NBS BUILDING SCIENCE SERIES 162

Ring-on-Ring Tests and Load Capacity of Cladding Glass

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ABSTRACT

Although ring-on-ring test results have been used in the past to obtain information on the strength of glass, no methodology has so far been developed in the literature explicitly relating such results to the load capacity of cladding glass. The main purpose of this report is to propose such a methodology. The proposed methodology makes use of recent advances in the modeling of the fracture mechanics behavior of glass and the calculation of stresses in plates exhibiting geometric nonlinearity. Evidence is presented which strongly suggests that the probability distribution of the load capacity of cladding glass panels whose failure is due to surface flaws can be estimated reliably on the basis of results of ring-on-ring tests used in conjunction with (a) numerical methods for the analysis of stresses in plates, and (b) information on the elastic and fracture mechanics behavior of glass currently available or that can be obtained routinely. Two interesting findings are noted. First, owing to the way in which results of ring-on-ring tests are utilized, the relatively large variabilities typical of fracture mechanics parameters, as well as the uncertainties with respect to the shapes of surface flaws, have a minor effect on the estimation of load capacities. Second, two-parameter Weibull distributions, previously used in the literature to model the strength of glass and the load capacity of cladding panels, are not consistent with experimental results. On the other hand, three-parameter Weibull distributions model the observed glass behavior credibly.

Keywords: buildings; engineering mechanics; failure; fracture mechanics; glass; loads (forces); probability theory; ring-on-ring tests; strength.

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1. INTRODUCTION

For most common construction materials (e.g., steel or concrete), the load capacity of structural members is determined on the basis of strength data obtained from tests conducted on small size standard specimens, rather than from the destructive testing of full-size members. The economy inherent in the use of small-size standard specimens is due not only to their lower cost as compared to the cost of full-size members, but also to the fact that strength data obtained by testing a sufficiently large number of such specimens can be used for designing a wide variety of structural members with different configurations, types of loading, and sizes.

Procedures for determining the load capacity of glass panels on the basis of strength data obtained by testing small standard specimens do not currently exist. For this reason design charts issued by glass manufacturers have traditionally been based on destructive tests performed on full-size glass panels (see, e.g., reference 1). It has been pointed out in the literature (see, e.g., reference 2), that such charts exhibit significant inconsistencies. Such inconsistencies are due at least in part to the relatively small numbers of panels (between 2 and 30--see references 4, 5, and 6) used in most of the tests on which the charts were based. Since the charts cover a wide range of panel sizes, the number of panels that would have to be subjected to destructive tests in order to develop dependable data for design could in practice be prohibitive.

It therefore is desirable to develop a methodology for estimating cladding panel design loads from strength data obtained by testing small standard specimens. The purpose of this report is to propose such a methodology. The methodology utilizes recent advances [3, 14], which enable the probability distribution of the load capacity of glass panels to be estimated by numerical methods on the basis of information on the strength, stiffness, and fracture mechanics properties of glass.

Information characterizing the elastic behavior of glass is available or can be obtained by well established test methods. The elastic parameters exhibit relatively small variability, so that uncertainties pertaining to their actual values do not significantly affect the estimation of design loads. In recent years test methods have been developed to obtain fracture mechanics parameters governing subcritical crack growth and strength degradation under load. Associated with the fracture mechanics parameters are large uncertainties, which include uncertainties with respect to the shape of surface flaws. However, it is shown in this work that the consequences of these uncertainties are minor from an engineering point of view.

The critical question, then, is whether it is possible to obtain from tests of small standard specimens the information on the strength of glass needed to estimate reliably the load capacity of cladding panels whose failure is due to surface flaws. Evidence presented in this work strongly suggests that, if the ring-on-ring testing method is employed, the answer to this question is affirmative.

2. ESTIMATION OF LOAD CAPACITY OF CLADDING PANELS

The purpose of this section is to review pertinent developments that make it possible to estimate numerically the probability distribution of the load capacity of cladding panels if the fundamental parameters characterizing the behavior of glass are known.

2.1 FRACTURE MECHANICS OF GLASS

The basic criterion for fracture is derived from the Griffith equilibrium expression, and may be written as

$$K_I = K_{IC} \tag{1}$$

where K_I = stress intensity factor, and K_{IC} = critical value of K_I. If equation 1 holds, the system reaches the state of instability wherein the rate of crack growth becomes for practical purposes infinite and failure occurs [8, 9]. K_{IC} is a property of the material and is determined experimentally. The stress intensity factor, K_I, is proportional to the actual stresses in the material in the presence of cracks causing stress concentrations. K_I can be expressed as follows [8, 9]:

$$K_I(t) = Y\sigma(t)\sqrt{c(t)} \tag{2}$$

where

Y = geometric shape factor,
σ = nominal stress (i.e., stress calculated by assuming the absence of cracks),
c = length of crack normal to the stress
t = time.

The geometric shape factor, Y, in equation 2 is a function of crack geometry and plate dimensions [10]. For semi-elliptical surface flaws (figure 1) for which $c/h \simeq 0$ and $a/b \simeq 0$, values of Y calculated in reference 10 for various ratios c/a are shown in table 2.1.

According to experiments reported in references 11, 12, and 13, the following relationship holds for the rate of subcritical crack growth in the region $K_I < K_{IC}$:

$$\frac{dc}{dt} = \mathcal{A} K_I^n(t) \tag{3}$$

The parameters $\mathcal{A}$ and n depend upon ambient humidity and temperature and are obtained experimentally. Equation 3 expresses quantitatively the fact that the cracks in an element of glass subjected to stress for some length of time will grow - albeit not catastrophically - provided that the stress is contained

Table 2.1 Dependence of Factor Y Upon Ratio c/a for $c/h \simeq 0$ and $a/b \simeq 0$

c/a	Y
0.00	1.985
0.25	1.827
0.50	1.581
0.75	1.353
1.00	1.163
1.10	1.157
1.20	1.149
1.30	1.139
1.40	1.127
1.50	1.116
1.60	1.103
1.70	1.090
1.80	1.077
1.90	1.064
2.00	1.050

within a certain range. This phenomenon is referred to as static or dynamic fatigue according to whether the stress is constant or time-dependent.

It follows from equations 1 and 2 that the strength of glass, S, i.e., the value of the nominal stress at which failure occurs, is

$$S(t) = \frac{K_{IC}}{Y\sqrt{c(t)}} \tag{4}$$

If K_I and c are eliminated from equations 2, 3, and 4 and the notation S(0) = S_i is used (S_i = initial strength), the following relationship is obtained:

$$S(t) = \left[S_i^{n-2} - \frac{1}{B}\int_0^t \sigma^n(\tau)\, d\tau\right]^{\frac{1}{n-2}} \tag{5}$$

where

$$1/B = \frac{n-2}{2} A Y^2 K_{IC}^{n-2} \tag{5a}$$

[11]. If we consider an area, A_k, over which the tension stress, $\sigma(t)$, is uniform and independent of direction, failure occurs within that area if

$$\sigma(t) \geqslant S(A_k, t) \tag{6}$$

where $S(A_k, t)$ = strength calculated by equation 5 in which S_i corresponds to the largest initial flaw within the area A_k, regardless of the direction of that flaw, that is

$$S_i = \frac{K_{IC}}{Y c_{max}^{1/2}(A_k, 0)} \tag{7}$$

and $c_{max}(A_k, 0)$ = length of largest flaw within A_k at time $t = 0$.

We now consider the case in which the state of stress is uniform over the area A_k, but the principal stresses within the area are unequal. The effect of normal stress is by far the strongest as far as crack propagation is concerned [9, p. 54], and that the effect of shear stresses may therefore be neglected (see, e.g., reference 14). The center of A_k is denoted by M_k, and the normal tension stresses at time t parallel to direction α_ℓ are denoted by $\sigma(M_k, \alpha_\ell, t)$. We define the sector $\Delta\alpha_\ell$, centered on the direction α_ℓ, and such that normal stresses at time t parallel to any radial direction contained within that sector may be assumed to differ negligibly from $\sigma(M_k, \alpha_\ell, t)$ [figure 2]. Failure within the area A_k will not necessarily be initiated by a flaw normal, or almost normal, to the largest principal stress, since the largest of these flaws may well be relatively small. Neither will failure be necessarily initiated by the largest flaw within the area A_k, since that flaw may well be perpendicular to a relatively low normal stress. Rather, failure will be initiated by the largest of the flaws normal to any radius within the sector $\Delta\alpha_\ell$ [whose length is denoted by $c_{max}(A_k, \Delta\alpha_\ell, t)$] to which there corresponds a strength, $S(A_k, \Delta\alpha_\ell, t)$, such that:

$$\sigma(M_k, \alpha_\ell, t) \geqslant S(A_k, \Delta\alpha_\ell, t) \tag{8}$$

For any sector, $\Delta\alpha_\ell$, the strength at time t, $S(A_k, \Delta\alpha_\ell, t)$, is calculated by equation 5 in which $\sigma(M_k, \alpha_\ell, \tau)$ is substituted for $\sigma(\tau)$ and the initial strength $S_i(A_k, \Delta\alpha_\ell)$ is substituted for S_i. The following relation holds:

$$S_i(A_k, \Delta\alpha_\ell) = \frac{K_{IC}}{Y c_{max}^{1/2}(A_k, \Delta\alpha_\ell, 0)} \tag{9}$$

where $c_{max}(A_k, \Delta\alpha_\ell, 0)$ = length of largest initial crack normal to any radius within the sector $\Delta\alpha_\ell$.

2.2 RELATION BETWEEN INITIAL STRENGTH AND THE FINAL STRENGTH CORRESPONDING TO A 60-SEC LOAD

A particular case of practical interest is that in which the load acting on the panel has constant value over a 60-sec time interval and is zero outside that interval[a]. The 60-sec load induces at point M_k a normal stress parallel to the direction α_ℓ, denoted by $\sigma_{60}(M_k, \alpha_\ell)$, which is constant throughout the duration of the load and equal to zero at all other times. Failure initiated by a flaw normal or almost normal to a radius within the sector $\Delta\alpha_\ell$ occurs at time $t = 60$ sec if the stress $\sigma_{60}(M_k, \alpha_\ell)$ is equal to the strength $S(A_k, \Delta\alpha_\ell, t = 60)$. Substituting $\sigma_{60}(M_k, \alpha_\ell)$ for $\sigma(t)$ and $S_i(A_k, \Delta\alpha_\ell)$ for S_i in equation 5, it follows that the failure stress is given by the relation

$$\sigma_{60}^{n-2}(M_k, \alpha_\ell)\,[\sigma_{60}^{2}(M_k, \alpha_\ell) + \frac{B}{60}] + \frac{B}{60}\, S_i^{n-2}(A_k, \Delta\alpha_\ell) \tag{10}$$

For soda-lime glass $\sigma_{60}^2(M_k, \alpha_\ell)$ is of the order of $10^3(\mathrm{MPa})^2$ or more, and $B/60$ is of the order of $1(\mathrm{MPa})^2$ or less, so that the failure stress may be written as

$$\sigma_{60}(M_k, \alpha_\ell) \simeq \frac{S_i^{\frac{n-2}{n}}(A_k, \Delta\alpha_\ell)}{(60\,\frac{1}{B})^{1/n}} \tag{11}$$

2.3 ESTIMATION OF 60-SEC LOAD CAUSING FAILURE OF A PANEL

The relationship between the load acting on a cladding panel and the stresses in the panel is generally nonlinear. This relationship can be obtained by using, for example, a finite-difference program such as that developed by Texas Tech University [15], and is commonly expressed in terms of the nondimensional quantities

$$LF = pb^4/Dh \tag{12}$$

$$SF = \sigma b^2 h/D \tag{13}$$

where LF and SF = loading and stress factors, respectively, p = uniform load per unit area, σ = stress, b = smaller side of rectangular plate, D = flexural rigidity, defined by:

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{14}$$

where E = modulus of elasticity, ν = Poisson's ratio, and h = thickness of the plate.

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[a] The 60-sec load is the standard reference load used in glass cladding design charts in the United States.

Once the relationship between loads and stresses is known, it is possible to obtain the 60-sec loads, $p_{60}^{k\ell}$, corresponding to the failure stresses σ_{60} (M_k, α_ℓ) calculated by equation 11 for each point M_k of a sufficiently dense grid and for each of a sufficient number of directions α_ℓ. The smallest of these loads, denoted by p_{60}, is the load causing failure (i.e., the load capacity) of the panel characterized by the set of initial strengths $S_i(A_k, \Delta\alpha_\ell)$. Calculations of the load capacity, p_{60}, can be carried out for a large number, M, of panels. The probability distribution of 60-sec load capacity can be estimated from the M values so obtained. A computer program for estimating this distribution if the initial strengths $S_i(A_k, \Delta\alpha_\ell)$ are specified is described in Appendix III. In the case of heat-strengthened or tempered glass, the procedure for estimating the probability distribution of the 60-second load capacity p_{60} is the same, except that stresses $\sigma_{60}(M_k,\alpha_e)-\sigma_R$ should be used in equation 11 in lieu of the stress, $\sigma_{60}(M_k,\alpha_\ell)$, where σ_R denotes the residual thermal stress, which can be determined by routine experimental procedures.

2.4 SPECIFICATION OF INITIAL STRENGTHS $S_i(A_k, \Delta\alpha_\ell)$

The initial strength $S_i(A_k)$ corresponding to the largest flaw within A_k, regardless of its direction, is commonly described by a Weibull distribution:

$$P[S_i(A_k)] = 1 - \exp\left\{ -\left[\frac{S_i(A_k) - \mu_s}{S_o(A_k)} \right]^m \right\} \tag{15}$$

From the assumption that the number of flaws of any given size is on the average proportional to the area A being considered, it follows that

$$P[S_i(A)] = 1 - \exp\left\{ -\left[\frac{S_i(A) - \mu_s}{S_o(A)} \right]^m \right\} \tag{16}$$

where

$$S_o(A) = \left(\frac{A_k}{A}\right)^{\frac{1}{m}} S_o(A_k) \tag{16a}$$

[25, pp. 5 and 10]. We will refer to equation 16 as the fundamental Weibull distribution of the strength of glass.

Similarly, from the assumption that the flaw orientations are uniformly distributed [i.e., that the number of flaws normal to the stress $\sigma_{60}(M_k, \alpha_\ell)$[a] is on the average equal to $\Delta\alpha_\ell/(\pi/2)$ times the number of flaws parallel to any direction], it follows that the probability distribution of the initial strength $S_i(A_k, \Delta\alpha_\ell)$ is

$$P[S_i(A_k, \Delta\alpha_\ell)] = 1 - \exp\left\{ -\left[\frac{S_i(A_k, \Delta\alpha_\ell) - \mu_s}{S_o(A_k, \Delta\alpha_\ell)}\right]^m\right\} \tag{17}$$

where

$$S_o(A_k, \Delta\alpha_\ell) = \left(\frac{\pi}{2\Delta\alpha_\ell}\right)^{1/m} S_o(A_k) \tag{18}$$

Assuming that equations 15 through 18 are valid, it follows that the initial strength $S_i(A_k, \Delta\alpha_\ell)$ can be specified by the probability distribution given by equation 17, provided that the parameters $S_o(A)$, μ_s, and m are known. These parameters can be obtained from ring-on-ring tests, which are described in the following section.

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[a] Or to the stress $\sigma_{60}(M_k, -\alpha_\ell) = \sigma_{60}(M_k, \alpha_\ell)$.

3. RING-ON-RING TESTS

3.1 PRINCIPLE AND DESCRIPTION OF RING-ON-RING TESTING

Ring-on-ring testing devices involve the creation of a state of uniform axisymmetric tension stress in the central portion of one of the faces of a circular plate. This can be accomplished by placing the plate on a segmented circular ring and by applying on its upper surface a load transmitted through a circular ring concentric with and having a smaller diameter than the segmented support.

As noted in reference 17, because of the high elastic modulus and hardness of glass, any nonperfect contact between rigid loading rings and the test sample can lead to deviations from axisymmetry in the stress field. Reference 17 describes a ring-on-ring device designed to eliminate such nonuniformities. Each ring consists of a closely wound coil. The load that each coil transmits to the plate is applied by a rubber diaphragm which covers a circular groove filled with fluid. The function of the fluid is to equalize the loading along the coils. Strain gage measurements, and measurements of strength of indented specimens, are reported in reference 17 for ringon-ring devices both of the rigid type and of the type just described. According to the results of reference 17, errors in the measurement of strengths due to the use of rigid ring-on-ring devices are less than 5 percent for annealed soda-lime float glass. The errors are considerably larger (about 20 percent) for thermally tempered crown glass.

The uniform stress in the central portion of the tension face of a circular plate subjected to a ring-on-ring loading test is

$$\sigma = \frac{3p}{4\pi h^2}\left[2(1+\nu)\,\ell n\,\frac{a}{b} + \frac{(1-\nu)(a^2-b^2)}{a^2}\,\frac{a^2}{R^2}\right] \tag{19}$$

where p = load, h = plate thickness, a = radius of the support ring, b = radius of the loading ring, R = radius of the disc, and ν = Poisson's ratio [18].

For the case b/a = 1/2, the distribution of the stress, σ, along a radius, r, is represented in figure 3 [19]. Note that, owing to local effects, the central portion of the plate within which the stress σ is given by Eq. 19 is defined by the relation $r \lesssim 0.705b$.

It follows from appendix I that the precision of strength estimates based on the testing of any given sample of ring-on-ring specimens increases with the number of specimens within that sample that have failure origins in the central, uniform stress area of the specimen. It would therefore be desirable to attempt the development of test devices aimed at inhibiting stress corrosion on the tension side of the nonuniform stress area of the specimen. This could be accomplished by immersing that side in, or covering it with, an inert agent such as water-free oil.

3.2 ESTIMATION OF INITIAL STRENGTHS FROM RING-ON-RING TEST RESULTS, AND INFLUENCE OF UNCERTAINTIES WITH RESPECT TO PARAMETERS B AND n UPON THE ESTIMATION OF DESIGN LOADS

Ring-on-ring tests yield the stress at the time of failure, $\sigma(t_f)$, which is equal to the strength at the time of failure, $S(t_f)$. To estimate the initial strength corresponding to $S(t_f)$, equation 5 is used. In the case of a ramp loading on a specimen that exhibits a linear load-stress relationship up to failure [21, 22], steps similar to those that led to equation 11 yield the relation

$$\sigma(t_f) \simeq \left[\frac{n+1}{t_f \frac{1}{B}}\right]^{1/n} S_i^{\frac{n-2}{n}} \qquad (20)$$

Equation 20 allows the estimation of S_i from measurements of $\sigma(t_f)$ and t_f. The estimates of S_i depend upon the values assumed for n and B.

We now consider a specimen subjected to a ramp-like load, for which the time to failure and the stress at the time of failure were found to be t_f and $\sigma(t_f)$, respectively. We seek the stress, σ_{60}, induced by a constant load with a 60-sec duration that would have caused failure of that same specimen. It follows from equation 11 (with $\Delta\alpha_\ell = \pi/2$) and equation 20 that

$$\sigma_{60} \simeq \frac{1}{\left[(n+1)\frac{60}{t_f}\right]^{1/n}} \sigma(t_f) \qquad (21)$$

Equation 21 shows that the ratio $\sigma_{60}/\sigma(t_f)$ is independent of B. Thus, even though the variability of B is fairly large, reflecting as it does uncertainties with respect to K_{IC}, n, A, and Y (see equation 5a and table 2.1), the effect of this variability upon the ratio $\sigma_{60}/\sigma(t_f)$ can be ignored.

Table 3.2 lists results of calculations which show that for ratios $t_f/60$ of the order of 0.5 to 2 the effect of the parameter n upon the ratio $\sigma_{60}/\sigma(t_f)$ is small[a]. The calculations were based on two values: n=16 and n=19.7 [23]. These values are consistent with results of tests conducted within the framework of this project on indented soda-lime specimens, in accordance with the method described in reference 24. The tests yielded the value n=15.95 when measurement results were not corrected for residual stress effects at the indented crack tips, and the value n=20.6 when the residual stress effects were accounted for. The magnitude of the residual stresses at the crack tips of a glass panel, and therefore the effective value of n that should be used in determining fatigue effects for the panel, is unknown. However, it is reasonable to assume that the latter will lie between the values n=15.95 or so

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[a] The choice of a proper loading rate can in practice ensure that the ratios $t_f/60$ are indeed of that order - see tables 3.3 and 3.4.

and n=20.6 or so. Hence the choice of the values n=16 and n=19.7 for the sensitivity calculations of table 3.2.

Table 3.2 Ratios $\sigma_{60}/\sigma(t_f)$ for t_f = 30 sec and t_f = 120 sec, Assuming n = 19.7 and n = 16.00

	n = 19.7	n = 16	Difference
t_f = 120 sec	0.888	0.875	1.5%
t_f = 30 sec	0.828	0.802	3%

Equation 21 and table 3.2 (which show, respectively, that the ratio $\sigma_{60}/\sigma(t_f)$ is independent of B, and that for any given stress, $\sigma(t_f)$, obtained by testing a ring-on-ring specimen, the corresponding 60-sec strength, σ_{60}, depends weakly upon n) suggest that estimates of the probability distribution $P(p_{60})$, of the 60-sec load capacity of a panel, p_{60}, inferred on the basis of results of ring-on-ring tests are also independent of B and weakly dependent upon n. This was confirmed by estimates of $P(p_{60})$ carried out for a 4 ft x 4 ft x 1/8 in. annealed glass panel in which several assumed sets of values of n and 1/B were used to convert strengths at time of failure, $\sigma(t_f)$, into initial strengths, S_i.

3.3 RESULTS OF RING-ON-RING TESTS

This section presents results of ring-on-ring tests conducted within the framework of this project. The ring-on-ring testing device (figure 4) consisted of rigid rings with radii a = 0.0603 m (support ring) and b = 0.0254 m (loading ring). The device was employed in combination with a 10K lbf Universal Testing Machine. All the specimens were subjected to ramp loads (i.e., loads increasing for practical purposes linearly with time). The glass used in the tests was new and was obtained from the same manufacturer and batch.

Tests were conducted in air on a set of 56 annealed float glass square specimens with side D = 7 in. (0.1792 m) and on a set of 29 annealed float glass circular specimens with radius R = 3.5 in (0.889 m). The nominal thickness was h = 1/4 in. (6 mm) for all specimens. The stresses at the center of the plates were calculated by equation 19, in which h was the measured (rather than the nominal) thickness for each specimen. Following reference 19, in the case of the square plates the parameter R in equation 19 was assumed to be equal to one-half the average of the edge and diagonal lengths. This was confirmed experimentally to within a few percentage points by strain-gage measurements of stresses on circular and square ring-on-ring specimens. The test results and the calculated values of the stresses at the center of the plates at the time of failure, $\sigma(t_f)$, are listed in tables 3.3 and 3.4. It is noted that the influence of humidity upon the results of tables 3.3 and 3.4 is negligible (see reference 14, p. 35).

If it is assumed that $n = 19.7$ and $1/B = 0.0738569(\mathrm{MPa})^{-2}s^{-1}$ (corresponding to $\mathcal{A} = 1.08\ (\mathrm{MPa})^{-n}\ m^{1-\frac{n}{2}}\ s^{-1}$ [23], $K_{IC} = 0.75$ MPa [23], and $Y = 1.12$ - see equation 5a), it follows from equation 20 that the initial strengths are

$$S_i = 0.7272\ t_f^{1/17.7}\ [\sigma(t_f)]^{19.7/17.7} \tag{22}$$

Statistical analyses of the initial strengths calculated by equation 22 from the stresses $\sigma(t_f)$ were carried out as indicated in appendix I. It was assumed that the area A referred to in appendix I is a circle with radius $r \simeq 0.705b = 0.705$ in. (0.0179 m), i.e., $A = \pi r^2 \simeq 1.56$ in.^{2} (0.001 m^2) (see figure 3).

Estimated parameters of the Weibull distributions of the initial strengths, S_i, from results of tests used in conjunction with equation 22 are listed in table 3.5, where the sample means, $\overline{S}_i$, standard deviations, $s(S_i)$, coefficients of variation, $s(S_i)/\overline{S}_i$, sample maximum, S_{imax}, and sample minimum, S_{imin}, of the data are also shown.

The initial strengths S_i calculated by equation 22 are nominal, rather than actual, since the values of B and n used therein are uncertain. However, it follows from the form of equations 11 and 20 and from the results of table 3.2 that, if the values of B and n used in these equations are the same, the effect of uncertainties with respect to the actual values of B and n largely cancels out when estimating the load capacity of glass panels.

Table 3.3 Test Data for Circular Plates

	h	t_f	p	r	RH%	$\sigma(t_f)$
1	0.5334	45	2358.5	0.76	67	47.32
2	0.5639	54	3453.2	0.76	67	61.99
3	0.5690	59	3419.9	1.27	71	62.06
4	0.5613	48	2839.1	1.78	71	51.44
5	0.5537	73	4534.6	1.27	71	84.44
6	0.5613	43.5	2109.3	2.03	66	38.22
7	0.5080	36	1637.6	1.27	66	36.23
8	0.5080	41.5	2073.7	2.29	66	45.87
9	0.5334	31	1713.3	2.03	60	34.38
10	0.5486	82	4694.8	2.29	60	89.05
11	0.5436	57	2536.5	0.00	67	49.00
12	0.5512	59	3924.9	1.27	67	73.75
13	0.5334	47	3021.6	1.02	67	60.63
14	0.5359	70	3742.5	1.52	67	74.39
15	0.5537	45	2460.9	1.27	72	45.82
16	0.5588	52	2963.7	1.27	72	54.18
17	0.5563	45	2336.3	2.29	72	43.10
18	0.5563	73	3898.2	2.03	70	71.91
19	0.5537	51	2269.5	1.52	70	42.26
20	0.5537	50	2216.1	2.29	70	41.26
21	0.5461	73	3804.8	1.52	70	72.83
22	0.5486	54	2594.4	2.03	70	49.21
23	0.5359	71	3675.7	0.51	70	73.07
24	0.5385	76	5255.5	2.54	60	103.46
25	0.5410	76	3960.5	2.54	72	77.25
26	0.5537	62	3956.1	2.79	71	73.66
27	0.5461	74	4000.6	2.79	67	76.58
28	0.5639	49	2523.2	3.30	66	45.30
29	0.5334	37	2055.9	3.30	67	41.25

where h = thickness in cm
t_f = load time in seconds
p = failure load in Newtons
r = distance from origin of fracture to center in cm
RH = relative humidity
$\sigma(t_f)$ = final strength in MPa

Table 3.4 Test Data for Square Plates

	h	p	t_f	r	RH%	$\sigma(t_f)$
1	0.5385	2647.8	57	2.29	74	50.09
2	0.5359	2287.3	54	0.76	74	43.69
3	0.5563	2429.7	48	0.76	74	43.07
4	0.5537	2621.1	61	2.03	66	46.90
5	0.5461	2242.8	49	1.79	66	41.26
6	0.5461	3911.6	73	2.29	72	71.95
7	0.5461	3773.6	81	1.52	72	69.42
8	0.5436	2883.6	74	1.27	72	53.53
9	0.5461	4427.8	95	0.76	66	81.45
10	0.5461	3017.1	73	0.51	66	55.50
11	0.5410	2451.9	67	1.79	62	45.96
12	0.5334	4561.3	100	1.52	70	87.95
13	0.5334	4525.7	100	1.79	70	87.26
14	0.5334	3163.9	80	1.27	70	61.00
15	0.5436	2309.6	63	1.02	63	42.88
16	0.5359	2919.2	75	2.29	62	55.76
17	0.5537	2558.8	67	1.27	66	45.79
18	0.5461	5157.6	97	1.27	66	94.87
19	0.5486	5736.1	106	1.52	66	104.56
20	0.5436	5006.3	101	2.29	62	92.94
21	0.5410	3840.4	73	1.52	62	71.98
22	0.5410	4583.5	93	2.29	62	85.91
23	0.5349	3119.5	73	1.52	62	59.59
24	0.5334	4401.1	93	2.03	68	84.85
25	0.5512	3408.7	84	2.03	68	61.55
26	0.5410	5402.3	104	2.29	62	101.26
27	0.5512	4458.9	93	0.76	62	80.51
28	0.5359	4579.1	103	1.27	62	87.47
29	0.5588	4570.2	92	1.02	62	80.29
30	0.5512	1984.7	51	1.79	66	35.84
31	0.5461	4423.3	98	2.03	66	81.37
32	0.5436	5816.2	117	2.54	66	101.98
33	0.5385	2558.8	56	2.54	62	48.41
34	0.5512	3399.8	82	2.54	70	61.39
35	0.5410	4783.8	103	2.54	62	89.66
36	0.5512	2656.7	73	2.54	68	47.97
37	0.5461	5388.9	102	2.54	68	99.13
38	0.5537	5647.1	107	2.54	68	101.05
39	0.5461	5660.4	111	2.54	68	104.12
40	0.5436	4147.4	89	2.54	66	76.99
41	0.5563	5024.1	101	2.54	62	89.06
42	0.5385	4104.2	101	2.54	62	96.56
43	0.5334	2233.9	65	2.54	62	43.07
44	0.5385	5526.9	110	2.54	68	104.56
45	0.5410	2710.1	55	2.79	66	50.80
46	0.5410	3088.3	63	2.79	66	57.89
47	0.5359	3964.9	94	2.79	62	74.74
48	0.5385	3333.1	84	2.79	70	63.06
49	0.5461	5611.5	114	2.79	70	103.22
50	0.5563	5117.5	100	2.79	62	90.72
51	0.5359	4699.2	87	3.05	66	89.76
52	0.5358	4294.3	97	3.05	62	82.03
53	0.5486	3288.6	80	3.05	62	59.94
54	0.5334	2621.1	74	3.30	68	50.54
55	0.5486	2843.6	74	3.56	68	51.83
56	0.5334	4325.4	97	3.56	62	83.40

where h = thickness in cm
t_f = load time in seconds
p = failure load in Newtons
r = distance from origin of fracture to center in cm
RH = relative humidity
$\sigma(t_f)$ = final strength in MPa

Table 3.5 Sample Statistics of Initial Strengths and Estimated Initial Strength Distribution Parameters Corresponding to Area A ≃ 1.56 in^2 for Tests in Air

								2-parameter Weibull[a]		3-parameter Weibull		
Case	Specimens (1)	Sample Size (2)	$\overline{S_i}$ (MPa) (3)	$s(S_i)$ (MPa) (4)	$s(S_i)/\overline{S_i}$ (5)	S_{imax} (MPa) (6)	S_{imin} (MPa) (7)	S_o (MPa) (8)	m (9)	S_o (MPa) (10)	μ_s (MPa) (11)	m (12)
1	Circular	29	87.63	31.0	0.354	141.6	46.2	121.5	3.10	72.8	46.9	1.34
2	Square	56	110.83	36.8	0.332	174.4	48.8	173.2	2.91	156.8	56.3	0.97
3	Circular and Square	85	102.9	36.5	0.355	174.4	46.2	156.4	2.79	118.1	47.7	1.29

[a] For 2-parameter Weibull distributions $\mu_s = 0$.

4. ESTIMATES OF THE LOAD CAPACITY OF GLASS PANELS

The methodology for estimating the 60-sec load capacity of glass panels proposed in this work consists of using the procedure described in section 2.3 in conjunction with equations 17 and 18. The parameters $S_o(A_k)$, m, and μ_s in these equations are obtained by fitting a Weibull distribution to the nominal initial strengths S_i calculated from the breaking strengths $\sigma(t_f)$ by using equation 20 (or a similar relation if the dependence of load on time is not linear). The values of the parameters B and n used in equation 20 must be the same as those of equation 11. Values of n used in the calculations should be based on experimental results obtained, e.g., by techniques described in reference 24.

This methodology was applied to a 4 ft x 4 ft x 1/8 in (1.22 m x 1.22 m x 3 mm) annealed glass panel simply supported on four sides, using the values E = 68.9 GPa, ν = 0.22, the fracture mechanics parameters listed in section 3.3, and the parameters of the Weibull distribution of the initial glass strength listed in table 3.5. The grid size and the angle $\Delta\alpha_\ell$ used in the numerical calculations were 7.62 mm x 7.62 mm and 18°, respectively. For each of the sets of two and three parameters listed in table 3.5 the load capacities, p_{60}, of 1000 of panels were estimated, and the values so obtained were fitted by two- and three-parameter Weibull distributions, respectively. The parameters of the best fitting distributions are listed in table 4.1, which also lists mean values, $\bar{p}_{60}$, standard deviations, $s(p_{60})$, coefficients of variation, $s(p_{60})/\bar{p}_{60}$, loads corresponding to a probability of failure of 8 in 1000, $p_{60}(0.008)$, and loads corresponding to a probability of failure of 0.5.

Estimates of the load capacity corresponding to a probability of failure of 8 in 1,000, $p_{60}(0.008)$, based on full-scale measurements are provided for design purposes in references 1 and 3. According to reference 1, $p_{60}(0.008) \simeq 26$ psf (1 psf = 47.9 Pa). According to reference 3, $p_{60}(0.008) \simeq 23$ psf. It is noted that to account for strength degradation in service, the value of reference 3 corresponds to strengths reduced by a factor of 2/3 with respect to those obtained in new glass [26]. Had this reduction not been effected, i.e., had the strength of new glass been used, to the value $p_{60}(0.008) \simeq 23$ psf there would have corresponded roughly the value $p_{60}(0.008) \simeq (3/2) \times 23 =$ 34.5 psf.

The values $p_{60}(0.008)$ estimated in this report on the basis of three-parameter Weibull distributions (col. 9 of table 4.1, Case I) are somewhat higher than the corresponding value based on references 1 and 3 (35.5 psf to 42.1 psf versus 26 psf to 34.5 psf.) These differences could be explained by two factors. First, the estimates obtained in this report do not take into account edge failures, which can reduce the load capacity of panels considerably. (For example, according to reference 7, 38 percent of the total number of panel failures reported therein originated at the edges.) Second, sampling errors in the estimation of the Weibull parameters of the strength may be present, since the estimated values of $p_{60}(0.008)$ are based on the testing of a limited number of ring-on-ring test specimens (from 29 for case 1 to 85 for case 3). Therefore, as shown in appendix II, the true values of $p_{60}(0.008)$ could well be as low as about 20 psf and as high as about 50 psf. Note also that the estimated coefficients of variation of the load capacity p_{60} obtained by using three-parameter Weibull distributions (0.09 to 0.105) are lower than the value indicated in reference 3 ($\simeq$ 0.22). On the other hand, as can be seen from table 4.2 which summarizes test results reported in reference 7, coefficients of variation of the load capacity of new glass panels obtained from any one manufacturer are close in at least four out of eight cases to those estimated herein.

It is concluded that the procedure proposed in this report provides credible estimates of the load capacity of panels experiencing surface failures provided that three-parameter Weibull distributional models are used. The precision of these estimates depends upon number of ring-on-ring test results on which they are based, as suggested by numerical experiments presented in appendix II.

A second conclusion of interest is that modeling the strength of glass and the load capacity of cladding panels by two-parameter Weibull distributions leads to a clear incompatibility between results obtained by testing ring-on-ring specimens on the one hand, and the documented behavior of cladding panels on the other. Indeed, the use of two-parameter distributions yields estimates of $p_{60}(0.008)$ of the order of 1 psf (see table 4.1). These are grossly incompatible with the values quoted previously from references 1 and 3. A similar conclusion was reached independently by Walker [27].

Table 4.1 Estimated Statistics and Weibull Distribution Parameters of the 1-min Load Capacity, p_{60}, for a 4 ft x 4 ft x 1/8 in Annealed Glass Panel Supported on Four Sides (Based on Tests in Air)

	Case	$\bar{p}_{60}$ (psf)	$s(p_{60})$ (psf)	$s(p_{60})/\bar{p}_{60}$	$(p_{60})_o$ (psf)	μ_{60} (psf)	m_{60}	$p_{60}(0.008)$ (psf)	$p_{60}(0.5)$ (psf)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
I[a]	1	42.8	3.86	0.090	9.80	34.14	2.41	35.5	42.6
	2	50.9	5.00	0.098	11.04	41.08	2.05	42.1	50.3
	3	45.4	4.78	0.105	12.20	34.57	2.42	36.2	45.1
II[b]	1	7.3	3.43	0.471	8.23	0	2.26	0.98	7.0
	2	9.5	4.87	0.515	10.71	0	2.06	1.03	9.0
	3	6.9	3.58	0.516	7.86	0	2.06	0.75	6.6

[a] Based on three-parameter Weibull distributions of load capacity p_{60} and three-parameter Weibull distributions of the strength of glass.

[b] Based on two-parameter Weibull distributions of load capacity p_{60} and two-parameter Weibull distributions of the strength of glass.

$\bar{p}_{60}$ = sample mean

$s(p_{60})$ = sample standard deviation

$(p_{60})_o$ = scale parameter

μ_{60} = location parameter

m_{60} = shape parameter

$p_{60}(0.008)$ = 1-min load capacity corresponding to probability of failure of panel of 8 in 1,000

$p_{60}(0.5)$ = 1-min load capacity corresponding to probability of failure of 0.5.

Table 4.2 Summary of Tests Reported in Reference 7

Manufacturer	A					B					C				
Rate of Loading	n	E	S	c.o.v(p)	c.o.v.(p_{60})	n	E	S	c.o.v(p)	c.o.v.(p_{60})	n	E	S	c.o.v(p)	c.o.v.(p_{60})
0.022 psi/sec	9	4	5	0.112	0.120	10	5	5	0.169	0.182	11	1	10	0.123	0.132
0.22 psi/sec	9	0	9	0.093	0.130	9	7	2	N.C.	N.C.	9	1	8	0.253	0.274
2.2 psi/sec	9	3	6	0.117	0.126	10	4	6	0.167	0.179	11	3	6	0.191	0.224

n = total number of panels tested

E = number of edge failures

S = number of surface failures

c.o.v.(p) = coefficient of variation of recorded failure loads (based on sample size S)

c.o.v.(p_{60}) = coefficient of variation of nominal 60-sec loads obtained from recorded failure loads by using equation 21 (based on sample size S)

N.C. = not calculated owing to small size of sample S.

5. SUMMARY AND CONCLUSIONS

A methodology was proposed for estimating the probability distribution of the load capacity of annealed glass panels whose failure is due to surface flaws. The methodology requires the calculation of stresses induced on the panel surface by the external loads and employs information on the modulus of elasticity, Poisson's ratio, the fracture mechanics parameters, and the probabilistic description of the glass strength. This description is obtained by using the ring-on-ring testing method. It is shown that owing to the way in which this method is used, errors in the estimation of the load capacity due to uncertainties with respect to the fracture mechanics parameters of glass and to the shape of surface flaws are largely cancelled. Results of ring-on-ring tests and of calculations based on those tests show that credible predictions of panel load capacities are obtained that are consistent with data available in the literature, provided that the probability distribution of the glass strength and the probability distribution of the panel load capacity are modeled by the three-parameter Weibull distribution. The two-parameter Weibull distribution appears to provide an incorrect model of the strength of glass and of the load capacity of glass panels.

The results obtained in this work strongly suggest that the proposed methodology can provide reliable estimates of the load capacity of glass panels whose failure is due to surface flaws. However, a definitive statement to this effect would require validation based on (1) larger ring-on-ring test samples than those used in this work, and (2) reliable statistics of the load capacity of panels manufactured from the same batch of glass as the ring-on-ring test specimens.

The topic of glass panel failures due to edge imperfections was not addressed in this report. As shown by the statistics in table 4.2, the ratio of such failures to the total number of failures is highly variable. It is suggested that the probability distribution of the load capacity of glass panels, regardless of type of failure, can be modeled from information on statistics of edge failures on the one hand and of surface failures on the other.

In view of their relatively low cost, ring-on-ring tests could be more economical than full-size panel tests, in spite of the relatively large number of specimens that would have to be tested in order to attain acceptable precisions of the estimates. The extent to which this is the case would have to be determined by studies based on more extensive test data than have been obtained within the framework of this project.

Ring-on-ring testing may be a desirable alternative to full-size panel tests not only for economical reasons, but also in situations where the amount of material available for testing is limited. This might be the case in studies of in-service strength degradation in which the material being tested consists of weathered window glass recovered from existing buildings.

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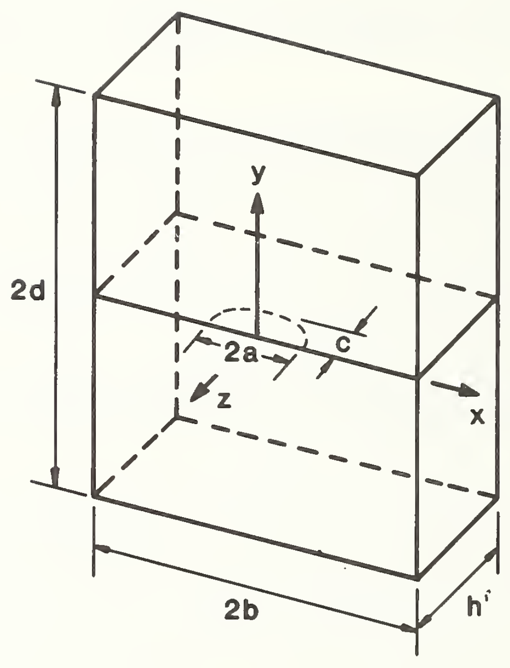

Figure 1. Notations

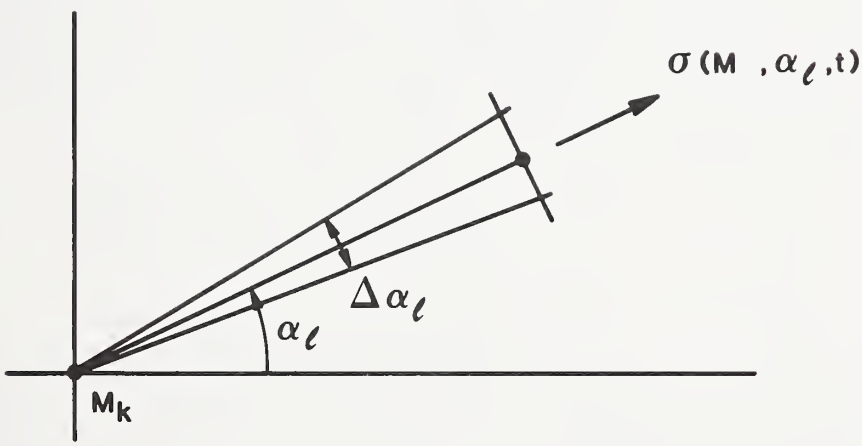

Figure 2. Notations

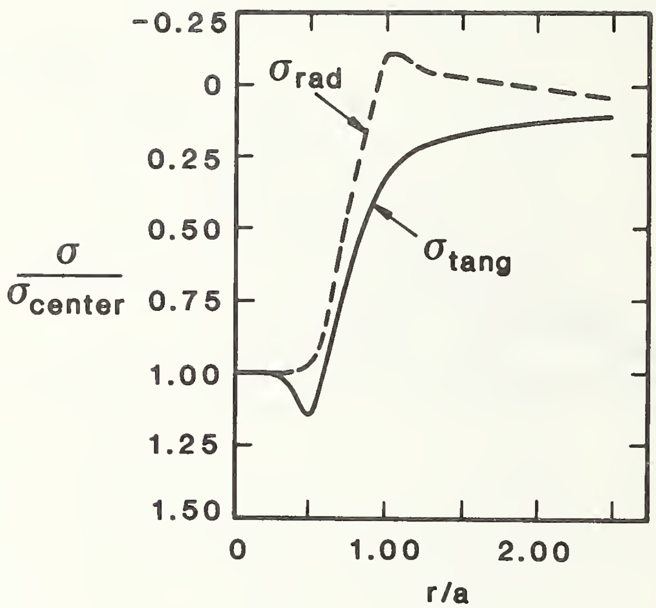

Figure 3. Tangential and Radial Stress Distribution (a = radius of support ring; b/a = 1/2; b = radius of loading ring)

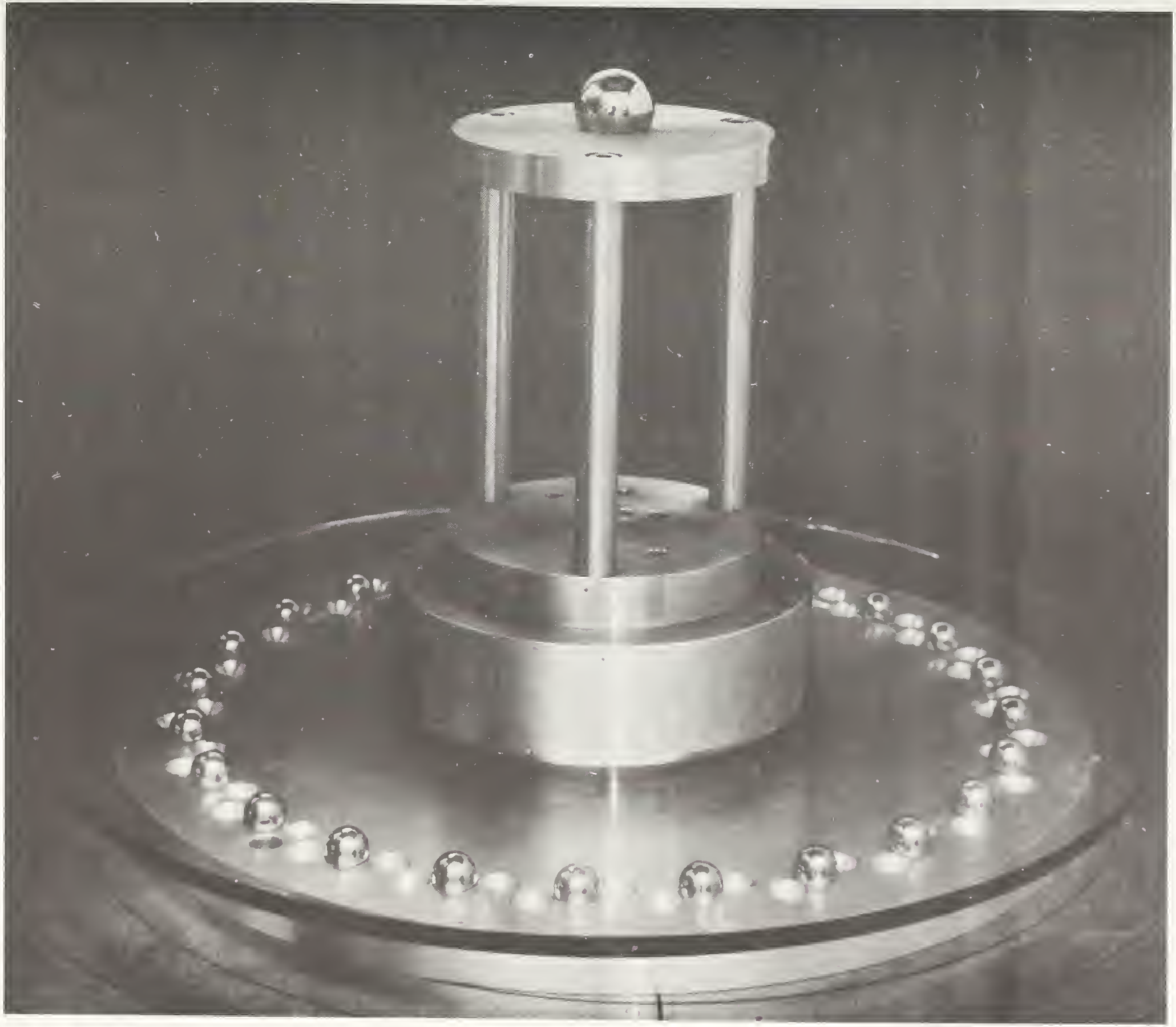

Figure 4. Ring-on-Ring Testing Device

APPENDIX I. MAXIMUM LIKELIHOOD ESTIMATION FOR THE THREE PARAMETER WEIBULL DISTRIBUTION BASED ON A SAMPLE WITH CENSORING DUE TO COMPETING RISKS

Let X be a random variable representing the tensile strength of glass cladding specimens within an area subjected to uniform axial bending. It is assumed that the distribution of X is Weibull that is,

$$F(x) = 1-\exp[-(\frac{x-\lambda}{\beta})^{\alpha}$$

where α, $\beta>0$, $\lambda \geqslant 0$, $x \geqslant \lambda$, and α, β, and λ are the three parameters of the distribution.

We assume that a number of specimens, say n, are tested and each of the specimens has the same Weibull distribution of tensile strengths X. However, what we observe at each trial is either the tensile strength at which the specimen fails or the tensile strength outside the area A. For a given specimen, let Q denote the strength at which failure occurs outside the area A. Then what we observe is one of the two events [A1-1, A1-2]

$$[X=x] \cap [X<Q] \text{ or } [Q=q] \cap [X \geqslant Q].$$

Then the likelihood, L, of the event

$$\bigcap_{i=1}^{k} [X_i \leqslant Q_i]\ [X_i = x_i] \bigcap_{j=k+1}^{n} [X_j > Q_j]\ [Q_j = q_j] \qquad \text{(A1.1)}$$

where $k \leqslant n$ is the random number of failures observed, has the form

$$L = C \prod_{i=1}^{k} f(x_i) \prod_{j=k+1}^{n} [1-F(q_j)] \qquad \text{(A1.2)}$$

where f is the density function, F is the distribution function, and

$$C = \binom{n}{k} = \frac{n!}{(n-k)!k!}$$

The logarithm of the likelihood function for equation A1.2 is

$$\ell n\ L = \ell n\ [C \prod_{i=1}^{k} f(x_i) \prod_{j=k+1}^{n} (1-F(q_j))]$$

and can be rewritten as

$$\ell n\, L = \ell n\, C + k \ell n \alpha - k \alpha \ell n \beta + (\alpha-1) \sum_{i=1}^{k} \ell n(x_i - \lambda)$$

$$- \sum_{i=1}^{k} \left(\frac{x_i-\lambda}{\beta}\right)^{\alpha} - \sum_{j=k+1}^{n} \left(\frac{q_j-\lambda}{\beta}\right)^{\alpha}$$

On differentiating equation A1.3 with respect to α, β, and λ in turn and equating the resulting expressions to zero, one obtains the maximum likelihood estimation equations for the three Weibull parameters [A1-3]. These three equations are nonlinear in the three parameters and can be solved iteratively by a Newton-Raphson method as discussed in [A1-1] and [A1-3].

References

A1-1 Harter, H. L., Moore, A. H., "Maximum-Likelihood Estimation of the Parameters of Gamma and Weibull Populations From Complete and From Censored Samples," Technometrics 7(4) 639-643 (1965).

A1-3 Lemon, G. H. "Maximum Likelihood Estimation for the Three Parameters Weibull Distribution Based on Censored Samples," Technometrics 17(2) 247-254 (1975).

A1-2 Whittaker, I. C. Besunner, P. M., A Reliability Analysis Approach to Fatigue Life Variability of Aircraft Structures, Air Force Materials Laboratory Tech. Report AFML-TR-69-85 (1969).

APPENDIX II. DEPENDENCE OF ESTIMATED 60-SEC LOAD CAPACITY, $p_{60}(0.008)$, UPON NUMBER OF RING-ON-RING SPECIMENS BEING TESTED

Estimates of the order of magnitude of the error in the estimation of the load capacity $p_{60}(0.008)$ were carried out as follows. It was assumed that the probability distribution of the strength can be modeled by a three-parameter Weibull distribution with scale, location, and shape parameters (corresponding to an area A = 1.56 in^2) S_o = 118.1 MPa, μ_s=47.7 MPa, and m = 1.27 (see table 3.5, case 3). It is further assumed that the probability distribution of the load capacity p_{60} is also Weibull with parameters to be determined.

The first step in obtaining the desired estimates consists of generating from this distribution by Monte Carlo simulation a sample of N values of the glass strength S. These N values can be viewed as strengths that would be measured if N ring-on-ring specimens were tested.

The second step is to fit a three-parameter Weibull distribution to these N values.

The third step is to estimate from this distribution the corresponding 60-sec load capacity $p_{60}(0.008)$ as shown in section 2.3.

These three steps were carried out 20 times for each of the sample sizes N = 15, 100, 250, 500, and 1,000. The results of the calculations are summarized in table A2.1, which shows for each value N the means of 20 estimates $p_{60}(0.008)$, their standard deviation $s[p_{60}(0.008)]$, and their minimum and maximum values, min $[p_{60}(0.008)]$ and max $[(p_{60}(0.008)]$. Table A2.1 suggests the order of magnitude of the sample size needed to obtains estimates of $p_{60}(0.008)$ with various precisions. Note that these estimates are tentative, since (1) they do not account for the effect of censoring discussed in appendix A1, and (2) they assume that the Weibull distribution parameters of the glass strength obtained from the testing of only 89 specimens are the "true" parameters.

Table A2.1 Estimated Means, Standard Deviations, Minimum Values, and Maximum Values Obtained from Sets of Estimated Values $p_{60}(0.008)$, Corresponding to Various Ring-on-Ring Test Sample Sizes n.

N	15	100	250	500	1000
$p_{60}(0.008)$	33.10	34.36	35.50	34.98	34.69
$s[p_{60}(0.008)]$	14.10	8.20	5.16	4.23	2.01
min $[p_{60}(0.008)]$	2.53	17.54	26.35	27.84	29.53
max $[p_{60}(0.008)]$	58.24	48.26	47.75	43.45	37.54

APPENDIX III. COMPUTER PROGRAM FOR ESTIMATING PROBABILITY DISTRIBUTION OF LOAD CAPACITY P60

```
C Computer program PSIXTY is a modified and corrected version
C of the program listed in "Wind Loading and Strength of
C Cladding Glass" by D.A. Reed and E. Simiu (NBS BSS 154).
C It estimates the probability of failure of square glass
C panes subjected to a constant sixty second pressure.  The
C initial strength values for each element on each pane are
C simulated from Weibull distribution input parameters.  The
C stress at the center of each element is obtained as a non-
C linear function of pressure by using the Texas Tech
C University program referred to in Section 2.3 of this
C report.  The stress-pressure relationship is summarized
C in subroutine LF of this program.
C
C To obtain failure statistics, numerical experiments are
C conducted. A user-defined number of panes (=N in main program)
C are loaded, and a distribution of the breaking pressure is
C determined.  The breaking pressure is different for each pane,
C since each pane has a random Weibull strength distribution.
C The pane is divided into square elemental
C areas, wherein each element has a random Weibull distribution
C strength assigned for each of the 20 directions. Symmetry is
C taken advantage of in this program.
C
C This program was developed for panes with size 48"x48"x1/8".
C However, with small modifications, it can be used for panes
C of any specified size.
C
C The basic procedure is:
C
C FOR EACH OF  N  PANES ...
C          GENERATE THE RANDOM STRENGTHS FOR EACH DIRECTION IN EACH
C           OF THE DISCRETE ELEMENTS OF THE PANE.
C
C          FOR EACH LOCATION, FIND THE 60-SECOND PRESSURE THAT WILL
C           CAUSE THE STRESS=STRENGTH.
C
C          FIND THE MINIMUM 60-SECOND PRESSURE FOR THE PANE.
C
C FIT THE N MINIMUM 60-SECOND PRESSURES TO A WEIBULL DISTRIBUTION.
C
C INPUT:  N=Number of panes to use.
C         SSHAPE,SSCALE,SLOC= Input Weibull strength distribution
C                              parameters.
C         SEED=Random seed for random # generator.
C
C OUTPUT: 60-second breaking pressure (psf) parameters of the
C          fitted Weibull distribution.
C         60-second breaking pressure statistics (mean,S.D.,C.O.V.)
C
C Subroutines called:
C
C   Internal: F
C             WEIB
C             PSIXTY
C             FDMIN
C             DIR
```

```
C            LF
C
C   External: GGUBS - an IMSL routine to generate uniform random #'s
C             SRTAD - an IMSL routine to sort inplace the elements
C                       of a vector.
C
      REAL MEAN

      DOUBLE PRECISION SEED,SEEDS

      COMMON /P60PAR/ PSHAPE,PSCALE,PLOC

      READ (5,100) N
      READ (5,110) SSHAPE,SSCALE,SLOC,AREA
      READ (5,120) SEED

  100 FORMAT (I10)
  110 FORMAT (4F10.0)
  120 FORMAT (D20.0)

      SEEDS = SEED

      WRITE (6,130) SSHAPE,SSCALE,SLOC,AREA,N,SEEDS
  130 FORMAT (//,
     1 46HC O M P U T E R   P R O G R A M   P S I X T Y,//,
     2 37H GLASS STRENGTH WEIBULL DISTRIBUTION:,/,
     3 21H SHAPE PARAMETER    =, F8.3,/,
     3 21H SCALE PARAMETER    =, F7.2,/,
     4 21H LOCATION PARAMETER =, F7.2,//,
     5 21H TEST SPECIMEN AREA =, F8.3,//,
     6 21H NUMBER OF PANELS   =, I5,//,
     7 21H RANDOM NUMBER SEED =, D20.10)

      CALL PSIXTY
     + (N,AREA,SSHAPE,SSCALE,SLOC,PSHAPE,PSCALE,PLOC,MEAN,STDDEV,COV,
     +         SEED)

      F8 = F(0.008)
      FM = F(0.5)

      WRITE (6,140) PSHAPE,PSCALE,PLOC,MEAN,STDDEV,COV,F8.FM
  140 FORMAT (//,
     1 19H PSIXTY PARAMETERS:,/,
     2 21H SHAPE PARAMETER    =, F10.3,/,
     3 21H SCALE PARAMETER    =, F 9.2,/,
     4 21H LOCATION PARAMETER =, F 9.2,//,
     5 21H MEAN               =, F 9.2,/,
     6 21H STANDARD DEVIATION =, F10.3,/,
     7 21H COV                =, F10.3,//,
     8 21H F(0.008)           =, F 9.2,/,
     9 21H F(0.5)  (MEDIAN)   =, F 9.2)

      END
      FUNCTION F(Z)

      COMMON /P60PAR/ PSHAPE,PSCALE,PLOC
```

```
      F = PLOC + PSCALE*(-ALOG(1.0 - Z))**(1.0/PSHAPE)

      RETURN
      END
      SUBROUTINE WEIB(N,M,MR,SSHP,SSCL,SLOC,E1,E2,E3,T,S1,S2,S3)
C     PROGRAM WEIBULL--COMPUTES THREE PARAMETER WEIBULL
C     MAXIMUM LIKELIHOOD ESTIMATES FOR BOTH LEFT AND
C     RIGHT SINGLE CENSORING

C     DATE:  JUNE 31,1981
C     DATE PROGRAM CHECKED OUT:  JULY 9,1981
C     PROGRAMMER:  JONATHAN W. MARTIN

C     REFERENCE:  HARTER,H. L.;MOORE,A.H.  MAXIMUM LIKELIHOOD
C          ESTIMATION OF THE PARAMETERS OF GAMMA AND WEIBULL
C          POPULATIONS FROM COMPLETE AND CENSORED SAMPLES.
C           TECHNOMETRICS 7:639-643;1965.  ERRATA,9:195;1967.

C      NOTATION FOR INPUT DATA:
C          N  =  SAMPLE SIZE (BEFORE CENSORING), N=1000 OR LESS
C                AS DIMENSIONED
C          SSCL=  0 IF THE SCALE PARAMETER THETA IS KNOWN
C          SSCL = 1 IF THE SCALE PARAMETER THETA IS UNKNOWN
C          SSHP = 0 IF THE SHAPE PARAMETER K IS KNOWN
C          SSHP = 1 IF THE SHAPE PARAMETER K IS UNKNOWN
C          SLOC = 0 IF THE LOCATION PARAMETER C IS KNOWN
C          SLOC = 1 IF THE LOCATION PARAMETER C IS UNKNOWN
C          T(I) = I-TH ORDER STATISTIC OF SAMPLE (I=1,N)
C                 (SUBSTITUTE BLANK CARDS FOR UNKNOWN CENSORED
C                 OBSERVATIONS)
C          M   =  NUMBER OF OBSERVATIONS REMAINING AFTER
C                 CENSORING N-M FROM ABOVE
C          C(1)=INITIAL ESTIMATE (OR KNOWN VALUE) OF
C          THETA(1) = INITIAL ESTIMATE(OR KNOWN VALUE) OF THETA
C          EK(1) = INITIAL ESTIMATE (OR KNOWN VALUE ) OF K
C          MR = NUMBER OF OBSERVATIONS CENSORED FROM BELOW,
C                 NORMALLY ZERO INITIALLY
C      NOTATION FOR OUTPUT DATA:

C         N,SSCL,SSHP,SLOC,M,C(1),THETA(1),EK(1)--SAME AS FOR INPUT
C         C(J) = ESIMATE FOR LOCATION PARAMETER AFTER J-1
C                ITERATIONS (OR KNOWN VALUE ) OF
C         THETA(J) = ESTIMATE OF SCALE PARAMETER AFTER J-1
C                ITERATIONS (OR KNOWN VALUE) OF THETA
C         EK(J) = ESTIMATE OF SHAPE PARAMETER AFTER J-1
C                ITERATIONS (OR KNOWN VALUE) OF K
C                (MAXIMUM VALUE OF J AS PRESENTLY DIMENSIONED
C                IS 550)
C         EL = NATURAL LOGARITHM OF LIKELIHOOD FOR C(J),THETA(J),
C              EK(J)

      DIMENSION THETA(5550),EK(5550)
      DOUBLE PRECISION C(5550),T(1000),SLK
       DIMENSION X(600),Y(600)
       REAL E1,E2,E3
```

```
C       INTEGER PSCL(2),PSHP(2),PLOC(2)
C       DATA PSCL/' YES',' NO '/,
C      1     PSHP/' YES',' NO '/,
C      2     PLOC/' YES',' NO '/
C      WRITE(6,*)N,M,MR,SSHP,SSCL,SLOC
C      WRITE(6,*)E1,E2,E3
C    4 READ(5,1)N,M,MR,SSHP,SSCL,SLOC
C     IF(N) 66,66,77
C   77 READ(5,5) EK(1),THETA(1),C(1)
       EK(1)=E1
       THETA(1)=E2
       C(1)=E3
C    5 FORMAT(3F5.0)
C      READ(5,121)(DATAS(J),J=1,18)
       ISCL = SSCL + 1
       ISHP = SSHP + 1
       ILOC = SLOC + 1
       EN = N
C      DO 2 I=1,N

       IF (M) 64,64,32
   32 EM =M
   31 ELNM = 0.
       JCOUNT = 0
       EMR = MR
       MRP = MR + 1
       NM = N - M + 1
       DO 34 I=NM,N
       EI = I
   34 ELNM = ELNM + ALOG(EI)
       IF(MR) 66,35,74
   74 DO 75 I=1,MR
       EI = I
   75 ELNM = ELNM - ALOG(EI)
   35 DO 30 J=1,550
       JCOUNT = JCOUNT + 1
       IF(J-1) 66,25,37
   37 JJ = J-1
       SK = 0.
       SL = 0.
       DO 6 I = MRP,M
    6 SK = SK + (T(I)-C(JJ))**EK(JJ)
       IF(SSCL) 7,7,8
    7 THETA(J) = THETA(JJ)
       GO TO 9
    8 IF(MR) 66,19,20
   19 THETA(J) = ((SK +(EN-EM)*(T(M)-C(JJ))**EK(JJ))/EM)
      1**(1./EK(JJ))
       GO TO 9
   20 X(1) = THETA(JJ)
       LS = 0
       DO 21 L=1,55
       LL = L - 1
       LP = L + 1
       X(LP) = X(L)
       ZRK = ((T(MRP)-C(JJ))/X(L))**EK(JJ)
```

```
      Y(L) = -EK(JJ)*(EM-EMR)/X(L) + EK(JJ)*SK/X(L)**(EK(JJ) + 1.)
     1+EK(JJ)*(EN-EM)*(T(M)-C(JJ))**EK(JJ)/X(L)**(EK(JJ)+1.)
     2-EMR*EK(JJ)*ZRK*EXP(-ZRK)/(X(L)*(1.-EXP(-ZRK)))
      IF(Y(L)) 53,73,54
   53 LS = LS -1
      IF(LS + L) 58,55,58
   54 LS = LS + 1
      IF(LS - L) 58.56,58
   55 X(LP) = .5*X(L)
      GO TO 61
   56 X(LP) = 1.5*X(L)
      GO TO 61
   58 IF(Y(L)*Y(LL)) 60,73,59
   59 LL = LL-1
      GO TO 58
   60 X(LP) = X(L) + Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
   61 IF(ABS(X(LP)-X(L))-1.E-4) 73.73,21
   21 CONTINUE
   73 THETA(J) = X(LP)
    9 EK(J) = EK(JJ)
      IF(SSHP) 12,12,11
   11 DO 17 I = MRP,M
   17 SL = SL + DLOG(T(I)-C(JJ))
      X(1)=EK(J)
      LS=0
      DO 51 L=1,55
      SLK = 0.
      DO 18 I=MRP,M
   18 SLK=SLK +(DLOG(T(I)-C(JJ))-ALOG(THETA(J)))*(T(I)-C(JJ))
     1**X(L)
      LL = L-1
      LP = L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-C(JJ))/THETA(J))**X(L)
      Y(L)=(EM-EMR)*(1./X(L)-ALOG(THETA(J)))+SL-SLK/THETA(J)
     1**X(L)+(EN-EM)*(ALOG(THETA(J))-DLOG(T(M)-C(JJ)))
     2*(T(M)-C(JJ))**X(L)/THETA(J)**X(L)
     3+EMR*ZRK*(ALOG(ZRK)/X(L))*EXP(-ZRK)/(1.-EXP(-ZRK))
      IF(Y(L)) 43,52,44
   43 LS = LS -1
      IF(LS+L) 47,45,47
   44 LS = LS + 1
      IF(LS-L) 47,46,47
   45 X(LP) = .5*X(L)
      GO TO 50
   46 X(LP) = 1.5*X(L)
      GO TO 50
   47 IF(Y(L)*Y(LL)) 49,52,48
   48 LL = LL - 1
      GO TO 47
   49 X(LP) = X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
   50 IF(ABS(X(LP)-X(L))-1.E-4) 52.52,51
   51 CONTINUE
   52 EK(J) = X(LP)
   12 C(J) = C(JJ)
      IF(SLOC) 25.25,14
```

```
 14 IF(1.-EK(J)) 16,78,78
 78 IF(SSCL+SSHP) 57,57,16
 16 X(1) = C(J)
    LS = 0
    DO 23 L=1,55
    SK1 = 0.
    SR = 0.
    DO 15 I=MRP,M
    SK1 = SK1 +(T(I)-X(L))**(EK(J)-1.)
 15 SR = SR+ 1./(T(I)-X(L))
    LL = L-1
    LP = L + 1
    X(LP) = X(L)
    ZRK = ((T(MRP) -X(L))/THETA(J))**EK(J)
    Y(L)=(1.-EK(J))*SR+EK(J)*(SK1+(EN-EM)*(T(M)-X(L))
   1**(EK(J)-1.))/THETA(J)**EK(J)-EMR*EK(J)*ZRK*
   2EXP(-ZRK)/((T(MRP)-X(L))*(1.-EXP(-ZRK)))
    IF(Y(L)) 39,24,40
 39 LS = LS-1
    IF(LS+L) 70,41,70
 40 LS = LS+1
    IF(LS-L) 70,42,70
 41 X(LP) = .5*X(L)
    GO TO  22
 42 X(LP) = .5*X(L)+.5*T(1)
    GO TO 22
 70 IF(Y(L)*Y(LL)) 72,24,71
 71 LL = LL - 1
    GO TO 70
 72 X(LP) = X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
 22 IF(ABS(X(LP)-X(L))-1.E-4) 24.24,23
 23 CONTINUE
 24 C(J) = X(LP)
    GO TO 25
 57 C(J) = T(1)
 25 IF(MR) 66.38,69
 38 DO 63 I = 1,M
    IF(C(J)+1.E-4-T(I)) 68,67,67
 67 MR = MR + 1
    WRITE(6,201)
201 FORMAT(' ',' LINE 200 IN PROGRAM--INITIAL LOCATION PARAMETER',
   1'ESTIMATE GREATER'/'THAN FIRST OBSERVED FAILURE TIME')

 63 C(1) = T(1)
 68 IF(MR) 66,69,31
 69 SK = 0.
    SL = 0.
    DO 36 I=MRP,M
    SK = SK+(T(I)-C(J))**EK(J)
 36 SL = SL+DLOG(T(I)-C(J))
    ZRK=((T(MRP)-C(J))/THETA(J))**EK(J)
    EL=ELNM+(EM-EMR)*(ALOG(EK(J))-EK(J)*ALOG(THETA(J)))+
   1(EK(J)-1.)*SL-(SK+(EN-EM)*(T(M)-C(J))**EK(J))/
   2(THETA(J)**EK(J))+EMR*ALOG(1.-EXP(-ZRK))
    IF(J-3) 30.27,27
 27 IF(ABS(C(J)-C(JJ))-1.E-4) 28.28,30
```

```
   28 IF(ABS(THETA(J)-THETA(JJ))-1.E-4) 29.29,30
   29 IF(ABS(EK(J)-EK(JJ))-1.E-4) 99,99,30
   30 CONTINUE

C          OUTPUT

  99    CONTINUE
      S1=THETA(JCOUNT)
      S2=EK(JCOUNT)
           S3=C(JCOUNT)
      GO TO 66
   64 CONTINUE
   66 RETURN
      END
      SUBROUTINE PSIXTY(NPANEL,AREA,CO,SO,SU,MP,PEST,SV
     1 ,XMEAN,ZSAM,COV,DSEED)
C
C  (SEE DATA STATEMENT)
C SIDEA = LENGTH OF THE SQUARE
C THE LENGTH OF THE SIDE OF THE SQUARE IS INPUT IN INCHES
C TH = THICKNESS OF THE PLATE
C THICKNESS INPUT IN INCHES
C MODULUS OF ELASTICITY (E) INPUT IN PSI
C
C INPUT:
C  NPANELS = # PANELS TO 'BREAK' TO FIT WEIBULL DISTR. TO
C SO,CO,SU = WEIBULL STRENGTH PARAMETERS FOR AN ELEMENTAL AREA =
C
C OUTPUT:
C XMEAN = MEAN P60
C ZSAM  = S.D. OF P60
C COV   = C.O.V. OF P60
C PEST  = SCALE PARAMETER OF WEIBULL MLE FIT OF P60 FOR NPANELS
C MP    = GAMMA  "         "     "
C SV    = LOCATION PARAMETER  "  "
C
C PROCEDURE:
C FOR EACH OF NPANEL PANES...
C            GENERATE 216 R.V. STRENGTHS (FROM UNDERLYING WEIBULL
C               DISTR.) 36 LOCATIONS W/ EACH 6 DIRECTIONS=216
C
C           FOR EACH LOCATION, IF SIGMA=STRENGTH, FIND CORRESPONDING
C            LOAD(P 60) WHICH GIVES THAT STRESS(STRENGTH), I.E. THAT
C              LOAD WHICH WILL BREAK IT FOR THAT LOCATIONS STRENGTH.
C
C            FIND THE MINIMUM LOAD (P60) FOR THIS PANEL.
C
C  FIT THE MINIMUM LOADS (P 60) FOR THE NPANEL PANES TO A WEIBULL
C   DISTRIBUTION.
C
C NOTE THAT SYMMETRY IS USED IN THIS PROCEDURE, I.E. THE PANE IS
C DIVIDED INTO FORU QUADRANTS (BY SYMMETRY) AND THIS QUADRANT IS
C  DIVIDED BY ITS AXIS OF SYMMETRY (THE DIAGONAL) IN TWO.
C
C THE NUMBERING IN SUBROUTINE DIR AND LF USES THE UPPER HALF OF
C LOWER LEFT QUADRANT.  THE CENTROID OF THE PANE IS NUMBERED
```

```
C NODE #1.  THE 24" SQUARE QUADRANT OF THE 48" SQUARE PANE IS
C DIVIDED INTO 8X8 3 INCH SQUARES. NODE # 1 IS THE CENTROID OF
C THE ENTIRE PANE AND IS THE CENTROID OF ELEMENT #1. NODE # 2 IS
C 3 INCHES LEFT OF NODE #1 AND IS THE CENTROID OF ELEMENT #2.
C ETC. TILL NODE # 8 IS 21 INCHES LEFT OF NODE #1 AND IS THE
C CENTROID OF ELEMENT #8. AGAIN, USING SYMMETRY, NODE #9 IS
C DIRECTLY UNDER NODE#2 AND IS THE CENTROID OF ELEMENT #9.
C NODE #10 IS 3 INCHES LEFT OF NODE #9 ETC UNTIL NODE # 15
C WHICH IS 18 INCHES LEFT OF NODE #9. THEN NODE # 16 IS DIRECTLY
C UNDER NODE #10. ETC. THEN NODE #22 IS DIRECTLY UNDER NODE#17.
C ETC. UNTIL THE LOWEST ROW WHICH HAS ONLY NODE #36.
C NOTE THAT EACH ELEMENT IS A THREE INCH SQUARE (THIS IS
C NECESSARY SINCE THE R.V. STRENGTH ASSUMES AN EQUAL ELEMENTAL
C AREA. THUS, ELEMENT #1 HAS NO COUNTERPART IN THE OTHER THREE
C QUADRANTS. LIKEWISE ELEMENT'S #2-8 EACH REPRESENT FOUR OTHER
C ELEMENTS IN THE PANE (ALSO WITH THE ELEMENTS ON THE DIAGONAL)
C THE REMAINING ELEMENTS ARE REPLICATES OF 8 OTHER ELEMENTS IN
C THE PANE.
C
      DOUBLE PRECISION SIG(36,6),SI(36,6),PEQ(36,6),A1(216),
     *RSIG(36,6),S2,BP,M,RN,RY,RNM,KIC,ROOT,COEFS,COEFP,
     *SCALE,SMIN,SMAX,RF,DSEED,E.MIN(1000)
      REAL F(216),ZSAM,COV,XMEAN,MP
      INTEGER IPOINT,NUPPER
      DIMENSION ICHEK(216,2)
      EQUIVALENCE (SI(36,6),PEQ(36,6))
      COMMON //IPLACE(216,2),ICOUNT
C
      DATA KIC,A,RN,RY/0.75D0,1.08,19.69D0,1.25D0/
      DATA TH,SIDEA,PR/0.125,48.0,0.21/
      DATA E/1D07/
      DATA RF,IPOINT/1.0D0,1/
C
C CALCULATIONS
C
      M=1/CO
      RNM=RN-2.
      ROOT=1./RN
C
C 1/B IS A PARAMETER USED IN EQN. 19 OF THE BSS REPORT
C RY=1.25; A=1.08; N=19.69 FROM [5] OF BSS 154
C SEE ABOVE DATA STATEMENT
C
      BP=(RNM*A*RY*RY*(KIC**RNM))/2.
C
C FLEX=FLEXURAL RIGIDITY
C
      FLEX=( E*TH*TH*TH )/( 12.*(1.-PR*PR ))
C
      S2=SIDEA*SIDEA
C
C CF. EQN (9) OF BSS 154
C THE AREA FOR WHICH WE ARE SIMULATING A STRENGTH IS THE
C ELEMENT SIZE, WHICH AS EXPLAINED ABOVE IS 1/16 OF SIDEA
C THUS, AREA= (SIDEA/16)**2
C HOWEVER, TAKING DIRECTION INTO ACCOUNT AS EXPLAINED BELOW.
```

```
C THIS AREA IS DIVIDED BY 10.0
C
      SELEM=(SIDEA/16.)**2
      SNEW=SO* (AREA/(SELEM/10.))**M
      COEFS=(S2*TH)/FLEX
      COEFP=(S2*S2)/(FLEX*TH)
C
C THESE COEFFICIENTS ARE USED FOR NONDIMNSIONALIZING
C PRESSURES AND STRESSES.
C
      SCALE=(1./(60.*BP))**ROOT
      ROOT=RNM*ROOT
C
C  OBTAIN THE DIRECTIONAL MULTIPLIERS FROM [9] OF BSS 154
C
      CALL DIR(RSIG)
C
C THEN CALCULATE THE STRESSES IN TERMS OF THE STRENGTHS...
C
C IF THE STRESSES ARE LESS THAN OR EQUAL TO
C ZERO, THEN OMIT THESE FROM CONSIDERATION...
C
C LOGIGAL UNIT 9 CONTAINS A FILE WITH PSEUDO-RANDOM
C NUMBERS WHICH FOLLOW A UNIFORM DISTRIBUTION ON
C (0,1). AN ALTERNATIVE METHOD WOULD BE TO GENERATE THE NUMBERS
C DIRECTLY FROM A SUBROUTINE.
C
C THEN GENERATE INITIAL STRENGTH VALUES
C
      DO 926KK=1,NPANEL
      I=0
      DO 12J=1,36
      IF(J.EQ.1) THEN
        N=1
      ELSEIF(J.EQ.9 .OR. J.EQ.16 .OR. J.EQ.22 .OR. J.EQ.27 .OR.
     +  J.EQ.31 .OR. J.EQ.34 .OR. J.EQ.36 .OR. (J.GE.2.AND.J.LE.8))THEN
        N=4
      ELSE
        N=8
      ENDIF
C
C SINCE THE STRENGTH IS RANDOMLY SIMULATED W/IN EACH ELEMENT
C WE CAN ARBITRARILY ORIENT THE COORD SYSTEM, S.T. THE MAXIMUM
C PRINCIPAL STRESS OCCURS @ ALPHA=0
C DIVIDING THE CIRCLE INTO 20 = 18 DEGREE ARCS, THE FIRST ARC
C IS REPEATED TWICE, @ ARC#1 AND 180 DEGREES AROUND THE CIRCLE.
C SIMILARLY, ARC#6, CORRESPONDING TO THE MINOR PRINCIPAL STRESS
C ALSO OCCURS TWICE. THE STRESSES IN THE ARCS BETWEEN ARC # 1 AND
C ARC # 6 OCCUR FOUR TIMES (BY SYMMETRY). THUS STRESS @ ARC#1
C OCCURS 1/10; STRESS @ ARC#6 OVVURS 1/10; REMAINING ARCS
C STRESS OCCURS 1/5 EACH; 1/10+1/10+4(1/5)=1.0
C USING THE ANALOGY OF EQN (11) IN BSS 154
C THE AREA CORRESPONDING TO SO IS 3 SQUARE INCHES DIVIDED
C BY 10.     ALSO, THE STRESSES CORRESPONDING TO ARC#2-5,
C ARE SIMULATED TWICE, BECAUSE THEY OCCUR TWICE AS OFTEN AS
C THOSE CORRESPONDING TO THE MAJOR AND MINOR PRINCIPAL STRESSES
```

```
C THUS....
C
      DO 12 K=1,6
      IF (K.NE.1 .AND. K.NE.6) N=N*2
C
C GGUBS is an IMSL routine to generate N random uniform #'s
C
      CALL GGUBS(DSEED,N,F)
      CALL FDMIN(F,N,FMIN)
       IF ( FMIN .EQ. 0 ) THEN
       FMIN=FMIN+0.00001
       ENDIF
C SI= STRENGTH (UNITS OF MPA)
C SIG=STRESS FACTOR (SF) (UNITS OF PSI) = F(SIGMA 60)
C SIG FROM EQN. (13) & (19)   0.00689 PSI PER MPA
C
C FOLLOWING LINE CHANGED FROM FMIN   TO  (1.-FMIN)
C ORIG. WAS OK SINCE ONLY ONE RANDOM # 0-1  &  A RANDOM UNIFORM #
C BETWEEN 0-1 IS EQUIVALENT TO RANDOM      1- RANDOM #
C C
      SI(J,K)=SNEW*((-ALOG((1.-FMIN)))**M) +SU
      SIG(J,K)=SCALE*( SI(J,K)**ROOT )
      SIG(J,K)= COEFS*( SIG(J,K)/( .00689*RSIG(J,K)))
      IF( SIG(J,K) .GT. 0)THEN
         I=I+1
C J==> POSITION
C K==> DIRECTION
         ICHEK(I,1)=J
         ICHEK(I,2)=K
      ENDIF
 12   CONTINUE
      ICOUNT=I
C
C SORT THE REMAINING STRESSES....
C
      DO 160 I=1,ICOUNT
       J=ICHEK(I,1)
       K=ICHEK(I,2)
      A1(I)=SIG(J,K)
 160  CONTINUE
C
      IC=ICOUNT
C
C SRTAD is an IMSL routine to sort inplace the IC elements of vector
C  A1 min to max.
C
      CALL SRTAD(A1,1,IC)
      SMIN=A1(1)
      SMAX=A1(ICOUNT)
      SMAX=RF*SMAX
C
C OMIT FROM CONSIDERATION THE FOLLOWING STRESS VALUES...
C
      GO TO (10,10,30)IPOINT
C
C FOLLOWING LOOP IS UNNECESSARY IF RF=0.0!
```

```
 10   I2=0
      DO 161 I=1,ICOUNT
       J=ICHEK(I,1)
       K=ICHEK(I,2)
      IF( SIG(J,K) .LE. SMAX )THEN
          I2=I2+1
          IPLACE(I2,1)=J
          IPLACE(I2,2)=K
      ENDIF
 161  CONTINUE
C
      ICOUNT=I2
C
       GO TO 15
C
 30   CONTINUE
      DO 162I=1,ICOUNT
        J=ICHEK(I,1)
        K=ICHEK(I,2)
        IF( SIG(J,K) .EQ. SMIN )THEN
          IPLACE(1,1)=J
          IPLACE(1,2)=K
        ENDIF
 162  CONTINUE
C
C CALCULATE LOAD FACTOR, LF ... PEQ=LF=F(SF).
C
 15     CALL LF(SIG,IPOINT,PEQ)
C
       GO TO (11,11,31)IPOINT
C
 11     JK=0
      DO 17 IV=1,ICOUNT
       J=IPLACE(IV,1)
       K=IPLACE(IV,2)
       JK=JK+1
      IF( PEQ(J,K) .LE. 0 )THEN
      PEQ(J,K)=1.0D20
      ENDIF
C A1=LF/COEFP = PRESSURE  (UNITS OF ?)
      A1(JK)=PEQ(J,K)/COEFP
 17   CONTINUE
C
C SORT VALUES ....
C
      CALL SRTAD(A1,1,ICOUNT)
      MIN(KK)=144.*A1(1)
      GO TO 926
C
C IF ONLY THE MINIMUM STRESS IS CHECKED,
C CALCULATE ONLY ONE VALUE OF PEQ......
C
 31   J=IPLACE(1,1)
      K=IPLACE(1,2)
      MIN(KK)=144.*PEQ(J,K)/COEFP
C
```

```
 926   CONTINUE
C
C      UNITS CHANGED TO PSF FOR CALCULATIONS
C MIN=P60(PSF)MIN OF PANEL'S P60
C
       CALL SRTAD(MIN,1,NPANEL)
C
       SUM1=0.0
       SUM2=0.0
       DO 3003I=1,NPANEL
3003   SUM1=SUM1+MIN(I)
       XMEAN=SUM1/NPANEL
       DO 3004I=1,NPANEL
3004   SUM2=SUM2+( MIN(I) - XMEAN )**2
       ZSAM=SQRT(SUM2/( NPANEL - 1 ))
       COV=ZSAM/XMEAN
C
       N=NPANEL
C SSHP,SSCL,SLOC=1 ===>SHAPE,SCALE,LOCATION PARAMETER UNKNOWN
C E1=INITIAL ESTIMATE OF SHAPE PARAMETER
C E2= "          "       " SCALE   "
C E3= "          "       " LOCATION  "
C
       SSHP=1.
       SSCL=1.
       SLOC=1.
       E1=2.3
       E2=10.
       E3=MIN(1) - 2.0
       IF(E3 .LE. 0.0) E3=0.001
       IF( SU .LE. 0.1) THEN
       E3 = 0.001  @      SLOC = 0.0
       ENDIF
 311    MR=0
C       WRITE(6,305)
 305   FORMAT(1X,3H305)
C
       CALL WEIB(N.N,MR,SSHP,SSCL,SLOC,E1,E2,E3.MIN,PEST,MP,SV)
C
C
       RETURN
       END
       SUBROUTINE FDMIN(F,N,FMIN)
       DIMENSION F(N)
C
       FMIN=1.E+10
       DO 10 I=1,N
       FMIN=AMIN1(FMIN,F(I))
 10    CONTINUE
       RETURN
       END
C
       SUBROUTINE DIR(RSIG)
       DOUBLE PRECISION PH,RSIG(36,6),C(6),SII(6),S2S1(36)
       DATA S2S1/1.0D0,.98D0,.94D0,.89D0,.79D0,.68D0,.52D0,.12D0,.97D0.
      *.93D0,.86D0,.77D0,.65D0,.47D0,.05D0,.89D0,.82D0,.71D0,.57D0..36D0,
```

```
     *-.09D0,0.73D0,0.61D0,0.45D0,
     *.21D0,-.24D0,.47D0,.28D0,.02D0,-.38D0,.83D0,-.17D0,-.52D0,
     *-0.39D0,-0.66D0,-.81D0/
      DO 5K=1,6
      PH=(K-1)*3.14159/10.
      C(K)=DCOS(PH)*DCOS(PH)
 5    SII(K)=DSIN(PH)*DSIN(PH)
      DO 10J=1,36
       DO 10K=1,6
        RSIG(J,K)=C(K) + S2S1(J)*SII(K)
 10     CONTINUE
      RETURN
      END
C
      SUBROUTINE LF(SIG,IPOINT,PEQ)
      DOUBLE PRECISION PEQ(36,6),SIG(36,6),LIF(25),L(36,25)
      COMMON // IPLACE(216,2),ICOUNT
C
      DATA ((L(J,I),I=1,24),J=1,6)/0,6.8,13.7,20.6,27.5,34.0,40.5,
     *46.6,53.0,58.3,63.0,104.0,129.0,149,167,182,195,207,218,
     *228,310,359,407,481,0,6.7,13.5,20.4,27.0,33.7,40.4,46.1,
     *52,57.7,63,103,130,151,170,186,199,211,222,233,319,364,410,
     *484,0,6.5,13,19.7,26.3,32.8,39,45,51,56.7,62,104.134,157,
     *177,194,209,223,235,246,333,384,430,497,0,6,12.3,18.5,24.9,
     *30.9,37.2,43,49,54.2,59,103,136,163,186,206,224,240,255,
     *268,370,432,484,550,0,5.5,11.2,16.8,22.6,28.3,33.8,39,45,50,
     *55,90,135,165,192,215,237,257,275,290,420,509,581,661,0,
     *4.7,9.5,14.3,19.2,24.1,28.9,34,39,43,48,90,125,157,186,
     *212,234,257,278,296,457,580,687,792/
C
      DATA ((L(J,I),I=1,24),J=7,11)/0,3.6,7.2,10.8,14.5,18.2,21.7,
     *25.5,29.3,33,37,71,102,130,156,180,203,224,244,263,426,
     *563,689,811,0,2.0,4,6.1,8.3,10,12,14,17,19,21,40,59,78,
     *94,109,125,139,153,166,279,377,471,563,0,6.7,13.6,20.4,
     *27.1,33.7,40.0,45.9,52.2,57.5,62.1,102.6,129.2,149.7,167.7,
     *183,196.4,209.2,220.6,232.4,319.8,375.9,428.4,504.2,0,6.6,
     *13.3,20,26.6,33,39.2,45.1,51.2,56.6,61.3,102.8,131.9,154.8,
     *174.9,192.3,207.5,221.3,233.7,245.7,337.1,397.8,454.0,530.8,
     *0,6.3,12.7,19,25.4,31.3,37.4,42.9,49,54.2,59.1,102.6,134.7,
     *161.5,184.7,205.1,223.3,239.8,254.6,268.3,373.5,441.5,
     *500.1,574.2/
C
      DATA ((L(J,I),I=1,24),J=12,15)/0,5.8,11.6,17.3,23.2,28.6,
     *34.4,39.6,44.9,50.4,54.9,98.9,134.2,164.5,191.5,215.4,
     *237.3,257.3,275.8,291.8,424.9,516.9,593.1,672.8,0,5,
     *10,15,20,24.7,29.6,34.4,39,43.6,47.9,89.4,125.1,156.9,
     *186,212.5,236.9,259.8,281.1,300.3,463.1,587.7,697,800.7,
     *0,3.9,7.9,10,15.5,19.2,23,26.6,30.1,33.8,37.4,71.2,101.9,
     *130.5,157.1,182,205.1,227.2,247.9,267.3,434.8,573.2,699.7,
     *820.9,0,2.5,4.9,7.3,9.4,11.8,13.9,16,18.2,20.2,22.2,41.9,
     *60.8,78.6,95.7,111.8,126.9,141.6,155.6,169.1,285.7,385.8,
     *479.8,571.5/
C
      DATA ((L(J,I),I=1,24),J=16,20)/0,6.6,13.3,19.9,26.4,32.7,38.9,
     *44.4,50.4,55.7,60.3,100.7,128.7,151.0,170.4,187,201.9,
     *216.1,230.2,242.8,349.3,423.7,490.2,572.4,0,6.5,12.8,19.3,
```

```
     *25.6,31.6,37.7,43.1,48.7,53.9,58.5,99.7,130.8,157.1,180.5,
     *201.1,219.9,236.9,252.8,266.6,381.3,463.6,537.5,621.6,0
     *,6.1,12.1,18.1,23.9,29.5,35.2,40.2,45.6,50.5,55,96.8,131.2,
     *161.7,189.4,214.4,237.3,258.3,278.2,295,435.7,534.2,620,
     *705.3,0,5.5,10.8,16.1,21.2,26.1,31.2,35.7,40.2,44.8,48.9,
     *88.6,123.6,156.1,186.3,214.4,240.5,264.7,287.8,308.5,
     *481.8,612.1,724.5,828.9,0,4.5,8.9,13.2,17.3,21.3,25.1,
     *28.9,32.7,36.2,39.5,72.1,102.4,131.5,159.3,186.1,210.8,
     *234.7,257.5,278.9,460.4,605.2,735.4,856.9/
C
      DATA ((L(J,I),I=1,24),J=21,24)/0,3.3,6.2,9.3,12.1,14.7,17.2,
     *19.5,21.9,24.1,26.1,45.6,63.7,81.5,99,116.2,132.2,148.2,
     *163.5,178.5,307.3,413.9,510.9,603.4,0,5.7,6.5,12.9,19.2,
     *25.3,31.2,37,42.4,47.8,52.7,57.3,97.3,127.3,153.0,175.0,
     *195,213,229,244,391.7,493.4,583.5,676.3,0,5.6,6.3,12.4,
     *18.4,24.2,29.8,35.3,40.4,45.6,50.4,54.7,94.6,126.9,156,
     *184,209,233,255,276,444.6,556.9,655.6,750.8,0,5.3,5.8,
     *11.4,16.9,22.2,27.3,32.3,36.9,41.6,45.9,50.2,88.1,121.6,
     *154,184,214,241,268,294,507.1,648.2,769.1,876.5/
C
      DATA ((L(J,I),I=1,24),J=25,28)/0,4.7,5,9.9,14.6,19.1,23.4,
     *27.6,31.5,35.2,39.1,42.4,74.6,103.9,133,161,190,217,
     *244,269,501.7,662.8,802.8,929.4,0,3.6,3.9,7.7,11.3,14.7,
     *17.8,20.9,23.6,26.4,28.9,31.3,52,70.2,88.1,106,124,141,
     *159,176,344.9,468.2,576.2,674.6,0,5.7,6.2,12.2,18.2,23.9,
     *29.3,34.7,39.7,44.6,49.3,53.6,92.4,124.1,180,205.3,228.2,
     *250.1,270.8,288.7,436.2,569,687.7,798.2,0,5.6,5.9,11.8,
     *17.3,22.7,27.9,32.9,37.6,42.3,46.6,50.9,88.1,120.6,181,
     *210.3,237.9,265.2,291.7,316.4,525.4,680.3,811.5,926.7/
C
      DATA ((L(J,I),I=1,24),J=29,32)/0,5.1,5.4,10.7,15.8,20.5,25.1,
     *29.6,33.8,38.1,41.8,45.5,78.5,82.3,165,193.9,221.8,249.7,
     *277.7,305.1,548.9,739.9,900.6,1041.0,0,4.2,4.5,8.9,12.9,
     *17,20.9,24.4,27.8,31.2,34.2,37,61.8,76.8,119.9,138.6,
     *156.9,175.5,194.2,213.2,395.9,553.2,688.4,806.5,0,5.4,
     *11.6,17.3,22.6,27.8,32.7,37.2,41.9,46.1,50,87,120,151,
     *172.9,210.2,237.9,265.6,292.3,318.1,538.5,700.9,827,923,0,
     *5.4,11.2,16.5,21.5,26.4,31.2,35.5,39.9,43.9,47.9,83,114.
     *144,172.8,201.7,230,258.286.4,314.7,582.5,809.3,998.4,
     *1157.6/
C
      DATA ((L(J,I),I=1,24),J=33,36)/0,4.7,9.9,14.7,19.2,23.6,27.8,
     *31.8,35.7,39.2,40,73,99,122,143.9,165.4,186.3,206.9,227.3,
     *248.2,456.5,658.9,843,1001.9,0,5.4,11.2,16.6,21.7,26.6,
     *31.4,36,40.6,44.8,48.8,85.5,118,148.5,178.3,207.8,236.9,
     *265.9,295,323.7,607.3,865.2,1085.7,1264.5,0,5.1,10.6,
     *15.8,20.7,25.6,30.1,34.6,39.2,43.3,47.3,83.5,114.5,143.3,
     *170.6,196.7,222.6,247.7,272.6,296.9,538.2,780.4,1015.6,
     *1229.8,0.,5.1,10.8,16.1,21.4,26.4,31.3,36,40.8,45.3,49.7,
     *89.6,124.4,156.7,187.9,217.2,246.4,274.7,302.4,328.9,
     *587.5,846.4,1113.9,1389.1/
C
      DATA (LIF(I),I=1,25)/0,24.8,49.7,74.5,99,124,149,174,199,
     *223,249,497,745,993,1242,1490,1738,1987,2235,2484,4967,
     *7450,9934,12417,100000/
C
```

```
C APPROXIMATIONS FOR THE LARGEST VALUES OF SIGMA
C ARE TAKEN FROM THE FOLLOWING REPORT :
C "PROPOSED METHOD FOR DETERMINING THE THICKNESS
C OF GLASS IN SOLAR COLLECTOR PANELS," BY
C DONALD MOORE, JET PROPULSION LABORATORY,
C CALIFORNIA INSTITUTE OF TECHNOLOGY, PASADENA,
C CALIFORNIA, MARCH, 1980.
C
      DO 6000J=1,29
6000  L(J,25)=1500
      DO 7000J=30,36
7000  L(J,25)=5000
C
C CALCULATE THE LOAD FACTOR FOR EACH STRESS
C
      DO 1000 II=1,ICOUNT
      IF( IPOINT .EQ. 3 )GO TO 300
        J=IPLACE(II,1)
        K=IPLACE(II,2)
      SS=SIG(J,K)
      GO TO 500
 300   J=IPLACE(1,1)
       K=IPLACE(1,2)
       SS=SIG(J,K)
 500  IF(SS .GT. L(J,25)) THEN
        PEQ(J,K)=LIF(25)
        GOTO 1000
      ENDIF
      IF(SS.EQ.0)THEN
        PEQ(J,K)=0
        GOTO 1000
      ENDIF
      IF(SS.LT. L(J,2)) THEN
        PEQ(J,K)=SS*(LIF(2)-LIF(1))/L(J,2)
        GOTO 1000
      ENDIF
       DO 200I=2,25
      IF(SS .GT. L(J,I-1) .AND. SS .LT. L(J,I)) THEN
       R=(SS-L(J,I-1))*(LIF(I)-LIF(I-1))/(L(J,I)-L(J,I-1))
     1 + LIF(I-1)
      PEQ(J,K)=R
      GOTO 1000
      ENDIF
      IF(SS .EQ. L(J,I)) THEN
        PEQ(J,K)=LIF(I)
        GOTO 1000
      ENDIF
 200  CONTINUE
       IF( IPOINT .EQ. 3 )GO TO 400
1000  CONTINUE
C
 400  RETURN
      END
```

APPENDIX IV. COMPUTER PROGRAM FOR ESTIMATING PARAMETERS OF GLASS STRENGTH DISTRIBUTION

```
*BATCH
C*******************************************************
C
C       PROGRAM COMPUTES THE MAXIMUM LIKELIHOOD ESTIMATES
C       FOR THE TWO OR THREE PARAMTER WEIBULL AND
C       FOR THE TWO PARAMETER LOGNORMAL FOR PROGRESSIVE CENSORED
C       SAMPLES
C
C       OUTPUT INCLUDES PARAMETRIC ESTIMATES, NUMBER OF SAMPLES
C        AND NUMBER OF FAILED SAMPLES.  FOR THE PURPOSES OF THIS
C        PROJECT, ONLY THE ESTIMATES OF THE WEIBULL DISTRIBUTION
C        ARE USED.
C
C       SUBROUTINE HAMLET IS USED TO OBTAIN TWO PARAMETERS
C       WHILE THIRD PARAMETER IS OPTIMIZED WITHIN THE MAIN
C       PROGRAM USING AN INCREMENTAL OPTIMIZING SEARCH (i.e.
C           FIRST SEARCH IN TEN PERCENT INTERVALS AND THEN TAKE
C           MOST PROMISING TWENTY PERCENT REGION AND SEARCH IN
C           TWO PERCENT INTERVALS, ETC.)
C        OPTIMUM OCCURS WHEN LOGARITHM OF LIKELIHOOD FUNCTION
C        IS A MAXIMUM
C
C        INPUT DATA:
C
C             N- NUMBER OF SAMPLES
C             NF- NUMBER OF FAILURES
C             IPROG- FLAG WHICH INDICATES IF PROGRESSIVE
C                    CENSORING IS IN EFFECT (1-YES;OTHER-NO)
C             ICR- NUMBER OF PARAMETERS USED IN FIT
C                  (0-TWO;<>0-THREE)
C             T(I)-FAILURE STRENGTHS OF SAMPLES AS CALCULATED FOR THE
C                  CENTRAL PORTION OF THE RING
C             G(I)-CENSORED VALUES OF STRENGTH (READ IN FROM SUBROUTINE)
C             IDC-FLAG USED TO DETERMINE STARTING POINT FOR
C                 OPTIMIZING THIRD PARAMETER (AS % OF 1ST
C                 STRENGTH VALUE)
C                 [3-95%; 4-85%; OTHER-99%]
C
C         LOGICAL UNIT ASSIGNMENTS:
C
C             5-DATAFILE(INPUT)
C             9-CONSOLE
C             6-DATAFILE(OUTPUT)
C             7-CONSOLE(USED BY SUBROUTINE)
C
C************************************************************
      COMMON /PARAM/ ALPHA,BETA,XBAR1,S,NG,XNF
      DIMENSION T(500),TLOG(500)
      REAL G(150),T1(150),G1(150),LSTAR(10),G2(150),JR
      DATA T/500*0.0/,TLOG/500*0.0/
      GFLAG=0
      READ(5,5) N,NF
    5 FORMAT(2I3)
      READ(5,5) IPROG,ICR
      AN = N
      DO 500 I = 1,NF
      READ (5,1) T(I)
    1 FORMAT(F15.0)
  500 CONTINUE
      CALL HAMLET(N,NF,T,IPROG,G,GFLAG)
      IF(ICR.EQ.0) GOTO 223
C
C        BEGIN OPTIMIZING SEARCH FOR THIRD PARAMETER
```

```
C
      WRITE(9,15)
   15 FORMAT( 'INPUT IDC (3-85%,4-95%)?')
      READ(9,*)IDC
      GFLAG=2
      DO 64 IV=1,NF
   64 T1(IV)=T(IV)
      NPG=N-NF
      DO 69 JV=1,NPG
   69 G1(JV)=G(JV)
      XLHI=99.0
      IF (IDC.EQ.3) XLHI=95.
      IF (IDC.EQ.4) XLHI=85.
      XLLO=0.0
      XLSTEP=-10.
   84 DO 93 JL=1,10
   93 LSTAR(JL)=0.0
      I5=0
C
C        HERE IS THE OPTIMIZATION LOOP
C
      DO 97 IR=11,1,-1
      JR=XLHI+I5*XLSTEP
      I5=I5+1
      C=(T1(1)*JR/100.)
      IF(C.LT.0.0) C=0.0
C
C        REEVALUATE FAILURE TIMES AND CENSORED TIMES
C
      DO 87 JQ=1,NF
   87 T(JQ)=T1(JQ)-C
      DO 77 JP1=1,NPG
   77 G2(JP1)=G1(JP1)-C
C
C        CALL HAMLET AGAIN TO COMPUTE NEW 1ST AND 2ND
C        PARAMETERS
C
      CALL HAMLET(N,NF,T,IPROG,G2,GFLAG)
      A=ALPHA
      B=BETA
      XNF=NF
      SUM1=0.0
      SUM2=0.0
      DO 74 JZ=1,XNF
      SUM1=SUM1+LOG(T(JZ))
   74 SUM2=SUM2+(T(JZ)/BETA)**ALPHA
      DO 76 J4=1,NPG
   76 SUM2=SUM2+(G2(J4)/BETA)**ALPHA
C
C        COMPUTE LOGARITHM OF LIKELIHOOD FUNCTION
C
  154 LSTAR(I5)=XNF*LOG(A)-XNF*A*LOG(B)+(A-1.0)*SUM1-SUM2
      WRITE(6,*)LSTAR(I5),A,B,C,SUM1,SUM2,NPG
      IF (I5.LE.1) GOTO 97
      IF(IDC.EQ.1) GOTO 97
      IF (LSTAR(I5).GT.LSTAR(I5-1)) GOTO 97
C
C        OPTIMUM HAS BEEN LOCATED; REDEFINE SEARCH INTERVAL
C
      XNLO=JR
      XNHI=XNLO+2.*ABS(XLSTEP)
      IF (XNHI.GT.XLHI) XNHI=XLHI
      IF (XNLO.EQ.XNHI) XNLO=XNHI+XLSTEP
      XLLO=XNLO
      XLHI=XNHI
      XLSTEP=(XLLO-XLHI)/10.
```

```
C
C       CHECK FOR PRECISION OF .0001 PERCENT
C
      C1=ABS(LSTAR(I5)-LSTAR(I5-1))
      C2=ABS(.000001*LSTAR(I5))
      IF (C1.LE.C2) GOTO 223
      GOTO 84
   97 CONTINUE
C
C       OUTPUT RESULTS
C
  223 WRITE(6,6057)N,NF,BETA,ALPHA,C,XBAR1,S
 6057 FORMAT(/////' NUMBER OF SPECIMENS LOADED', I29//
     1' NUMBER OF SPECIMENS FAILED',I29//
     2' WEIBULL SCALE PARAMETER',F30.5//
     3' WEIBULL SHAPE PARAMETER',F30.5//
     6' LOCATION PARAMETER',F30.5//
     4' LOGNORMAL SCALE PARAMETER',F27.5//
     5' LOGNORMAL SHAPE PARAMETER',F27.5)
      WRITE(6,6058)N,NF,(T(I),I=1,NF)
 6058 FORMAT(2I3,1000(/F10.0))
      STOP
      END
      SUBROUTINE HAMLET(N,NF,F,IPROG,G,GFLAG)
      COMMON /PARAM/ ALPHA,BETA,XBAR1,S,NG,XNF
      REAL F(150),G(150),X(150)
      REAL MUU
      VUA = 1.
      X31 = 9.
      ZC=0
      GIG = 1.
      IF(IPROG.EQ.1) GIG=0
      NFF = NF
      XNF = NF
      XN = N
      NG = N - NF
      NGG=NG
      IF(NG.EQ.0) GO TO 5110
      IF(GFLAG.NE.0) GOTO 5110
      DO 5130 I = 1,NG
      IF(IPROG.EQ.1) GOTO 5120
      G(I) = F(NF)
      GOTO 5130
 5120 READ(5,5121) G(I)
 5121 FORMAT(F10.0)
 5130 CONTINUE
5110  CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                  C
C     ORDER FAILURE AND CENSORED TIME DATA         C
C                                                  C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      BAC = -10.
      GNG = NG
      IF (GNG-.01) 1100, 1100, 1109
1109  IF (GIG - .01) 1106, 1106, 1100
 1106 NG1=NG-1
      DO 1112 I=1, NG1
      I1=I+1
      DO  1115 J=I1,NG
      IF (G(I) - G(J)) 1115, 1115, 1118
1118  GMID = G(J)
      G(J) = G(I)
      G(I) = GMID
1115  CONTINUE
      WRITE(7.4887) G(I)
```

```
 4887 FORMAT('0','  G(I)',F10.3)
1112  CONTINUE
      WRITE(7,4887) G(NG)
1100  NFM1 =NF - 1
      DO 3 I=1,NFM1
      I1=I+1
      DO 1 J=I1,NF
      IF (F(I)-F(J)) 1,1,2
2     FMID=F(J)
      F(J)=F(I)
      F(I)=FMID
1     CONTINUE
      X(I)=ALOG10(F(I))                                              LOGGING
3     CONTINUE
      X(NF) = ALOG10(F(NF))
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                  C
C    FIRST TWO ORDERED FAILURE TIMES               C
C                                                  C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      FN1 = F(1)
      FN2 = F(2)
      FNL = F(NF)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                  C
C     UNBIASED ML POINT ESTIMATES FOR LOG NORMAL   C
C     POP. PARA. ARE CALCULATED BELOW.             C
C                                                  C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      XSUM=0.                                                        SIGMA
      X2SUM=0.                                                       XBAR
      DO 4 I=1, NF
      XSUM=XSUM+X(I)
      X2SUM=X2SUM+X(I)*X(I)
4     CONTINUE
      XN=N
      XNF=NF
      FNF = XNF
      FN= N
      FNG = FN - FNF
      XBAR=XSUM/XNF
      XBARL = XBAR
      S2=(X2SUM-XSUM*XBAR)/(XNF-1.)
      S=SQRT(S2)
      S=S+.00001
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                 C
C     UNBIASED ESTIMATES OF LOG NORMAL MODEL ARE  C
C     GIVEN BELOW FOR CENSORED CASE               C
C                                                 C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C     THIS SECTION FOR HAMLET --- CALLED INSERT C
      IF (FN - FNF - .01) 9100, 9100, 9103
 9107 SIG =.557/ALPHA
      MUU = BETA * .56 ** (1. / ALPHA)
C     AAA
C     BBB
      MUU = ALOG10(MUU)
C THE BELOW WORK IS COMMON TO HAMLET AND LOG7
8048  IF (GIG - 0.1) 8020, 8020, 8024
C
C      LOG NORMAL MLE -- SINGLE CENSORING
C
8024  EPS =(ALOG10(G(1)) - MUU) / SIG
      ZE = CNORML  (EPS, -150., 0.0, 1.0)
      FEZE = EXP(-(EPS *EPS/2.)) * 0.3989423
```

```
      ZE = FEZE / (1.- ZE)
      ZEE = FNG * ZE
      EASY = ZEE * EPS
      AI = ZEE * (ZE -EPS)
      BI = ZEE + EPS * AI
      CI = EPS * (ZEE + BI)
      GO TO 8028
C
C      LOG  NORMAL MLE -- PROGRESSIVE CENSORING
C
8020  ZEE = 0.
      EASY = 0.
      AI = 0.
      BI = 0.
      CI = 0.
      DO 8032 ICE = 1, NG
      EPS = (ALOG10(G(ICE)) - MUU) / SIG
      FEZE = EXP(-(EPS *EPS/2.)) * 0.3989423
      ZE = CNORML  (EPS, -150., 0.0, 1.0)
      ZE = FEZE/  (1. - ZE)
      ZEE = ZEE + ZE
      EASY = EASY + EPS * ZE
      AP=ZE * (ZE - EPS)
      BP = ZE + EPS * AP
      CP = EPS * (ZE + BP)
      AI = AI + AP
      BI = BI + BP
      CI = CI + CP
8032  CONTINUE
8028  TST = (XBAR - MUU) / SIG
      TST2 = TST * TST
      SR2 =(S2 / (SIG * SIG)) * (1. - 1./FNF)
      FL = TST + ZEE / FNF
      GL = TST2 + SR2 - 1. + EASY/FNF
      PL = - (1. + AI/FNF) / SIG
      QL = - (2. * TST + BI/FNF)/SIG
      RL = - (3. * (TST2 + SR2) - 1. + CI/FNF) / SIG
      DL = PL * RL -QL * QL
      EH= (GL * QL - FL * RL) / DL
      EK= (FL * QL - GL * PL) / DL
 4016 CONTINUE
      IF (ABS(EK) - SIG/2. - .02) 4008, 4008, 4012
4012  EK = EK/2.
      GO TO 4016
4008  IF (ABS(EH) - 0.5) 4020, 4020, 4024
4024  EH = EH/2.
      GO TO 4008
4020  CONTINUE
      BAB = -6.
      TE1 = SIG +EK
      TE2 = MUU +EH
      ZC = ZC + 1.
      IF (ZC-53.) 4000, 4000, 4004
 4004 CONTINUE
      WRITE(7,4005)
 4005 FORMAT('SEE LINE 329 IN HAMLET')
      GO TO 308
4000  CONTINUE
      IF (TE1) 8040, 8040, 8044
8040  SIG = SIG/2.
      BAB = 6.
      GO TO 8058
8044  SIG = TE1
8058  IF(TE2) 8050, 8050, 8054
8050  MUU=MUU/2.
      BAB=6.
```

```
      GO TO 8060
8054  MUU = TE2
8060  IF (BAB) 8049, 8048, 8048
8049  ERROR = ABS (EH)+ ABS (EK)
      AERR = 0.0004
      IF (ERROR - AERR) 8064, 8064, 8048
8064  XBAR = 10. ** MUU
      S=SIG
C THE ABOVE WORK IS COMMON TO HAMLET AND LOG7
      XBARL = MUU
      BNF = XBAR
      ANF = 2. * SIG
      IF (BAC) 9111, 9111, 9404
C
C      MLE WEIBULL PARAMETERS TOO LARGE--USE DEFAULT
C      PARAMETERS
C
9404  ALPHA = 0.557 / S
      BEEL = XBARL + 0.2506 / ALPHA
      BETA = 10. ** BEEL
      GO TO 9111
C
C
C      COMPUTE NEW FANG PARAMETERS
C
C
9100  AL = 0.557/S
      BEEL=XBAR + 0.2506816/AL
      BEE= 10. ** BEEL                                                  BE
      XBAR= 10. ** XBARL
      WUM=0.                                                            NWFGLD
      WUMH=0.
      DO 133 I=1, NF
      WUM=WUM + F(I)
      WUMH= WUMH + 1./F(I)
133   CONTINUE
      ESS = WUM/XNF
      REC = WUMH/XNF
      R = 1./REC
      BNF = SQRT(ESS * R)                                               BNF
      ANF = SQRT (2. * (ESS/BNF - 1.))
CMLE OF WBL PARAS FOLLOW-FIRST SECTION CALCS SHAPE--- LAST SECTION FOR NEW FANG
      ALO= AL * VUA
      GO TO 12
9103  ALO = F(1) / (F(2) - F(1) + 5)
      AL = 0.557/S
      BEEL = XBAR + 0.2506 / AL
      BEE = 10. ** BEEL
      ALO = AL
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                  C
C     UNBIASED ESTIMATES OF WEIBULL MODEL ARE       C
C     GIVEN BELOW FOR THE COMPLETE AND CENSORED
C     CASE                                         C
C                                                  C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
12    IF (ALO*X(NF)-150.) 207,207,208
207   SFA= 0.
      SFAX= 0.
      SFAX2 = 0.
      DO 5 I=1, NF
        P = F(I) ** ALO
      SFA= SFA + P
      SFAX= SFAX + P * X(I)*2.302585
      SFAX2= SFAX2 + P * X(I) * X(I) *2.302585*2.302585
5     CONTINUE
```

```
      SGA=0.                                                            411.
      SGAX=0.                                                           412.
      SGAX2=0.                                                          413.
      IF (GNG - .01) 1130, 1130, 1133                                   414.
1133  IF (GNG - 7.01) 1136, 1136, 1139                                  415.
1139  IF (GIG - .01) 1136, 1136, 1142                                   416.
1142  P= G(1) ** ALO                                                    417.
      GLOGG = ALOG (G(1))                                               418.
      SGA= GNG * P                                                      419.
      SGAX= SGA * GLOGG                                                 420.
      SGAX2= SGAX * GLOGG                                               421.
      GO TO 1130                                                        422.
1136  DO 1148 I=1, NG                                                   423.
      P= G(I) **ALO                                                     424.
      GLOGG= ALOG (G(I))                                                425.
      SGA= SGA + P                                                      426.
      SGAX= SGAX + GLOGG * P                                            427.
      SGAX2= SGAX2 + P * GLOGG * GLOGG                                  428.
1148  CONTINUE                                                          429.
1130  SFA= SFA + SGA                                                    430.
      SFAX = SFAX + SGAX                                                431.
      SFAX2 = SFAX2 + SGAX2                                             432.
      FALO= SFAX/SFA - 1./ALO - XBARL*2.302585                          433.
      SFA2=2*ALOG(SFA)                                                      434
      DALO1=ALOG(SFAX2)-ALOG(SFA)
      DALO2=2*ALOG(SFAX)-SFA2
      DFALO=(10.**DALO1)-(10.**DALO2)+1./(ALO*ALO)
32    IF (ABS(DFALO) - .00001) 30, 30, 31                               436.
30    DFALO= 10. * DFALO                                                437.
      GO TO 32                                                          438.
31    CORR= -FALO/DFALO                                                 439.
      IF (ABS(CORR) - .0001) 8, 8, 9                                    440.
9     TRY= ALO + CORR                                                   441.
      IF (TRY) 10, 10, 11                                               442.
10    ALO= ALO/2.                                                       443.
      GO TO 12                                                          444.
11    ALO= TRY                                                          445.
      GO TO 12                                                          446.
8     ALPHA= ALO + CORR                                                 447.
      BETA = (SFA/XNF) ** (1./ALPHA)                                    448.
      GO TO 223                                                         449.
208   CONTINUE                                                          450.
      IF (FNG - 0.1) 209, 210, 210                                      451.
C                                                                       452.
C      STATEMENT 12 POSITIVE--WEIBULL PARAMETERS ABORTED                453.
C                                                                       454.
  210 WRITE(7,211)
  211 FORMAT('SEE LINE 453 IN HAMLET')
      GO TO 308
209   CONTINUE                                                          457.
      ALPHA=AL                                                          458.
      BETA=BEE                                                          459.
      BAC = 8.                                                          460.
223   CONTINUE                                                          461.
      IF(FNG - .01) 9111,9111,9107
C                                                                       463.
C      DETERMINE IF COMPLETE SAMPLE:  IF NOT FIND                       464.
C      CENSORED LOG NORMAL PARAMETERS                                   465.
C                                                                       466.
9111  CONTINUE                                                          467.
      ALPHA = ALPHA/VUA                                                 468.
      NL=NFF                                                            469.
      NW=NFF                                                            470.
      XBAR1 = ALOG(XBAR)                                                488.1
      S = S*ALOG(10.)                                                   488.2
308   CONTINUE                                                          504.
```

```
      RETURN
      END
      FUNCTION CNORML (XH,XL,XM,XS)
C     S00887   CNORML   PS-497    CHIANG E. C. H.   661004 6600
      P(X)=.5*(1.-(1./(1.+.14112821*X+.08864027*X**2
     1+.02743349*X**3-.00039446*X**4+.00328975*X**5
     2)**8))
C             IF NM=1,X1 AND X2 SHOULD BE STANDARDIZED TO N(0,1)
      X1=(XH-XM)/XS
      X2=(XL-XM)/XS
      Z1=X1/1.414213567
      X1 = Z1
      Z2=X2/1.414213567
      IF(Z1*Z2)1,2,3
C     WHEN Z1 AND Z2 HAVE DIFFERENT SIGN
1     Z2=ABS(Z2)
      CNORML=P(Z2)+P(Z1)
      GO TO 100
C     TO FIND WHETHER Z1 OR Z2 IS 0
2     CNORML=P(ABS(Z1+Z2))
      GO TO 100
C     WHEN Z1 AND Z2 HAVE THE SAME SIGN
3     Z2=ABS(Z2)
      Z1=ABS(Z1)
      CNORML=ABS(P(Z2)-P(Z1))
100   RETURN
      END
$BEND
```

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Ring-on-Ring Tests and Load Capacity of Cladding Glass

5. AUTHOR(S) E. Simiu, D. A. Reed, C. W. C. Yancey, J. W. Martin, E. M. Hendrickson, A. C. Gonzalez, M. Koike, J. A. Lechner, M. E. Batts

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11. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)*

Although ring-on-ring test results have been used in the past to obtain information on the strength of glass, no methodology has so far been developed in the literature explicitly relating such results to the load capacity of cladding glass. The main purpose of this report is to propose such a methodology. The proposed methodology makes use of recent advances in the modeling of the fracture mechanics behavior of glass and the calculation of stresses in plates exhibiting geometric nonlinearity. Evidence is presented which strongly suggests that the probability distribution of the load capacity of cladding glass panels whose failure is due to surface flaws can be estimated reliably on the basis of results of ring-on-ring tests used in conjunction with (a) numerical methods for the analysis of stresses in plates, and (b) information on the elastic and fracture mechanics behavior of glass currently available or that can be obtained routinely. Two interesting findings are noted. First, owing to the way in which results of ring-on-ring tests are utilized, the relatively large variabilities typical of fracture mechanics parameters, as well as the uncertainties with respect to the shapes of surface flaws, have a minor effect on the estimation of load capacities. Second, two-parameter Weibull distributions, previously used in the literature to model the strength of glass and the load capacity of cladding panels, are not consistent with experimental results. On the other hand, three-parameter Weibull distributions model the observed glass behavior credibly.

12. KEY WORDS *(Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)*

buildings; engineering mechanics; failure; fracture mechanics; glass; loads (forces); probability theory; ring-on-ring tests; strength.

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