Structural Deflections.
A Literature and State-of-the-Art Survey
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Structural Deflections.
A Literature and
State-of-the-Art Survey

Building Science Series

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SI Conversion Units

In view of the present accepted practice in this country for building technology, common U. S. units of measurement have been used throughout this paper. In recognition of the position of the United States as a signatory to the General Conference on Weights and Measures, which gave official status to the metric SI system of units in 1960, assistance is given to the reader interested in making use of the coherent system of SI units by giving conversion factors applicable to U. S. units used in this paper.

Length

1 in = 0.0254 meter (exactly)
1 ft = 0.3048 meter (exactly)

Mass

1 lb (1bm) = 0.4536 kilogram

Force

1 kip = 4448 newton

Stress

1 psf = 47.88 newton/meter$^2$
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Structural Deflections
A Literature and State-of-the-Art Survey
T. V. Galambos, P. L. Gould, M. K. Ravindra, H. Suryoutomo,
and
R. A. Crist

A literature survey and state-of-the-art study was compiled using approximately 225 primary source documents, research papers and texts. Over 800 documents were scanned to arrive at the primary source documents. The problem of structural deflections is discussed and reviewed in its component areas of static and dynamic deflections as related to forcing functions and structural characteristics. Also the interactions of major structural deflections with building structures subsystems and human occupants is reviewed. Emphasis is placed on serviceability limit states of deflections. Detailed comparisons of human response to structural vibrations are also made. This report is broad in scope and covers the areas of analysis, design and experimentation.

Key Words: Analysis; deflection; design; dynamic; experimental; human sensitivity; loading functions; specifications; static; structural engineering; subsystems; vibration.

1. Introduction

This literature survey and state-of-the-art study* encompasses a broad area of the structural engineering field; deflections of building structures. Other types of structures such as towers, bridges, etc. are briefly mentioned but primarily in the context of information that may be applicable to building structures. This study was compiled using approximately 225 primary source documents, research papers, and texts. Over 800 documents were scanned to arrive at the primary source document.

The report was assembled considering building structure deflection in a sequence that would be used when considering the solution of a structural deflection problem, i.e., separation of static and dynamic deflections and then considering each of these as a three component problem of load--structural characteristics--response (deflection). This sequence is developed and defined for the respective components throughout the report. Also the interactions of major structural deflections with building structures subsystems and human occupants is reviewed.

In this report an attempt is made to identify serviceability criteria for the present practice of building construction, to determine the theoretical and experimental basis for these requirements and to evaluate the range of applicability of the existing criteria. Primary emphasis is on the identification and documentation of existing serviceability provisions. The bibliographical listings are not necessarily a listing of all available literature but rather a listing of what was considered by the authors to be the most pertinent. This report is broad in scope and covers the areas of analysis, design and experimentation.

In design and construction in the past, deflections have been relatively small compared to member sizes or average building size, thus, they have not been a dominant concern for the structural designer. Within this decade the rapid increase of building construction costs have been pressing the industry for more economical methods of construction; new materials have become available which result in low effective material modulus and overall reduced structural stiffness; and more sophisticated and accurate methods of design and analysis are being used. For these reasons deflections have become more significant than in the past with respect to design control. Serviceability limit states have become progressively more important. With an emphasis on serviceability, the structural designer is confronted not only with a more complex situation of designing for deflection as well as strength, but, he is also required to solve an interdisciplinary problem. Serviceability of a building structure has a direct correlation to the comfort of the occupants of a structure. The aspects of human comfort as related to structural deflection involve the knowledge of psychologists, bioengineers, the medical profession, only to name a few. Professionals with diverse backgrounds such as structural engineers and psychologists have to find a common ground of communication so that problems can be rationally approached. New vocabularies in each field have to be mastered and understood.

Structural deflections are difficult to discuss unless they are broken down to more specific subjects. For this report serviceability is of primary concern. A serviceable structure is a structure which meets the needs for which it is intended in everyday use. Deflections of a structure at a serviceable level are not generally those deflections which occur at a collapse or an ultimate limit state. Some brief mention of deflections at ultimate limit states will be made in this report with respect to ductility as it applies to life safety and earthquake loading. However, most of the emphasis will be on serviceability referenced to small recurrence interval loadings and to continued functional use of a structure.

The problem of deflections in building structures as viewed in this report is divided into components as shown in figure 1. The primary division is the separation of static and dynamic deflections. Traditionally, many dynamic structural design problems were reduced to a pseudo dynamic problem by use of "equivalent" static loads. Advancement of technology now permits the actual consideration of the dynamic structural problem. The primary difference between the static and dynamic problem is the consideration of the dimension of time in the dynamic case. Dynamic deflections corresponding to dynamic loads are time dependent, i.e., they have a describable time history. Strictly speaking for the general case, all loads and deflections are time dependent and thus dynamic. However, in the more practical sense, if time parameters of loading and structural characteristics are considered, a difference can be made between the static and dynamic cases. A static load, thus causing a static deflection, is one which is slowly applied and released. Slowly refers to the duration of time of application and release of load compared to the natural period of the structure. If the ratio of load application or release duration to natural period is large, the load and corresponding deflection can be considered static. Conversely, if the ratio of load application or release duration to natural period is small, the load and corresponding deflection are dynamic and the dimension of time has to be considered. It also can be stated that in the dynamic case, inertial forces are significant and must be considered to create dynamic equilibrium. An order of magnitude estimate for static and dynamic loads which cause corresponding static and dynamic deflections in building structures can be given. Dynamic loads are applied and released in the order of seconds or less whereas
Figure 1. Problem Statement, Structural Deflections
static loads are applied or released in the order of minutes or greater. Another type of dynamic, i.e., time dependent load, has to be considered in a greater order of magnitude time domain. Long term, in the order of months and years, deflections due to sustained (static) loading attributed to creep of the constituent materials of the structure are important to the serviceability of a structure. These deflections are more accurately understood by the study of the constituent materials. The subject of creep deformations is important but due to the scope of this work has not been emphasized in this report.

Once the differentiation is made between static and dynamic deflections, it becomes clear why traditional deflection design requirements are inadequate in a general sense. The structural design practice has been dominated by deflection criteria such as; the vertical deflection of a beam or floor for a given load shall not exceed the span divided by 360, or the horizontal deflection of a high-rise structure subjected to a lateral load shall not exceed 0.002 times the total height of the structure. In the case of the requirement for floors and beams where occupants are responding to a time dependent deflection, the L/360 criterion has little rational justification. It has been adequate in the past because of the relatively stiff floors using high effective modulus materials. Now that new materials (lower moduli and higher stresses) and more flexible systems are being used, the dimension of time has to be considered in the deflection criteria. In the case of the lateral deflection criteria, pseudo dynamic loads (equivalent static loads) have been used to calculate deflections from structural analysis methods which did not consider the dynamic properties of the structure and did not consider all elements of the structure which contribute to its stiffness. This causes a two-way confusion. The loading is not realistic and the analysis model of the structure is inaccurate, thus, a deflection criterion on this basis has as its only rationale that it has been adequate through experience. When past experience is somewhat nullified by changes in structural design methods and materials, the deflection criteria then becomes inadequate. These two typical deflection requirements have been used as a panacea for a very complex aggregation of design considerations. They should not be expected to be broadly applicable.

Further breakdown of each of the static and dynamic deflection problems can be made. Considering the static deflection problem, possibly the less complex of the two, the problem can be stated in engineering terms as shown on figure 1.

\[ \{p\} = [K] \{y\} \]

From a given load, \(\{p\}\), and given structural characteristics, \(K\), a system deflection, \(\{y\}\), can be determined. Alternatively, with two of the three components of the problem known the other can be determined. The fourth component of this engineering statement is interaction with the occupants of the structure and the subsystem. (figure 1). It is by far the most complex and difficult to define of the four components. The subsystems considered are the partitions, windows, doors, mechanical equipment, etc. Human response is expressed in subjective terms such as "desirable" and "undesirable" which are difficult to define. Even though the subsystem response is objective, it has not been clearly defined in the past what system response characteristics affect the function of the subsystems.

The dynamic deflection problem is shown on figure 1. It can be expressed in block diagram from as

\[ F(t) \quad \rightarrow \quad G(t) \quad \rightarrow \quad Y(t) \]

In general terms, the major components are; the dynamic load or forcing function, \(F(t)\), the structural characteristics or transfer function,
The forcing function can be random or transient such as that created by foot traffic or vehicles. Steady state loading is generally caused by reciprocating mechanical equipment; wind load is both random and transient but it is referred to here separately because of its importance. Earthquake loading is also random and transient, however, as previously mentioned, this type of loading will not be considered extensively. It is applicable to the ultimate limit state more than the serviceability limit state. The possibility should not be overlooked that small recurrence interval, low intensity earthquakes could be considered in the category of serviceability.

Dynamic structural characteristics involve the identification of the natural frequency of a structure (mass and stiffness) and damping. The dynamic structural characteristics are more complex than just determining the natural frequencies of a structural frame. The dynamic structural characteristics are the properties of the entire structure which interact (in the mechanical as well as the mathematical sense) with the forcing function to result in a structural system response. Non-structural elements (partitions, walls, etc.) have a significant effect on these properties. Also it is possible that the interaction of the structure and its foundation may affect the structural characteristics. The system response is a time dependent deflection.

Similar to the static deflection problem, there is an interaction of the dynamic system deflections (figure 1) with the human occupants (human response) and subsystems. Human response is considered in three general divisions, whole body vibration, audial and visual. Whole body vibration is a general category which does not designate the specific cues that sense whole body vibration. Specific cues are the local excitation of internal organs, inner ear, skin, bone, muscle, etc. Human sensitivity (figure 1) has been found to cover a wide range of motion parameters such as acceleration, velocity, damping and total duration of vibration exposure. The classification of human response parameters in terms of quality of response start at a perception threshold and cover a range to an intolerable or damaging level of vibration. The environment of a human while responding to vibration is another important variable affecting the quality of human response to vibration. Visual response to vibrations, such as the sway of high-rise structures observed from one structure to another or the local motion of equipment within a structure, affects the human response. Audible response to vibrations such as the creaking of a structure or the banging of elevator counterweights is also a factor to be considered in human response.

Subsystem response is a parallel area of concern in the dynamic interaction problem. The system deflections must allow for the proper functioning of the subsystems. Measures of the proper functioning of the subsystem are local damage and serviceability.

This breakdown of the structural deflection problem is principally concerned with serviceability. It has divided the problem into its major categories to lead to a rational approach of the problem. Some of these categories have been studied extensively and major contributions have been made to the technology. Other areas have had fragmented or limited study and require further research and assimilation. Subsequently, this report will elaborate on the details of this problem breakdown as shown...
in figure 1. As the literature is reviewed, it will be evident where gaps of knowledge exist. These points will be discussed individually within each section.

It is appropriate to point out how a rational understanding of the structural deflection problem shown in figure 1 can lead to a properly designed structure with respect to deflections. The understanding and solution of the interaction problems lead to allowable deflections. The iteration loop of design applicable to deflections is then completed. It is formed by three major components; structural characteristics, specified load, and allowable deflection.

Many different deflection criteria would be required for satisfactory behavior if all probable loading situations are considered for a particular structure. Several are evident from figure 1.

Static system deflection is controlled to limit:

- human response to static deflection
- subsystem response to static deflection

Dynamic system deflection is controlled to limit:

- dynamic whole body vibration
- audible perception of motion
- dynamic visual perception to motion
- dynamic subsystem response.

Although these are not all that are possible, they appear to be the major categories. Each case can result in a deflection requirement. It is possible that each structure is unique with respect to which of these cases will be significant. Categories may be combined as more is learned of each. Again, it is evident why a single deflection specification, which was derived from tradition without rationale, cannot be expected to result in adequate deflection criteria.
3. Static Deflections

3.1 General

This section covers the computation, measurement and the assessment of the effects of static deflection on structural systems and subsystems. Those deflections which are associated with lateral building movement are discussed separately in Section 4.3. Appendixes A and B include a summary of various code requirements with explicit reference to the respective codes.

3.2 Effects of Deflections on Structures.

The possible undesirable effects of excessive structural deflections have been noted by Allen [2]*. These effects include 1) cracking of primary structural members which may provide a means for the transmission of unwanted sound, moisture and cold air and promote corrosion, as well as being unsightly; 2) cracking or crushing of non-structural components such as partitions; 3) lack of fit of doors and windows; 4) walls out of plumb; 5) eccentricity of loading due to rotation; 6) unsightly droopiness; and 7) ponding. Allen suggests that many deflection-caused problems can be alleviated by alternate design solutions such as flexible joints, etc.

The American Concrete Institute (ACI) Committee 435 report [1] lists the following reasons for limiting deflections: 1) Sensory acceptability (visual, tactile, auditory); 2) Serviceability of structure (surfaces which should drain water, floors which should remain plane, members supporting sensitive equipment); 3) Effect on nonstructural elements (walls, ceilings, adjacent building elements supported by other members); 4) Effect on structural elements (deflections causing instability of primary structure, deflections causing different force system or change in stresses in some other element, deflections causing dynamic effects). In this report examples of each of the items noted are given along with deflection limitations.

A comprehensive examination of the effect of deflections on structures is contained in the report, Deformations, Committee No. 4 of Comité Europeen du Beton [8]. In this report, the effect of deflections on structures are separated into deflections which do and do not effect the overall stability of structures. Those deflections which affect the structural stability are further classified into 1) static; and 2) dynamic effects. Those deflections which do not effect structural stability are classified as 1) psychological or esthetic effects; 2) effects which may produce damage in other non load-bearing structural units; 3) indirect effects on the stability of other structural members or structures; and 4) effect of deflections on the serviceability of the structure.

The American Institute of Timber Construction Standard [19] states the reasons why a deflection limitation requirement is desirable: 1) Possible damage to attached or connected materials such as plaster and roofing; 2) Effect on the function of the completed structure such as

* Numbers in brackets refer to literature references. References are grouped in the respective report sections and are listed in alphabetical order in each section.
vibration and springiness; 3) The acceptable final shape or position of the completed structure or member such as roof pitch and pockets affecting drainage or interior appearance; and 4) Effect on door clearances or effect on sash and glass below a member.

For wood floor structures, Vermeyden [20] had traced the requirements for deflection limitations to a prevention of 1) sagging or cracking of ceilings and jamming of doors; and 2) discomfort. He discusses the effect of long-term deflection as well as instantaneous elastic deflections.

With regard to aluminum structures, the Report on the Structural Use of Aluminum by the Institution of Structural Engineers [14] states that the deflection of a member shall not be such as to impair the strength, functioning or appearance, or to cause damage to the finish of any part of the structure.

Some structural deflection phenomena are often long-term time-dependent and consequently may become evident only after some years. As such, the construction sequence, inelastic material properties, climatic effects and pattern of usage may determine if a given structure will suffer from deflection induced problems. Furthermore, when the designer considers the relative importance of deflection criteria in building design, he should discern between those factors which can lead to catastrophic failure, such as ponding and corrosion, as opposed to those items which simply cause inconvenience, such as jammed windows. A quantifiable method of discerning between failure and unserviceability type criteria has been noted by Davenport [7].

3.3 Quantitative Limitations on Deflections.

Quantitative limitations on deflections are most often given in terms of a fraction of the span length, L. A summary of this type of limitation is given by Allen [3]. In this commentary on the various standards pertinent to the National Building Code of Canada, 1970, various types of roof, floor and wall members are considered along with the most used construction materials, timber, reinforced concrete, steel and aluminum. The general range of values is from L/180 to L/360. These deflection limitations refer to dead load, live load, creep and ponding deflections when applicable. Representative specific values are given in Appendix B.

An early study (1948) of the behavior of houses was done by Whittmore et al at the National Bureau of Standards [21]. Insight of these authors was expressed through the realization that the the L/360 limitation was quite arbitrary. Absolute limitations of deflection for various types of loading (compressive, transverse, concentrated, impact and racking) and elements (walls-load and non-load bearing, floors and roofs) are given. As an example, for transverse loading on floors, 2-inches of deflection is allowed for a 40 lb/ft² superimposed static load. Also a permanent set, for the same condition, of 1-inch is allowed. The rationale for these large allowable deflections is that the actual loading on the floor may not be more than 8 lb/ft². Many qualifications would have to be made before this type of criteria could be used.

A comprehensive summary of deflection limitations for reinforced concrete construction is provided in the ACI Committee 435 report [1]. In this report, deflection limitations are given for the various classifications noted in the previous section. The limitations are given in the usual fraction of span but, in addition, absolute deflection limits are specified with respect to the effect of deflections on non-structural elements. Also, the rationale of using the depth/span ratio as an indirect quantitative index of allowable deflection is developed. A notable feature of the suggested limitations is the specification of the portion of the total deflection upon which each limitation is based.
A great many state and local codes in the United States incorporate the Building Code Requirements for Reinforced Concrete, ACI 318-71 [2]; the Specification for the Design, Fabrication and Erection of Structural Steel, AISC [11]; the American Institute of Timber Construction Specification, AITC [19]; the Specifications for Aluminum Structures [4]; and the Standard Specifications and Load Tables of the Steel Joist Institute, SJI [16]. These standards thus should reflect the general state-of-the-art practice with respect to these building materials. The pertinent deflection limitations are summarized in Appendix A and are discussed in the subsequent paragraphs. The model codes summarized in Appendix A also incorporate these provisions. A comparative review of the model codes is given by Buchert, Mulner and Rubey [6].

The ACI Building Code gives deflection limitations in terms of a fraction of the span for the following member types: 1) Flat roofs not supporting or attached to non-structural elements likely to be damaged by large deflections; 2) Floors not supporting or attached to non-structural elements likely to be damaged by large deflections; 3) Roof or floor construction supporting or attached to non-structural elements likely to be damaged by large deflections; and 4) Roof or floor construction supporting or attached to non-structural elements not likely to be damaged by large deflections. As a second means of controlling deflections, minimum thickness requirements are provided for solid one-way slabs, ribbed one-way slabs and beams which are not supporting or attached to partitions or other construction likely to be damaged by large deflections. Also, minimum thickness requirements are provided for nonprestressed two-way construction.

The AISC specification states that beams and girders supporting floors and roofs shall be proportioned with due regard to the deflection produced by the design loads but only specifies a quantitative limitation of L/360 for members supporting plastered ceilings. However, in the commentary, guidelines are suggested for the depth/span ratio as a function of the yield point stress, $F_y$, as a means for controlling deflection. For fully stressed beams and girders in floors, the suggested value of the depth ratio is $F_y/800$ while for roof purlins $F_y/1000$ (excluding flat roofs) is given.

Another deflection-related phenomenon treated in detail by the AISC specification is ponding, defined therein as the retention of water due solely to the deflection of flat roof framing. The code provisions are based on the spans and moments of inertia of both the primary and secondary members and of the steel deck supported on secondary members. These relations are based on equations given by Marino [12]. In addition, graphical design aids are provided in the commentary when a more exact determination of required flat roof framing stiffness is required. The AISC specification also limits the total bending stress to 0.80 $F_y$ in primary and secondary members due to the weight of the ponded water plus dead and gravity live loads. It is noted that stresses due to wind or seismic forces need not be included in a ponding analysis.

The AITC specification requires the simultaneous satisfaction of deflection limitations of L/240 for total load and L/360 for live load.

The Specification for Aluminum Structures suggests the use of the effective width concept when deflection at design loads is critical.

The SJI specification gives deflection limitations of S/360 for floors and roofs with plastered ceiling attached or suspended and L/240 for other roofs.

A comparison of European standards and specifications for reinforced concrete is provided by the Committee 4 report of Comité Européen du Beton [8] where information from Austria, Belgium, Switzerland, Germany,
Denmark, Spain, England, Italy, Portugal, Poland, Sweden, Turkey and Yugoslavia is presented. The information from this report regarding allowable limits of deformation is included in Appendix B.

A survey of European and North American practice for timber members is provided by Vermeyden [20]. In addition to the Dutch regulations for the limiting deflections of wood beams, he also surveys those from Germany, Switzerland, Austria, Denmark, France, Great Britain and Canada. He notes that in all cases the limitations are based on elastic deflections but that total load is used in some cases while live load is used in others. The specific deflection limitations contained in this paper are summarized in Appendix B.

Another literature survey with similar information to that contained in Vermeyden's report was conducted by Onysko [13]. Deflection limitations from this report are given in Appendix B.

3.4 Computation of Deflections.

When numerically computed values of deflections are specified and presumably correlated with the presence (or absence) of certain undesirable effects in buildings, it is preferable to have a rational and standardized basis of computation. The possibility for achieving this standardization is dependent on the complexity of the structural framing system including connections, foundations and the materials of construction.

With respect to the problems introduced by complex framing systems and the ensuing representation by the mathematical model for which the deflections are computed, an example of the poor correlation between computed and observed deflections of a structural frame is reported by Wiss and Curth [22]. Section 4.3 discusses the available computer-based techniques of structural analysis which enable improved mathematical models of structures to be considered.

The problems introduced by constituent material considerations are principally encountered in the computation of deflections for reinforced concrete members and structures. These problems are enumerated in great detail in ACI Committee 435 report [1] and appear to be a result of several factors: 1) the assumption that concrete carries no tension for strength calculations which may be unduly conservative for deflection calculations; 2) the assessment of the end restraint introduced by monolithic construction; 3) the non-linear material behavior in the working load range prior to and beyond cracking; 4) the historical sequence of loading including construction and load test sequences; 5) the contribution of the transformed area of the reinforcing steel to the stiffness in uncracked sections and the contribution of the compression steel to the stiffness in cracked sections; and 6) long-term deflections due to shrinkage and creep. The difficulties involved in considering any one of these factors makes the precise determination of deflections in reinforced concrete structures a somewhat academic exercise which suggests that limitations on computed deflections should be rather broad.

With respect to European practices, a comparative study of various assumptions for elastic modulus, effect of sustained loading and effective moment of inertia is contained in Committee 4 report of Comite European du Beton [8] along with some suggested overall methods of computing deflections.

For timber structures, creep under sustained load must also be considered in addition to elastic deformations. A method for computing deflections in timber structures including creep effects is proposed by Vermeyden [20].
With regard to structures constructed of structural steel or aluminum, it appears that the accepted methods of elastic analysis are applicable with the basic assumption of linear elastic behavior in the working load range. Some further elaboration, however, is needed for steel structures designed by the method of plastic design. Since this method considers the structural behavior at the collapse load and many of the deflection limitations found in the various codes are based on structural serviceability at working loads, the determination of deflections prior to the formation of the "last hinge" and collapse will not usually be sufficient to check serviceability criteria. This aspect of plastic design has been investigated by the Joint WRC-ASCE Committee on Plasticity Related to Design [10]. Some correlations between working load and ultimate deflections are reported. It appears that an additional elastic analysis under service loads would be required if the approximate deflections calculated by the method suggested in the report exceed the prescribed deflection limitations.

Since deflection limitations are becoming an increasingly important consideration in the design process, some authors have proposed design methods in which the structural members are directly proportioned to satisfy deflection requirements and then checked for strength, a procedure opposite to that is usually followed. Such methods are proposed by Stevens [17] and Blakey [5]. Certainly these methods are rational and can provide safe and serviceable structures; however, this has revealed that the presently available limitations on deformations are not nearly as precise as those on strength and it does not seem logical to base a design on such criteria at the present time.

3.5 Field Observations.

A comprehensive survey of existing data for 98 buildings, 58 with no damage and 40 of which have been damaged as a consequence of foundation settlement is reported by Skempton and MacDonald [15]. Although the authors are concerned with foundation settlement induced deflections, this work has bearing on other types of deflection, especially the tentative values for damage limits. The types of damage are classified as follows: 1) structural (involving the frame); 2) architectural (involving only the panel walls, floors or finishes); and 3) functional. From the study of frame buildings with infill panels, it is evident that architectural damage such as cracking of wall panels is likely to occur at distortions smaller than those which cause structural damage. It appears that these authors found that a limiting differential settlement of L/300 represents a reasonable value which, if exceeded, is likely to result in architectural damage. This criteria is based on the data obtained from the observed settlements and additional unpublished test data.

Supporting data to the previously cited conclusions by Skempton and MacDonald is given by Thomas [18] with a particular emphasis on brickwork.

An investigation into the causes of large deflections of electrically heated concrete floors is described by Jenkins, Plowman and Haseltine [9]. The deflections are attributed to aggregate shrinkage, slab creep, and creep with the latter being the most significant. It was recommended that compression reinforcement be used in slabs and that the allowable span/depth ratios given in the British Standard Code of Practices; The Structural Use of Reinforced Concrete in Buildings, CP 114 (1957) be revised. The CP 114 (1969) (Appendix B) designates a deflection limitation for simply supported beams of L/d < 20.
4. Dynamic Deflections

4.1 General

Dynamic deflections, as previously discussed, are a result of a dynamic load acting on a structure or element. Dynamic loads extend over a broad range of time durations. They can be transient and short in duration, i.e., less than the natural period of a structure or cyclical and long in duration, i.e., steady state. Depending on the relationship between the time characteristics of the loading function and the natural period of the structure, the structure may deflect less than if the same load were applied statically or it may deflect many times the deflection of the same load applied statically (resonance).

The review of the literature for dynamic deflections is first covered in a broad scope by discussing current design practice, and then discussing forcing functions, structural systems, dynamic structural characteristics, damage to structures and vibration isolation. Because of the importance of particular areas of dynamic deflections, detailed discussions will be made on the specific areas of floor vibrations, drift, and human perception and response to the motion of structures.

4.1.1 Current Design Practice.

An excellent state-of-the-art report which includes an extensive list of references has been presented by Steffens [30]. Since Steffens' report covers most of the available literature on the subject of structural vibration up to 1964, this report will focus mostly on the work done after 1964.

Present structural design practice to prevent destructive or disturbing vibrations varies in sophistication from suggestions of simple arbitrary deflection limits to recommending a dynamic analysis of the structure.

The specifications of the American Institute of Steel Construction [29] read "Beams and girders supporting large open floor areas free of partitions and other sources of damping, where transient vibration due to pedestrian traffic might not be acceptable, shall be designed with due regard for vibration."

No specific recommendations for design considering vibrations have been given either in the American National Building Standard [24] or in the ACI Building Code [23].

Perhaps the National Building Code of Canada [28] is the most progressive of all present codes in this respect, however, no clear guidelines are given. Following are some of the clauses in it relating to vibrations.

Structural members shall be designed so that their deflections and vibrations under expected service loads will be acceptable with regard to

a) the intended use of the building or member

b) possible damage to nonstructural members and materials

c) possible damage to the structure itself.
The reference velocity pressure, $q$, for the design of structural members for deflection and vibration shall be based on a probability of being exceeded in any one year of 1 in 10 (as against a probability of 1 in 30 for the design of structural members for strength).

Buildings whose height is greater than four times their minimum effective width or greater that 400 ft and other buildings whose light weight, low frequency and low damping properties make them susceptible to vibration shall be

a) designed by experimental methods for the danger of dynamic overloading and vibration and the effects of fatigue, or

b) designed using a dynamic approach to the action of wind must.

Theoretical models have been proposed for the dynamic analysis of structures. The adequacy of theoretical models has been discussed by Ward [31] in relation to the investigation carried out by the National Research Council of Canada. Many uncertainties exist in input to analysis, for example, the dynamic structural characteristics. In addition, the limits that constitute excessive vibrations which cause minor structural damage and physical disturbance are yet to be defined quantitatively.

Traditionally, in typical civil engineering structures, resonance has not been a dominant problem. Whenever resonance was a possibility due to machinery housed in the structure, proper care in machinery operation and in the design of the structure eliminated its occurrence. Resonance of structural floors caused by people moving on the floors is claimed not to occur if the natural frequency of the floor is greater than about 5 Hz [28]. This condition is satisfied by most floor systems but may be a problem for long-span and low frequency systems.

For low-rise buildings the structural response to wind is not as significant as for high-rise buildings. Hence most research efforts have been directed to the study of wind effects on tall buildings.

The dynamic response of structures to earthquakes has been extensively studied. The recommended practices in various countries for seismic design are discussed by Beles and Ifrim [25], Korchinsky [27], Ferry-Borges and Ravara [26], Wiegel [32], Wiggins and Moran [33] and in the commentary of the National Building Code of Canada [28]. The various earthquake design provisions concern principally the ultimate limit states rather than the serviceability limit states and thus this topic will not be extensively considered in this report.

Structural response to other types of dynamic excitation such as sonic booms, shock waves and traffic are special situations and as such they must be considered individually.

Minor damage due to the sway of tall buildings and physical discomfort due to transient floor vibrations have been considered in present design practice through simple static deflection limitations.
4.1.2 Forcing Functions.

Most dynamic loading sources that cause structural vibrations are classified in two groups, natural and man-made. Dynamic loads resulting from wind and earthquakes belong to the first group. Man-made sources of vibration are sonic booms, air shock waves, machinery, traffic and disturbances caused by construction processes.

The effects of wind, particularly the turbulent characteristics of it, are important in the design of tall structures, suspension bridges, etc. Present practice makes use of a dynamic approach [37, 38, and 39] or model testing in wind tunnels for large structures. The nature of wind and its treatment in design are discussed by Ferry-Borges [34], Davenport [35], and Scruton and Flint [36].

The problem of seismic structural vibrations has received much attention. Present practice allows routine design using seismic coefficients [24, 28 and 42]. For tall buildings and for buildings with irregular layouts, design based on dynamic analysis is recommended. A survey of present knowledge in earthquake engineering is given by Wiegel [43]. The interaction of structure-foundation system in the seismic response of structures is discussed by Parmell et al [40] and more recently by Sarrazin et al [41]. Sarrazin's approach is more general than Parmelee's by using nondimensional parameters and root mean square response to a white noise input simulating an earthquake environment. Also Sarrazin assumes a different geometry in the mathematical model. The essential feature of earthquake resistant design is to determine the maximum response and its direction for a chosen forcing function representing an earthquake. The validity of the analysis depends on the accuracy of the choice of the design earthquake and of the structural parameters such as damping, ductility (resistance function) and the natural period of the structure. It should again be emphasized that consideration of earthquake loading relates primarily to the ultimate rather than the serviceability limit states.

The nature of the sonic boom problem and its effect on structures has been discussed by Hubbard [44], Lowery and Andrews [45], McKinley [47] and others [46, 48, and 49]. Although it was regarded that general structural damage due to sonic boom was unlikely, these studies were undertaken after a series of allegations of damage to window glass. As a result of McKinley's study [47], it was suggested that windowpanes designed to resist wind pressures of 10 psf are not likely to break when exposed to booms from jets operating under control. The reported damage [45, 46, and 47] may have been caused by a number of other reasons such as improper fitting of panes to the frames and rusting of metal window frames.

Blasting operations produce earth vibrations that are imposed upon structures and buildings. Crandell [51], Wiss [55] and others [50, 52, 53 and 54] have studied the effects of such vibrations on structures. Resonance appears to be unlikely; minor damage such as plaster cracking is often incorrectly attributed to this source of vibration. Wiss [55] has indicated safe limits on the sizes of explosives charges to preclude damage to nearby structures.

In a report of the Australian Bureau of Mineral Resources, Geology and Geophysics, Anthony [56] points out that the vibrations from machinery are not likely to cause damage beyond a radius of 20 feet. Many economically feasible methods are available for isolating the structural components from machinery-induced vibrations. Novak [57] has studied the influence of the non-linear properties of soils on vibrations of machine foundations.
Vibrations from traffic have limited effects on adjoining structures as a study by the British Railway [58] has shown. The Australian report [56] indicates that the traffic movements (including trolleys and trains) are not likely to cause damage beyond a radius of about 40 feet.

4.1.3 Structural Systems.

The effects of vibrations on low-rise buildings have been studied mainly in the context of earthquake loadings. However, transient vibrations due to moving people are important regardless of building height. A literature review of the performance of wood-joist floor systems is made by Onysko [59] wherein the problems of dynamic loading leading to floor resonance and human sensitivity to vibration are discussed. Section 4.4 of this report is devoted entirely to the review of the literature on these topics.

Tall buildings are subjected to vibrations from both wind and earthquake loadings. The natural frequencies of tall buildings are likely to be lower than 1.0 Hz [66]. This low frequency, and the fact that the modern tall buildings tend to have low damping, make the effect of both wind and earthquake loadings significant. Davenport [62 and 63] has suggested design criteria for all buildings for wind loading. These criteria attempt to account for most of the significant wind effects, namely, collapse, damage to masonry and finishes, damage to windows and cladding, fatigue damage and comfort of occupants. Dynamic analysis taking into account wind gusts and model testing in a boundary layer wind tunnel simulating the actual terrain are the only sophisticated design approaches available today. The response of tall buildings to turbulent wind action consists of a fluctuation about a mean deflected position with oscillations usually occurring at a frequency equal to the fundamental frequency of the building. Davenport [62] and Chang [60] have given expressions for the maximum acceleration in terms of the vibrational characteristics of the building. Robertson [67] has also stressed the importance of the establishment of rational acceptance standards for building sway. Goldberg and Herness [64] have formulated a method of calculating the normal modes and associated natural frequencies of multistory buildings considering both shear and bending deformations in the floor and stiffening walls. Some dynamic deflection measurements of multistory buildings have been reported by Korchinskiy [66].

Vibration of towers have been examined by Chiu et al [61] and Ishizaki and Katsura [65]. The procedure followed by these authors is to formulate a mathematical model of the tower and to study the effects of variations in vertical wind profile, mean wind velocity and its standard deviation.

The effects of slenderness and flexibility of multistringer steel highway bridges has been studied with respect to strength and serviceability by Wright and Walker [69]. A conclusion reached was that a more flexible bridge tends to be stronger in resistance to yielding of a stringer. Flexibility of bridges was related to pedestrian comfort and it was found that substantial changes in flexibility only moderately affected human response.

Tajimi et al [68] report the vibration characteristics of a nuclear reactor enclosure structure. The structure was analyzed for factors such as natural period, damping etc., to check the validity of static-design earthquake coefficient. Subsequent field measurements have shown good agreement with the analytical results.

A thorough description of the state-of-art in foundation vibrations is given by Whitman [72]. The theory idealizing the foundation as a
mass supported by an elastic half-space is gaining increasing support over the traditional approach of a single degree-of-freedom, mass-spring-dashpot system. The major problem still remains of estimating the constitutive relationships for the soil to be used in the elastic half-space analysis. Whitman includes a complete list of pertinent references. Hsieh [70] has presented a method of analyzing the coupled modes of vibration of a foundation resting directly on soil using a mathematical model with six-degrees of freedom. Theoretical work and background information on foundation vibrations is contained in the book by Richart, Hall and Woods [71].

4.1.4 Natural Frequency.

The natural period or its reciprocal, the natural frequency, of a structure is one of the fundamental parameters used in the determination of the response of a structure to dynamic loads. Various techniques are known by which a building structure may be caused to vibrate to determine its natural period of vibration. Some examples of these are: firing attached rockets [80], pulling the structure with a cable and then releasing it, and having a person move his body back and forth in synchronism with the natural period of vibration of a structure [75]. Alternatively, the response of the buildings at certain frequencies of artificially applied forced vibration or from random wind excitation [74] may be studied to determine the natural period of the buildings. Janney and Wiss [77], and Rea, et al [81] have also reported dynamic tests on structures to determine the natural period.

As a result of experiments, Steffens [30] presents several empirical formulas which make it possible to calculate the natural period of a building from its dimensions. The height of a building appears to be the most significant parameter governing the natural period of a building. The natural period varies from 0.1 second for low buildings to 10 seconds for very tall buildings. Davenport [63] has reported that the natural period of the Empire State Building in New York is about 8 seconds.

The Uniform Building Code [42] and the American National Standards Institute [24] recommend that the natural period of a building be calculated using the simple relationship

\[
T = \frac{0.05h_n}{\sqrt{D}}
\]

or for moment-resisting space frames,

\[
T = 0.10 N
\]

where

\(T\) = natural period in seconds

\(h_n\) = height of the structure

\(D\) = dimension of the building parallel to the lateral force

and

\(N\) = number of stories.

Blume [73] has proposed a pseudo-stiffness procedure to determine the natural periods, modes and stiffness taking the joint rotations and soil-structure interaction into account. Keintzel [78] has considered the
effect of structural types (high walls without openings, frames and high walls with openings or frames filled with masonry) on the natural periods. Rubinstein [85] and Rubinstein and Hurty [82] have considered the effect of joint rotation and axial deformation on the periods of multi-story framed structures. They recommend that all joints within a given floor level should be assumed to undergo equal rotation and that the axial deformation in the columns may be neglected. Considerable reduction in computation time is achieved with a small sacrifice in accuracy (6 percent).

Actual measurements of natural periods of tall buildings have shown that the empirical formulas may provide a reasonable estimate of the natural period. Wiss and Curth [84] compare the measured natural period of a 56-story building with those determined by equations from the Uniform Building Code [42] or ANSI A58.1-72 [24]. The structure contains a shear wall core to the 42nd floor and floor slabs and unbraced columns above the 42nd floor.

<table>
<thead>
<tr>
<th>Formula</th>
<th>Long Direction</th>
<th>Short Direction</th>
</tr>
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<tbody>
<tr>
<td>T = 0.1 N</td>
<td>5.6</td>
<td>5.6</td>
</tr>
<tr>
<td>$T = \frac{0.05h_n}{\sqrt{D}}$</td>
<td>2.9</td>
<td>3.0</td>
</tr>
<tr>
<td>Measured</td>
<td>3.7</td>
<td>4.0</td>
</tr>
</tbody>
</table>

The natural periods of vibration of structural elements such as beams, floors and slabs is fairly straightforward either theoretically or experimentally. Steffens [30] gives expressions for natural periods of different types of beams under concentrated and uniformly distributed loads. He reports that the natural periods of steel beams will be in the range 0.02 to 0.20 seconds, and for floors and slabs the range is 0.05 to 0.10 seconds. For wood-joint floor systems, Onysko [59] observes natural periods of 0.16 seconds. Jacobson [76] has theoretically determined the natural periods of uniform cantilever beams considering flexure and shear in a beam and the elastic yielding of a rigid support. Masur [79] has given a theoretical solution for the natural frequencies of rigid frames.

4.1.5 Damping.

Observations of the free vibration of a real system reveals that the amplitude of vibration decays with time. This behavior is attributed to the action of damping in the system. The damping forces act in opposition to the motion, doing negative work on the system and dissipating energy from the system.

There are three major categories of damping. If it is assumed that the damping is proportional to velocity, it is known as viscous damping and the amplitudes of successive free oscillations reduce in geometric progression. If the damping is the result of solid body friction, then the amplitudes of successive free vibrations decrease in arithmetical progression. This is known as Coulomb friction damping. In a real structure, some of the energy loss may be attributed to the internal friction of the material. This is called structural damping. Although it is difficult to predict the magnitude of various types of damping, experiments suggest [88] that damping increases as the driving force increases. It was also noted that damping in each mode of vibration was proportional to the resonant frequency of the corresponding mode.
Another type of damping which is important in structures is the aerodynamic damping which is considered in the dynamic response of structures from wind loadings. This damping is sometimes negative which causes an instability of the structure known as "galloping." Some of the reported failures of icecovered transmission cables and suspension bridges are caused by galloping. Fortunately, for typical building structures aerodynamic damping is negligible in comparison with the mechanical damping associated with structures (viscous, coulomb and structural). However, it is possible that local problems caused by galloping can exist for isolated members such as columns, braces, struts and truss systems. Damping of a real system is a complex phenomenon involving all mechanical damping types. It is common practice to approximate the damping of a structural system by equivalent viscous damping.

Steffens [30] has discussed various factors and coefficients used for damping. Of these, the critical damping coefficient \( c_0 \) and the damping ratio \( D \) are important. The critical damping coefficient, \( c_0 \), is defined as the least value of the damping coefficient, \( c \), required to prevent oscillation of a system. The damping ratio relates the actual damping to the critical damping value, i.e., \( D = c/c_0 \). Steffens has also given the damping ratios for steel, rubber, concrete and other materials.

Davenport [62] suggests the following values of damping ratio, \( D \), for tall buildings:

- Concrete: \( 0.01 < D < 0.02 \)
- Steel: \( 0.005 < D < 0.01 \)

Englekirk and Matthiesen [85] report that a damping ratio of 0.035 was measured in the vibration test of an eight story reinforced concrete building. Nielson [88] found the damping ratio to be between 0.005 and 0.01 for a nine-story steel frame building. Reed [89] reports the largest range of damping for tall structures, \( 0.004 < D < 0.07 \), which he obtained from a survey of the literature. Jennings and Duroiwa [86] and Laxan [87] have conducted experimental investigations to study the effect of intensity and history of motion and soil interaction on the damping characteristics of structures. Damping of structural elements can be improved by coating their connecting surfaces with a thin layer of plastic material or by inserting mechanical damping devices [87].

Steffens [30] has outlined the methods of determining damping in any given elastic system. Analysis of the results of free vibration tests or examination of the response of the structure (at given frequencies) to forced vibration will give an approximation of the damping of a structure.

4.1.6 Damage to Structures.

Damage of structures from vibrations may range from collapse to minor damage such as cracking of plaster and window glass or interference with the operation of equipment.

Comparatively few cases have been reported in the literature where severe vibration, except for earthquakes, has been the most likely cause of structural damage, although complaints alleging damage to structures as the result of vibrations are made frequently. Steffens [30] has dealt with this topic in detail. He concludes that structural damage due to dynamic response is likely only under earthquake conditions. He also points out the inadequacies of the mathematical models used to determine the response of structures to dynamic loads (i.e., earthquake loads), uncertainties regarding dynamic characteristics of the structure (i.e., damping) and the uncertainties in loadings (e.g., lack of strong motion.
data). Some or all of these factors contribute to the observation that buildings designed to withstand an acceleration of 0.1 g (g = 32.2 ft/sec²) are often found to be almost unharmed by accelerations as large as 0.3 g. Analysis of the strong motion data from the San Fernando Earthquake of February 1971 [95], where accelerations up to 1 g were recorded, may shed some light on the actual earthquake resistance of buildings.

Minor damage such as cracking of plaster, window glass and damage to brickwork are often incorrectly attributed to severe vibration [90, 91 and 92]. It was pointed out in these references that to cause damage to plaster ceilings by vibration, amplitudes have to be in the order of 0.1 in. or accelerations should be in the order of 1 g. These amplitudes are not often experienced for structures in service even with blasting, pile-driving or forge-hammer operations. The probability of fatigue damage due to low level vibration is very small for most structural materials.

As the loading causing vibration (e.g. wind and earthquake) and the response of the structure are stochastic in nature, design has to be based on a specified but nonzero risk. Davenport [62] has suggested a set of recurrence intervals for various unserviceability limits such as breakage of windows and cladding, comfort of occupants and structural fatigue for the design of tall buildings to resist wind.

Various criteria [90 and 91] for assessing the vibration damage of structures have been suggested. Koch [93] has classified the intensities of vibration and their probable damaging effects on structures. Investigation on vibration effects of blasting has yielded some information as to the safe charges of blast and criteria for damage [94].

4.1.7 Vibration Isolation.

The reduction and isolation of harmful vibration as a result of operation of industrial plant and machinery can be achieved [96] by correct siting of machinery, proper balancing and the use of antivibration mountings or special foundations.

Isolation of structures from vibration can be attained by the provision of trenches, using sand and gravel under the foundations, using suitable building materials and type of construction, inserting special antivibration pads under beam supports and column bases or mounting the complete building on a suitable spring system. Steffens [30] has described each of these methods and has cited a number of cases where each method was used in practice.

Den Hartog et al [97] have described a method to isolate a large structure from shock vibration by suspending the foundation system within an underground cavity. Vibration isolators are used to reduce the effect of machine vibrations on buildings and Eherhart [98 and 99] has theoretically examined the vibration transmissibility of different types of foundations. Nelson [100] and Waller [102] have described the use of rubber bearing pads and viscoelastic material to damp structural vibration.

Waller [101] has emphasized the significance of the damping characteristics of a structure in controlling its vibration. He has described the use of vibration absorbers in the 260 meter high chimney at Drax Power Station, England. He suggests the possibility of using this technique to control vibration in tall buildings.
4.2 Floor Vibrations.

4.2.1 Introduction.

Floors are flexible structural systems and they will vibrate when excited by a dynamic force. When the excitation ceases, the vibration will then dampen and cease. Floors may be excited by human activity, by machinery located on the floor, or any other activity which could impart direct dynamic forces to the floor. Indirect excitation may be transmitted to the floor from the structural frame which may be vibrated by an earthquake, wind, vibrating machinery or various transportation vehicles.

Most problems with vibrating floors occur when the vibrations are perceptible or annoying to the occupants. Because human sensitivity to motion is very keen, the stresses and deformations corresponding to perceptible vibrations are relatively small and no structural distress is expected. A possible exception to this could be floors which support machinery where distress may be caused by fatigue or resonance. Steady state vibratory forces may create stresses which cause fatigue failures. Resonance should be avoided where floors are exposed to steady state forcing functions.

The floor vibration problem has the following aspects: 1) Determination of what is perceptible and annoying vibration to human occupants (Section 4.4); 2) development of adequate mathematical models to represent the floor systems; 3) development of procedures to analyze and design acceptable floor systems with respect to vibration; and 4) determination of economically feasible repair procedures for unacceptable floors.

4.2.2 Analytical and Experimental Studies on Floor Systems.

Floor systems are treated as 1) beam and girder assemblies, 2) grid systems, 3) isotropic plates and 4) orthotropic plates. The study of the vibration of beams, grids and plates is a part of mechanics and the fundamentals are treated in any number of texts on the theory of vibrations. With finite element procedures and computers, structures or structural elements can be analyzed as long as the system and its dynamic load are realistically defined. The search of the literature indicates that there is an adequate number of references on plate or grid vibrations available so that the major aspects of the analysis of floor vibration can be determined.

The following specific problems have been treated analytically in the literature:

1) Beam and girder systems [105, 106, and 120].
2) Rectangular slabs and plates with various edge conditions [108, 109, 112, 114, 116, 117, 120, 122 and 123].
3) Stiffened plates [110 and 118].
4) Orthotropic plates [113].
5) Beams and slab system [118].
6) Slab on rigid and elastic columns [115].
7) Grid systems [120].
8) Sandwich plates [103, 107 and 124].
9) Plates not rectangular in plan [108, 111 and 119].
10) Tapered plates [104].

In addition, plate and beam vibration solutions are tabulated in a Russian handbook [121]. Solutions for buckled plates, prestressed plates and beam-and-slab bridges are also given in the literature. A great
many types of vibrating plate problems have been solved, but they are concerned with applications in other technologies (aircraft space vehicles, etc.) than building technology.

Bleich [105] considers the floor system to be made up of a set of beams and girders orthogonally connected to each other. Two subsystems are considered; the beams supported by infinitely stiff girders which frame at right angles to the beams and the girders which support infinitely stiff beams. He shows that by combining the dynamic properties of the subsystems (which are now simple beams for which solutions are available), the dynamic properties of the total grid system can be obtained. Finally, a set of simultaneous equations are developed and the determinant of the coefficients of the equations gives the desired natural frequencies.

Rogers [120] gives the complete dynamic solution (i.e., natural frequencies and mode shapes) to three problems: 1) Floor systems with a central girder and simply supported beams framing into this girder at right angles; 2) rectangular, simply supported isotropic elastic slab; and 3) orthogonal gridwork. He also describes Bleich's method [105] thoroughly. Examples are worked for each case and the procedure is readily usable for design calculations.

Burckhardt [106] presents nomographs and charts to rapidly determine the natural frequencies of floor beams. The floor is idealized and represented by beams of uniform stiffness with various end conditions.

Mackey and Ying [115] present a method of analysis for floor systems, considering floors as plates resting on supporting columns. The fourth order partial differential equation involved is solved by the normal mode method. A simple frequency equation in terms of the number of "symmetrically situated" columns is derived and some typical cases are solved numerically. The stiffness of the floor system on elastic interior columns is also discussed and mode shapes due to free and forced vibrations are briefly considered.

Floor systems are idealized by Lenzen, Dorsett and Sokolowski [113] as rectangular orthotropic plates, and solutions are presented for the free undamped vibration (natural frequencies), for the case of forced harmonic vibration, and for static loading. The theoretical results are compared with experimental results and a simplification involving the concept of "effective number of joists" is advanced.

Steffens [30] gives a broad review of many vibration problems. The subjects of floor vibration and frequency of beams and slabs is covered. Steffens notes that tests in Norway (1958) on timber joist floors resulted in the recommendation that the deflection of the floor should not exceed 0.034 in. under a concentrated load of 220 lbs. Most domestic floors were stated to have resonant frequencies in the range of 10 to 30 Hz. Excessive floor vibrations can be reduced by changing the properties of the floor system. Approximate natural frequency formulas are given for beams. Reference is made to the literature for natural frequencies of slabs.

Ohmart [118] determined the response of a solid web steel beam-concrete slab floor system to dynamic impact by considering the floor system as stiffened simply supported rectangular plate. The floor section was treated as a system of discrete elements, i.e., a plate and beams. A series of beam and slab test floors were constructed and tested to verify the theoretical analysis. Excellent comparisons between theory and experiment were obtained for the fundamental natural frequency and fair correlation was observed for the maximum displacement.
From the current state-of-the-art of dynamic analysis it appears that many practical and relevant problems have been solved and that techniques exist to solve most problems. There are two things missing: 1) design aids to assist the design engineer in computing frequencies and amplitudes and 2) the treatment of the problem of the combined structural frame and floor systems.

4.2.3 Experiments on Steel Joist Floor Systems (Lenzen).

Theoretical treatments of the dynamics of plates and floor systems generally use idealized models of the systems and boundary conditions. In reality, floor systems are highly complex with boundary conditions which are often difficult to define. They are constructed of diverse elements such as, slabs, beams, girders, joists, flooring, partitions, insulation, ducts and ceilings all of which interact in a complex manner. In buildings designed for human occupancy such as offices, stores, schools, hospitals, etc., vibrating floors infrequently caused problems with the types of construction used up until a few years ago. Recent availability of high strength steels at a cost almost equal to that of the cost of ASTM A36 steel has resulted in much lighter structural floors. For example, the most popular open web steel joists are the H-series bar joists which are made of steel with a minimum yield stress of 50 ksi. Lighter floors, larger areas without partitions and lighter ceilings have resulted recently in some floors with unsatisfactory vibrations. Both the AISC Specifications [11] and the National Building Code of Canada [28] require that annoying vibration of floors be avoided without defining what annoying vibration is and without guiding the designer to the appropriate design computations.

Steel joist-concrete slab floors are one of the lightest forms of floor construction. The Steel Joist Institute has sponsored a significant amount of research on this problem at the University of Kansas under the direction of Lenzen [125 through 132] which was performed in the period 1959 to 1970. Some of this work has been reported in a series of reports in the University of Kansas "Studies in Engineering Mechanics." Since considerable research on the topic of steel joist-concrete floors was performed by Lenzen, it is necessary to examine this work in detail to give it proper perspective in the state-of-the-art in floor vibrations.

4.2.3.1 Dynamic Response Experiments.

The first phase of the floor vibration work performed for the Steel Joist Institute is given in reference 129. A test floor consisting of a 2 1/2 in. thick slab resting on six steel joists about 15 ft. long and spaced at 20 in. on center was constructed. Three sets of steel joists, 8, 10 and 12 in. deep, were used in the study. The joist ends were simply supported and the slab edge, at right angles to the longitudinal axis of the joists, was free. The dynamic excitation of the floor was achieved by subjecting it to; 1) steady state vibration and 2) impact vibration caused by dropping an iron ball or a human heel drop. The heel drop was performed by a man standing on his toes, relaxing, thus allowing both heels to impact the floor. The measured natural frequencies indicated that the slab and joist acted as a composite system even though no intentional shear connecting devices were present. It was shown that the natural frequency of such floors can be predicted adequately by considering the beam to consist of the steel joist and an effective concrete slab. The report also discusses the measured frequencies in 46 different floors in service and construction. Various degrees of conformance with theoretical predictions were noted. Human response to vibrating floors was also discussed and is presented in Section 4.4 of this report.
Lenzen [129] discussed various features of damping and artificial damping devices. A test floor, with a clear span of 24 ft., joist spacing of 24 in., joist depth of 12 in., and with 9 joists was constructed to study damping devices. The efficiency of insulation, bridging, prestressing and a variety of cable installations in increasing damping was found to be negligible, but external mass-spring-dashpot devices were found to be excellent in enhancing the damping characteristics of the floor system. Lyons [131] gives a detailed account of the investigation of the structural and external damping devices used by Lenzen.

Floors in twenty buildings with composite and non-composite slabs on solid web steel beams were subjected to impact with a mechanical impact device and with a human heel drop [128]. The dynamic response of the floor system (amplitude, frequency, damping) was recorded and analytical studies were made of the dynamic properties of the floor system. A great deal of data on framing, floor dimensions, records of response etc., are given in the report. In two of the twenty buildings it was noted that the occupants complained of annoying vibrations of the floors. The other floors appeared to have sufficient damping and it was suggested that spray-on fire proofing may contribute to damping. The comparison of the measured and predicted amplitudes due to impact indicated that the T-beam analogy is not a good method for these floors and an orthotropic plate model is suggested for a better analysis. The report [128] contains a great deal of data which might be used for a more in-depth analysis in the future.

The second phase of the Steel Joist Institute sponsored research project on floor vibration at the University of Kansas is given in reference 127. A test floor, larger than and different from the other floors, was constructed and subjected to a variety of tests. The floor consisted of a concrete slab and 25 joists 32 ft. long and spaced at 24 in. on center. All four sides of the floor were simply supported. Excitation of the floor system consisted of a shaker producing steady-state vibration or a mechanical device producing impact. Relatively long duration impact was also produced by a human jumping on the floor. Static and dynamic deflections and strains as well as accelerations were measured.

Dynamic properties of the test floor were determined by experiment for the following configurations:

1) 2 in. slab, simple bridging
2) 2 in. slab, no bridging
3) 2 in. slab, full bridging
4) 4 in. slab, full bridging
5) 4 in. slab, no bridging
6) 4 in. slab, only half the floor acting
7) 4 in. slab, only half the floor acting and ends of slab removed.

The conclusions from these tests were the following:

1) The slab and joist floor system acts as a "unit, i.e., the floor must be considered to be composite even though no mechanical shear connectors were present.
2) Bridging does not affect the dynamic characteristics of these floors.
3) The concept of an "effective floor size" was experimentally demonstrated. Floors larger than the effective floor size behave essentially the same way as if their size were equal to this area.
4) Addition of mass and stiffness by increasing the slab thickness results in changed dynamic characteristics. Reduction of amplitude is greater than the increase of the frequency, thus human response was stated to be more favorable.

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5) Cracked floors and slabs with cut-outs have an unfavorable vibrational performance.

6) "Beating" which is noticeable and unpleasant occurs in light floors.

In addition to the experiment results, formulas for the determination of the effective floor size, the impact amplitude and the frequency are given, and it is shown that close agreement exists between the experimental observations and the theoretical predictions. The theory is based on the assumptions of orthotropic plate behavior [113].

The dynamic response of three concrete slab-steel joist floor systems was studied at the structures laboratory of the University of Kansas (to be published). The largest of these had a length of approximately 33 ft and the floor was approximately 50 ft wide with joists running longitudinally along the 33 ft length. A simple support was provided on all four sides of the rectangular floor. The other floors were 15 ft and 24 ft long and about 12 ft and 20 ft wide with the joists running along the lengths of the slabs. The two sides supporting the joists were simple supports and the other two edges were free.

The three joist-slab floor systems were subjected to a great many tests to study the effects of the following variables on the dynamic response: slab thickness, joist size and depth, end conditions, beam-versus-slab action, bridging, dynamic damping devices, dynamic excitation devices, ceilings, joist spacing and presence of humans.

Parallel with the experimental program, an analytical study was performed to investigate the theoretical dynamic response of three steel joist-concrete slab floor systems. Various analytical models were used, and the most sophisticated of these considered the floor system to be an orthogonally anisotropic composite plate.

Another phase of the research performed by Lenzen (to be published) was on the problem of damping by artificial means, i.e., the use of damping devices. Many internal and external damping systems were investigated. It was found that the most effective device was an external friction system which proved to be the predecessor of various such devices now commercially available.

In the previous unpublished work a comparison was made of the laboratory results to many (both vibrationally adequate and inadequate) floor systems in service and under construction. This work indicates that there was a correlation between the human response studies, the field observations, the laboratory tests and the analytical studies. This interplay yielded considerable insight into the problem of vibrating floor systems.

4.2.3.2 Damping.

In view of the importance, considerable effort was devoted by Lenzen [129] to determine the damping characteristics of steel joist-concrete slab floors. Because of the complexity of this problem, no conclusive results were obtained, however, certain observations were made:

1) Bridging of all types will not improve the damping characteristics and thus the presence or absence of bridging in the final floor makes no difference as far as vibration abatement is concerned.
2) Addition of superimposed dead weight did not improve the response of the floors.
3) Multiple sizes and types of joists did not improve the response.
4) People were excellent dampers and the presence of groups of people on a floor, as contrasted to only one or two people, effectively damped the vibrations.

5) An increase in slab thickness improved the vibration characteristics of a floor by increasing the effective stiffness of the composite section, thus reducing the amplitude. The corresponding increase in frequency was not large, thus this is an effective means of avoiding undesirable vibration if no additional damping is present from partitions.

6) Normal construction provided sufficient damping through such features as walls, partitions, flooring, floor covering, ceilings, etc., so that transient vibrations were usually in the non-perceptible or barely perceptible range.

7) If unusual conditions existed, such as large school rooms or churches, in which enough damping was not inherent in the construction, damping could be provided by damping units available commercially. In such cases a careful analysis of the vibration problem may be warranted.

8) Any loss of the integrity of the concrete slab increased the vibration. Cutting away the slab from the ends of the joists was especially undesirable. A severely cracked slab could reduce the stiffness of the composite joist-slab section by as much as 20 percent.

9) The floor system may not be the only cause of its objectionable vibration. Joists supported by flexible beams can interact with the beams therefore the interaction of the total system must be considered. The problem can be avoided by providing stiffer support beams.

4.2.3.3 Conclusions.

The extended scope of the research on the dynamic behavior of steel joist-concrete slab floor systems performed by Lenzen provides some data for an understanding of vibrating floors.

One of the achievements of the research was that it was possible to correlate theoretical predictions of dynamic behavior with the corresponding measured behavior in the laboratory and in the field. The correlation was excellent with respect to the laboratory test floors, where good knowledge of all parameters existed, and it was also satisfactory in the tests made on most floors in the field. Such satisfactory correlation gives confidence to the analytical determination of the dynamic parameters of floor systems in the design stage.

The combined experimental and analytical studies led to the following insights into the dynamic behavior of joist-slab floor systems:

1) Under human impact loading all joist-slab floors act as composite floors, even though no positive shear connection is provided and the strength is governed by only the joists. The reason for this is that under the relatively small loads and deflections friction between the slab and the joist provides essentially composite behavior. This composite action results in an increase in the stiffness of the floor system. Positive verification of the composite behavior under vibratory loads was provided only for normal-weight concrete, although the same is estimated to be true for light-weight concrete slabs. Limited tests showed, however, that no increase in stiffness can be expected from gypsum concrete floors.

2) The joist-slab floor system acts as a two-way plate system under vibratory loads. Since the stiffness is different in each direction, i.e., the stiffness of the composite section along the length parallel to the joists and the stiffness of the slab alone perpendicular to the joists, the analysis of dynamic behavior was achieved by assuming an orthotropic plate. Such an analysis verified the experimentally observed existence of an "effective floor area" which participated in the vibratory motion after impact. For floor areas larger than this effective area it was found that only the effective floor area was actively participating
in vibration. Equations for determination of the effective floor area, the frequency and the amplitude are given in Appendix C.

While the dynamics of floor systems have been explained by developing analytical models, the reaction of humans to vibrating floors is not easily quantified. Human reaction depends on many parameters. This topic is discussed in Section 4.4 of this report.

4.3 Drift.

The term drift is used to describe the lateral deflection of a building generally when subjected to a lateral loading such as wind and earthquake or to thermal gradients such as from solar heating. This section on drift covers the topics of effects, limitations, calculation and measurements.

It is apropos to clarify the use of terms referring to the direction of motion of a structure with respect to the wind direction. Three commonly used pairs of terms appear in the literature and are summarized here.

<table>
<thead>
<tr>
<th>Motion perpendicular</th>
<th>Motion parallel</th>
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<tbody>
<tr>
<td>to wind</td>
<td>to wind</td>
</tr>
<tr>
<td>across-wind</td>
<td>along-wind</td>
</tr>
<tr>
<td>lift</td>
<td>drag</td>
</tr>
<tr>
<td>transverse</td>
<td>longitudinal</td>
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Drift deflections are of primary concern in the design of tall buildings. The many aspects of the design of tall buildings has been the subject of a joint effort of the American Society of Civil Engineers and the International Association for Bridge and Structural Engineering on the Planning and Design of Tall Buildings. A conference held at Lehigh University, August 1972, produced conference proceedings which assembled this effort. Specifically, drift deflections are considered in the proceeding references [134, 135 and 136] of this conference. The major aspects of these references are covered within subsequent discussions in this section of other more detailed references.

4.3.1 Effects of Lateral Loading on Structures.

The undesirable effects of excessive structural deflections have been classified into four broad categories in the ACI Committee 435 report [133]: 1) sensory acceptability; 2) serviceability of the structures; 3) effect on non-structural elements; and 4) effect on structural elements. Of these, drift limitations are given only regarding the category of effect on non-structural elements.

In the ACI Committee 442 report [156], lateral deflection is considered as a serviceability criterion for the lateral load design of high-rise buildings. Particular attention is called to the effect of drift on the stability and cracking of members.

Davenport [139 and 141], has included among the major factors governing design for wind: 1) failure due to instability of the frame; 2) failure due to yielding with excessive permanent deformations; 3) failure due to widespread damage to exterior cladding; 4) unserviceability due to excessive deflections causing cracking of walls and degradation of the structural skin and mechanical systems; 5) excessive sway accelerations
causing discomfort of occupants; and 6) breakage of windows. All of these may be related to the drift of the structure.

With respect to seismic loading, Hisada [146] has classified the problems associated with drift as: 1) restriction of damage to the non-structural components such as glass panels, curtain wall panels, plaster walls and other partitions; and 2) protection from motion sickness or discomfort.

Reed [155] presents a summary, as well as new data, for response of high-rise structures to wind loading and human response to this type of motion. Structural response to wind loading is discussed here and human response is discussed in Section 4.4 of this report. Reed presents calculation methods and analytical models from various other authors. Consideration is given to the identification of the wind loading on the structure and structural characteristics such as damping and natural frequency. A comprehensive and broad coverage of various methods and models are given. Emphasis is given to a probabilistic approach to the forcing functions as well as the structural response. Reed observed, from an examination of previous data and calculations and his calculations, that total displacement in the drag direction is often greater than total lift displacement, whereas at the same time lift acceleration is greater than drag acceleration. Also it has been observed that lift deflections are sometimes greater than drag deflections. This points to the important fact that there is a very high probability that structural motion normal to the wind is greater than the along-wind component. Reed considers both the across-wind and along-wind components in his study.

Reed found that drift deflections have been limited to a range of 0.0015 to 0.0030 times the total height of a structure depending upon whether the buildings were masonry or curtain-wall towers. The lower allowables were applied to the curtain-wall type structures. Acceleration calculations and measurements motivated observations that: lift accelerations may be 2 to 4 times the drag component, analytically predicted drag accelerations are 2 to 3 times the field or wind tunnel measurements, and twist motions are of the same magnitude as lift components. Reed points out that across-wind or lift response is a significant consideration in determining drift deflection or response. Details of this phenomenon have been studied by Novak, Davenport, Wooten, Scruton, Vickery, and Clark [150, 151, 162, and 165]. It is not the intention of this survey to present the methods for prediction of lift response, but rather to draw particular attention to the subject when lateral vibrations induced by wind loading are being considered for high-rise structures.

4.3.2 Quantitative Limitations on Drift.

In numerous building codes, standards, recommended design procedures, as well as in textbooks and research reports, the limitations on drift are specified as a fraction of the total building height. The value of the fraction appears to be loosely correlated with observations on the performance of structures designed and constructed with theoretical drift limitations. Values ranging from 1/200, mentioned by Khan, [147] to 1/1200 noted by Frischmann and Prabhu [145] are given in the literature. Fleming [144] has provided a summary of early work in this area.

The origin of this now familiar form of limitation was attributed to Spurr [158] according to Frischmann and Prabhu [145]. From 1930 to the present, the acceptable value has decreased somewhat. This is probably due to the changing technology of high-rise building construction which has seen the progressive elimination of masonry partitions and other elements which added considerable stiffness to the structure. These
components traditionally have not been included in the mathematical model used to compute the drift deflection.

A recent survey of seismic design codes by Hisada [146] revealed that drift limitations are specified for design earthquake loads in the Mexican and New Zealand seismic codes. Allowable values of drift deflection from 1/400 to 1/500 are given with an increase of twice the value if adequate clearances for all non-structural components are provided. In the commentary of the Uniform Building Code [160] a value of 1/200 is recommended. In Japan there is no limitation given in the Building Standard Law [146] but 2 cm per story is usually taken as the limit of the drift computed by dynamic analysis for high-rise buildings. Provisions on the separation of buildings to avoid contact during earthquake loading are given in seismic codes in the U.S.S.R., Venezuela, Mexico and Portugal [146] and U.S.A. [160].

The ACI Committee 442 report [156] comments that a deflection limit of 1/500 recommended by ACI Committee 435 [133] has resulted in the design of modern buildings that appear to have been satisfactory with respect to the following effects of sway under wind loading: 1) the stability of the individual columns as well as the structure as a whole; 2) the integrity of nonstructural partitions and glazing; and 3) the comfort of the occupants of buildings. It is pointed out that the method of computing the drift as well as the assumptions used in the calculations vary considerably among practicing engineers. This aspect is discussed further in Section 4.3.3 of this report.

Another limitation, reported by Philcox [153], that has been correlated with performance is the height-depth ratio of the building which is limited to 10 for a single shear wall and to 15 for a core. The depth is apparently the dimension parallel to the wind direction.

It has also been recognized that limiting the total deflection at the top of a structure may not be adequate. An additional limitation of 0.15 inches per story is given in the ACI Committee 435 report [133]. Davenport [139] reported that cracking of partitions in a 40 story building may occur if the drift exceeds 0.25 in. per story.

Since the limitations for drift essentially reflect an attempt to provide factors of safety for both collapse and serviceability criteria, it is likely that no single limitation is adequate. Davenport [139] has noted that the risk of incurring collapse should be extremely small and much less than the risk of unserviceability. Also, he notes that a distinction should be made between a structure which gives warning of collapse and one which collapses suddenly. He suggested mean occurrence rates per annum for the following: 1) catastrophic failure; 2) incipient, non-catastrophic failure; 3) window breakage, wall cracking; and 4) excessive acceleration. It would appear that future performance criteria should include probabilistically-based considerations of damage and risk.

4.3.3 Calculation of Drift.

As noted in the preceding section, the drift limitations expressed as a fraction of the overall building height have seemingly become more restrictive in recent years. This is attributed to a reduction in the stiffening provided by the non-structural elements. Also, the tremendous improvements in design and analysis techniques, facilitated by the computer, have influenced the correlation between actual building performance and design calculations. Assumptions previously implicit in the mathematical models are now explicitly included. Large, Carpenter and Morris [148] enumerate the contributory elements to the theoretical drift as: 1) web drift (bending of girders, bending of columns, deformation of connections
and bracing); and 2) chord drift (direct lengthening and shortening of columns). Also, the so-called secondary moments due to the direct column loads acting through the lateral displacements (P-Delta effect) can be important.

All elementary methods of rigid frame analysis permit the computation of deflections due to the bending of the girders and columns assuming fully continuous connections. The effect of so-called semirigid connections and secondary moment has been examined by Williams and Radland [163]. The deflection due to column shortening can be readily computed [143]. With the computer-based analysis techniques available, the modeling of a structure to include most of the structurally effective components is possible and it is expected that improved mathematical models should provide for closer correlation between observed behavior and design calculations.

With reference to the relative contributions of the web drift or shear racking and the chord drift or column shortening portions of the total drift, Khan [147] has shown that the web drift can be controlled and set to a predetermined magnitude by providing adequate column and girder stiffness at each floor. Because it is principally a function of the overturning moment, the chord component is more difficult to affect without simply increasing the column areas.

Secondary moments and the corresponding additional deflections have been examined with respect to limit design methods. Modern standards [159 and 137] consider the secondary moment in the design of beam-columns. The computation of sway deflections in unbraced frames including secondary moment effects is discussed by Driscoll [143].

4.3.4 Measurements of Drift Deflections.

Probably the first scientific measurements of the drift of a tall structure were performed by Gustave Eiffel on the top of the Eiffel Tower between 1893 and 1895. His summarized results are given in English by Parmelee [152]. Among other observations, Eiffel noted that high velocity gusts (implying short duration) had a smaller effect on the displacement of the top than those caused by the lower velocity continuous wind and that the measured displacements were less than the computed values. This he attributed to a lack of reality in the analysis assumptions. Parmelee [152] also reports that in 1994 the deflection of the Monadnock Building in Chicago, a sixteen story bearing wall building, was measured during a windstorm. The deflections were observed by transits and checked by observing the oscillation of plumb-bobs suspended in the stairway shaft. The two observations agreed well and gave an east and west vibration amplitude of 1/4 to 1/2 inch.

In order to verify a popular design method devised by Spurr [158], a scale model of a 55 story three-panel symmetrical bent was constructed and tested at Ohio State University. The static load test results, presented by Large, Carpenter and Morris [148], verified that the Spurr method was accurate for the design of regular symmetrical bents, but that the method was inadequate for irregularities such as fixed column bases, tall basement stories and two-story entrances. This is still an important result even for contemporary design since the Spurr method is based on a prescribed drift.

The continuous observations of the Empire State Building in New York are described by Rathburn [154]. Among these observations are a record of the pressure distributions on the structure and the movements of the top of the building.
The velocity and direction of the wind were obtained from an anemometer 1263 ft. above the street. The author questioned the accuracy of the anemometer for recording the velocity of the wind because of the presence of the building itself and other buildings in the area. The pressure and pressure distribution of the wind was studied by thirty manometers. Pressure distribution at the 36th, 55th and 75th floors are given for a 90 MPH NNW wind, 50 to 60 MPH NW wind and no wind. A lack of correlation in the measurements was indicated.

The motion at the top of the building was found to consist of two types: 1) a mean deflection from the vertical under the action of a steady wind; and 2) an oscillating motion under gusting which had different periods in each axis of the structure. The motion was studied using a vertical collimator on the 6th floor to observe a target on the 86th floor. Also a plumb-bob was suspended from near this target to near the sixth floor. The author reported that the collimator proved to be very useful in studying the vibrations while the plumb-bob was more useful in obtaining the mean deflections.

The reported plumb-bob observations are grouped into three classes based on the wind velocity: 1) less than 20 MPH; 2) 20-30 MPH; and 3) more than 30 MPH. The maximum deflection in the east direction under 90 MPH NNW wind was 10.7 inches. It was noted that the building did not return to the original position at rest indicating the considerable amount of inelastic material surrounding the structural frame. An interesting conclusion was that the rigidity of the building including the masonry exterior was stated to be approximately 350 percent more than the structural steel frame.

Investigations conducted by the National Research Council of Canada directed primarily toward measuring the natural periods of buildings are summarized by Wiss and Curth [164]. They also comment on the work of Rathburn [154]. Using these studies as background, they describe an investigation designed to measure the dynamic response of a high-rise concrete frame building to wind loading. The authors designate the steady component of the total measured lateral deflection as drift and the oscillations as vibration.

Seismic type instrumentation was selected to make motion measurements. A suitable system for measuring the vibrations in a high-rise building is stated as: 1) the natural period of the unit should be adjustable between 15 sec. and 70 sec.; and 2) the unit should be capable of measuring horizontal displacements up to 10 in. single amplitude [164]. A horizontal seismic pendulum was developed for the purpose of measuring the vibrations and a vertically hung pendulum was designed as a tiltmeter to measure drift. Two perpendicular horizontal pendulums and a two-component tiltmeter were placed in the concrete frame structure at the NE corner of the 55th (top) floor. An identical set of instruments was located at the SW corner of same floor. Tiltmeters were also located on the 42nd and 3rd floors.

Data were recorded for a period of 30 days using an oscillograph. The average daily winds ranged from 5 mph to 20 mph with daily peak gust velocities between 14 and 48 mph (U.S. Weather Bureau data). Measured wind velocities above the top of the building ranged from 0 mph to 70 mph during the recording period. A number of difficulties due to temperature fluctuations were encountered in calibrating the instruments and are reported. The largest magnitude of the vibration recorded was a peak-to-peak displacement of 1.0 in. during the 70 mph wind velocity at the top of the building. The drift at the 70 mph wind speed was not directly determined because of the time characteristics of the high wind velocities compared to the pendulum. The authors developed a formula for determining the drift based on changes in velocity from 10 mph to 20 mph. The extrapolation of data with the formula resulted in an estimated drift of 1.5 in. for the 70 mph wind.
Also, a comparison was made between the drift computed by the structural designer and the corresponding value calculated by the formula based on the test data [164]. The structural designer computed a deflection of 8.9 in. at the top which corresponded to wind velocity at the top of 103 mph as calculated from the equivalent static pressure specified in the Chicago Building Code. Using the 103 mph velocity in the drift formula, a drift of 3.3 in. was calculated which is 27 percent of the value computed by the structural engineer. It was noted that the building had shear and core wall elements as well as columns to resist the wind loading which made it difficult to calculate drift deflections. It appears that an inaccurate mathematical structural model and loading function lead to an inaccurate drift calculation. Also, the accuracy of measurement devices needs to be considered as pointed out by Dalgliesh and Ward [138] in a discussion of Wiss and Curth [164] research.

Mackintosh [149] has reported that for frames tested with large lateral loadings, the difference in measured plastic compared to elastic drift due to beam flexure is much greater than would be expected.

An interesting set of aeroelastic wind tunnel studies to assess the relative aerodynamic merits of different building shapes is reported by Robertson [157]. Aeroelastic models of six building shapes (triangle, modified triangle, square, cross, 2:1 rectangle and circle), each with the same floor area, height, density and frequency of transverse vibration, were studied in a simulated city environment of a boundary layer wind tunnel. Envelopes of peak deflection (static plus dynamic) at the top of the models are plotted and the circular cross-section appears to have the least overall deflection while the triangle and the 2:1 rectangle have the largest deflections in the principal directions. No analysis of these results are presented in the reference.

Davenport et al [142] describe the analysis of measurements made of drift deflections for the 100 story John Hancock Center in Chicago. They also determined natural periods of vibration for the two principal axes of the structure. A comparison was made between actual dynamic deflections and those predicted by the "gust loading factor" approach given by Davenport [140] and Vickery [161]. Measurements of the fluctuating response were reported to agree with the gust load factor approach. The fraction of critical damping was approximately constant for the two axes of the structure at 0.009 and 0.006 for a wide range of deformations from fractions of an inch to one inch. The largest movements were found to be normal to the wind direction.

4.4 Human Perception and Response to Motion of Structures.

The study of human response to motion is part of a general area which is often termed human engineering. Humans are subject to motion during most of their lives while walking, travelling, working and even sleeping. Some motion causes pleasure and a feeling of well being, while other types of motion cause annoyance, discomfort, sickness, or even death. There is considerable literature available on the general subject and the limits of various levels of response such as perception, annoyance, tolerance, etc., to various types of motion in ships, planes, cars, trains, space vehicles, etc., are well known.

A special area of the general study of human response to motion is human response to the motion of building structures. The state of knowledge in this area is by no means complete. In fact, not really enough is known to be able to make definitive suggestions to the designers and builders of structures. This section describes pertinent references and makes comparisons of various vibration criteria.
Deflection and vibration may disturb or hamper the functional performance of the subsystems of a building, i.e., elevators jamming, machinery or instruments functioning improperly, windows popping out, etc. The resulting human cost is annoyance, discomfort and inconvenience resulting in economic loss. If the tolerance of the subsystems and humans is known, the structure may be designed so that deflections are not likely to be large enough to cause more than very infrequent malfunction or human discomfort.

"Large" deflections and "violent" vibrations may also impair the safety of the structure and result in its failure or collapse. With the usual ductile structures used in modern construction, the deformations corresponding to this limit state are several orders of magnitude larger than those human occupants are willing to tolerate on a day-to-day basis. Of course, the ultimate limit states involving deflections must be considered in design. Thus, the human responds to deflections and vibrations which are usually much smaller than those which result in collapse.

A seldom discussed aspect of human response to deflection and vibration of structures is fear from collapse if motion is experienced. People generally believe that structures are rigid and that visible or felt deformation portends failure or at least an unsafe situation. This psychological factor leads to various degrees of fear or discomfort which exacerbate the physiological effects of motion.

Awareness of static deflection is based on the sense of vision (sagging ceiling or floor, cracked plaster or cracked walls). Adverse reaction results if the deflection can be sensed by vision. How much deflection can be detected by eye? Perhaps the traditional rule of L/360 is related to this effect. Could psychological research give an answer. There was no indication of an answer in the literature which was examined.

Awareness of vibratory deflection is based on the kinesthetic senses and on vision. Visual effects are direct, i.e., a floor is seen to vibrate, and indirect, i.e., chandeliers sway or relative motion is observed. Vibratory motion is sensed by the human kinesthetic perception through the inner ear, or by other parts of the body. Severe vibration can also be accompanied by noise, although even small vibrations may result in audible effects, i.e., tinkling glassware in a display case.

A great deal is known about the response of humans to vibratory motion, but the present knowledge of human reaction to the motion of buildings is still quite rudimentary. Guignard [169] in a review paper, had the following to say about the subject in 1971:

The human sensitivity to mechanical vibration extends both above and below the range of hearing and is indicated by a variety of receptor organs distributed throughout the body. In the most sensitive range (below about 30 Hz), the threshold of sensation may be as low as 1 cm/sec². The social reaction to traffic induced vibration of structures, however, may be modified by many physical and psychological factors. Much of the published work on the human response to vibration has been concerned with relatively intense and continuous vibration such as is felt in vehicles or near industrial machinery, and most of the existing recommendations as to criteria and limits of human exposure to vibration are related to those fields. It is suggested that there is need for more basic research into the factors determining human reactions to vibration of relatively low intensity, of the kind induced in buildings and other structures excited by traffic or industrial operations outside.
4.4.1 Previous State-of-the-Art Reviews.

Human reaction to vibration is important and work has been done on this subject. Reviews of the state-of-the-art have been made from various points of view. Eight such reviews are given in this section. Each of these is relatively short and the benefit of another compilation combining all of these is rather doubtful, thus only a brief statement is made about each.

Historically significant reports related to these state-of-the-art reviews are given in the list of references [167, 171 through 174, 176, 179 and 181]. These are listed for the convenience to the reader if specific details are desired.

Cope [166] reviewed the status of knowledge on the response of humans to moving vehicles. He gives 19 references and the report was published in 1960. The reviews by Hornick and Lefritz [170] and by Richards [177] deal with the same subject.


The most up-to-date and pertinent review, as far as response to motion of buildings is concerned, is by Guignard [169]. He points out the need for fundamental research in this area, noting that most of the information on human response to vibration was developed for much greater vibratory intensities than is tolerable on a day-to-day basis for buildings occupied by people.

The report of Onysko [175] is a broad review of structural requirements of wood joist floor systems including the requirements for strength and deflection. The date of the report is January 1970, and 97 references are listed and discussed. From a structural designer's point of view this is the most significant literature review for floors of those cited here. Onysko discusses the code requirements, the background for code provisions, past research and current problems. The following quote from this report indicates the conclusion reached that more research is needed.

While it has been demonstrated that deflection under a concentrated load correlates reasonably well with acceptability, this criterion alone does not yield the necessary guidance to provide acceptable floors. It has been seen that the vibration of floors affects their acceptability. Thus, it may be expected that a performance specification founded on human perception to vibration will provide the necessary guidance. It is in this area that work is necessary, not only to define how the variables affect acceptability, but also to indicate how floor designs might be improved.

Steffens [178] discusses the various scales used for measuring human reaction to vibration, but dwells mainly on the effects of vibration on the structure itself. He lists 172 references covering the time period from about 1900 through 1965, but only a few of these refer to work on human response. The references are mostly English and German, although some Japanese and East European work is covered. There is a notable absence of Russian literature.
The review by Wright and Green [180] specifically cover the state of knowledge of human response to motion up to 1959. The various scales of response are described and 70 references are cited.

The clear conclusion from these eight reviews is that there is no means enough known to formulate valid rules on performance criteria regarding human response to vibration for the design of building structures.

4.4.2 Physiological Responses.

Many studies have been performed to examine the response of the human body and its subsystems to different types of motion. The references listed for this section [182 through 197] should be considered to be representative samples rather than as a complete list. The work has concentrated on the dynamics of human organs and the body as a whole, and physiological reactions to relatively severe vibrations and motions. Much has been learned about the reaction of humans to motions encountered in moving and vibrating vehicles; however, this work is only indirectly applicable to motion of building structures.

4.4.3 Subjective Responses of Humans.

The references of this section deal with the human subjective response to vibration. Typically, human subjects were placed on devices which provided a controlled and programmed vibratory environment. The test subjects then answered questions about what they felt and how they reacted. Generally the human test subjects were subjected to sinusoidally vibrating situations, thus the results apply essentially to long term vibration (steady-state) and not necessarily to transient vibrations which are also encountered in building structures. These studies demonstrate that the scales of human response as developed by Reiher and Meister [212] and Goldman [204] are valid for defining response to steady-state vibration which can be expected in vehicles or in structures with vibrating machinery. Whether or not these scales are valid for designing structures subjected to random and transient dynamic loading for human occupancy has been repeatedly questioned.

The majority of the references listed in this section [198, 199, 201, 202, 205 and 207 through 211] have been reviewed by Wright and Green [180] or Steffens [178]; therefore, they will not be reviewed here. Four references have not been reviewed in the previously mentioned surveys and will be discussed here in some detail. These references are significant because they were the only ones which report work performed specifically to measure the response of humans to building vibration.

The research by Blume [200] measured the response of occupants in buildings which are subjected to vibrations originating from underground nuclear blasts. Tests used eight subjects on a suspended platform (pendulum) to determine the threshold level of horizontal motion perception. These tests were correlated with motion perceptions reported by observers in high-rise buildings in Las Vegas, Nevada, during underground nuclear tests. Threshold of perception was presented as a curve relating acceleration to period. A statistically derived equation for the acceleration at the threshold of perception was given as

\[
\text{mean perception threshold (m)} = 0.00245 + 0.00025 \times \text{frequency}
\]

where \( g \) is the acceleration of gravity. The standard deviation about the mean expression line was 0.0012 \( g \). The range of applicability of
Blume's equation is for a frequency range of 33.3 Hz to 0.2 Hz or, in terms of period, from 0.03 sec. to 5 sec. The ranges of the formula were put together as a composite from results presented earlier by Goldman and von Gierke [168] in the high frequency range, from results of the pendulum tests on eight subjects in the laboratory (medium frequency), and from results of the Las Vegas building observations (low frequency).

The results of Blume's work give further information on the response of humans to low frequency, low amplitude horizontal vibration. The mean acceleration at the threshold of perception range from 0.0025 g to 0.0108 g over the whole applicable frequency domain. This compares with the range of 0.004 g to 0.0075 g reported by Khan and Parmelee [206], but the latter did not correlate their findings with frequency. Blume also notes that there is significant difference between individuals and that the body attitude is less significant. This is essentially the same conclusion as that reached by Khan and Parmelee. No physiological and psychological data were given on the eight subjects. The number of subjects was small and no consideration was given to the effect of damping and duration of vibration.

Khan and Parmelee [206] and Chen and Robertson [203] studied the response of humans to wind gust induced vibrations in tall multi-story structures. Khan and Parmelee made three sets of observations. The first concerned an analytical study of the maximum acceleration of four tall buildings (two 50 story, one 60 story and one 100 story buildings) under gusts deviating 20 percent from the maximum wind velocity for 5 sec. Maximum accelerations were found to vary from 0.002 g to 0.004 g. The second report concerned tests on 30 human subjects on a rotating platform. Body posture (standing, sitting, lying), face direction (four orthogonal directions relative to direction of acceleration) and acceleration were the variables. It was found that body posture and face direction had less effect than differences between individuals. The barely perceptible level of acceleration was found to be from 0.004 g to 0.0075 g, and 0.02 g was disturbing to all subjects. The final set of observations contained acceleration data taken in one building (860 Lake Shore Drive, Chicago; a 26 story apartment building) at one location (top story) during a wind storm. The maximum acceleration measured during a 70 mph wind was 0.0042 g.

Chen and Robertson [203] tested 112 subjects to determine the threshold of horizontal motion perception. Tests were performed in a horizontally oscillating room at periods of 5, 10 and 15 seconds. The following variables were considered: period, body orientation (four directions with respect to direction of motion), body movement (standing, walking, sitting) and expectancy level (three levels; total ignorance of the test, expectation of motion, and previous experience of motion in test chamber). The data from the subjective responses of the test subjects were analyzed statistically and they were presented in the form of curves relating cumulative frequency to perception thresholds in g's. All of the test variables were found to be significant. This contrasts with Blume's observation that body posture was not significant. However, Blume's subjects did not move and all can be assumed to have expected the motion. Chen and Robertson demonstrated that the perception threshold in g's is highest for walking subjects and the lowest for sitting subjects. It appears that perception thresholds are smaller when subjects are anticipating motion or have experienced it previously.

Additional tests have been performed by Wiss, Janney and Elstner, in cooperation with Parmelee on many subjects to study the effect of damping. This work has not been published, however, findings relating amplitude, frequency and damping to human perception and response have been developed.
4.4.4 Observed Human Response.

The references 213 through 220 are representative of research relating to the ability of test subjects to see, track and respond in a vibrating environment. This work is especially significant for transportation vehicles, but the motions are more severe than those expected in a building structure, thus are only indirectly applicable.

The previously referenced research on human response to motion was performed essentially on subjects in a laboratory controlled environment. Some of the results were planned to be used in the design of buildings. Data on observed human response to motion of structures in service, which is necessary to make a successful transformation from laboratory controlled human response scales to building design, is difficult and expensive.

Lenzen (to be published, refer to Section 4.2.3.1 of this report) built several full scale concrete slab-steel joist floor systems. He tested subjects standing or sitting in chairs on the test floors as these floors were excited by impact, and then recorded the impressions and reactions of the test subjects. Also data on subjective responses were obtained by interviewing people who had experienced vibrating floors in actual buildings in service and during construction. The human response characteristics were coordinated with dynamic analyses of the corresponding floor systems and limits of human tolerance were established. No documentation as to subjective response was presented, only conclusions are given. Lenzen concluded that the most important parameter affecting the response of humans to transient motion is the rate at which the amplitude of deflection due to the initial impact is damped out. It was demonstrated that if the deflection amplitude decayed to about 20 percent of its initial value at impact in five cycles or less, the human subjects felt only the initial impact. Perception to oscillations and unpleasant reactions were nonexistent. If the vibration persisted for more than five cycles, the oscillations were felt.

In the case where no inherent structural damping exists due to partitions or other items, Lenzen [225] suggested a modification of the Reiher and Meister [212] human response curves for a design guide of floor systems. These curves are also presented in the review report by Wright and Green [180]. The deflection amplitude-frequency domain is subdivided by curves into subdomains of "not perceptible, slightly perceptible, distinctly perceptible, strongly perceptible, disturbing and very disturbing." The same subdivisions are retained by Lenzen, but the deflection amplitude scale (ordinate) is referenced to the initial amplitude upon impact, and the magnitude of the amplitude is increased to 10 times the Reiher and Meister scale. This increase was made to empirically account for transient vibration rather than steady state. The boundary line between the slightly perceptible and the distinctly perceptible domains was taken to be the design limit for floors supporting human activity. Furthermore, this limiting line was shifted upward parallel to its original position a distance equal to 20 percent of the original distance between the original lines representing the two boundaries of the distinctly perceptible region. This shift is assumed to empirically incorporate the fact that all floor systems possess some damping.

While the modification of the Reiher and Meister human response curves to account for transient vibration in a damped structure appears to be somewhat arbitrary, a great deal of circumstantial evidence was presented for justification. This evidence is taken from interviews of people subjected to vibrations of floors in the laboratory and in the field, and from dynamic analyses of floor systems which had been judged as satisfactory or unsatisfactory in service.
It is difficult to draw quantitative conclusions from the Lenzen vibration work, but qualitative aspects of the problem become apparent from studying his results: 1) the important differences between transient and steady state vibration; 2) the importance of damping; and 3) the difficulty of transforming human response in the laboratory to actual field requirements. Observations of unsatisfactory floors in the field were subjectively correlated with interview data on laboratory floors.

Onysko [175], in his literature survey, reviews in considerable detail studies made by Russel [226], Hansen [224] and Vermeyden [228] on the human subjective response to wood-joist floors. These sought to discover whether or not the test floors were acceptable to the test subjects when the floors were subjected to various load intensities. It was quite difficult to determine what the tests results meant. It appeared that a relationship between deflection and occupant comfort was developed. These results were then used to develop allowable deflection criteria.

Chang [221] gives a graph which can be used to ascertain human comfort in buildings subjected to wind loading, but it is not clear from the paper where this information comes from and how it was developed. An article by Crandell [222] gives curves for human and structural response to vibration. He notes that safe vibrations for structures are two orders of magnitude larger than the vibrations felt by humans. Davenport [223] considers the dynamic response of a tall building and attempts a correlation with thresholds of human response.

Reed [227] recently studied high-rise structures with respect to wind loading and human comfort. Linear and angular acceleration were identified as the response parameters for humans in tall buildings. Time-rate of change of acceleration (jolt or jerk) may also be significant. This suggests that acceleration response is a function of frequency. The fundamental natural frequencies observed of tall structures was found to range from .07 to .3 Hz. Items other than motion were observed to be significant and to influence human response, such as, noise, structural creaking and groaning, local motion of fixtures within a structure and the world moving outside. The latter refers to perceptible relative visual motion as viewed from one structure to another. An important observation is made that perceptible motion is not necessarily annoying motion. A review is made of the cues which manifest external human response, i.e., human mechanisms which transduce vibration to human perception.

Reed's experimental program included a subjective survey of human response to the motion of two high-rise buildings located in widely separated geographic areas of the United States each subjected to strong winter storms. Motion measurements were taken in one of the structures and were inferred for the other from wind tunnel studies and calculation. Table 1 gives the pertinent information for each structure, the storm data and the structural response. The height of both buildings is approximately 550 feet and the fundamental natural frequencies were approximately 0.17 Hz. The average magnitudes of motion were 0.002 g (rms, root-mean-square) and 0.005 g (rms) during the storm peaks which lasted 20 to 30 minutes. The structures and storm intensities were very similar. Occupants of the structures were surveyed by personal interviews regarding their response to the motion of the structures during the storms. A large portion of those interviewed rated motion sensation and motion sickness symptoms as significant. The motion sickness symptom was rated the highest of the two. The interviews indicated that a learning mechanisms is possible, i.e., there is an ability of people to adapt to the motion of high-rise structures.

Perception of motion was studied by Reed on a probabilistic basis from structures in service by determining the percent of people objecting
<table>
<thead>
<tr>
<th>Statistic</th>
<th>Building A</th>
<th>Building B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>550' +</td>
<td>550' +</td>
</tr>
<tr>
<td>Structural System</td>
<td>Moment resisting frames</td>
<td>Exterior tube</td>
</tr>
<tr>
<td>First natural period</td>
<td>5 - 6 sec.</td>
<td>6 - 7 sec.</td>
</tr>
<tr>
<td>Age</td>
<td>&lt; 10 yr.</td>
<td>&lt; 10 yr.</td>
</tr>
<tr>
<td>Building density</td>
<td>9 - 10 pcf</td>
<td>9 - 10 pcf</td>
</tr>
<tr>
<td>Environment</td>
<td>Urban, near ocean</td>
<td>Urban, near ocean</td>
</tr>
<tr>
<td>Building type</td>
<td>Office</td>
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</tr>
<tr>
<td>Average number of occupants</td>
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</tr>
<tr>
<td>Length of perceptible motion</td>
<td>6 hr</td>
<td>5 hr</td>
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<tr>
<td>during work day</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length of storm peak</td>
<td>30 min</td>
<td>20 min</td>
</tr>
<tr>
<td>Average rms motion level</td>
<td>0.001 g (rms)</td>
<td>0.002 g (rms)</td>
</tr>
<tr>
<td>during entire perceptible</td>
<td></td>
<td></td>
</tr>
<tr>
<td>period</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average rms motion level</td>
<td>0.002 g (rms)</td>
<td>0.005 g (rms)</td>
</tr>
<tr>
<td>during storm peak</td>
<td></td>
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</tr>
</tbody>
</table>
to the motion resulting from a storm when given the number of storms that could occur in one year. Typical results ranged from 0.4 to 25.6 percent of the people objecting to 2 to 5 storms per year. This approach is proposed to be superior to the extrapolation of laboratory experiments. Reed's research is based on all environmental and psychological factors that interact with motion response. A general conclusion made for the structures studied is that people are sensitive to motion and do not consider perceptible motion acceptable except on rare occurrences. When applied to the design problem, a probabilistic approach is taken which relates a perceptible acceleration magnitude to a desirability of its occurrence. It was found through interviews with owners, developers and engineers that a reasonable limit of 2 percent of the occupants of a structure could object to motion in one year without interferring significantly with the overall operation of a structure. This objection rate is then related to a return period of a severe storm for the given acceleration level of the structure investigated.

4.4.5 Vibration Clauses in Standards.

Traditionally the possible problems with vibrating buildings were avoided by specifying live load (superimposed static load) deflection limits in structural standards. This topic was discussed in detail in Sections 3. and 4.3. Generally this has worked through experience with respect to human response to structural vibrations. The application of these specifications made computations quite simple such that load tables of allowable live loads for presumably adequate structural systems could be compiled. The codes generally do not distinguish between various uses of the buildings, except that the Dutch Code TAB 1955 [extracted from 20] for wood floors has a live load deflection limit of 1/500 of the span for floors not subject to considerable vibration while a limit of 1/800 of the span is prescribed for floors subject to considerable vibration.

With the increased use of high strength materials, larger spans and generally more flexible construction, it became apparent that the simple live load deflection limits did not always suffice. Khan and Parmelee [206] report some buildings which were found to be unsatisfactory in high velocity gusty windstorms (horizontal deflections), and Lenzen [128] occasionally refers to unsatisfactory floors. It appears that the live load deflection limits as specified in codes provide a guide for satisfactory performance for vibratory loads encountered in buildings most of the time, but that this is not an infallible procedure. As a result, modern standards [29] do require that

Beams and girders supporting large open floor areas free of partitions or other sources of damping, where transient vibrations due to pedestrian traffic might not be acceptable, shall be designed with due regard for vibration.

Similar provisions are contained in the Canadian Standards [28] where the Commentary also contains an approximate formula for computing peak acceleration due to wind gusts, and a perceptible acceleration of about 0.01 g is indicated. The Guide Criteria for the Design and Evaluation of Operation Breakthrough Housing Systems [230] recommends that
Transient vibrations induced by human activity should decay to 0.2 of their initial displacement-amplitude within a time not to exceed 1/2 second.

Furthermore, steady state vibration is to be isolated, or, where this is not possible, a human perception curve (deflection amplitude versus frequency), based on the Lenzen modified Reiher and Meister [225] curves, should be satisfied.

The International Organization for Standardization currently has a draft standard, Guide for the Evaluation of Human Exposure to Whole-Body Vibration ISO/DIS 2631 [231] which is within committee for review and approval. This guide is written with a general approach for application to many vibratory environments. As quoted from the Guide, it is applicable to

Vibrations transmitted to the body as a whole through the supporting surface; namely, the feet of a standing man, the buttocks of a seated man, or the supporting area of a reclining man. This kind of vibration is usual in vehicles, in vibrating buildings and in the vicinity of working machinery.

The Guide is applicable to a frequency range of 1 to 80 Hz for steady state, periodic vibrations, random vibrations with a distributed frequency spectrum and continuous shock excitation if the energy is contained within the 1 to 80 Hz band. Vibration directions are three dimensional and apply to the human for the foot-to-head, right-to-left side and back-to-chest orientations. All allowable vibrations are given in terms of frequency and rms (root-mean-square) acceleration. Therefore, interpretations of the given allowable vibrations can be made for vertical floor vibrations and transverse or drift deflections. An adjustment formulation of the Guide applicable to building structures has been proposed by Splittgerber [229 and 232]. Several classifications of commercial and residential occupancies are categorized. These categories are: 1) hospitals and sanatoriums; 2) private homes situated in a district with prevailing residential buildings; 3) private homes situated in a district in which residential buildings and industrial plants are mixed; and 4) private homes situated in a district with prevailing industrial plants. An allowable vibration level is designated for night and day occupancy for each of these four occupancy classifications. Generally the night time allowable are lower than the day time. Vibration allowables were established for vibratory inputs of steady state, interrupted (less than 2 hours without pause) and short time (1 to 3 transient vibrations in a 24-hour period). One band of acceptable vibration is designated with the subjective title of "reduced comfort boundary to avoid considerable annoyance to occupants" [223].

The Guide [231] is strictly qualified in that it may not be extrapolated outside the frequency band of 1 to 80 Hz. Below 1 Hz symptoms of kinetosis (motion sickness) appear which do not relate to frequency, intensity and duration of vibration exposure in a manner to that of the higher frequency vibrations. Above 80 Hz local factors such as precise direction of vibration and area of application become significant factors. These qualifications are significant with respect to indicating regions of vibration frequencies which manifest significantly different human responses.

It is apparent that codes and standards are moving cautiously toward a specific set of requirements with regard to human response to structural motion. This, of course, is necessary, but certainly not enough guidance
is given to the designer to be reasonably sure of avoiding annoying vibration. Until more is known about this difficult topic, this dilemma will be present and the designer must use his best judgement.

4.4.6 Comparison of Data for Human Response to Structural Vibrations.

The literature review has revealed several sources of information from which the structural designer can obtain guidance for the control of deflections to minimize discomfort of building occupants. The information is diverse and many times does not give consistent results when the various data are compared.

For comparison purposes two categories can be formed: 1) human response to vertical vibrations, 2) and human response to horizontal or lateral vibrations. Vertical vibrations refer to floor vibrations and lateral vibrations refer to drift deflections such as that induced by wind load.

The literature is not consistent in presenting the magnitudes of vibration thresholds and sensitivities. Thus a base parameter of root-mean-square (rms) acceleration will be used so that comparisons can be made. Some data is presented in the form of deflection or half amplitude deflection. This data was transformed to rms acceleration by

\[ a_{\text{peak}} = 4\pi^2 f^2 \Delta \]  

(1)

where

\( \Delta \) = half amplitude deflection

\( f \) = frequency, Hz

and

\[ a_{\text{rms}} = \frac{1}{\sqrt{2}} a_{\text{peak}} \]  

(2)

General vibration specifications such as the ISO Guide [231] present allowable vibrations in dB (decibel) levels above or below a basic specified acceleration. The dB is a power ratio defined as

\[ 1\text{dB} = 20 \log \frac{a_1}{a_2} \]  

(3)

where the \( a_i \) (i = 1, 2) are the basic or allowable acceleration magnitudes.

Human response to structural vibrations changes depending upon the vibration time history, i.e., steady or transient. Steady state has an explicit definition of continued sinusoidal vibration or random vibration that can be described in a continuous manner. Transient vibrations have been ambiguously described in the literature. Each author has a separate definition, none of which appear to have a common denominator. Comparison of steady state vibration sensitivity from several sources will be made which apply to vertical floor vibrations induced by forcing functions emanating from mechanical equipment, etc. Transient vertical vibrations in floors will then be considered. Finally attention will be given to transient horizontal (lateral) vibrations induced by wind loads.
The ISO Guide [231] is presented in rms acceleration units. The allowable magnitude of acceleration as proposed by Splittgerber [232] is given as a dB (decibel) level above or below a basic ISO Guide acceleration allowable. Figure 2 shows the allowable upper and lower boundaries for steady state vertical vibration for day and night occupancy of private homes situated in a district with prevailing residential buildings determined from the ISO Guide and adjusted according to Splittgerber. For this particular vibration classification, no differentiation was made between day and night occupancy, however, a differentiation is made for other housing classifications between day and night. The vibration levels for these boundaries shown on figure 2 are for a foot-to-head orientation, \( a_z \), applicable to the standing or sitting human.

Also shown on figure 2 are the perception band widths from the original work of Reiher and Meister [212]. These perception bands range from a perception threshold to an unpleasant level. The data from Reiher and Meister is in the form of half-amplitude deflection, thus it was transformed to rms acceleration by equations (1) and (2). Another set of curves from Goldman [204] are shown on figure 2 to complete the steady state vibration comparison. Goldman's curves are average perception levels from a perception threshold to unpleasant and represent much of the data on human perception to the time of Goldman's report (1948). The bounds of curves on figure 2 represent the rms acceleration and frequency ranges over which they were established by the respective authors. The Goldman summary data and the Reiher and Meister data compare consistently which is not unexpected as the Reiher and Meister data was included in the Goldman summary. However, the Goldman data does extend over a broader frequency range. The ISO Guide with the Splittgerber adjustment falls below the perceptible range of both Goldman and Reiher and Meister for \( f > 4 \) Hz. This indicates that for this frequency range the ISO Guide requires that floor vibrations not be perceptible assuming the Goldman and Reiher and Meister data are valid. For \( 1 < f < 4 \) Hz the ISO Guide is below the Goldman average perceptible curve and trends in a similar manner to the Goldman curve. Reiher and Meister data do not extend to this frequency range. Data for steady state vertical vibrations appear consistent within the various subjective classifications, i.e., perceptible, unpleasant, etc., and trend in a similar manner with respect to frequency versus acceleration.

As previously mentioned, transient vibrations have been ambiguously defined in the literature. The adjustments recommended by Splittgerber for the ISO Guide, which are applicable to vertical transient vibrations, use two different states of transient vibrations, interrupted and short time. Also a differentiation is made between day and night occupancy. Figure 3 shows three different bands of allowable vertical transient vibration for the ISO Guide-Splittgerber references; short-time day, interrupted day, and short-time and interrupted night. Each of these decrease in allowable acceleration level in the respective order listed.

Lenzen [127] uses another definition of transient vibration which considers the characteristics of one single transient pulse rather than an aggregation of pulses over a given period of time such as Splittgerber. Lenzen's transient vibration specification is a modification of the original Reiher and Meister work [212] for steady state vibrations. All levels of Lenzen's perception bands are shown on figure 3. Lenzen's data is in half-amplitude displacement and was transformed to rms acceleration according to equations (1) and (2). Lengths of curves represent the frequency range over which his modification extends. The trends of the ISO Guide Splittgerber and Lenzen data for \( f > 8 \) Hz are similar; however, for \( f < 8 \) Hz there is a divergence of the two. That is, the allowable perception level increases for the ISO Guide with respect to Lenzen's allowables as frequency decreases. The allowable short-time day ISO Guide specification for \( f > 8 \) Hz corresponds to Lenzen's slightly perceptible range which does appear to be a consistent relation; however, at \( f = 1 \) Hz the Lenzen
Legend

- Goldman [209]
- Reiher and Meister [212]
- ISO, Splittgerber [231, 232]

Figure 2. Human Response Thresholds, Steady State Vertical Vibration
Figure 3. Human Response Thresholds, Transient Vertical Vibration
allowable is an order of magnitude (1 x 10) below the ISO Guide-Splittgerber allowable for a similar perception range. Except for \( f < 4 \) Hz, the ISO Guide-Splittgerber interrupted day and short-time, interrupted night-fall approximately an order of magnitude below the Lenzen slightly perceivable range. No definite conclusion can be drawn except that, relatively speaking, either Lenzen's allowables are too high for slightly perceivable boundaries or the Splittgerber allowables are too restrictive for the interrupted day and short-time, interrupted night boundaries.

Horizontal vibrations are considered with respect to wind loading on high-rise structures. The vibration direction induced in the human is generally considered as side-to-side or back-to-chest for a standing or sitting human. The horizontal vibrations are considered transient for these comparisons however, each author's definition of the nature of transient vibration is different. Transient vibration infers that the structure is responding to either gust loading or the short time (such as the Splittgerber designation) lateral vibration as it oscillates from the excitation of the rise and fall of the peak of a storm activity. All data are compared on a basis of frequency and rms acceleration. Where data were in the form of peak acceleration or deflection, transformations were made consistent with equations (1) and (2).

Blume [200] compiled data from others and their own experiments. They determined a median for a perception threshold for horizontal vibration. Blume's tests exposed the subjects to an oscillatory motion of which the total duration was not designated, however, it was observed that the subjects responded to the initial acceleration peak. This threshold is shown on figure 4 with boundaries of + s, the unbiased estimation of the standard deviation. The data compilation covers a large frequency range of \( 0.2 < f < 30 \) Hz. Most high-rise structures fall in the lower portion of the band of \( f < 1 \) Hz. For this lower region of the data, the threshold of perception is approximately constant with respect to frequency. The median threshold is between 0.0015 and 0.002 g (rms) and the estimated standard deviation is approximately 0.001 g (rms).

Splittgerber [232] recommends adjustments to the ISO Guide for horizontal vibrations similar in manner to the way in which they were applied to vertical vibrations. Horizontal vibrations or accelerations, \( \ddot{x} \), refer to back-to-chest and right-to-left side orientation with respect to vibration input to the human. These levels of allowable vibration are shown in figure 4 for short-time day and interrupted day for private homes situated in a district with prevailing residential buildings. The short-time day would be applicable to gust loading response, whereas the interrupted day would be applicable to response from the rise and fall of the peak of a storm activity. The ISO Guide-Splittgerber allowable vibrations are not applicable for \( f < 1 \) Hz. The trends of the ISO and Blume data appear consistent and the bounds are similar and coincident for some of the band width of allowables. The ISO Guide-Splittgerber short-time day allowable is approximately one order of magnitude higher than the Blume data and the ISO Guide-Splittgerber interrupted day allowable. The ISO Guide-Splittgerber allowables indicated that humans are less sensitive to short time vibrations than the longer exposure period of an interrupted vibration.

Chen and Robertson [203] presented low frequency threshold human response data for horizontal vibrations for \( 0.067 < f < 0.2 \) Hz. The mean of the perception threshold (50th percentile threshold) is shown on figure 4 with boundaries of plus and minus the unbiased estimated standard deviation. The Chen and Robertson data is offset on the acceleration axis by a factor of approximately 2 with respect to the Blume data and the Chen and Robertson data are higher. The trend of their data is divergent from the trend of the Blume data, this is, for decreasing frequency, the perception threshold is decreasing for the Blume data and increasing for the Chen and Robertson
Figure 4. Human Response Thresholds, Transient Horizontal Vibration
data. The two sets of data do not overlap but are adjoining, therefore, it is impossible to determine any transitional trends for the connecting frequency ranges. The Chen and Robertson data indicate an estimated standard deviation of approximately 0.001 g which is similar to the estimated standard deviation of the Blume data.

Reed [227] presented an approach to human perception that was different from all other literature reviewed. Perception levels were determined from structures in service. Two discrete points were determined from these high-rise structures subjected to strong storms. Several different subjective evaluations resulting in various terms referring to motion perception were made. A nuisance-comfort evaluation was made through personal interviews. It was found for the two structures that 37.6 percent of the people felt that the motion was a nuisance and/or uncomfortable for the lower level acceleration, 0.002 g, (figure 4) and 60.5 percent felt the motion to be a nuisance and/or uncomfortable for the higher level acceleration, 0.005 g (figure 4). It should be noted that one point of the Reed data falls on the median of the perception threshold of Robertson data and the other point of the Reed data falls close to the median of the perception threshold of the Blume data. However, Reed's discrete points do not agree with the Chen and Robertson and Blume data in that the latter two are perception thresholds whereas the Reed data is an uncomfortable region. Subjective classifications are difficult to conclusively compare, however, these do appear to be inconsistent.

The data for the low frequencies, f < 1 Hz, for the horizontal vibration is insufficient to determine whether it is inconsistent or that there is a change of trend of acceleration response. In a gross sense, the data could indicate an increase in perception level as frequency decreases, a similar trend as that indicated for the transient and steady state vertical vibrations at the lower frequencies (figures 2 and 3). Much information needs to be obtained for perception thresholds of horizontal vibrations for f < 1 Hz. This frequency region will be difficult to interpret due to the predominance of kinetosis (motion sickness) which both the ISO Guide [231] and Reed [227] have observed.

Chang [221] presented data and specifications for human response to horizontal vibrations; however, these data are not used here for comparisons as their basis is questionable. Chang's acceleration sensitivity levels were determined from tests performed by Parks and Snyder [211] for 1 < f < 27 Hz for subjects strapped to a seat and subjected to vertical vibrations. Transformation of this narrow band vertical vibration data to broadband horizontal vibration specifications is questionable. Reed [227] also points out that a review and reinterpretation of Parks' and Snyder's report resulted in a large change in the threshold values at f = 1 Hz for vertical vibrations. This draws even more question to the Chang specification.

An overall comparison is made (figure 5) of perception levels for transient vertical, transient horizontal and steady state vertical vibrations. Extreme boundaries of similar perception thresholds were determined from figures 2, 3 and 4. Each of the three classes of vibrations with the respective extreme boundaries were then plotted on figure 5. The boundaries are broad; trend similarly in some areas and are divergent in others. Divergence of extremes occur in all three vibration categories for f < 5 Hz. Divergence is especially evident for the transient vertical vibrations. Acceleration perception levels for transient vertical vibrations are higher compared to steady state by an order of magnitude. The acceleration levels generally translate down without a change in frequency (figure 5) from transient to steady state thresholds. Generally comparing transient vertical and transient horizontal vibrations, the perception level for horizontal is approximately one order of magnitude below the vertical.
Figure 5. Comparison of Human Perception Levels to Vibration

RMS Acceleration, \( a \times g \times 10^x \)

Frequency, \( f \), Hz

Steady State Vertical Vibration

Transient Vertical Vibration

Transient Horizontal Vibration

10^{-2} 10^{-1} 2 3 5 7 10^2 10^3 10^4

10^{-1} 10^2 2 3 5 7 10^1 10^2 10^3 10^4
There is also a general trend of a shift of the frequency to the left for the horizontal with respect to the vertical transient vibrations. That is, there is both a reduction of acceleration as well as frequency for a given threshold of perception for transient horizontal vibration as compared to transient vertical vibration. The "appendage" of transient horizontal vibration response (Figure 5) for $f < 0.2$ Hz trends similar to the transient vertical vibration response.

The broad band of variance for threshold levels, trends of human response with respect to frequency, and discontinuities and gaps in data demonstrate the need for considerable study in the domain of human response to structural vibrations. Considerably more questions arise in the area of human response to transient vibrations than to steady state vibrations. Definition of a transient load in itself is a very complex and difficult task. These comparisons were not intended to compile all available data, but to present selected data and research representing a broad expanse of time from 1931 to the present.

4.5 Discussion.

The problem of designing building structures for deflection which are as economical as the state-of-the-art of construction permits and are satisfactory with respect to serviceability, functionally, and aesthetic and human requirements is composed of three parts: 1) the definition of the excitation (forcing function); 2) the definition of the structural characteristics and 3) the definition of the deflection or response requirements. These three are vitally connected and one cannot consider each individually without regard to the other two. Each of the three parts of the problem are random variables and a truly satisfactory answer cannot be obtained without a probabilistic formulation, coupled with cost optimization. Methodologies for such a solution exist, although a realistic practical solution is still far off. Some parts of the puzzle are already available: wind loads and gusts have been defined probabilistically, dynamic analyses of complex structures have been performed and some indication of human response to motion is known. Many pieces are still missing: what should the forcing motion be for human activity on a floor system; what is the statistically quantified human response to damped vibration; what is the actual dynamic character of a building as compared to what can be modeled; what role does the psychology of fear have in the human response to motion of buildings, etc.?

Much needs to be determined about human response to motion in buildings. But more importantly, a definition of the total problem must first be formulated, each missing piece must be identified, and then it must be decided to what extent it is worthwhile to get a complete answer to each question.

By examination of the literature in the area of human response, it is clear that previous work is either inapplicable, i.e., it is concerned with motions well above that acceptable in buildings or incomplete (no psychological evaluation), inadequate consideration for damping, no clearly defined test conditions, etc. It is possible to obtain the required answers with a good deal of effort in a broad research program involving cooperation between psychologists, physiologists, engineers, builders and statisticians. Ultimately such a program is necessary to provide an adequate and scientifically based guide for the totally functional building structure.
5. References

Static Deflections


2. ACI Committee 318, Building Code Requirements for Reinforced Concrete (ACI 318-71), American Concrete Institute, Detroit, Michigan, 1971.


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Dynamic Deflections

Current Design Practice


Forcing Functions - Wind


Forcing Functions - Seismic


Forcing Functions - Sonic Boom


Forcing Functions - Shock Wave


Forcing Functions - Machinery


Forcing Functions - Traffic


Structural Systems - Houses and Apartments


Structural Systems - Tall Buildings


Structural Systems - Foundations


Natural Frequency


Damping


Damage to Structures - Risk


Damage to Structures - Assessment


Vibration Isolation


Floor Vibrations

Analytical Studies on Floor Systems


Experiments on Steel Joist Floor Systems (Lenzen)


Drift


137. "Building Code Requirements for Reinforced Concrete (ACI-318-71)", American Concrete Institute, Detroit, Michigan, 1971.


Human Perception and Response to Motion

State of the Art Reviews


Physiological Responses


Subjective Responses of Humans


Observed Human Response

Performance in a Vibrating Environment


Response to Building Vibration


Vibration Clauses in Standards


233. Private correspondence, Dr. H. Splittgerber to Dr. R. Crist, July 1972.
APPENDIX A

Summary of Vertical Deflection Limitations in U. S. Codes and Standards

National Standards
Model Codes
State and Local Codes
<table>
<thead>
<tr>
<th>Standard</th>
<th>Deflection</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>American Concrete Institute, 318-71, Section 0.5</td>
<td>Stiffness limits in L/d, span/depth</td>
<td></td>
</tr>
<tr>
<td>Beams or ribbed one-way slabs</td>
<td>Solid one-way slabs</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>20</td>
<td>Simply supported</td>
</tr>
<tr>
<td>18.5</td>
<td>24</td>
<td>Propped cantilever</td>
</tr>
<tr>
<td>21</td>
<td>28</td>
<td>Both ends continuous</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>Cantilever</td>
</tr>
<tr>
<td>Deflection limits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L/180</td>
<td>flat roofs, immediate live load</td>
<td></td>
</tr>
<tr>
<td>L/360</td>
<td>floors, immediate live load</td>
<td></td>
</tr>
<tr>
<td>L/430</td>
<td>floors and roofs, damage due to</td>
<td></td>
</tr>
<tr>
<td></td>
<td>deflection undesirable, longtime</td>
<td></td>
</tr>
<tr>
<td></td>
<td>sustained loads + immediate live</td>
<td></td>
</tr>
<tr>
<td></td>
<td>load</td>
<td></td>
</tr>
<tr>
<td>L/240</td>
<td>floors and roofs, damage due to</td>
<td></td>
</tr>
<tr>
<td></td>
<td>deflection unlikely; longtime</td>
<td></td>
</tr>
<tr>
<td></td>
<td>sustained and immediate live loads.</td>
<td></td>
</tr>
<tr>
<td>STANDARD</td>
<td>DEFLECTION LIMITS</td>
<td>COMMENTS</td>
</tr>
<tr>
<td>-------------------------------------------------------------------------</td>
<td>-------------------</td>
<td>----------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>L/240</td>
<td>Floors and roofs with plaster ceiling attached</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Other cases.</td>
</tr>
<tr>
<td>CODES</td>
<td>DEFLECTION LIMITS</td>
<td>COMMENTS</td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>-------------------</td>
<td>-----------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>L/240</td>
<td>Unplastered roofs</td>
</tr>
<tr>
<td></td>
<td>L/360</td>
<td>Unplastered floors</td>
</tr>
<tr>
<td>Uniform Building Code, 1970 - UBC-ICBO</td>
<td>L/360</td>
<td>Trussed rafters under live load supporting finished ceiling</td>
</tr>
<tr>
<td></td>
<td>L/240</td>
<td>Roof members supporting plaster or floor members</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Live load</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total load.</td>
</tr>
<tr>
<td>CODE</td>
<td>DEFLECTION LIMITS</td>
<td>CONTENTS</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>-------------------</td>
<td>--------------------------------------------</td>
</tr>
<tr>
<td>Alaska</td>
<td>No specific provisions</td>
<td></td>
</tr>
<tr>
<td>Code of Ordinances, City of</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anchorage, Alaska, 1967</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Connecticut</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic Building Code, State of</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Connecticut, 1971</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>L/360</td>
<td>Working load for floor and roof assemblies.</td>
</tr>
<tr>
<td></td>
<td>L/240</td>
<td>Plastered</td>
</tr>
<tr>
<td></td>
<td>L/180</td>
<td>Unplastered floor.</td>
</tr>
<tr>
<td>District of Columbia</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D. C. Building Code (1972)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>L/180</td>
<td>Floor and roof assemblies under</td>
</tr>
<tr>
<td></td>
<td></td>
<td>approved working load</td>
</tr>
<tr>
<td></td>
<td>L/240</td>
<td>Unplastered roofs</td>
</tr>
<tr>
<td></td>
<td>L/360</td>
<td>Unplastered floor</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Plastered Construction</td>
</tr>
<tr>
<td>Code</td>
<td>Deflection Limits</td>
<td>Comments</td>
</tr>
<tr>
<td>------</td>
<td>------------------</td>
<td>----------</td>
</tr>
<tr>
<td>Florida Epcot Building Code - Ready Creek Improvement District (Walt Disney World), 1970</td>
<td>L/360 L/240</td>
<td>Roof member supporting plaster or floor member. Live load only Live load + dead load including creep.</td>
</tr>
</tbody>
</table>

All roofs shall be designed with sufficient slope or camber to assure adequate drainage after the dead load deflection due to initial set and long-time deformation has taken place, except that roofs may be constructed level if the dead load deflection pockets due to initial set and long-time deformation are not over 1/2-inch deep below the drainage invert, and at no point deflect more than 1/2 inch for a 5 psf live load on all or alternate spans. Cantilever members drained at the unsupported end need not meet the deflection requirements.
<table>
<thead>
<tr>
<th>CODE</th>
<th>DEFLECTION LIMITS</th>
<th>CONTENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hawaii</strong>&lt;br&gt;The revised Ordinance at Honolulu, 1961</td>
<td>Same as Uniform Building Code, 1970 - UBC - ICBO</td>
<td></td>
</tr>
<tr>
<td><strong>Indiana</strong>&lt;br&gt;Building Rules and Regulations&lt;br&gt;Building Council of Indiana&lt;br&gt;Vol. I., 1969</td>
<td>L/360</td>
<td>Floor joists supporting plastered ceilings under full live load and dead load excluding plaster.</td>
</tr>
<tr>
<td><strong>Municipal Code of Evansville, Ind.</strong>&lt;br&gt;1962</td>
<td>No specific provisions</td>
<td></td>
</tr>
<tr>
<td><strong>Minnesota</strong>&lt;br&gt;Minneapolis City Charter and Ordinances, 1949</td>
<td>L/250</td>
<td>Concentrated live load deflection of formed steel roofing.</td>
</tr>
<tr>
<td><strong>Missouri</strong>&lt;br&gt;Revised Code of St. Louis, Vol. II, 1960</td>
<td>L/180, L/240, L/360</td>
<td>Floor and roof assemblies under approved working load. Unplastered roof, Unplastered floor, Plastered Construction</td>
</tr>
<tr>
<td>CODE</td>
<td>DEFLECTION LIMITS</td>
<td>COMMENTS</td>
</tr>
<tr>
<td>------------------------------------------</td>
<td>-------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>New Jersey</td>
<td></td>
<td>Working Load</td>
</tr>
<tr>
<td>The Building Code of Title 12,</td>
<td></td>
<td>Plastered roof and floor assemblies</td>
</tr>
<tr>
<td>Administrative Code - NJAC 1971</td>
<td></td>
<td>Unplastered floor construction</td>
</tr>
<tr>
<td></td>
<td>L/360</td>
<td>Unplastered roof construction</td>
</tr>
<tr>
<td></td>
<td>L/240</td>
<td></td>
</tr>
<tr>
<td></td>
<td>L/180</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Recovery 75% after removal of first application. No increase after second application.</td>
</tr>
<tr>
<td>CODE</td>
<td>DEFLECTION LIMITS</td>
<td>CONTENTS</td>
</tr>
<tr>
<td>------</td>
<td>-------------------</td>
<td>----------</td>
</tr>
</tbody>
</table>
| New York  
State Building Construction Code  
Applicable to General Building  
Construction, State of New York.  
December, 1964 | L/360  
L/240  
L/180  
Residual deflection from first application of load ≤ 25% of maximum deflection under load. Total residual deflection after second application = 1.1 residual deflection from first application. | Imposed Test Load  
To be plastered  
Not to be plastered  
Promenade roof not to be plastered  
1 1/2 times imposed Load |
| State Building Construction Code  
Applicable to Multiple Dwellings,  
State of New York.  
December, 1964 | Same as for General Building Construction. |
<table>
<thead>
<tr>
<th>CODE</th>
<th>DEFLECTION LIMITS</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York (Continued) State Building Construction Code Applicable to One-and Two-Family Dwellings, State of New York December, 1964</td>
<td>Same as for General Building Construction.</td>
<td>Members supporting walls or partitions constructed of frangible materials under dead load and live load including the effects of creep and shrinkage.</td>
</tr>
<tr>
<td>City of New York Building Code (1968)</td>
<td>L/360</td>
<td>Members supporting glass panels under design wind load.</td>
</tr>
<tr>
<td></td>
<td>L/360</td>
<td>Loaded with Live Load Live Load plus K times the dead load. (factor K varies from 0 to 2 depending on the composition of the member)</td>
</tr>
<tr>
<td>STATE AND LOCAL CODES</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CONTENTS</td>
<td>DEFORMATION LIMITS</td>
<td>CODE</td>
</tr>
<tr>
<td>Allowable or design live test load</td>
<td>L/180</td>
<td>Ohio Building Code, 1970</td>
</tr>
<tr>
<td>Structural panels, decks or members</td>
<td>L/350</td>
<td>Ohio Building Code for Dwelling Houses, 1972</td>
</tr>
<tr>
<td>If plastered ceilings are attached.</td>
<td>L&lt;sup&gt;2&lt;/sup&gt;/180-t; Cantilever beams and slabs</td>
<td>City of Akron, Ohio 1960</td>
</tr>
<tr>
<td>It is probably the depth, (not found in reproduced part of code).</td>
<td>L&lt;sup&gt;2&lt;/sup&gt;/10,000t; Flat slabs (L = the longest span)</td>
<td></td>
</tr>
<tr>
<td>Code</td>
<td>Deflection Limits</td>
<td>Comments</td>
</tr>
<tr>
<td>------</td>
<td>-------------------</td>
<td>----------</td>
</tr>
<tr>
<td>City of Akron, Ohio (Continued)</td>
<td>( L^2/10,000 ) - Beams and slabs continuous at the supports for the direction of the principal reinforcement. L/180</td>
<td>Floor construction intended to support or be attached to partitions or other construction likely to be damaged by large deflections at the floor.</td>
</tr>
<tr>
<td>City of Memphis, Tennessee Building Code, 1967</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wisconsin</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wisconsin Administrative Code, 1972</td>
<td>( L/360 )</td>
<td>Beams and girder supporting floors and plastered ceilings under design live load.</td>
</tr>
</tbody>
</table>
APPENDIX B

Summary of Vertical Deflection Limitations
in
Foreign Codes and Standards
<table>
<thead>
<tr>
<th>CODE OR STANDARD</th>
<th>DEFLECTION LIMITS</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SAA Steel Structures Code, CAI - 1968</td>
<td>Must not impair strength or serviceability; limits to be responsibility of engineer.</td>
<td></td>
</tr>
<tr>
<td><strong>Australia [13]</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Austria [8]* (this provision is not clear in the English translation)</td>
<td>L/600, Average</td>
<td>Concentrated loading 75 lbs.</td>
</tr>
<tr>
<td>Reinforced Concrete, ONB 4001 1965</td>
<td>L/450, Minimum</td>
<td></td>
</tr>
<tr>
<td></td>
<td>L/450, Minimum</td>
<td></td>
</tr>
<tr>
<td></td>
<td>L/300 or</td>
<td></td>
</tr>
<tr>
<td></td>
<td>One-way: L/D ≤ 35</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Two-way (unrestrained): L/D ≤ 50</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Two-way (restrained): L/D ≤ 60</td>
<td></td>
</tr>
</tbody>
</table>

* Numbers in brackets refer to literature references. References are grouped in the respective report sections and are listed in alphabetical order in each section.
<table>
<thead>
<tr>
<th>CODE OR STANDARD</th>
<th>DEFLECTION LIMITS</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Timber Floor Joists,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CEN 10050-4:1994-62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belgium</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reinforced Concrete [8]</td>
<td></td>
<td>General instructions</td>
</tr>
<tr>
<td></td>
<td>Same as Germany</td>
<td></td>
</tr>
</tbody>
</table>

Reinforced concrete structures must have sufficient rigidity so that no deformation takes place which would create severe conditions for decorative elements applied to the concrete surface or with respect to drainage or crack formation in partition walls or similar damage. Further, one should not fail to consider an increase in deformation under sustained load. General stipulations should be made that deformations should create no distortion in the structure or its equipment as a consequence of load, creep, shrinkage or temperature changes, which may attain different intensities for the different materials present in the structure.
<table>
<thead>
<tr>
<th>CODE OR STANDARD</th>
<th>DEFLECTION LIMITS</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Canada</strong>&lt;br&gt;CSA Standard, A23.3 - 1970&lt;br&gt;Reinforced Concrete Structures,&lt;br&gt;Section 6.</td>
<td>Stiffness limits, in terms of L/d, span/depth.&lt;br&gt;<strong>Beams</strong>&lt;br&gt;20 23 26 10 L/360&lt;br&gt;<strong>Slabs</strong>&lt;br&gt;25 30 35 12 (one-way)</td>
<td>Simply supported&lt;br&gt;Simply supported&lt;br&gt;Fixed ended&lt;br&gt;Cantilever&lt;br&gt;Live load deflection for floors with partitions.&lt;br&gt;For all types of structures&lt;br&gt;Members supporting floors&lt;br&gt;Plastered&lt;br&gt;Asphaltic roof coverings&lt;br&gt;Sheet metal roofs.</td>
</tr>
<tr>
<td>CODE OR STANDARD</td>
<td>DEFLECTION LIMITS</td>
<td>CONTENTS</td>
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<tr>
<td>-----------------</td>
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<td>----------</td>
</tr>
<tr>
<td>Canada (Continued)</td>
<td></td>
<td>Total Load</td>
</tr>
<tr>
<td>CSA Standard 086-1970</td>
<td>L/360</td>
<td>In the presence of plaster or ceramic materials</td>
</tr>
<tr>
<td>&quot;Timber Structures&quot;, Section 3.4.1.2</td>
<td>L/180</td>
<td>All floors if no material in which cracks may be caused by deflections are attached.</td>
</tr>
<tr>
<td>GSA 0152 - 1964 [13], Residential Standards</td>
<td>L/180, L/360</td>
<td>Concentrated loading 175 lbs. Uniformly distributed loading for spans over 24 in.</td>
</tr>
<tr>
<td>Denmark</td>
<td></td>
<td>Live load</td>
</tr>
<tr>
<td>Timber Floors DS 415, Article 12 [20]</td>
<td>L/500</td>
<td>Residential construction with light partitions not considered part of live load.</td>
</tr>
<tr>
<td>CODE OR STANDARD</td>
<td>DEFLECTION LIMITS</td>
<td>COMMENTS</td>
</tr>
<tr>
<td>------------------</td>
<td>-------------------</td>
<td>----------</td>
</tr>
<tr>
<td>England</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BS 449, Part 1, 1970</td>
<td>Deflection must not impair use of building - functional and aesthetic</td>
<td>Live load deflection limit for all beams.</td>
</tr>
<tr>
<td>&quot;The use of Structural Steel in Buildings&quot;, Section 15</td>
<td>L/360</td>
<td>General instructions</td>
</tr>
<tr>
<td>Reinforced Concrete [8]</td>
<td>Reinforced concrete structures should have sufficient rigidity to prevent deformations and deflections, which can affect the strength or service capacity of the structure or produce cracks in partition walls or in surface details.</td>
<td></td>
</tr>
<tr>
<td>CP 114: Part 2: 1969</td>
<td>Stiffness limit given only, in terms of L/d, span/depth</td>
<td></td>
</tr>
<tr>
<td>&quot;Reinforced Concrete in Buildings&quot;</td>
<td></td>
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</tr>
<tr>
<td>Section 309</td>
<td></td>
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<tr>
<td>Beams</td>
<td>One way slabs</td>
<td>Two way slabs</td>
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<td></td>
<td>Simply supported</td>
<td>Continuous</td>
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<td></td>
<td></td>
<td>Cantilever.</td>
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<tr>
<td>CODE OR STANDARD</td>
<td>DEFLECTION LIMITS</td>
<td>COMMENTS</td>
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<tr>
<td>England (Continued)</td>
<td></td>
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</tr>
<tr>
<td>&quot;Prestressed Concrete in Buildings&quot;, Section 320</td>
<td>Stiffness limits, in terms of L/d, span/depth</td>
<td>Simply supported</td>
</tr>
<tr>
<td>CP 116: Part 2: 1969</td>
<td></td>
<td>Continuous</td>
</tr>
<tr>
<td>&quot;Precast Concrete&quot;, Section 314, 334</td>
<td></td>
<td>Cantilever</td>
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<td>Beams</td>
<td>One-way slabs</td>
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<td>12</td>
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<tr>
<td></td>
<td>Deflection must not impair strength or efficiency - functional and aesthetic.</td>
<td></td>
</tr>
<tr>
<td>Timber Floors,</td>
<td></td>
<td>Total Load</td>
</tr>
<tr>
<td>CP 112: Part 2: 1971</td>
<td>L/333</td>
<td></td>
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<tr>
<td>CODE OR STANDARD</td>
<td>DEFLECTION LIMITS</td>
<td>COMMENTS</td>
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<tr>
<td>France</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reinforced concrete [8]</td>
<td>L/500</td>
<td>Door and window lintels, beams and joists in slabs under masonry and</td>
</tr>
<tr>
<td></td>
<td></td>
<td>portion walls, joists under slab and roof supports; working load.</td>
</tr>
<tr>
<td></td>
<td>L/1000</td>
<td>Flexural members of floors and roofs not carrying walls; live load</td>
</tr>
<tr>
<td></td>
<td>L/500</td>
<td>(snow and traffic)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Flexural members of floors and roofs not carrying walls; working load.</td>
</tr>
<tr>
<td>CODE OR STANDARD</td>
<td>DEFLECTION LIMITS</td>
<td>COMMENTS</td>
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<tr>
<td><strong>France</strong> (Continued) Timber Floors, NF P21-202, Art. 7, 1946 [20]</td>
<td><strong>Height/Span Ratio</strong>&lt;br&gt; L/500 1/12&lt;br&gt;L/500 - L/300 1/12 - 1/24&lt;br&gt; &gt; L/300 &lt; 1/24</td>
<td><strong>Total Load</strong>&lt;br&gt;If justified by the importance of the structure, the permanent load contribution is multiplied by a factor of 2 in computing the total load to be used in the deflection computation.</td>
</tr>
<tr>
<td><strong>Germany</strong> DIN 1050, 1946 Steel buildings, Section 13</td>
<td><strong>Floor Beams:</strong> L/300&lt;br&gt;<strong>Roof Beams:</strong> L/200&lt;br&gt;<strong>Beams in Rigid Frames:</strong> L/500&lt;br&gt;<strong>Encased Beams:</strong> L/200&lt;br&gt;<strong>Composite Beams:</strong> L/300&lt;br&gt;<strong>Cantilever Beams:</strong> L/250</td>
<td></td>
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<tr>
<td>CODE OR STANDARD</td>
<td>DEFLECTION LIMITS</td>
<td>COMMENTS</td>
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<tr>
<td>Germany (Continued)</td>
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<tr>
<td>Reinforced Concrete [8]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Timber Floor Joists</td>
<td></td>
<td></td>
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<tr>
<td>DIN 1052, Article 58, 1969 [20]</td>
<td>Same as Austria</td>
<td></td>
</tr>
<tr>
<td></td>
<td>L/300</td>
<td>Dwelling and office areas</td>
</tr>
<tr>
<td></td>
<td>L/200</td>
<td>Roof joists, rafters etc.</td>
</tr>
<tr>
<td></td>
<td>L/150</td>
<td>Collar ties</td>
</tr>
<tr>
<td>Holland</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Timber Floor Joists</td>
<td></td>
<td></td>
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<tr>
<td>T.G.B. 1955, Article 33 [20]</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>L/500 for L ≤ 5.25 'l (17'-3'')</td>
<td>Live Load</td>
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<tr>
<td></td>
<td></td>
<td>Floors not subjected to</td>
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<td></td>
<td></td>
<td>considered vibration.</td>
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<tr>
<td></td>
<td></td>
<td>Floors subjected to considerable</td>
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<tr>
<td></td>
<td></td>
<td>vibration.</td>
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<tr>
<td>CODE OR STANDARD</td>
<td>DEFLECTION LIMITS</td>
<td>COMMENTS</td>
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<tr>
<td>Norway</td>
<td></td>
<td>Same as Germany</td>
</tr>
<tr>
<td>Timber floor joists</td>
<td></td>
<td>Span-to-Depth ratio</td>
</tr>
<tr>
<td>NS 446, 1957 [20]</td>
<td></td>
<td>Main Beams: 15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Secondary Beams: 20</td>
</tr>
<tr>
<td>Poland</td>
<td></td>
<td>One-Way Slabs</td>
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<tr>
<td>Reinforced Concrete [3]</td>
<td></td>
<td>Simply Supported: 35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Restrained: 40</td>
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<tr>
<td></td>
<td></td>
<td>Two-Way Slabs</td>
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<tr>
<td></td>
<td></td>
<td>Simply Supported: 45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Restrained: 50</td>
</tr>
<tr>
<td>CODE OR STANDARD</td>
<td>DEFLECTION LIMITS</td>
<td>COMMENTS</td>
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<td>------------------</td>
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<tr>
<td>Sweden</td>
<td></td>
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<tr>
<td>Reinforced Concrete [8]</td>
<td>Slab Thickness</td>
<td></td>
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<tr>
<td></td>
<td>( d_{\text{min}} = \sqrt[3]{\frac{7 M_f}{\sigma_{bz}}} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( M_f = ) maximum span moment</td>
<td></td>
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<tr>
<td></td>
<td>( \sigma_{bz} = ) allowable tensile stress of concrete in flexure.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>L/D Ratios</strong></td>
<td></td>
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<tr>
<td></td>
<td>Beams: 24</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Flat slabs with caps: 32</td>
<td></td>
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<tr>
<td></td>
<td>Roofs: 40</td>
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<tr>
<td></td>
<td>Deformations should not be of a magnitude creating a probability of undesirable effects in the structure or its equipment.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Beams</strong></td>
<td></td>
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<tr>
<td></td>
<td>( L = ) distance between points of zero moment</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( L = ) longer span</td>
<td></td>
</tr>
<tr>
<td>Code or Standard</td>
<td>Deflection Limits</td>
<td>Contents</td>
</tr>
<tr>
<td>------------------</td>
<td>-------------------</td>
<td>----------</td>
</tr>
<tr>
<td>Switzerland</td>
<td>L/400</td>
<td>Live load</td>
</tr>
<tr>
<td>Standards for Steel Structures, S.I.A. - 1956. Timber Floor Joists S.I.A. No. 164 Article 24 [20]</td>
<td>Similar to Germany except for special conditions such as junctions with plasterboard where the boundary values will be determined accordingly.</td>
<td></td>
</tr>
<tr>
<td>Turkey</td>
<td>Two-Way Slabs, Simply-supported: L/D ≤ 50 Two-Way Slabs, Restrained or Continuous: L/D ≤ 60</td>
<td></td>
</tr>
<tr>
<td>CODE OR STANDARD</td>
<td>DEFLECTION LIMITS</td>
<td>COMMENTS</td>
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<tr>
<td>Yugoslavia</td>
<td></td>
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<tr>
<td>Reinforced Concrete [8]</td>
<td>Rectangular or T-Beams</td>
<td>L/D ≤ 20</td>
</tr>
<tr>
<td></td>
<td>Slabs</td>
<td></td>
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<tr>
<td></td>
<td>One-way: L/D ≤ 30</td>
<td></td>
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<tr>
<td></td>
<td>Two-way: L/D ≤ 35</td>
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</table>

L = distance between points of zero moment.
Steel floor joist vibration formulas were developed by Lenzen [113].* The steel joist-concrete slab floor system was assumed to act as a two-way plate system under vibratory excitation from human activity. Only an equivalent number of joists, $N$, are assumed to participate in the vibration, where

$$N = 1 + 2 \sum_{i=1}^{n} \cos \pi \frac{x_i}{2x_o} \quad (1C)$$

where $n = 1, 2, \ldots, n$, number of joist in floor under consideration

$N = \text{equivalent number of full effective joists}$

$x_i = \text{distance from the center joist to the joist under consideration}$

$x_o = \text{distance from the center joist to the edge of the effective floor}$, where

$$x_o = \frac{3}{4} \sqrt{2} \frac{\varepsilon L}{\pi} \quad (2C)$$

where $L = \text{length of joist and } \varepsilon = \left( \frac{D_x}{D_y} \right)^{1/4}$

$D_x = \text{flexural stiffness perpendicular to the joist (slab only)}$

$D_y = \text{flexural stiffness parallel to the joists (composite section)}.$

The flexural stiffness perpendicular to the joists is the stiffness of a unit width of the concrete slab or

$$D_x = \frac{E_c t^3}{12} \quad (4C)$$

where $E_c = \text{modulus of elasticity of concrete}$

$t = \text{slab thickness}$

The flexural stiffness parallel to the joists is determined from the formula

$$D_y = \frac{E I_c}{b} \quad (5C)$$

where $I$ is the moment of inertia of a composite section consisting of one steel joist and the concrete slab of width, $b$, where $b$ is the effective slab width which usually is equal to the joist spacing. $I$ is determined by converting the slab to an equivalent steel element by reducing its width, $b$, to $b_e = b \frac{E_c}{E}$.

* Numbers in brackets refer to literature references. References are grouped in the respective report sections and are listed in alphabetical order in each section.
The floor will behave dynamically as a simply supported equivalent flexural beam consisting of \( N \) joists, thus its dynamic properties can be determined much more easily than the dynamic properties of an orthotropic slab. The beam has an equivalent flexural stiffness of

\[
I_{\text{equ}} = N I_c
\]  

(6C)

and an equivalent mass of

\[
m_{\text{equ}} = \frac{w N}{g}
\]  

(7C)

where \( w \) is the weight per unit length of one joist and the slab of width equal to the joist spacing, and \( g \) is the acceleration due to gravity.

The natural frequency of the equivalent beam can be determined from the flexure formula for a beam

\[
f_n = \frac{\pi}{2L^2} \sqrt{\frac{E I_{\text{equ}}}{m_{\text{equ}}}} = \frac{\pi}{2L^2} \sqrt{\frac{E I_c g}{w}}
\]  

(8C)

Human response to vibrating floors depends not only on the frequency of vibration but also on the amplitude of the deflection due to impact. Lenzen used: 1) an impactor, consisting of a cylindrical steel weight of 3155 lbs in a vertical frame; and 2) a heel drop on the floor by a human. These two methods of excitation gave distinctly different responses in the floors. The first impact, due to the mechanical impactor, was approximated by a rectangular pulse having a force of 794 lbs and a duration of 0.01 seconds, \((F = 794 \text{ and } t_d = 0.01)\). The corresponding maximum amplitude at the center of the effective floor area is given as \( A_0 \) where

\[
A_0 = \left[ \frac{4FL^3}{\pi N E I_c} \right] \text{ for } t_o \leq t_d
\]  

(9C)

and

\[
A_0 = \left[ \frac{4FL^3}{\pi N E I_c} \right] \sin \pi f_n t_d \text{ for } t_o > t_d
\]  

(10C)

where

- \( A_0 \) = first maximum amplitude, in.,
- \( F \) = force of rectangular impulse, lbs.,
- \( L \) = length of joist, in.,
- \( N \) = equivalent number of full effective joists
- \( I_c \) = moment of inertia of one joist and its slab, acting compositely, in
- \( E \) = modulus of elasticity of steel, \( E = 29 \times 10^6 \text{ psi} \)
- \( f_n \) = natural frequency (eq. 8C)
\[ t_d = \text{duration of impulse} \]
\[ t_o = \text{time to occurrence of maximum amplitude}, \]
\[ t_o = \frac{1}{2 f_n} \]  

(11C)

The impact due to the human heel drop was approximated by a triangular impulse of duration, \( t_d \), and maximum force, \( F \), and the values of \( t_d = 0.05 \) seconds and \( F = 606 \) lbs were used. The maximum amplitude at time \( t_o \) is

\[ A_o = \left[ \frac{2 F L^3}{\pi f_n E N I_c} \right] \left( 2 - \frac{t_o}{t_d} \right) \quad \text{for } t_o \leq t_d \]  

(12C)

where

\[ t_o = \left[ \frac{1}{\pi f_n} \right] \tan^{-1} \left( 2 \pi f_n t_d \right) \]  

(13C)

and

\[ A_o = \left[ \frac{2 F L^3}{\pi^4 E N I_c} \right] \left( \frac{1}{2 \pi f_n t_d} \right) \times \]

\[ \sqrt{2 \left[ 1 - 2 \pi f_n t_d \sin 2 \pi f_n t_d - \cos 2 \pi f_n t_d \right] + \left[ 2 \pi f_n t_d \right]^2} \]

for \( t_o > t_d \)  

(14C)

where

\[ t_o = \left( \frac{1}{2 \pi f_n} \right) \tan^{-1} \left[ \frac{1 - \cos 2 \pi f_n t_d}{\sin 2 \pi f_n t_d - 2 \pi f_n t_d} \right] \]  

(15C)
A literature survey and state-of-the-art study was compiled using 233 primary source documents, research papers, and texts. Over 800 documents were scanned to arrive at the primary source documents. The problem of structural deflections is discussed and reviewed in its component areas of static and dynamic deflections as related to forcing functions and structural characteristics. Also the interactions of major structural deflections with building structures subsystems and human occupants is reviewed. Emphasis is placed on serviceability limit states of deflections. Detailed comparisons of human response to structural vibrations are also made. This report is broad in scope and covers the areas of analysis, design and experimentation.
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