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Analysis of Reinforced Concrete Beams Subjected to Fire

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NBS Building Science Series no. 76

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ANALYSIS OF REINFORCED CONCRETE BEAMS SUBJECTED TO FIRE

Bruce Ellingwood

James R. Shaver

Methods for analytically predicting the behavior of simply supported reinforced concrete beams subjected to fire are presented. This is generally a two-step process involving a thermal analysis followed by a stress analysis. This study emphasizes the latter, wherein the determination of moment-curvature-time relationships for the beam cross section incorporates the temperature-dependent strength degradation in the steel and concrete as well as thermal and creep strains. The sensitivity of the predictions to various phases of analytical modeling is investigated to establish the parameters most important for the prediction of beam behavior and to indicate where additional data should be gathered. A comparison of predicted behavior with that observed in fire tests shows excellent agreement when realistic reinforcement temperature histories are used.

Key Words: Creep; fire endurance; fire tests; reinforced concrete; sensitivity analysis; steel; structural mechanics; uncertainty.

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NOTATION

E_s, E_c	-	Young's modulus for steel and concrete, respectively.
n, K	-	strain hardening exponent, strength coefficient.
M, M_{wl}, M_u	-	moment; working load and ultimate load moments, respectively.
P	-	axial thrust.
T, T_{ij}, T_o, T_k	-	temperature; temperature in discrete element ij; initial temperature; temperature in bar k.
Z	-	Zener-Holloman creep constant.
d, d_k	-	depth to reinforcement from top of beam; depth to bar k.
e_T, e_o, e_I	-	total strain; total strain at top of beam; imposed strain.
f_c, f_c'	-	concrete stress, ultimate concrete strength.
f_s, f_y	-	steel stress; steel yield stress.
ρ_b'	-	balanced reinforcement ratio.
ρ_w, ρ_f	-	reinforcement ratios for T-beams defined by ACI 318-71 [2].
t	-	time.
x,y	-	beam cross section coordinates measured from top of beam.
α_c, α_s	-	coefficients of thermal expansion for concrete and steel, respectively.
$\Delta H/R$	-	activation energy of creep divided by gas constant.
α_m, T_m, α_o	-	constants for determining temperature dependency of α_c and α_s.
β	-	constant used to determine $\Theta(T)$; $\beta = \Delta H/R$.
ε, σ	-	strain; stress.
$\varepsilon_{cth}, \varepsilon_{sth}, \varepsilon_{cr}, \varepsilon_{ps}$	-	thermal strain in concrete, thermal strain in steel, creep strain in steel, prestressing strain in steel, respectively.
ε_c	-	mechanical concrete strain at top of beam at x_R.
ε_{to}	-	constant related to primary creep strain.
$\varepsilon_o, \varepsilon_p$	-	constants used in the constitutive relations for concrete and steel, respectively.
θ	-	temperature-compensated time.
ϕ	-	unit rotation (curvature).

1.0 INTRODUCTION

The provision for sufficient fire resistance and reserve load-carrying capacity for reinforced concrete structural elements is an important problem in engineering design and is required by most building codes. Currently, this resistance to fire is determined primarily on the basis of the performance of an element subjected to the American Society for Testing and Materials (ASTM E-119) [3]* fire test. Although the standard fire test may not be representative of an actual fire, it is generally recognized as being necessary to provide a basis for comparison between various designs and to satisfy the need for reproducibility in test data. The suitability of prospective designs is thus likely to be judged by their performance in this standard test in the foreseeable future.

Criteria for the fire-resistant design of reinforced or prestressed members are difficult to develop. Because of the considerable cost in conducting a standard fire test of even a simple beam, it would generally not be economical to test a sufficiently large number of specimens to determine experimentally fire endurance for a spectrum of possible designs. Although it is possible to determine fire resistance with a limited experimental data base by interpolation, this procedure relies to a great extent on judgment and experience and, moreover, provides little indication of the sensitivity of member endurance to various designer-controlled parameters. An alternative is to use thermomechanical models to predict analytically the behavior of reinforced concrete members subjected to fire. In this context, the limited data extant would be used to define experimental constants employed in the models and to update and improve the analysis procedure itself. This procedure furnishes a logical basis not only for interpreting such test data as is available but also for extrapolating, within reasonable limits, to situations not covered by the data base.

This methodology is described in the following sections. The analysis of a reinforced concrete beam section subjected to a non-linear strain distribution produced by a time-dependent temperature history is discussed. A sensitivity analysis of the analytical

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* Numbers in brackets indicate literature references on page 41.

models is presented in order to show the effect of inaccuracies in data reduction and to provide some guidance as to where acquisition of additional data would most readily pay off. A computer program used to perform these analyses is documented in detail, including the preparation of input data, flow charts of the computational algorithm, and a source listing of the computer code. The methodology described herein is believed to provide a framework for systematically developing fire-resistant design procedures in the long term.

2.0 METHODS OF ANALYSIS

At room temperature, the ultimate capacity of a properly designed reinforced concrete beam will exceed the sustained or service load moment by a prescribed margin of safety. During the fire, however, the ultimate capacity will degrade to the point where it is less than the service load, at which point the beam will fail. Accordingly, the reserve moment capacity as a function of temperature or time and the period that the beam can sustain its working load are of particular interest. In this analysis, the moment-curvature-time relationship for a reinforced and/or prestressed concrete beam section subjected to fire is developed. The effects of the resulting thermal expansion of the concrete and steel, creep in steel, and progressive deterioration of the materials at elevated temperatures are incorporated into the strength calculations. An accompanying finite element thermal analysis program [15] is used to determine the temperature distribution on the beam cross section. This thermal analysis is made prior to the strength analysis.

Because of the non-linear nature of the stress-strain equations for concrete and for steel reinforcement when stressed above the proportional limit, closed-form expressions for the moment-curvature relations for a reinforced concrete beam section are difficult to obtain. Therefore, an iterative procedure is employed wherein a strain distribution is assumed on the cross section, the stresses are determined from the appropriate constitutive relations, and a resultant thrust and moment are computed from these stresses; i.e.

$$\int \sigma(x,y)\ dA = P$$

$$\int \sigma(x,y)\ y\ dA = M \qquad (1)$$

$$\int \sigma(x,y)\ x\ dA = 0$$

If the thrust and moment computed from Eq. 1 equal the applied loads on the beam, the resulting moment and curvature are recorded; otherwise, the assumed strain distribution is modified, and the process is repeated until the calculated and applied loads converge.

Although the above computation procedure is easily visualized in a reinforced concrete beam at room temperature, prolonged exposure to fire induces a nonlinear thermal strain distribution on the cross section and causes the material strengths to degrade and the steel (and, to a lesser extent, the concrete) to creep under sustained load conditions.

These factors affect the load-deformation characteristics of the beam, as described in the following.

Fig. 1 shows the distribution of strains on reference coordinate x_R of a beam cross section which has been exposed to fire. It might be noted that the reference coordinate x_R is not necessary in a room temperature analysis because the strain distribution across the beam width is, or is assumed to be, constant. Under elevated temperatures, however, the strain distribution becomes nonlinear in both the depth and width direction. Under the assumption that plane sections remain plane after deformation, the total strain $e_T(y)$ is defined by [10],

$$e_T(y) = e_o + \phi y \tag{2}$$

in which e_o is the total strain at the top of the beam and ϕ is the unit rotation at the section (y is measured from the top of the beam). In what follows, extensional strain will have positive sign.

The total strain is the sum of two components, i.e.,

$$e_T(y) = e_I(x,y) + \varepsilon(x,y). \tag{3}$$

Note that although e_T is a function of y only, e_I and ε are functions of x and y because of the nonlinearity in the temperature distribution across the width and depth of the section. The imposed strain component $e_I(x,y)$ consists of thermal strains in the concrete, ε_{cth}, and steel, ε_{sth}, and the creep strain in the steel, ε_{cr} (to be discussed in Eqs. 7 and 8). The mechanical strain $\varepsilon(x,y)$ is required to establish equilibrium, subject to the compatibility requirement expressed by Eq. 2. The stresses on the cross section are computed from the mechanical strains using the appropriate constitutive relationship. Eqs. 2 and 3 show this mechanical strain is given by

$$\varepsilon(x,y) = e_o + \phi y - e_I(x,y). \tag{4a}$$

It may be observed from Eq. 4a that two parameters, e_o and ϕ, are sufficient to describe uniquely the strain distribution on the cross section once the imposed strains

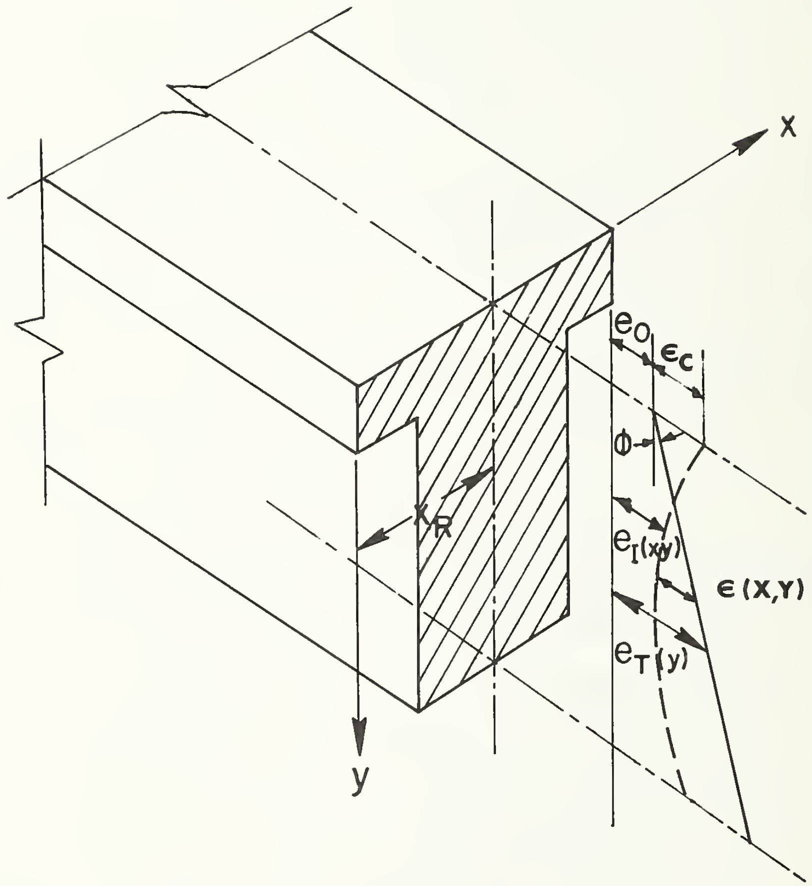


FIG. 1 - STRAINS ON REFERENCE COORDINATE X_R OF BEAM SECTION

are determined. These two parameters may be adjusted using a self-accelerating method [19] for the iterative solution of equations in order to satisfy equilibrium, Eq. 1. Rather than to iterate with e_o directly, it is convenient to employ the more conventional parameter ε_c which denotes the mechanical limiting concrete compressive strain at the top of the beam and at the reference width coordinate x_R (see Fig. 1).

To express e_o in terms of ε_c, note that at $(x,y) = (x_R,0)$, $\varepsilon(x_R,0) = \varepsilon_c$ and thus

$$e_o = \varepsilon_c + e_I(x_R,0) \tag{4b}$$

where $e_I(x_R,0)$ is the imposed (thermal) strain at the top of the beam. In general, then, the mechanical strain is given by

$$\varepsilon\ (x,y) = \varepsilon_c + \phi y - [e_I(x,y) - e_I(x_R,0)]. \tag{5}$$

It is convenient in performing the strength computations (and, indeed, necessary for all but the simplest cases) to discretize the beam cross section with a series of small elements rather than to treat it as a continuum. The forces can then be obtained by summation. Using this approach, the strain at the centroid of a concrete element furnishes a stress which is assumed to be constant over that particular element. In particular, the strain in the concrete element with centroidal coordinates (x_i,y_j) is

$$\varepsilon_c(x_i,y_j) = \varepsilon_c + \phi y_j - [\varepsilon_{cth}\ (x_i,y_j) - \varepsilon_{cth}\ (x_R,0)]. \tag{6a}$$

The strain in the steel reinforcing bar with depth d_k would be

$$\varepsilon_s(d_k) = \varepsilon_c + \phi d_k - [\varepsilon_{sth}(d_k) + \varepsilon_{cr}(d_k) - \varepsilon_{cth}(x_R,0)]. \tag{6b}$$

If the beam is prestressed in addition to or instead of being reinforced,

$$\varepsilon_s(d_k) = \varepsilon_c + \phi d_k - [\varepsilon_{sth}(d_k) + \varepsilon_{cr}(d_k) - \varepsilon_{cth}(x_R,0)] + \varepsilon_{ps}(d_k) \tag{6c}$$

in which ε_{ps} is prestressing strain in steel element k.

The thermal strains in the concrete and steel, indicated in Eq. 6, are computed from the temperature distribution on the cross section. The temperature within each concrete element is assumed to be constant, and is determined by averaging the nodal temperatures calculated during the thermal analysis. The steel temperatures may either be computed from the adjacent nodal temperatures in the surrounding concrete mass or may be read from cards or tape at each calculation time step.

The thermal strains for concrete and steel, respectively, are

$$\varepsilon_{cth}(x_i,y_j) = \int_o^{T_{ij}} \alpha_c(T)\, dT \qquad \text{for concrete}$$

$$\varepsilon_{sth}(d_k) = \int_o^{T} \alpha_s(T,d_k)\, dT \qquad \text{for steel} \tag{7}$$

where α_c and α_s are coefficients of thermal expansion of concrete and steel, dependent on temperature T.

The creep strain in the steel is determined using the Harmathy-Dorn theory [9, 12] along with a time-hardening rule for determining the creep under a variable stress history. The procedure for computing the creep increment corresponding to a small time interval is illustrated in Fig. 2. The primary and secondary stages of creep are described by Harmathy's creep equation

$$d\varepsilon_{cr}/d\theta = Z \coth^2 (\varepsilon_{cr}/\varepsilon_{to}) \tag{8a}$$

or, in integrated form,

$$\varepsilon_{cr}/\varepsilon_{to} - \tanh \varepsilon_{cr}/\varepsilon_{to} = Z\theta/\varepsilon_{to} \tag{8b}$$

in which Z and ε_{to} are material-dependent functions of stress, and $\theta(t)$ is the temperature-compensated time [9],

$$\theta(t) = \int_o^t \exp [- \Delta H/RT(\tau)]\, d\tau. \tag{9}$$

Term ΔH is the activation energy for creep, R is the gas constant, and T(t) is the temperature as a function of time. The time-hardening rule appears appropriate for studying creep at elevated temperatures [6, 16], and assumes that the creep strain increment during a given

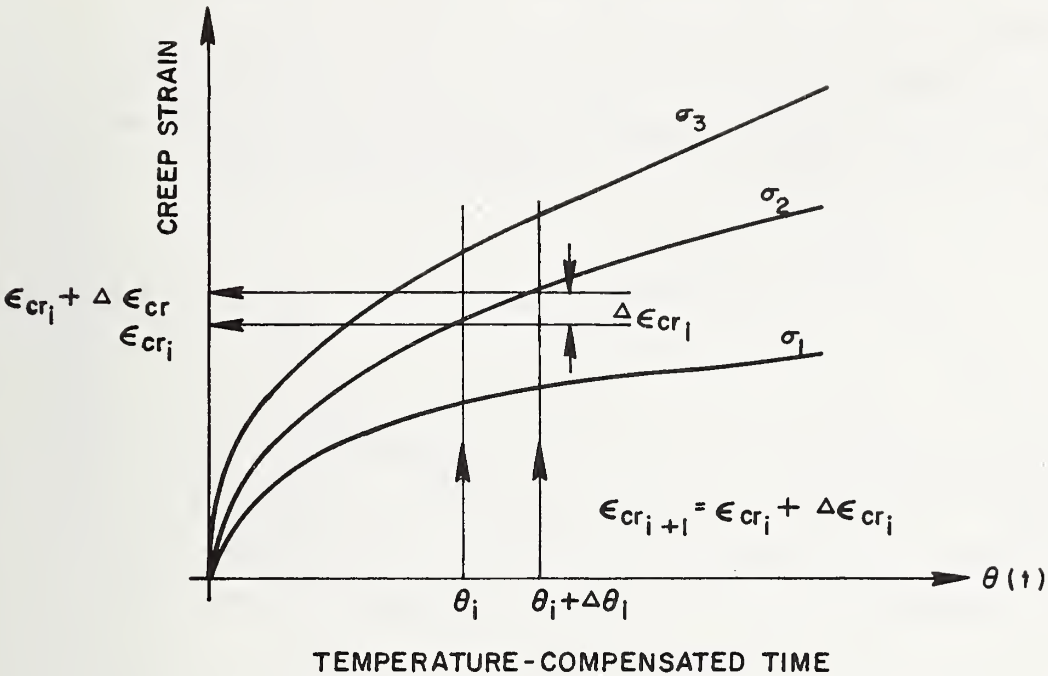


FIG. 2 - COMPUTATION OF CREEP STRAIN INCREMENT WITH TIME-HARDENING RULE

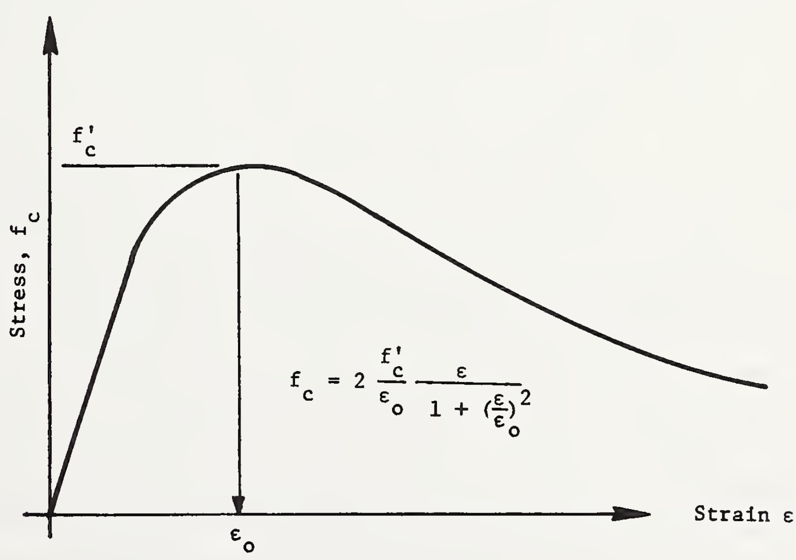


FIG. 3 - STRESS-STRAIN CURVE FOR CONCRETE

time increment at a constant stress is dependent only on the accumulated temperature-compensated time up to the start of that increment.

Finally, the stresses on the cross section are computed from the strains defined by Eq. 6, using the appropriate temperature-dependent constitutive relations for the materials. In the present study, the stress-strain curve for the concrete is given by [8]

$$f_c = \frac{2f'_c}{\varepsilon_o} \frac{\varepsilon}{1 + (\varepsilon/\varepsilon_o)^2} \tag{10}$$

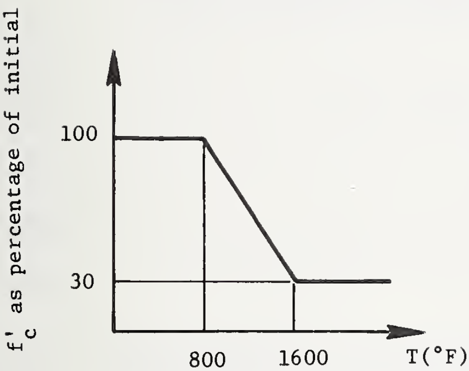
which is illustrated in Fig. 3, and in which f'_c is the compressive strength and ε_o is a constant. By taking the derivative, it may be seen that $2f'_c/\varepsilon_o = E_c$, where E_c is the initial tangent modulus of elasticity, and thus $f_c = f(f'_c, E_c)$. The idealized dependence of f'_c and E_c on temperature is illustrated in Fig. 4, where it may be seen that these properties degrade at elevated temperatures [1, 4, 5].

In similar fashion, either the common elastic-perfectly plastic curve or a strain-hardening model similar to the Ramberg-Osgood relation may be used to determine the stress-strain curve for the steel. The latter relation is given by

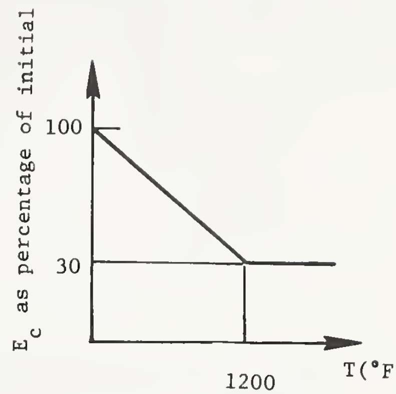
$$\begin{aligned} f_s &= E_s\varepsilon, \quad \varepsilon \le \varepsilon_p \\ f_s &= E_s\varepsilon - K\ (\varepsilon - \varepsilon_p)^n, \quad \varepsilon > \varepsilon_p \end{aligned} \tag{11}$$

and is shown in Fig. 5; K, n, and ε_p are experimental constants. Eq. 11 has the capability for modeling the "rounded" stress-strain characteristics observed in steel at elevated temperatures. The modulus E_s, and the yield strength f_y, are temperature-dependent [13, 14, 20] as shown in Fig. 6 for grade 40 reinforcement. K, n, and ε_p can be computed knowing f_y and E_s.

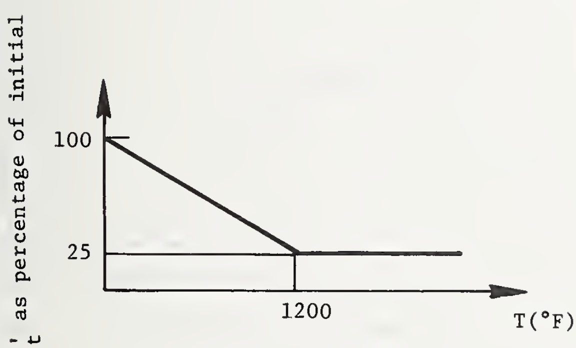
In this study, the ultimate moment capacity of the beam was given by the maximum point on the moment-curvature relationship. The corresponding compressive strain in the concrete, ε_c, was typically in the range 0.003 - 0.005 for the under-reinforced beams considered.



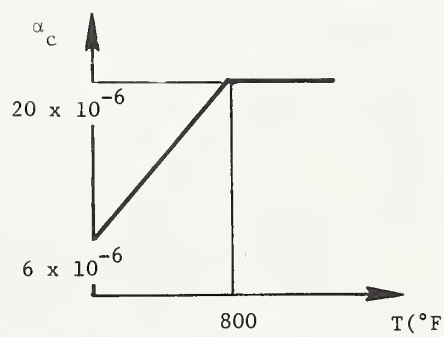
(4a) Compressive Strength



(4b) Initial Modulus



(4c) Tensile Strength



(4d) Coefficient of Thermal Expansion

FIG. 4 - NORMAL WEIGHT CONCRETE PROPERTIES AT ELEVATED TEMPERATURES

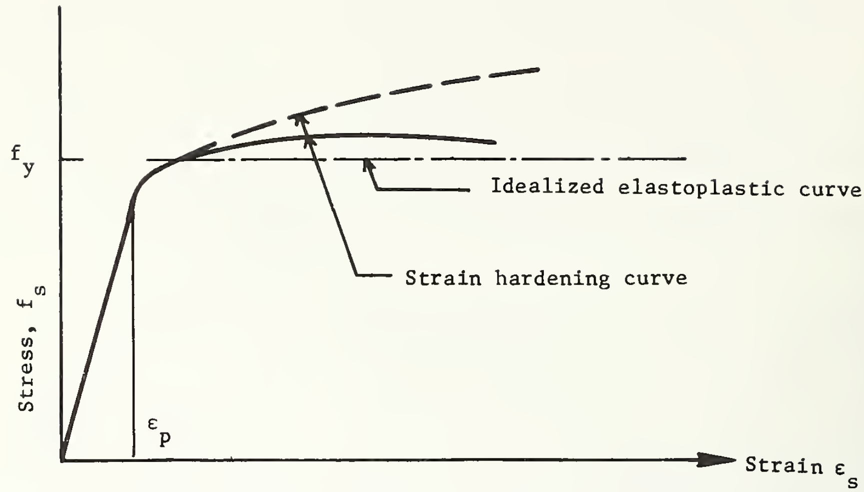
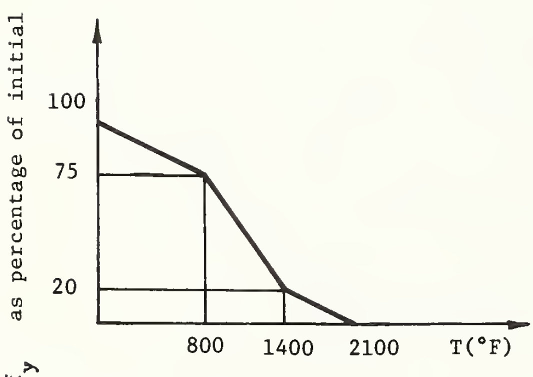
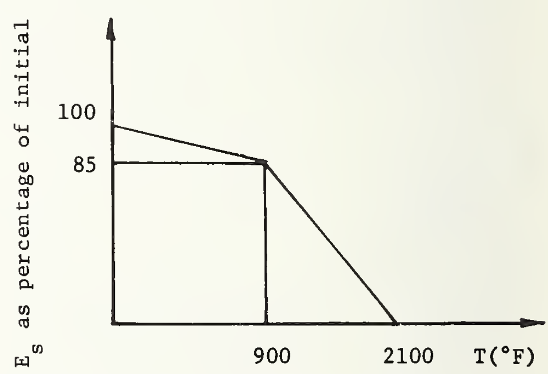


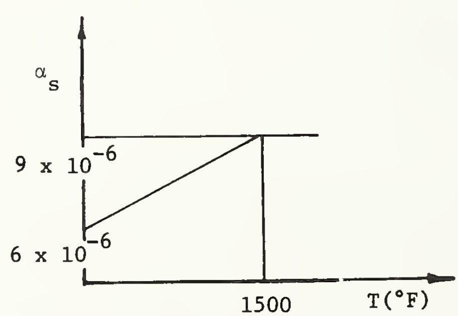
FIG. 5 - STRESS-STRAIN CURVE IDEALIZATIONS FOR REINFORCEMENT



(6a) Yield Stress



(6b) Modulus of Elasticity



(6c) Coefficient of Thermal Expansion

FIG. 6 - REINFORCING STEEL PROPERTIES AT ELEVATED TEMPERATURES

The accuracy with which the temperature history in the reinforcement can be established is especially important, since the strength of a lightly reinforced beam depends primarily on the strength of its reinforcing steel, and both the constitutive relations and predictions of creep in the steel are affected by the temperatures. In the present algorithm, the steel temperatures may be either computed or may be read independently at each calculation time stage. The latter method provides a capability for directly utilizing experimental data obtained from thermocouples attached to the steel during a fire test in making strength predictions. Indeed, when the measured temperatures are used in the strength analysis described above, the resulting predictions of behavior agree quite closely with test observations.

The temperature distribution on the beam cross section may be determined from a finite element thermal analysis in which the appropriate heat boundary conditions are specified. Since an examination of thermal analyses was considered to be outside the scope of the present study, the temperature distributions were estimated from an available two-dimensional thermal analysis program [15] which, for simplicity, treats the beam section as homogenous, and in which the steel and the heat flow along the member are not explicitly considered. Temperatures are thus determined at discrete nodal points. The temperature in a reinforcing bar is then calculated from those at the three nodes adjacent to it, weighting each nodal temperature by the area coordinate of the bar with respect to that particular node. Although area coordinates provide a natural way to weight the nodal temperatures, steel temperatures which are estimated from those in the surrounding concrete can be as much as 40 percent too high. The reasons for this discrepancy are that the reinforcement in the actual beam acts as a heat sink and longitudinal heat conductor [5], and that the moisture in the surrounding concrete condenses around the steel, providing a layer of insulation around the bar. In large reinforced sections, moreover, the heat sink caused by the steel would significantly perturb the thermal distribution in the surrounding concrete mass. Therefore, the available capacity of the beam would be severely under-estimated using the above thermal analysis without correction. This will be shown quantitatively in a subsequent example where it will also be shown that the computed and measured temperatures will agree if the calculated temperature increments are scaled by an appropriate constant. However, this scaling factor depends on beam geometry, reinforcing arrangement, and temperature history, and its empirical nature introduces additional uncertainty.

A thermal analysis has recently become available [4,5] that allows sections with several different materials and thermal properties to be modeled, and which promises to remove a substantial amount of the uncertainty in steel temperature calculation. However, the assumption therein that perfect thermal contact is maintained between two adjacent materials may not hold in the case of prestressed beams where shielded cables are used. Moreover, although the section is assumed to remain uncracked, the random flexural cracking which occurs in beams would tend to raise the temperature in the reinforcement. The importance of accurate steel temperatures in predicting beam behavior and ultimate strength therefore mandates additional study in this area.

Since the beam section is discretized during the finite element thermal analysis, the same grid system is employed in the strength analysis in the current study. In general, however, this would not be necessary. The coarseness of the mesh will decrease the accuracy of the temperature estimates as well as the calculated stress distribution and moment capacity. Discretization problems were outside the scope of the present study, and have not been explored in detail. The selection of an appropriate mesh relies to a great extent on engineering judgment. A finer mesh should be selected if the thermal or mechanical strain gradients are expected to be sharp. For example, if the neutral axis falls within the top two element rows of a discretized beam section, the estimate of the concrete compressive stress block will be very crude since the stress is assumed to be constant over any element.

Similarly, the temperature-time history must be discretized for both thermal and strength calculations. The choice of too large a time step will cause an error in the estimates of thermal and creep strains, particularly at higher temperatures later in the test. For the case studies discussed in the sequel, a 10 minute interval has been used with success.

3.0 MODELING UNCERTAINTIES

Since the behavior of reinforced concrete beams subjected to fire load is a complex phenomenon, it is essential to identify potential sources of solution error that may arise as a result of the mathematical modeling. These sources would include the creep model, thermal strain analysis, and the stress-strain curve idealization, including the rate of degradation of material properties with increasing temperature. While the models employed herein are felt to be suitable for this study, the sensitivity analysis must be sufficiently detailed that the results obtained can be realistically interpreted. Although the ultimate moment capacity of an under-reinforced simply supported concrete beam may not be particularly sensitive to errors in creep or thermal strains, this is not true for its curvature (deflection) at failure. Furthermore, if the beam is partially restrained, these factors may be important. Therefore, these limitations and sources of uncertainty in the models and their parameters are discussed in the following paragraphs.

3.1 Creep Model

The creep model, Eq. 8, perhaps constitutes the largest single source of uncertainty [6,7,12,17] and warrants extended attention. All creep analyses [17] attempt to relate the creep strain ε_{cr}, its time rate $d\varepsilon_{cr}/dt$, applied stress σ, temperature T and time t in some parametric equation, i.e.,

$$F_1(\varepsilon_{cr}, d\varepsilon_{cr}/d\theta, \sigma) = 0 \qquad (12a)$$

and when time and temperature are combined into one normalizing parameter θ, the temperature-compensated time of Equation 9, this becomes

$$F_2\ (\varepsilon_{cr},\ d\varepsilon_{cr}/d\theta, \sigma) = 0\ . \qquad (12b)$$

In addition to the state variables, these functions contain certain empirical constants, which must be determined experimentally.

The determination of these experimental constants constitutes a significant part of the uncertainty in the creep analysis. For example, the experimental parameter ε_{to} in Eq. 8 determines the amount of primary creep that occurs. However, since the test specimens are loaded and heated to test stress and temperature over a finite time, it may be difficult to differentiate between instantaneous recoverable inelastic deformation in the specimen, and primary creep deformation, which is irrecoverable [7,17]. Moreover, it should be expected that ε_{to} would depend not only on σ but on $d\sigma/dt$ as well, since the anelastic strain is dependent on the loading rate. Thus, it should not be too surprising that ε_{to} has been observed to be a poorly reproducible quantity which exhibits considerable scatter [12,13]. The parameter Z in Eq. 8 describes the steady-state secondary stage creep rate. Inasmuch as the most reliable and reproducible test results have been generated for the steady state portion of the creep curves more confidence may be placed in values of Z reported in the literature than in values of ε_{to}. Under certain service conditions, however, the material may become structurally unstable, in which case Z would be a function of temperature as well as stress. Moreover, the hypothesis of functional similarity underlying Eq. 8 may not be valid for a broad range of stress in all materials [17].

The temperature-compensated time parameter $\theta(t)$ described in Eq. 9 has some theoretical basis in statistical mechanics. The key parameter in evaluating $\theta(t)$ is $\beta = \Delta H/R$ which is experimentally determined and is commonly assumed to be constant at all temperatures. Although ΔH has been shown to be insensitive to material structure for pure metals at temperatures exceeding one-half the melting point [9], the situation for alloys is more complicated [17]. In general, ΔH is dependent on the structural state of the material and its prior deformation history; if a phase change in the metal occurs due to elevated temperature, ΔH would be expected to change accordingly. It seems apparent that β should actually vary over the range of temperatures encountered during the fire test, and indeed there is some evidence [17] that $d\beta/dT > 0$. Unfortunately, there is insufficient data at present to estimate this functional relationship.

An appreciation of the numerical effect of variations in the creep parameters on the predicted creep strain may be gained by examining the total differential of Eq. 8b, i.e.,

$$d\varepsilon_{cr} = (\coth \varepsilon_{cr}/\varepsilon_{to} - \mathrm{csch}^2\, \varepsilon_{cr}/\varepsilon_{to})\, d\varepsilon_{to}$$

$$+ \coth^2 \varepsilon_{cr}/\varepsilon_{to}\, (\theta dZ + Z d\theta) \qquad (13a)$$

or

$$d\varepsilon_{cr} = C_1\, d\varepsilon_{to} + C_2\, dZ + C_3 d\theta. \qquad (13b)$$

We might observe that the coefficient C_1 is bounded by $0 < C_1 < 1$, and thus a variation in ε_{to} will cause a change of the same magnitude in ε_{cr}. The effect of variations in Z and θ is not as clear, as Z ranges from 10^{14} to 10^{25}/hr [13] in Grade 40 reinforcement while θ varies from 0 to 10^{-18}hr during a four hour duration of a standard ASTM fire test. If Z errs by an order of magnitude, then $d\varepsilon_{cr}$ will err by a similar amount, as shown by the second term in Eq. 13b. In spite of the general predictability of the secondary creep stage, existing data indicates that this error is a distinct possibility at certain stress levels [13,18].

Decomposing the third term in Eq. 13b, we observe that if T is essentially constant,

$$\frac{d\theta}{\theta} \approx -\frac{\beta}{T}\frac{(d\beta)}{\beta} \tag{14a}$$

For the steels and temperatures of interest in the current study, $\beta \approx 70000$ [13] and $T = 1000°F$ (538°C); thus $\beta/T \approx 70$, and

$$\frac{d\theta}{\theta} \approx -70\frac{d\beta}{\beta} \tag{14b}$$

It may be concluded that an error of 10 percent in the estimate of β will result in a 700 percent error in the temperature-compensated time increment which, in turn, may cause an error of an order of magnitude in the estimate of the creep strain increment according to Eq. 13. In connection with this sensitivity and the temperature dependency noted earlier, it is important to note that the experimental values of β are usually obtained over a fairly narrow temperature range, say 800°F - 1100°F (427°C - 593°C), while the temperature in the steel reinforcement may range over 1000°F (538°C) during a fire test. In some instances, the uncertainty in β may tend to limit the usefulness of $\dot{\theta}(t)$ as a temperature-time normalizing parameter.

There are additional limitations and restrictions in the use of Eq. 12 itself in predicting creep behavior under non-steady stress and temperature conditions. For example, although it is tacitly assumed in developing Eq. 12 that the stress remains constant during the entire creep test it is actually the load that remains constant. The continual reduction in area during testing due to accumulations of plastic deformation causes the stress to increase. This seemingly fine point requires that the usefulness of Eqs. 12 in predicting creep under service loads be restricted to small deformations where creep strains do not exceed a few percent.

When the stress is strongly time-dependent during service, one must assume that the creep strain increment is controlled by a time-hardening or strain-hardening mechanism or a combination of the two to predict creep deformation under these conditions [16]. This requires rather broad assumptions regarding the temperature-dependent viscoelastic properties of the material. The strain-hardening mechanism implies that the material is structurally stable and the creep increment depends on the amount of prior deformation; satisfactory agreement with experiment has been found under conditions of variable stress but essentially constant temperature. The time-hardening mechanism is analogous to the flow of a a nonlinear viscous fluid with time-dependent viscosity, and seems appropriate at elevated temperatures when the stress levels change only slightly. It should be realized that in a reinforced concrete beam subjected to fire, the actual state is somewhere in between these two extremes. In any event, the assumption that the stress and temperature are constant for a particular creep curve, upon which basis creep increments are calculated for a variable stress and temperatures, ignores the finite loading and heating times necessary to conduct the tests from which such curves are derived.

In sum it is apparent that additional experimental creep data is required, especially with regard to ε_{to} and β, before the same degree of confidence can be placed in the creep analysis as in the remaining portions of the methodology.

3.2 Analysis of Thermal Strains

Thermal strains are analyzed according to Eq. 7 for both concrete and steel. The principal source of uncertainty arises from the expression for the temperature-dependent thermal expansion $\alpha(T)$. The scatter in $\alpha(T)$ is considerable for concrete [5], and somewhat less for steel [20]. Typically, $\alpha(T)$ increases approximately linearly up to some T_m beyond which it is assumed to remain constant;

$$\alpha\,(T) = [\alpha_o + \frac{d\alpha}{dT}\,(T - T_o)], \quad T \leq T_m \tag{15}$$

$$= \alpha_m\,, \quad T > T_m$$

where α_o is typically about $6 \times 10^{-6}/°F$ ($3.3 \times 10^{-6}/C°$). For concrete, $T_m = 800°F$ ($427°C$) and at this temperature, $8 \times 10^{-6}/°F < \alpha_m < 22 \times 10^{-6}/°F$ ($4.4 \times 10^{-6}/°C < \alpha_m < 12.2 \times 10^{-6}/°C$).

Using Eqs. 7 and 15, this variation in thermal expansion would result in a difference in computed thermal strains of 0.005 and 0.0106 in those elements having temperatures of 800°F (427°C) and 1200°F (629°C) respectively. For steels, at T_m = 1500°F (815°C), $9 \times 10^{-6}/°F < \alpha_m < 12 \times 10^{-6}°F$ (or $5 \times 10^{-6}/°C < \alpha_m < 6.7 \times 10^{-6}/°C$), implying a difference of 0.0013 in computed thermal strain when the reinforcement is at 1200°F (649°C). However, the effect of these variations on the ultimate (reserve) moment capacity of the beams under the fire load considered in this study was found to be slight, since the steel was observed to have yielded well before the beam capacity was reached.

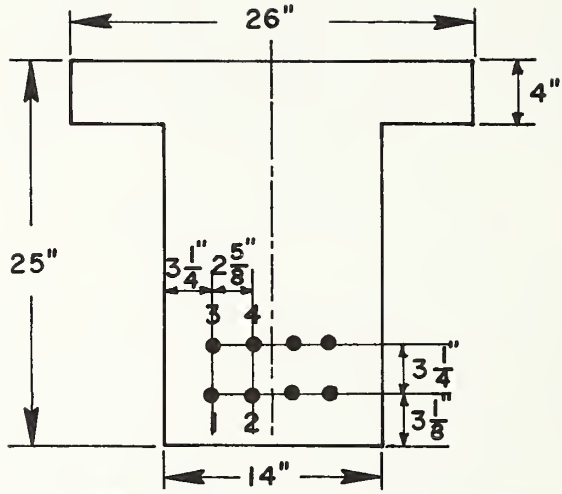
3.3 Constitutive Relations

The reinforcement stress-strain curve idealization and the modeling of the temperature-dependent reinforcement strength degradation have a significant effect on the predicted moment capacity of the reinforced concrete beam, as will be shown subsequently.

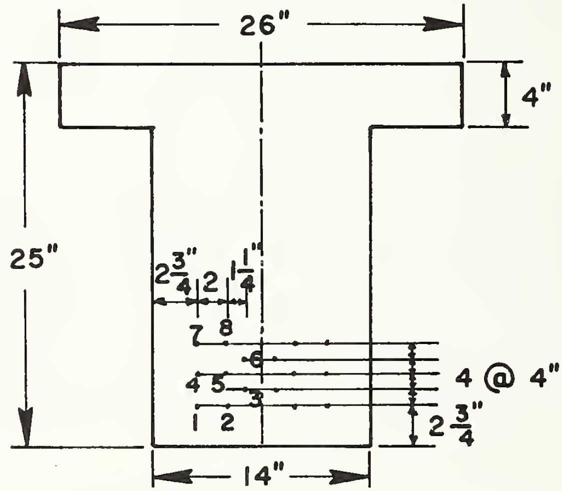
In particular, an examination of available data on the degradation of yield stress with temperature [5,13,14,20] revealed that the scatter in reinforcement yield stress increased with temperature. At 800°F (427°C), for example, yield stresses reported for intermediate grade steels ranged from 45 percent to 85 percent of their room temperature values. In turn, this implies that the unpredictability in moment capacity will also increase with temperature. On the other hand, the form of the concrete stress-strain curve and the temperature-dependent material properties of the concrete were not found to be especially influential for the lightly reinforced beams considered in this study, although they would be expected to be important in reinforced concrete members subjected to large axial thrusts. Moreover, although the small tension load-carrying capacity of concrete is included in the numerical analysis, its effect on the load-deformation behavior of the beams is negligible, except at extremely small loads.

4.0 CASE STUDIES

A lightly reinforced T-beam section fabricated with normal weight concrete, shown in Fig. 7, was chosen for the case studies herein. A series of simply supported beams spanning 40 ft (12.2 m) with similar cross sections were tested by Portland Cement Association



(7a) - REINFORCING CONFIGURATION



(7b) - PRESTRESSING CONFIGURATION

FIG. 7 - BEAM CROSS SECTIONS ANALYZED

(PCA) using the standard ASTM E-119 [3] fire. In this test series, the grade of reinforcement, concrete strength, and the effect of prestressing compared to reinforcing were considered. The usefulness of the present analysis will be demonstrated by its ability to predict the behavior of the above beams under fire test. It might be emphasized that the effect of end restraint on beam strength was not considered in this study. Restricting the scope to determining the load-carrying capacity of a cross-section of course limits the applicability of the analysis to simply supported members, where the section investigated is that having the maximum applied moment.

4.1 T-Beam with Grade 40 Reinforcement

In the first illustration using the section shown in Fig. 7a, the reinforcement consists of 8 No. 10 Grade 40 deformed bars with a nominal yield strength of 40 ksi (276 MN/m^2). The reinforcement ratio of $\rho_w - \rho_f = 0.0086$ is considerably less than the maximum allowable [2] value of $0.75\rho_b' = 0.055$. The corresponding beam tested by PCA [11] sustained a test moment of 4770 in-kips (0.539 MN-m) for 310 minutes, at which time the test furnace control failed; the projected beam endurance was about 6 hours. The test moment was 54 percent of the ultimate beam capacity at room temperature. At the time of test, the concrete compressive strength was $f_c' = 7230$ psi (49.9 MN/m^2), while the actual steel yield strength was $f_y = 46$ ksi (317 MN/m^2).

The temperature distribution on the cross section was determined as a function of time from a thermal analysis provided by Issen [15]. The cross section was discretized in 1-inch (2.54 cm) squares, and the strains and stresses on the cross section, as well as its load carrying capacity, were calculated at 10 minute time increments, beginning with a temperature of 70°F (21°C) at time zero, until the ultimate capacity fell below the sustained load moment. (This discretization was used for all case studies considered herein.) About 1 1/2 minutes was needed to perform the thermal analysis on an UNIVAC 1108 (Exec 8) System, followed by 1 1/2 minutes to perform the strength calculations for the entire fire test of this beam. Beam symmetry allowed consideration of only one-half the cross section; this is reflected in the value of moment in the strength-time curves which follow.

Fig. 8 describes the temperature history in the reinforcement which is computed directly from the temperatures of the surrounding concrete mass. As might be expected, the calculated bar temperatures are dependent upon the amount of concrete cover provided, reaching about 1400°F (760°C) in bar 1 after 300 minutes and about 750°F (399°C) in bar 4 which is better protected. The actual temperatures in the reinforcement at midspan which were monitored with thermocouples for the duration of this test are shown in Fig. 9 (reproduced from Ref. 11) for comparison. The computed steel temperatures are considerably higher than the actual values for reasons discussed earlier. After 5 hours of test, for example, the measured temperatures range from 70 percent to 85 percent of those calculated, with the lower percentages for the hotter bars. Moreover, the measured temperatures tend to increase more rapidly than predicted during the early stages of the fire.

To compensate for this discrepency in a reasonably simple manner, an empirical scaling factor is introduced by which the increments in calculated steel temperatures are multiplied so that, on the average, the computed and measured values agree. In the present analysis, this factor is taken as a constant for simplicity. However, it is clear from inspecting Fig. 8 and 9 that the scaling factor is dependent on the amount of bar cover and elapsed time and, moreover, would be expected to be influenced by the type of reinforcement and concrete moisture content as well.

The effect of the method selected to determine the steel temperature history on the estimated beam strength is shown in Fig. 10, where the degradation in ultimate moment capacity resulting from progressive material deterioration is illustrated. In these and all subsequent calculations, the coefficient of thermal expansion in Fig. 6c was used to calculate the thermal strains, while in computing the creep, the following experimental [13] parameters were substituted in Eqs. 8 and 9:

$$\Delta H/R = 75000$$

$$\varepsilon_{to} = 1.7 \times 10^{-10} \, (f_s)^{1.75}$$

$$Z = \begin{cases} 0.026 \, f_s^{\,4.7} \, , & f_s < 15000 \\ 1.23 \times 10^{-16} \exp\,[.0003 \, f_s], & f_s > 15000 \end{cases}$$

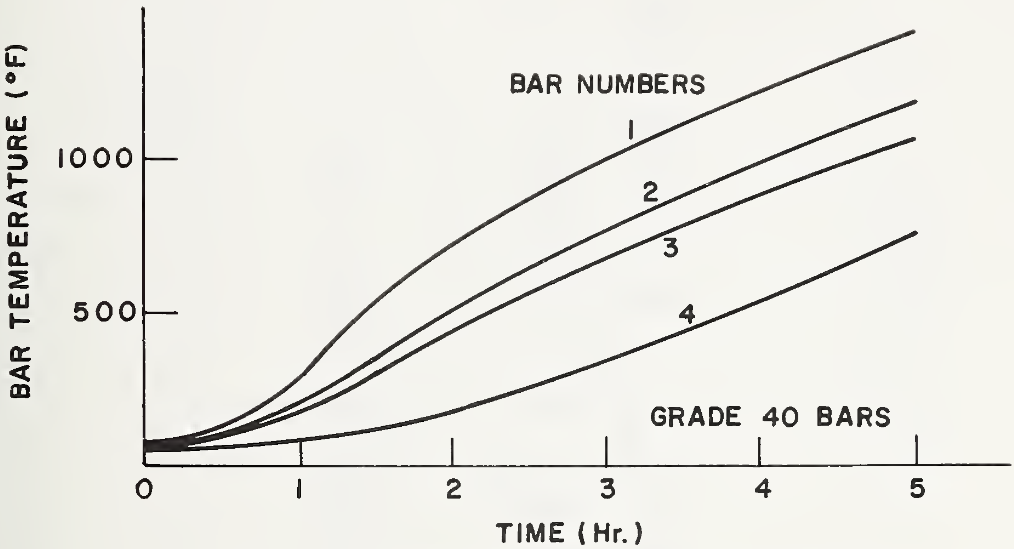


FIG. 8 - COMPUTED TEMPERATURES IN REINFORCEMENT

(Cross section shown in Fig. 7a)

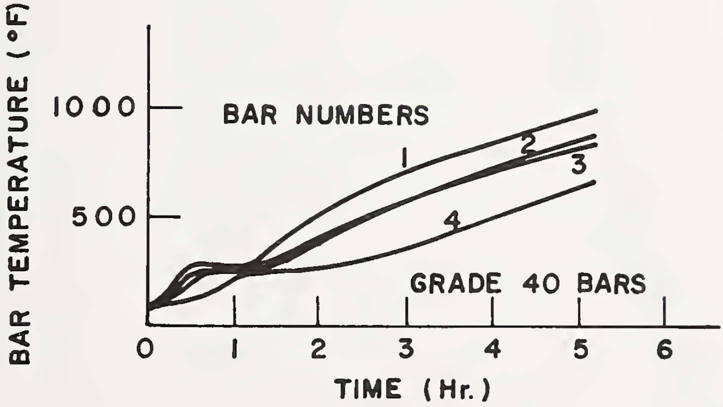


FIG. 9 - MEASURED TEMPERATURES IN REINFORCEMENT

(Cross section shown in Fig. 7a)

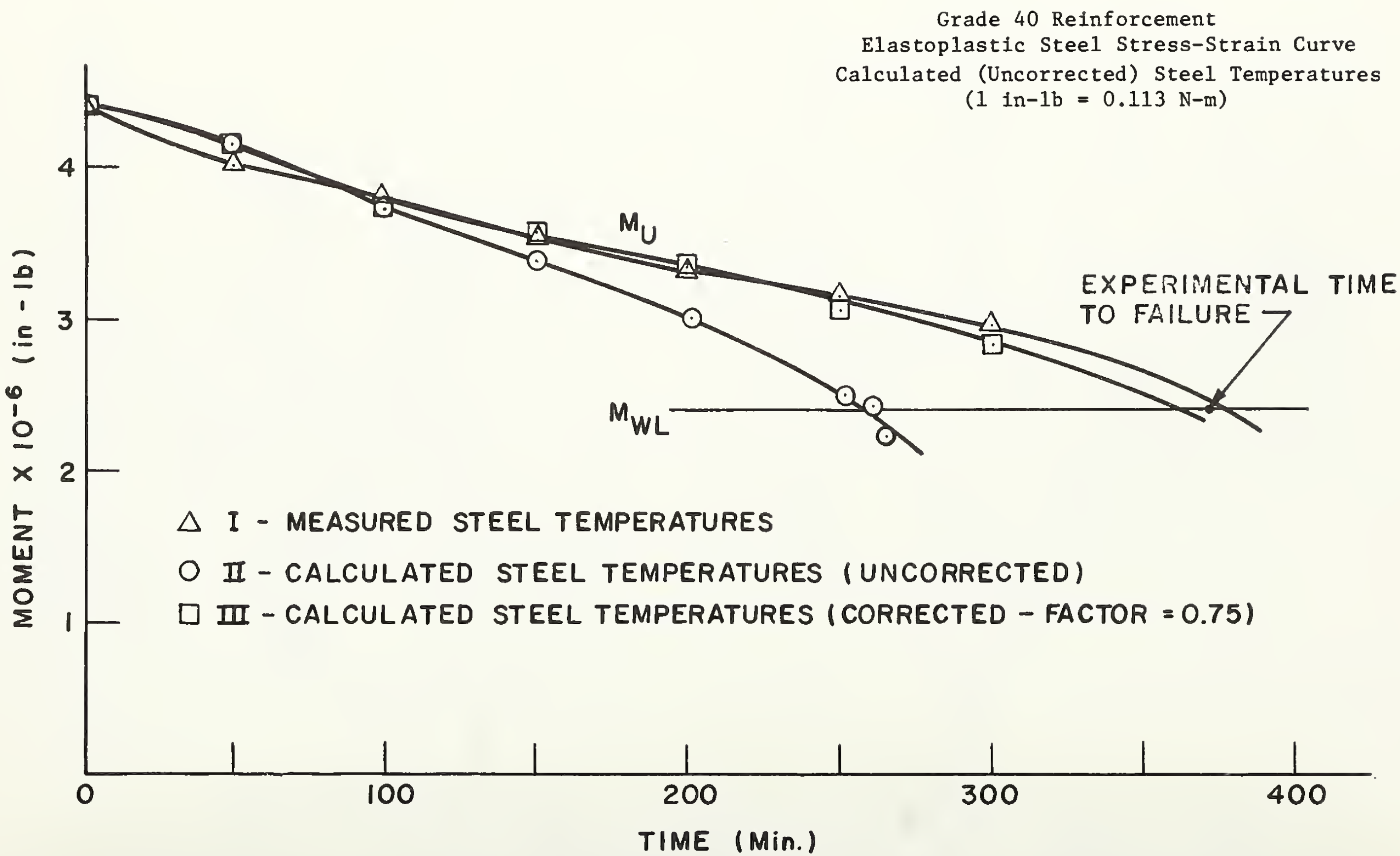


FIG. 10 - EFFECT OF STEEL TEMPERATURE COMPUTATION ON BEAM ENDURANCE

Using the measured temperatures (Fig. 9) and an elastoplastic stress-strain curve with the reported [11] yield stress of f_y = 46 ksi (317 MN/m^2) the extrapolated duration predicted for the beam is about 375 minutes (curve I) which agrees quite closely with the 6 hour duration projected in Ref. 11. When the uncorrected calculated steel temperature history of Fig. 8 is used instead, but all other parameters remain fixed, the predicted beam endurance (curve II) becomes about 270 minutes. An error of roughly 40 percent in estimating temperature thus causes an error of 30 percent in the predicted test duration for this beam. Clearly, a failure to determine the temperature history in the reinforcement accurately may limit the usefulness of the analysis in interpreting experimental data.

A notable improvement in predicting beam behavior is obtained when an empirical scaling factor of 0.75 is applied to the calculated steel temperature, as shown by curve III in Fig. 10. In spite of some local irregularities, the general agreement between curves I and III is quite close, indicating that judicious use of the scaling factor can yield reasonable predictions when the actual temperature data is unavailable or would be difficult to obtain. Although the factor is 0.75 for this particular beam, additional studies would be required before this result can be generalized to other beam geometries and reinforcement arrangements. It might be emphasized, however, that since the error induced by using the uncorrected (scaling factor of unity) calculated steel temperature is in the conservative direction, the resulting predictions would still be useful for purposes of design and for parametric sensitivity studies. In the absence of any experimental data from which the scaling factor could be deduced, a value of unity should be assigned.

The steel stress-strain curve idealization also has an important effect in the predicted reserve capacity above working load, $M_u - M_{wl}$. The effect of the elastoplastic and strain hardening stress-strain models on predicted strength is compared in Fig. 11 by curves I and II, where the steel yield stress has been chosen at its nominal value of 40 ksi (276 MN/m^2) and the uncorrected calculated temperatures in Fig. 8 have been employed. Observe from curves I and II that while the reserve capacities of the beam prior to failure may differ considerably, the durations range from 240 to 295 minutes. Considering the uncertainties involved, this range should not be too surprising. It might also be noted that the elastoplastic model gives reasonable but conservative estimates, and is much easier to apply in design and parametric sensitivity studies, since the family of stress-strain-temperature curves can be completely specified by two rather than four temperature-dependent material constants.

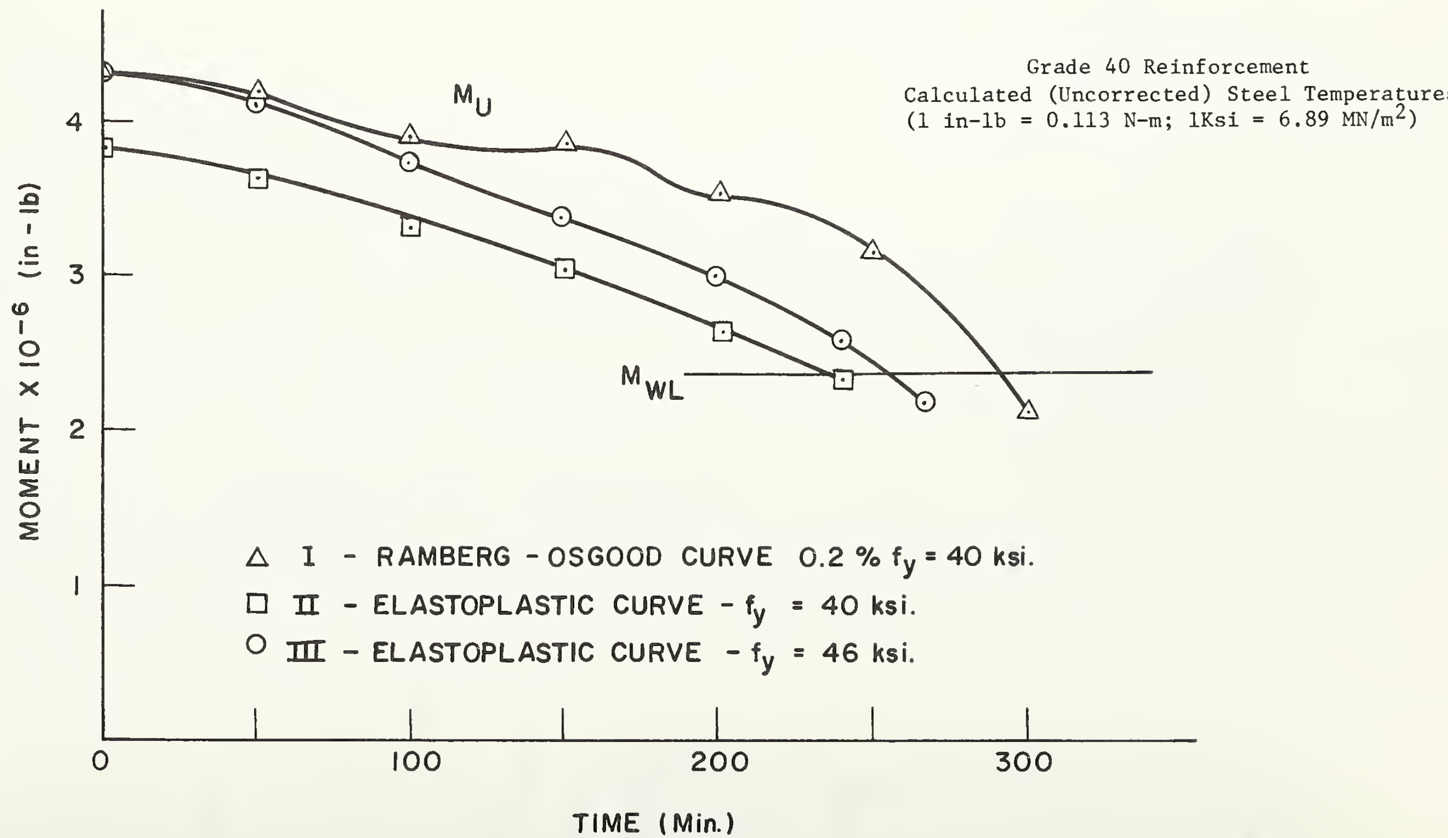


FIG. 11 - EFFECT OF STEEL STRESS-STRAIN IDEALIZATION ON BEAM ENDURANCE

The slight irregularity in the ultimate moment capacity computed with the strain hardening stress-strain curve occurring at 150-160 minutes is caused by slight error in the definition of experimental constants K(T) and n(T) in the vicinity of its slope transition at 800°F (427°C). Although this can easily be removed by recalculating the constant and/or using more points to define n and K vs. T, it provides another illustration of the sensitivity of the ultimate moment calculations to stress-strain curve modeling. The coincidence of curves I and III in Fig. 11 at room temperature (t = 0) is fortuitous, and results from the strain hardening modeled by the reinforcement constitutive relation which was used to obtain curve I. Thus, while the nominal yield stress is 40 ksi (276 MN/m^2), the actual reinforcement stresses at ultimate depend on the reinforcement strains. Here, the reinforcement ratio was such that these strains were 0.0206 in/in and 0.025 in/in, with corresponding stresses of 45.5 and 46.1 ksi (314 and 318 MN/m^2) in the two layers of reinforcement (see Fig. 7); hence, the equality of curves I and III at t = 0.

The strength of a lightly-reinforced beam is governed primarily by the yield stress of the reinforcement. The consequence of using a nominal design value for f_y instead of the actual value is illustrated by comparing curves II and III in Fig. 11 in which f_y = 40 ksi (276 MN/m^2), and f_y = 46 ksi (317 MN/m^2), respectively. It is well known that f_y exhibits some scatter and that nominal design values are chosen accordingly so that the likelihood of actually encountering a strength less than nominal is quite small. In statistical terms, if the mean of f_y is 46 ksi (317 MN/m^2), the nominal value might correspond approximately to the 5 percentile value of its probability distribution. This inherent variability would be expected to limit the degree of test reproducibility observed from a series of nominally identical beams tested under controlled conditions.

Uncertainty in the yield stress is not the only determinant of observed scatter in experimental or field data, however. The depth to the centroid of the steel reinforcement, d, not only influences the moment capacity but also indirectly controls the temperature elevation in the bars for a beam with fixed geometry, since larger depths would mean less concrete bar cover. This may be seen in Fig. 12, where the predictions for the PCA [11] beam (curve III of Fig. 10), which had a minimum concrete cover of 2 1/2 in (6.35 cm), are compared to those for beams with 5 3/8 in (13.7 cm) and with 1 in (2.54 cm) cover. As would be expected, the beam with the maximum d (minimum cover) yields the largest margin of reserve capacity at room temperature. Because of the limited fire protection provided,

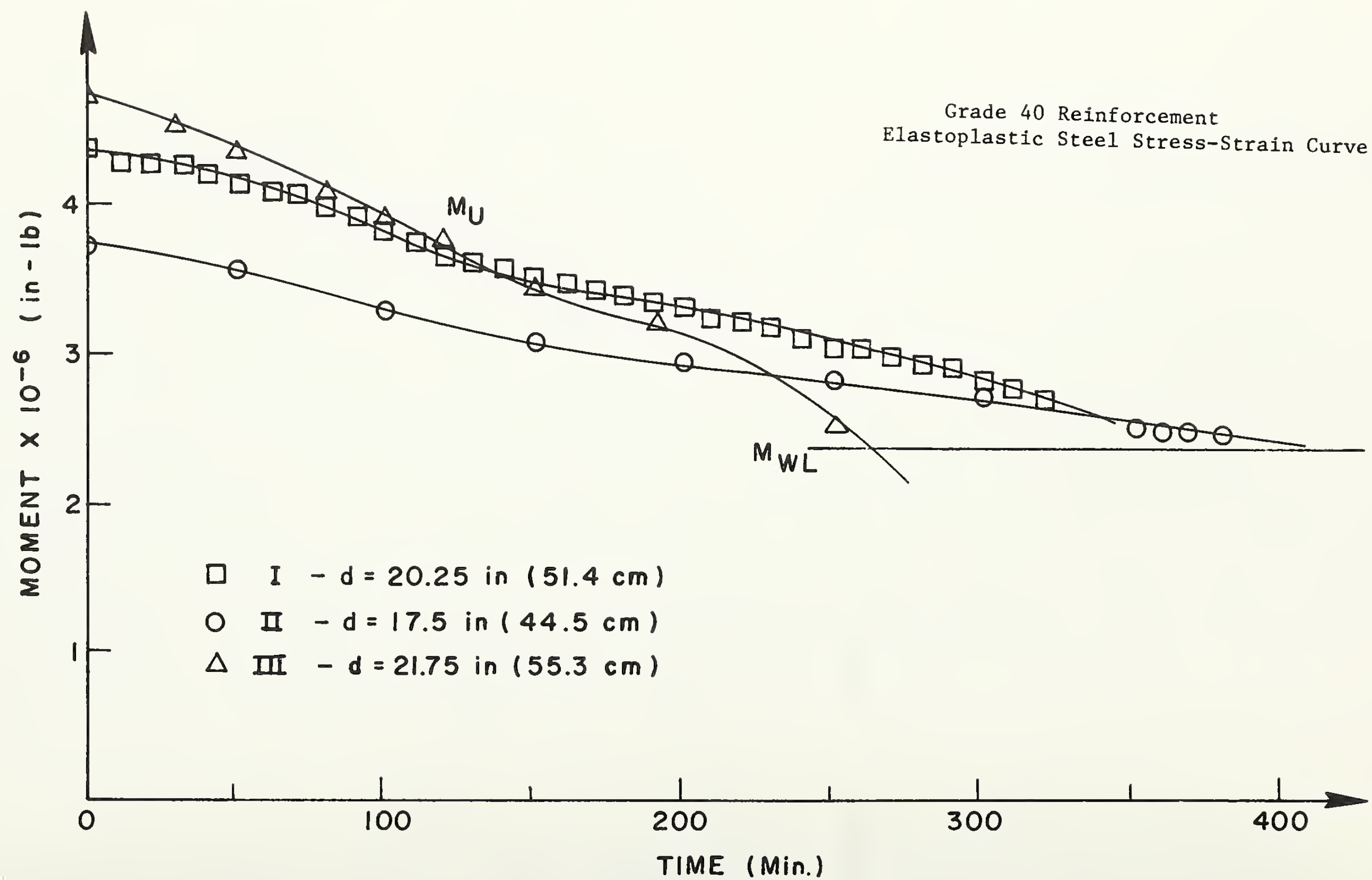
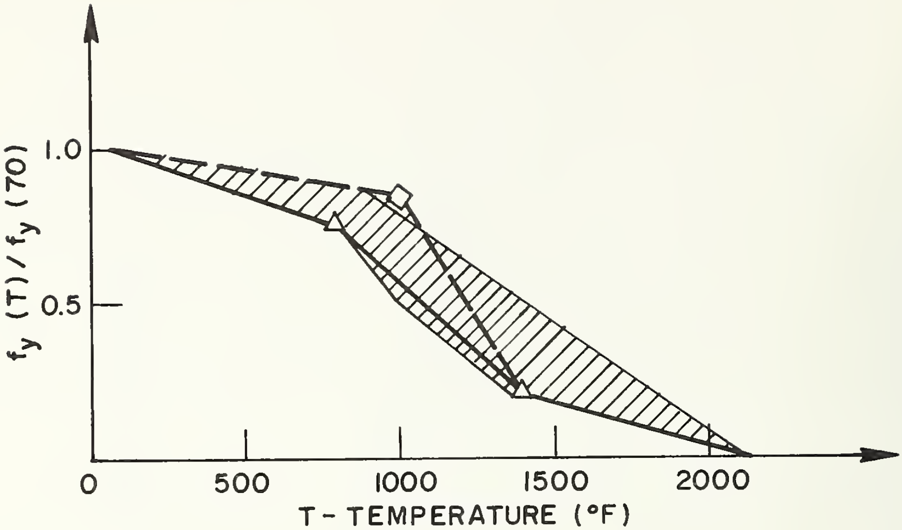


FIG. 12 - EFFECT OF BAR COVER ON BEAM ENDURANCE

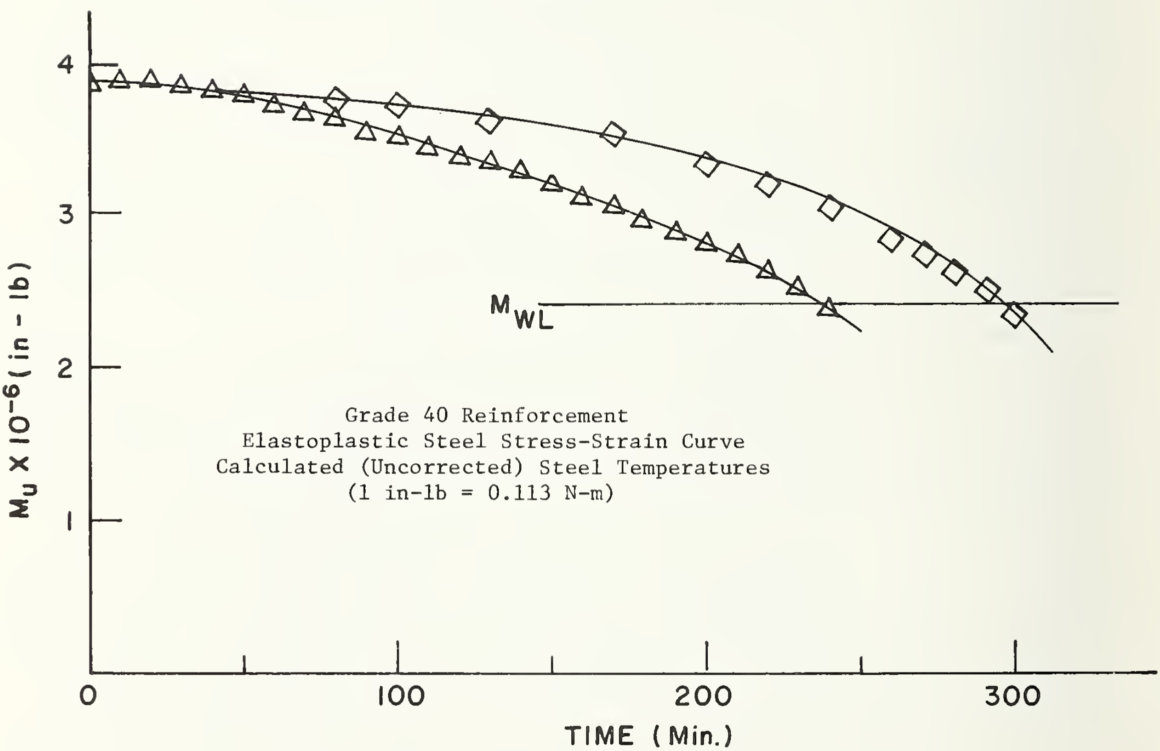
however, the bars also heat more rapidly and to higher temperatures with the consequence that after 2 1/2 hours its reserve strength is reduced to that of the beam with 2 1/2 in (6.35 cm) cover. Conversely, the predicted response of the beam with 5 3/8 in (13.7 cm) cover was most favorable in the long run because of its relatively lower steel temperatures. While the uncertainty in steel yield stress is in large part unavoidable due to the inherent variability of the material, the uncertainty in bar placement arises from workmanship and, to a certain extent, can be controlled by good construction practice. The potential lack of cover that would result from the tendency for d to exceed its design value due to construction loads might adversely affect the response of the beam to fire in the long term.

Families of curves such as those in Figs. 10, 11, and 12 may be used not only for planning and interpreting experiments but also to construct design aids to be used in dimensioning fire-resisting structural members. For example, one such requirement might be that the beam still carry 25 percent in excess of its service live load after 3 hours of fire; satisfying this criteria would entail the selection of a certain amount of concrete cover in conjunction with a given reinforcement ratio and yield strength. However, a non-dimensional analysis does not appear possible since the thermal analysis is geometry-dependent, and on each of a family of cross sections deemed to be representative of most design cases it will be necessary to first perform a thermal and then a stress analysis prior to establishing the design aids.

The predictions of moment capacity are also sensitive to the rate of steel strength degradation. In Fig. 13a, the scatterband for the temperature-dependent degradation in yield strength is shown with two piecewise-linear curves that might reasonably be chosen to model this behavior. For illustrative purposes, the uncorrected calculated steel temperatures were used. The differences in predicted ultimate moment capacity using these two models along with the elastoplastic stress-strain curve for the steel is apparent from Fig. 13b, where it is observed that yield strength degradation to 85 percent at 1000°F (538°C) instead of 75 percent at 800°F (427°C) results in approximately one extra hour of beam endurance. Figs. 10 through 13 emphasize the need for reasonably precise knowledge of the relations for the reinforcement if credible analytical predictions of beam behavior are to be made.



(13a) Temperature-Dependent Degradation in Steel Yield Strength



(13b) Degradation in Moment Capacity of Beam

FIG. 13 - EFFECT OF STEEL STRENGTH DEGRADATION ON BEAM ENDURANCE

The behavior of the thermal and creep strains in the steel is described in Fig. 14, using the strain hardening idealization along with the (uncorrected) calculated bar temperatures. These strains were employed in the analysis used to obtain curve I in Fig. 11. The selection of a scaling factor of unity serves to accentuate certain effects of thermal and creep strains in the reinforcement on structural response that will be discussed. The thermal strains increase quite regularly, in accordance with the temperature increase in the bars. During the first portion of the test, the creep strains are essentially zero, but after the bar temperatures exceed approximately 800°F (538°C) the creep strains exceed 0.02. However, the creep analysis is only valid for small strains and, moreover, the temperature in bar 1 exceeds 1300°F (705°C) after about 260 minutes of test, implying that $\Delta H/R = 75000$ may no longer be appropriate. Therefore, the validity of the creep calculations beyond this stage is somewhat uncertain. While these factors will not significantly affect the calculation of ultimate strength since the beam is under-reinforced, they will cause the deflection of the beam at working load to be overestimated. It might be noted that the creep behavior shown in Fig. 14, where a rapid strain increase follows a long period of very slow accumulation, has been observed in other experiments (viz. Fig. 2, Ref. 12).

Anomalies in creep strains are clearly reflected in the calculated values of working load stress in the reinforcement; these in turn, determine the creep increments in the subsequent time interval. Therefore, any error in estimating creep strain or working load stress will tend to compound with time. The working load stresses in the steel are clearly nonconstant with time, as shown in Fig. 15. Inasmuch as bar 1 is hottest, its strength properties degrade most rapidly, its stress at working load decreases and the additional load is picked up by bars 2, 3, and 4. Later, at about 220 minutes, bar 2 and 3 also become quite hot and began shedding their load, with the net result that bar 4, which remains relatively cool, is loaded into the strain-hardening range. A slight instability in the stress-time curves has been observed to occur in a number of instances when the creep strain in one of the bars exceeds about 0.025, and shows up here beginning at about 240 minutes. This is typically accompanied by a sudden drop in the mechanical stress in the corresponding bar. The exact cause for this behavior must be determined from additional studies.

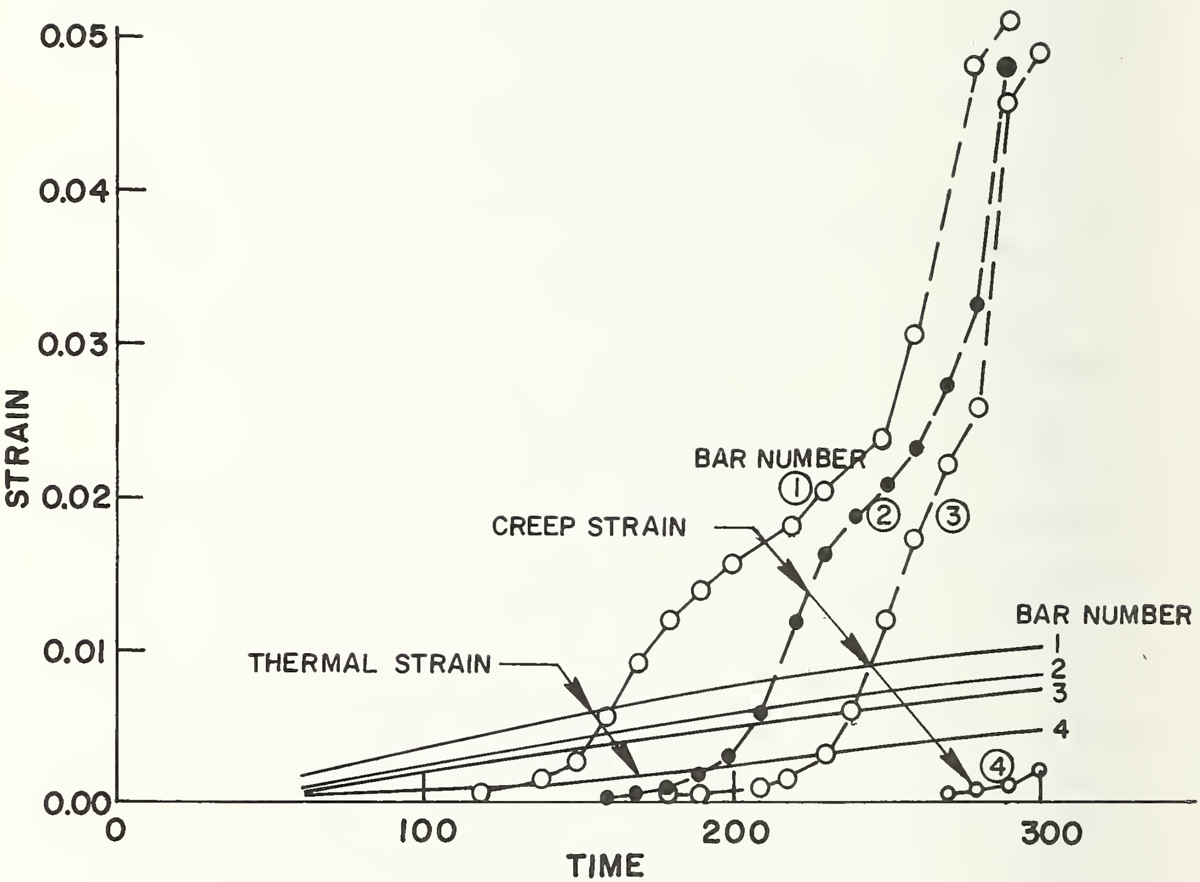


FIG. 14 - CREEP STRAINS AND THERMAL STRAINS IN REINFORCEMENT

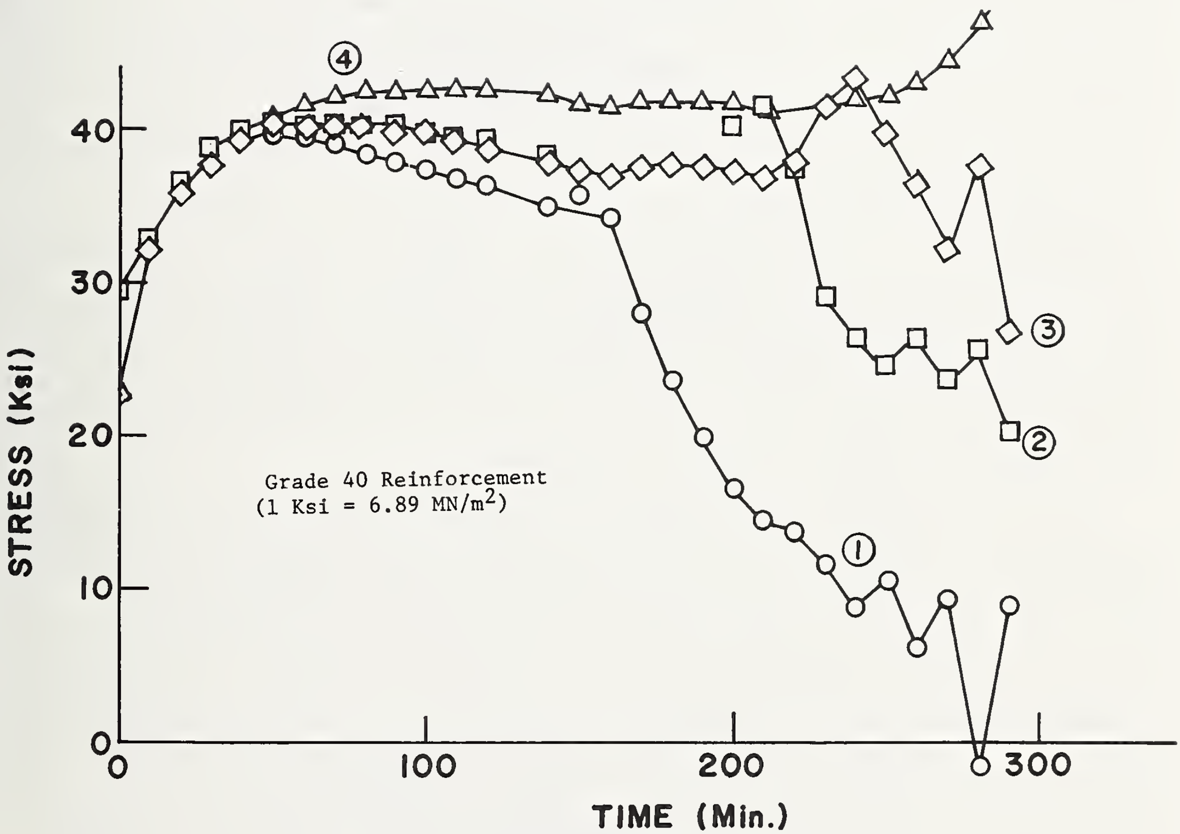


FIG. 15 - STRESSES IN REINFORCEMENT AT WORKING LOAD

It should be remarked here that when the corrected calculated steel temperatures (or measured temperatures) are used, the predicted creep behavior is much less dramatic, the creep strains at failure being on the order of 0.01 - 0.02. Nevertheless, it is necessary to have some feeling for the consequences of high steel temperatures and creep in terms of the analysis, since these factors cannot be precluded in all beam configurations that might be of interest.

The thermal expansion of the concrete was ignored in some previous [11] strength calculations, where it was argued that since the temperatures in the concrete compressive zone were much lower than the tension reinforcement, the concrete thermal strains would be unimportant. The effect of ignoring ε_{cth} in computing the beam capacity in this case study is shown in Fig. 16. It is apparent that when the elastoplastic stress-strain idealization is used, ε_{cth} has very little influence on the calculated moment capacity, while the effect is somewhat more pronounced with the strain hardening model. However, the predicted beam endurance is the same regardless of whether ε_{cth} is included or not. Therefore, the decision on whether to include this parameter should depend on the intent and desired accuracy of the analysis. In view of the sensitivity of the solution to other sources of uncertainity, the benefits derived from including concrete thermal expansion are judged to be marginal, at least in simply supported beams, provided that reserve capacity and endurance, rather than beam distortion are the primary factors of interest.

Finally, an example of the variability in creep strain predictions resulting from uncertainty in the value of $\beta = \Delta H/R$ is shown in Fig. 17 for bar 1, which has the highest temperature. The concrete thermal expansion was suppressed in these calculations, and the uncorrected steel temperatures were used for illustrative purposes. It is seen that decreasing $\Delta H/R$ from 75000 to 70000 causes the creep strain increment between 230 and 240 minutes to increase from 0.0097 to 0.0487. The very large imposed strain which may result forces the mechanical stress in the corresponding bar to go into compression in order to maintain strain compatibility. Not only does this affect the load-deformation-time relationship for the beam, it also implies that a reversal of inelastic deformation should occur for the compressively stressed bar during the subsequent time increment. Unfortunately, the computation of creep strain reversals is still clouded with uncertainty [17], and numerical results obtained under such circumstances should therefore be viewed with some suspicion.

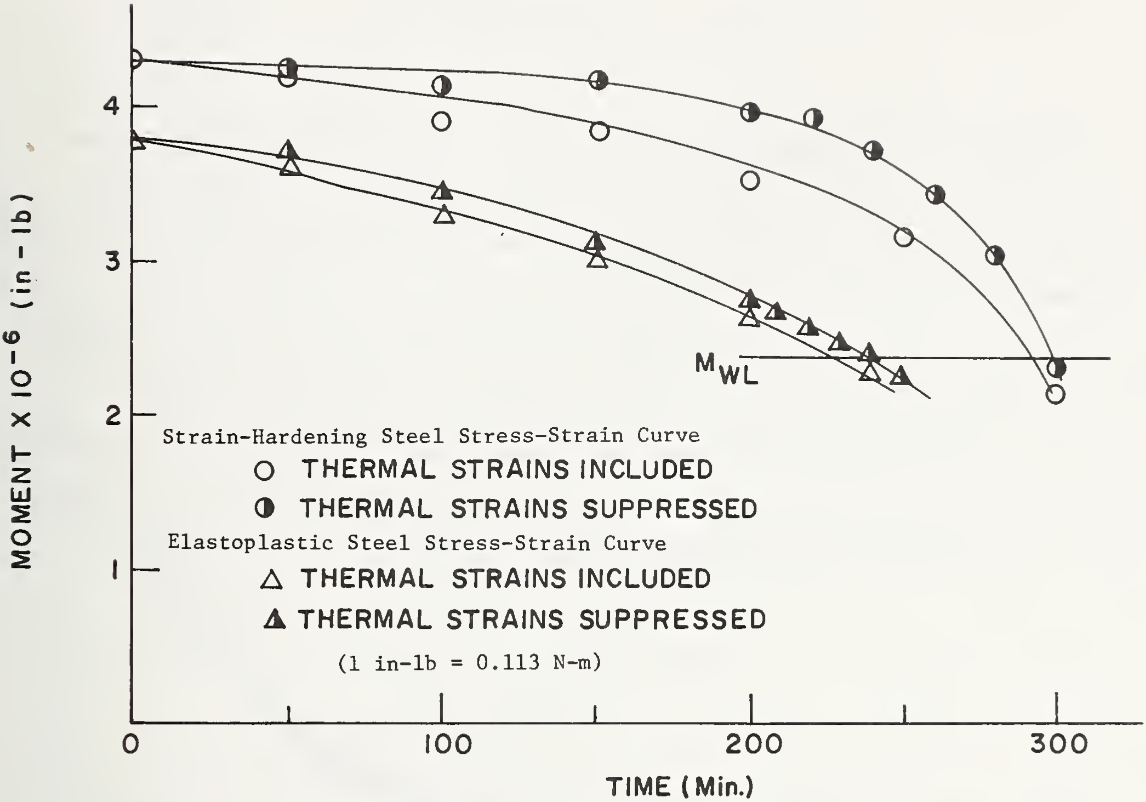


FIG. 16 - EFFECT OF CONCRETE THERMAL EXPANSION ON MOMENT CAPACITY

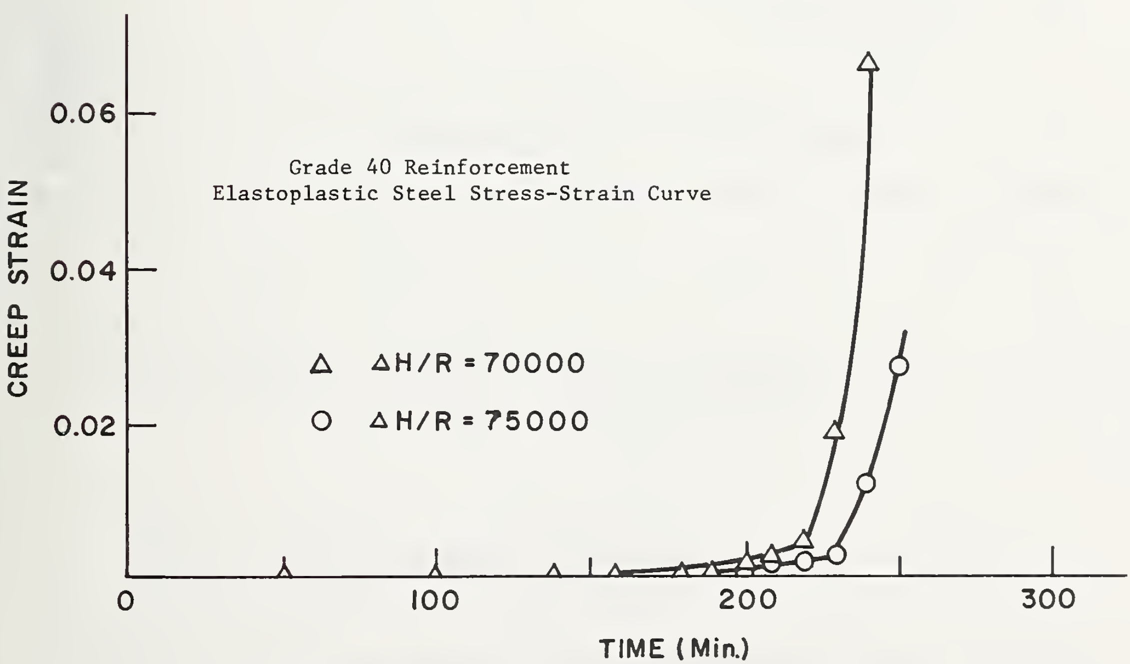


FIG. 17 - SENSITIVITY OF CREEP STRAIN IN BAR 1 TO $\Delta H/R$

4.2 T-Beam with Grade 60 Reinforcement

The second case study considers the section shown in Fig. 7a which contains 8 No. 9 Grade 60 bars with a nominal yield strength of 60 ksi (413 MN/m^2). The bar locations are the same as in the previous case study. The corresponding beam tested by PCA [11] sustained a test moment of 5250 in-kips (0.594 MN-m) for 373 minutes, at which time the beam failed. At the time of test, the compressive strength of the concrete was $f_c' = 5910$ psi (41 MN/m^2), while the actual yield strength of the reinforcement was $f_y = 66$ ksi (454 MN/m^2).

The time-dependent degradation in ultimate moment capacity predicted from the thermomechanical analysis is shown in Fig. 18. An elastoplastic stress-strain curve for the reinforcement was used, along with the following experimental creep constants [13] in Eqs. 8 and 9:

$$\Delta H/R = 65000$$

$$\varepsilon_{to} = 1.25 \times 10^{-7} f_s$$

$$Z = \begin{cases} 267.7 \; f_s^{3.25} , & f_s \leq 15000 \\ 3.69 \times 10^{-14} \exp(0.00022 f_s) , & f_s > 15000 \end{cases}$$

The use of the uncorrected calculated reinforcement temperatures has approximately the same effect on the calculated capacity and duration that was observed with the Grade 40 reinforced beam (cf. Figs. 10 and 18). A divergence between the two solutions occurs at about 150 minutes, and the predicted endurance is about 100 minutes less for the beam in which the uncorrected calculated steel temperatures were employed, being 245 minutes instead of the 355 minutes obtained when measured temperatures were used. This again demonstrates the ability of the analysis to accurately predict beam response, provided that an accurate reinforcement temperature history is available, and the need for such data if credible results are to be obtained.

4.3 Prestressed T-Beam

A prestressed beam was chosen for the final case study. The section geometry and pre-stressing arrangement are shown in Fig. 7b. The corresponding beam tested by PCA [11]

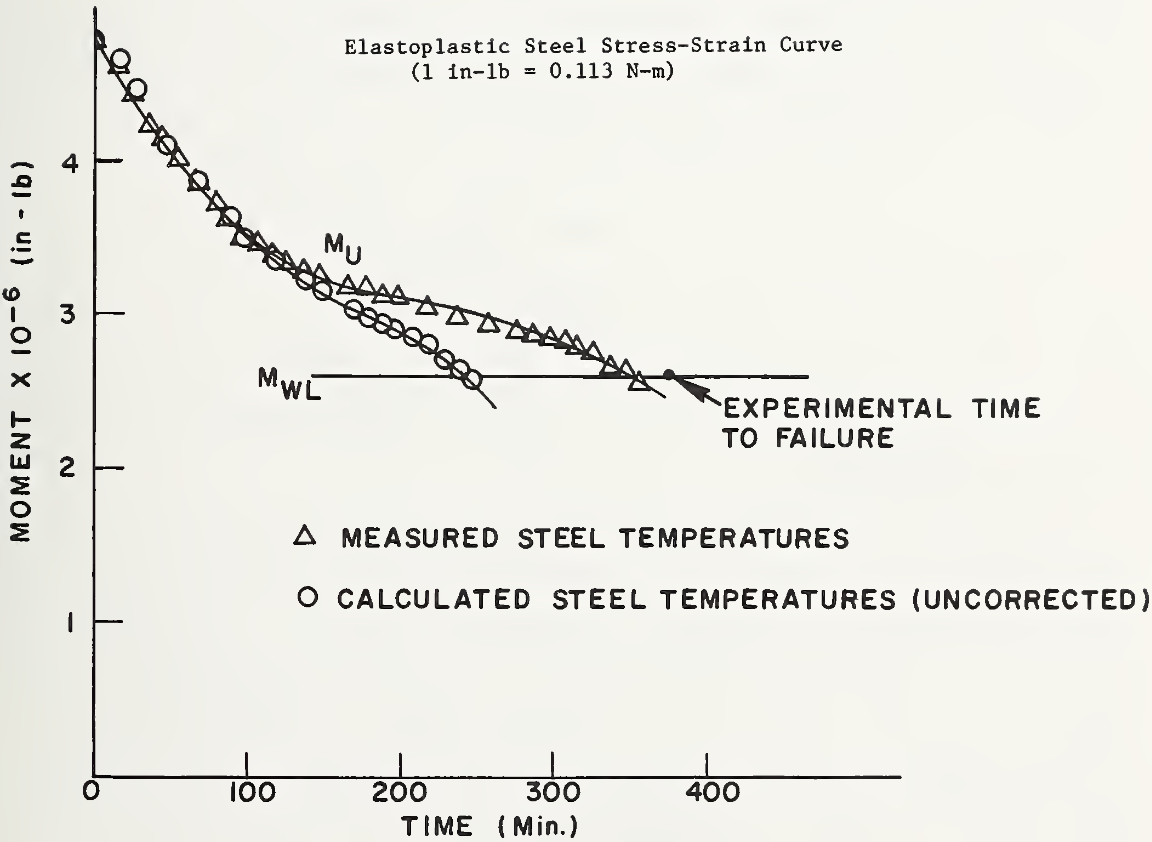


FIG. 18 - PREDICTED BEAM CAPACITY WITH GRADE 60 REINFORCEMENT

contained 16 tendons of cold-drawn 1/2 in (1.27 cm) diameter 7-wire strand, with a nominal 0.2% yield stress of 225 ksi (1550 MN/m^2), which were pretensioned to 175 ksi (1206 MN/m^2) prior to placing the concrete. The corresponding prestrain (Eq. 6c) is $\varepsilon_{ps} = 0.0064$. The concrete compressive strength at test was $f_c' = 5940$ psi (41 MN/m^2) while the strand yield stress was 236 ksi (1625 MN/m^2). The observed time to failure was 237 minutes with a service load moment of 5030 in-kips (0.569 MN-m).

The strain hardening model was chosen for the stress-strain relationship for the strand because of its observed rounded nature. The values of K and n at room temperature were K = 39743000 psi (274027 MN/m^2) and n = 1.113. The yield stress degrades at a somewhat more rapid rate at elevated temperatures for the ultra high strength steels used in strand [13] than for ordinary steels. Rather than to use the degradation model shown in Fig. 6a, therefore, the yield stress ratio was set at 0.75 at 600°F (316°C) and 0.25 at 1000°F (538°C). The following experimental creep constants were used [13]:

$$\Delta H/R = 55000$$

$$\varepsilon_{to} = 3.3 \times 10^{-6} f_s^{0.67}$$

$$Z = \begin{cases} 64 f_s^3, & f_s < 25000 \\ 8.21 \times 10^{13} \exp[0.0001 f_s], & f_s \geq 25000 \end{cases}$$

The predicted time-dependent degradation in ultimate moment capacity for this prestressed beam is shown in Fig. 19. The temperature history in the steel measured during the corresponding test [11] was used in performing the calculations. The agreement between predicted and observed endurance again appears to be reasonable with the difference being easily attributed to the uncertainties in material response and modeling discussed in detail in connection with the first case study. The rather precipitous drop in capacity that occurs at 200 minutes is caused by an acceleration in the accumulated creep strains in tendons 1, 4, and 7 (see Fig. 7b), which have the least amount of cover and whose temperatures approach 900°F (483°C) after 3 hours of test. Similar behavior has been noted earlier, cf. Fig. 14 and curve I in Fig. 11. In the present case study, the predicted creep strains in the hotter bars exceeded 0.03 at 190 minutes; the limit of applicability of the small deformation theory, therefore, is clearly being approached.

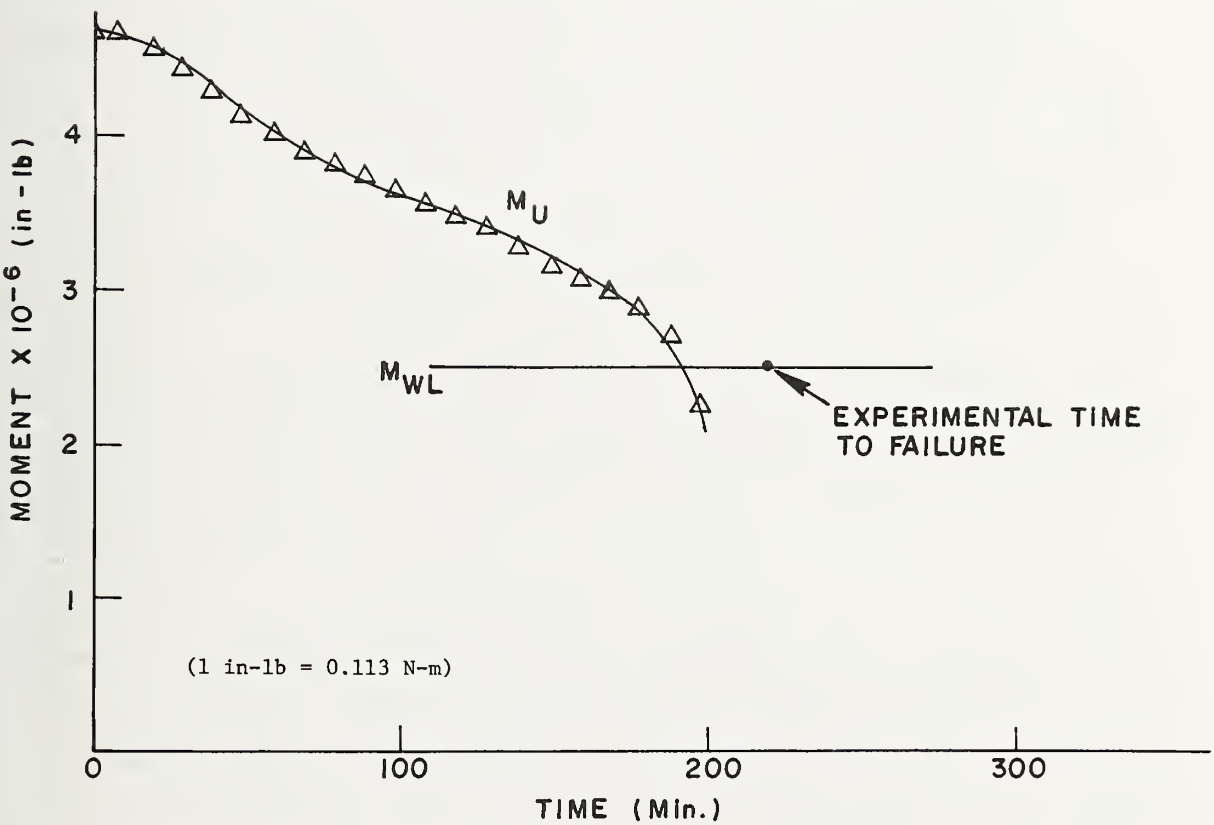


FIG. 19 - PREDICTED MOMENT CAPACITY OF PRESTRESSED BEAM

5.0 CONCLUSIONS AND RECOMMENDATIONS

In this study, methods for analytically predicting the behavior of simply supported reinforced concrete beams subjected to fire have been discussed. The stress analysis of a beam cross section was considered, incorporating the temperature-dependent strength degradation of the steel and concrete, as well as the thermal and creep strains. The main objectives of the analysis were to determine (1) the reserve strength of the beam above working load as a function of elapsed time, and (2) the time it could sustain that load under fire. These predictions gave reasonable agreement with experimental data [11], provided that the material properties and temperature history in the reinforcement were accurately defined.

It should be apparent that the ability of any analytical model to predict structural behavior is dependent not only on the model itself but also on the accuracy with which its parameters can be defined. The sensitivity analysis performed herein revealed that the primary factors affecting the predicted behavior are the uncertainties in the various temperature-dependent material parameters and in the calculation of bar temperatures. Therefore, if accuracy in analytically reproducing the results of a fire test is of particular interest, it is essential that the structural parameters in the analysis be the same as those in the experiment. Such reproducibility may not always be possible because of the inability to estimate some quantities accurately and the inherent randomness in others.

In many cases, particularly in developing design standards, it would be prudent from a safety viewpoint to select conservative values for the parameters (thus, for example, although the average yield strengths of intermediate grade reinforcement may be 46 ksi (317 MN/m^2) a value of 40 ksi (276 MN/Mm^2) is used in design). Naturally, such assumptions lead to a conservative estimate of the fire endurance for the beam. The question of how conservative to choose such design parameters may best be answered using a statistical methodology in which the various factors contributing to possible beam unreliability can be analyzed systematically.

Although the findings in this study are based on limited data and thus must be considered as preliminary in nature, the following specific conclusions and recommendations can be presented:

1. The single most important factor affecting the predicted beam behavior is the estimated (or calculated) temperature history in the steel reinforcement or prestressing. The computation of these temperatures directly from those in the surrounding concrete yields a very conservative result in terms of predicted beam endurance.

2. The temperature-dependent material properties of the reinforcement are significant in establishing the reserve capacity of the beam as a function of time and in predicting beam endurance. Concrete properties do not appear to be especially important, however, provided that the section does not carry axial load. In fact, the exclusion of concrete thermal expansion seemed to have little effect insofar as fire endurance calculations were concerned.

3. The inherent (random) variability of the reinforcement strength and uncertainty in placing the reinforcement arising from careless workmanship would tend to limit the reproducibility of beam fire tests and the predictability of member behavior in service.

4. The creep model is the weak link in the analysis. This is not because of its concept, which is as easily justified and defended as any available alternative, but because of the uncertainty in the definition of the model parameters and the sensitivity of the creep predictions to them. One must bear in mind the limitations of the creep model and the conditions under which its parameters were derived.

5. The elastoplastic curve appears to be appropriate and sufficient for modeling the stress-strain relation for reinforcing steel. The reinforcement strength properties at all temperatures can thus be completely specified with two temperature-dependent material constants. There appears to be little advantage in using the strain hardening model with its two additional required constants, unless strain hardening commences immediately upon yielding.

6. It appears feasible to use this analysis for developing criteria for the design of fire-resistant beams, provided that reasonable temperature and material property estimates can be obtained. Families of curves derived from analyses similar to those illustrated in Figures 10, 11, 12, 18 and 19, could be employed to determine the effect of such factors as bar cover and reinforcement ratio on fire resistance. Since the present analysis tacitly assumes that the parameters are single-valued, a thorough analysis of modeling and statistical uncertainties should be performed before specific design aids can be advanced.

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APPENDIX A

INSTRUCTIONS FOR PROGRAM USAGE AND INPUT DATA PREPARATION

This appendix contains the specific instructions for preparation of the card input needed to use this program. The required data is broken into (4) major blocks:

I. Beam description and analysis control parameters;

II. Cross-section geometry;

III. Material property parameters;

IV. Time and/or temperature increments and output requests.

These input blocks are read by the program in the sequence given. It should be noted at the outset that the program was written in FORTRAN V and executed on a UNIVAC 1108 (Exec 8) system.

A.1 Beam Description and Analysis Control Parameters

The input block containing beam description and analysis control parameters requires input for the following four (4) subgroups.

A) Beam description: (2 cards required)

Each card may contain up to 78 alphanumeric characters used to describe the beam being analyzed in Cols. 1-78 on each card.

B) Control Words: (1 card required)

Two alphanumeric words containing six characters each are located in columns 1-13 with a space between each word in column 7.

1) Control Word 1: (Cols. 1-6)

REINFO - means that the beam contains reinforcing steel;

PRESTR - means that the beam contains prestressed steel;

REPRST - means that the beam contains both reinforcing and prestressed steel.

2) Control Word 2: (Cols. 8-13)

RAMOSG - means that the steel stresses are to be computed using the Ramberg-Osgood idealization

FLATT - means that the steel stresses are to be computed using the elasto-plastic idealization

C) Working Load Moment: (1 card required)

The working load moment in (in-lb) is given as a floating point number in columns 1 to 10.

D) Convergence Criteria: (1 card required)

Two numeric words are required and are located in columns 1-20.

1) Criteria Word 1: (Cols. 1-10)

An integer which specifies the maximum number of iterations allowed for convergence. The minimum value is recommended as 20 since the initial iteration cycle is generally long.

2) Criteria Word 2: (Cols. 11-20)

A floating point number which specifies the tolerance within which the resultant axial thrust must fall for convergence. One (1.0) pound has been used with usually less than four cycles required for convergence within this tolerence.

A.2 Cross-Section Geometry

The input block for cross-section geometry contains two subgroups. The cross-section discretization used by this program is the one generated and used by the finite element thermal analysis in determining the temperature distribution on the cross-section. This discretization is made available to the program via an output tape from the thermal analysis which also contains the temperature distribution on the cross-section at given time increments. In order to properly read the grid from the output tape two (2) parameters are required.

A) Cross-Section Geometry: (1 card required)

This card contains four (4) integers located in columns 1-40 which specify the cross-section geometry parameters.

1) Control Word 1: (Cols. 1-10)

This integer specifies the number of reinforcing and/or prestressed steel elements in the cross-section.

2) Control Word 2: (Cols. 11-20)

An integer which is the minimum row number for the grid input to the thermal analysis.

3) Control Word 3: (Cols. 21-30)

An integer which is the maximum row number for the grid input to the thermal analysis.

4) Control Word 4: (Cols. 31-40)

An integer which specifies the reference line χ_R for computation of the imposed (thermal) strain. As an example, in the case of a symmetric cross-section this reference line could be the axis of symmetry and the reference integer would then be the column number used by the thermal analysis to specify the grid line which lies along the axis of symmetry.

B) Steel Geometry: (1 card required for each reinforcing bar and/or prestressed strand)

Each bar or strand card contains three (3) floating point numbers in columns 1 to 30 with an integer in columns 31-40. The first number in Cols. 1-10 is the x-coordinate in inches for the bar with reference to the coordinate axis used to input the grid to the thermal analysis. The second number located in columns 11 to 20 specifies the y-coordinate in inches and the third floating point number in Cols. 21-30 gives the area of the bar in square inches. The integer located in Cols. 31-40 specifies whether the steel is a reinforcing bar or a prestressing strand. A zero (0) is input for a reinforcing bar and a one (1) for prestressed steel.

A.3 Material Property Parameters

The input block for the material property parameters is broken into two (2) subgroups with one for the concrete material properties and the other for the properties of the reinforcing and/or prestressed steel. Each subgroup in turn has two (2) parts: (1) Input for the material properties at the base (room) temperature and (2) Input for the degradation or change in material properties as a function of temperature. A third data group for the creep properties of the steel and prestressing strains is also required.

The degradation or change in all material properties as a function of temperature for both steel and concrete is modeled by a piece-wise linear curve in the program. Input data required by the program for handling the material properties in this manner are the total number of end points for linear segments plus the ordinate and abcissa values for each end point. Fig. 4a exhibits a typical piece-wise linear curve which in this case shows the degradation of concrete strength with temperature. This curve is comprised of three(3) segments and requires information at four (4) end points for its complete description.

In the event that a material property is required at a temperature which is greater than the maximum temperature value input, an error message is written and the value at the maximum temperature is used.

In addition, seven constants are required to compute the stress-dependent primary creep parameter, ε_{to}, and the secondary creep rate, Z, which appear in Eq. 8. Specifically,

$$\varepsilon_{to} = C_1 (f_s)^{C_2}$$

$$Z = C_3 (f_s)^{C_4}, \quad f_s \leq C_7$$

$$Z = C_5 \exp (C_6 f_s), \quad f_s > C_7$$

The expression for Z depends on whether the steel stress exceeds a transition stress C_7. Finally, the computation of temperature-compensated time Eq. 9 requires definition of

$$C_8 = \Delta H/R$$

A) Concrete Material Properties:

1) Base Temperature Properties: (1 card required)

This card contains three (3) floating point numbers in Cols. 1-10, 11-20 and 21-30 which are, respectively, the maximum compressive strength for the concrete at the base temperature, its modulus of elasticity, and the constant 0.85, which is defined in ACI 318-71, Sect. 10.2.7 [2].

2) Degradation or change in material properties with temperature. The following required sequence of input is used for concrete:

a) Compressive Strength

b) Tensile Strength

c) Modulus of Elasticity

d) Thermal Expansion of Concrete

Each input consists of one or more cards with the following format:

Card No. 1 (required): Contains an integer in columns 1 to 10 which gives the number of end points required to specify the piece-wise linear curve. The x-y coordinates for the first three end points are placed on the card beginning in column 21. Floating point numbers are used to specify the x and y coordinates of the end points in that order. Each coordinate is contained in a field of ten (10) columns.

Card Nos. 2, 3, etc. (if necessary): These cards contain the x-y coordinates for the remaining end points. Again, floating point numbers in fields of ten (10) columns beginning in column 1 are used to specify the x and y coordinates, respectively. If the y coordinate for the final point on the curve is in the field given by columns 71-80 on any card, then a blank card must follow before the next input group.

B) Reinforcing and/or Prestressed Steel Properties

1) Base Temperature Properties: (1 card required)

This card contains two (2) floating point numbers in Cols. 1-10 and 11-20 which are respectively, the yield strength of the steel and the modulus of elasticity for the steel at the base temperature.

2) Degradation or change of material properties with temperature. The following required sequence of input is used for steel:

a) Yield strength

b) Strain hardening exponent for strain hardening constitutive model

c) Strength coefficient for strain-hardening constitutive model

d) Modulus of Elasticity

e) Coefficient of Thermal Expansion for Steel

The input format for each material property change is the same as that given in the concrete segment. The information for the strain-hardening constitutive model is not required if the elasto-plastic idealization is used to model the steel stress-strain relationship.

3) Creep Properties and Prestressing Strains

a) Creep Properties (2 cards required):

The eight (8) coefficients are placed, four (4) to a card, in fields on 10 columns each using the Fortran E format. The input order of these coefficients is the same as that given by the subscripts attached to the coefficients presented in the introduction to this section.

b) Prestressing Strains

Prestressing strains, when required, are input in the same order as used for the steel coordinates. They are input as floating point numbers in fields of 10 columns with eight (8) strains to a card. In the case where both reinforced and prestressed steel are present in the beam, zero (0) strain must be input for the reinforcement bars. This information is not required for a beam containing only reinforcing steel.

A.4 Time and/or Temperature Increments and Output Requests

The input block for the time and/or temperature increments and output requires data for three (3) subgroups.

Four types of output are available during each computation at a specified time. As a minimum, the elapsed time from the beginning computation and the ultimate moment capacity for the cross-section at that time increment are always printed. The other three types of output are optional and returned only during the computation at a given time if an integer one (1) is specified in the appropriate input field. If the optional output is not wanted then an integer (0) is input on the card. The following is a description of the optional output.

Output A) The steel temperatures and steel material properties at that temperature including the accumulated creep of the steel from time zero.

Output B) The assumed top concrete strain at the reference line, the curvature which satisfies equilibrium and compatibility for the assumed strain, the moment capacity for that strain and the number of iterations needed to satisfy the convergence criterion are given for each strain. In addition, the stress and strains in the steel are given at working load and ultimate moment.

Output C) This includes all of the information from Output B plus the total stress and strain in the concrete and steel and the mechanical stress and strain in the concrete and steel for each assumed top concrete strain. It should be noted that a request for Output C gives Output B and thus Output B should not be requested on the input card.

A) Temperature and time parameters: (one card required)

This card contains: (1) a floating point number in columns 1-10 which gives the base (room) temperature for the initial calculation, (2) a six character alphanumeric word in columns 15-20 which specifies whether the steel temperatures are calculated from the temperature of the surrounding concrete (word must be CALCST) or read from cards (word must be READST), (3) a floating point number in columns 21-30 which allows for adjustment of the steel temperatures when they are determined from the temperature of the surrounding concrete and (4) an integer in columns 31-40 which specifies the number of time increments starting with the base temperature for which the moment-curvature of the beam cross-section is to be determined.

B) Time and Output Requests: (one card required for each time a computation is wanted)

Each card contains a floating point number in columns 1-10 which specifies the time of the calculation and three (3) integers in columns 11-20, 21-30 and 31-40, which specify the optional output (A, B, or C, respectively) wanted with each time calculation.

C) Steel Temperature: (one or more cards is required for each time a computation is wanted if READST is input)

Each card contains a maximum of eight (8) bar or strand temperatures as floating point numbers in fields of ten (10) columns beginning with column 1. If there are less than 8 bars or strands than only one (1) card is needed to input the steel temperatures at each time increment.

The following is a sample of the input data that was used to obtain curve I in Fig. 10 of the main body of the report.

```
  BEAM 84 - 40Y REINFORCEMENT - UNITS=(LB,IN,MIN,DEGREES-F)
  BLANK CARD
REINFO FLATT
  2385000.
         20         1.
          4         1       26        14
     9.125      3.125     1.270
    11.687      3.125     1.270
     9.125      6.375     1.270
    11.687      6.375     1.270
     7230.   4970000.       .85
          4                 70.        1.0       800.        1.0      1600.        0.3
     2100.        0.2
          3                 70.        1.0      1200.        .25      2100.  .25
                       BLANK CARD
          3                 70.        1.0      1200.        0.4      2100.        0.3
                     BLANK CARD
          3                 70.    .000006       800.    .000020      2100.    .000020
                     BLANK CARD
    46000.  29000000.
          4                 70.        1.0       800.        .75      1400.        .20
     2100.        0.0
          4                 70.     1.0207       800.      1.013      1200.    1.00523
     2100.        1.0
          4                 70.  30603000.       800.   25591000.     1200.  18660000.
     2100.        0.0
         1.7E-10          1.75E0          2.6E-02          4.7E0
         1.23E16         3.0E-04           1.5E04          7.5E04
          3                 70.        1.0       900.        .85      2100.        0.0
                     BLANF CARD
          3                 70.    .000006      1500.    .000009      2100.    .000012
                     BLANK CARD
     92.95     READST       1.0        39
        0.          1         1
       10.          1         1
       20.          1         1
       30.          1         1
       40.          1         1
       50.          1         1
       60.          1         1
```

Additional time and output request data,
as indicated by the above.

```
     150.2      121.6     107.2     92.95
     228.8      185.9     171.6     107.2
     278.8      257.4     228.8     128.7
     286.0      264.6     257.4     150.2
     271.7      257.4     257.4     178.8
     264.6      257.4     250.2     214.5
```

Additional bar temperature data corresponding
to each time step, as indicated by the above.

```
END OF
```

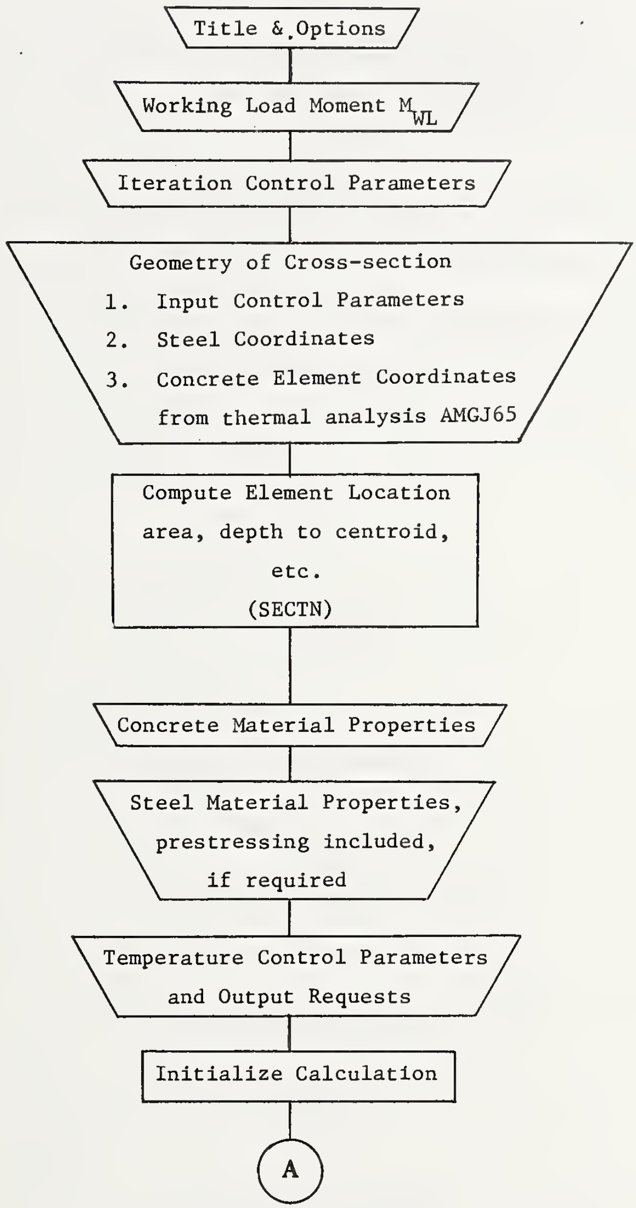
APPENDIX B

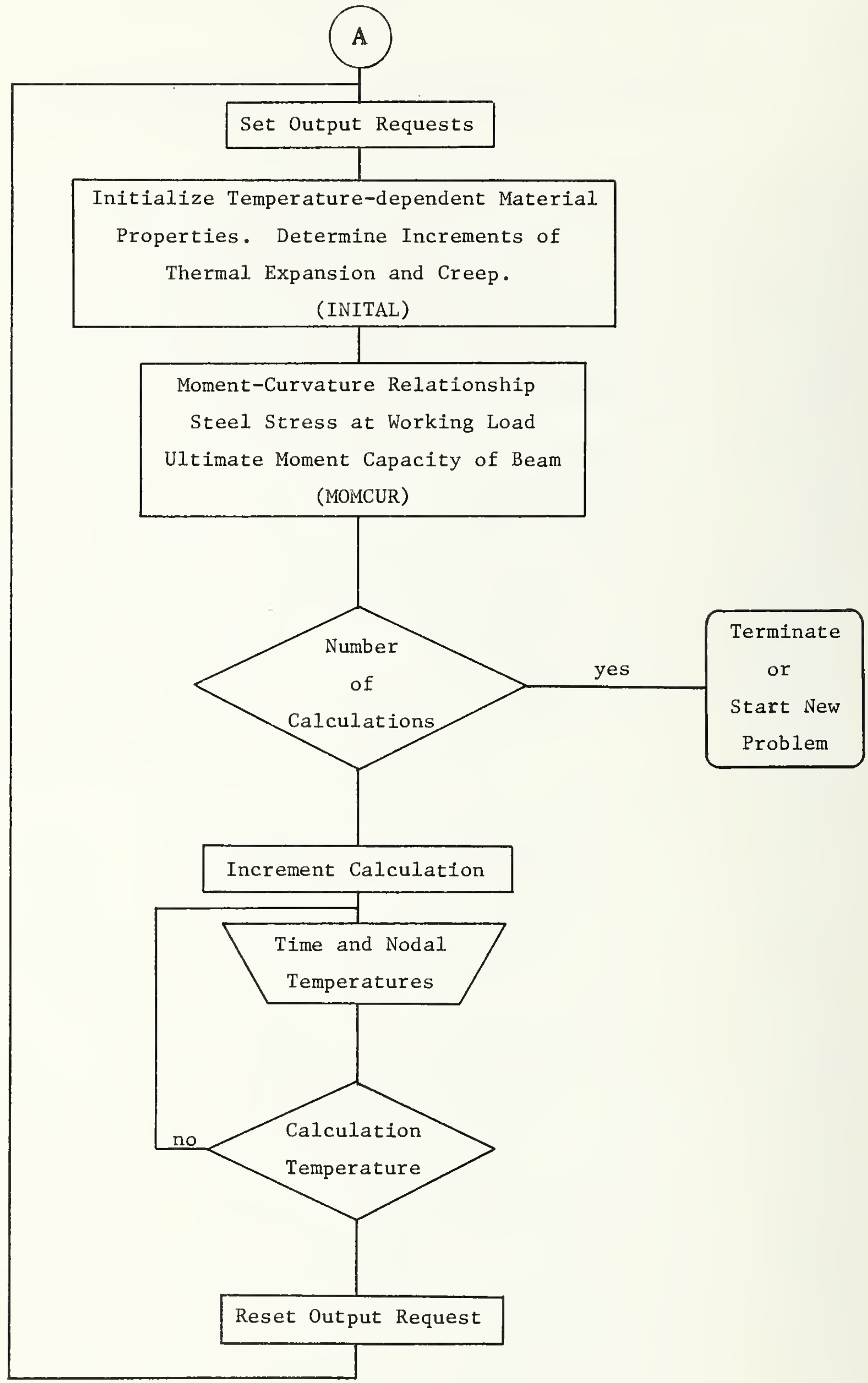
COMPUTER PROGRAM DOCUMENTATION

This appendix contains the source listing for the computer program used to perform the analysis of a reinforced concrete beam section subjected to a non-linear temperature distribution described in Section 2 of this report. In addition to the source listings for the main program and subroutines, flow diagrams are given for the main program, which controls the main input and sequencing of calculations, and subroutine MOMCUR, which controls the computation of the moment-curvature relationship for a given temperature distribution. The program which is written in FORTRAN V is restricted in use since it requires input from a thermal analysis of the cross-section under investigation. Those source statements in the main program which require information from the thermal analysis are located at line 44 through line 50 and line 138. These statements would have to be modified in order to accept information (cross-section discretization and nodal point temperatures) from a thermal analysis different from the one used in this study [15].

MAIN PROGRAM

Controls Input and General Sequence of Calculations




A
Set Output Requests
Initialize Temperature-dependent Material Properties. Determine Increments of Thermal Expansion and Creep.
(INITAL)
Moment-Curvature Relationship
Steel Stress at Working Load
Ultimate Moment Capacity of Beam
(MOMCUR)
Number of Calculations
yes
Terminate or Start New Problem
Increment Calculation
Time and Nodal Temperatures
no
Calculation Temperature
Reset Output Request

```
C     MAIN PROGRAM  CONTROLS INPUT, CALCULATION SEQENCE AND OUTPUT
C
      DIMENSION TITLE(26),TCODE(30,50),TIMEP(100),INOUT(100,4)
      COMMON/GEOM/JRANGE,IMIN(50),IMAX(50),JMIN,JMAX,NEQ(50),
     1 XX(30,50),YY(30,50),XS(20),YS(20),BPYC(20,3),NLS(20,4),YDEP
      COMMON/STL/NSS,SDEPT(20),STEMP(20),STEM1(20),NSMAT(20),FY(20),
     1 AK(20),AN(20),AMODS(20),ALPHAS(20),EPST(20),EPSPST(20),
     2 SSTRN(20),TSTS(20),SAREA(20),STRESS(20),EPSW(20),FSW(20),
     3 EPSULT(20),FSULT(20)
      COMMON/CONC/CDEPT(29,49),CAREA(29,49),CTEMP(29,49),CTEM1(29,49),
     1 FPC(29,49),FPT(29,49),AMODC(29,49),ALPHAC(29,49),CEPT(29,49),
     2 ALDTCT,CSTRN(29,49),TSTC(29,49),CSTRES(29,49)
      COMMON/CRP/CREEP(20),TCMPT(20),Z(20),EPTO(20),EPSC1(20),
     1EPSC2(20),CZ(8,2)
      COMMON/MATIN/FC,EC,NFC,RFCT(10,2),NFT,RFT(10,2),NEC,RECT(10,2),
     1 NALC,ALPHC(10,2),FYS,ES,NYS,RYS(10,2),NAN,ANT(10,2),
     2 NK,AKT(10,2),FYSPS,NYSP,RYSPS(10,2),NANPS,ANPST(10,2),
     3 NKPS,AKPST(10,2),NES,FES(10,2),NALS,ALPHS(10,2)
      COMMON/TTEMP/TIME,TIMP,TEMP(30,50),TEMPIN,TFCTR
      COMMON/INDEX/NTINC,IREF,TOL,NITER,SIDEAL,STCAL,IEPSC,ISTRSA,ICONV
      DOUBLE PRECISION TCMPT
      REWIND 10
C
C  INPUT PROGRAM CONTROL PARAMETERS
C
      READ (5,1250) (TITLE(I),I=1,26)
10    WRITE (6,1250) (TITLE(I),I=1,26)
C  READ IN PROGRAM OPTIONS AS ALPHANUMERIC DATA
      READ(5,1251) BMTYPE,SIDEAL
      READ(5,1194) BMWL
      WRITE(6,1300) BMWL
1300  FORMAT(1X,'WORKING LOAD MOMENT =',E10.5,' IN-LB')
      READ(5,1221) NITER, TOL
C
C  INPUT BEAM GEOMETRY DESCRIPTION
C
      READ(5,1252) NSS, JMIN, JMAX, IREF, IGEOM
      JRANGE = JMAX - JMIN
      READ(5,1190) (XS(I),YS(I),SAREA(I),I=1,NSS)
      WRITE(6,1301)
1301  FORMAT(10X,'STEEL COORDINATES'/13X,'XS',13X,'YS',10X,'AREA')
      WRITE(6,1302) (XS(I),YS(I),SAREA(I),I=1,NSS)
1302  FORMAT(3E15.4)
C  READ IN THE NODAL PT.DATA FROM AMGT65
      JRAN=JMAX-JMIN+1
      READ (10) (IMAX(J),IMIN(J),NEQ(J),J=1,JRAN),BW
      DO 690 J=1,JRAN
          IRAN=NEQ(J)
          READ (10) (XX(I,J),YY(I,J),TCODE(I,J),ICODE,I=1,IRAN)
690       CONTINUE
      CALL SECTN
      IF(IGEOM .EQ. 1) CALL GEOMOT
```

```
C
C   INPUT CONCRETE MATERIAL PROPERTIES
C
      READ(5,1190) FC, EC, XF
      WRITE(6,1303) FC,EC
1303  FORMAT(5X,'FC =',E15.5,5X,'EC =',E15.5)
      FC = XF * FC
      READ(5,1191) NFC,((RFCT(I,J),J=1,2),I=1,NFC)
      READ(5,1191) NFT,((RFT(I,J),J=1,2),I=1,NFT)
      READ(5,1191) NEC,((RECT(I,J),J=1,2),I=1,NEC)
      READ(5,1191) NALC,((ALPHC(I,J),J=1,2),I=1,NALC)
C
C    INPUT STEEL MATERIAL  PARAMETERS
C
      IF(BMTYPE .EQ. 'PRESTR') GO TO 12
C  REINFORCING STEEL PROPERTIES
      READ(5,1194) FYS, ES
      READ(5,1191) NYS,((RYS(I,J),J=1,2),I=1,NYS)
      READ(5,1191) NAN,((ANT(I,J),J=1,2),I=1,NAN)
      READ(5,1191) NK,((AKT(I,J),J=1,2),I=1,NK)
      READ(5,1186)  (CZ(I,1),I=1,8)
      WRITE(6,1304)FYS,ES
1304  FORMAT(5X,'FYS =',E15.5,5X,'ES =',E15.5)
      IF(BMTYPE .EQ. 'REINFO') GO TO 11
C   NSMAT TELLS WHICH STEEL ELEMENTS ARE PRESTRESSED
      READ(5,1189)  (NSMAT(I),I=1,NSS)
      GO TO 13
12    DO 1188 I = 1, NSS
1188  NSMAT(I) = 1
C   PRESTRESSING CABLE ONLY
13    READ(5,1194) FYSPS, ES
      READ(5,1191) NYSP,((RYSPS(I,J),J=1,2),I=1,NYSP)
      READ(5,1191) NANPS,((ANPST(I,J),J=1,2),I=1,NANPS)
      READ(5,1191) NKPS,((AKPST(I,J),J=1,2),I=1,NKPS)
      READ(5,1186)  (CZ(I,2),I=1,8)
      WRITE(6,1305) FYSPS,ES
1305  FORMAT(5X,'FYS(PS) =',E15.5,5X,'ES(PS) =',E15.5)
      GO TO 14
11    DO 1187 I = 1, NSS
1187  NSMAT(I) = 0
C  ES(T)/ES(70) AND ALPHA(S) ARE THE SAME FOR
C   BOTH REINFORCING AND PRESTRESSING
14    READ(5,1191) NES,((RES(I,J),J=1,2),I=1,NES)
      READ(5,1191) NALS,((ALPHS(I,J),J=1,2),I=1,NALS)
C  STRAINS IN PRESTRESSING CABLE
      IF(BMTYPE .EQ. 'REINFO') GO TO 15
      READ(5,1211)(EPSPST(I),I=1,NSS)
      DO 16 I=1,NSS
      IF (NSMAT(I) .EQ. 0) GO TO 16
      WRITE(6,1306) I,EPSPST(I)
1306  FORMAT(5X,'BAR',I2,2X,'PRESTRESSED TO STRAIN',E15.5)
  16  CONTINUE
```

```
C
C  INPUT TIME AND/OR TEMPERATURE PARAMETERS AND OUTPUT REQUESTS
C
  15  READ(5,1192) TEMPIN,STCAL,TFCTR,NCALC
      WRITE(6,1307) TEMPIN,NCALC
1307  FORMAT(1X,'STARTING TEMP =',F8.2,5X,'NCALC =',I5)
      IF(STCAL .EQ. 'CALCST') WRITE(6,1308) TFCTR
      IF(STCAL .EQ. 'READST') WRITE(6,1309)
1308  FORMAT(1X,'STEEL TEMPERATURE IS CALCULATED FROM SURROUNDING'/
     1' CONCRETE TEMPERATURE WITH ADJUSTMENT FACTOR OF',F8.2)
1309  FORMAT(1X,'STEEL TEMPERATURE IS INPUT AT EACH TIME INCREMENT')
      DO 2000 I=1,NCALC
2000  READ(5,1170) TIMER(I),(INOUT(I,J),J=1,4)
C
C   CALCULATION SEQUENCE ( MOMENT- CURVATURE-TIME RELATIONSHIPS )
C
      NTINC = 1
      TIME = 0.0
      ISTEMP = 0
      IEPSC = 0
      ISTRSA = 0
C  SET OUTPUT REQUESTS
1091  IF(INOUT(NTINC,1) .EQ. 1) ISTEMP = 1
      IF(INOUT(NTINC,2) .EQ. 1) IEPSC = 1
      IF(INOUT(NTINC,3) .EQ. 1) ISTRSA = 1
      CALL INITAL
      IF(ISTEMP .EQ. 1) CALL TEMPOT
      CALL MOMCUR(BMWL,$1090)
      IF(NTINC .EQ. NCALC) GO TO 1090
      NTINC = NTINC + 1
      TIMP = TIME
C  TIMER IS TIME AT WHICH CALCULATION IS PERFORMED, CORRES-
C  PONDING TO SOME, BUT NOT NECESSARILY ALL, OF THE TIMES
C  OUTPUT FROM THE THERMAL ANALYZER
1092  READ(10) TIME, TEMP
      IF(ABS(TIMER(NTINC)-TIME) .GE. .05) GO TO 1092
      ISTEMP = 0
      IEPSC = 0
      ISTRSA = 0
      GO TO 1091
1090  READ(5,1250) (TITLE(I),I=1,26)
      IF(TITLE .EQ. 'END OF') GO TO 1150
      GO TO 10
C
C  END OF CALCULATION SEQUENCE
C
1150  STOP
1250  FORMAT(13A6/13A6)
1251  FORMAT(2(A6,1X),I6,1X,A6)
1170  FORMAT(F10.0,4I10)
1221  FORMAT(I10,F10.0)
1252  FORMAT(5I10)
1160  FORMAT(16I5)
1211  FORMAT(8F10.0)
1190  FORMAT(3F10.0)
1191  FORMAT(I10,10X,6F10.0/(8F10.0))
1194  FORMAT(2F10.0)
1186  FORMAT(4E16.4)
1189  FORMAT(20I2)
1192  FORMAT(F10.0,4X,A6,F10.0,I10)
      END
```

```
      SUBROUTINE SECTN
      COMMON/GEOM/JRANGE,IMIN(50),IMAX(50),JMIN,JMAX,NEQ(50),
     1 XX(30,50),YY(30,50),XS(20),YS(20),BRYC(20,3),NLS(20,4),YDEP
      COMMON/STL/NSS,SDEPT(20)
      COMMON/CONC/CDEPT(29,49),CAREA(29,49)
C  FIND DEPTH OF EACH BAR FROM TOP
780   YMAX=0.
      YMIN=10000.
      JRAN=JMAX-JMIN+1
      DO 790 J=1,JRAN
          IRAN=NEQ(J)
          DO 790 I=1,IRAN
          YMAX=AMAX1(YMAX,YY(I,J))
          YMIN=AMIN1(YMIN,YY(I,J))
790       CONTINUE
      YDEP=YMAX-YMIN
      WRITE (6,1260) YMAX,YMIN,YDEP
1260  FORMAT(' YMAX = ',E10.5/' YMIN = ',E10.5/' YDEP = ',E10.5)
      DO 800 N=1,NSS
          SDEPT(N)=YMAX-YS(N)
800       CONTINUE
C CALCULATE THE AREA OF EACH QUAD ACROSS EACH HORIZONTAL STRIP AND
C FIND DEPTH OF EACH CONCRETE ELEMENT
       DO 830 J = 1, JRANGE
       IR = IMAX(J) - IMIN(J)
       DO 830 I = 1, IR
              IU=I+IMIN(J)-IMIN(J+1)
              YN=(YY(I,J)+YY(I+1,J)+YY(IU,J+1)+YY(IU+1,J+1))/4.0
              CDEPT(I,J)=YMAX-YN
              CALL TAREA (XX(I,J),YY(I,J),XX(I+1,J),YY(I+1,J),XX(IU+1,J+
     21),YY(IU+1,J+1),A1)
              CALL TAREA (XX(I,J),YY(I,J),XX(IU+1,J+1),YY(IU+1,J+1),XX(I
     2U,J+1),YY(IU,J+1),A2)
              CAREA(I,J)=A1+A2
830           CONTINUE
      AREA=0
      DO 860 J=1,JRANGE
          IAA=IMAX(J)-IMIN(J)
          DO 850 I=1,IAA
              AREA=AREA+CAREA(I,J)
850           CONTINUE
860       CONTINUE
      WRITE (6,870) AREA
870   FORMAT (' BEAM AREA= ',E10.5)
      IF(STCAL .EQ. 'READ') GO TO 880
C LOCATE THE 4 NEAREST NODES TO EACH BAR
      CALL CQUAD
880   CONTINUE
      RETURN
      END



      SUBROUTINE TAREA (X1,Y1,X2,Y2,X3,Y3,AREA)
C THIS CALCULATES THE AREA OF A TRIANGLE .IF PTS 1,2,AND 3 ARE
C SUPPLIED IN A COUNTER CLOCKWISE SEQUENCE THE AREA WILL BE POSITIVE
      AREA=0.5*(X1*(Y2-Y3)-X2*(Y1-Y3)+X3*(Y1-Y2))
      RETURN
      END
```

```
      SUBROUTINE CQUAD
C  THIS SUBROUTINE IDENTIFIES WHERE EACH BAR IS LOCATED WRT NODE OR
C     QUADRILATERAL ELEMENT
      COMMON/GEOM/JRANGE,IMIN(50),IMAX(50),JMIN,JMAX,NEQ(50),
     1 XX(30,50),YY(30,50),XS(20),YS(20),BRYC(20,3),NLS(20,4),YDEP
      COMMON/STL/NSS
      DIMENSION X(5),Y(5),XI(4),YI(4)
C        NLS(N,K) LABLES STEEL    N =BAR NUMBER
C                                 K = 1 BAR NUMBER
C                                   =2 I OF REFERENCE NODE
C                                   =3 J OF REFERENCE NODE
C                                   =4 BAR LOCATION
C                               IF  NLS(N,4)=1 BAR IS AT NODE I,J
C                                           =2 BAR IS IN LH SIDE OF QUAD
C                                           =3 BAR IS IN RH SIDE OF QUAD
C        TEST FOR BAR LOCATION IN CONCRETE ELEMENT
      DO 220 NN=1,NSS
         SX=XS(NN)
         SY=YS(NN)
C TEST EACH QUADRILATERAL IN SEQUENCE
              DO 190 J=1,JRANGE
                 IR = IMAX(J) - IMIN(J)
                  DO 180 I=1,IR
                     IT=I+IMIN(J)-IMIN(J+1)
                     X(1)=XX(I,J)
                     X(2)=XX(I+1,J)
                     X(3)=XX(IT+1,J+1)
                     X(4)=XX(IT,J+1)
                     X(5)=X(1)
                     Y(1)=YY(I,J)
                     Y(2)=YY(I+1,J)
                     Y(3)=YY(IT+1,J+1)
                     Y(4)=YY(IT,J+1)
                     Y(5)=Y(1)
                     DO 20 M=1,4
                        CALL TAREA (X(M),Y(M),X(M+1),Y(M+1),SX,SY,AREA
     2)
                        IF (AREA.LT.0) GO TO 180
20                      CONTINUE
C  BAR IS WITHIN THIS QUADRILATERAL. CHECK IF AT A NODE POINT
                     DO 30 L=1,4
      IF(ABS(SX-X(L)) .GE. 1.E-06) GO TO 30
      IF(ABS(SY-Y(L)) .GE. 1.E-06) GO TO 30
                        GO TO (40,50,60,70), L
30                      CONTINUE
                     GO TO 90
C IDENTIFY NODE WHERE BAR IS LOCATED
CNOTE THAT NODE LABELS ARE TRUE LABELS AND NOT NORMALIZED LABELS
40                   NLS(NN,2)=I+IMIN(J)-1
                     NLS(NN,3)=J+JMIN-1
                     GO TO 80
50                   NLS(NN,2)=I+IMIN(J)
                     NLS(NN,3)=J+JMIN-1
                     GO TO 80
60                   NLS(NN,2)=IT+IMIN(J+1)
                     NLS(NN,3)=J+JMIN
                     GO TO 80
70                   NLS(NN,2)=IT+IMIN(J+1)-1
                     NLS(NN,3)=J+JMIN
80                   NLS(NN,1)=NN
                     NLS(NN,4)=1
                     GO TO 120
```

```
C IDENTIFY WHICH SIDE OF QUAD BAR IS IN
90                     CALL TAREA (X(1),Y(1),X(3),Y(3),SX,SY,AREA)
                       IF (AREA.GE.0) GO TO 100
C BAR IS IN RH SIDE OF QUADRILATERAL
                       NLS(NN,4)=3
                       GO TO 110
100                    NLS(NN,4)=2
110                    NLS(NN,1)=NN
                       NLS(NN,2)=I+IMIN(J)-1
                       NLS(NN,3)=J+JMIN-1
120                    NXY=NLS(NN,4)
                       GO TO (130,140,150), NXY
130                    BRYC(NN,1)=1
                       BRYC(NN,2)=0
                       BRYC(NN,3)=0
                       GO TO 220
140                    CALL TAREA (X(1),Y(1),X(3),Y(3),X(4),Y(4),AREA)
                       AREA1=AREA
                       XI(1)=X(3)
                       YI(1)=Y(3)
                       XI(2)=X(4)
                       YI(2)=Y(4)
                       XI(3)=X(1)
                       YI(3)=Y(1)
                       XI(4)=X(3)
                       YI(4)=Y(3)
                       GO TO 160
150                    CALL TAREA (X(1),Y(1),X(2),Y(2),X(3),Y(3),AREA)
                       AREA1=AREA
                       XI(1)=X(2)
                       YI(1)=Y(2)
                       XI(2)=X(3)
                       YI(2)=Y(3)
                       XI(3)=X(1)
                       YI(3)=Y(1)
                       XI(4)=X(2)
                       YI(4)=Y(2)
160                    DO 170 II=1,3
                          CALL TAREA (XI(II),YI(II),XI(II+1),YI(II+1),SX
    2,SY,AREA)
                          BRYC(NN,II)=AREA/AREA1
170                       CONTINUE
        GO TO 220
180                    CONTINUE
190                 CONTINUE
         WRITE (6,210) NN
210      FORMAT ('0 E R R O R  CANNOT LOCATE BAR',I5,)
         CALL EXIT
220      CONTINUE
      RETURN
      END
```

```
      SUBROUTINE INITAL
C
      COMMON/ TTEMP/TIME
      WRITE(6,102)
 102  FORMAT(1H0)
      WRITE(6,103) TIME
 103  FORMAT(10X,'ELAPSED TIME =',F8.2,' MINUTES')
C   INITIALIZE, COMPUTE MATERIAL PARAMETERS, TEMPERATURES IN
C   CONCRETE AND STEEL ELEMENTS, AND NON-EQUILIBRATING STRAINS
C   IN SECTION.  THIS SET OF COMPUTATIONS NEED BE PERFORMED
C   ONLY ONCE EVERY TIME INCREMENT.
C
C   TEMPERATURE DISTRIBUTION ON CROSS SECTION
      CALL TEMSET
C   MATERIAL PROPERTIES FOR THIS TEMPERATURE DISTRIBUTION
      CALL MATPRP
C   IF TEMPERATURE IS ELEVATED, COMPUTE THERMAL EXPANSION
C   OF STEEL AND CONCRETE, AND CREEP STRAINS IN REINFORCEMENT
C   THERMAL STRAINS
      CALL THSTRN
C   CREEP STRAINS IN THE STEEL
      CALL CREEP
101   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TEMSET
C     TEMSET SETS UP THE TEMPERATURES IN STEEL AND CONCRETE ELEMENTS
      COMMON/GEOM/JRANGE,IMIN(50),IMAX(50)
      COMMON/STL/NSS,SDEPT(20),STEMP(20),STEM1(20)
      COMMON/CONC/CDEPT(29,49),CAREA(29,49),CTEMP(29,49),CTEM1(29,49)
      COMMON/TTEMP/TIME,TIMP,TEMP(30,50),TEMPIN,TFCTR
      COMMON/INDEX/NTINC,IREF,TOL,NITER,SIDEAL,STCAL
C   IF NTINC = 1, ROOM TEMPERATURE CALCULATIONS
      IF(NTINC .EQ. 1) GO TO 40
      DO 10 I = 1, NSS
10    STEM1(I) = STEMP(I)
      IF(STCAL .EQ. 'READST') GO TO 20
C CALT CALCULATES THE TEMPERATURE OF EACH BAR
      CALL CALT
      GO TO 70
C     IF(STCAL = READ), STEEL TEMPERATURES ARE READ SEPARATELY
20    READ (5,30) (STEMP(N),N=1,NSS)
30    FORMAT (16E10.5)
70    DO 71 J = 1, JRANGE
      IRANGE = IMAX(J)-IMIN(J)
      DO 71 I = 1, IRANGE
      CTEM1(I,J) = CTEMP(I,J)
      IA = I+IMIN(J)-IMIN(J+1)
71    CTEMP(I,J) = .25*(TEMP(I,J)+TEMP(I+1,J)+TEMP(IA,J+1)+
     1TEMP(IA+1,J+1))
      RETURN
C
C ROOM  TEMPERATURE ASSIGNMENT
40    DO 50 N=1,NSS
         STEMP(N)=TEMPIN
50       CONTINUE
      DO 60 J=1,JRANGE
      NI = IMAX(J)-IMIN(J)
         DO 60 I=1,NI
      CTEMP(I,J) = TEMPIN
60       CONTINUE
      RETURN
      END

      SUBROUTINE CALT
C   THIS ELEMENT CALCULATES THE STEEL TEMPERATURES FROM CALCULATED
C   TEMPERATURES
      COMMON/GEOM/JRANGE,IMIN(50),IMAX(50),JMIN,JMAX,NEQ(50),
     1 XX(30,50),YY(30,50),XS(20),YS(20),BRYC(20,3),NLS(20,4)
      COMMON/STL/NSS,SDEPT(20),STEMP(20)
      COMMON/TTEMP/TIME,TIMP,TEMP(30,50),TEMPIN,TFCTR
      DO 60 N=1,NSS
         NXY=NLS(N,4)
         J=NLS(N,3)-JMIN+1
         I=NLS(N,2)-IMIN(J)+1
         IT=I+IMIN(J)-IMIN(J+1)
         GO TO (10,20,30), NXY
10       STEMP(N)=TEMP(I,J)
         GO TO 40
20       STEMP(N)=BRYC(N,1)*TEMP(I,J)+BRYC(N,2)*TEMP(IT+1,J+1)+BRYC(N,3
     2)*TEMP(IT,J+1)
         GO TO 40
30       STEMP(N)=BRYC(N,1)*TEMP(I,J)+BRYC(N,2)*TEMP(I+1,J)+BRYC(N,3)*T
     2EMP(IT+1,J+1)
40       STEMP(N)=(STEMP(N)-TEMPIN)*TFCTR+TEMPIN
60       CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MATPRP
C     SUBROUTINE COMPUTES AMODC, FPC, ALPHAC FOR CONCRETE
C     AND AMODS, YIELD STRENGTH, STRENGTH COEFFICIENT, HARDENGIN
C     EXPONENT, AND ALPHAS FOR STEEL BARS AND STRAND.
C
      COMMON/GEOM/JRANGE,IMIN(50),IMAX(50)
      COMMON/STL/NSS,SDEPT(20),STEMP(20),STEM1(20),NSMAT(20),FY(20),
     1 AK(20),AN(20),AMODS(20),ALPHAS(20)
      COMMON/CONC/CDEPT(29,49),CAREA(29,49),CTEMP(29,49),CTEM1(29,49),
     1 FPC(29,49),FPT(29,49),AMODC(29,49),ALPHAC(29,49)
      COMMON/MATIN/FC,EC,NFC,PFCT(10,2),NFT,PFT(10,2),NEC,RECT(10,2),
     1 NALC,ALPHC(10,2),FYS,ES,NYS,RYS(10,2),NAN,ANT(10,2),
     2 NK,AKT(10,2),FYSPS,NYSP,RYSPS(10,2),NANPS,ANPST(10,2),
     3 NKPS,AKPST(10,2),NES,RES(10,2),NALS,ALPHS(10,2)
      COMMON/INDEX/NTINC
      IF(NTINC .GT. 1) GO TO 101
C     TIME = 0 -- ROOM TEMPERATURE -- SECTION PROPERTIES CONSTANT
      FT = 5.*SQRT(FC)
      DO 110 J = 1, JRANGE
      IRANGE = IMAX(J) - IMIN(J)
      DO 110 I = 1, IRANGE
      AMODC(I,J) = EC
      FPC(I,J) = FC
      FPT(I,J) = FT
110   ALPHAC(I,J) = 0.
      DO 111 I = 1, NSS
      IF(NSMAT(I) .EQ. 1) GO TO 112
C   NSMAT(I) = 0 - ORDINARY RE-BAR
      FY(I) = FYS
      AK(I) = AKT(1,2)
      AN(I) = ANT(1,2)
      GO TO 113
C   NSMAT(I) = 1 - PRESTRESSING CABLE
112   FY(I) = FYSPS
      AK(I) = AKPST(1,2)
      AN(I) = ANPST(1,2)
113   AMODS(I) = ES
111   ALPHAS(I) = ALPHS(1,2)
      RETURN
```

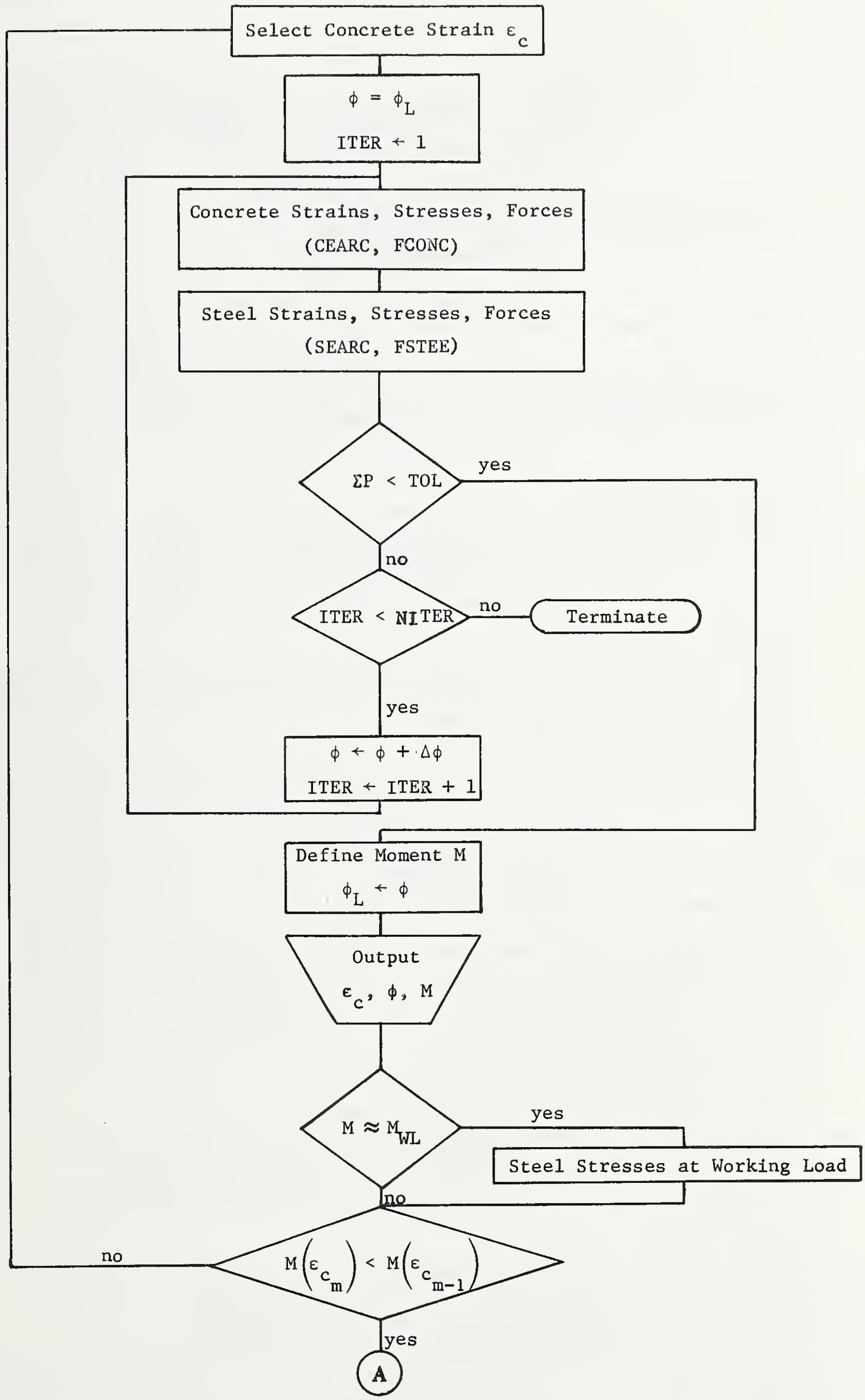
```
C    TIME .NE. 1 - ELEVATED TEMPERATURE - CALCULATE MATERIAL
C   PROPERTIES FOR EACH CONCRETE ELEMENT AND EACH STEEL BAR
101     CONTINUE
C    PROPERTIES FOR CONCRETE
        DO 120 J = 1, JRANGE
        IRANGE = IMAX(J) - IMIN(J)
        DO 120 I = 1, IRANGE
        CTA = (CTEMP(I,J)+CTEM1(I,J))/2.
        AMODC(I,J) = EC*VALUE(NEC,PECT,CTA)
        FPC(I,J) = FC*VALUE(NFC,RFCT,CTA)
        FPT(I,J) = FT*VALUE(NFT,RFT,CTA)
        ALPHAC(I,J) = VALUE(NALC,ALPHC,CTA)
120     CONTINUE
C    PROPERTIES FOR STEEL - BOTH REINFORCING AND PRESTRESSING
        DO 121 I = 1, NSS
        STA = (STEMP(I)+STEM1(I))/2.
        IF(NSMAT(I) .EQ. 1) GO TO 122
C   NSMAT(I) = 0 - ORDINARY RE-BAR
        FY(I) = FYS*VALUE(NYS,RYS,STA)
        AK(I) = VALUE(NK,AKT,STA)
        AN(I) = VALUE(NAN,ANT,STA)
        GO TO 123
C    NSMAT(I) = 1 - BAR I IS PRESTRESSING CABLE
122     FY(I) = FYSPS*VALUE(NYSP,RYSPS,STA)
        AK(I) = VALUE(NKPS,AKPST,STA)
        AN(I) = VALUE(NANPS,ANPST,STA)
C   BOTH REIINFORCING AND PRESTRESS HAVE SAME EXPRESSIONS
C    FOR MODULUS, COEFFICIENT OF THERMAL EXPANSION
123     AMODS(I) = ES*VALUE(NES,RES,STA)
        ALPHAS(I) = VALUE(NALS,ALPHS,STA)
121     CONTINUE
        RETURN
        END
```

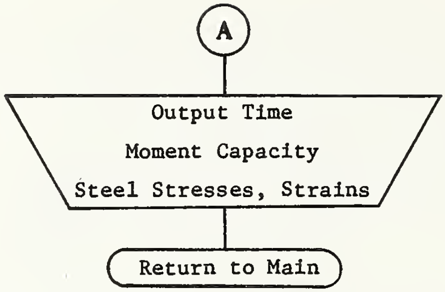
```
      FUNCTION VALUE(NP,PAR,T)
C   FUNCTION DETERMINES MATERIAL PROPERTY VALUE FOR TEMPERATURE T
      DIMENSION PAR(10,2)
      I = 1
104   IF(PAR(I,1)-T) 101,102,103
102   VALUE = PAR(I,2)
      RETURN
103   VALUE = PAR(I-1,2)+(PAR(I,2)-PAR(I-1,2))*(T-PAR(I-1,1))/
     1(PAR(I,1)-PAR(I-1,1))
      RETURN
101   I = I + 1
      IF(I .LE. NP) GO TO 104
      VALUE = PAR(NP,2)
      WRITE(6,105) T, PAR(NP,1)
105   FORMAT(5X,'ERROR - BOUNDS OF CURVE DESCRIBING MATERIAL
     1PARAMETER EXCEEDED.   T =',E12.5,5X,'TEMP(NP) =',E12.5)
      RETURN
      END
```

```
      SUBROUTINE THSTRN
C THIS ROUTINE COMPUTES THE THERMAL STRAINS IN CONCRETE & STEEL
      COMMON/GEOM/JRANGE,IMIN(50),IMAX(50)
      COMMON/STL/NSS,SDEPT(20),STEMP(20),STEM1(20),NSMAT(20),FY(20),
     1 AK(20),AN(20),AMODS(20),ALPHAS(20),EPST(20)
      COMMON/CONC/CDEPT(29,49),CAREA(29,49),CTEMP(29,49),CTEM1(29,49),
     1 FPC(29,49),FPT(29,49),AMODC(29,49),ALPHAC(29,49),CEPT(29,49),
     2 ALDTCT
      COMMON/INDEX/NTINC,IREF
C  SINCE AT ROOM TEMPERATURE THSTRN IS NOT CALLED,
C  CEPT AND EPST MUST BE INITIALIZED WHEN NTINC = 1
      IF(NTINC .GT. 1) GO TO 72
      ALDTCT = 0.0
      DO 68 J = 1, JRANGE
      IR = IMAX(J)-IMIN(J)
      DO 68 I = 1, IR
68    CEPT(I,J) = 0.
      DO 69 N = 1, NSS
69    EPST(N) = 0.
       RETURN
C
C  THERMAL EXPANSION IN CONCRETE
72    DO 70 J = 1, JRANGE
      IRANGE = IMAX(J) - IMIN(J)
      DO 70 I = 1, IRANGE
      DCEPT = ALPHAC(I,J)*(CTEMP(I,J)-CTEM1(I,J))
70    CEPT(I,J) = CEPT(I,J) + DCEPT
C  THERMAL EXPANSION AT TOP OF BEAM
      I1 = IREF - IMIN(JRANGE)
      I2 = IREF - IMIN(JRANGE-1)
      SL = CEPT(I2,JRANGE-1)-CEPT(I1,JRANGE)
      SL = SL/(CDEPT(I2,JRANGE-1)-CDEPT(I1,JRANGE))
      ALDTCT = CEPT(I1,JRANGE)-SL*CDEPT(I1,JRANGE)
C
C   THERMAL EXPANSION IN THE REINFORCEMENT
      DO 71 N = 1, NSS
      DSEPT = ALPHAS(N)*(STEMP(N)-STEM1(N))
71    EPST(N) = EPST(N) + DSEPT
C
      RETURN
      END
```

```
      SUBROUTINE CREEP
C*THE CREEP ANALYSIS IS BASED ON THE DORN-HARMATHY THEORY AND ON THE    *
C*     TIME HARDENING RULE                                               *
C     *******************************************************************
      COMMON/STL/NSS,SDEPT(20),STEMP(20),STEM1(20),NSMAT(20),FY(20),
     1 AK(20),AN(20),AMODS(20),ALPHAS(20),EPST(20),EPSPST(20),
     2 SSTRN(20),TSTS(20),SAREA(20),STRESS(20),EPSW(20),FSW(20)
      COMMON/CRP/CREEP(20),TCMPT(20),Z(20),EPTO(20),EPSC1(20),
     1EPSC2(20),CZ(8,2)
      COMMON/TTEMP/TIME,TIMP
      COMMON/INDEX/NTINC
      DIMENSION DELH(20)
      DOUBLE PRECISION TCMPT,DELH,TAV,DEP
C  WORKING STRESS IN REINFORCEMENT IS REQUIRED FOR THIS ROUTINE
C  WHEN NTINC = 1, PARAMETERS MUST BE INITIALIZED
      IF(NTINC .GT. 1) GO TO 16
      DO 17 I = 1, NSS
      TCMPT(I) = 0.0
      CREEP(I) = 0.0
      EPSC1(I) = 0.
      K = NSMAT(I)+1
17    DELH(I) = CZ(8,K)
       RETURN
C  DETERMINE HARMATHY-DORN CONSTANTS
16    DO 10 I = 1, NSS
      AFSW  = ABS(FSW(I))
      K = NSMAT(I)+1
      EPTO(I) = CZ(1,K)*AFSW**CZ(2,K)
      IF(AFSW - CZ(7,K)) 12,12,13
12    Z(I) = CZ(3,K)*AFSW**CZ(4,K)
      GO TO 10
13    Z(I) = CZ(5,K)*EXP(CZ(6,K)*AFSW)
10     CONTINUE
C  INITIALIZE EPSC1 WHEN NTINC .NE. 1
       DO 18 N = 1, NSS
      Y = Z(N)*TCMPT(N)/EPTO(N)
      IF(Y .GT. .29) GO TO 19
      X  = CBRT(Y*5.088)
      GO TO 18
19    X = -2.*(1.+Y)
      X = Y+1.-2.*EXP(X)
18    EPSC1(N) = X*EPTO(N)*ABS(FSW(N))/FSW(N)
C  COMPUTE CREEP INCREMENT FOR THIS TIME INTERVAL AND
C  TOTAL ACCUMULATED CREEP STRAIN
      DO 220 N=1,NSS
130       TAV=.5*(STEMP(N)+STEM1(N))+460.
          DEP=DELH(N)/TAV
          TCMPT(N)=TCMPT(N)+DEXP(-DEP)*(TIME-TIMP)/60.
150       Y=Z(N)*TCMPT(N)/EPTO(N)
          IF (Y.GT..29) GO TO 160
          X=CBRT(Y*5.088)
          GO TO 170
160   X = -2.*(1.+Y)
      X = Y+1.-2.*EXP(X)
170   EPSC2(N) = X*EPTO(N)*ABS(FSW(N))/FSW(N)
      DEPSC = EPSC2(N) - EPSC1(N)
      IF(DEPSC .LT. 0.) DEPSC = 0.0
220   CREEP(N) = CREEP(N) + DEPSC
      RETURN
      END
```

Determines Moment-Curvature Relation for Fixed Time Temperature Using Concrete Strain, ε_c






```
      SUBROUTINE MOMCUR(BMWL,$)
C
C    SUBROUTINE CALCULATES MECHANICAL EQUILIBRATING STRAINS
C    IN CONCRETE AND STEEL AT A GIVEN TEMPERATURE, AND
C    CALCULATES MOMENT-CURVATURE RELATION FROM A SET OF
C    TABULATED CONCRETE STRAINS
C
      DIMENSION BM(50)
      COMMON/GEOM/JRANGE,IMIN(50),IMAX(50),JMIN,JMAX,NEQ(50),
     1 XX(30,50),YY(30,50),XS(20),YS(20),BRYC(20,3),NLS(20,4),YDEP
      COMMON/STL/NSS,SDEPT(20),STEMP(20),STEM1(20),NSMAT(20),FY(20),
     1 AK(20),AN(20),AMODS(20),ALPHAS(20),EPST(20),EPSPST(20),
     2 SSTRN(20),TSTS(20),SAREA(20),STRESS(20),EPSW(20),FSW(20),
     3 EPSULT(20),FSULT(20)
      COMMON/TTEMP/TIME
      COMMON/INDEX/NTINC,IREF,TOL,NITER,SIDEAL,STCAL,IEPSC,ISTRSA,ICONV
      DOUBLE PRECISION PHI,PHIWL,PHIL,DELT,GMMA,GML,GMWL
       IF(NTINC .NE. 1) GO TO 1069
      PHIWL = 0.0004 / YDEP
1069  NSTEP = 1
      PHIL = PHIWL
      BMP = 0.0
      MM = 0
3000  ICONV = 0
1074  EPSC = EPS(NTINC,NSTEP,MM,EPSWL)
      PHI = PHIL
      ITER = 1
C  ASSUME PHI AND ITERATE THROUGH ITER .LE. NITER CYCLES TO
C  FIND PHI(THRUST=0)
1072  CALL CEARC(EPSC,PHI)
      CALL FCONC(SUMC,SUMT,SUMM)
      CALL SEARC(EPSC,PHI)
      CALL FSTEE(SUMC,SUMT,SUMM)
      THRUST = SUMC + SUMT
      IF(ABS(THRUST) .LE. TOL) GO TO 1071
C  HORIZONTAL FORCES DO NOT BALANCE - ADJUST PHI AND TRY AGAIN
      IF(ITER .GE. NITER) GO TO 1073
      PHI = PHI - DELT(ICONV,MM,ITER,EPSC,PHI,THRUST,PHIL,GML,
     1GMWL,GMMA)
      ITER = ITER + 1
      GO TO 1072
C   SOLUTION FOR PHI EXCEEDS NITER TRIALS ALLOWABLE.
1073  WRITE(6,1001)
1001  FORMAT(1X,'SOLUTION FOR PHI DOES NOT CONVERGE')
      IF(ICONV .EQ. 1) CALL EXIT
      WRITE(6,1101)
1101  FORMAT(10X,'PHI',9X,'P',8X,'PHIA',8X,'PA',9X,'GMA',9X,'DPHI')
      ICONV = 1
       GO TO 1074
```

```
C    DEFINE BENDING MOMENT
1071  BM(NSTEP) = SUMM
C  ASSUMED PHI FOR EPS(NSTEP+1)
      PHIL = PHI
      GML = GMMA
      IF(ISTRSA .EQ. 1) CALL STSAOT(NSTEP,EPSC,PHI,SUMM,ITER)
      IF(IEPSC .EQ. 1) CALL EPSCOT(MM,NSTEP,EPSC,PHI,SUMM,ITER)
C    CHECK IF CALCULATED MOMENT APPROXIMATES WORKING LOAD
C    MOMENT BMWL
      IF(BM(NSTEP) .LT. BMWL) GO TO 1077
C  CHECK FOR FIRST ASSUMED STRAIN (NSTEP = 1) IN TIME INCREMENT
      IF(NSTEP .NE. 1) GO TO 1090
      MM = -1
      GO TO 1074
1090  IF(BMP .GE. BMWL) GO TO 1075
C    COMPUTE WORKING STRESSES IN STEEL AT BMWL
      CBMWL = (BMWL-BMP)/(BM(NSTEP)-BMP)
      DO 1076 N = 1, NSS
      EPSW(N) = EPSW(N) + (SSTRN(N)-EPSW(N))*CBMWL
1076  FSW(N) = STESTR(EPSW(N),FY(N),AK(N),AN(N),AMODS(N),SIDEAL)
      PHIWL = PHIWL + CBMWL * (PHI - PHIWL)
      EPSWL = EPSC
      GMWL = GMMA
      IF(IEPSC .EQ. 1 .OR. ISTRSA .EQ. 1) CALL STWLOT
      GO TO 1075
1077  DO 1078 N = 1, NSS
1078  EPSW(N) = SSTRN(N)
      PHIWL = PHI
      MM = 1
1075  IF(BM(NSTEP) .GE. BMP .OR. BM(NSTEP) .LE. 0.0) GO TO 1079
      WRITE(6,1003) TIME, BM(NSTEP-1)
1003  FORMAT(5X,'TIME =',F10.4,5X,'ULTIMATE MOMENT CAPACITY =',E15.5)
      IF(IEPSC .EQ. 1 .OR. ISTRSA .EQ. 1) CALL STULOT
      IF(BM(NSTEP-1) .GT. BMWL) RETURN
      WRITE(6,1086)
1086  FORMAT(5X,'*****BEAM CANNOT CARRY WORKING LOAD*****')
      RETURN 2
1079  BMP = BM(NSTEP)
       DO 1081 N = 1, NSS
       EPSULT(N) = SSTRN(N)
 1081  FSULT(N) = STRESS(N)
      NSTEP = NSTEP + 1
      GO TO 3000
      END
```

```
      FUNCTION EPS(NTINC,NSTEP,MM,EPSWL)
C   THIS FUNCTION DETERMINES THE INCREMENT FOR EPSC
C   BASED ON THE PREVIOUS EPSCWL AND WHETHER THE
C   COMPUTED BM IS .GT. OR .LT. BMWL (MM = -1 OR MM = 1)
C
C   ONCE EPSCWL IS LOCATED EPSC IS INCEREMENTED UNTIL
C   THE ULTIMATE BM IS REACHED
C        .
      IF(MM .NE. 0) GO TO 200
      IF(NTINC .EQ. 1 ) GO TO 100
      IF(NSTEP .EQ. 1) EPSL = EPSWL
      GO TO 200
 100  IF(NSTEP .EQ. 1) EPSL = -0.0004
 200  IF(MM .GT. 0) GO TO 300
      EPS = EPSL + 0.0002
      EPSL = EPS
      RETURN
 300  IF(EPSL .LE. -0.0009) GO TO 400
      EPS = EPSL - 0.0002
      EPSL = EPS
      RETURN
 400  EPS = EPSL - 0.0005
      EPSL = EPS
      RETURN
      END
```

```
      SUBROUTINE CEARC(EPSC,PHI)
C THIS ELEMENT FINDS THE STRESS IN EACH ELEMENT ATTRE LOCATING THE
C       APPROPPIATE TEMPERATURE LEVEL
       DOUBLE PRECISION PHI
      COMMON/GEOM/JRANGE,IMIN(50),IMAX(50)
      COMMON/CONC/CDEPT(29,49),CAREA(29,49),CTEMP(29,49),CTEM1(29,49),
     1 FPC(29,49),FPT(29,49),AMODC(29,49),ALPHAC(29,49),CEPT(29,49),
     2 ALDTCT,CSTRN(29,49),TSTC(29,49),CSTRES(29,49)
C   ALL DISTANCES EXCEPT YY ARE MEASURED FROM THE TOP OF
C   THE BEAM
      DO 220 J = 1,  JRANGE
       IRANGE = IMAX(J) - IMIN(J)
      DO 220 I = 1, IRANGE
      CSTRN(I,J) = EPSC + PHI * CDEPT(I,J) - (CEPT(I,J)-ALDTCT)
      TSTC(I,J) = CSTRN(I,J) + CEPT(I,J)
       CSTRES(I,J) = CONSTR(CSTRN(I,J),FPC(I,J),FPT(I,J),AMODC(I,J))
 220   CONTINUE
      RETURN
      END
```

```
      FUNCTION CONSTR(X,FC,FT,EMODC)
C
C  FUNCTION COMPUTES CONCRETE ELEMENT STRESS, GIVEN STRAIN
C
      R = -2.*FC/EMODC
      ECULT = 1.75*R
      IF (X .LT. ECULT) GO TO 13
      IF(X .GT. 0.0) GO TO 10
      IF(X) 11,12,12
11    CONSTR = EMODC*X/(1.+(X/R)**2)
      RETURN
12    CONSTR = EMODC*X
      RETURN
13    RE = (ECULT/R)**2
      A = EMODC*ECULT/(1.+RE)
      SLA = EMODC*(1.-RE)/(1.+RE)**2
      CONSTR = A + SLA*(X-ECULT)
      IF(CONSTR .LE. 0.) RETURN
C  STRESS IN ELEMENT IS ZERO IF IT EXCEEDS CRUSHING STRAIN
C  OR TENSILE STRENGTH
 10    CONSTR = 0.0
       RETURN
      END
```

```
      SUBROUTINE FCONC (SUMC,SUMT,SUMM)
C THIS ELEMENT FINDS THE  FORCE AND MOMENT FOR EACH ELEMENT
      COMMON/GEOM/JRANGE,IMIN(50),IMAX(50)
      COMMON/CONC/CDEPT(29,49),CAREA(29,49),CTEMP(29,49),CTEM1(29,49),
     1 FPC(29,49),FPT(29,49),AMODC(29,49),ALPHAC(29,49),CEPT(29,49),
     2 ALDTCT,CSTRN(29,49),TSTC(29,49),CSTRES(29,49)
      SUMC = 0.
      SUMT = 0.
      SUMM = 0.
      DO 50 J=1,JRANGE
      IIRAN = IMAX(J)-IMIN(J)
          DO 40 I=1,IIRAN
              FORCE=CSTRES(I,J)*CAREA(I,J)
              IF (FORCE) 10,40,20
10            SUMC=SUMC+FORCE
              GO TO 30
20            SUMT=SUMT+FORCE
30            SUMM=SUMM+FORCE*CDEPT(I,J)
40            CONTINUE
50        CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SEARC(EPSC,PHI)
C  DETERMINES STRESS IN EACH STEEL ELEMENT OF BEAM SECTION
C
       DOUBLE PRECISION PHI
      COMMON/STL/NSS,SDEPT(20),STEMP(20),STEM1(20),NSMAT(20),FY(20),
     1 AK(20),AN(20),AMODS(20),ALPHAS(20),EPST(20),EPSPST(20),
     2 SSTRN(20),TSTS(20),SAREA(20),STRESS(20)
      COMMON/CONC/CDEPT(29,49),CAREA(29,49),CTEMP(29,49),CTEM1(29,49),
     1 FPC(29,49),FPT(29,49),AMODC(29,49),ALPHAC(29,49),CEPT(29,49),
     2 ALDTCT.
      COMMON/CRP/CREEP(20)
      COMMON/INDEX/NTINC,IREF,TOL,NITER,SIDEAL
C  ROOM TEMPERATURE - NO IMPOSED STRAINS OTHER THAN
C  PRESTRESS, IF ANY
C  ELEVATED TEMPERATURE CALCULATIONS REQUIRE THERMAL EXPANSION AND CREEP
 STRAINS
      DO 102 I = 1, NSS
      SSTRN(I) = EPSC + PHI * SDEPT(I) - (EPST(I) + CREEP(I) - ALDTCT)
      IF(NSMAT(I) .EQ. 0) GO TO 101
      SSTRN(I) = SSTRN(I) + EPSPST(I)
101   TSTS(I) = SSTRN(I) + EPST(I) +CREEP(I)
102   STRESS(I)=STESTR(SSTRN(I),FY(I),AK(I),AN(I),AMODS(I),SIDEAL)
      RETURN
      END



      FUNCTION STESTR(X,YS,AK,AN,ES,SIDEAL)
C
C   FUNCTION COMPUTES REINFORCEMENT STRESSES, GIVEN
C  MECHANICAL STRAIN.
C
      IF(SIDEAL .EQ. 'FLATT') GO TO 16
      AX = X
      EPL = 0.75 * YS / ES
      IF(AX)10,10,11
10    AX = -AX
11    EU = (ES/(AK*AN))**(1./(AN-1.)) + EPL
      IF (AX .GT. EU) GO TO 13
      IF (AX .GT. EPL) GO TO 12
      STESTR = AX * ES
      GO TO 15
13    AX = EU
12    STESTR = ES*AX - AK*(AX-EPL)**AN
  15  IF(X .LE. 0.) STESTR = -STESTR
      RETURN
C
C  THIS PORTION INSERTED FOR NSITIVITY STUDY OF STRESS-STRAIN
C   IDEALIZATION
C
16    AX = ABS(X)
      EY = YS / ES
      IF( AX .GE. EY ) GO TO 17
      STESTR = AX * ES
      GO TO 18
17    STESTR = YS
18    IF ( X .LT. 0.0 ) STESTR = - STESTR
      RETURN
C
      END
```

```
      SUBROUTINE FSTEE (SUMC,SUMT,SUMM)
C      THIS ELEMENT CALCULATES THE FORCE IN THE STEEL AND THE MOMENT
C      CARRIED BY THE STEEL
      COMMON/STL/NSS,SDEPT(20),STEMP(20),STEM1(20),NSMAT(20),FY(20),
     1 AK(20),AN(20),AMODS(20),ALPHAS(20),EPST(20),EPSPST(20),
     2 SSTRN(20),TSTS(20),SAREA(20),STRESS(20)
C  SUMM, SUMT, AND SUMM HAVE BEEN INITIALIZED IN FCONC
C
      DO 40 N = 1, NSS
      FORCE = STRESS(N)*SAREA(N)
          IF (FORCE) 10,40,20
10        SUMC=SUMC+FORCE
          GO TO 30
20        SUMT=SUMT+FORCE
30        SUMM=SUMM+FORCE*SDEPT(N)
40        CONTINUE
      RETURN
      END




      DOUBLE PRECISION FUNCTION DELT(ICONV,MM,IE,EPSC,PHI,P,
     1DELTMX,GML,GMWL,GAMMA)
C
C   PROGRAM DETERMINES OPTIMAL INCREMENT FOR ZEROING THRUST
C   ON BEAM USING SELF-ACCELLERATING ITERATION.
C   REFERENCE - J. TRAUB, ITERATIVE METHODS FOR SOL'N OF EQNS, P. 186
C
C  IF FIRST STRAIN, SELECT INITIAL VALUES
      COMMON/INDEX/NTINC
       DOUBLE PRECISION PHI,APHI,GAMMA,BETA,BTP,DELTMX,GML,GMWL
      IF(IE .GT. 1) GO TO 101
C   DEFINE INITIAL VALUE OF ACCELLERATING PARAMETER BETA,
C   UTILIZING FACT THAT DP/D(PHI) .GT. 0
      IF(MM .EQ. 0) GO TO 104
      BETA = -1. /GML
      GO TO 102
 104  IF(NTINC .EQ. 1) GO TO 103
      BETA = -1. / GMWL
      GO TO 102
 103  BETA = -1.0
      GO TO 102
101   BETA = -1./GAMMA
102   BTP = BETA * P
      APHI = PHI + BTP
      CALL CEARC(EPSC,APHI)
      CALL FCONC(SUMC,SUMT,SUMM)
      CALL SEARC(EPSC,APHI)
      CALL FSTEE(SUMC,SUMT,SUMM)
      PA = SUMC + SUMT
      GAMMA = (PA - P)/BTP
      DELT = P / GAMMA
      IF(DABS(DELT) .GT. DELTMX) DELT = DELTMX * DABS(DELT) / DELT
      IF(ICONV .EQ. 1)WRITE(6,100)PHI,P,APHI,PA,GAMMA,DELT
100   FORMAT(D12.5,E12.4,D12.5,E12.4,2D12.4)
      RETURN
      END
```

```
      SUBROUTINE OUTPUT
      COMMON/GEOM/JRANGE,IMIN(50),IMAX(50),JMIN,JMAX,NEQ(50),
     1 XX(30,50),YY(30,50),XS(20),YS(20),BRYC(20,3),NLS(20,4)
      COMMON/STL/NSS,SDEPT(20),STEMP(20),STEM1(20),NSMAT(20),FY(20),
     1 AK(20),AN(20),AMODS(20),ALPHAS(20),EPST(20),EPSPST(20),
     2 SSTRN(20),TSTS(20),SAREA(20),STRESS(20),EPSW(20),FSW(20),
     3 EPSULT(20),FSULT(20)
      COMMON/CONC/CDEPT(29,49),CAREA(29,49),CTEMP(29,49),CTEM1(29,49),
     1 FPC(29,49),FPT(29,49),AMODC(29,49),ALPHAC(29,49),CEPT(29,49),
     2 ALDTCT,CSTRN(29,49),TSTC(29,49),CSTRES(29,49)
      COMMON/CRP/CREEP(20),TCMPT(20),Z(20),EPTO(20)
      COMMON/TTEMP/TIME
      DOUBLE PRECISION TCMPT,PHI
C
      ENTRY GEOMOT
C
      JRAN = JMAX - JMIN + 1
      WRITE(6,710)
  710 FORMAT(9X,'J',6X,'IMAX',6X,'IMIN',7X,'NEQ')
      DO 730 J =1,JRAN
      M = J + JMIN - 1
  730 WRITE(6,720) M,IMAX(J),IMIN(J),NEQ(J)
  720 FORMAT(4I10)
      WRITE(6,740)
  740 FORMAT(4X,'I',4X,'J',13X,'XX',13X,'YY')
      DO 750 J= 1,JRAN
      IRAN = NEQ(J)
      DO 750 I =1,IRAN
      JJ = J + JMIN - 1
      II = I + IMIN(J) - 1
  750 WRITE(6,760) II,JJ,XX(I,J),YY(I,J)
  760 FORMAT(2I5,2E15.5)
      DO 240 N=1,NSS
          WRITE (6,230) (N,K,NLS(N,K),K=1,4)
 230      FORMAT (4(' NLS(',I2,',',I2,')=',I2))
 240      CONTINUE
      WRITE (6,250)
 250  FORMAT ('         ')
      DO 270 N=1,NSS
          WRITE (6,260) (N,K,BRYC(N,K),K=1,3)
 260      FORMAT (3(' BRYC(',I2,',',I1,')=',E10.5))
 270      CONTINUE
      WRITE (6,250)
      RETURN
C
      ENTRY TEMPOT
C
      WRITE(6,200)
200   FORMAT(7X,'BAR',5X,'TEMP')
      WRITE(6,201) (N,STEMP(N), N=1,NSS)
201   FORMAT(I10,F10.4)
      WRITE(6,300)
 300  FORMAT(7X,'BAR',10X,'FY',7X,'AMODS',10X,'AK',10X,'AN')
      WRITE(6,301) (N,FY(N),AMODS(N),AK(N),AN(N),N=1,NSS)
 301  FORMAT(I10,4E12.4)
      WRITE(6,400)
 400  FORMAT(7X,'BAR',7X,'EPSTH')
      WRITE(6,401) (N, EPST(N), N=1,NSS)
 401  FORMAT(I10,E12.5)
      WRITE(6,280)
 280  FORMAT(7X,'BAR',11X,'Z',8X,'EPTO',9X,'TCT',7X,'CRP')
      WRITE(6,271) (N,Z(N),EPTO(N),TCMPT(N),CREEP(N),N=1,NSS)
 271  FORMAT(I10,2E12.4,D12.5,E12.4)
      RETURN
```

```
      ENTRY STSAOT(NSTEP,EPSC,PHI,SUMM,ITER)
C
      WRITE(6,1186) TIME,EPSC
1186  FORMAT('0'/10X,'TOTAL STRAINS AT TIME',F6.1,' EPSC',E12.4)
      DO 413 J = 1,JRANGE
      IR = IMAX(J) - IMIN(J)
413   WRITE(6,500) J,(TSTC(I,J),I=1,IR)
      WRITE(6,1187)
1187  FORMAT(7X,'BAR',4X,'TOTAL STEEL STRAIN')
      WRITE(6,1188) (N,TSTS(N),N=1,NSS)
1188  FORMAT(I10,E12.4)
      WRITE(6,1189)
1189  FORMAT('0'/10X,' MECHANICAL CONCRETE STRAINS')
      DO 414 J = 1,JRANGE
      IR = IMAX(J) - IMIN(J)
414   WRITE(6,500) J,(CSTRN(I,J),I=1,IR)
       WRITE(6,1084)
 1084  FORMAT(1H0/  10X,'CONCRETE STRESSES')
       DO 412 J = 1, JRANGE
       IR = IMAX(J) - IMIN(J)
  412   WRITE(6,500) J, (CSTRES(I,J),I=1,IR)
 500   FORMAT(I5,(7E10.4))
      WRITE(6,1190)
1190  FORMAT('0'/10X,'MECHANICAL STEEL STRAINS & STRESSES')
      WRITE(6,1191)
1191  FORMAT(7X,'BAR',6X,'STRAIN',6X,'STRESS') .
      WRITE(6,1083) (N,SSTRN(N),STRESS(N),N=1,NSS)
      WRITE(6,1192)
1192  FORMAT('0')
      GO TO 3000
C
      ENTRY EPSCOT(MM,NSTEP,EPSC,PHI,SUMM,ITER)
C
      IF(MM .NE. 0) GO TO 4000
3000  WRITE(6,2002)
2002  FORMAT(11X,'EPSC',12X,'PHI',9X,'MOMENT',4X,'CYCLES')
4000  WRITE(6,1002) EPSC,PHI,SUMM,ITER
1002  FORMAT(E15.5,D15.5,E15.5,I10)
      RETURN
C
      ENTRY STWLOT
C
      WRITE (6,101)
101   FORMAT(10X,' WORKING STRESSES & STRAINS IN STEEL')
      WRITE(6,102)
102   FORMAT(7X,'BAR',8X,'FSWL',7X,'EPSWL')
      WRITE(6,1083) (N,FSW(N),EPSW(N), N=1,NSS)
      WRITE(6,2002)
      RETURN
C
      ENTRY STULOT
C
       WRITE(6,1082)
 1082  FORMAT(7X,'BAR',6X,'EPSULT',7X,'FSULT')
       WRITE(6,1083) (N,EPSULT(N),FSULT(N), N=1,NSS)
 1083  FORMAT(I10,2E12.4)
      RETURN
      END
```

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16. ABSTRACT (*A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.*)

Methods for analytically predicting the behavior of simply supported reinforced concrete beams subjected to fire are presented. This is generally a two-step process involving a thermal analysis followed by a stress analysis. This study emphasizes the latter, wherein the determination of moment-curvature-time relationships for the beam cross section incorporates the temperature-dependent strength degradation in the steel and concrete as well as thermal and creep strains. The sensitivity of the predictions to various phases of analytical modeling is investigated to establish the parameters most important for the prediction of beam behavior and to indicate where additional data should be gathered. A comparison of predicted behavior with that observed in fire tests shows excellent agreement when realistic reinforcement temperature histories are used.

17. KEY WORDS (*six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons*)

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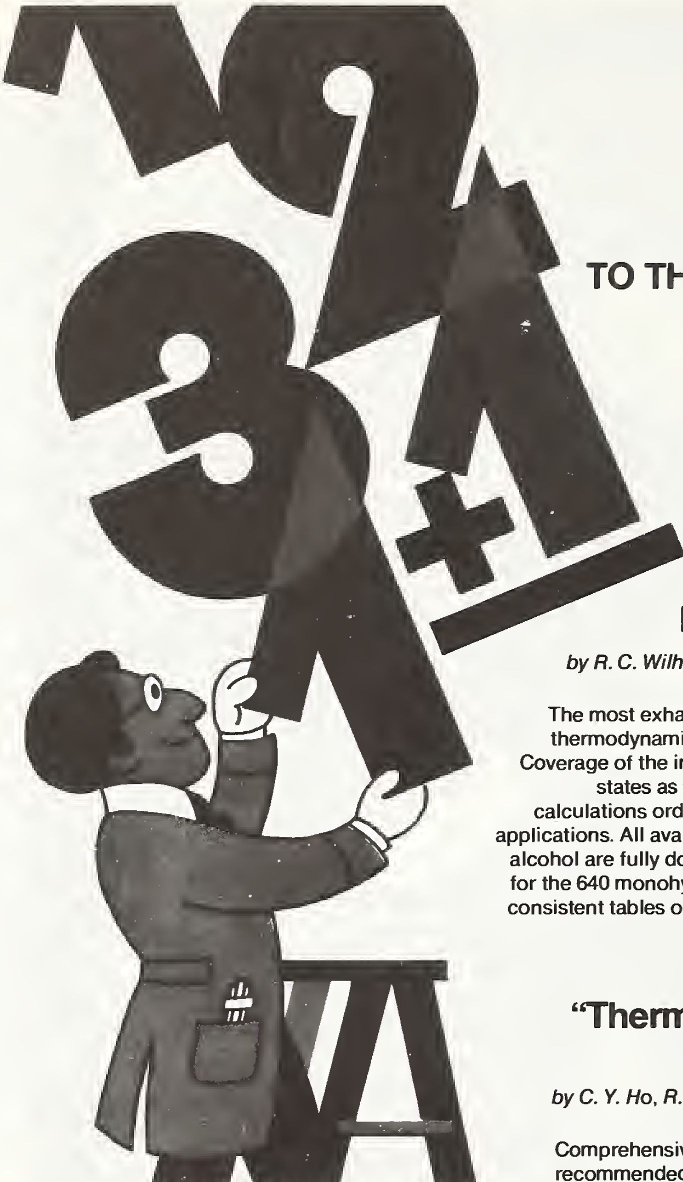

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