) \quad , \tag{95}$$

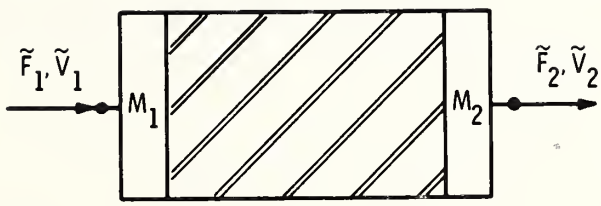


Fig. 28 Antivibration mount with end plates of masses M_1 and M_2 to which the boundaries of a uniform rod-like sample of rubberlike material are attached.

and

$$\upsilon_{22} = [c. - \gamma_2(n^* \ell)s.] \quad , \tag{96}$$

where $\gamma_1 = M_1/M_R$ and $\gamma_2 = M_2/M_R$. These four-pole parameters relate equally well to a rodlike mount of significant lateral dimensions when n^* is replaced by the complex wavenumber N^* specified in Sec. 4.2.

Of considerable interest is the simplicity of the four-pole parameter υ_{21}, the reciprocal of which describes a blocked quasi transfer impedance that is independent of the values of M_1 and M_2; thus

$$\frac{1}{\upsilon_{21}} = \left.\frac{\tilde{F}_2}{\tilde{V}_1}\right|_{\tilde{V}_2 = 0} = - \frac{\mu_R^*}{\sin n^* \ell} = \frac{j\omega M_R}{n^* \ell \sin n^* \ell} \quad . \tag{97}$$

Further, at frequencies well below the initial wave-effect frequency ω_1 in the mount, $\sin n^* \ell \rightarrow n^* \ell$ and

$$\frac{1}{\upsilon_{21}} = \frac{\omega(\rho A \ell)}{j\ell^2} \left(\frac{E^*}{\omega^2 \rho}\right) = \frac{(AE^*/\ell)}{j\omega} \quad , \tag{98}$$

which is the impedance of a simple spring of complex stiffness $K^* = (AE^*/\ell)$. If this stiffness is symbolized by $K^* = K(1 + j\delta_K)$, then

$$\frac{1}{\upsilon_{21}} = \frac{K}{\omega} (\delta_K - j) \tag{99}$$

and measurement of the magnitudes of the imaginary part and of the ratio of the imaginary to the real part (tan phase angle) of $1/\upsilon_{12}$ will yield K/ω and the reciprocal of the damping factor δ_K, respectively. The larger the value of δ_K, the greater the accuracy to which the phase angle and, hence, δ_K can be measured. This measurement approach, which will be referred to and applied subsequently, was proposed and utilized in Refs. 58 and 219, where the permissible upper bound of measurement was said to be the frequency $0.25\ \omega_1$.

6.3 Resilient Mounting on Nonrigid Substructures

Examined now is the vibration response of a nonrigid substructure of arbitrary impedance Z_T that lies beneath the antivibration mounting considered in the foregoing. The mount and the substructure are characterized by the four-pole parameters υ_{ij} (Eqs. 93-96) and α_{ij}, respectively. Initially, the item supported by the mount is assumed to remain masslike at all frequencies. The entire assembly is shown diagrammatically in Fig. 29(a). The same item of mass M is shown rigidly mounted in the reference assembly of Fig. 29(b), where it generates an untenably large vibration of the substructure.

The same exciting force $\tilde{F}_1$ is considered to act upon or to be generated within M in both Figs. 29(a) and (b). This force gives rise to a transmitted force $\tilde{F}_{12}$ at the point of juncture of the mount and the substructure in Fig. 29(a), and to a force $\tilde{F}_{12R}$ at the same location on the substructure in Fig. 29(b). The companion velocities are $\tilde{V}_{12}$ and $\tilde{V}_{12R}$, respectively. Beneath the substructure, the output forces and velocities are $\tilde{F}_2$, $\tilde{V}_2$, and $\tilde{F}_{2R}$, $\tilde{V}_{2R}$. In prior discussions of the vibration of nonrigid substructures (Refs. 12, 13, 19, 24, 59, 159, 160, 162, 164-167, 173, 174, 177, 178, 188, 193, 210, 214, 220-226) attention is devoted either to the mount transmissibility $T = |\tilde{F}_{12}/\tilde{F}_1|$, or to the mount response ratio $R = |\tilde{V}_{12}/\tilde{V}_{12R}|$ -- or its reciprocal,

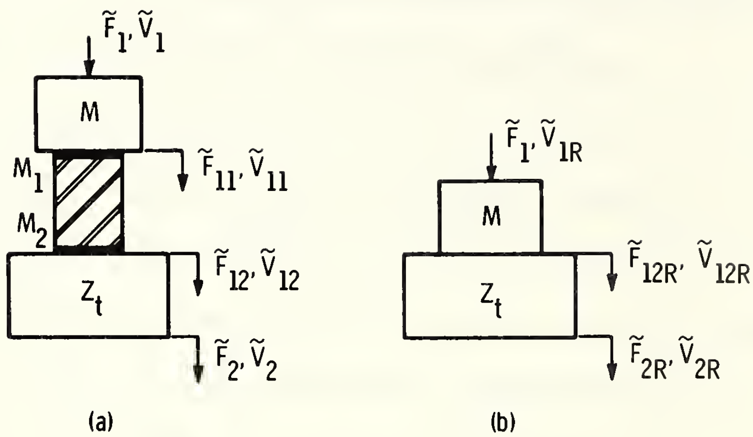


Fig. 29 (a) Antivibration mount of Fig. 28 isolating the vibration of a mounted item of mass M from a nonrigid substructure of arbitrary impedance Z_T, and (b) the rigid attachment of M to the substructure at the same location as in (a).

mount effectiveness $E = R^{-1}$. Consequently, it is appropriate now to evaluate the quantities T and R in general terms using the four-pole parameter techniques described in the foregoing.

The forces and velocities experienced by the mounted item and substructure in Fig. 29(a) are readily understood to be related by the equation

$$\begin{bmatrix} \tilde{F}_1 \\ \tilde{V}_1 \end{bmatrix} = \begin{bmatrix} 1 & j\omega M \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \upsilon_{11} & \upsilon_{12} \\ \upsilon_{21} & \upsilon_{22} \end{bmatrix} \begin{bmatrix} \tilde{F}_{12} \\ \tilde{V}_{12} \end{bmatrix} \quad , \tag{100}$$

where the matrix product can be written as the third matrix

$$\begin{bmatrix} \bar{\upsilon}_{11} & \bar{\upsilon}_{12} \\ \upsilon_{21} & \upsilon_{22} \end{bmatrix}$$

in which $\bar{\upsilon}_{11}$, $\bar{\upsilon}_{12}$, υ_{21}, and υ_{22} are again defined by Eqs. 93-96, the only change being that the mass ratio γ_1 in Eqs. 93 and 94 for $\bar{\upsilon}_{11}$ and $\bar{\upsilon}_{12}$ is redefined as

$$\gamma_1 = (M + M_1)/M_R \approx M/M_R \quad . \tag{101}$$

It is evident from Eq. 100 that

$$\begin{aligned} \tilde{F}_1 &= (\bar{\upsilon}_{11}\tilde{F}_{12} + \bar{\upsilon}_{12}\tilde{V}_{12}) \\ &= (\bar{\upsilon}_{11}Z_T + \bar{\upsilon}_{12})\ \tilde{F}_{12}/Z_T = (\bar{\upsilon}_{11}Z_T + \bar{\upsilon}_{12})\ \tilde{V}_{12} \quad . \end{aligned} \tag{102}$$

Likewise, from reference to Fig. 29(b), it can be stated that

$$\begin{bmatrix} \tilde{F}_1 \\ \tilde{V}_{1R} \end{bmatrix} = \begin{bmatrix} 1 & j\omega M \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{F}_{12R} \\ \tilde{V}_{12R} \end{bmatrix} \tag{103}$$

and, consequently, that

$$\tilde{F}_1 = (\tilde{F}_{12R} + j\omega M\tilde{V}_{12R}) = (Z_T + j\omega M)\ \tilde{V}_{12R} \quad . \tag{104}$$

From Eqs. 102 and 104 it is possible to write down the transmissibility and response ratio of the mounting system directly; thus

$$T = \left|\frac{\tilde{F}_{12}}{\tilde{F}_1}\right| = \left|\frac{Z_T}{(\overline{\upsilon}_{11} Z_T + \overline{\upsilon}_{12})}\right| \tag{105}$$

and

$$R = \left|\frac{\tilde{V}_{12}}{\tilde{V}_{12R}}\right| = \left|\left(\frac{Z_T + j\omega M}{\overline{\upsilon}_{11} Z_T + \overline{\upsilon}_{12}}\right)\right| \quad . \tag{106}$$

(Note that Eqs. 102, 104, and 105 could equally well have been stated from inspection of Eqs. 71 and 72.) Response ratio, the magnitude of the substructure velocity observed when M is resiliently mounted to the velocity observed when M is attached rigidly to the foundation, provides a measure of the vibration reduction that the mounting affords -- the smaller the value of R, the larger the reduction in substructure velocity and the more beneficial the mounting. Note that, because

$$Z_T = \tilde{F}_{12}/\tilde{V}_{12} = \tilde{F}_{12R}/\tilde{V}_{12R} \quad , \tag{107}$$

the response ratio could have been defined equally well as the ratio $|\tilde{F}_{12}/\tilde{F}_{12R}|$ of the forces exerted on the substructure in the resiliently and rigidly mounted cases. Note also that R is not as small as T unless $Z_T >> j\omega M$; that is to say, if $j\omega M$ is comparable with, or greater than Z_T, the mount will be <u>less</u> effective than predicted by its transmissibility curve. Physically, this reflects the fact that the beneficial action of the antivibration mount in Fig. 29(a) will be countered, to some extent, by the greater freedom of the foundation to respond to a given applied force than was possible in Fig. 29(b). Thus, the foundation in Fig. 29(a) is no longer relieved of part of the applied force by the inertia of the mass M -- an acute disadvantage if M is large, as it may well be. In this circumstance, it has been suggested (Ref. 59) that the response of the substructure be restrained by an auxiliary mass having a significant fraction of the mass M of the mounted item. The relevant expressions for transmissibility and response ratio follow immediately from Eqs. 105 and 106 if the mass ratio $\gamma_2 = M_2/M_R$ that appears in the four-pole parameter $\bar{\upsilon}_{12}$ of these equations is redefined as

$$\gamma_2 = (M_2 + m)/M_R \approx m/M_R \quad . \tag{108}$$

One further result follows from consideration of the matrix equation for the substructure,

$$\begin{bmatrix} \tilde{F}_{12} \\ \tilde{V}_{12} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} \tilde{F}_2 \\ \tilde{V}_2 \end{bmatrix} \quad , \tag{109}$$

and its constituent equations

$$\tilde{F}_{12} = \alpha_{11}\tilde{F}_2 + \alpha_{12}\tilde{V}_2 \tag{110}$$

and

$$\tilde{V}_{12} = \alpha_{21}\tilde{F}_2 + \alpha_{22}\tilde{V}_2 \quad . \tag{111}$$

Thus, the output velocity $\tilde{V}_2$ can be eliminated from these equations to yield

$$\tilde{F}_2 = \tilde{V}_{12} \; (\alpha_{22}Z_T - \alpha_{12}) \quad ; \tag{112}$$

likewise,

$$\tilde{F}_{2R} = \tilde{V}_{12R} \; (\alpha_{22}Z_T - \alpha_{12}) \quad . \tag{113}$$

Consequently, an additional definition can be stated for response ratio, which now has the triple significance

$$R = \left|\frac{\tilde{V}_{12}}{\tilde{V}_{12R}}\right| = \left|\frac{\tilde{F}_2}{\tilde{F}_{2R}}\right| = \left|\frac{\tilde{F}_{12}}{\tilde{F}_{12R}}\right| \quad . \tag{114}$$

The new definition of R describes the ratio $|\tilde{F}_2/\tilde{F}_{2R}|$ of the forces that are transmitted to the <u>termination of the substructure</u> in the resiliently and rigidly mounted cases. Further, a companion force transmissibility across the entire system can logically be defined and determined from Eqs. 102 and 112 as

$$T_{overall} = \left|\frac{\tilde{F}_2}{\tilde{F}_1}\right| = \left|\left(\frac{\alpha_{22}Z_T - \alpha_{12}}{\overline{\upsilon}_{11}Z_T + \overline{\upsilon}_{12}}\right)\right| \quad . \tag{115}$$

This significant quantity differs from both R and T as previously specified.

The results of one independent calculation of $T_{overall}$ by the present author for a rectangular platelike substructure with an aspect ratio of 0.5 are plotted in Fig. 30 as the dashed curve. The mounted item is driven by a vibratory force $\tilde{F}_1$ and is supported by four antivibration mounts located symmetrically about the plate center, each at coordinates of one-third the length and breadth of the plate from the nearest plate corner. The mounting points have the same driving-point impedance and experience the same velocity. The output force $\tilde{F}_2$ comprises four discrete forces at the plate corners plus distributed forces along the simply supported plate boundaries. Transmissibility $T_{overall} = |\tilde{F}_2/\tilde{F}_1|$ has been calculated in terms of the previously utilized frequency ratio $\Omega = \omega/\omega_o$, where ω_o is now the natural frequency of the mounting system calculated as though the platelike substructure was ideally rigid. The mounted item is four times more massive than the substructure, and the fundamental plate resonance is assigned the frequency $4\omega_o$. The mounts and the substructure have the damping factors 0.05 and 0.01, respectively. The transmissibility curve at high frequencies is characterized by many plate resonances; moreover, the number of resonances that are excited can detrimentally increase if the mounts are located at other, less favorable positions.

The overall transmissibility across an identical mounting system to which lumped masses have been added to each mount location is shown by the solid curve. The total added mass m is equal to that of the mounted

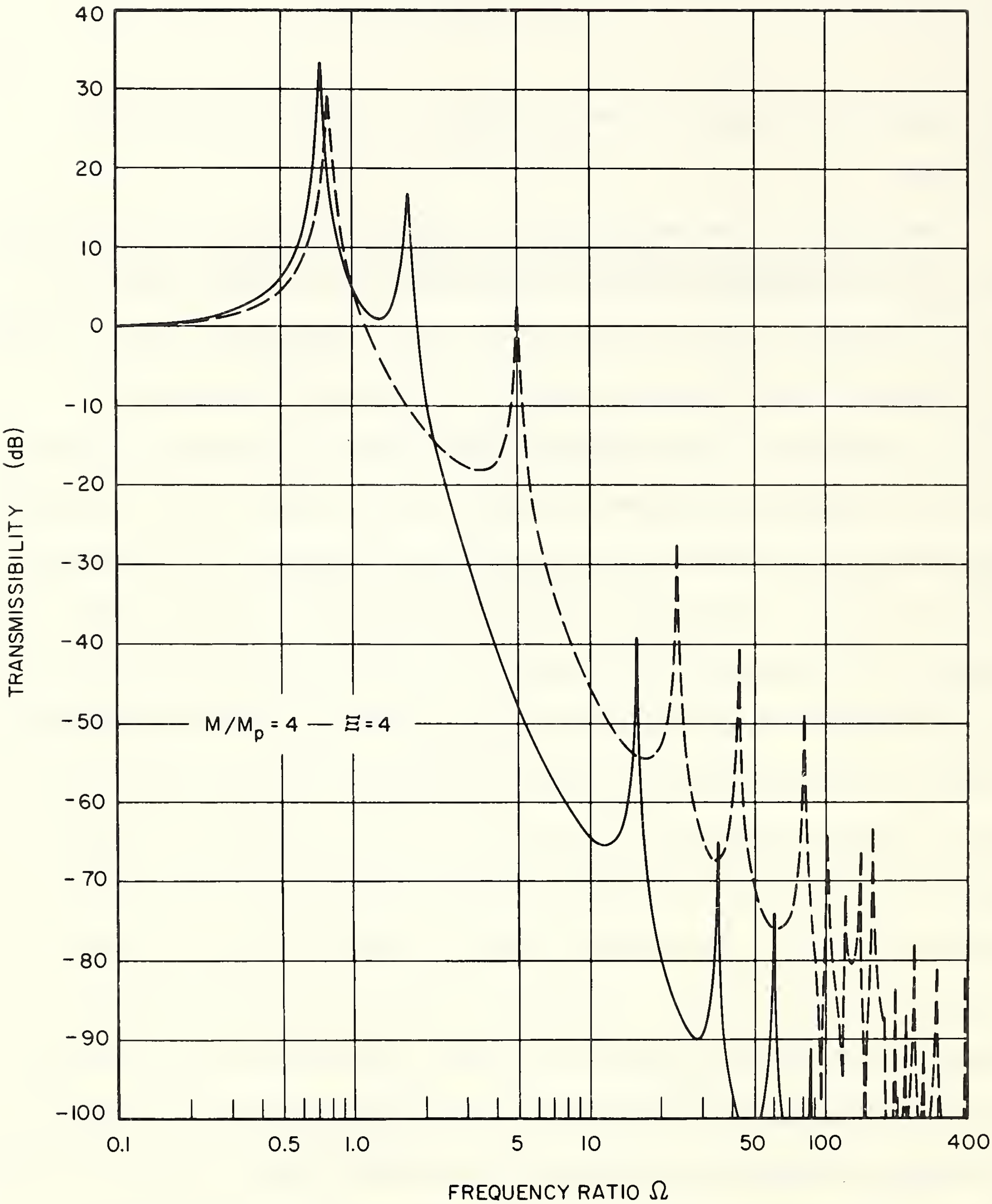


Fig. 30 $T_{overall}$ for an item of mass M that is resiliently mounted at each corner to a rectangular platelike substructure with simply supported boundaries and an aspect ratio of 0.5; M is four times more massive than the plate. The antivibration mounts are symmetrically and favorably located about the plate center, and are terminated on the plate by lumped masses of total mass m = M. The damping factors of the mounts and the platelike substructure are 0.05 and 0.01, respectively. The dashed curve shows $T_{overall}$ for the same mounting system without the loading masses (m = 0).

item (m = M). Use of such heavy mass loading is necessary if the level of the transmissibility curve is to be reduced significantly. For added mass equal to 0.25 M, the resultant transmissibility curve would lie approximately halfway between the solid and dashed curves at frequencies above the fundamental plate resonance ($\Omega \approx 3$). Such small added mass as 0.05 M would be ineffectual in reducing transmissibility much below the level of the dashed curve, except at very high frequencies where the impedance of the loading masses m would eventually predominate the plate impedance.

6.4 Nonrigidity of Mounted Item

To conclude this Section, it is appropriate to demonstrate how readily the effects of nonrigidity in the mounted item can be accounted for in the preceding four-pole equations. Thus, if the mounted item is characterized by the four-pole parameters φ_{ij}, then the forces and velocities experienced by the mounted item and the substructure in Fig. 29(a) will be related, not by Eq. 100, but as follows:

$$\begin{bmatrix} \tilde{F}_1 \\ \tilde{V}_1 \end{bmatrix} = \begin{bmatrix} \upsilon'_{11} & \upsilon'_{12} \\ \upsilon'_{21} & \upsilon'_{22} \end{bmatrix} \begin{bmatrix} \tilde{F}_{12} \\ \tilde{V}_{12} \end{bmatrix} \quad , \tag{116}$$

where

$$\upsilon'_{11} = (\varphi_{11}\upsilon_{11} + \varphi_{12}\upsilon_{21}) \quad , \tag{117}$$

$$\upsilon'_{12} = (\varphi_{11}\upsilon_{12} + \varphi_{12}\upsilon_{22}) \quad , \tag{118}$$

$$\upsilon'_{21} = (\varphi_{21}\upsilon_{11} + \varphi_{22}\upsilon_{21}) \quad , \tag{119}$$

and

$$\upsilon'_{22} = (\varphi_{21}\upsilon_{12} + \varphi_{22}\upsilon_{22}) \quad . \tag{120}$$

In these equations, υ_{11}, υ_{12}, υ_{21}, and υ_{22} are defined precisely as before by Eqs. 93-96 in which the initial definition of $\gamma_1 = M_1/M_R$ pertains. Because Eqs. 100 and 116 are closely similar, the expressions that were derived previously for transmissibility can be restated, by inspection, as follows:

$$T = \left|\frac{\tilde{F}_{12}}{\tilde{F}_1}\right| = \left|\frac{Z_T}{(\upsilon'_{11}Z_T + \upsilon'_{12})}\right| \tag{121}$$

and

$$T_{overall} = \left|\frac{\tilde{F}_2}{\tilde{F}_1}\right| = \left|\left(\frac{\alpha_{22}Z_T - \alpha_{12}}{\upsilon'_{11}Z_T + \upsilon'_{12}}\right)\right| \quad . \tag{122}$$

Response ratio can also be restated simply by noting from Eq. 116 that

$$\tilde{F}_1 = (\upsilon'_{11}Z_T + \upsilon'_{12})\ \tilde{V}_{12} \quad , \tag{123}$$

and by noting from the relation between the forces and velocities experienced by the mounted item and substructure in Fig. 29(b) that

$$\begin{aligned} \tilde{F}_1 &= (\varphi_{11}\tilde{F}_{12R} + \varphi_{12}\tilde{V}_{12R}) \\ &= (\varphi_{11}Z_T + \varphi_{12})\ \tilde{V}_{12R} \quad ; \end{aligned} \tag{124}$$

in consequence,

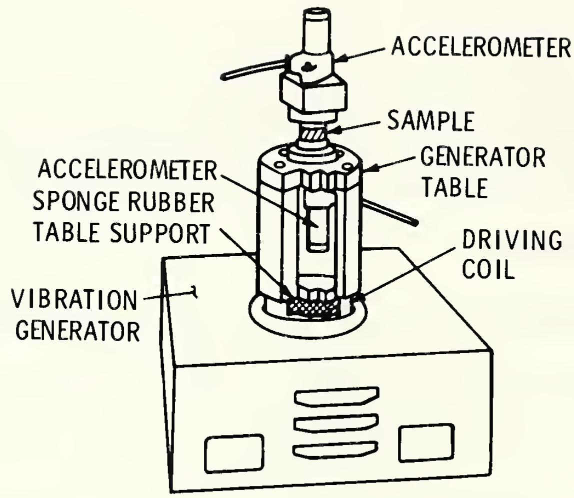
$$R = \left|\frac{\tilde{V}_{12}}{\tilde{V}_{12R}}\right| = \left|\left(\frac{\varphi_{11}Z_T + \varphi_{12}}{\upsilon'_{11}Z_T + \upsilon'_{12}}\right)\right| \quad . \tag{125}$$

7. Experimental Determination of Transmissibility

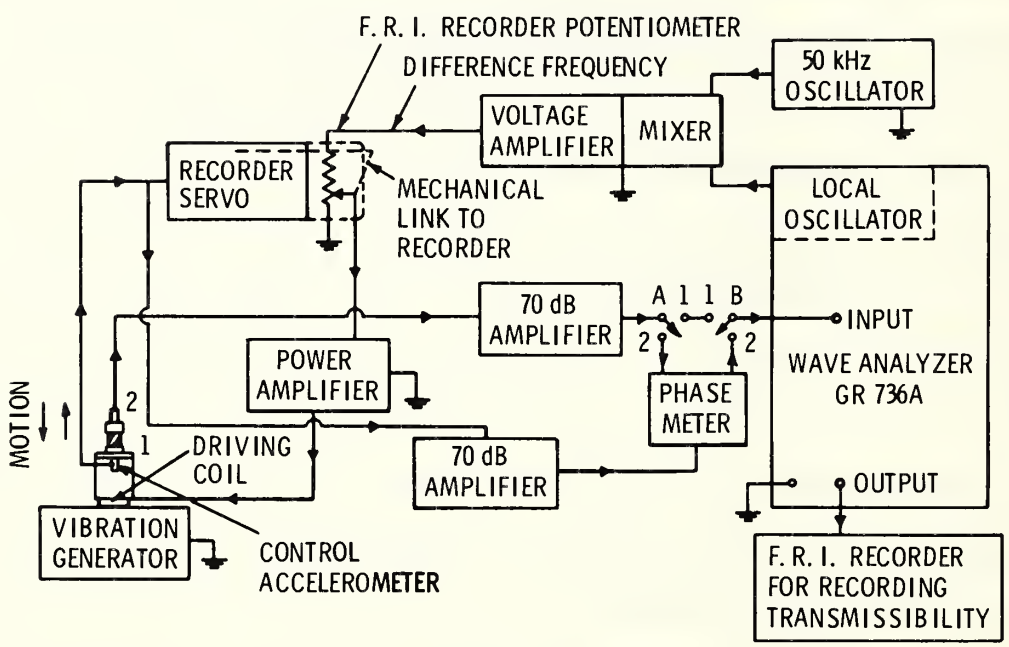
7.1 Direct Measurement

Reported throughout the prior literature are transmissibility measurements that have been obtained in one of two ways based on the simple sketches of Fig. 31 (Refs. 7, 13, 21, 68, 78, 117, 118, 135, 150, 152, 157-159, 161, 170, 194, 198, 208, 227-230). No other methods of transmissibility measurement are known to have been used previously or described elsewhere in the literature. Almost exclusively, apparatus has been built to simulate the simple mounting system of Fig. 13(a), the foundation and mounted item of which vibrate with the amplitudes $\tilde{x}_1$ and $\tilde{x}_2$. Transmissibility has been recorded as the readily measurable ratio of the companion accelerations; that is, $T = |(j\omega)^2\tilde{x}_2/(j\omega)^2\tilde{x}_1| = |\tilde{x}_2/\tilde{x}_1|$. The design of a representative experiment to establish T in this way, and a block diagram of the associated electronics, are reproduced from Ref. 158 in Fig. 31. Only three early German workers (Refs. 7, 157, and 194) chose to build apparatus to record transmissibility as the force ratio $T = |\tilde{F}_2/\tilde{F}_1|$, thus simulating the simple mounting system of Fig. 13(b).

Experiments to determine the transmissibility across the compound mounting system are described in Refs. 78, 157, 159, and 161; attention is confined to the simple system elsewhere. It is remarkable that Ref. 78, apparently overlooked in the many years since its publication in 1931, should have introduced the theory of the compound system and have confirmed it by experiment. References that describe the <u>results</u> of transmissibility



(a)



(b)

Fig. 31 (a) Apparatus and (b) electronic equipment used in a direct measurement of mount transmissibility. (Ref. 158.)

measurements made primarily on rubber antivibration mounts have been listed previously in Sec. 4.1.

A criticism of virtually all of the foregoing transmissibility measurements is that they were obtained from small-scale or model experiments in which the mounts experience a smaller static load than the one for which they are rated. Consequently, the natural frequency ω_o of the mounting system is often appreciably higher, and the strain in the mount appreciably lower, than would be the case in practice. The only exception appears to be the relatively low-frequency measurement of transmissibility that is described in Ref. 150. It is readily apparent that care is necessary in any "vibrating-foundation" measurement of transmissibility [as in Figs. 13(a) and 31(a)] to design the foundation so that its fundamental resonant frequency lies adequately above the frequency range of measurement.

7.2 Four-Pole Technique (Indirect Measurement)

It is evident that consideration could well be given to the determination of transmissibility by a four-pole technique. The proposed measurements would utilize an apparatus that has been designed to record the driving-point impedance and quasi transfer impedance of antivibration mounts subjected to significant static loads. The apparatus, which is described in Ref. 58, sandwiches between the top plate and base of a Universal Tension and Compression Testing Machine the following sequence of components (Fig. 32): a rubber pad, a small vibration generator, an impedance head, the antivibration mounting under test, an aluminum support block, and a piezoelectric force gage.

The antivibration mount is held by the support block in the manner likely to be encountered "in service," preferably contacting the lower end plate of the mount over the largest possible area. Mounts of different

UNIVERSAL TENSION AND COMPRESSION TESTING MACHINE

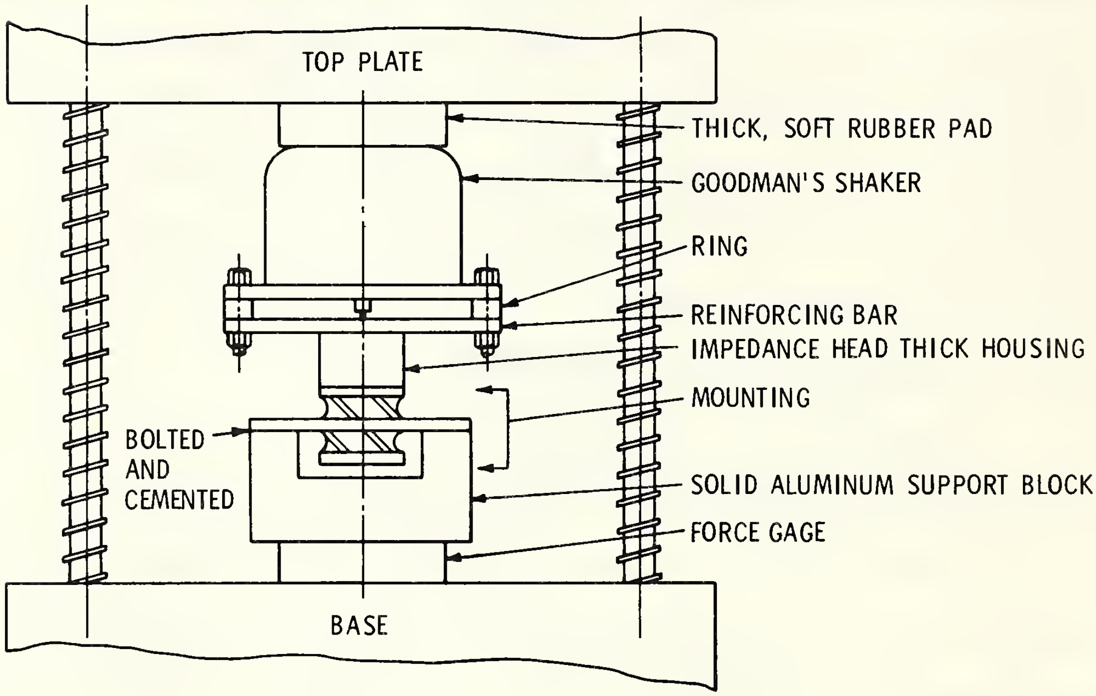


Fig. 32 Proposed apparatus for the indirect measurement of mount transmissibility by a four-pole technique. (Ref. 58.)

designs would be mated with other support blocks of appropriate outline. The blocks should have the greatest possible rigidity and could well be machined from alumina rather than aluminum (Ref. 231).

It is convenient to designate the forces and velocities at the input and output terminal pairs of the mount in Fig. 32 as $\tilde{F}_{11}$, $\tilde{V}_{11}$ and $\tilde{F}_{12}$, $\tilde{V}_{12}$, respectively, and once again to characterize the mount performance by the four-pole parameters υ_{11}, υ_{12}, υ_{21}, and υ_{22}. Because the output terminal pair of the mount is rigidly blocked ($\tilde{V}_{12} = 0$), Eqs. 66 and 68 show that

$$\upsilon_{11} = \left.\frac{\tilde{F}_{11}}{\tilde{F}_{12}}\right|_{\tilde{V}_{12} = 0} \tag{126}$$

and

$$\upsilon_{21} = \left.\frac{\tilde{V}_{11}}{\tilde{F}_{12}}\right|_{\tilde{V}_{12} = 0} \quad . \tag{127}$$

The quantities $\tilde{F}_{11}$ and $\tilde{V}_{11}$ are readily measured by the impedance head of Fig. 32, with suitable electronic cancellation of the small integral mass under the force gage in the head. The quantity $\tilde{F}_2$ is readily measured by the lower force gage at frequencies adequately below the resonant frequency of the gage and aluminum support block (> 5 kHz). Hence, the frequency-dependent values of the parameters υ_{11} and υ_{21} can be established by straight forward measurement. Importantly, the parameters can be established when there is significant static strain in the mount--which is introduced, to the extent required, by the Universal Tension and Compression Testing Machine.

If a basic comparison is required between the transmissibility curves of various antivibration mounts in the simple system of Fig. 13, for which the foundation impedance is extremely large and the mounted item of mass M is ideally rigid, then it suffices to have knowledge of the parameters υ_{11} and υ_{21} for each mount of interest. Thus, inspection of Eqs. 100 and 105, which relate to the more general mounting system of Fig. 29(a), shows that, in the foregoing circumstances (foundation impedance $Z_T = \infty$), the force transmissibility

$$T = \left|\frac{1}{\upsilon_{11}}\right| = \left|\frac{1}{(\upsilon_{11} + j\omega M\, \upsilon_{21})}\right| \quad , \tag{128}$$

a quantity that involves υ_{11} and υ_{21} only. Moreover, knowledge of these parameters is adequate to predict transmissibility and response ratio from Eqs. 105 and 106 when Z_T is finite and has any measured or hypothetical frequency dependence; it is also adequate to predict $T_{overall}$ from Eq. 115 provided that the four-pole parameters α_{12} and α_{22} of the foundation are also known.

By contrast, if it is wished to compare or predict the performance of various antivibration mounts in completely general terms -- as is essential when the mounted item is nonrigid and Eqs. 117-122 pertain -- then one of the remaining parameters υ_{12} and υ_{22} must be determined, the value of the other following from the general relation of Eq. 74. An exception would be when the mount is symmetrical (when it does not matter which side of the mount is input or output as in Fig. 28 when $M_1 = M_2$), in which case

$$\upsilon_{22} = \upsilon_{11} \tag{129}$$

and

$$\upsilon_{12} = (\upsilon_{11}^2 - 1)/\upsilon_{21} \quad . \tag{130}$$

Otherwise, it becomes necessary to reserve the mount such that its input and output terminals are interchanged, a requirement that may call for the use of a new and differently shaped support block (Fig. 32). If the forces and velocities at the new (reversed) input and output terminal pairs of the mount are $\tilde{F}_{11r}$, $\tilde{V}_{11r}$ and $\tilde{F}_{12r}$, $\tilde{V}_{12r}$, respectively, then reference to Eq. 81 shows that

$$\upsilon_{22} = \left|\frac{\tilde{F}_{11r}}{\tilde{F}_{12r}}\right|_{\tilde{V}_{12r} = 0} \quad ; \tag{131}$$

whence

$$\upsilon_{12} = (\upsilon_{11}\upsilon_{22} - 1)/\upsilon_{21} \quad . \tag{132}$$

In this reversed situation, a measurement to confirm the previously determined value of υ_{21} can be made according to the companion definition of Eq. 80:

$$\upsilon_{21} = \left|\frac{\tilde{V}_{11r}}{\tilde{F}_{12r}}\right|_{\tilde{V}_{12r} = 0} \quad . \tag{133}$$

Note that, although the four-pole parameters υ_{12} and υ_{22} can be determined readily when the output terminals are free ($\tilde{F}_{12}$ or $\tilde{F}_{12r} = 0$), it is no longer possible to impose a static strain on the mount, as it is when the output terminal pair is blocked, so that a prime advantage of the four-pole measurement method is lost.

Other relations for the mount that are of interest here can simply be deduced from Eqs. 66, 68, 80, and 81 and stated as follows:

$$\text{driving-point impedance} = \left.\frac{\tilde{F}_{11}}{\tilde{V}_{11}}\right|_{\tilde{V}_{12} = 0} = \frac{\upsilon_{11}}{\upsilon_{21}} \quad , \tag{134}$$

$$\text{reversed driving-point impedance} = \left.\frac{\tilde{F}_{11r}}{\tilde{V}_{11r}}\right|_{\tilde{V}_{12r} = 0} = \frac{\upsilon_{22}}{\upsilon_{21}} \quad , \tag{135}$$

$$\text{and quasi transfer impedance} = \left|\frac{\tilde{F}_{12}}{\tilde{V}_{11}}\right|_{\tilde{V}_{12} = 0} = \left|\frac{\tilde{F}_{12r}}{\tilde{V}_{11r}}\right|_{\tilde{V}_{12r} = 0} = \frac{1}{\upsilon_{21}} \quad . \tag{136}$$

Quasi transfer impedance is so named to distinguish it from the quantity that is usually referred to as transfer impedance; namely, the quantity

$$\left.\frac{\tilde{F}_{11}}{\tilde{V}_{12}}\right|_{\tilde{F}_{12} = 0} = \left.\frac{\tilde{F}_{11r}}{\tilde{V}_{12r}}\right|_{\tilde{F}_{12r} = 0} = \upsilon_{12} \quad . \tag{137}$$

The apparatus of Fig. 32 is designed, and is ideally suited, to measure the driving-point impedance and quasi transfer impedance of antivibration mounts under significant static loads. The results of such measurements on a rubber mount (Ref. 58) are shown in Figs. 33 and 34. It is evident in the first figure that the magnitude of the mount driving-point impedance Z (dashed curve) is springlike at low frequencies, and is masslike at higher frequencies where the impedance $j\omega M$, of the upper end plate (refer to Fig. 28) predominates that of the rubber in the mount. An impedance

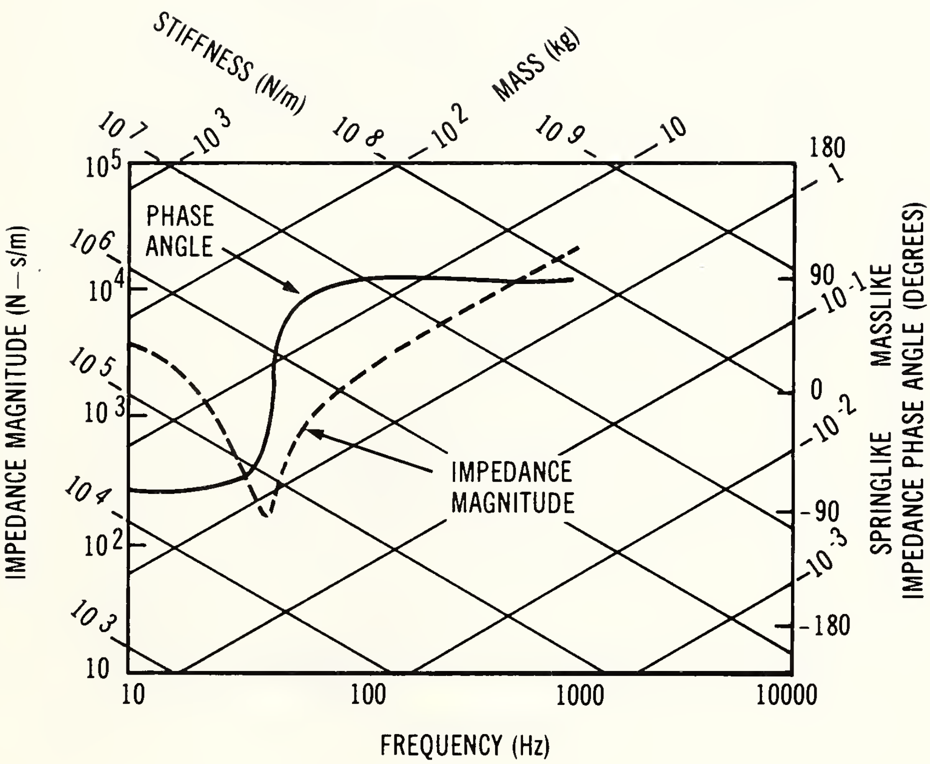


Fig. 33 Driving-point impedance of a rubber mount (magnitude and phase) as obtained with the apparatus of Fig. 32. (Ref. 58.)

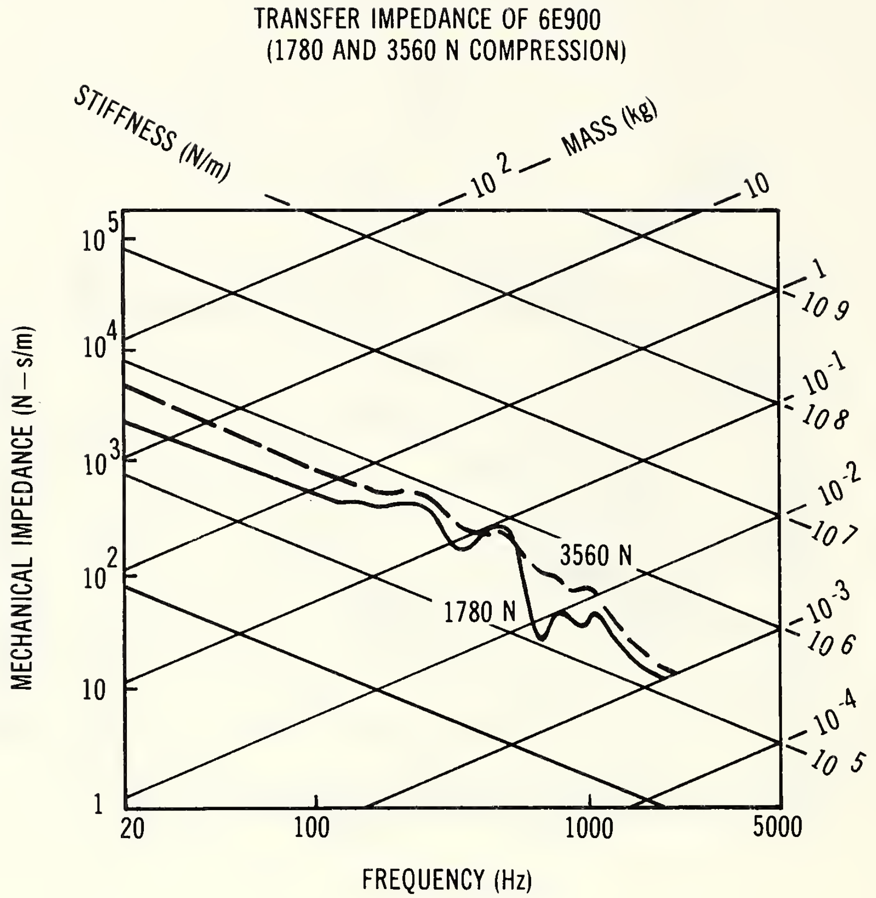


Fig. 34 Quasi transfer impedance of a rubber mount subjected to static loads of 1780 and 3560 N (180 and 360 kg-force, 400 and 800 lb-force) as obtained with the apparatus of Fig. 32. (Ref. 58.)

minimum occurs at intervening frequencies where the end plate and the rubber resonate. The foregoing behavior can be predicted through use of Eqs. 93, 95, and 134, which show that

$$Z = \frac{\upsilon_{11}}{\upsilon_{21}} = \frac{j\omega M_R}{(n^* \ell)} \left[\frac{c. - \gamma_1 (n^* \ell)\ s.}{s.} \right] \quad , \tag{138}$$

where $\gamma_1 = M_1/M_R$. Consequently, at low frequencies,

$$Z_{LF} = \left(\frac{K^*}{j\omega} + j\omega M_1 \right) = \frac{1}{j\omega} \left(K^* - \omega^2 M_1 \right) \quad , \tag{139}$$

where K^* is the complex mount stiffness (Sec. 6.2). Although slight wave effects in the rubber occur at approximately 200 and 450 Hz, they are masked by the impedance $j\omega M_1$; however, they are evident in the plot of the more sensitive phase measurements shown by the solid curve.

Figure 34 describes the magnitude of the quasi transfer impedance $TZ = |\upsilon_{21}|^{-1}$ when the mount is subjected to static loads of 180 and 360 kg (400 and 800 lb). This figure is reproduced from Ref. 58, where it is stated that the performance of mounts with rated loads as large as 4540 kg (10,000 lb) has been evaluated with the apparatus of Fig. 32 at frequencies up to 5 kHz. As explained in Refs. 58, 219, and in Sec. 6.2, the quasi transfer impedance $|\upsilon_{21}|^{-1} = K^*/j\omega$ provides a measure of the complex mount stiffness, readily enabling the real part of this stiffness and, with precision phase measurement (Ref. 110), the mount damping factor (Eq. 99) to be determined through a broad frequency range. In Fig. 34, the <u>mean</u> levels of transfer impedance essentially decrease in inverse proportion to frequency, even though wave resonances are superimposed on the curves at higher frequencies. Note that the wave resonances are least severe when the mount is loaded most heavily. Finally, note that the value of the important four-pole parameter υ_{11} of Eq. 128 can be determined, independently of its definition in Eq. 126, as the quotient of the mount driving-point and quasi-transfer impedances as recorded by the apparatus of Fig. 32; thus,

$$\frac{Z}{TZ} = \frac{(\upsilon_{11}/\upsilon_{21})}{(1/\upsilon_{21})} = \nu_{11} \quad . \tag{140}$$

8. Future Work

To conclude, it is appropriate to discuss several areas in which additional research would appear to be timely and beneficial. These areas can be listed as follows:

1. Search for New Antivibration Mount Material. In Sec. 3, the conclusion is reached that high-damping materials would have greater application in antivibration mountings if they could be produced such that their dynamic moduli remained constant, or increased only slowly with frequency. A search for such materials could focus either (a) on single materials or (b) on suitable mechanical combinations of pairs of materials in individual antivibration mounts -- so-called parallel mountings (Ref. 59). In case (a), the "single" materials might well comprise a polymer blend of materials such as polyvinyl chloride (PVC) and polybutadiene acrylonitrile that prove to be unusually compatible, exhibiting one rather than two transition frequencies.

2. Practical Three-Element Mounting. The so-called three element mounting is discussed in Sec. 3. Although this combination of two springs and one

dashpot has been widely analyzed and discussed in the literature, the mounting appears never to have been realized in practice. This is unfortunate because the heavily damped steady-state and transient responses of the mounting are often superior to those of a heavily damped rubber mounting (for example, see Ref. 59).

3. Static Strain Dependence of Antivibration Mount Performance. Information on how static strain influences the dynamic properties or rubbers, and rubber antivibration mountings, is sparse and, at times, seemingly contradictory. Such information should follow readily from the transmissibility experiment based on four-pole parameter techniques that is described in Sec. 7.2 (refer also to Eq. 99, Sec. 6.2).

4. Experimental Measurements. Experimental confirmation could well be obtained of analyses of the transmissibility across a mounting system in which the mounted item had variable and controllable nonrigidity (which could be introduced by the use of intentionally nonrigid flanges or feet, as discussed in Sec. 4.3). Such a mounting configuration could be extended by the introduction of a nonrigid and mathematically tractable substructure, so that measurements of response ratio R and $T_{overall}$ could also be compared with prediction.

5. Extension of the Investigation. A limitation to this report, and to most of the literature concerned with vibration isolation and antivibration mountings, is that attention is focussed on the translational (vertical) motion of the systems concerned. In reality, system excitation and response can well be multidirectional and rocking motion will occur, giving rise to the transmission of bending moments as well as forces between the mounted item, the mount, and any nonrigid substructure, and to the need to account for bending moments and angular deflections in the foregoing analyses. When these more complicated situations are encountered, four-pole

theory becomes inadequate, and a transmission-matrix theory of wider applicability must be used; in fact, the components of the mounting system must be viewed at least as eight terminal systems, in which case reliance would be placed on 4 x 4 transmission matrices rather than on the 2 x 2 that typify the simpler four-pole theory. Such analyses, and the experimental confirmation of their results and conclusions, present both a challenge and an opportunity.

Acknowledgments

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16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

The results of a search and critical evaluation of the literature pertinent to both the use and the characterization of the performance of antivibration mountings for the control of noise and vibration are described. First to be discussed are the static and dynamic properties of rubberlike materials that are suited for use in antivibration mountings. This is followed by analyses of the simple (one-stage) mounting system and its subsequent, impaired performance when second-order resonances occur either in the isolator (wave effects) or in the structure of the mounted item itself (nonrigid supporting feet). A discussion is then given to the performance of the compound or two-stage mounting system which possesses superior isolation properties for high frequencies. Next, the four-pole parameter technique of analysis is described and applied, in general terms, to the characterization of the performance of an antivibration mounting with wave effects for both the cases where either the supporting foundation or mounted item are nonrigid. The adopted methods for the direct measurement of antivibration-mounting performance are described, followed by an explanation of how this same experimental determination of transmissibility can also be made using an indirect measurement technique based upon four-pole parameter analysis considerations. Finally, recommendations for future work in various areas of research on antivibration mountings are given.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)*

Antivibration mounting; damping; dynamic properties; industrial engineering; isolation; machinery and equipment; mechanical impedance; mechanical vibrations; noise control; transmissibility; vibrations; vibration isolation.

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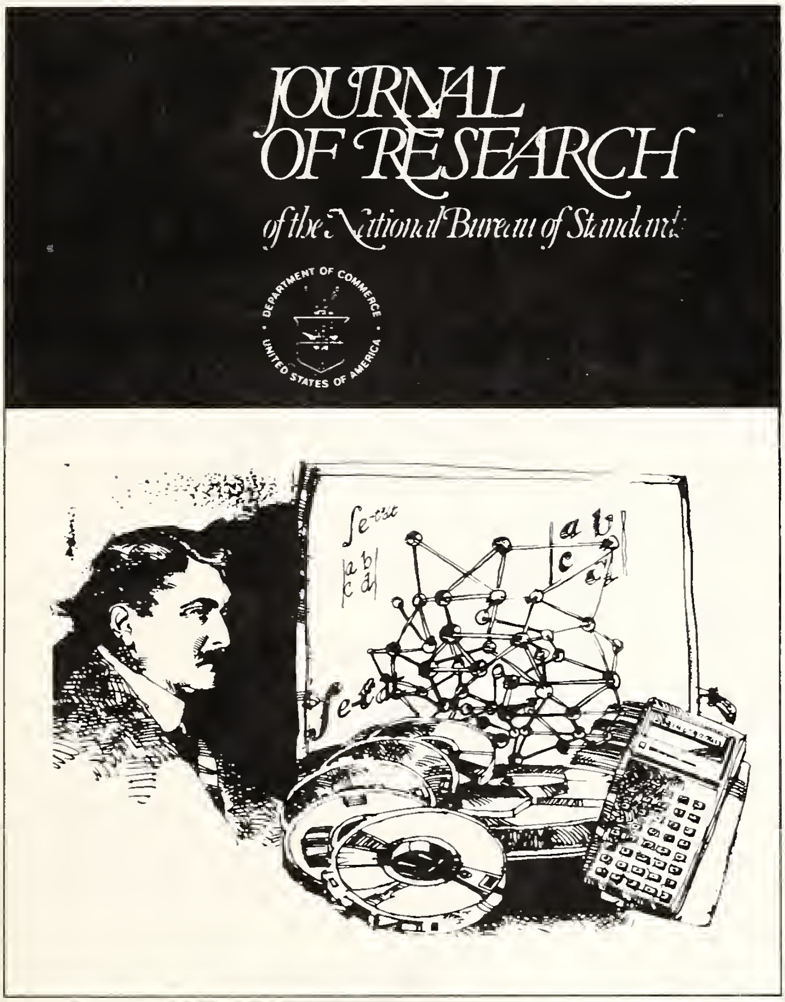

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