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UNITED STATES DEPARTMENT OF COMMERCE

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Interrelations Between Cement and Concrete Properties, Part 4

Shrinkage of Neat Portland Cement Pastes and Concretes

R. L. Blaine, H. T. Arni, and D. N. Evans

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Contents

	Page
Section 9. Shrinkage of hardened portland cement pastes..... R. L. Blaine, H. T. Arni and D. N. Evans,	1
Section 10. Shrinkage and Expansion of Concrete..... R. L. Blaine and H. T. Arni	37

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Section 9. Shrinkage of Hardened Portland Cement Pastes

R. L. Blaine, H. T. Arni, and D. N. Evans

The relationship between cement properties and drying shrinkage of partially hydrated neat cement pastes of normal consistency made of a large number of portland cements of different types and composition were studied by fitting multivariable regression equations with the aid of a digital computer. The time of cracking of annular specimens restrained from shrinking by a steel core varied from 0.1 to more than 40 hours. The principal variables which appeared to have the greatest effect were the fineness and K_2O and C_4AF contents of the cements. The fineness and K_2O contents of the cements appeared to have the greatest effect on the nonrestrained shrinkage of prisms at the time of cracking of the annular specimens. The shrinkage of neat cement bars after 1, 6, 27 days, and 6 months in laboratory air differed greatly with different cements. The principal variables appeared to be the C_3A , SO_3 , and C_3A/SO_3 ratios. Other commonly determined variables were associated with each of the above test values. The trace elements, other than Na_2O and K_2O , were not generally associated with the shrinkage or cracking characteristics.

Key Words: Chemical composition of portland cements; cracking of portland cements; cracking resistance of cements; portland cement; shrinkage of portland cements; trace elements.

CONTENTS

	Page		Page
1. Introduction.....	1	5.8. Ratio of nonrestrained shrinkage of bars at time of cracking of the annular specimens to the nonrestrained shrinkage of the bars at 28 days, (SHBR/SH28) or (SB/S28)...	23
2. Materials.....	2	5.9. Results of duplicate determinations.....	27
3. Tests and nomenclature.....	2	5.10. Comparative test-values on other hydraulic cements.....	27
4. Statistical treatment.....	4	5.11. Comparison with cements from other sources.....	28
5. Results of tests.....	4	6. Discussion.....	28
5.1. Nonrestrained shrinkage after 24 hr drying.....	4	6.1. Nonrestrained shrinkage versus age.....	28
5.2. Nonrestrained shrinkage after 6 days drying.....	8	6.2. Trends of variables.....	28
5.3. Nonrestrained shrinkage after 28 days drying.....	9	6.3. Shrinkage characteristics of neat cement pastes versus compressive strength of 1:2.75 (cement to sand) mortars.....	31
5.4. Nonrestrained shrinkage after 6 months drying.....	11	7. Summary and conclusions.....	33
5.5. Time of cracking of annular specimens restrained from shrinkage.....	14	8. References.....	35
5.6. Nonrestrained shrinkage of prisms at time of cracking of annular specimens.....	19		
5.7. Ratio of nonrestrained shrinkage at time of cracking of annular specimens to the nonrestrained shrinkage at 7 days, (SB/S7)...	21		

1. Introduction

Many hundreds of articles have been published relative to the drying shrinkage of portland cements, mortars, and concretes. Many of these have been reviewed, discussed, and referenced in the various international symposia on the chemistry of cements,—in Stockholm, in 1938 [1],¹ in London in 1952 [2], and in Washington in 1960 [3]. These many articles attest, to some extent, to both the importance and complexity of the phenomenon.

These reports of investigations of the many variables associated with drying shrinkage in-

cluded information on the effects of fineness, tri-calcium aluminate, gypsum, and the alkalies in the cements. Little or no information was presented on the effects of other differences in the composition of the cements, or of possible effects of trace elements on the drying shrinkage. The shrinkage was generally recognized to result from the drying or changes in moisture content of the colloidal material or gel formed as the hydration product of the cement.

At the time the present study of the interrelation between cement and concrete properties was initiated, a test method had been proposed [4] for determining amount of shrink-

¹ Figures in brackets refer to literature references at the conclusion of this section, p. 35.

age of the nonrestrained bars and the time of cracking of annular specimens of partially hydrated neat cement pastes. This test method appeared, at that time, to offer a means for evaluating shrinkage and cracking characteristics of the different cements. Shrinkage tests were, however, also made on concrete specimens, as information was required on the de-

gree of correlation of tests made on the neat pastes and those made on the concretes.

This section of this series of articles deals primarily with the variables associated with the shrinkage characteristics of the neat cement pastes. The studies of shrinkage of concretes made from these cements will be presented in the following section.

2. Materials

The portland cements used in this study have previously been described [5, 6, 7]. Briefly, they consisted of 199 commercially manufactured portland cements of the different types, and obtained from different areas of the USA, with a few obtained from other countries. These were classified primarily on the basis of their chemical composition and physical tests, and

included 82 Type I, 68 Type II, 20 Type III, 3 Type IV, and 12 Type V cements. Also included were 14 air-entraining cements. The chemical analyses and other tests previously reported in previous sections of this series of articles [6, 7, 8, 9, 10, 11, 12] indicated that the cements had a fairly broad range of compositions and properties.

3. Tests and Nomenclature

Neat portland cement pastes were mixed using a mechanical mixer specified for mixing hydraulic cement mortars [13]. The percentage mixing water used with 1000 g of cement was the same as that previously required [6] for normal consistency of each of the cement samples. The mixing time was the same as that required for hand mixing. The first 30 s were at the No. 1 speed and next 60 s at the No. 2 speed [13]. A determination was then made of the Vicat penetration. This paste was then returned to the mixing bowl and the whole mass remixed for 15 s before filling the molds. Two types of molds were used,—one for $1 \times 1 \times 10$ -in (effective-gage-length) specimens for nonrestrained shrinkage² measurements, and the second for annular specimens for determining the time of cracking of specimens restrained from shrinkage by a chromium-plated steel disc 1 in thick and 3.25 in in diameter, see figure 9-1³.

The cement paste was tamped into the lightly greased molds, troweled flush with the tops of the molds, covered with plastic, and then cured at 73 ± 1 °F and 95 to 100 percent relative humidity for 24 hours. The specimens were then removed from the molds and exposed to air in the laboratory at 73 °F and 50 ± 5 -percent relative humidity. The length of each prism was measured when removed from the mold and a line of conducting paint was placed near the outer edge on each annular specimen.

² The term "nonrestrained" or "free" shrinkage as used in this article indicates that no external restraint or force was used. It does not imply that the intrinsic internal restraints dependent on the size and shape of the specimens, the amount of nonhydrated cement in the paste, and the rate of evaporation of water from the specimens do not exist.

³ The mold dimensions and test procedures were modified from those proposed in the RILEM specification to permit use of available prism molds. The cross sectional areas and the areas of the surfaces exposed to the laboratory air were approximately the same for the bars and annular specimens.

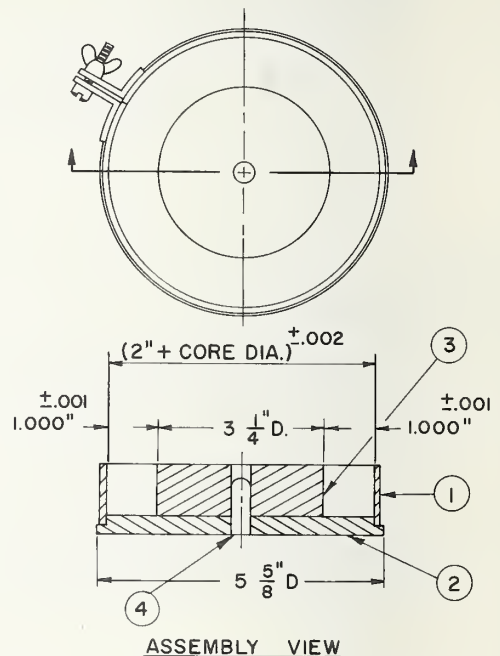


FIGURE 9-1. Mold for use in making annular specimens.

The removable brass ring (1) and base plate (2) held the chrome plated steel disc (3) in place with the center (4) during placing and tamping of the neat cement paste in the annular space.

Up to six pairs of prisms and annular specimens were supported in a rack (see figure 9-2) and arranged so that the length changes of the prisms could be measured continuously by means of the dial gages which were graduated to 0.0001 in. The electronic detecting device (see figure 9-3) was used to turn off an electric clock when an annular specimen with a steel core broke because of drying shrinkage.

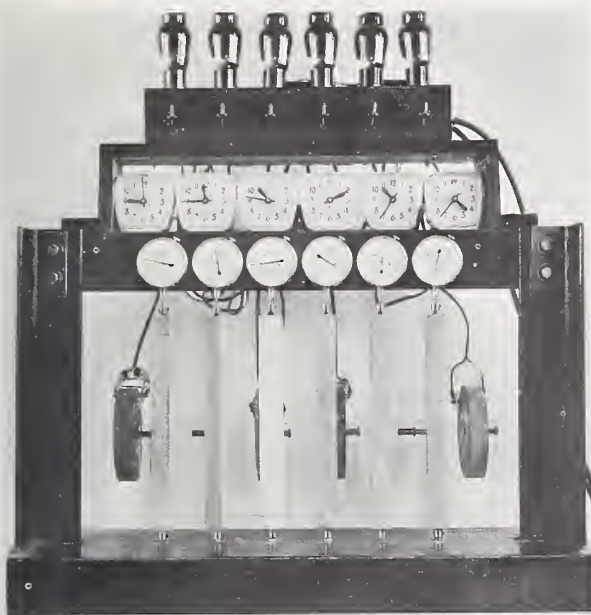


FIGURE 9-2. Rack for supporting bars and annular specimens for non-restrained and restrained shrinkage tests.

The length changes of the prisms or bars were measured at 15 and 30 min, 1, 3, 6, and 24 hr after removal from the molds or when it was noted that one of the clocks had stopped. An automatic camera was used to photograph the equipment with the dial gages and clocks every hour after the regular work day. The length measurements of the prisms at the time of breaking of the annular specimens were interpolated from the readings or from the pictorial records. At 24 or 48 hr after removal from moist-air storage (depending on the time of cracking of the annular specimen) the prisms were removed from the rack, measured with a standard comparator, and then placed on another rack in the laboratory at 73 °F and 50-percent relative humidity, and length measurements made at 7 and 28 days, 6 months, and 1 year.

The rack holding the specimens after they were first removed from the molds and moist storage was maintained in one position in the laboratory for the duration of tests on all the specimens. The rate of air-flow and humidity may have varied to some extent during the months required to make the tests. No attempt

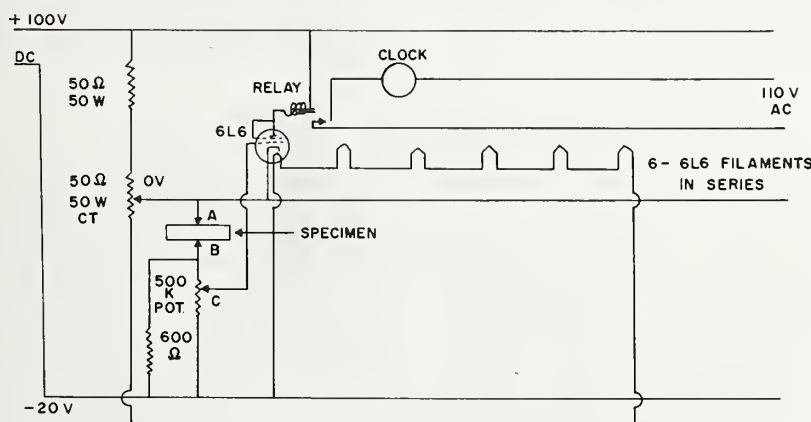


FIGURE 9-3. Diagram of electrical circuit used for stopping the electric clocks at time of cracking of the annular specimens.

With a specimen of low electrical resistance in place between terminals A & B, there is a drop of 20 V across the two parallel resistors of 600 Ω and 500K. The potentiometer (c) can then be adjusted to apply any desired grid bias from 0 to -20 Volts, thus controlling the plate current through the 6L6 tube. This is set at a point so as to just keep the relay closed. Upon breaking of the sample the resistance between points A & B increases. This causes point B to become more negative with respect to point A, which in turn increases the negative bias applied to the grid of the 6L6, resulting in a decrease of current through the tube, opening the relay, and stopping the clock.

was made to control the CO₂ content of the air surrounding the specimens. However, there is no reason to believe that it varied greatly from day to day.

With the large amount of data collected it was necessary to limit that which could be presented in this section. The notations or abbreviations used to designate the variables measured are as follows:

- SH24, percentage shrinkage of prisms at 24 hr;
- SH07, percentage shrinkage of prisms at 7 days;
- SH28, percentage shrinkage of prisms at 28 days;
- SH6M, percentage shrinkage of prisms at 6 months;

TOBR, time in hours for annular specimen to crack;

SHBR, percentage shrinkage of the prisms at the time of cracking of the annular specimen;

SB/S7, = SHBR/SH07 (see above);

SB/S28, = SHBR/SH28 (see above).

Other notations and abbreviations are the

same as those used in previous sections in this series of articles⁴.

Duplicate tests were made on different days using 20 portland cements obtained separately in order to evaluate the reproducibility of values for the nonrestrained shrinkage of the prisms and time of cracking of annular specimens.

4. Statistical Treatment

The statistical treatment of the shrinkage data was the same as that used in previous sections of this series of articles and it was described in detail in part 1, section 1 [5]. Briefly, these techniques included preliminary plots of the dependent versus major independent variables. After it had been determined which of the independent variables appeared significantly related to the dependent variable when used in a multivariable regression equation, the residuals of the equation were fitted by a least squares method to other single independent variables and the reduction in variance calculated. If any of the additional independent variables indicated a significant reduction in variance, they were tried in the multivariable equation and retained if the coef./s.d. ratio was greater than 1⁵.

5. Results of Tests

5.1. Nonrestrained Shrinkage After 24 Hr. Drying

The results of preliminary plots of the nonrestrained shrinkage of the neat cement prisms at various ages versus individual independent variables are presented in figure 9-4. The lines drawn through the 12 pairs of averages to indicate the trend, and the significance of the number of points in the opposite quadrants as well as the "quadrant sum" have previously been discussed in section 1, subsections 4.1 and 5 [5]. It may be noted that for the 24-hr shrinkage (SH24), SiO₂, Al₂O₃, Fe₂O₃, SO₃, K₂O, total alkali, C₃A, C₂S, C₄AF, fineness, and air, 1:4 mortars) all had "quadrant-sum" absolute val-

ues equal to or greater than 11, a value which may be considered significant at the 5-percent probability level. Multivariable regression equations are considered of the greatest significance in evaluating the association of the different independent variables to the dependent variable. Some of the limitations on interpretation of multivariable regression equations, as well as other statistical techniques used in this series of articles, have been discussed in section 1, subsections 4.2, 4.3, and 5. [5].

The frequency distribution of the 24-hr percentage-shrinkage values are presented in table 9-1. Each of the types of cement, as classified,⁶ had a considerable range of shrinkage values, and there was considerable overlapping of the values obtained with the different types.

Selected equations⁷ relating the percentage-shrinkage values of the AE + NAE cements at 24 hr to various independent variables are presented in table 9-2. Equation 1 presents the relationship for shrinkage to W₁, the percentage water required for normal consistency of

⁴Included among those notations etc. are the use of C₃A, C₃S, C₂S, and C₄AF for the calculated potential compound composition of the cements, viz, tricalcium aluminate, tricalcium silicate, dicalcium silicate and tetracalcium aluminoferrite. Also used are Loss for loss on ignition, Insol for insoluble residue, and APF for air-permeability fineness. Also used are AE + NAE to designate air-entraining plus non-air-entraining cements and NAE for the non-air-entraining cements. The compressive strength in psi of 1:2.75 (cement to sand) mortars cured for 1, 3, 7, 28 days, and 1 year are designated by ST01, ST03, ST07, ST28, and ST1Y respectively.

⁵The statistical terms used in this section are the same as those used in previous sections. For example, S.D. refers to the estimated standard deviation calculated from the residuals of a fitted equation, or the estimated standard deviation about an average. Also, as in previous sections, s.d. refers to the estimated standard deviation of the coefficient of an independent variable used in a fitted equation; coef./s.d., the ratio of the estimated coefficient (of an independent variable used in an equation) to its estimated standard deviation; and "F" = Fisher's ratio of variances. As indicated in previous sections a coef./s.d. ratio greater than one was considered to be of sufficient significance to warrant further investigation. D.F. is used to designate the number of degrees of freedom.

⁶As previously indicated [6], the cements were classified primarily on the basis of the chemical analysis and physical tests such as air-entrainment, fineness, etc.

⁷The equations selected were primarily those which indicated the association with dependent variables of commonly determined variables having coef./s.d. ratios greater than 1 when used in multivariable equations, and also indicated which of minor and trace elements might also have had an effect.

SHRINKAGE TIME	NOTE*	INDEPENDENT VARIABLES																	
		Si O ₂	Al ₂ O ₃	Fe ₂ O ₃	Ca O	Mg O	SO ₃	Na ₂ O	K ₂ O	TOTAL ALKALI	C ₃ A	C ₃ S	C ₂ S	C ₄ AF	$\frac{A}{F}$	$\frac{S}{A+F}$	AIR, P FINE	WAGN, FINE	AIR 1:4 MOR
24 HOURS	(1)	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{6}{6}$	$\frac{8}{4}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{12}{0}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{6}{6}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{10}{2}$	
	(2)																		
	(3)	NL	L?	NL	NO	L?	NL	L?	NL	NL	NL	L?	NL	NL	NL	NO	L?	L?	L?
	(4)	-15	+11	-12	-2	+6	+18	+3	+24	+11	+14	+9	-15	-16	+13	-6	+13	+11	+17
6 DAYS	(1)	$\frac{10}{2}$	$\frac{12}{0}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{12}{0}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{6}{6}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{10}{2}$
	(2)																		
	(3)	NL	L	L?	NL	L?	NL	L?	NL	NL	NL	NO	L?	L?	L?	L?	L?	L?	L?
	(4)	-14	+24	-8	-10	+8	+16	+10	+24	+16	+14	+1	-7	-10	+9	-5	+8	-1	+17
27 DAYS	(1)	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{12}{0}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{6}{6}$	$\frac{12}{0}$	$\frac{9}{1}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{6}{6}$	$\frac{8}{4}$	$\frac{10}{2}$
	(2)																		
	(3)	NL	L?	L?	L?	L?	L?	L?	NL	NL	NL	NO	NL	L?	L?	L?	NO	NO	L?
	(4)	-15	+12	-7	-8	+6	+15	+9	+24	+17	+14	+4	-24	-13	+18	-10	+3	0	+13
6 MONTHS	(1)	$\frac{12}{0}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{6}{6}$	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{12}{0}$	$\frac{8}{4}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{6}{6}$	$\frac{8}{4}$	$\frac{10}{2}$
	(2)																		
	(3)	NO	L?	L?	NO	L?	L?	L?	L	L	L?	NO	L?	L?	L?	L?	NO	NO	L?
	(4)	-24	+7	-10	-7	+12	+12	+7	+20	+17	+9	-3	-24	-6	+14	-9	+2	-2	+12

FIGURE 9-4. Results of plotting on the "Y" axis the percentage shrinkage in laboratory air at 24 hours, 6 days, 27 days, and 6 months of neat portland cement pastes of normal consistency cured for 24 hours in moist air versus various independent variables on the "X" axis.

* Note (1) Ratio of number of plotted points in pairs of diametrically opposite quadrants.

Note (2) General trend of lines drawn through plotted points.

Note (3) Apparent nature of relationship. L = linear; NL = nonlinear; NO = no apparent relationship; and L? = nature of relationship not determinable.

Note (4) Quadrant sum in corner test (see reference 5).

TABLE 9-1. Frequency distribution of cements with respect to percentage shrinkage of neat cement pastes of normal consistency after 24 hr drying in laboratory air following 24 hr moist-air curing (SH24)

Type cement	Percentage shrinkage														Total
	0.03 to 0.04	0.04 to 0.05	0.05 to 0.06	0.06 to 0.07	0.07 to 0.08	0.08 to 0.09	0.09 to 0.10	0.10 to 0.11	0.11 to 0.12	0.12 to 0.13	0.13 to 0.14	0.14 to 0.15	0.15 and over		
	Number of cements														
I.....	-----	5	22	22	8	10	1	2	1	3	1	1	1	77	
IA.....	-----	-----	1	2	1	3	1	-----	-----	-----	-----	-----	-----	8	
II.....	-----	13	17	19	9	-----	-----	1	-----	1	-----	-----	-----	60	
IIA.....	-----	-----	-----	-----	2	-----	-----	-----	-----	-----	-----	-----	-----	2	
III.....	-----	-----	1	7	4	1	4	1	-----	-----	1	-----	-----	19	
IIIA.....	-----	-----	-----	-----	-----	1	-----	-----	1	-----	-----	-----	-----	2	
IV, V.....	1	3	6	2	2	-----	-----	-----	-----	-----	-----	-----	-----	14	
Total.....	1	21	47	52	26	15	6	4	2	4	2	1	1	182	

the neat pastes. The estimated S.D. value obtained in eq 1 was lower than the estimated S.D. of the cements about their average (note 1 of table 9-2). By reference to table 9-34⁸ it may be noted that the relationship was significant at the 1-percent probability level.

Equation 2 of table 9-2 presents the relationships for some commonly determined variables plus the C₃A/SO₃ ratio. The use of C₃A, SO₃, and C₃A/SO₃ as independent variables was found in preliminary trial equations to result

in lower S.D. values than when the C₃A/SO₃ ratio was not used. The variables APF, Ba, P, Zn, and Cr had coef./s.d. ratios greater than 1 when included with the variables in eq 2 (eq 3), and the reduction in variance was significant at the 1-percent probability level (table 9-34). Of these added variables, however, only APF and Cr had coef./s.d. ratios greater than 2.

Equations 3A and 3B, calculated with the "odds" and "evens" in the array of cements, had coef./s.d. ratios less than one for the "odds" or "evens" or both, for C₃A, APF, Ba, P, and Zn. As indicated in previous sections, this may occur when (1) the coef./s.d. ratio is not significant in the equation for all the values, or

⁸ A tabulation of the reduction in variance or "F" values for all equations compared is presented in table 9-34, page 27. Presented also are the critical values for "F" which, if equalled or exceeded, indicate significance at the 1-percent and 5-percent levels for the number of degrees of freedom (D.F.) involved in each comparison.

TABLE 9-2. *Coefficients for equations relating the percentage shrinkage at 24 hr (after the NAE cements to various in*

Equation	Note	Const.	C ₃ A	C ₄ AF	Al ₂ O ₃	Fe ₂ O ₃
1-----	(1)	SH24 = -0.01241 s.d. = (0.01734)	-----	-----	-----	-----
2-----	(1)	SH24 = -0.00317 s.d. = (0.01558)	-0.00514 (0.00173)	-0.001457 (0.000465)	-----	-----
3-----	(1)	SH24 = -0.01903 s.d. = (0.01579)	-0.00464 (0.00171)	-0.001307 (0.000455)	-----	-----
3A-----	(2)	SH24 (odd) = -0.01123 s.d. = (0.02211)	-0.00608 (0.00228)	-0.001979 (0.000735)	-----	-----
3B-----	(3)	SH24 (even) = -0.02046 s.d. = (0.02466)	*-0.00273 (0.00290)	-0.000837 (0.000594)	-----	-----
4-----	(1)	SH24 = -0.02801 s.d. = (0.01859)	-----	-----	-0.01094 (0.00375)	-0.00431 (0.00114)
5-----	(1)	SH24 = -0.03981 s.d. = (0.01855)	-----	-----	-0.01217 (0.00371)	-0.00450 (0.00112)
5A-----	(2)	SH24 (odd) = -0.02792 s.d. = (0.02441)	-----	-----	-0.01425 (0.00465)	-0.00559 (0.00183)
5B-----	(3)	SH24 (even) = -0.05720 s.d. = (0.03540)	-----	-----	-0.01215 (0.00761)	-0.00403 (0.00147)

¹ 163 cements; Avg = 0.0643; S.D. = 0.01427.² 82 cements.³ 81 cements.

* Coef./s.d. ratio less than 1.

TABLE 9-3. *Coefficients for equations relating the percentage shrinkage at 24 hr (after the cements, to various inde*

Equation	Note	Const.	C ₃ A	C ₄ AF	Al ₂ O ₃	Fe ₂ O ₃	Na ₂ O
1-----	(1)	SH24 = -0.00196 s.d. = (0.01738)	-----	-----	-----	-----	-----
2-----	(1)	SH24 = -0.00182 s.d. = (0.01557)	-0.005779 (0.001744)	-0.001531 (0.000466)	-----	-----	+0.00733 (0.00546)
3-----	(1)	SH24 = -0.01970 s.d. = (0.01581)	-0.005532 (0.001715)	-0.001367 (0.000451)	-----	-----	+0.01004 (0.00531)
3A-----	(2)	SH24 (odd) = -0.01592 s.d. = (0.02145)	-0.005957 (0.002247)	-0.001455 (0.000643)	-----	-----	*+0.00304 (0.00945)
3B-----	(3)	SH24 (even) = -0.01472 s.d. = (0.02538)	-0.004410 (0.002904)	-0.001007 (0.000706)	-----	-----	+0.00982 (0.00633)
4-----	(1)	SH24 = -0.01726 s.d. = (0.01874)	-----	-----	-0.01292 (0.00370)	-0.00455 (0.00116)	+0.00593 (0.00534)
5-----	(1)	SH24 = -0.03589 s.d. = (0.01856)	-----	-----	-0.01328 (0.00367)	-0.00461 (0.00111)	+0.00874 (0.00516)
5A-----	(2)	SH24 (odd) = -0.02438 s.d. = (0.02426)	-----	-----	-0.01290 (0.00475)	-0.00431 (0.00161)	*+0.00112 (0.00938)
5B-----	(3)	SH24 (even) = -0.06950 s.d. = (0.03376)	-----	-----	-0.01852 (0.00683)	-0.00429 (0.00167)	+0.00907 (0.00597)

¹ 151 cements; Avg = 0.06328; S.D. = 0.01366.² 76 cements.³ 75 cements.

* Coef./s.d. ratio less than 1.

(2) when a preponderance of the cements with very high or very low values for an independent variable happens to fall within one of the groups. Zn and P are particularly vulnerable to the second of the above contingencies because of the comparatively small number of cements for which nonzero values for these two elements were reported.

Equations 4 and 5 of table 9-2 were calculated using the oxides instead of the calculated potential compounds. The use of the trace elements in eq 5 in addition to the commonly determined variables in eq 4 resulted in a reduction in variance significant at the 5-percent probability level (see table 9-34). Again, only APF and Cr. had coef./s.d. ratios greater than 2. In eqs 5A and 5B which were calculated for

the "odds" and "evens", APF, Ba, Zn, and P had coef./s.d. ratios less than 1 in one or the other of the equations.

A comparable series of equations is presented in table 9-3 for NAE cements. The coefficients of the independent variables are in reasonable agreement with those in table 9-2. Rb had a coef./s.d. greater than 1 in eq 5, and in the equations for the "odds" and "evens", (eqs 3A, 3B, 5A, and 5B) Na₂O, Ba, and P had coef./s.d. ratios less than 1 in one or the other of the smaller groups. The sign of the coefficient for Zn was positive in eqs 3A and 5A and negative in eqs 3, 3B, 5 and 5B. In none of these instances were the coefficients highly significant.

Using the independent variables and their coefficients from eq 3 of table 9-3, and the

initial 24 hr moist-air storage) of neat-cement bars of normal consistency made of AE + dependent variables (SH24)

Na ₂ O	K ₂ O	SO ₃	C ₃ A/SO ₃	Al ₂ O ₃ /SO ₃	APF	Ba	P	Zn	Cr	W ₁	S.D.
										+0.00310 (0.00070)	0.01353
+0.00978 (0.00546)	+0.01729 (0.00428)	+0.03889 (0.00760)	+0.00925 (0.00310)								0.01087
+0.01220 (0.00533)	+0.02006 (0.00448)	+0.03665 (0.00799)	+0.00876 (0.00304)		+0.0000047 (0.0000020)	+0.04551 (0.03263)	+0.01079 (0.00815)	-0.0346 (0.0306)	+0.4462 (0.2059)		0.01049
+0.01130 (0.00908)	+0.03013 (0.00696)	+0.03574 (0.01083)	+0.01037 (0.00407)		*+0.0000030 (0.0000033)	+0.05713 (0.04112)	+0.02026 (0.01138)	*-0.0027 (0.0451)	+0.5257 (0.3129)		0.01117
+0.01019 (0.00645)	+0.01141 (0.00599)	+0.03272 (0.01348)	+0.00622 (0.00515)		+0.0000047 (0.0000026)	*-0.06883 (0.09156)	*-0.00861 (0.01204)	*-0.0435 (0.0445)	+0.4258 (0.2787)		0.00959
+0.00950 (0.00524)	+0.01917 (0.00440)	+0.04310 (0.01014)		+0.02078 (0.00656)	+0.0000035 (0.0000020)						0.01064
+0.01165 (0.00519)	+0.01963 (0.00442)	+0.04603 (0.00993)		+0.02288 (0.00643)	+0.0000042 (0.0000020)	+0.05175 (0.03228)	+0.01122 (0.00806)	-0.0390 (0.0302)	+0.4531 (0.2033)		0.01036
+0.01084 (0.00880)	+0.02922 (0.00686)	+0.04520 (0.01243)		+0.02389 (0.00801)	*+0.0000030 (0.0000033)	+0.06262 (0.04071)	+0.02134 (0.01125)	*-0.0101 (0.0444)	+0.5561 (0.3079)		0.01101
+0.00930 (0.00630)	+0.01143 (0.00590)	+0.05558 (0.02083)		+0.02517 (0.01340)	+0.0000036 (0.0000027)	*-0.07342 (0.08952)	*-0.00845 (0.01185)	-0.0552 (0.0447)	+0.4378 (0.2745)		0.00945

initial 24 hr moist-air storage) of neat-cement bars of normal consistency made of NAE dependent variables (SH24)

K ₂ O	SO ₃	C ₃ A/SO ₃	Al ₂ O ₃ /SO ₃	APF	Ba	P	Zn	Cr	Rb	W ₁	S.D.
										+0.00264 (0.00070)	0.01310
+0.01730 (0.00436)	+0.03907 (0.00764)	+0.01020 (0.00310)									0.01064
+0.02004 (0.00452)	+0.03506 (0.00791)	+0.01012 (0.00302)		+0.0000046 (0.0000020)	+0.05233 (0.03194)	+0.01302 (0.00803)	-0.03234 (0.02979)	+0.4422 (0.2074)			0.01020
+0.02536 (0.00683)	+0.03388 (0.01024)	+0.01031 (0.00396)		+0.0000045 (0.0000032)	+0.06645 (0.03715)	+0.02752 (0.01119)	+0.06508 (0.06129)	+0.5193 (0.2870)			0.01058
+0.01482 (0.00605)	+0.03301 (0.01394)	+0.00381 (0.00506)		+0.0000041 (0.0000028)	*-0.08642 (0.08693)	*-0.00841 (0.01192)	-0.05719 (0.03401)	+0.3252 (0.3087)			0.00960
+0.01655 (0.00431)	+0.04762 (0.00955)		+0.02272 (0.00649)								0.01062
+0.01604 (0.00486)	+0.04577 (0.00979)		+0.02400 (0.00632)	+0.0000044 (0.0000020)	+0.06093 (0.03154)	+0.01148 (0.00801)	-0.03266 (0.02947)	+0.3713 (0.2099)	+0.8259 (0.5065)		0.01004
+0.02377 (0.00751)	+0.04012 (0.01232)		+0.02158 (0.00807)	+0.0000045 (0.0000033)	+0.07151 (0.03736)	+0.02648 (0.01171)	+0.06876 (0.06146)	+0.4604 (0.3082)	*+0.4243 (0.8129)		0.01060
+0.00966 (0.00619)	+0.06589 (0.01928)		+0.03545 (0.01195)	+0.0000030 (0.0000027)	*-0.07405 (0.08275)	*-0.00734 (0.01136)	-0.06432 (0.03289)	+0.3388 (0.2960)	+0.9599 (0.6391)		0.00915

ranges of these variables, values were calculated for their contributions and ranges of their contributions to the 24-hr percentage-shrinkage values. These calculated values, as presented in table 9-4, are estimates based on a single equation, and it may be noted that somewhat different values would be obtained by use of other equations⁹.

As in previous sections, the calculated-contribution values must also take into consideration the constant in the equation.

An increase in C₃A and C₄AF appears to be associated with lower shrinkage values but the use also of the SO₃ and the C₃A/SO₃ ratio in the equation may have affected the coefficient

TABLE 9-4. Calculated contributions of independent variables to the percentage shrinkage at 24 hr. (after the initial 24 hr. in moist-air storage) of neat-cement bars of normal consistency (SH24)

Independent variable	Ranges of variables, percent	Coefficients from eq 3 table 9-3	Calculated contributions to SH24	Calculated ranges of contributions to SH24
			Const. = -0.020	
C ₃ A-----	0 to 15	-0.00553	0 to -0.083	0.083
C ₄ AF-----	4 to 17	-0.00137	-0.005 to -0.023	0.018
Na ₂ O**-----	0 to 0.75	+0.0100	0 to +0.008	0.008
K ₂ O-----	0 to 1.1	+0.0200	0 to +0.022	0.022
SO ₃ -----	1.2 to 3.6	+0.0351	+0.042 to +0.126	0.084
C ₃ A/SO ₃ -----	0.4 to 10.1	+0.0101	+0.004 to +0.102	0.098
APF-----	*2500 to 5500	+0.0000046	+0.012 to +0.025	0.013
Ba**-----	0 to 0.2	+0.0523	0 to +0.010	0.010
P**-----	0 to 0.5	+0.0130	0 to +0.006	0.006
Zn**-----	0 to 0.2	-0.0323	0 to -0.006	0.006
Cr-----	0 to 0.02	+0.442	0 to +0.009	0.009

*cm²/g.

**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

for the C_3A . Differences in the C_3A and SO_3 appear to be important contributors to differences in shrinkage.

5.2. Nonrestrained Shrinkage After 6 Days Drying

It may be noted in figure 9-4 that SiO_2 , Al_2O_3 , SO_3 , K_2O , total alkali, C_3A , and (air, 1:4 mortar) all had absolute "quadrant-sum" values greater than 11 when the shrinkage at 7 days (the first 24 hr in moist storage) was plotted versus the individual independent variables.

The frequency distributions of the 7-day percentage-shrinkage values of the neat pastes of normal consistency of the different types of

cement are presented in table 9-5. There was a considerable range of values, especially for the Type I cements, and an overlapping of the values for the cements within the various types as classified.

Equations selected from those computed to determine the various independent variables associated with the shrinkage at 7 days (the first 24 hr in moist-air storage) of neat cement bars of normal consistency made of AE + NAE cements, are presented in table 9-6. Equation 1 relates the shrinkage to the percentage water used for normal consistency of the neat pastes. The coef./s.d. ratio for the W_1 was less than 2, and the reduction in variance was not significant at the 5-percent probability level (table 9-34).

TABLE 9-5. Frequency distribution of cements with respect to percentage shrinkage of neat cements of normal consistency after 6 days drying in laboratory air after 24 hr moist-air storage (SHO7)

Type cement	Percentage shrinkage												Total
	0.080 to 0.100	0.100 to 0.120	0.120 to 0.140	0.140 to 0.160	0.160 to 0.180	0.180 to 0.200	0.200 to 0.220	0.220 to 0.240	0.240 to 0.260	0.260 to 0.280	0.280 to 0.300	0.300 and over	
	Number of cements												
I.....	2	7	20	22	11	6	1	2	1		2	3	77
II.....	1	15	15	14	9	4	1	1					60
IIA.....				1	1								2
III.....		1	2	9	3	2	1	1					19
IIIA.....					1	1							2
IV, V.....	1	3	5	4	1								14
Total.....	4	26	43	52	29	13	4	5	1		2	3	182

TABLE 9-6. Coefficients for equations relating the percentage shrinkage at 7 days (the first cements, to various inde

Equation	Note	Const.	C_3A	C_4AF	Al_2O_3	Fe_2O_3	CaO
1.....	(1)	SHO7 s.d. = +0.0803 = (0.0356)					
2.....	(1)	SHO7 s.d. = +0.1147 = (0.0176)	*-0.00010 (0.00086)	-0.001344 (0.001108)			
3.....	(1)	SHO7 s.d. = -0.0118 = (0.0353)	-0.01389 (0.00383)	-0.001382 (0.001038)			
4.....	(1)	SHO7 s.d. = -0.0225 = (0.0344)	-0.01423 (0.00371)	-0.001525 (0.001007)			
4A.....	(2)	SHO7 (odd) s.d. = -0.0143 = (0.0421)	-0.01266 (0.00465)	-0.002051 (0.001409)			
4B.....	(3)	SHO7 (even) s.d. = -0.0405 = (0.0704)	-0.01702 (0.00745)	*-0.001282 (0.001626)			
5.....	(1)	SHO7 s.d. = +0.4203 = (0.1291)			*-0.00111 (0.00273)	-0.006786 (0.003052)	-0.004490 (0.001878)
6.....	(1)	SHO7 s.d. = +0.1362 = (0.1317)			-0.03343 (0.00848)	-0.005471 (0.002918)	-0.002810 (0.001966)
7.....	(1)	SHO7 s.d. = +0.1890 = (0.1268)			-0.03553 (0.00814)	-0.006824 (0.002822)	-0.003900 (0.001904)
7A.....	(2)	SHO7 (odd) s.d. = +0.3419 = (0.2024)			-0.03072 (0.01053)	-0.010100 (0.004255)	-0.005834 (0.003107)
7B.....	(3)	SHO7 (even) s.d. = +0.0187 = (0.1772)			-0.06140 (0.01645)	-0.005482 (0.004054)	-0.003327 (0.0052.002)

¹ 165 cements; Avg=0.1481; S.D.=0.02813.

² 83 cements.

³ 82 cements.

* Coef./s.d. ratio less than 1.

Only C₃A, C₄AF, and SO₃ were used as independent variables in eq 2. The coef./s.d. ratio for C₃A was less than 1. It may be noted that in eq 3, where C₃A/SO₃ and four other additional independent variables were used, the coefficient for C₃A was highly significant, and that the coefficient for SO₃ more than doubled. The reduction in the S.D. value was highly significant (see table 9-34). Coef./s.d. ratios for C₄AF and APF, however, were less than two. By use of the trace elements, Ba, P, and Rb in eqs 4 and 7, a further significant reduction in the S.D. was attained, although P had a coef./s.d. ratio less than 2. In the equations for the "odds" and "evens" (eqs 4A and 4B), P and Rb had coef./s.d. values less than one in one or the other, as may also be noted in eqs 7A and 7B, where the oxides were used instead of the calculated compounds. Equations 5, 6, and 7, calculated using the oxides, also indicate that the coefficients for Al₂O₃ and SO₃ are affected by inclusion of the Al₂O₃/SO₃ ratio.

A comparable series of equations for the NAE cements is presented in table 9-7. The coefficients obtained were fairly similar to those of the previous table except that the coefficient for (air, 1:4 mortar) was of questionable significance when the AE cements were not included. The coef./s.d. ratio for (air, 1:4 mortar) was less than 1 in eqs 4A and 4B in table 9-7. It was also less than one when this variable was included with those in eq 7. Of the minor constituents, only P had coef./s.d. ratios of less than 1 in the equations for the "odds" or

"evens" and Rb had a coef./s.d. ratio greater than 1 in all instances.

Using the coefficients of eq 4 of table 9-7 and the ranges of the independent variables, values were calculated for their contributions and ranges of contributions to the percentage shrinkage at 7 days. These calculated values are presented in table 9-8. Increases in C₃A and C₄AF were associated with lower shrinkage values when SO₃ and C₃A/SO₃ were included as independent variables. All the other independent variables with coef./s.d. ratios greater than 1 had positive coefficients. Variations of C₃A, SO₃ and C₃A/SO₃ appeared to have important effects. Although the coefficient for (air, 1:4 mortar) was of doubtful significance when only the NAE cements were used in the calculations, it was probably significant when the AE cements were included as in table 9-6.

5.3. Nonrestrained Shrinkage After 28 Days Drying

The preliminary plots of the 28-day-shrinkage values (the first 24 hr in moist air) of the neat cement pastes of normal consistency versus various individual independent variables are indicated in figure 9-4. It may be noted that SiO₂, Al₂O₃, SO₃, K₂O, total alkali, C₃A, C₂S, C₄AF, A/F, and (air, 1:4 mortar) had "quadrant-sum" absolute values greater than 11.

The frequency distribution of the 28-day percentage-shrinkage values of the prisms

*24 hr in moist-air-storage) of neat-cement bars of normal consistency made of AE+NAE
pendent variables (SHO7)*

Na ₂ O	K ₂ O	SO ₃	C ₃ A/SO ₃	Al ₂ O ₃ /SO ₃	Air, 1:4 mortar	APF	Ba	P	Rb	W ₁	S.D.
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	+0.00274 (0.00144)	0.02790
-----	-----	+0.02495 (0.00617)	-----	-----	-----	-----	-----	-----	-----	-----	0.02659
+0.02825 (0.01249)	+0.04011 (0.01023)	+0.06799 (0.01787)	+0.02410 (0.00689)	-----	+0.00143 (0.00055)	+0.0000050 (0.0000045)	-----	-----	-----	-----	0.02404
+0.03017 (0.01215)	+0.03165 (0.01073)	+0.06685 (0.01727)	+0.02484 (0.00664)	-----	+0.00171 (0.00054)	+0.0000072 (0.0000044)	+0.1675 (0.0698)	+0.02401 (0.01662)	+2.632 (1.115)	-----	0.02317
+0.03384 (0.02147)	+0.03349 (0.01578)	+0.06063 (0.02147)	+0.02209 (0.00818)	-----	+0.00139 (0.00082)	+0.0000099 (0.0000069)	+0.1601 (0.0808)	+0.04794 (0.02185)	*+1.292 (1.639)	-----	0.0236
+0.02890 (0.01590)	+0.02939 (0.01567)	+0.08050 (0.03558)	+0.02988 (0.01366)	-----	+0.00187 (0.00076)	*+0.0000046 (0.0000064)	+0.1715 (0.1619)	*-0.01194 (0.02917)	+3.336 (1.615)	-----	0.02385
-----	-----	+0.02176 (0.00623)	-----	-----	-----	-----	-----	-----	-----	-----	0.02622
+0.02225 (0.01229)	+0.03300 (0.01080)	+0.09222 (0.02272)	-----	+0.05723 (0.01499)	+0.00137 (0.00055)	+0.0000046 (0.0000045)	-----	-----	-----	-----	0.02392
+0.02409 (0.01185)	+0.02354 (0.01113)	+0.09462 (0.02173)	-----	+0.06167 (0.01439)	+0.00161 (0.00053)	+0.0000071 (0.0000044)	+0.2011 (0.0696)	+0.02851 (0.01649)	+2.432 (0.099)	-----	0.02285
+0.03060 (0.02109)	+0.02576 (0.01622)	+0.07894 (0.02678)	-----	+0.05163 (0.01798)	+0.00105 (0.00084)	+0.0000116 (0.0000068)	+0.2051 (0.0829)	+0.05712 (0.02205)	*+0.989 (1.685)	-----	0.02347
+0.02009 (0.01513)	+0.01828 (0.01612)	+0.17189 (0.04657)	-----	+0.11148 (0.03020)	+0.00186 (0.00072)	*+0.0000014 (0.0000063)	+0.2016 (0.1527)	*+0.01327 (0.02772)	+3.223 (1.531)	-----	0.02266

TABLE 9-7. Coefficients for equations relating the percentage shrinkage at 7 days (the cements, to various

Equation	Note	Const.	C ₃ A	C ₄ AF	Al ₂ O ₃	Fe ₂ O ₃	CaO
1-----	(1)	SHO7 s.d. = +0.0873 = (0.0361)					
2-----	(1)	SHO7 s.d. = +0.1223 = (0.0180)	*-0.00047 (0.00087)	-0.001504 (0.001121)			
3-----	(1)	SHO7 s.d. = -0.0125 = (0.0350)	-0.01666 (0.00383)	-0.001841 (0.001033)			
4-----	(1)	SHO7 s.d. = -0.0316 = (0.0346)	-0.01649 (0.00372)	-0.001917 (0.001003)			
4A-----	(2)	SHO7 (odd) s.d. = -0.0320 = (0.0467)	-0.01754 (0.00520)	-0.001925 (0.001748)			
4B-----	(3)	SHO7 (even) s.d. = +0.0358 = (0.0590)	-0.00715 (0.00625)	-0.002268 (0.001225)			
5-----	(1)	SHO7 s.d. = +0.4202 = (0.1332)			*-0.00193 (0.00230)	-0.006827 (0.003142)	-0.004370 (0.001935)
6-----	(1)	SHO7 s.d. = +0.1503 = (0.1342)			-0.03578 (0.00849)	-0.006085 (0.002957)	-0.003113 (0.001988)
7-----	(1)	SHO7 s.d. = +0.2262 = (0.1271)			-0.04005 (0.00801)	-0.008091 (0.002811)	-0.004495 (0.001899)
7A-----	(2)	SHO7 (odd) s.d. = +0.1438 = (0.2030)			-0.04181 (0.01065)	-0.007077 (0.004921)	-0.003232 (0.003054)
7B-----	(3)	SHO7 (even) s.d. = +0.3954 = (0.1603)			-0.02224 (0.01544)	-0.010587 (0.003282)	-0.006087 (0.002271)

¹ 153 cements; Avg=0.1463; S.D.=0.02748. ² 77 cements. ³ 76 cements. * Coef./s.d. ratio less than 1.

TABLE 9-8. Calculated contributions of independent variables to the percentage shrinkage at 7 days (the first 24 hr. in moist-air-storage) of neat-cement bars of normal consistency. (SHO7)

Independent variable	Ranges of variables, percent	Coefficients from eq 4 table 9-7	Calculated contributions to SHO7	Calculated ranges of contributions to SHO7
C ₃ A-----	0 to 15	-0.0165	Const. = -0.032 0 to -0.248	0.248
C ₄ AF**----	4 to 17	-0.00192	-0.008 to -0.033	0.025
Na ₂ O-----	0 to 0.75	+0.0313	0 to +0.023	0.023
K ₂ O-----	0 to 1.1	+0.0383	0 to +0.042	0.042
SO ₃ -----	1.2 to 3.6	+0.0722	+0.086 to +0.260	0.174
C ₃ A/SO ₃ ----	0.4 to 10.1	+0.0281	+0.011 to +0.284	0.273
Air, 1:4 mortar**	1 to 21	+0.00144	+0.001 to +0.030	0.029
APF-----	*2500 to 5500	+0.0000088	+0.012 to +0.048	0.026
Ba-----	0 to 0.2	+0.183	0 to +0.037	0.037
P**-----	0 to 0.5	+0.0262	0 to +0.013	0.013
Rb-----	0 to 0.01	+2.667	0 to +0.027	0.027

*cm²/g.

**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

made of the different cements is presented in table 9-9. It may be noted that a considerable range of values was obtained with each of the different types of cement and especially with those classified as Type I cements. The highest percentage-shrinkage values were obtained with the Type I or IA cements but some of the Type I cements were also in the group with the lowest percentage shrinkage.

A selected group of equations is presented in table 9-10 to indicate the variables found to be associated with the 28-day percentage-shrinkage values of the AE + NAE cements. Equation 1 indicates no apparent relationship between the percentage mixing water for normal consistency and the percentage shrinkage at 28

days. The use of variables in eq 2 of this table, resulted in a significant reduction in the S.D. values (see table 9-34) although the C₃A and C₄AF had coef./s.d. ratios less than 1. With the use of the 5 additional variables in eq 3, the variables C₃A, and C₃A/SO₃ were highly significant, and C₄AF had a coef./s.d. ratio greater than 1. The additional use of the trace elements, Ba, P, Zn, and Rb in eqs 4 and 6 resulted in a reduction in variance significant at the 1-percent probability level. It may be noted that in equations calculated for the "odds" and "evens" in the array of cements (eqs 4A, 4B, 6A, and 6B) there were instances where C₄AF, SiO₂, Fe₂O₃, MgO, APF, (air, 1:4 mortar), P, Zn, and Rb had coef./s.d. ratios less than 1 in one or more of the equations for the smaller groups.

A comparable series of equations for the NAE cements is presented in table 9-11. In these equations the (air, 1:4 mortar) had coef./s.d. ratios less than 1 in trial equations and was not included among the independent variables. The coefficients for the other independent variables in table 9-11 appear to be in reasonable agreement with corresponding coefficients in table 9-10 where the AE cements were included.

Contributions and ranges of contributions to the shrinkage at 28 days of the neat cement bars of normal consistency are presented in table 9-12. The coefficients in eq 3 of table 9-11 were used as a basis for these calculations. The three variables C₃A, SO₃, and C₃A/SO₃ when used together in a multivariable equation had the highest values for the calculated ranges of

first 24 hr in moist-air storage) of neat-cement bars of normal consistency made of NAE independent variables

Na ₂ O	K ₂ O	SO ₃	C ₃ A/SO ₃	Al ₂ O ₃ /SO ₃	Air, 1:4 mortar	APF	Ba	P	Rb	W _i	S.D.
										+0.00239 (0.00146)	0.02733
		+0.02266 (0.00630)									0.02633
+0.02475 (0.01322)	+0.04064 (0.01013)	+0.08038 (0.01685)	+0.02814 (0.00686)		+0.002116 (0.001253)						0.02367
+0.03133 (0.01274)	+0.03833 (0.01091)	+0.07218 (0.01721)	+0.02807 (0.00660)		+0.001444 (0.001217)	+0.0000088 (0.0000044)	+0.1828 (0.0684)	+0.02623 (0.01653)	+2.667 (1.104)		0.02255
+0.02664 (0.02410)	+0.04722 (0.01767)	+0.06717 (0.02410)	+0.02976 (0.00890)		*+0.001141 (0.002186)	+0.0000124 (0.0000079)	+0.1053 (0.0867)	+0.04855 (0.02428)	+2.615 (1.786)		0.02483
+0.03270 (0.01411)	+0.02753 (0.01347)	+0.04171 (0.02917)	+0.01167 (0.01148)		*+0.000685 (0.001399)	+0.0000069 (0.0000054)	+0.4800 (0.1437)	*-0.01348 (0.02373)	+1.795 (1.395)		0.01973
		+0.01943 (0.00638)									0.02598
+0.01907 (0.01309)	+0.03609 (0.01122)	+0.09353 (0.02275)		+0.05995 (0.01490)	+0.001950 (0.001249)	+0.0000058 (0.0000046)					0.02356
+0.02928 (0.01149)	+0.03082 (0.01104)	+0.10146 (0.02135)		+0.06712 (0.01411)		+0.0000090 (0.0000044)	+0.2233 (0.0682)	+0.03461 (0.01610)	+2.511 (1.087)		0.02224
+0.02383 (0.02184)	+0.04504 (0.01726)	+0.09417 (0.02752)		+0.06910 (0.01805)		+0.0000125 (0.0000078)	+0.1391 (0.0872)	+0.05939 (0.02295)	+2.363 (1.752)		0.02444
+0.02927 (0.01224)	+0.01372 (0.01345)	+0.06602 (0.04359)		+0.03899 (0.02853)		+0.0000080 (0.0000053)	+0.5488 (0.1373)	*-0.01109 (0.02259)	+1.644 (1.326)		0.01877

TABLE 9-9. Frequency distribution of cements with respect to percentage shrinkage of neat cements of normal consistency after 27 days drying in laboratory air following 24 hr moist-air curing (SH28)

Type cement	Percentage shrinkage													Total
	0.14 to 0.16	0.16 to 0.18	0.18 to 0.20	0.20 to 0.22	0.22 to 0.24	0.24 to 0.26	0.26 to 0.28	0.28 to 0.30	0.30 to 0.32	0.32 to 0.34	0.34 to 0.36	0.36 to 0.38	0.38 and over	
	Number of cements													
I.....	2	5	11	14	17	6	10	4	-----	1	2	2	3	77
IA.....	-----	-----	-----	3	1	1	1	1	-----	-----	1	-----	-----	8
II.....	3	8	12	14	7	6	5	3	2	-----	-----	-----	-----	60
IIA.....	-----	-----	-----	-----	1	1	-----	-----	-----	-----	-----	-----	-----	2
III.....	-----	-----	3	2	7	2	4	1	-----	-----	-----	-----	-----	19
IIIA.....	-----	-----	-----	1	-----	-----	1	-----	-----	-----	-----	-----	-----	2
IV, V.....	2	1	2	7	1	-----	1	-----	-----	-----	-----	-----	-----	14
Total.....	7	14	28	41	34	16	22	9	2	1	3	2	3	182

contributions to the 28-day shrinkage-values. Other independent variables, such as the alkalis and Ba, also appear associated with the shrinkage values.

5.4. Nonrestrained Shrinkage After 6 Months Drying

In the preliminary plots of the 6-months shrinkage versus various individual independent variables it may be noted in figure 9-4 that SiO₂, MgO, SO₃, K₂O, total alkali, C₃S, A/F, and (air, 1:4 mortar) had absolute "quadrant-sum" values greater than 11. The frequency distributions of the different types of cements with respect to the shrinkage

of neat cements of normal consistency after 6 months drying are presented in table 9-13. As in the frequency-distribution tables at the earlier ages (tables 9-1, 9-4 and 9-9) there was a spread of values of the cements within each of the types, especially for the Type I cements, and an overlapping of the values obtained for the cements of the different types. Equations selected from those computed to determine the various independent variables associated with the 6-month-shrinkage values of AE + NAE cements are presented in table 9-14. With the use of commonly determined variables in eqs 1 or 3 there was a highly significant reduction in the S.D. values, (see table 9-34) and, with the additional trace elements,

TABLE 9-10. *Coefficients for equations relating the percentage shrinkage at 28 days (the first cements, to various inde*

Equation	Note	Const.	C ₃ A	C ₄ AF	SiO ₂	Al ₂ O ₃	Fe ₂ O ₃	MgO	Na ₂ O
1-----	(1)	SH28 s.d. = +0.1935 = (0.0508)							
2-----	(1)	SH28 s.d. = +0.1297 = (0.0525)	*-0.00373 (0.00567)	*-0.00059 (0.00160)					
3-----	(1)	SH28 s.d. = +0.0119 = (0.0488)	-0.01643 (0.00521)	-0.001491 (0.001392)				+0.003261 (0.002317)	+0.07957 (0.01674)
4-----	(1)	SH28 s.d. = -0.0129 = (0.0475)	-0.01756 (0.00496)	-0.001545 (0.001329)				+0.004797 (0.002292)	+0.08660 (0.01614)
4A-----	(2)	SH28 (odd) s.d. = -0.0182 = (0.0577)	-0.01657 (0.00600)	-0.003002 (0.001769)				+0.008882 (0.003351)	+0.07326 (0.02791)
4B-----	(3)	SH28 (even) s.d. = -0.0365 = (0.0976)	-0.02032 (0.01032)	*-0.000222 (0.000222)				*+0.002062 (0.003448)	+0.09162 (0.02169)
5-----	(1)	SH28 s.d. = +0.2455 = (0.1387)			-0.00751 (0.00409)	-0.03898 (0.01108)	-0.00822 (0.00354)		+0.07956 (0.01585)
6-----	(1)	SH28 s.d. = +0.1299 = (0.1627)			-0.00580 (0.00464)	-0.04133 (0.01108)	-0.00779 (0.00354)	+0.003964 (0.002477)	+0.07996 (0.01573)
6A-----	(2)	SH28 (odd) s.d. = +0.3095 = (0.2447)			-0.01057 (0.00731)	-0.03685 (0.01372)	-0.01272 (0.00486)	+0.006216 (0.003616)	+0.06422 (0.02713)
6B-----	(3)	SH28 (even) s.d. = -0.2049 = (0.2645)			*-0.00066 (0.00652)	-0.06752 (0.02260)	*-0.00310 (0.00548)	*-0.003007 (0.003676)	+0.08453 (0.02068)

¹ 165 cements; Avg=0.2247; S.D.=0.03976.² 83 cements.³ 82 cements.

* Coef./s.d. ratio less than 1.

TABLE 9-11. *Coefficients for equations relating the percentage shrinkage at 28 days (the cements, to various inde*

Equation	Note	Const.	C ₃ A	C ₄ AF	SiO ₂	Al ₂ O ₃	Fe ₂ O ₃
1-----	(1)	SH28 s.d. = +0.1256 = (0.0533)	-0.00616 (0.00584)	*-0.00096 (0.00163)			
2-----	(1)	SH28 s.d. = +0.2251 = (0.0477)	-0.01761 (0.00520)	-0.002184 (0.001382)			
3-----	(1)	SH28 s.d. = -0.0228 = (0.0466)	-0.02040 (0.00491)	-0.002171 (0.001295)			
3A-----	(2)	SH28 (odd) s.d. = -0.0192 = (0.0651)	-0.02061 (0.00701)	*-0.002131 (0.002357)			
3B-----	(3)	SH28 (even) s.d. = +0.0492 = (0.0703)	-0.00912 (0.00737)	-0.002686 (0.001456)			
4-----	(1)	SH28 s.d. = +0.2616 = (0.1400)			-0.00831 (0.00416)	-0.04420 (0.01119)	-0.008559 (0.003566)
5-----	(1)	SH28 s.d. = +0.1389 = (0.1601)			-0.00627 (0.00460)	-0.04723 (0.01093)	-0.008560 (0.003455)
5A-----	(2)	SH28 (odd) s.d. = +0.2407 = (0.2606)			-0.01002 (0.00787)	-0.05521 (0.01562)	-0.009743 (0.003340)
5B-----	(3)	SH28 (even) s.d. = +0.2215 = (0.2154)			*-0.00405 (0.00519)	*-0.00953 (0.01929)	-0.010306 (0.003670)

¹ 153 cements, Avg=0.2228; S.D.=0.03937.² 77 cements.³ 76 cements.

* Coef./s.d. ratio less than 1.

24 hr in moist-air storage) of neat-cement bars of normal consistency made of AE + NAE
pendent variables (SH28)

K ₂ O	SO ₃	C ₃ A/SO ₃	Al ₂ O ₃ /SO ₃	APF	Air, 1:4 mortar	Ba	P	Zn	Rb	W ₁	S.D.
										*+0.0013 (0.0021)	0.03983
	+0.04390 (0.02544)	+0.01095 (0.01041)									0.03829
+0.08193 (0.01434)	+0.07160 (0.02432)	+0.03038 (0.00939)		+0.0000072 (0.0000060)	+0.00087 (0.00074)						0.03217
+0.06879 (0.01476)	+0.07198 (0.02310)	+0.03279 (0.00892)		+0.0000106 (0.0000058)	+0.00112 (0.00071)	+0.2887 (0.0957)	+0.04454 (0.02202)	-0.09902 (0.09034)	+2.910 (1.477)		0.03050
+0.07933 (0.02065)	+0.06632 (0.02794)	+0.03077 (0.01061)		+0.0000160 (0.0000087)	*+0.00071 (0.00105)	+0.3222 (0.1085)	+0.07274 (0.02754)	*+0.02507 (0.16517)	*+1.845 (1.127)		0.02967
+0.06265 (0.02287)	+0.08792 (0.04916)	+0.03841 (0.01885)		*+0.0000074 (0.0000089)	+0.00122 (0.00103)	+0.2746 (0.2294)	*+0.00272 (0.04030)	-0.1283 (0.1189)	+3.210 (2.214)		0.03244
+0.08333 (0.01264)	+0.08251 (0.02923)		+0.05713 (0.01950)								0.03217
+0.06610 (0.01480)	+0.08873 (0.02995)		+0.06750 (0.01929)	+0.0000070 (0.0000062)	+0.00122 (0.00071)	+0.3321 (0.0941)	+0.04125 (0.02191)		+3.000 (1.473)		0.03043
+0.06999 (0.02150)	+0.06113 (0.03508)		+0.05064 (0.02275)	+0.0000108 (0.0000096)	*+0.00087 (0.00105)	+0.3331 (0.1073)	+0.07419 (0.02759)		*+1.713 (2.135)		0.02967
+0.05808 (0.02220)	+0.18688 (0.06774)		+0.12983 (0.04293)	*+0.0000040 (0.0000091)	+0.00130 (0.00099)	+0.3765 (0.2134)	*-0.00819 (0.03840)		+3.446 (2.140)		0.03136

first 24 hr in moist-air storage) of neat-cement bars of normal consistency made of NAE
pendent variables (SH28)

MgO	Na ₂ O	K ₂ O	SO ₃	C ₃ A/SO ₃	Al ₂ O ₃ /SO ₃	APF	Ba	P	Zn	Rb	S.D.
			+0.04961 (0.02601)	+0.01447 (0.01064)							0.03826
	+0.08421 (0.01636)	+0.09857 (0.01361)	+0.07168 (0.02399)	+0.03121 (0.00929)		+0.0000094 (0.0000061)					0.03180
+0.005386 (0.002268)	+0.09205 (0.01543)	+0.07790 (0.01449)	+0.07884 (0.02260)	+0.03679 (0.00877)		+0.0000133 (0.0000057)	+0.3194 (0.0933)	+0.04893 (0.02137)	-0.09635 (0.08742)	+3.021 (1.450)	0.02948
+0.003992 (0.003523)	+0.08144 (0.03045)	+0.09711 (0.02339)	+0.06935 (0.03229)	+0.03789 (0.01219)		+0.0000167 (0.0000107)	+0.2128 (0.1232)	+0.07777 (0.03130)	*+0.07214 (0.13900)	*+2.412 (2.430)	0.03365
+0.008647 (0.002813)	+0.08983 (0.01526)	+0.05078 (0.01676)	+0.04069 (0.03419)	+0.01778 (0.01354)		+0.0000111 (0.0000063)	+0.8193 (0.1830)	*+0.00447 (0.02873)	-0.14922 (0.10943)	+2.455 (1.636)	0.02301
	+0.07881 (0.01600)	+0.08667 (0.01291)	+0.08868 (0.02933)		+0.06275 (0.01950)						0.03171
+0.003919 (0.002478)	+0.08770 (0.01516)	+0.07347 (0.01453)	+0.09543 (0.02920)		+0.07393 (0.01881)	+0.0000094 (0.0000062)	+0.3441 (0.0935)	+0.04897 (0.02138)	+0.10668 (0.08722)	+2.934 (1.452)	0.02940
*+0.002059 (0.003967)	+0.08159 (0.02951)	+0.09010 (0.02346)	+0.09186 (0.03802)		+0.08284 (0.02437)	*+0.0000110 (0.0000113)	+0.2521 (0.1219)	+0.08061 (0.03074)	*+0.06314 (0.13602)	*+2.166 (2.383)	0.03292
+0.007472 (0.003026)	+0.08648 (0.01512)	+0.04950 (0.01697)	*+0.01441 (0.05792)		*+0.01200 (0.03714)	+0.0000095 (0.0000069)	+0.8444 (0.1859)	*+0.00241 (0.02916)	-0.14377 (0.11173)	+2.482 (1.670)	0.02332

TABLE 9-12. Calculated contributions of independent variables to the percentage shrinkage at 28 days (the first 24 hr in moist-air-storage) of neat-cement bars of normal consistency

Independent variable	Ranges of variables, percent	Coefficients from eq 3 table 9-11	Calculated contributions to SH28	Calculated ranges of contributions to SH28
			Const. = -0.023	
C ₃ A	0 to 15	-0.0204	0 to -0.306	0.306
C ₄ A**	4 to 17	-0.00217	-0.009 to -0.037	0.028
MgO	0 to 5	+0.00539	0 to +0.027	0.027
Na ₂ O	0 to 0.75	+0.0920	0 to +0.069	0.069
K ₂ O	0 to 1.1	+0.0779	0 to +0.086	0.086
SO ₃	1.2 to 3.6	+0.0788	+0.095 to +0.283	0.188
C ₃ A/SO ₃	0.4 to 10.1	+0.0368	+0.015 to +0.372	0.357
APF	*2500 to 5500	+0.0000133	+0.033 to +0.073	0.040
Ba	0 to 0.2	+0.319	0 to +0.064	0.064
P	0 to 0.5	+0.0489	0 to +0.024	0.024
Zn**	0 to 0.2	-0.0964	0 to -0.019	0.019
Bb	0 to 0.01	+3.021	0 to +0.030	0.030

*cm²/g.

**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

Ba, Rb, and Ni in eqs 2 and 4, a further significant reduction was obtained. The coef./s.d. ratio for Rb, however, was less than 2. Equations calculated for the "odds" and "evens" in the array of cements (eqs 2A, 2B, 4A and 4B) indicated that C₃A, C₃S, MgO, C₃A/SO₃, Al₂O₃/SO₃, and Rb had coef./s.d. ratios less than 1 in one or more of the equations for the smaller groups.

A comparable series of equations for the NAE cements is presented in table 9-15. The coefficients of the different independent variables agreed fairly well with those of table 9-14 where the AE cements were included. The variables C₃A, C₃S, and Rb had coef./s.d. ratios of less than 1 in equation 2A where the "odds" in the array of cements were used in the computations.

TABLE 9-13. Frequency distribution of cements with respect to percentage shrinkage of neat cements of normal consistency after 6 months drying in laboratory air after 24 hr moist-air storage (SH6M)

Type cement	Percentage shrinkage												Total
	0.16 to 0.20	0.20 to 0.24	0.24 to 0.28	0.28 to 0.32	0.32 to 0.36	0.36 to 0.40	0.40 to 0.44	0.44 to 0.48	0.48 to 0.52	0.52 to 0.56	0.56 to 0.60	0.60 and over	
	Number of cements												
I	2	17	22	14	13	1	4	1	1	2	1	77	
IA			4	1	2		1					8	
II	4	17	22	5	10	2						60	
IIA			1	1								2	
III		2	9	5	3							19	
IIIA			1	1								2	
IV, V	1	4	7	2								14	
Total	7	40	66	29	28	3	5	0	1	2	0	1	182

Coefficients of the independent variables in eq 2 of table 9-15 and their ranges were used to calculate their contributions and their possible ranges of contributions to the 6-month-shrinkage values of the neat cement bars for NAE cements. These are presented in table 9-16. The independent variables, C₃A/SO₃, Na₂O, K₂O, and Ba, apparently resulted in the greatest calculated ranges of the contributions to the 6-month-shrinkage values.

5.5 Time of Cracking of Annular Specimens Restrained from Shrinkage

The frequency distributions of the time of cracking of the annular specimens made with the different portland cements, each using the water required for normal consistency, are presented in table 9-17. The specimens were stored in an atmosphere of 95 to 100 percent relative humidity for the first 24 hr. The time of cracking ranged from a few minutes to more than 40 hr after removal from the molds. The Type I cements had a broad distribution of

values with a large number cracking within a relatively short time. More than half of the specimens made with Type III cements also cracked within 2.5 hr. The median values of the Types II, IV, and V cements indicated that a longer time period was required for the annular specimens made with these cements to crack. There was, however, considerable overlapping of the distributions of the time of cracking of the cements of the different types.

Preliminary plots of the time of cracking of the annular specimens versus various individual independent variables are presented in figure 9-5. Absolute values of "quadrant sum" equal to or greater than 11 were obtained for SiO₂, Al₂O₃, Fe₂O₃, SO₃, K₂O, total alkali, C₃A, C₂S, C₄AF, and Al₂O₃/Fe₂O₃.

An outstanding feature of the distribution of the time-of-cracking data is the large concentration at the early end of the scale. More than half of the breaks occurred in 10 hr or less. Thus the distribution curve of the data is strongly skewed to the right, i.e., the "hump" of the curve is near the left end with a long "tail" stretching far out to the right.

TABLE 9-14. Coefficients for equations relating the percentage shrinkage at 6 months (the first 24 hr in moist-air storage) of neat-cement bars of normal consistency made of AE + NAE cements, to various independent variables (SH6M)

Equation	Note	Const.	C ₄ A	C ₃ S	CaO	Na ₂ O	K ₂ O	MgO	C ₃ A/SO ₃	Al ₂ O ₃ /SO ₃	APF	Ba	Rb	Ni	S.D.
1	(1)	SH6M s.d. = +0.0028 = (0.0395)	-0.003249 (0.002795)	+0.000622 (0.000620)	---	+0.1305 (0.0220)	+0.1271 (0.0198)	+0.00467 (0.00330)	+0.01000 (0.00538)	---	+0.00001092 (0.00000879)	---	---	---	0.04692
2	(1)	SH6M s.d. = +0.0199 = (0.0379)	-0.004572 (0.002550)	+0.000627 (0.000566)	---	+0.1377 (0.0201)	+0.1162 (0.0198)	+0.00836 (0.00306)	+0.01529 (0.00499)	---	+0.00001950 (0.00000810)	+0.5445 (0.1157)	+3.584 (2.026)	+2.519 (1.194)	0.04257
2A	(2)	SH6M (odd) s.d. = +0.0639 = (0.0624)	*-0.001479 (0.004239)	*+0.000385 (0.000892)	---	+0.1838 (0.0327)	+0.1135 (0.0336)	*+0.00424 (0.00352)	*+0.00769 (0.00885)	---	+0.00001609 (0.00001240)	+0.3443 (0.2433)	+4.514 (3.133)	+4.930 (2.217)	0.04827
2B	(2)	SH6M (even) s.d. = -0.0033 = (0.0488)	-0.004288 (0.003106)	+0.001047 (0.000739)	---	+0.0895 (0.0242)	+0.1082 (0.0235)	+0.01091 (0.00365)	+0.01499 (0.00587)	---	+0.00002093 (0.00001066)	+0.5682 (0.1244)	*+1.879 (2.498)	+2.354 (1.362)	0.03524
3	(1)	SH6M s.d. = -0.4413 = (0.2970)	---	---	+0.00843 (0.00453)	+0.1328 (0.0218)	+0.1304 (0.0179)	+0.00943 (0.00435)	---	+0.01075 (0.00680)	+0.00001149 (0.00000759)	---	---	---	0.04683
4	(1)	SH6M s.d. = -0.6951 = (0.2731)	---	---	+0.01099 (0.00415)	+0.1417 (0.0199)	+0.1220 (0.0182)	+0.01485 (0.00404)	---	+0.01910 (0.00643)	+0.00002029 (0.00000703)	+0.5486 (0.1159)	+3.478 (2.021)	+2.577 (1.189)	0.04245
4A	(2)	SH6M (odd) s.d. = -0.4654 = (0.4437)	---	---	+0.00802 (0.00686)	+0.1887 (0.0326)	+0.1193 (0.0293)	+0.00949 (0.00741)	---	*+0.01078 (0.01330)	+0.00001689 (0.00001156)	+0.3518 (0.2415)	+4.674 (3.138)	+4.778 (2.183)	0.04779
4B	(2)	SH6M (even) s.d. = -0.8738 = (0.3581)	---	---	+0.01340 (0.00531)	+0.0940 (0.0234)	+0.1249 (0.0230)	+0.01829 (0.00476)	---	+0.02047 (0.00668)	+0.00002537 (0.00000893)	+0.5622 (0.1239)	*+1.453 (2.453)	+2.617 (1.385)	0.03535

† 171 cements; Avg = 0.2769; S.D. = 0.05914. ‡ 85 cements. * Coef./s.d. ratio less than 1.

TABLE 9-15. Coefficients for equations relating the percentage shrinkage at 6 months (the first 24 hr in moist-air storage) of neat-cement bars of normal consistency made of NAE cements, to various independent variables (SH6M)

Equation	Note	Const.	C ₄ A	C ₃ S	CaO	Na ₂ O	K ₂ O	MgO	C ₃ A/SO ₃	Al ₂ O ₃ /SO ₃	APF	Ba	Rb	Ni	S.D.
1	(1)	SH6M s.d. = +0.0805 = (0.0409)	-0.004233 (0.002913)	+0.000645 (0.000639)	---	+0.1337 (0.0225)	+0.1355 (0.0206)	+0.00494 (0.00341)	+0.01133 (0.00534)	---	+0.00001341 (0.00000909)	---	---	---	0.04714
2	(1)	SH6M s.d. = +0.0039 = (0.00575)	-0.006027 (0.002618)	+0.000669 (0.000575)	---	+0.1392 (0.0202)	+0.1233 (0.0204)	+0.00879 (0.00311)	+0.01748 (0.00506)	---	+0.00002241 (0.00000823)	+0.5747 (0.1152)	+3.886 (2.065)	+2.617 (1.192)	0.04205
2A	(2)	SH6M (odd) s.d. = +0.0038 = (0.0559)	*+0.002184 (0.004602)	*+0.000161 (0.000890)	---	+0.1777 (0.0326)	+0.1079 (0.0331)	+0.00988 (0.00515)	+0.01227 (0.00927)	---	+0.00002638 (0.00001172)	+0.7243 (0.2197)	*+2.880 (3.065)	+2.575 (2.046)	0.04477
2B	(2)	SH6M (even) s.d. = -0.0076 = (0.0593)	-0.007528 (0.003408)	+0.001431 (0.000813)	---	+0.1129 (0.0270)	+0.1272 (0.0272)	+0.00908 (0.00402)	+0.01915 (0.00643)	---	+0.00001778 (0.00001269)	+0.4792 (0.1407)	+3.988 (2.903)	+2.990 (1.488)	0.04024
3	(1)	SH6M s.d. = -0.4514 = (0.3048)	---	---	+0.00850 (0.00466)	+0.1348 (0.0224)	+0.1351 (0.0185)	+0.00968 (0.00452)	---	+0.01021 (0.00694)	+0.00001262 (0.00000793)	---	---	---	0.04714
4	(1)	SH6M s.d. = -0.7285 = (0.2764)	---	---	+0.01139 (0.00420)	+0.1417 (0.0201)	+0.1243 (0.0188)	+0.01539 (0.00413)	---	+0.01916 (0.00644)	+0.00002154 (0.00000722)	+0.5780 (0.1159)	+3.563 (2.062)	+2.662 (1.191)	0.04207
4A	(2)	SH6M (odd) s.d. = -0.5163 = (0.4075)	---	---	+0.00782 (0.00631)	+0.1823 (0.0324)	+0.1202 (0.0287)	+0.01476 (0.00667)	---	+0.01705 (0.01165)	+0.00002641 (0.00001064)	+0.7421 (0.2257)	+3.295 (3.042)	+2.224 (2.002)	0.04472
4B	(2)	SH6M (even) s.d. = -0.9808 = (0.4096)	---	---	+0.01537 (0.00606)	+0.1129 (0.0268)	+0.1247 (0.0264)	+0.01718 (0.00563)	---	+0.02132 (0.00799)	+0.00001951 (0.00001091)	+0.4694 (0.1394)	+3.391 (2.903)	+3.139 (1.500)	0.04036

† 159 cements; Avg = 0.2754; S.D. = 0.05990. ‡ 79 cements. * Coef./s.d. ratio less than 1.

TABLE 9-16. Calculated contributions of independent variables to the percentage shrinkage at 6 months (the first 24 hr in moist-air-storage) of neat-cement bars of normal consistency (SH6M)

Independent variable	Ranges of variables, percent	Coefficients from eq 2 table 9-15	Calculated contributions to SH28	Calculated ranges of contributions to SH6M
C ₃ A	0 to 15	-0.00603	Const. = +0.004	0 to -0.090
C ₃ S*	30 to 70	+0.000669	+0.020 to +0.047	0.027
Na ₂ O	0 to 0.75	+0.139	0 to +0.104	0.104
K ₂ O	0 to 1.1	+0.123	0 to +0.135	0.135
MgO	0 to 5	+0.00879	0 to +0.044	0.044
C ₃ A/SO ₃	0.4 to 10.1	+0.0175	+0.007 to +0.177	0.170
APF	*2500 to 5500	+0.0000224	+0.056 to +0.123	0.067
Ba	0 to 0.2	+0.575	0 to +0.115	0.115
Rb*	0 to 0.01	+3.686	0 to +0.037	0.037
Ni	0 to 0.02	+2.617	0 to +0.052	0.052

*cm²/g.

**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

The multivariate analysis used throughout this series of articles is based on the assumption that the underlying distribution curve of the data is the familiar bell-shaped normal curve, with the hump in the middle and approximately symmetrical tails stretching out on both sides. In cases where both the data themselves and logical considerations related to the underlying phenomenon indicate that this is not the case, some other way of evaluating the relationships involved must be found if the results are to be meaningful.

Frequently, when data are not normally distributed, the log or square root or some other transformation of the original data fits the normal distribution more closely and can be handled with the usual statistical techniques

TABLE 9-17. Frequency distribution of the cements with respect to the time required for the annular specimens of neat cement to crack (TOBR)

Type cement	Time of cracking, hours													Total
	0 to 2.5	2.5 to 5.0	5.0 to 7.5	7.5 to 10.0	10.0 to 12.5	12.5 to 15.0	15.0 to 17.5	17.5 to 20.0	20.0 to 22.5	22.5 to 25.0	25.0 to 27.5	27.5 to 30.0	30.0 and over	
	Number of cements													
I.....	19	12	8	6	8	9	3	4	3	1	2	1	1	77
IA.....	3	1	1	1	3	5	14	4	2	1	2	1	1	8
II.....	1	7	7	9	7	5	14	4	2	1	2	2	2	59
IIA.....				1			1							2
III.....	10	1	4	1	2			1						19
IIIA.....	2													2
IV, V.....				1	3	3	2	2			1		2	14
Total.....	35	21	19	19	23	17	20	11	5	1	4	3	3	181

SHRINKAGE TEST	NOTE*	INDEPENDENT VARIABLES																	
		Si O ₂	Al ₂ O ₃	Fe ₂ O ₃	CaO	MgO	SO ₃	Na ₂ O	K ₂ O	TOTAL ALKALI	C ₃ A	C ₃ S	C ₂ S	C ₄ AF	$\frac{A}{F}$	$\frac{S}{A+F}$	AIR,P. FINE.	WAGN. FINE.	AIR 1:4 MOR.
TO BR	(1)	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{10}{2}$	$\frac{6}{6}$	$\frac{8}{4}$	$\frac{10}{2}$	$\frac{9}{3}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{8}{4}$
	(2)																		
	(3)	L	NL	L?	NO	L?	NL	NL	NL	NL	NL	L?	NL	L?	NL	L?	NL	L?	NL
	(4)	+17	-14	+14	+2	-9	-16	-6	-19	-16	-15	-8	+13	+16	-17	+9	-7	-6	-9
SH BR	(1)	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{12}{0}$	$\frac{12}{0}$	$\frac{9}{3}$	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{6}{6}$	$\frac{9}{3}$
	(2)																		
	(3)	L	NL	L?	L?	L?	NL	L?	L	L	NL	NO	L?	L?	L	L?	NL	NO	L?
	(4)	+13	-11	+7	+7	-16	-14	-2	-24	-24	-9	-2	+6	+6	-6	+8	-12	-4	-7
SHBR/SHO7 or SB/S7	(1)	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{12}{0}$	$\frac{8}{4}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{12}{0}$
	(2)																		
	(3)	L	NL	L?	L?	L?	L	L?	L	L	L?	NL	L?	L?	L?	L?	NL	L?	L
	(4)	+20	-11	+9	+7	-13	-24	-6	-19	-20	-11	-3	+14	+16	-10	+13	-16	-6	-24
SHBR/SH28 or SB/S28	(1)	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{12}{0}$	$\frac{10}{2}$	$\frac{8}{4}$	$\frac{6}{6}$	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{8}{4}$	$\frac{9}{3}$	$\frac{8}{4}$	$\frac{10}{2}$
	(2)																		
	(3)	L	L?	NL	L	L	L	NL	L	L	NL	NO	NL	L?	L?	L?	NL	L?	L
	(4)	+12	-12	+13	+4	-10	-16	-9	-24	-20	-12	-6	+12	+11	-11	+11	-11	-8	-16

FIGURE 9-5. Results of plotting (1) the time of cracking of the annular specimens, (2) the percentage shrinkage of prisms at the time of cracking of the annular specimens, and (3) (4) the ratios of the percentage unrestrained shrinkage at the time of cracking of the annular specimens to the unrestrained shrinkage at 7 and 28 days, respectively, on the "Y" axis versus various independent variables on the "X" axis.

* Note (1) Ratio of number of plotted points in pairs of diametrically opposite quadrants.

Note (2) General trend of lines drawn through plotted points.

Note (3) Apparent nature of relationship. L = linear; NL = nonlinear; NO = no apparent relationship; and L? = nature of relationship not determinable.

Note (4) Quadrant sum in corner test (see reference 5).

[18]. Of several different functions of the time-of-break-data tried, the square root gave the best approximation to a normal distribution. Table 9-18 gives the frequency distribution of the square root of the time-of-break-data by types.

TABLE 9-18. Frequency distribution of cements with respect to the square root of the time required for the annular specimens of neat cement to crack (TOBR)^{0.5}

	Square root of time of break in hours									
Type	0 to 0.75	0.75 to 1.50	1.50 to 2.25	2.25 to 3.00	3.00 to 3.75	3.75 to 4.50	4.50 to 5.25	5.25 to 6.00	6.00 to 6.75	Total
	Number of cements									
I.....	4	15	12	13	15	10	6	1	1	77
IA.....		3	1	1	3					8
II.....	1		7	14	13	19	3	2		59
IIA.....				1		1				2
III.....	4	6	2	3	3	1				19
IIIA.....		2								2
IV, V.....				1	6	4	1	2		14
Total.....	9	26	22	33	40	35	10	5	1	181

Selected equations relating various independent variables with the square root of time of

break are given in table 9-19 for AE + NAE cements and in table 9-20 for NAE cements alone.¹⁰ Equation 1 of table 9-19 indicates that specimens made with cements with the higher water requirements for normal consistency (W₁) had a tendency to crack earlier than those with lower water requirements. The relationship of commonly determined variables to the time of break is presented in eq 2. The S.D. value was reduced very significantly (see table 9-34). With the addition of the independent variables MgO, Ba, Rb, and Zr in eq 3 a further significant reduction in variance was attained. In the equations for the "odds" and "evens", SO₃ in eq 3A and Rb and Zr in eq 3B had coef./s.d. ratios less than 1. The use of the oxides SiO₂ and Fe₂O₃ in eqs 4 and 5 resulted in S.D. values almost identical with those attained in eqs 2 and 3 when the calculated compounds C₃S and C₄AF were used.

¹⁰ Since the analysis of this subsection is based on the square root of the time-of-break-data, the various statistical quantities calculated such as coefficients, standard deviations, etc., cannot be used directly as quantitative measures of significance associated with the original untransformed data. However, since the square root transformation is order-preserving¹¹, relative magnitudes and trends of these measures may be used to support the general remarks about relationship given here. It is interesting that, although the coefficients and standard deviations were changed by the transformation of the data, coef./s.d. ratios were almost the same for the square root as for the untransformed data.

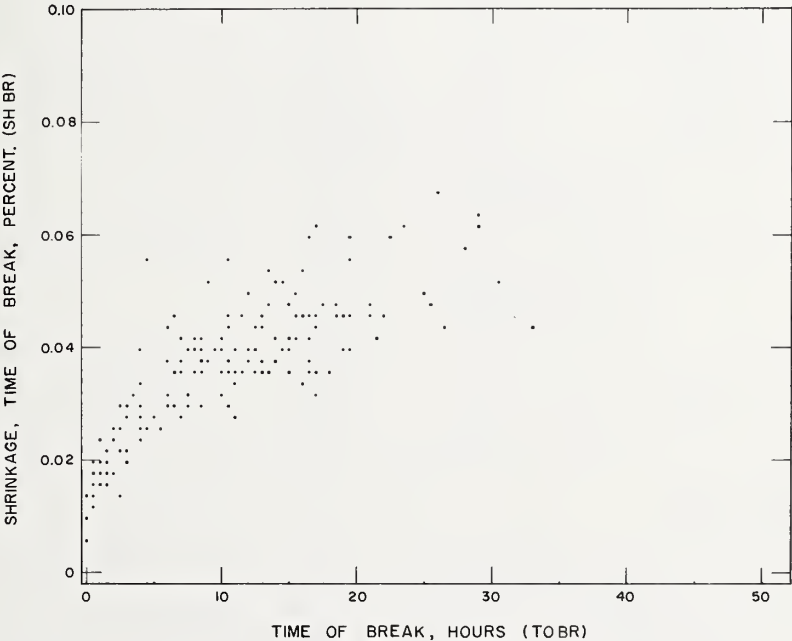


FIGURE 9-6. Results of plotting the percentage shrinkage of the non-restrained bars at the time of cracking of the annular specimens versus the time of cracking of the annular specimens.

(Both types of specimens were made of neat portland cement pastes of normal consistency and cured for 24 hours in moist air prior to exposure to laboratory air.)

Corresponding equations for the NAE cements are presented in table 9-20. The coefficients for the different independent variables and the coef./s.d. ratios were in reasonable

agreement with those of table 9-19 where the AE cements were included.

Coefficients of the variables in eq 3 of table 9-20 were used with their ranges to calculate

TABLE 9-19. Coefficients for equations for AE+NAE cements, relating the square root of time of breaking (TOBR)^{0.5} (in hours) of the annular specimens made of neat-cement of normal consistency (after initial 24 hr moist-air storage) to various independent variables

Equation	Note	Const.	C ₂ S	C ₄ AF	SiO ₂	Fe ₂ O ₃	MgO	SO ₃	Na ₂ O	K ₂ O	APF	Loss	Ba	Rb	Zr	W ₁	S.D.
1	(1)	(TOBR) ^{0.5} s.d. = +8.74 = (1.61)	---	---	---	---	---	---	---	---	---	---	---	---	---	---	1.250
2	(1)	(TOBR) ^{0.5} s.d. = +5.82 = (0.78)	+0.0254 (0.0107)	+0.1332 (0.0307)	---	---	---	-0.432 (0.247)	-1.55 (0.41)	-2.72 (0.35)	-0.000877 (0.000171)	+0.529 (0.141)	---	---	---	-0.225 (0.065)	0.882
3	(1)	(TOBR) ^{0.5} s.d. = +6.32 = (0.76)	+0.0301 (0.0102)	+0.1388 (0.0293)	---	---	-0.1155 (0.0597)	-0.389 (0.235)	-1.61 (0.39)	-2.34 (0.38)	-0.000979 (0.000165)	+0.563 (0.138)	-8.37 (2.19)	-89.5 (38.0)	-2.82 (1.66)	---	0.837
3A	(2)	(TOBR) ^{0.5} (odd) s.d. = +5.86 = (1.06)	+0.0318 (0.0147)	+0.1576 (0.0455)	---	---	-0.1081 (0.0852)	*-0.203 (0.340)	-1.71 (0.69)	-1.72 (0.58)	-0.001115 (0.000260)	+0.589 (0.213)	-7.94 (4.89)	-138.7 (53.4)	-3.46 (1.95)	---	0.838
3B	(2)	(TOBR) ^{0.5} (even) s.d. = +6.91 = (1.14)	+0.0222 (0.0152)	+0.1376 (0.0402)	---	---	-0.1051 (0.0914)	-0.462 (0.359)	-1.78 (0.53)	-2.92 (0.59)	-0.000966 (0.000227)	+0.580 (0.193)	-7.75 (2.60)	*-40.9 (57.6)	*+3.53 (4.09)	---	0.848
4	(1)	(TOBR) ^{0.5} s.d. = +1.63 = (1.77)	---	---	+0.1982 (0.0647)	+0.4590 (0.0923)	---	---	-1.43 (0.41)	-2.61 (0.35)	-0.001039 (0.000149)	+0.549 (0.143)	---	---	---	---	0.882
5	(1)	(TOBR) ^{0.5} s.d. = +0.99 = (1.69)	---	---	+0.2433 (0.0625)	+0.4955 (0.0878)	---	---	-1.47 (0.39)	-2.38 (0.37)	-0.001127 (0.000144)	+0.619 (0.140)	-8.80 (2.16)	-73.3 (37.8)	-2.69 (1.64)	---	0.835
5A	(2)	(TOBR) ^{0.5} (odd) s.d. = +1.48 = (2.42)	---	---	+0.2140 (0.0905)	+0.5307 (0.1362)	---	---	-1.67 (0.67)	-1.65 (0.54)	-0.001212 (0.000218)	+0.595 (0.212)	-8.24 (4.80)	-119.4 (53.0)	-3.24 (1.89)	---	0.833
5B	(2)	(TOBR) ^{0.5} (even) s.d. = +1.42 = (2.47)	---	---	+0.2317 (0.0911)	+0.5032 (0.1187)	---	---	-1.67 (0.51)	-3.00 (0.56)	-0.001131 (0.000196)	+0.682 (0.193)	-8.40 (2.53)	-27.2 (56.2)	*+3.51 (4.04)	---	0.836

1 170 cements; Avg = 2.912; S.D. = 1.294.

2 85 cements.

* Coef./s.d. ratio less than 1.

TABLE 9-20. Coefficients for equations for NAE cements, relating the square root of time of breaking (TOBR)^{0.5} (in hours) of the annular specimens made of neat-cement of normal consistency (after initial 24 hr moist-air storage) to various independent variables

Equation	Note	Const.	C ₂ S	C ₄ AF	SiO ₂	Fe ₂ O ₃	MgO	SO ₃	Na ₂ O	K ₂ O	APF	Loss	Ba	Rb	Zr	W ₁	S.D.
1	(1)	(TOBR) ^{0.5} s.d. = +8.53 = (1.68)	---	---	---	---	---	---	---	---	---	---	---	---	---	---	1.264
2	(1)	(TOBR) ^{0.5} s.d. = +5.74 = (0.81)	+0.0249 (0.0111)	+0.1407 (0.0321)	---	---	---	---	-1.51 (0.42)	-2.76 (0.36)	-0.000912 (0.000179)	+0.510 (0.147)	---	---	---	-0.225 (0.068)	0.896
3	(1)	(TOBR) ^{0.5} s.d. = +6.24 = (0.77)	+0.0307 (0.0105)	+0.1455 (0.0301)	---	---	-0.1475 (0.0615)	-0.266 (0.242)	-1.54 (0.40)	-2.27 (0.40)	-0.001022 (0.000170)	+0.544 (0.142)	-8.94 (2.21)	-96.5 (39.4)	-2.89 (1.68)	---	0.839
3A	(2)	(TOBR) ^{0.5} (odd) s.d. = +6.54 = (1.15)	+0.0293 (0.0100)	+0.1987 (0.0510)	---	---	-0.2473 (0.1022)	*-0.140 (0.385)	-1.72 (0.69)	-1.92 (0.60)	-0.001248 (0.000303)	+0.572 (0.218)	-17.58 (5.84)	-113.1 (58.4)	-3.71 (2.02)	---	0.882
3B	(2)	(TOBR) ^{0.5} (even) s.d. = +6.19 = (1.10)	+0.0271 (0.0149)	+0.1047 (0.0403)	---	---	-0.0883 (0.0810)	*-0.241 (0.335)	-1.41 (0.52)	-2.61 (0.59)	-0.000996 (0.000216)	+0.695 (0.202)	-7.01 (2.44)	-83.1 (56.3)	+12.12 (7.85)	---	0.808
4	(1)	(TOBR) ^{0.5} s.d. = +1.73 = (1.84)	---	---	+0.1901 (0.0672)	+0.4834 (0.0955)	---	---	-1.38 (0.42)	-2.60 (0.37)	-0.001037 (0.000156)	+0.528 (0.148)	---	---	---	---	0.893
5	(1)	(TOBR) ^{0.5} s.d. = +1.78 = (1.78)	---	---	+0.2165 (0.0651)	+0.5102 (0.0898)	---	---	-1.39 (0.40)	-2.17 (0.40)	-0.001143 (0.000149)	+0.585 (0.143)	-9.56 (2.21)	-85.3 (39.4)	-3.00 (1.67)	---	0.837
5A	(2)	(TOBR) ^{0.5} (odd) s.d. = +2.36 = (2.67)	---	---	+0.2054 (0.0980)	+0.6758 (0.1485)	---	---	-1.62 (0.69)	-1.69 (0.60)	-0.001329 (0.000243)	+0.615 (0.217)	-18.65 (5.75)	-101.3 (56.8)	-3.81 (2.00)	---	0.873
5B	(2)	(TOBR) ^{0.5} (even) s.d. = +1.84 = (2.44)	---	---	+0.2065 (0.0889)	+0.3774 (0.1192)	---	---	-1.27 (0.50)	-2.58 (0.57)	-0.001098 (0.000195)	+0.758 (0.204)	-7.47 (2.41)	-69.8 (55.4)	+11.57 (7.78)	---	0.799

1 158 cements; Avg = 2.954; S.D. 1.304.

2 79 cements.

* Coef./s.d. ratio less than 1.

their contributions to the time of breaking of the annular specimens as well as the ranges of these contributions. The results of these calculations are presented in table 9-21. Differences in fineness and K₂O appear to have the greatest effect on the time of breaking with restrained shrinkage. Increases in fineness, K₂O, Na₂O, Ba, and possibly MgO, SO₃, Zr, and Rb appear associated with a decrease in the time of breaking. Increases in loss on ignition and C₄AF appear associated with an increase in the time of breaking.

TABLE 9-21. Calculated contributions to the time of breaking (TOBR)^{0.5} of the annular specimens (after initial 24 hr moist-air-storage) of neat-cement of normal consistency

Independent variable	Ranges of variables, percent	Coefficients from eq 3 table 9-20	Calculated contributions to (TOBR) ^{0.5}	Calculated ranges of contributions to (TOBR) ^{0.5}
C ₂ S	5 to 50	+0.0307	Const. = +6.24 +0.15 to +1.54	1.39
C ₄ AF	4 to 17	+0.1455	+0.58 to +2.47	1.89
MgO	0 to 5.0	-0.1475	0 to -0.74	0.73
SO ₃ **	1.2 to 3.6	-0.266	-0.32 to -0.96	0.64
Na ₂ O	0 to 0.75	-1.54	0 to -1.15	1.15
K ₂ O	0 to 1.1	-2.27	0 to -2.50	2.50
APF	*2500 to 5500	-0.001022	-2.55 to -5.62	3.07
Loss	0.3 to 3.6	+0.544	+0.16 to +1.96	1.80
Ba	0 to 0.2	-8.94	0 to -1.79	1.79
Rb	0 to 0.01	-96.5	0 to -0.96	0.96
Zr**	0 to 0.5	-2.89	0 to -1.44	1.44

*cm²/g.
**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

5.6. Nonrestrained Shrinkage of Prisms at Time of Cracking of Annular Specimens

A plot of the percent shrinkage of the non-restrained bars at the time of cracking of the annular specimens versus the time of cracking of the annular specimens restrained from shrinkage is presented in figure 9-6. Some of the annular specimens cracked at a time when the nonrestrained specimens had shrunk very little. With other cements, the neat pastes were apparently more capable of adjusting to the stresses, or the shrinkage occurred at a slower rate, and a longer period of time was required to crack the specimens restrained from shrinkage. No measurements were made of the loss of weight of the specimens with time of exposure to the laboratory air.

The frequency distributions (for the different types of cements) of the percentage shrinkage of the nonrestrained bars at the time of cracking of the restrained annular specimens are presented in table 9-22¹¹. The range of the percentage-shrinkage values was quite broad for each of the different types of cements as classified, and there was an overlapping of the values for the cements within the different types.

¹¹ Since these data appear to have a reasonably normal distribution, they are analyzed in the usual way.

TABLE 9-22. Frequency distribution of cements with respect to percentage shrinkage of neat cement bars at the time of breaking of the annular specimens (SHBR)

Type cement	Percentage shrinkage													Total
	0.005 to 0.010	0.010 to 0.015	0.015 to 0.020	0.020 to 0.025	0.025 to 0.030	0.030 to 0.035	0.035 to 0.040	0.040 to 0.045	0.045 to 0.050	0.050 to 0.055	0.055 to 0.060	0.060 to 0.065	0.065 and over	
	Number of cements													
I		3	13	6	10	3	10	9	11	3	3	2	1	74
IA			1	1	1	2	2				1			8
II		2		2	3	9	15	14	7	3	2	1		58
IIA							1					1		2
III	2	2	4	2	3	1	2	1		1	1			19
IIIA				1	1									2
IV, V					2		4	2	3	2			1	14
Total	2	7	18	12	20	15	34	26	21	9	7	4	2	177

The results of the preliminary plots of the nonrestrained shrinkage of the bars at the time of cracking of the annular specimens which were restrained versus the various individual independent variables are presented in figure 9-5. The absolute value of the quadrant sum was 11 or greater for SiO₂, Al₂O₃, MgO, SO₃, K₂O, total alkali, and fineness. Comparing the signs of the quadrant sums as well as the general trend of the lines drawn through the plotted points with those obtained for the shrinkage at 24 hr (SH24) in the top group of figures in figure 9-4, it may be noted that, where comparable, the slopes and signs were different for

practically every one of the variables. Equations relating various independent variables to the nonrestrained shrinkage of the neat cement bars at the time of cracking of the annular specimens for AE + NAE cements are presented in table 9-23. The relationship to the percentage water required for normal consistency, W₁, in eq 1 was not highly significant. With the use of commonly determined variables in eqs 2 or 4 there was a highly significant reduction in the S.D. value (see table 9-34). The use of the trace elements Rb, Ti, Cu, and V in eq 3 or Rb, Cu, and V in eq 5 resulted in a reduction of the S.D. value significant at the 5-

TABLE 9-23. Coefficients for equations for AE + NAE cements relating the percentage shrinkage of neat-cement bars at the time of cracking of the annular specimens to various independent variables (SHBR)

Equation	Note	Const.	C ₃ A	Al ₂ O ₃	Na ₂ O	K ₂ O	APF	MgO	Loss	Rb	Ti	Cu	V	W ₁	S.D.
1.	SHBR s.d.	= +0.08956 = (0.01620)	---	---	---	---	---	---	---	---	---	---	---	-0.001383 (0.000653)	0.01265
2.	SHBR s.d.	= +0.09210 = (0.00591)	-0.000433 (0.000262)	---	-0.01694 (0.00449)	-0.02525 (0.00399)	-0.0000118 (0.0000016)	-0.00121 (0.00067)	+0.00518 (0.00154)	---	---	---	---	---	0.00967
3.	SHBR s.d.	= +0.09775 = (0.00688)	-0.000641 (0.000271)	---	-0.01531 (0.00445)	-0.02464 (0.00446)	-0.0000123 (0.0000016)	-0.00155 (0.00068)	-0.00472 (0.00154)	-0.5682 (0.4489)	+0.00654 (0.00647)	-0.2549 (0.1057)	-0.0972 (0.0450)	---	0.00946
3A.	SHBR (odd) s.d.	= +0.10626 = (0.00836)	-0.000591 (0.000356)	---	-0.00850 (0.00608)	-0.01849 (0.00597)	-0.0000138 (0.0000018)	-0.00287 (0.00088)	+0.00472 (0.00179)	-1.4118 (0.5515)	-0.01756 (0.00942)	-0.3151 (0.1480)	*+0.0278 (0.0663)	---	0.00831
3B.	SHBR (even) s.d.	= +0.08531 = (0.01127)	-0.000725 (0.000411)	---	-0.01898 (0.00648)	-0.03395 (0.00719)	-0.0000101 (0.0000016)	*-0.00007 (0.00110)	+0.00674 (0.00293)	*+0.6375 (0.6997)	+0.02306 (0.00873)	-0.2028 (0.1492)	-0.1105 (0.0614)	---	0.00991
4.	SHBR s.d.	= +0.09407 = (0.00679)	---	-0.000912 (0.000836)	-0.01729 (0.00451)	-0.02601 (0.00405)	-0.0000119 (0.0000016)	-0.00124 (0.00067)	+0.00509 (0.00154)	---	---	---	---	---	0.00972
5.	SHBR s.d.	= +0.10147 = (0.00767)	---	-0.001284 (0.000865)	-0.01625 (0.00448)	-0.02539 (0.00454)	-0.0000124 (0.0000016)	-0.00164 (0.00068)	+0.00450 (0.00155)	-0.5542 (0.4550)	---	-0.2274 (0.1051)	-0.0492 (0.0449)	---	0.00957
5A.	SHBR (odd) s.d.	= +0.10443 = (0.00981)	---	-0.001288 (0.001130)	-0.00954 (0.00622)	-0.01971 (0.00608)	-0.0000138 (0.0000018)	-0.00266 (0.00091)	+0.00500 (0.00183)	-1.4751 (0.5665)	---	-0.2734 (0.1508)	*+0.0210 (0.0668)	---	0.00852
5B.	SHBR (even) s.d.	= +0.09663 = (0.01263)	---	-0.001527 (0.001386)	-0.02300 (0.00667)	-0.03228 (0.00769)	-0.0000103 (0.0000029)	*-0.00095 (0.00111)	+0.00503 (0.00301)	*+0.5692 (0.7371)	---	-0.1662 (0.1554)	-0.0845 (0.0953)	---	0.01013

1 169 cements; Avg = 0.03553; S.D. = 0.01278.

2 85 cements.

3 84 cements.

* Coef./s.d. ratio less than 1.

TABLE 9-24. Coefficients for equations for NAE cements relating the percentage shrinkage of neat-cement bars at the time of cracking of the annular specimens to various independent variables (SHBR)

Equation	Note	Const.	C ₃ A	Al ₂ O ₃	Na ₂ O	K ₂ O	APF	MgO	Loss	Rb	Cu	V	W _i	S.D.
	(1)	SHBR s.d. = +0.0665 = (0.0167)											-0.001255 (0.000675)	0.01262
	(1)	SHBR s.d. = +0.0936 = (0.0060)	-0.000468 (0.000264)		-0.01644 (0.00449)	-0.02429 (0.00406)	-0.0000120 (0.0000016)	-0.00163 (0.00068)	+0.00471 (0.00154)					0.00948
	(1)	SHBR s.d. = +0.1006 = (0.0066)	-0.000673 (0.000274)		-0.01496 (0.00449)	-0.02290 (0.00462)	-0.0000124 (0.0000016)	-0.00202 (0.00068)	+0.00414 (0.00154)	-0.6045 (0.4570)	-0.2483 (0.1041)	-0.0555 (0.0437)		0.00928
A	(2)	SHBR (odd) s.d. = +0.0990 = (0.0092)	-0.000449 (0.000401)		-0.01196 (0.00835)	-0.02200 (0.00731)	-0.0000127 (0.0000021)	-0.00206 (0.00105)	+0.00366 (0.00207)	-0.8785 (0.6510)	-0.2014 (0.1519)	*-0.0018 (0.0792)		0.00948
B	(3)	SHBR (even) s.d. = +0.1027 = (0.0106)	-0.000957 (0.000411)		-0.01793 (0.00584)	-0.02684 (0.00677)	-0.0000121 (0.0000028)	-0.00183 (0.00095)	+0.00530 (0.00259)	*-0.2631 (0.6912)	-0.3829 (0.1894)	-0.0798 (0.0558)		0.00950
	(1)	SHBR s.d. = +0.0955 = (0.0069)		-0.000914 (0.000844)	-0.01686 (0.00452)	-0.02532 (0.00413)	-0.0000121 (0.0000016)	-0.00163 (0.00068)	+0.00464 (0.00155)					0.00954
	(1)	SHBR s.d. = +0.1019 = (0.0077)		-0.001252 (0.000876)	-0.01567 (0.00450)	-0.02412 (0.00470)	-0.0000125 (0.0000016)	-0.00197 (0.00069)	+0.00409 (0.00156)	-0.5976 (0.4643)	-0.2135 (0.1040)	-0.0447 (0.0445)		0.00940
A	(2)	SHBR (odd) s.d. = +0.0972 = (0.0111)		*-0.000255 (0.001260)	-0.01309 (0.00839)	-0.02431 (0.00735)	-0.0000128 (0.0000021)	-0.00182 (0.00108)	+0.00373 (0.00209)	-0.9342 (0.6587)	-0.1726 (0.1511)	*+0.0180 (0.0804)		0.00956
B	(3)	SHBR (even) s.d. = +0.1000 = (0.0118)		-0.002462 (0.001332)	-0.01809 (0.00593)	-0.02670 (0.00696)	-0.0000116 (0.0000028)	-0.00184 (0.00096)	+0.00485 (0.00261)	*-0.2049 (0.7017)	-0.3435 (0.1698)	-0.0731 (0.0567)		0.00964

1 157 cements; Avg = 0.03552; S.D. = 0.01272.

2 79 cements.

3 78 cements.

* Coef./s.d. ratio less than 1.

percent probability level. In the equations for the "odds" and "evens" (eqs 3A, 3B, 5A, and 5B) the coef./s.d. ratios of MgO, Rb, and V were less than 1 in one or the other of the equations for the smaller groups of cements, and the coefficient for Ti had a reversal of sign in eq 3B.

Corresponding equations for the NAE cements are presented in table 9-24. The coefficients and coef./s.d. ratios of the independent variables are generally in accord with those of the previous table where the AE cements were included. In table 9-24, the coef./s.d. ratio for Ti was less than 1 when included with the other variables in eq 3. As in the previous table, the use of commonly determined variables in eqs 2 and 4 resulted in a highly significant reduction in the S.D. values. The additional use of the trace elements in eqs 3 and 5 did not result in highly significant reductions in the S.D. values (see table 9-34). The equations for the "odds" and "evens" (eqs 3A, 3B, 5A and 5B) indicated that the coef./s.d. ratios of Al_2O_3 , Rb, and V were less than 1 in one or the other of the pairs of equations for the smaller groups of cement.

Using the values for the coefficients of the independent variables in eq 3 of table 9-24 and the ranges of these variables, calculations were

made of their possible contributions to SHBR and the ranges of their contributions. These calculated values are presented in table 9-25. Differences in fineness and K_2O appear to have the greatest effect on the nonrestrained shrinkage of the bars at the time of breaking of the restrained annular specimens. Increases in Na_2O , C_3A , MgO and Cu may also be associated with lower values for SHBR but increases in the loss on ignition with higher values.

5.7. Ratio of Nonrestrained Shrinkage at Time of Cracking of Annular Specimens to the Nonrestrained Shrinkage at 7 Days, (SB/S7)

The frequency distributions of the different types of cements with respect to the ratios of the nonrestrained percentage-shrinkage values (SHBR/SHO7) or SB/S7 are presented in table 9-26. The shrinkage ratios varied over a wide range,—from 0.02 to over 0.54—indicating that the annular specimens of some cements cracked when very little shrinkage had occurred in the bars, and others only after considerably more shrinkage had occurred. This difference is, however, further complicated by the fact that the rate of shrinkage was not the same for all cement pastes as was noted in figure 9-7.

The results of plotting SB/S7 versus the various independent variables are presented in Fig. 9-5. The variables SiO_2 , Al_2O_3 , MgO, SO_3 , K_2O , total alkali, C_3A , C_2S , C_4AF , $S/(A + F)$, air permeability fineness and (air, 1:4 mortar) all had absolute values of 11 or more for the "quadrant-sum."

Selected equations are presented in table 9-27 to indicate the relationships of the combinations of independent variables to the ratios of the nonrestrained shrinkage at the time of cracking of the annular specimens divided by the nonrestrained shrinkage at 7 days for the AE + NAE cements. The relationship in eq 1 indicates that the shrinkage ratios are probably lower with higher water-requirements for normal consistency. Table 9-34 indicates that the

TABLE 9-25. Calculated contributions to the percentage shrinkage of neat-cement bars at the time of cracking of the annular specimens (SHBR)

Independent variable	Ranges of variables, percent	Coefficients from eq 3 table 9-24	Calculated contributions to SHBR	Calculated ranges of contributions to SHBR
			Const. = +0.101	
C_3A -----	0 to 15	-0.000673	0 to -0.010	0.010
Na_2O -----	0 to 0.75	-0.01496	0 to -0.011	0.011
K_2O -----	0 to 1.1	-0.0229	0 to -0.025	0.025
APF-----	*2500 to 5500	-0.0000124	-0.025 to -0.068	0.043
MgO-----	0 to 5.0	-0.00202	0 to -0.010	0.010
Loss-----	0.3 to 3.6	+0.00414	+0.001 to +0.015	0.014
Rb**-----	0 to 0.01	-0.6045	0 to -0.060	0.060
Cu-----	0 to 0.005	-0.2483	0 to -0.012	0.012
V**-----	0 to 0.1	-0.0555	0 to -0.006	0.006

*cm²/g.

**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

TABLE 9-26. Frequency distribution of the cements with respect to the ratio of the shrinkage of the neat cement bars at the time of cracking of the annular specimens to their shrinkage at 7 days (SHBR/SHO7 or SB/S7)

Type cement	Shrinkage ratio														Total
	0.02 to 0.06	0.06 to 0.10	0.10 to 0.14	0.14 to 0.18	0.18 to 0.22	0.22 to 0.26	0.26 to 0.30	0.30 to 0.34	0.34 to 0.38	0.38 to 0.42	0.42 to 0.46	0.46 to 0.50	0.50 to 0.54	0.54 and over	
	Number of cements														
I	1	10	10	8	8	5	10	7	4	2	2	1	4	1	73
IA		1	2	1	1	2			1						8
II		2		7	4	9	10	12	8	3	3				58
IIA					1					1					2
III	2	4	4	2	1	3	1	1							19
IIIA			1	1											2
IV, V					2	2	2	2	2	1		2	1		14
Total	3	17	17	19	17	21	23	22	15	8	5	3	5	1	176

TABLE 9-27. Coefficients for equations for AE+NAE cements relating the ratio of the shrink of the bars at 7 days (SHBR/SHO7), or

Equation	Note	Const.	C ₂ S	C ₄ AF	CaO	Fe ₂ O ₃
1-----	(1)	SB/S7 = +0.6197 s.d. = (0.1418)	-----	-----	-----	-----
2-----	(1)	SB/S7 = +0.6546 s.d. = (0.0687)	*+0.000803 (0.000923)	+0.00578 (0.00276)	-----	-----
3-----	(1)	SB/S7 = +0.6754 s.d. = (0.0666)	+0.001293 (0.000916)	+0.00843 (0.00276)	-----	-----
3A-----	(2)	SB/S7 (odd) = +0.7251 s.d. = (0.0931)	*+0.001006 (0.001243)	+0.00676 (0.00401)	-----	-----
3B-----	(2)	SB/S7 (even) = +0.6727 s.d. = (0.1056)	*+0.001121 (0.001445)	+0.00844 (0.00413)	-----	-----
4-----	(1)	SB/S7 = +1.827 s.d. = (0.679)	-----	-----	-0.01645 (0.00985)	*+0.00657 (0.01072)
5-----	(1)	SB/S7 = +1.980 s.d. = (0.655)	-----	-----	-0.01819 (0.00950)	+0.01433 (0.01055)
5A-----	(2)	SB/S7 (odd) = +1.797 s.d. = (0.884)	-----	-----	-0.01512 (0.01287)	*+0.01079 (0.01504)
5B-----	(2)	SB/S7 (even) = +1.893 s.d. = (1.036)	-----	-----	-0.01707 (0.01500)	*+0.01526 (0.01568)

¹ 167 cements; Avg = 0.2514; S.D. = 0.1125.² 84 cements.³ 83 cements.

* Coef./s.d. ratio less than 1.

relationship was significant at the 5-percent probability level.

The use of commonly determined variables in eq 2 resulted in a highly significant reduction in the S.D. value. C₂S was included although the coef./s.d. ratio was less than 1 in eq 2 but greater than 1 in eq 3. With the additional use of the trace elements, Rb, V, Ni, and Cu in eq 3, a further significant reduction in the S.D. value was obtained although none of the coefficients of any of the individual trace elements was by itself highly significant. The coef./s.d. ratio of C₂S, Rb, V, and Cu was less than 1 in eqs 3A or 3B, or both when the smaller groups of cements were used in computing the equations.

The use of the oxides in eq 4 instead of the potential compounds also resulted in a significant reduction in the S.D. value (see table

9-34). With the addition of the trace elements in eq 5 a further significant reduction was obtained. In the equations for the "odds" and "evens" (eqs 5A and 5B) the coef./s.d. ratios of Fe₂O₃, SO₃, Rb, and Cu were less than 1 in one or both of the equations for the smaller groups of cements and the sign of the coefficient for V was reversed in eq 5A.

A corresponding series of relationships computed for the NAE cements is presented in table 9-28. Equation 1 indicates the relationship of the shrinkage ratios to the water requirements for normal consistency of the neat cement pastes. Equations 2 and 4 present the relationships of commonly determined variables and eqs 3 and 5 the commonly determined variables with the addition of trace elements. In eq 5, SO₃ was also used. A highly significant

SO ₃	MgO	Na ₂ O	K ₂ O	APF	Rb	V	Ni	Cu	W ₁	S.D.
									-0.01488 (0.00572)	0.1106
	-0.01571 (0.00546)	-0.1745 (0.0371)	-0.2560 (0.0307)	-0.0000821 (0.0000127)						0.0783
	-0.01965 (0.00536)	-0.1653 (0.0362)	-0.2533 (0.0348)	-0.0000872 (0.0000123)	-5.748 (3.667)	-0.5124 (0.3763)	-3.663 (2.176)	-1.798 (0.853)		0.0752
	-0.02217 (0.00764)	-0.1523 (0.0457)	-0.2436 (0.0502)	-0.0000946 (0.0000165)	-10.406 (5.003)	*+0.6035 (0.6778)	-5.118 (3.322)	-2.040 (1.237)		0.0731
	-0.02032 (0.00832)	-0.2346 (0.0662)	-0.2644 (0.0537)	-0.0000802 (0.0000196)	*+0.899 (5.567)	-1.0049 (0.4935)	-5.515 (3.260)	*-0.765 (1.354)		0.0782
-0.0367 (0.0228)	-0.02672 (0.00860)	-0.1869 (0.0381)	-0.2563 (0.0340)	-0.0000739 (0.0000142)						0.0778
-0.0331 (0.0222)	-0.03135 (0.00838)	-0.1773 (0.0372)	-0.2607 (0.0379)	-0.0000822 (0.0000141)	-5.715 (3.651)	-0.3812 (0.3634)	-3.772 (2.175)	-1.819 (0.851)		0.0749
*+0.0127 (0.0320)	-0.03253 (0.01216)	-0.1632 (0.0475)	-0.2508 (0.0539)	-0.0000945 (0.0000187)	-10.687 (5.025)	+0.6916 (0.6646)	-5.248 (3.362)	-1.890 (1.239)		0.0732
-0.0408 (0.0328)	-0.03044 (0.01250)	-0.2425 (0.0665)	-0.2724 (0.0590)	-0.0000703 (0.0000230)	*+1.013 (5.551)	-0.8362 (0.4853)	-5.295 (3.248)	*-0.941 (1.358)		0.0780

reduction in the S.D. values was attained by using the additional independent variables in the equations (see table 9-34). It may be noted in tables 9-27 and 9-28 that the coefficients for Na₂O, K₂O, fineness, and MgO were all highly significant. An increase in each of these independent variables appeared associated with lower SB/S7 ratios.

The coefficients of the independent variables in eq 3 table 9-28 were used together with their approximate ranges of values to calculate their contributions to the SB/S7 ratio and the ranges of these contributions. These values are presented in table 9-29. Differences in fineness and K₂O were associated to the greatest extent with differences in the SB/S7 ratios. Differences in Na₂O, MgO, C₄AF, and Cu may also be associated with differences in the SB/S7 ratios.

5.8. Ratio of Nonrestrained Shrinkage of Bars at Time of Cracking of The Annular Specimens to the Nonrestrained Shrinkage of the Bars at 28 Days, (SHBR/SH28) or (SB/S28)

The frequency distributions of the shrinkage-ratio values (SB/S28) of the different types of cements are presented in table 9-30. A fairly broad distribution of the shrinkage-ratio values was obtained with each of the types of cement as classified. There was also an overlapping of the values in the different types. However, the median value of the Type III cements was lowest. The median value of the Type I cements was slightly higher, and the Types II, IV and V had still higher median values.

The results of the preliminary plots of SB/S28 versus various individual independent vari-

TABLE 9-28. Coefficients for equations for NAE cements relating the ratio of the shrinkage of the neat-cement bars at the time of cracking of the annular specimens to the shrinkage of the bars at 7 days (SHBR/SHOT) or SB/S7, to various independent variables

Equation	Note	Const.	C ₂ S	C ₄ AF	CeO	SO ₃	MgO	Na ₂ O	K ₂ O	APF	Rb	V	Ni	Cu	W ₁	S.D.
1	(1)	SB/S7 s.d. = +0.5924 = (0.1477)	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-0.01364 (0.00597)	0.1113
2	(1)	SB/S7 s.d. = +0.6619 = (0.0688)	*+0.000883 (0.000930)	+0.00624 (0.00278)	-----	-----	-0.01850 (0.00552)	-0.1721 (0.0373)	-0.2525 (0.0313)	-0.0000851 (0.0000129)	-----	-----	-----	-----	-----	0.0771
3	(1)	SB/S7 s.d. = +0.6776 = (0.0665)	+0.001391 (0.000922)	+0.00871 (0.00277)	-----	-----	-0.02214 (0.00539)	-0.1594 (0.0365)	-0.2421 (0.0361)	-0.0000895 (0.0000125)	-6.403 (3.730)	-0.5154 (0.3732)	-3.418 (2.163)	-1.785 (0.842)	-----	0.0738
3A	(2)	SB/S7 (odd) s.d. = +0.7230 = (0.0941)	*+0.000875 (0.001381)	+0.00784 (0.00374)	-----	-----	-0.02546 (0.00811)	-0.1478 (0.0503)	-0.2413 (0.0529)	-0.0000955 (0.0000166)	-6.076 (5.142)	+0.8467 (0.7130)	-6.811 (3.393)	-1.214 (1.111)	-----	0.0731
3B	(2)	SB/S7 (even) s.d. = +0.6325 = (0.0695)	+0.001737 (0.001357)	+0.00929 (0.00441)	-----	-----	-0.01800 (0.00760)	-0.1730 (0.0569)	-0.2534 (0.0528)	-0.0000794 (0.0000203)	-6.509 (5.622)	-0.9084 (0.4655)	-3.257 (3.145)	-2.736 (1.375)	-----	0.0762
4	(1)	SB/S7 s.d. = +2.025 = (0.492)	-----	-----	-0.01895 (0.00746)	-----	-0.03109 (0.00753)	-0.1904 (0.0377)	-0.2837 (0.0312)	-0.0000923 (0.0000118)	-----	-----	-----	-----	-----	0.0770
5	(1)	SB/S7 s.d. = +2.637 = (0.501)	-----	-----	-0.02722 (0.00752)	-0.02932 (0.02180)	-0.03987 (0.00740)	-0.1832 (0.0368)	-0.2661 (0.0388)	-0.0000882 (0.0000144)	-6.168 (3.738)	-0.3763 (0.3626)	-3.357 (2.173)	-1.584 (0.834)	-----	0.0740
5A	(2)	SB/S7 (odd) s.d. = +2.244 = (0.727)	-----	-----	-0.02086 (0.01098)	-0.03745 (0.03499)	-0.03941 (0.01126)	-0.1666 (0.0503)	-0.2489 (0.0376)	-0.0000908 (0.0000190)	-6.721 (5.198)	+0.8353 (0.6932)	-6.477 (3.468)	*-1.007 (1.106)	-----	0.0737
5B	(2)	SB/S7 (even) s.d. = +2.920 = (0.747)	-----	-----	-0.03184 (0.01103)	*-0.02681 (0.02987)	-0.03896 (0.01051)	-0.2010 (0.0580)	-0.2922 (0.0550)	-0.0000809 (0.0000245)	*-4.922 (5.583)	-0.6880 (0.4577)	-3.715 (3.117)	-2.671 (1.365)	-----	0.0761

1 155 cements; Avg = 0.2554; S.D. = 0.1128.

2 78 cements.

3 77 cements.

* Coef./s.d. ratio less than 1.

TABLE 9-29. Calculated contributions to the ratio of the shrinkage of the neat-cement bars at the time of cracking of the annular specimens to their shrinkage at 7 days, SHBR/S_{HO}7 or SB/S7

Independent variable	Ranges of variables, percent	Coefficients from eq 3 table 9-28	Calculated contributions to SB/S7	Calculated ranges of contributions to SB/S7
			Const. = +0.678	
C ₂ S**	5 to 50	+0.001391	+0.007 to +0.069	0.062
C ₄ AF	4 to 17	+0.00871	+0.035 to +0.148	0.113
MgO	0 to 5	-0.02214	0 to -0.111	0.111
Na ₂ O	0 to 0.75	-0.1594	0 to -0.120	0.120
K ₂ O	0 to 1.1	-0.2421	0 to -0.266	0.266
APF	*2500 to 5500	-0.0000895	-0.224 to -0.492	0.268
Rb**	0 to 0.01	-6.403	0 to -0.064	0.064
V**	0 to 0.1	-0.5154	0 to -0.052	0.052
Ni**	0 to 0.02	-3.418	0 to -0.068	0.068
Cu	0 to 0.05	-1.785	0 to -0.089	0.089

*cm²/g.
**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

ables are presented in figure 9-5. "Quadrant sums" with absolute values of 11 or more were obtained for the independent variables SiO₂, Al₂O₃, Fe₂O₃, SO, K₂O, C₃A, C₂S, C₄AF, A/F, S/(A + F), APF, and (air, 1:4 mortar).

Selected equations indicating the relationship of various independent variables to the ratio of the nonrestrained shrinkage of the bars at the time of cracking of the annular specimens to the nonrestrained shrinkage of the bars at 28 days are presented in table 9-31 for the AE + NAE cements. The use of the percentage water required for normal consistency in eq 1 did not result in a highly significant reduction in the S.D. value (see table 9-34). Using commonly determined variables found to have coef./s.d. ratios greater than 1 in eqs 2 and 4, highly significant reductions in the S.D. values

TABLE 9-30. Frequency distribution of the cements with respect to the ratio of the shrinkage of neat cement bars at the time of cracking of the annular specimens to their shrinkage at 28 days (SHBR/SH28 or SB/S28)

Type cement	Shrinkage ratio													Total
	0.02 to 0.06	0.06 to 0.10	0.10 to 0.14	0.14 to 0.18	0.18 to 0.22	0.22 to 0.26	0.26 to 0.30	0.30 to 0.34	0.34 to 0.38	0.38 to 0.42	0.42 to 0.46	0.46 to 0.50	0.50 and over	
	Number of cements													
I	8	15	11	12	11	8	1	5	1				1	73
IA		3	1	3		1								8
II	1	3	7	13	22	7	4	1						58
IIA				1			1							2
III	5	5	3	3	2	1								19
IIIA		1	1											2
IV, V			2	3	2	3	2	2						14
Total	14	27	25	35	37	20	8	8	1	0	0	0	1	176

were obtained, and with the additional use of the trace elements Rb, Ni, V, and Cu in eqs 3 and 5, further significant reductions in the S.D. values were obtained. In the equations for the "odds" and "evens" (eqs 3A, 3B, 5A, and 5B) all of the trace elements as well as C₂S had coef./s.d. ratios less than 1 in one or the other of the equations calculated with the smaller groups of cements.

A similar series of equations is presented in table 9-32 for the NAE cements. The use of commonly determined variables in eq 2 resulted in a highly significant reduction in the S.D. value, and the addition of the trace elements Rb, Ni, V, and Cu resulted in a further significant reduction in the S.D. value (see table 9-34). Comparing eqs 4 and 5 it may be noted that the coef./s.d. ratio for CaO was less than 1 when MgO was not used as an independent variable in equation 4 but was greater than 3 in eq 4 when MgO was used. The use of commonly determined variables in eqs 4 or 5 resulted in highly significant reductions in the

S.D. values and the additional use of the trace elements in eq 6 resulted in a further significant reduction in the S.D. value. The eqs calculated for the "odds" and "evens" in the array of cements (eqs 3A, 3B, 6A, and 6B) indicated that C₂S, Rb, Ni, and Cu had coef./s.d. ratios less than 1 in one or the other of the pairs of equations, and that the coefficient for V had a change of sign in eqs 3A and 6A.

In tables 9-31 and 9-32 the coefficients for fineness, K₂O, and Na₂O were the most highly significant and an increase in these variables was associated with a decrease in the SB/S28 ratio.

Using the coefficients of eq 3 of table 9-32 and the approximate ranges of values, calculations were made of the contributions and ranges of contributions of the various independent variables. The results of these calculations are presented in table 9-33. Fineness values and K₂O content of the cements had the greatest calculated ranges of contributions to the SB/S28 ratios.

TABLE 9-31. Coefficients for equations for AE + NAE cements relating the shrinkage of the neat-cement bars at the time of cracking of the annular specimens to the shrinkage of the bars at 28 days, (SHBR/SH28), or (SB/S28), to various independent variables

Equation	Note	Const.	C ₂ S	C ₄ AF	CaO	MgO	Na ₂ O	K ₂ O	APF	Rb	Ni	V	Cu	W ₁	S.D.
1	(1)	SB/S28 s.d. = +0.3782 = (0.0652)	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-0.00867 (0.00384)	0.07397
2	(1)	SB/S28 s.d. = +0.4233 = (0.0451)	+0.000644 (0.000607)	+0.00339 (0.00183)	-----	-0.00983 (0.00360)	-0.1371 (0.0239)	-0.1775 (0.0200)	-0.0000494 (0.0000083)	-----	-----	-----	-----	-----	0.05146
3	(1)	SB/S28 s.d. = +0.4376 = (0.0438)	+0.000972 (0.000602)	+0.00504 (0.00183)	-----	-0.01242 (0.00353)	-0.1306 (0.0233)	-0.1746 (0.0230)	-0.0000530 (0.0000081)	-3.932 (2.401)	-2.292 (1.444)	-0.3560 (0.2474)	-1.180 (0.563)	-----	0.04941
3A	(2)	SB/S28 (odd) s.d. = +0.4679 = (0.0620)	*+0.000594 (0.000831)	+0.00401 (0.00262)	-----	-0.01368 (0.00500)	-0.1342 (0.0365)	-0.1560 (0.0338)	-0.0000592 (0.0000109)	-7.145 (3.261)	*-1.554 (2.239)	+0.3616 (0.4642)	-1.404 (0.819)	-----	0.04812
3B	(2)	SB/S28 (even) s.d. = +0.4252 = (0.0669)	+0.000940 (0.000935)	+0.00572 (0.00270)	-----	-0.01260 (0.00530)	-0.1443 (0.0327)	-0.1911 (0.0344)	-0.0000463 (0.0000128)	*+0.416 (2.697)	-4.487 (2.064)	-0.6588 (0.3099)	*-0.667 (0.821)	-----	0.05084
4	(1)	SB/S28 s.d. = +1.484 = (0.319)	-----	-----	-0.01505 (0.00483)	-0.01985 (0.00473)	-0.1516 (0.0239)	-0.1998 (0.0197)	-0.0000541 (0.0000074)	-----	-----	-----	-----	-----	0.05068
5	(1)	SB/S28 s.d. = +1.778 = (0.317)	-----	-----	-0.01882 (0.00475)	-0.02473 (0.00472)	-0.1485 (0.0232)	-0.2041 (0.0229)	-0.0000594 (0.0000074)	-3.684 (2.366)	-2.649 (1.421)	-0.2708 (0.2368)	-1.032 (0.545)	-----	0.04876
5A	(2)	SB/S28 (odd) s.d. = +1.582 = (0.448)	-----	-----	-0.01590 (0.00680)	-0.02499 (0.00713)	-0.1472 (0.0361)	-0.1747 (0.0327)	-0.0000620 (0.0000099)	-7.369 (3.119)	*-1.868 (2.199)	+0.3803 (0.4398)	-1.235 (0.784)	-----	0.04708
5B	(2)	SB/S28 (even) s.d. = +1.831 = (0.475)	-----	-----	-0.01663 (0.00707)	-0.02496 (0.00668)	-0.1661 (0.0324)	-0.2258 (0.0340)	-0.0000543 (0.0000119)	*+1.117 (3.644)	-4.675 (2.016)	-0.5619 (0.3007)	*-0.568 (0.700)	-----	0.05040

1 167 cements; Avg = 0.1635; S.D. = 0.07487. 2 84 cements. 3 83 cements. * Coef./s.d. ratio less than 1.

TABLE 9-32. Coefficients for equations for NAE cements relating the ratio of the shrinkage of the neat-cement bars at the time of cracking of the annular specimens to the shrinkage of the bars at 28 days, (SHBR/SH28), or (SB/S28), to various independent variables

Equation	Note	Const.	C ₂ S	C ₄ AF	CaO	MgO	Na ₂ O	K ₂ O	APF	Rb	Ni	V	Cu	W ₁	S.D.
1	(1)	SB/S28 s.d. = +0.3651 = (0.0974)	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-0.008071 (0.004013)	0.07465
2	(1)	SB/S28 s.d. = +0.4294 = (0.0453)	+0.000686 (0.000612)	+0.00379 (0.00185)	-----	-0.01176 (0.00365)	-0.1374 (0.0240)	-0.1766 (0.0204)	-0.0000518 (0.0000085)	-----	-----	-----	-----	-----	0.05072
3	(1)	SB/S28 s.d. = +0.4402 = (0.0440)	+0.001019 (0.000609)	+0.00529 (0.00184)	-----	-0.01407 (0.00357)	-0.1292 (0.0236)	-0.1695 (0.0239)	-0.0000548 (0.0000082)	-4.126 (2.457)	-2.083 (1.442)	-0.3532 (0.2466)	-1.141 (0.558)	-----	0.04874
3A	(2)	SB/S28 (odd) s.d. = +0.4835 = (0.0637)	*+0.000894 (0.000913)	+0.00461 (0.00263)	-----	-0.01490 (0.00550)	-0.1019 (0.0423)	-0.1960 (0.0371)	-0.0000605 (0.0000113)	-5.864 (3.530)	-4.960 (2.325)	+0.6105 (0.4678)	-0.933 (0.817)	-----	0.04981
3B	(2)	SB/S28 (even) s.d. = +0.4024 = (0.0630)	+0.001275 (0.000848)	+0.00454 (0.00272)	-----	-0.01249 (0.00476)	-0.1403 (0.0296)	-0.1659 (0.0331)	-0.0000449 (0.0000126)	*-2.363 (3.469)	*-1.657 (1.921)	-0.5682 (0.2961)	-1.804 (0.853)	-----	0.04729
4	(1)	SB/S28 s.d. = +0.5554 = (0.2515)	-----	-----	*-0.000998 (0.003848)	-----	-0.1396 (0.0253)	-0.2108 (0.0212)	-0.0000576 (0.0000082)	-----	-----	-----	-----	-----	0.05320
5	(1)	SB/S28 s.d. = +1.557 = (0.320)	-----	-----	-0.01596 (0.00485)	-0.02228 (0.00481)	-0.1518 (0.0239)	-0.2003 (0.0200)	-0.0000568 (0.0000077)	-----	-----	-----	-----	-----	0.04991
6	(1)	SB/S28 s.d. = +1.864 = (0.3181)	-----	-----	-0.02000 (0.00479)	-0.02703 (0.00479)	-0.1465 (0.0233)	-0.2004 (0.0237)	-0.0000615 (0.0000076)	-3.979 (2.415)	-2.514 (1.415)	-0.2685 (0.2351)	-1.022 (0.541)	-----	0.04797
6A	(2)	SB/S28 (odd) s.d. = +1.697 = (0.440)	-----	-----	-0.01719 (0.00669)	-0.02557 (0.00718)	-0.1162 (0.0413)	-0.2176 (0.0359)	-0.0000645 (0.0000103)	-6.407 (3.451)	-5.224 (2.279)	+0.7924 (0.4419)	-0.789 (0.785)	-----	0.04875
6B	(2)	SB/S28 (even) s.d. = +1.913 = (0.473)	-----	-----	-0.02097 (0.00698)	-0.02683 (0.00657)	-0.1602 (0.0298)	-0.2061 (0.0328)	-0.0000563 (0.0000117)	*-1.576 (3.364)	-2.312 (1.868)	-0.5095 (0.2849)	-1.786 (0.827)	-----	0.04639

1 155 cements; Avg = 0.1655; S.D. = 0.07538. 2 78 cements. 3 77 cements. * Coef./s.d. ratio less than 1.

TABLE 9-33. Calculated contributions to the ratio of the shrinkage of the neat-cement bars at the time of cracking of the annular specimens to their shrinkage at 28 days, SHBR/SH28 or SB/S28

Independent variable	Ranges of variables, percent	Coefficients from eq. 3 table 9-32	Calculated contributions to SB/S28	Calculated ranges of contributions to SB/S28
C ₂ S**	5 to 50	+0.001019	Const. = +0.440	0.046
C ₄ AF	4 to 17	+0.00529	+0.005 to +0.051	0.069
MgO	0 to 5	-0.01407	+0.021 to +0.090	0.070
Na ₂ O	0 to 0.75	-0.1292	0 to -0.070	0.097
K ₂ O	0 to 1.1	-0.1695	0 to -0.097	0.186
APF	*2500 to 5500	-0.0000548	0 to -0.186	0.164
Rb**	0 to 0.01	-4.126	-0.137 to -0.301	0.041
Ni**	0 to 0.02	-2.083	0 to -0.041	0.042
V**	0 to 0.1	-0.3532	0 to -0.042	0.035
Cu	0 to 0.05	-1.141	0 to -0.035	0.057

*cm²/g.

**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

TABLE 9-34. "F" values for significance of reduction of variance due to added variables

Table	Equations*	"F" ratio	D.F.	Critical "F" Ratio	
				$\alpha=0.01$	$\alpha=0.05$
9-2-----	0,1	10.20	2:161	4.74	3.05
	0,2	17.84	7:156	2.75	2.07
	2,3	3.30	5:151	3.14	2.27
	0,4	17.27	8:155	2.62	2.00
	4,5	3.12	4:151	3.44	2.44
9-3-----	0,1	7.59	2:149	4.75	3.05
	0,2	14.98	7:144	2.77	2.07
	2,3	3.54	5:139	3.15	2.27
	0,4	15.12	7:144	2.77	2.07
	4,5	3.85	6:138	2.94	2.16
9-6-----	0,1	2.37	2:163	4.74	3.05
	0,2	5.91	4:161	3.43	2.44
	2,3	9.99	4:157	3.43	2.44
	3,4	5.00	3:154	3.90	2.68
	0,5	5.98	5:160	3.13	2.27
9-7-----	0,1	1.84	2:151	4.75	3.05
	0,2	4.41	4:149	3.44	2.44
	2,3	9.84	4:145	3.44	2.44
	3,4	4.69	4:141	3.44	2.44
	0,5	4.64	5:148	3.14	2.27
9-10-----	0,1	0.71	2:163	4.74	3.05
	0,2	3.58	5:160	3.14	2.27
	2,3	14.33	5:155	3.14	2.27
	3,4	5.36	4:151	3.44	2.44
	0,5	12.01	8:156	2.62	2.01
9-11-----	0,1	2.80	5:148	3.14	2.27
	1,2	23.08	3:145	3.90	2.68
	2,3	5.74	5:140	3.15	2.27
	0,4	11.36	8:145	2.63	2.01
	4,5	4.95	6:139	2.94	2.16
9-14-----	0,1	13.58	8:163	2.63	2.01
	1,2	12.67	3:160	3.89	2.68
	0,3	15.53	7:164	2.75	2.08
	3,4	12.86	3:161	3.89	2.68
9-15-----	0,1	13.22	8:151	2.63	2.01
	1,2	13.92	3:148	3.90	2.68
	0,3	14.96	7:152	2.76	2.08
	3,4	13.95	3:149	3.90	2.68
9-19-----	0,1	7.09	2:168	4.72	3.05
	0,2	25.5	8:162	2.60	2.00
	2,3	5.47	4:158	3.43	2.43
	0,4	29.0	7:163	2.75	2.07
	4,5	7.14	3:160	3.90	2.67
9-20-----	0,1	6.08	2:156	4.75	3.05
	0,2	23.1	8:150	2.63	2.00
	2,3	6.27	4:146	3.43	2.44
	0,4	26.6	7:151	2.76	2.08
	4,5	6.22	4:147	3.44	2.43

TABLE 9-34. "F" values for significance of reduction of variance due to added variables—Continued

Table	Equations*	"F" ratio	D.F.	Critical "F" Ratio	
				$\alpha=0.01$	$\alpha=0.05$
9-23-----	0,1	2.75	2:167	4.74	3.05
	0,2	19.03	7:162	2.75	2.08
	2,3	2.82	4:158	3.43	2.44
	0,4	18.59	7:162	2.75	2.08
	4,5	2.71	3:159	3.89	2.68
9-24-----	0,1	2.25	2:155	4.75	3.05
	0,2	18.95	7:150	2.76	2.08
	2,3	3.18	3:147	3.90	2.68
	0,4	18.44	7:150	2.76	2.08
	4,5	2.50	3:147	3.90	2.68
9-27-----	0,1	3.89	2:165	4.74	3.05
	0,2	20.75	9:158	2.52	1.93
	2,3	4.32	4:154	3.44	2.44
	0,4	23.77	8:159	2.63	2.01
	4,5	4.14	4:155	3.44	2.44
9-28-----	0,1	3.10	2:153	4.75	3.05
	0,2	26.25	7:148	2.76	2.08
	2,3	4.38	4:144	3.44	2.44
	0,4	30.61	6:149	2.94	2.16
	4,5	3.47	5:144	3.15	2.27
9-31-----	0,1	3.04	2:165	4.74	3.05
	0,2	27.64	7:160	2.75	2.08
	2,3	4.39	4:156	3.44	2.44
	0,4	33.91	6:161	2.92	2.16
	4,5	4.23	4:157	3.44	2.44
9-32-----	0,1	2.52	2:153	4.75	3.05
	0,2	27.77	7:148	2.76	2.08
	2,3	4.07	4:144	3.44	2.44
	0,4	32.24	5:150	3.15	2.27
	0,5	34.09	6:149	2.93	2.16
	5,6	4.07	4:145	3.44	2.44

*Equation 0 refers to the variance for the values themselves with no fitted equation for the AE+NAE cements.

5.9. Results of Duplicate Determinations

The results of a series of duplicate determinations made on 20 non-air-entraining cements are presented in table 9-35. The range of values with this group of cements was not so great as was obtained with those of the larger study. The S.D. values of the duplicate determinations are naturally much smaller than those obtained for the calculated equations. The duplicate determinations, though made on different days, did not extend over so long a period of time as the tests made on the larger group of cements. If laboratory conditions varied over the longer time period this may also have contributed to higher S.D. values.

5.10. Comparative Test-Values on Other Hydraulic Cements

The restrained and nonrestrained shrinkage tests were also made on two commercial portland-pozzolan cements and seven portland-blast-furnace-slag cements using the same test method previously described with neat cement pastes of normal consistency. The average time of cracking of the annular specimens was 11.3 hr with a range of 1.6 to 27.4 hr. The average shrinkage of the nonrestrained bars at the time of cracking of the annular specimens was 0.043 percent with a range of 0.016 percent to 0.074

TABLE 9-35. Comparison of estimated standard deviation values of duplicate determinations with those of the calculated equations

Test	Range of values for cements used for duplicate determinations	Estimated S.D. for duplicate determinations	Estimated S.D. values for equations	Eq and table in text
Shrinkage 24 hr.....	0.047% to 0.082%	0.002	0.010	3;9-3
Shrinkage 7 days.....	0.126% to 0.176%	0.003	0.024	4;9-7
Shrinkage 28 days.....	0.185% to 0.276%	0.004	0.029	3;9-11
Time of cracking annular specimens*	0.3 hr to 2.33 hr	0.87	5.36	
Shrinkage (nonrestrained) at time of cracking annular specimens.....	0.015% to 0.058%	0.002	0.009	3;9-24
Shrinkage time of break divided by nonrestrained shrinkage at 7 days.....	0.082 to 0.413	0.014	0.075	3;9-28
Shrinkage time of break divided by nonrestrained shrinkage at 28 days.....	0.051 to 0.264	0.009	0.049	3;9-32

* These values were calculated on the time-of-break rather than the square root of the time of break.

percent. The average shrinkage at 28 days was 0.240 percent with a range of 0.174 percent to 0.339 percent. The values for shrinkage of the nonrestrained bars and time of cracking of the annular specimens were within the ranges of the values obtained with the portland cements.

5.11. Comparison with Cements from Other Sources

Mather [19] has reported values for the non-restrained shrinkage and time of cracking of neat cements used in the Portland Cement Association long-time study of cement perfor-

mance in concrete [20]. Annular specimens made of these cements had an average time of breaking of 13.5 hr with a range of 0.4 to 31.8 hr. The 28-day nonrestrained shrinkage of these cements averaged 0.207 percent with a range of 0.147 to 0.313 percent. The average time of breaking of annular specimens made of these cements was somewhat greater, (13.5 versus 10.4 hr) indicated in this report, and the percentage shrinkage at 28 days somewhat lower for these cements, (0.207 versus 0.225) indicated in this report. The averages and ranges of values reported for both groups of cements were in reasonable agreement.

6. Discussion

6.1. Nonrestrained Shrinkage Versus Age

The average values for percentage shrinkage of the neat portland cement bars of normal consistency after curing for 24 hr in moist air, and then exposed to laboratory air were presented in tables 9-2, 9-6, 9-10 and 9-14 for 1, 7, 28 days and 6 months. The average values for the NAE cements are also presented graphically in figure 9-7. Also presented in this figure are percentage-shrinkage values for six individual cements at 1 hr to 1 year (on a log scale) to indicate the approximate range of values obtained. (Using the log scale for time resulted in the concavity of these curves.) Approximately 85 percent of the 1-year shrinkage had occurred at 28 days and there appeared to be very little shrinkage of most of the specimens after 6 months. The annular specimens of the three cements represented by the upper three curves (labeled 1, 2, and 3) cracked at 0.1, 0.6, and 1.1 hr, whereas those represented by the lower three curves (labeled 4, 5, and 6) cracked at 22, 14, and 17 hr respectively.

It is generally recognized that variations of CO₂ content in the laboratory air may have been a contributing factor to the shrinkage values which were obtained. However, the specimens were all made in one laboratory during a period of about 3 months, and carbonation may be expected to affect all specimens by about the same amount and would not have affected the relative values obtained.

6.2. Trends of Variables

Single equations from each of the tables relating the variables associated with values obtained for shrinkage at the various ages, shrinkage ratios of the prisms, and time of cracking of the annular specimens, all made of NAE cements, are presented in table 9-36. The equations are presented vertically instead of horizontally, and the coef./s.d. ratios, and calculated ranges of contributions of the independent variables are presented as in previous sections.

It may be noted in table 9-36 that an increase in the C₃A was probably associated with a decrease in SHBR but the effect was of minor importance. An increase in C₃A was associated with lower shrinkage at 1, 7, and 28 days when used with C₃A/SO₃ and SO₃ as well as other additional variables. An increase in the C₃A content is normally associated with greater shrinkage but, as indicated by Lerch [14] and Picket [15], the shrinkage of mortars and concretes made of high C₃A cements may be substantially reduced by use of an optimum gypsum content. The interaction of the three independent variables C₃A, C₃A/SO₃ and SO₃ in the equations may have accounted for the negative sign for the C₃A coefficient. The effect of differences in C₃A, C₃A/SO₃, and SO₃ on shrinkage of the neat bars was relatively great. It may be noted that for the neat cement bars, moist cured for 24 hr, an increase in the C₃A/

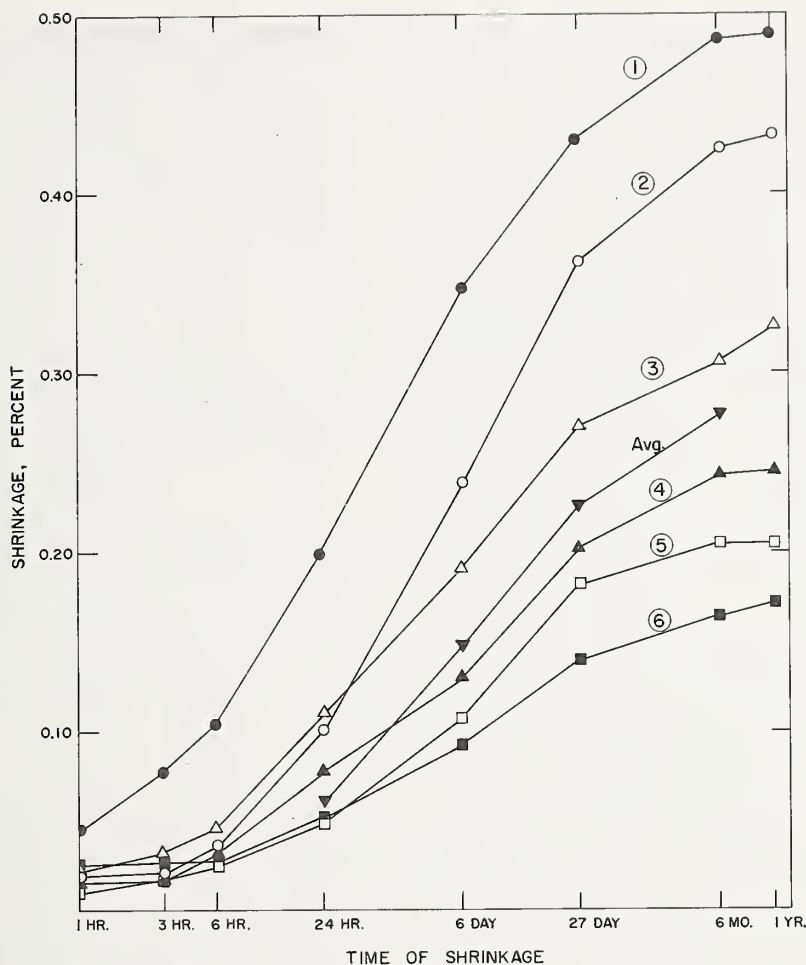


FIGURE 9-7. Results of plotting the average percentage shrinkage of neat portland cement pastes of normal consistency cured for 24 hours in moist air and then exposed to laboratory air for periods up to 6 months.

Also presented are the percentage-shrinkage values of six individual cements (labeled 1, 2, 3, 4, 5, and 6) versus time, selected to show the range of values obtained.

SO_3 ratio and an increase in SO_3 were both associated with greater shrinkage, the $\text{C}_3\text{A}/\text{SO}_3$ at 1, 7, 28 days and 6 months, and the SO_3 at 1 and 7 days.

The samples of the cements used for these shrinkage tests were obtained at a time when the upper limits of SO_3 were lower than they are in present specifications. It is therefore probable that the amounts of SO_3 were below the optimum values for most, if not all, of the cements. It would, therefore, be of questionable value to use the coefficients of SO_3 , $\text{C}_3\text{A}/\text{SO}_3$, and C_3A with the SO_3 values permitted in present specifications. Neither C_3A , $\text{C}_3\text{A}/\text{SO}_3$ nor SO_3 had coef./s.d. ratios greater than 1 in the equations for the time of cracking of the annular specimens or the shrinkage ratios, $\text{SB}/\text{S7}$ or $\text{SB}/\text{S28}$. The effect of C_3A and SO_3 as well as other independent variables associated with the shrinkage of concrete specimens will be dis-

cussed in the next section of this series of articles (see page 37).

In table 9-36 it may be noted that an increase in the C_4AF may be associated with a decrease in shrinkage at 24 hr, and possibly with an increase in the shrinkage ratios $\text{SB}/\text{S7}$ and $\text{SB}/\text{S28}$. An increase in C_4AF also appeared to be associated with an increase in the time of cracking of the annular specimens.

Differences in the C_3S and C_2S contents of the cements were not significantly related to the shrinkage values or ratios obtained for the different cements. An increase in C_2S may possibly be associated with a longer time period for the restrained annular specimens to break.

Higher MgO values appear associated with lower SHBR values and lower shrinkage ratios but possibly with greater shrinkage of the neat cement bars at 28 days and 6 months.

TABLE 9-36. Coefficients, coef./s. d. ratios, and calculated ranges of contributions of independent variables associated with nonrestrained shrinkage at various ages of neat cement bars of normal consistency, time of break of annular specimens, and shrinkage ratios for NAE cements

Equation	3	3	4	3	2	3	3	3
Table	9-24	9-3	9-7	9-11	9-15	9-20	9-28	9-32
Dependent Variable----	SHBR	SH24	SHO7	SH28	SH6M	(TOBR) ^{0.5}	SB/S7	SB/S28
Constant-----	+0.101	-0.020	-0.032	-0.023	+0.004	+6.24	+0.678	+0.440
C ₃ A, coef-----	-0.00067	-0.00553	-0.0165	-0.0204	-0.0060			
Coef./s.d.-----	2.5	3.2	4.4	4.2	2.3			
Calculated range *-----	0.001	0.083	0.248	0.306	0.090			
C ₄ AF, coef-----		-0.00137	** -0.00192	** -0.00217		+0.1455	+0.0087	+0.0053
Coef./s.d.-----		3.0	1.9	1.7		4.8	3.1	2.9
Calculated range-----		0.018	0.025	0.028		1.89	0.113	0.069
C ₂ S, coef-----					** +0.00067			
Coef./s.d.-----					1.2			
Calculated range-----					0.027			
C ₂ S, coef-----						+0.0307	** -0.0014	** +0.00102
Coef./s.d.-----						2.9	1.5	1.7
Calculated range-----						1.39	0.062	0.046
C ₂ A/SO ₃ , coef-----		+0.0101	+0.0281	+0.0368	+0.0175			
Coef./s.d.-----		3.4	4.3	4.2	3.5			
Calculated range-----		0.098	0.273	0.357	0.170			
SO ₃ , coef-----		+0.0351	+0.0722	+0.0788		** -0.266		
Coef./s.d.-----		4.4	4.2	2.5		1.1		
Calculated range-----		0.084	0.174	0.188		0.64		
MgO, coef-----	-0.00202			+0.0054	+0.00879	-0.1475	-0.0221	-0.0141
Coef./s.d.-----	3.0			2.4	2.8	2.4	4.1	3.9
Calculated range-----	0.010			0.044	0.044	0.73	0.111	0.070
Na ₂ O, coef-----	-0.0150	** +0.0100	+0.3133	+0.0921	+0.1392	-1.54	-0.1594	-0.1292
Coef./s.d.-----	3.3	1.9	2.5	6.0	6.9	3.8	4.4	5.5
Calculated range-----	0.011	0.008	0.023	0.069	0.104	1.15	0.120	0.097
K ₂ O, coef-----	-0.0229	+0.0200	+0.0383	+0.0779	+0.1233	-2.27	-0.2421	-0.1695
Coef./s.d.-----	5.0	4.4	3.5	5.4	6.0	5.7	6.7	7.1
Calculated range-----	0.025	0.022	0.042	0.086	0.135	2.50	0.266	0.186
APF, coef-----	-0.0000124	+0.0000046	+0.0000088	+0.0000133	+0.0000224	-0.001002	-0.000090	-0.0000548
Coef./s.d.-----	8.0	2.3	2.0	2.7	2.7	6.0	7.2	6.7
Calculated range-----	0.043	0.013	0.026	0.040	0.067	3.07	0.268	0.164
Air, 1:4 mortar, coef-----			** +0.00144					
Coef./s.d.-----			1.2					
Calculated range-----			0.029					
Loss, coef-----	+0.0041					+0.544		
Coef./s.d.-----	2.7					4.1		
Calculated range-----	0.010					1.80		
Ba, coef-----		** +0.0523	+0.1828	+0.3194	+0.575	-8.94		
Coef./s.d.-----		1.6	2.7	3.4	5.0	4.0		
Calculated range-----		0.010	0.037	0.064	0.115	1.79		
Cr, coef-----		+0.4422						
Coef./s.d.-----		2.1						
Calculated range-----		0.009						
Cu, coef-----	-0.2483						-1.785	-1.141
Coef./s.d.-----	2.4						2.1	2.0
Calculated range-----	0.012						0.089	0.057
Ni, coef-----					+2.617		** -3.418	** -2.083
Coef./s.d.-----					2.2		1.6	1.4
Calculated range-----					0.052		0.068	0.042
P, coef-----		** +0.0130	** +0.0262	+0.0489				
Coef./s.d.-----		1.6	1.6	2.3				
Calculated range-----		0.006	0.013	0.024				
Rb, coef-----	** -0.6045		+2.667	+3.021	** +3.686	-96.5	** -6.403	** -4.126
Coef./s.d.-----	1.3		2.4	2.1	1.8	2.4	1.7	1.7
Calculated range-----	0.060		0.027	0.030	0.037	0.96	0.064	0.041
V, coef-----	** -0.0555						** -0.515	** -0.3532
Coef./s.d.-----	1.3						1.4	1.4
Calculated range-----	0.006						0.052	0.035
Zn, coef-----		** -0.0323		** -0.0964				
Coef./s.d.-----		1.1		1.1				
Calculated range-----		0.006		0.019				
Zr, coef-----						** -2.89		
Coef./s.d.-----						1.7		
Calculated range-----						1.44		

* The calculated range was computed from the range of values for the cements times the coefficient for each independent variable.

** The independent variable is probably not significant as the coef./s.d. ratio is less than 2.

The results reported in table 9-36 confirm previously reported information that alkalis are associated with shrinkage. An increase in Na_2O was associated with lower nonrestrained-shrinkage values at the time of cracking of the restrained-shrinkage annular specimen, as well as the shorter period of time of cracking of the annular specimens. An increase in Na_2O was also associated with higher shrinkage values at 28 days and 6 months and possibly also at 7 days.

An increase in K_2O was also associated with lower SHBR values and lower TOBR values and with higher shrinkage at all ages of 1 day to 6 months. Differences in K_2O values and fineness of the cements appear to be the major factors causing early cracking of the annular specimens made of neat cements.

An increase in the fineness also appears associated with lower nonrestrained shrinkage values of the bars at the time of cracking of the annular specimens as well as the shrinkage ratios. Increases in fineness may possibly be associated with greater shrinkage values at ages 1 day to 6 months, but the coef./s.d. ratios obtained were significant only at the 5-percent probability level.

An increase in the loss on ignition may be associated with an increase in the time required for the annular specimens to crack, and an increase in the nonrestrained shrinkage of the bars at the time of cracking of the annular specimens.

Of the various trace elements in the cements, only Ba appeared highly significant in its relationship to values obtained in any of the tests. An increase in Ba appeared associated with an increase in shrinkage values at 28 days and 6 months and probably at 7 days. Increases in Ba are probably also associated with a shorter period of time for cracking of the annular specimens. Of the other trace elements, increases in Cr may be associated with increased 24-hr shrinkage, increases in Cu with decrease in nonrestrained shrinkage at the time of cracking of the annular specimens, increases in Ni with increases in 6-month shrinkage of the bars, increased P with increased 28-day shrinkage and increased Rb with increased 7- and 28-day nonrestrained shrinkage.

6.3. Shrinkage Characteristics of Neat Cement Pastes Versus Compressive Strength of 1:2.75 (Cement to Sand) Mortars

Two reports by A. de Sousa Coutinho [16, 17] indicated that with neat cement pastes, a greater cracking tendency was associated with higher strength and that the reverse was true with concretes. The findings by A. S. Coutinho may have contributed to the withdrawal of the proposed neat-cement restrained-shrinkage test from the RILEM specifications. No strength

tests, either tensile or compressive, were made on the neat cement pastes used for the non-restrained shrinkage tests or the time of cracking of the annular specimens. It therefore appeared desirable to determine the relationship of shrinkage values, ratios, and the time of cracking to the available compressive-strength data of the 1:2.75 (cement to graded Ottawa sand) mortars. These equations for the NAE cements are presented in table 9-37. Also presented are the reduction in variance or " F " values resulting from the calculation of a constant and the coefficient of an independent variable. An " F " value greater than 3.05 is significant at the 5-percent probability level and, if greater than 4.7, at the 1-percent probability level. It may be noted in table 9-37 that an increase in compressive-strength values of the mortar at 1, 3 and 7 days, (eqs 1, 2, and 3) was associated with the 24-hr shrinkage of the neat cement bars. Higher compressive-strength values of the mortars at 1 year were associated with lower shrinkage values for the neat cement bars at 7 and 28 days and at 6 months (eqs 10, 15 and 20). (The compressive-strength values of the 2-in cubes were all determined with moist specimens that had not been subjected to any drying cycles).

Higher compressive-strength values for the mortars at 1, 3, 7 and 28 days appeared associated with lower values of nonrestrained shrinkage of the neat cement bars at the time of cracking of the annular specimens, (eqs 21, 22, 23, and 24) as well as a shorter time for the annular specimens to crack, (eqs 26, 27, and 28). Equations 25 and 30 indicate that higher strength values for the mortars at 1 year may possibly be associated with higher nonrestrained shrinkage of the neat cements at the time of cracking of the annular specimens and a greater time before the annular specimens cracked. Equations 31 through 40, relating the nonrestrained-shrinkage ratios of the neat cements to the compressive strengths of the mortars, also indicate the reversal of the sign of the coefficient for the 1-year compressive-strength relationship.

The relationships in table 9-37 indicate that variables associated with the shrinkage tests of the neat cement pastes may also be associated with the compressive strengths. Referring to section 7 of this series and table 7-77 of section 7 [11] it was noted that, although higher values for fineness and C_3A were associated with higher compressive strengths at the early ages, higher values for C_3A were associated with lower strength at 1 year (ST1Y). Differences in fineness and C_3A values were of major significance in the equations for strength of the mortars at 1 and 3 days but not in the equations for shrinkage of the neat cements at 24 hr. A comparison of the effect of the alkalis Na_2O and K_2O in tables 7-77 and 9-36 are also of

TABLE 9-37. Relationship between shrinkage, shrinkage ratios, and time of cracking of neat portland cement pastes, and the compressive strength in psi of 1:2.75 (cement to sand) mortars at different ages

Equation	Note	Dependent variables	Independent variables						S.D.	"F"
			Const.	STO1	STO3	STO7	ST28	ST1Y		
1	(1)	SH24 =	+0.0541 s.d. (0.0036)	+0.0000121 (0.0000034)					0.0178	6.4
2		SH24 =	+0.0476 s.d. (0.0044)		+0.0000084 (0.0000019)				0.0175	9.4
3		SH24 =	+0.0466 s.d. (0.0052)			+0.0000058 (0.0000015)			0.0177	7.4
4		SH24 =	+0.0571 s.d. (0.0089)				+0.00000172 (0.00000171)		0.0184	0.5
5		SH24 =	+0.0897 s.d. (0.1117)					-0.0000038 (0.0000018)	0.0182	2.1
6	(2)	SHO7 =	+0.1414 s.d. (0.0075)	+0.0000089 (0.0000070)					0.0370	0.8
7		SHO7 =	+0.1338 s.d. (0.0093)		+0.0000075 (0.0000041)				0.0368	1.6
8		SHO7 =	+0.1323 s.d. (0.0108)			+0.0000054 (0.0000032)			0.0369	1.4
9		SHO7 =	+0.1609 s.d. (0.0179)				*-0.0000021 (0.0000034)		0.0372	0.2
10		SHO7 =	+0.2174 s.d. (0.0233)					-0.0000107 (0.0000037)	0.0363	4.2
11	(3)	SH28 =	+0.2200 s.d. (0.0102)	*+0.0000072 (0.0000097)					0.0508	0.3
12		SH28 =	+0.2094 s.d. (0.0128)		+0.0000081 (0.0000056)				0.0505	1.1
13		SH28 =	+0.2058 s.d. (0.0148)			+0.0000064 (0.0000043)			0.0505	1.1
14		SH28 =	+0.2424 s.d. (0.0245)				*-0.0000030 (0.0000047)		0.0508	0.2
15		SH28 =	+0.3537 s.d. (0.0311)					-0.0000201 (0.0000049)	0.0484	8.4
16	(4)	SH6M =	+0.2681 s.d. (0.0125)	*+0.0000082 (0.0000118)					0.0619	0.2
17		SH6M =	+0.2587 s.d. (0.0156)		+0.0000080 (0.0000068)				0.0617	0.7
18		SH6M =	+0.2560 s.d. (0.0181)			+0.0000061 (0.0000053)			0.0617	0.6
19		SH6M =	+0.2970 s.d. (0.0298)				*-0.0000041 (0.0000057)		0.0619	0.2
20		SH6M =	+0.4340 s.d. (0.0378)					-0.0000250 (0.0000059)	0.0588	8.9
21	(5)	SHBR =	+0.0484 s.d. (0.0023)	-0.0000134 (0.0000021)					0.0114	19.8
22		SHBR =	+0.0505 s.d. (0.0029)		-0.0000070 (0.0000012)				0.0116	15.6
23		SHRB =	+0.0530 s.d. (0.0034)			-0.00000534 (0.00000099)			0.0117	14.7
24		SHBR =	+0.0527 s.d. (0.0060)				-0.00000340 (0.00000116)		0.0124	4.3
25		SHBR =	+0.0143 s.d. (0.0082)					+0.00000331 (0.00000128)	0.0125	3.3
26	(6)	TOBR =	+17.81 s.d. (1.38)	-0.00769 (0.00129)					6.86	17.8
27		TOBR =	+20.25 s.d. (1.69)		-0.00456 (0.00073)				6.80	19.4
28		TOBR =	+22.38 s.d. (1.98)			-0.00364 (0.00057)			6.77	20.28
29		TOBR =	+22.78 s.d. (3.57)				-0.00243 (0.00069)		7.31	6.30
30		TOBR =	-2.78 s.d. (4.88)					+0.00206 (0.00077)	7.42	3.64
31	(7)	SB/S7 =	+0.3593 s.d. (0.0208)	-0.000110 (0.000020)					0.1037	15.9
32		SB/S7 =	+0.3793 s.d. (0.0261)		-0.000058 (0.000011)				0.1051	13.4
33		SB/S7 =	+0.4048 s.d. (0.0306)			-0.0000461 (0.0000089)			0.1051	13.5
34		SB/S7 =	+0.3680 s.d. (0.0547)				-0.0000227 (0.0000105)		0.1121	2.3
35		SB/S7 =	+0.0128 s.d. (0.0721)					+0.0000378 (0.0000113)	0.1099	5.6
36	(8)	SB/S28 =	+0.2363 s.d. (0.0139)	-0.0000724 (0.0000130)					0.0691	15.5
37		SB/S28 =	+0.2493 s.d. (0.0174)		-0.0000383 (0.0000075)				0.0701	13.0
38		SB/S28 =	+0.2661 s.d. (0.0204)			-0.0000303 (0.0000059)			0.0700	13.1
39		SB/S28 =	+0.2453 s.d. (0.0364)				-0.0000155 (0.0000070)		0.0745	2.5
40		SB/S28 =	-0.0095 s.d. (0.0476)					+0.0000277 (0.0000075)	0.0725	6.8

* Coef./s.d. ratio less than 1.

¹ 160 cements, Avg = 0.0658, S. D. = 0.0184

² " " = 0.1501, " = 0.0371

³ " " = 0.2670, " = 0.0507

⁴ " " = 0.2761, " = 0.0618

⁵ 159 cements, Avg = 0.0353, S. D. = 0.01267

⁶ " " = 10.28, " = 7.54

⁷ " " = 0.2515, " = 0.1134

⁸ " " = 0.1654, " = 0.0754

interest. Differences in Na_2O were not associated with early strength in equations in table 7-77 but K_2O was. In equations of table 9-36 differences in K_2O content was of major significance in the time of cracking of the annular specimens. Increases in both Na_2O and K_2O were associated with decreases in the nonrestrained shrinkage at the time of cracking of the annular specimens.

The literature on shrinkage and cracking of portland cement pastes, mortars and concretes has indicated that many variables are involved. Among these are the size and shape of the specimens, the amount of hydration product, the gel-space ratios [21], the moisture gradient within a specimen, the moisture diffusivity, the ability of the structural units in the hydration products to adjust to the stresses produced by

shrinkage called "creep" or semi-plastic flow, and the amount of restraint offered by unhydrated cement particles in the cement paste, or the aggregates in the mortars or concretes.

The neat cement pastes of normal consistency, with a water-cement ratio of about 0.25, hydrated for only 24 hr, may be expected to have a considerable amount of unhydrated cement, and the unhydrated particles would be closely adjacent to each other. This would result in a considerable restraint to shrinkage in the paste, or at least more so than in concrete where the water-cement ratio may be more than twice as great. Shrinkage tests made on concrete are usually made after a longer curing period and under these conditions, the amount and the nature of the hydration product may also differ from that at 24 hr.

7. Summary and Conclusions

(1) Determinations were made of the drying-shrinkage of $1 \times 1 \times 10$ -in (effective gage length) bars made of neat cement pastes. The specimens made of 183 cements were stored for 24 hr in moist air and then exposed to laboratory air at 50 percent relative humidity. Tests were also made of the time of cracking of annular specimens made of the same pastes and restrained from shrinking by a steel disc. The storage conditions were the same as for the bars. Multivariable regression equations were used to determine which of the independent variables or combinations of variables, such as fineness and composition of the cements, as well as trace elements in the cements were associated to a significant degree with the test-values obtained. The test values used as dependent variables were the percentage shrinkage at 1, 7, 28 days, and 6 months, the percentage shrinkage of the bars at the time of cracking of the annular specimens, and the time of cracking of the annular specimens restrained from shrinking. Also used as dependent variables were the ratios of the nonrestrained shrinkage at the time of cracking of the annular specimens to the nonrestrained shrinkage at 7 and 28 days.

(2) The frequency distributions of the cements with respect to percentage shrinkage at 1, 7, 28 days, and 6 months indicated a wide distribution of values especially for the Type I cements. There was an overlapping of the percentage-shrinkage values of the cements in the various types as classified. It is then not possible to say that a cement, because it conforms with the chemical composition and physical requirements of any one type of cement, will therefore have greater or less shrinkage than a cement meeting requirements for some other type.

(3) The time of cracking of the annular specimens ranged from 0.1 hr to more than 40 hr after being removed from moist storage and placed in laboratory air. The average time of cracking was about 10 hr. Eighty-five percent of the Types III and IIIA, 59 percent of the Types I and IA, 41 percent of the Types II and IIA, and 7 percent of the Types IV and V cracked in less than 10 hr. The cements classified as Type I had the greatest range of values, but there was a great deal of overlapping of the values obtained for the cements of the different types.

(4) The percentage shrinkage of the bars at the time of cracking of the annular specimens ranged from about 0.010 percent to more than 0.065 percent with an average of about 0.035 percent. About 76 percent of the Types III and IIIA, 43 percent of the Types I and IA, 27 percent of the Types II and IIA, and 14 percent of the Types IV and V had shrinkage values less than the average. This indicates, to some extent, the overlapping of values obtained with the different types of cement.

(5) The ratios of the percentage shrinkage of the neat cement bars at the time of cracking of the annular specimens to the shrinkage at 7 or 28 days were calculated as possible indications of plastic deformation or creep of the hydrated pastes. A considerable range of values was obtained, with overlapping of the values for the cements of the different types. The majority of the Types III and I cements had values below the average which was not the case with the Types II, IV, and V cements.

(6) An increase in the percentage water required for normal consistency of the neat cement pastes was associated with an increase in the shrinkage at 24 hr, but not at later ages. An increase in the percentage water required for normal consistency of the neat cement

pastes was also associated with a decrease in the time of cracking of the annular specimens.

(7) Multivariable regression equations were calculated to determine which of the available independent variables were associated with the various determined shrinkage values or characteristics. Typical equations were compiled in table 9-36¹².

(7.1) Higher C_3A/SO_3 ratios were associated with higher shrinkage values at 1, 7, 28 days, and 6 months when used together with C_3A and SO_3 as independent variables. Under these conditions, higher SO_3 values were also associated with higher shrinkage, but higher C_3A values were associated with lower shrinkage values. Differences in values of these independent variables showed the greatest association with differences in shrinkage values at 1, 7, and 28 days for all of the variables studied. None of these independent variables or combinations were significantly associated with the time of cracking of the annular specimens nor the shrinkage ratios.

(7.2) An increase in C_4AF was associated with a decrease in shrinkage at 24 hr and an increase in the time required for the annular specimens to crack.

(7.3) Differences in C_3S and C_2S of the cements were apparently not associated with the shrinkage values obtained. An increase in C_2S is probably associated with an increase in the time required for the annular specimens to crack.

(7.4) An increase in MgO may possibly be associated with an increase in shrinkage at 7 and 28 days and with a decrease in shrinkage of the bars at the time of cracking of the annular specimens.

(7.5) An increase in Na_2O was associated with an increase in shrinkage at 28 days and 6 months and a decrease in the time of cracking of the annular specimens and the nonrestrained shrinkage when the annular specimens cracked.

(7.6) An increase in K_2O was associated with an increase in shrinkage at all ages and with a decrease in the time of cracking of the annular specimens, and with a decrease of the nonrestrained shrinkage at the time of cracking of the annular specimens.

(7.7) An increase in the fineness of the cements was associated with a decrease in the nonrestrained shrinkage at the time of cracking of the annular specimens and with a decrease in the time required for the annular specimens to crack and possibly with an increase in shrinkage at all ages.

(7.8) An increase in the loss-on-ignition values was possibly associated with an increase in the time of cracking of the annular specimens

and an increase in the nonrestrained shrinkage at the time of cracking.

(7.9) The use of trace elements found to have a coef./s.d. ratio greater than 1 in the equations resulted, in most instances, in a reduction in variance significant at the 1-percent probability level. Except for Ba, (and Na_2O and K_2O , as previously mentioned), none of the individual trace elements were associated with the shrinkage characteristics to a highly significant degree. An increase in Ba is probably associated with an increase in shrinkage at 28 days and 6 months and possibly at 7 days. Increases in Ba and Rb are possibly associated with a decrease in the time required for the annular specimens to crack. Cr, Cu, Ni, P, and Rb may possibly be associated with shrinking at one or more ages.

(8) The use of the major oxides instead of the calculated potential compound-composition values of the cements (and also using the Al_2O_3/SO_3 ratios) resulted in equations with approximately the same reduction in variance as when the compound-composition values were used.

(9) Equations computed for all cements, i.e., the air-entraining cements together with the non-air-entraining cements, resulted in coefficients and coef./s.d. ratios of the independent variables in reasonable accord with those obtained when only the non-air-entraining cements were used.

(10) An increase in compressive strength of the 1:2.75 (cement to Ottawa sand) mortars at 1 day was associated with an increase in shrinkage values at 24 hr of the bars made of neat cement paste of normal consistency. An increase in compressive-strength values of these mortars at early ages was associated with a decrease in the time required for the annular specimens to crack and a decrease in nonrestrained shrinkage at the time of cracking of the annular specimens. An increase in the compressive-strength values of these mortars at 1 year was associated with an increase in the nonrestrained shrinkage of the neat cement bars at 7, 28 days, and 6 months.

(11) The estimated standard deviations of results of duplicate determinations made on different days were much lower than those computed from the equations using available independent variables. It therefore appears that other independent variables or combinations of independent variables may contribute to the shrinkage characteristics of hydrated neat portland pastes.

(12) The averages and ranges of values for 28-day shrinkage, and time of breaking with restrained shrinkage, of the cements used in this investigation were in reasonable agreement with those reported for the PCA long-time study of cement performance in concrete.

¹² Remarks about relations with time of break of annular specimens are based on the square root of time of break. See footnote 10, subsection 5.5.

In addition to the acknowledgements indicated in previous sections of this series of articles, special acknowledgements are made to M. R. DeFore and A. C. Figlia, who conducted

most of the tests, and to Paul Worksman for studies of the reproducibility of the shrinkage values and characteristics.

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Section 10. Shrinkage and Expansion of Concrete

R. L. Blaine and H. T. Arni

The relationship between the cement characteristics and the drying shrinkages of concretes made of a large number of cements of different types and compositions, as well as the subsequent expansions when the concretes were rewetted, were studied by fitting multivariable regression equations with the aid of a digital computer. Specimens $6 \times 8 \times 16$ inches, were made using 5.5 bags of cement (nominal) per cubic yard with a water/cement ratio of 0.635, as well as with a slump of 5 ± 1 inches. Increases in C_3A , C_4AF , fineness of the cement, and air content of the concrete were associated with increases in both shrinkage on drying and expansion on rewetting of the concretes. Increases in Na_2O , SO_3 , and ignition loss of the cements were associated with decreases in the shrinkage values of the concretes. None of the individual trace elements, except possibly Rb and SrO, were associated to a highly significant degree with shrinkage or expansion characteristics of the concretes. The use of trace elements in equations with other more commonly determined variables resulted in a significant reduction in variance. There was no relationship between the shrinkage of concrete specimens moist cured 14 days and then dried in laboratory air for eight weeks, and the shrinkage of neat cements, moist cured only 24 hours and then dried in laboratory air. Higher values for dynamic Young's modulus of elasticity of the concretes were associated with lower shrinkage and expansion values.

Key Words: Cement composition; expansion-shrinkage ratios; portland cement concrete; shrinkage of concrete; trace elements.

CONTENTS

	Page		Page
1. Introduction.....	37	b. Residual shrinkage of series A concretes made having a slump of 5 ± 1 in. (ADSX).....	54
2. Materials.....	38	6.5. Ratio of percentage expansion divided by the percentage shrinkage.....	57
2.1. Cements.....	38	a. Ratio of expansion divided by the shrinkage of series O concretes made with a water/cement ratio of 0.635 (OREX = OEXP/OSHR).....	57
3. Methods of test.....	38	b. Ratio of expansion divided by the shrinkage of concretes made having a slump of 5 ± 1 in. (AREX = AEXP/ASHR).....	57
3.1. Preparation of concretes.....	38	6.6. The ratio of the residual shrinkage after rewetting to the initial shrinkage (OREX and AREX).....	61
3.2. Test specimens.....	38	6.7. Shrinkage of tops and bottoms of concrete specimens as cast.....	64
3.3. Storage of specimens.....	39	7. Discussion.....	67
3.4. Shrinkage and expansion measurements.....	39	7.1. Comparison of equations for the dependent variables.....	67
3.5. Weight loss and gain.....	40	7.2. The effect of trace elements.....	69
4. Abbreviations.....	40	7.3. Agreement of duplicate determinations of percentage shrinkage and percentage expansion.....	71
5. Statistical analyses.....	40	7.4. Shrinkage and expansion of concrete versus shrinkage characteristics of neat cement.....	75
6. Results of tests.....	41	7.5. Shrinkage and expansion of concrete versus the dynamic Young's modulus of elasticity.....	75
6.1. Cement content of concretes.....	41	7.6. Top and bottom shrinkage and expansion.....	
6.2. Drying shrinkage of concretes.....	42	7.7. Comparative test values on other hydraulic cements.....	76
a. Drying shrinkage of series O concretes having a water/cement ratio of 0.635 (OSHR).....	42	8. Summary and conclusions.....	76
b. Drying shrinkage of series A concretes having a 5 ± 1 in. slump (ASHR).....	43	9. References.....	77
6.3. Expansion of concretes as a result of rewetting.....	47		
a. Expansion of series O concretes made with a water/cement ratio of 0.635, air dried, and then rewetted (OEXP).....	47		
b. Expansion of series A concretes made with a slump of 5 ± 1 in., air-dried and then rewetted (AEXP).....	50		
6.4. Residual shrinkage or difference between the percentage shrinkage during drying and the percentage expansion with subsequent rewetting.....	51		
a. Residual shrinkage of series O concretes made with a water/cement ratio of 0.635 (ODSX).....	51		

1. Introduction

As indicated in the previous section of this series of articles, there have been a great number of studies and articles published relative to the drying shrinkage of cement and concrete.

L'Hermite [1]¹ has listed many of these, and has presented not only an excellent summary,

¹ Figures in brackets refer to literature references at the end of this section, p. 77.

but also theories on the relationships of the various types of volume change. There are many recognized variables that affect the drying shrinkage of concrete, including the water/cement ratio and the kind of aggregates used, as well as the amount of curing or hydration of the cement prior to exposure of specimens to drying. Other variables may include the size and shape of the test specimens as well as the temperature and humidity at which they are dried.

L'Hermite has indicated [1] that different types of hydraulic cements may have quite different shrinkage characteristics, and that the influence of chemical composition of cement clinker on shrinkage and creep has not yet been studied. Swayze has indicated [2] that the effect of the type of cement on ultimate shrinkage of concrete is insignificant, but he did find considerable differences in shrinkage with both neat cements and mortars. The relationship, or lack of relationship, between the shrinkage of neat cement and the shrinkage of concrete has long been a matter of controversy. The differing observations of L'Hermite and Swayze appeared to justify a further study of this phenomenon. The previous section dealt with the

variables which were associated with the shrinkage of neat cement paste; this section deals with the shrinkage of the concretes made with the same 199 portland cements.

Portland cement concretes that have been dried expand when rewetted but the amount of expansion is usually less than the original contraction. The variables associated with the expansion and the residual contraction of concretes have not been well established. Because of this lack of knowledge studies were also made of the expansion and the residual contraction of the concretes as well as various shrinkage-expansion ratios using the same specimens, employing the statistical procedures which have been used throughout this series of articles.

Because of the limited quantities of the different cements, it was not possible to study all of the known variables. The investigation was limited to the determination of the effects on shrinkage and expansion of the physical and chemical properties of the cements and the air contents of two series of concretes, one made with a constant water/cement ratio and the other with a 5 ± 1 -in slump.

2. Materials

2.1. Cements

The cements used in this investigation have previously been described. The frequency distributions of the results of chemical analyses as well as spectrochemical analyses were reported in section 1, part 1 of this series of articles [3]. Other tests with these cements have been reported in parts 2 and 3 of this series of articles [4] [5], and the variables as-

sociated with the shrinkage characteristics of the neat portland cement pastes were presented in the previous section (see page 1). Most of the 199 commercial portland cement were obtained from different areas of the United States, but a few were from other countries. The cements were classified as to type, principally on the basis of chemical and physical properties².

3. Methods of Test

3.1. Preparation of Concretes

The details of the proportioning and mixing of the concretes were described in part 1, section 1 of this series of articles [3].

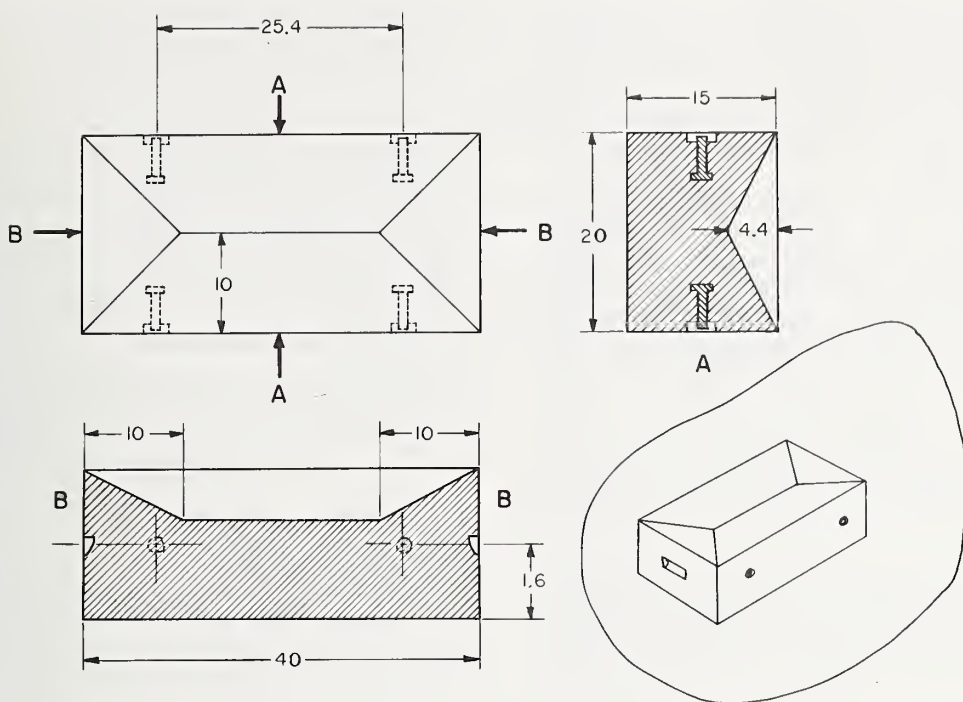
Two series of concretes were made with these cements. One series, (Series O), was made with a constant water/cement ratio (w/c) of 0.635. A second series, (Series A), was made in which the water was varied, if necessary, to obtain a concrete with a 5 ± 1 -in slump. In both series, the cement-to-aggregate weight ratio was constant. The quantities of cement and aggregate per unit volume of concrete varied to some extent from the nominal $5\frac{1}{2}$ bags of cement per cubic yard as designed, primarily because of variations of the entrained and entrapped air. The variations of the cement content will be reported in a later

subsection. The mixing and placing of the concretes were performed at 73 °F.

3.2. Test Specimens

The $6 \times 8 \times 16$ -in concrete blocks used for the shrinkage and expansion measurements are shown in figure 10-1. They were cast in plastic-lined, paraffined-hardwood molds. The sides and bottoms of the molds were bolted together using heavy graphite grease in all joints to prevent leakage or loss of water. No oil or grease was used on the plastic liner. Metal frames fastened to the molds were used to hold the gage points ($5/16 \times 1\frac{1}{2}$ -in phosphor-bronze bolts) in position, 10 in, center to center, in

² On the basis of this classification there were 82 Type I, 8 Type IA, 68 Type II, 3 Type IIA, 20 Type III, 3 Type IIIA, 3 Type IV, and 12 Type V cements.



NOTE: ALL DIMENSIONS IN CENTIMETERS

FIGURE 10-1. Design and dimensions of specimen used for determining shrinkage and expansion values of concretes made from different cements.

(The purpose of the depression in the one side was to retain water and ice when specimens were placed with this side up in later outdoor exposure tests).

the top and bottom of the concrete block as cast until the molds could be stripped. (The concrete blocks were made for outdoor exposure tests after completion of the laboratory measurements, and the purpose of the 2-in depression in the one side was to hold rain, melted snow, and ice in the weathering test).

Two specimens were made from each of the cements, one of each of the two series of concrete, the one with a water/cement ratio of 0.635 and the other with a 5 ± 1 -in slump. Each specimen was cast with a 6×16 -in side as a base. The 8-in depth of concrete was placed in three layers, and each layer was tamped 25 times with a $\frac{5}{8}$ -in diameter bullet-nosed tamping rod. The sides of the mold were spaded after each layer, and the mold was tapped vigorously to remove entrapped air.

3.3. Storage of Specimens

The concrete specimens remained in the molds covered with wet burlap for the first 24 hr. After stripping the molds, drilling the gage holes, and making length and weight measurements, the specimens were placed in a fog room at 100 percent relative humidity for 13 days. The concrete specimens were then placed on end with five sides exposed to laboratory air at

73 °F and 50 ± 5 -percent relative humidity for 8 weeks. They were then immersed in water at 73 °F for 4 weeks.

3.4. Shrinkage and Expansion Measurements

The gage holes were drilled in the phosphor-bronze bolts after the molds were stripped at about 24 hr. A 10-in gage-length Whittemore-type strain gage together with an invar-steel reference bar was used to measure changes in the distances between the gage holes in both the tops of the specimens as cast as well as the gage holes in the bottoms of specimens as cast. The first measurements were made as soon as possible after removal from the molds. Measurements were also made after the concrete specimens had been stored as indicated in par. 3.3. The length changes as determined by means of the Whittemore gage were multiplied by 10 to arrive at values for percentage length change.

The percentage shrinkage was calculated from the difference between the measurements made after storage in the fog room and those made after the additional 8 weeks in laboratory air. The calculated values for percentage shrinkage will generally be referred to simply as shrinkage or shrinkage values. The percent-

age expansion was calculated from the differences between the measurements made after the 8 weeks in laboratory air and those made after the subsequent 4 weeks in water. The calculated values for percentage expansion will generally be referred to simply as expansion or expansion values.

The averages of the top and bottom measurements were used in calculating the shrinkage and expansion value unless otherwise stated. The values reported were for a single specimen except for those cements where the use of a w/c of 0.635 resulted in a slump of 5 ± 1 in, in which case the average of the two specimens was used.

3.5. Weight Loss and Gain

At the times that length measurements were made, the specimens were also weighed. The

percentage weight loss was calculated from the weight after the moist-air storage and the weight after the 8-week drying in laboratory air. The percentage weight-gain was calculated from the weight of the specimen after drying in laboratory air for 8 weeks and the weight after the subsequent 4 weeks in water.

It was noted that there was generally a slight weight gain (and also expansion) of the specimens in the 13 days of moist-air storage. This gain may have occurred because of inadequate moist curing during the first 24 hr, or because of loss of weight during the period after the molds were stripped and the gage holes drilled and initial measurements made and before the specimens were placed in the fog room.

4. Abbreviations

The abbreviations, notations, etc. used in this section for chemical and physical properties are the same as those used in previous sections of this series of articles³.

In this section, the prefix "O" of the four-letter titles for the various dependent variables refers to the concretes of Series O in which a water/cement ratio of 0.635 was used. The prefix "A" refers to the concretes of Series A which had a 5 ± 1 -in slump. A summary of the abbreviations for the dependent variables used in this section are as follows:

For Series O concretes made with a water/cement ratio of 0.635:

OSHR = percentage shrinkage after 56 days drying in laboratory air following 14 days curing in a fog room.

OEXP = percentage expansion after 28 days water storage following the 14 days moist-air curing and 56 days drying in laboratory air.

ODSX = OSHR minus OEXP = residual shrinkage.

OREX = OEXP/OSHR.

OEXR = (OSHR minus OEXP)/OSHR = ODSX/OSHR.

ODSH = percentage shrinkage of the top of the specimen as cast minus the percentage shrinkage of the bottom as cast.

For Series A concretes having a 5 ± 1 -in slump the letter "A" is used instead of "O" as ASHR, AEXP, ADSX, AREX, AEXR, and ADSH for the dependent variables.

5. Statistical Analyses

The statistical techniques used to find and evaluate the independent variables associated with the shrinkage, expansion, the differences, and the ratios, have been described in a previous section of this series on "Materials and Techniques" [3]. The statistical treatment was the same as that used in all previous sections of this series of articles. Multiple regression equations were calculated by a least-squares method using various independent variables to determine which showed an association with each of the dependent variables. As in previous sections equations were calculated for both the AE + NAE cements and the NAE cements. Calculations were made using only commonly determined variables and also using these to-

gether with minor and trace elements.

Ratios of reduction in variance to original variance ("F" ratios) obtained by fitting equations were calculated for two kinds of cases: (1) for equations including a few main independent variables as compared to the original data on the dependent variables, or (2) for equations in which additional independent variables were included as compared to a previous equation with fewer variables⁴. The calculated "F" ratios and critical "F" values which must be equalled or exceeded for significance at the

³These abbreviations include the use of C₃A, C₃S, C₂S, and C₄AF for the calculated potential compounds, tricalcium aluminate, tricalcium silicate, dicalcium silicate, and tetracalcium aluminoferrite, respectively. Also used are Insol for insoluble residue, Loss for loss on ignition, APF for air permeability fineness, Wagn for Wagner Turbidimeter fineness, AE + NAE refers to air-entraining plus non-air-entraining cements, and NAE to the non-air-entraining cements.

⁴Statistical terms and notations employed in this section are the same as those used in previous sections of this series of articles. For example, S.D. refers to the estimated standard deviation calculated from the residuals of a fitted equation, or the estimated standard deviation about the average. Also, as in previous sections, s.d. refers to the estimated standard deviation of a coefficient of an independent variable used in a fitted equation. The term coef./s.d. is the ratio of the estimated coefficient (of an independent variable used in an equation) to its estimated standard deviation. "F" designates Fisher's ratio of variances and D.F. is used to designate the number of degrees of freedom. As indicated in previous sections, a coef./s.d. ratio greater than one was considered to be of sufficient significance to warrant further investigation.

$\alpha = 0.05$ and $\alpha = 0.01$ probability levels are summarized in a table (table 10-44).

After equations had been developed in which independent variables were significantly associated with the dependent variables, the residuals of these equations were fitted by a least-squares method to other single independent variables and the reduction in variance calculated. If any of the additional independent variables indicated a significant reduction in variance, they were tried in the equation and retained if the coef./s.d. ratio was greater than 1.

Equations were also computed for the "odds" and "evens" in the array of cements. Comparisons were made of the coefficients of the variables in the two groups of data, and these were compared to the coefficients and coef./s.d. ratios computed for all the cements.

Although shrinkage and expansion measurements were made on all 199 cements, the calculations of the equations presented in this section were limited to those for which trace-element determinations had been made. The calculated relationships of the shrinkage and expansion of the concretes to the dynamic mod-

ulus of $3 \times 4 \times 16$ -in concrete specimens, or to the shrinkage and cracking characteristics of neat cement pastes, included all the cements for which values were available except the three white cements and the cement having a high autoclave expansion. A few other cements that had large deviations from the calculated relationships, for no apparent reason, were also excluded.

Equations presented in this article were selected from a large number of trial equations indicating the calculated relationship of various independent variables to the shrinkage and expansion values, their differences, and their ratios⁵. A summary of the relationships indicated by the equations for both series of concretes will be given in the discussion in subsection 7.

Some of the limitations on interpretations of multivariable regression equations as well as other statistical techniques used in this series of articles have been discussed in section 1, subsections 4.2., 4.3., and 5 [3]. Other limitations and problems of interpretation have also been discussed in other subsections of this series of articles.

6. Results of Tests

6.1. Cement Content of Concretes

The frequency distribution of the cements with respect to the cement content of the Series O concretes made with a water/cement ratio of 0.635 is presented in table 10-1. In table 10-2 is presented similar information with respect to the Series A concretes made with a slump of 5 ± 1 in. It is apparent, from these tables, that concretes made with the air-entraining cements had cement contents considerably lower than the originally designed mix. The cement contents were also, in most instances, lower than those of the concretes made with the non-air-entraining cements. The possible effects of differences of cement content on shrinkage and expansion will be discussed in a later subsection. The subject is introduced at this time in

view of the fact that cement content may be a variable which must be considered in the analyses of the results of shrinkage and expansion measurements made on these concrete specimens. With a fixed cement to aggregate weight ratio, the entrained and entrapped air affected the weight per cubic foot of the freshly mixed concrete. The air content also affected the calculated cement factor. Of the two related variables (air content and cement factor), the air content was used as one of the independent variables in equations relating various independent variables to each of the dependent variables.

⁵ These equations were selected primarily to indicate the association with commonly determined independent variables having coef./s.d. ratios greater than one when used in multivariable equations, and also to indicate which of minor and trace elements may have an effect.

TABLE 10-1. Frequency distribution of cements with respect to cement content, in bags per cubic yard, of concretes made with a water-cement ratio of 0.635

Type cement	Cement content, bags per cubic yard of concrete													Total
	4.90 to 4.95	4.95 to 5.00	5.00 to 5.05	5.05 to 5.10	5.10 to 5.15	5.15 to 5.20	5.20 to 5.25	5.25 to 5.30	5.30 to 5.35	5.35 to 5.40	5.40 to 5.45	5.45 to 5.50	5.50 to 5.55	
	Number of cements													
I.....									2	20	38	21	1	82
IA.....	1		1	4	2									8
II.....								1	4	5	18	39	1	68
IIA.....			1		1		1							3
III.....									1	2	10	7		20
IIIA.....					2		1							3
IV, V.....										2	8	5		15
Total.....	1	0	2	4	5	0	2	1	7	29	74	72	2	199

TABLE 10-2. Frequency distribution of cements with respect to cement content in bags per cubic yard of concretes with a 5 ± 1 in. slump

Type cement	Cement content, bags per cubic yard													Total
	4.90 to 4.95	4.95 to 5.00	5.00 to 5.05	5.05 to 5.10	5.10 to 5.15	5.15 to 5.20	5.20 to 5.25	5.25 to 5.30	5.30 to 5.35	5.35 to 5.40	5.40 to 5.45	5.45 to 5.50	5.50 to 5.55	
	Number of cements													
I.....	1		1		3	3			5	27	31	16		79
IA.....														8
II.....		1				1	1	1	1	6	24	31	3	66
IIA.....														3
III.....						1		2	2	3	13	2		20
IIIA.....														3
IV, V.....										1	5	9		15
Total.....	1	1	1	0	3	5	1	3	8	37	73	58	3	194

6.2. Drying Shrinkage of Concretes

a. Drying Shrinkage of Series O Concretes having a water/cement ratio of 0.635 (OSHR)

The frequency distributions of the cements of the different types with respect to shrinkage of the Series O concretes are presented in table 10-3. The concretes were all made with a water/cement ratio of 0.635, moist cured for 14 days, and then placed in laboratory air for 56 days. The concretes made of the air-entraining cements generally had greater shrinkage values than the overall average of 0.030. There was a fairly broad distribution of values and an overlapping of the values for concretes made of the different types of NAE cements. The values for the AE cements also overlapped the values for the NAE cements.

Equations relating the shrinkage of the Series O concrete specimens made of AE + NAE cements to various independent variables are presented in table 10-4. Equation 1 indicates that there was a significant relationship between the shrinkage and the percentage weight loss after 56 days exposure to laboratory air. (See also table 10-44)⁶.

⁶ Eq 0 as used in table 10-44 refers to the S.D. value of the corresponding dependent variable as indicated in footnotes in the tables of equations.

A number of commonly determined variables were found to have coef./s.d. ratios greater than 1 when used in a multivariable equation. These are presented in eq 2, and, by reference to table 10-44, it may be noted that a highly significant reduction in variance was attained. With the additional use of the trace elements SrO, Zn, and Zr in eq 3 there was a further significant reduction in the S.D. value. The equations calculated for the "odds" and "evens" in the array of cements (eqs 3A and 3B) indicated that C_3S , Na_2O , and Zn had coef./s.d. ratios less than 1 in one or the other of the smaller groups and that Zr had a reversal of the sign of the coefficient⁷.

With the use of Al_2O_3 and Fe_2O_3 in eq 4 instead of the potential compounds as in eqs 2 and 3, a highly significant reduction in the S.D. value was also attained (see also table

⁷ If a number of cements having higher than normal values for an independent variable are included in one of the two groups (the "odds" or "evens") by the arbitrary division of the cements, this distribution may result spurious coefficients for the variable. If the coefficients of both "odds" and "evens" of an independent variable are significant, a greater confidence can be placed in the equation, and in the association of that independent variable with the dependent variable. If the coefficient for a variable in either the "odds" or "evens" is not greater than its s.d., some consideration may still be given to the larger groups of cements as indicating a possible relationship.

TABLE 10-3. Frequency distribution of the cements with respect to shrinkage of concretes made with a water-cement ratio of 0.635 moist-cured for 14 days, then dried in laboratory air for 56 days (OSHR)

Type cement	Percentage shrinkage										Total
	0.014 to 0.017	0.017 to 0.020	0.020 to 0.023	0.023 to 0.026	0.026 to 0.029	0.029 to 0.032	0.032 to 0.035	0.035 to 0.038	0.038 to 0.041	0.041 to 0.044	
	Number of cements										
I.....	2	2	2	5	11	23	18	14	5		82
IA.....						1		4	1	2	8
II.....	1	1	4	9	19	11	13	4	4	2	68
IIA.....								2			2
III.....			2	2	4	1	5	5	1		20
IIIA.....						1		1		1	3
IV, V.....		1	2	4	3	1	1	1		2	15
Total.....	3	4	10	20	37	38	37	31	11	7	198

10-44). With the additional use of the trace elements SrO, Zr, and Zn in eq 5 there was a reduction in variance, significant at the 5-percent probability level. The coef./s.d. ratio for Zn was less than 1 in eq 5A for the "odds" in the array of cements and the coef. for Zr had a reversal of sign. The coefficients of neither were highly significant in eq 5 where all cements were included in the calculations.

Table 10-5 presents a corresponding series of equations for the shrinkage of Series O concretes made of the NAE cements. The air content did not have a coef./s.d. ratio greater than 1 in the equations calculated for these cements, but this variable was highly significant in the previous table where the AE cements were involved. The shrinkage was probably significantly related to the percentage weight loss of the concretes made of the different cements as indicated in eq 1 of table 10-5. The use of commonly determined independent variables in eqs 2 and 4 resulted in highly significant reductions in the S.D. values. The additional use of the trace elements SrO, Zr, and Zn in eqs 3 and 5 resulted in a possibly further reduction in the S.D. values (see also table 10-44) significant at the 5-percent probability level. It may be noted that in equations for the "odds" and "evens" (eqs 3A, 3B, 5A, and 5B) that C_3S , Fe_2O_3 , SrO, Zr, and Zn each had coef./s.d. ratios less than 1 in one or the other of the equations for the smaller groups of cements. The coefficients of these independent variables were generally not highly significant when the larger groups of cements were used in the calculations in eqs 3 and 5.

Coefficients of the independent variables in eq 3 of table 10-5 and the approximate ranges of these variables were used to determine their calculated contributions to the shrinkage of the constant w/c concretes, as well as the ranges of such contributions. These are presented in table 10-6⁸. These calculated values are estimates based on a single equation, and somewhat different values would be obtained by use of other equations.

It may be noted in tables 10-4, 10-5 and 10-6 that increases in C_3A , C_4AF , and fineness of the cements were associated with increases in shrinkage values. Increases in Na_2O , SO_3 , and loss on ignition were associated with decreases in the shrinkage values. It is indicated in table 10-6 that differences in C_3A , fineness, and SO_3 show the greatest calculated effect on the shrinkage of the concretes. As previously indicated, the coefficient for the air content was highly significant when air-entraining cements

were included in the calculated equations as in table 10-4.

b. Drying Shrinkage of Series A Concretes having a 5 ± 1 -in. Slump (ASHR)

The frequency distributions of the cements of the different types with respect to shrinkage of the Series A concretes (ASHR) are presented in table 10-7. There was a broad distribution of results and an overlapping of the cements of the different types. It was previously indicated in table 2-12 of section 2 of this series of articles [3], that the air-entraining cements, Types IA, IIA, and IIIA, had slump values greater than 5 ± 1 in when made with a constant water/cement ratio. Although the water/cement ratio for these cements was reduced for the Series A concretes, most of these concretes were, as shown in table 10-3, in the upper half of the frequency distribution.

The frequency distribution of the water/cement ratios required for the Series A concretes was previously presented in table 2-17 in section 2 of this series of articles [3]. The w/c ranged from about 0.55 to 0.70.

Equations are presented in table 10-8 for AE + NAE cements indicating the relationships between the shrinkage of the Series A concretes and the various independent variables. Equation 1 gives no significant indication of a relationship between the water/cement ratio required for the concretes with a 5 ± 1 -in slump and the shrinkage. There was a relationship, significant at the 5-percent probability level, between the shrinkage and the percentage weight-loss of the specimens of the different concretes. Using commonly determined independent variables in eqs 3 and 5 resulted in highly significant reductions in the S.D. values (see also table 10-44). The additional use of the trace elements SrO, Zr, and Zn in eq 4 or Co, SrO and Zn in eq 6 resulted in reductions of the S.D. values, each significant at the 5-percent probability level. All of these trace elements had coef./s.d. ratios less than 1 in one or the other of the equations for the "odds" or "evens", eqs 4A, 4B, 6A, and 6B. The coef./s.d. for C_3S was also less than 1 in eq 4B.

Corresponding equations for the NAE cements are presented in table 10-9. The coef./s.d. ratio for "air content" was less than 1 in equations for the NAE cements. The use of w/c as an independent variable in eq 1 did not result in a significant reduction of variance. The use of commonly determined variables in eqs 3 and 5 resulted in highly significant reductions in the S.D. value. The additional use of trace elements SrO, Zr, and Zn in eq 4 or Co, Zr and Sn in eq 6 resulted in a reduction of the S.D. value at the 5-percent probability level (see table 10-44). In equations for the "odds" and "evens" there were instances where each

⁸ Corresponding tables for equations calculated for NAE cements, using the calculated potential compounds, other commonly determined variables, and trace elements having coef./s.d. ratios greater than 1 will be presented for other dependent variables. A summary table (table 10-45) is also presented for greater ease in comparing similar equations for AE + NAE cements.

TABLE 10-4. Coefficients for equations for AE+NAE cements relating OSHR, the percent nominal 5½ bags of cement per cubic yard and a water

Equation	Note	Const.	C ₃ A	C ₂ S	C ₄ AF	Al ₂ O ₃
1-----	(1)	OSHR = +0.0208 s.d. = (0.0028)	-----	-----	-----	-----
2-----	(1)	OSHR = +0.0130 s.d. = (0.0040)	+0.00136 (0.00013)	-0.0000765 (0.0000529)	+0.000926 (0.000174)	-----
3-----	(1)	OSHR = +0.0129 s.d. = (0.0039)	+0.00123 (0.00014)	-0.0000835 (0.0000517)	+0.000898 (0.000171)	-----
3A-----	(2)	OSHR (odd) = +0.0178 s.d. = (0.0056)	+0.00126 (0.00020)	-0.0001413 (0.0000684)	+0.000847 (0.000241)	-----
3B-----	(3)	OSHR (even) = +0.0076 s.d. = (0.0058)	+0.00111 (0.00022)	*+0.0000746 (0.0000890)	+0.000998 (0.000260)	-----
4-----	(1)	OSHR = +0.0098 s.d. = (0.0033)	-----	-----	-----	+0.00358 (0.00035)
5-----	(1)	OSHR = +0.0104 s.d. = (0.0033)	-----	-----	-----	+0.00323 (0.00038)
5A-----	(2)	OSHR (odd) = +0.0114 s.d. = (0.0048)	-----	-----	-----	+0.00345 (0.00055)
5B-----	(3)	OSHR (even) = +0.0100 s.d. = (0.0050)	-----	-----	-----	+0.00307 (0.00055)

¹ 177 cements; Avg=0.03028; S.D.=0.005832.

² 89 cements.

³ 88 cements.

* Coef./s.d. ratio less than 1.

TABLE 10-5. Coefficients for equations for NAE cements relating OSHR, the percentage 5½ bags of cement per cubic yard and a water-cem

Equation	Note	Const.	C ₃ A	C ₂ S	C ₄ AF	Al ₂ O ₃
1-----	(1)	OSHR = +0.0214 s.d. = (0.0028)	-----	-----	-----	-----
2-----	(1)	OSHR = +0.0157 s.d. = (0.0040)	+0.00135 (0.00013)	-0.0000775 (0.0000534)	+0.000866 (0.000172)	-----
3-----	(1)	OSHR = +0.0164 s.d. = (0.0040)	+0.00123 (0.00014)	-0.0000860 (0.0000522)	+0.000846 (0.000170)	-----
3A-----	(2)	OSHR (odd) = +0.0159 s.d. = (0.0055)	+0.00124 (0.00020)	-0.0001790 (0.0000638)	+0.001023 (0.000253)	-----
3B-----	(3)	OSHR (even) = +0.0143 s.d. = (0.0058)	+0.00105 (0.00021)	*+0.0000814 (0.0000918)	+0.000687 (0.000233)	-----
4-----	(1)	OSHR = +0.0123 s.d. = (0.0034)	-----	-----	-----	+0.00354 (0.00035)
5-----	(1)	OSHR = +0.0127 s.d. = (0.0033)	-----	-----	-----	+0.00323 (0.00038)
5A-----	(2)	OSHR (odd) = +0.0076 s.d. = (0.0049)	-----	-----	-----	+0.00341 (0.00055)
5B-----	(3)	OSHR (even) = +0.0170 s.d. = (0.0049)	-----	-----	-----	+0.00293 (0.00054)

¹ 165 cements, Avg=0.02979 percent; S.D.=0.005636.

² 83 cements.

³ 82 cements.

* Coef./s.d. ratio less than 1.

age shrinkage after 56 days drying following 14 days moist curing of concretes having a -cement ratio of 0.635, to various independent variables

Fe ₂ O ₃	Na ₂ O	SO ₃	Loss	APF	Air content	Zr	SrO	Zn	Percent wt. loss	S.D.
									+0.00355 (0.00106)	0.005668
	-0.00683 (0.00192)	-0.00853 (0.00115)	-0.00369 (0.00065)	+0.00000589 (0.00000080)	+0.00105 (0.00018)					0.004108
	-0.00822 (0.00198)	-0.00894 (0.00114)	-0.00363 (0.00065)	+0.00000613 (0.00000078)	+0.00100 (0.00018)	+0.0126 (0.0081)	+0.00854 (0.00407)	-0.0198 (0.0098)		0.004005
	-0.01027 (0.00246)	-0.00961 (0.00172)	-0.00502 (0.00121)	+0.00000708 (0.00000125)	+0.00101 (0.00024)	-0.0679 (0.0486)	+0.00772 (0.00616)	*-0.0145 (0.0199)		0.003942
	*-0.00275 (0.00361)	-0.00863 (0.00164)	-0.00317 (0.00085)	+0.00000510 (0.00000107)	+0.00108 (0.00027)	+0.0093 (0.0090)	+0.00860 (0.00604)	-0.0222 (0.0119)		0.004106
+0.000702 (0.000436)	-0.00648 (0.00191)	-0.00836 (0.00114)	-0.00372 (0.00065)	+0.00000549 (0.00000075)	+0.00106 (0.00018)					0.004121
+0.000848 (0.000429)	-0.00783 (0.00197)	-0.00876 (0.00114)	-0.00365 (0.00065)	+0.00000568 (0.00000074)	+0.00102 (0.00018)	+0.0119 (0.0081)	+0.00857 (0.00409)	-0.0190 (0.0099)		0.004025
+0.000813 (0.000616)	-0.01008 (0.00251)	-0.00908 (0.00174)	-0.00537 (0.00122)	+0.00000632 (0.00000123)	+0.00101 (0.00024)	-0.0847 (0.0490)	+0.00671 (0.00627)	*-0.0112 (0.0202)		0.004022
+0.000988 (0.000645)	-0.00363 (0.00345)	-0.00864 (0.00164)	-0.00317 (0.00085)	+0.00000550 (0.00000096)	+0.00104 (0.00027)	+0.0103 (0.0089)	+0.00793 (0.00600)	-0.0221 (0.0119)		0.004100

shrinkage after 56 days drying following 14 days moist curing of concretes having a nominal ent ratio of 0.635, to various independent variables

Fe ₂ O ₃	Na ₂ O	SO ₃	Loss	APF	Zr	SrO	Zn	Percent wt. loss	S.D.
								+0.00283 (0.00106)	0.005502
	-0.00665 (0.00190)	-0.00874 (0.00114)	-0.00376 (0.00065)	+0.00000593 (0.00000080)					0.004022
	-0.00795 (0.00196)	-0.00917 (0.00113)	-0.00367 (0.00066)	+0.00000619 (0.00000078)	+0.0118 (0.0080)	+0.00784 (0.00414)	-0.0209 (0.0096)		0.003920
	-0.00906 (0.00235)	-0.00923 (0.00164)	-0.00484 (0.00103)	+0.00000778 (0.00000128)	+0.0441 (0.0204)	*+0.00312 (0.00605)	-0.0309 (0.0116)		0.003732
	-0.00393 (0.00357)	-0.00895 (0.00167)	-0.00344 (0.00089)	+0.00000610 (0.00000105)	*+0.0017 (0.0093)	+0.01367 (0.00619)	*-0.0129 (0.0174)		0.004013
+0.000558 (0.000429)	-0.00627 (0.00189)	-0.00856 (0.00113)	-0.00380 (0.00065)	+0.00000554 (0.00000076)					0.004034
+0.000710 (0.000422)	-0.00749 (0.00195)	-0.00896 (0.00113)	-0.00371 (0.00066)	+0.00000575 (0.00000074)	+0.0111 (0.0080)	+0.00709 (0.00417)	-0.0202 (0.0097)		0.003941
+0.001365 (0.000659)	-0.00821 (0.00243)	-0.00875 (0.00171)	-0.00475 (0.00108)	+0.00000683 (0.00000129)	+0.0425 (0.0214)	*+0.00171 (0.00631)	-0.0285 (0.0121)		0.003905
*+0.000120 (0.000569)	-0.00453 (0.00350)	-0.00905 (0.00166)	-0.00334 (0.00088)	+0.00000502 (0.00000094)	*+0.0028 (0.0092)	+0.01266 (0.00609)	*-0.0140 (0.0175)		0.004007

TABLE 10-6. Calculated contributions of independent variables to the percentage shrinkage after 56 days drying following 14 days moist curing of concretes having a nominal 5½ bags of cement per cubic yard and a water-cement ratio of 0.635, and the calculated ranges of such contributions (OSHR)

Independent variables	Range of variables (percent)	Coefficients from eq (3) table 10-5	Calculated contributions to OSHR	Calculated ranges of contributions to OSHR
C ₃ A	0 to 15	+0.00123	Const. = +0.0164	0.0184
C ₃ S**	20 to 65	-0.000086	0 to +0.0184	0.0043
C ₄ AF	4 to 17	+0.000846	-0.0017 to -0.0060	0.0110
Na ₂ O	0 to 0.75	-0.00795	+0.0034 to +0.0144	0.0060
SO ₃	1.2 to 3.0	-0.00917	0 to -0.0060	0.0165
Loss	0.3 to 3.3	-0.00367	-0.0110 to -0.0275	0.0110
APF	*2500 to 5500	+0.0000619	-0.0011 to -0.0121	0.0185
Zr**	0 to 0.5	+0.0118	+0.0155 to +0.0340	0.0059
SrO**	0 to 0.4	+0.00784	0 to +0.0059	0.0031
Zn	0 to 0.2	-0.0209	0 to +0.0031	0.0042
			0 to -0.0042	

*cm²/g.

**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

TABLE 10-7. Frequency distribution of the cements with respect to shrinkage of concrete having a nominal 5½ bags of cement per cubic yard and a 5 ± 1 in. slump, moist-cured for 14 days, then dried in laboratory air for 56 days (ASHR)

Type cement	Percentage shrinkage											Total
	0.014 to 0.017	0.017 to 0.020	0.020 to 0.023	0.023 to 0.026	0.026 to 0.029	0.029 to 0.032	0.032 to 0.035	0.035 to 0.038	0.038 to 0.041	0.041 to 0.044	0.044 to 0.047	
	Number of cements											
I	1	3	2	3	13	20	18	13	4	1	-----	78
IA	-----	3	4	14	16	9	3	3	2	-----	-----	8
II	-----	-----	-----	-----	-----	-----	9	5	4	2	-----	66
IIA	-----	1	1	3	1	3	4	5	1	-----	-----	2
III	-----	-----	-----	-----	-----	-----	-----	-----	2	-----	-----	20
IIIA	-----	-----	-----	-----	-----	1	-----	-----	-----	1	1	3
IV, V	-----	-----	1	6	4	1	-----	1	1	-----	-----	15
Total	1	7	8	26	35	34	34	27	14	5	1	192

TABLE 10-8. Coefficients for equations for AE+NAE cements relating ASHR, the percent nominal 5½ bags of cement per cubic yard and a

Equation	Note	Const	C ₃ A	C ₃ S	C ₄ AF	Al ₂ O ₃	Fe ₂ O ₃
1	(1)	ASHR = +0.0363 s.d. = (0.0113)	-----	-----	-----	-----	-----
2	(1)	ASHR = +0.0218 s.d. = (0.0029)	-----	-----	-----	-----	-----
3	(1)	ASHR = +0.0118 s.d. = (0.0040)	+0.001358 (0.000135)	-0.0000835 (0.0000535)	+0.000942 (0.000176)	-----	-----
4	(1)	ASHR = +0.0129 s.d. = (0.0040)	+0.001221 (0.000146)	-0.0000903 (0.0000524)	+0.000918 (0.000174)	-----	-----
4A	(2)	ASHR (odd) = +0.0192 s.d. = (0.0060)	+0.001162 (0.000206)	-0.0001644 (0.0000711)	+0.000861 (0.000264)	-----	-----
4B	(3)	ASHR (even) = +0.0046 s.d. = (0.0058)	+0.001159 (0.000228)	*+0.0000590 (0.0000902)	+0.000997 (0.000243)	-----	-----
5	(1)	ASHR = +0.0083 s.d. = (0.0034)	-----	-----	-----	+0.00357 (0.00036)	+0.000778 (0.000441)
6	(1)	ASHR = +0.0099 s.d. = (0.0033)	-----	-----	-----	+0.00315 (0.00038)	+0.000992 (0.000436)
6A	(2)	ASHR (odd) = +0.0113 s.d. = (0.0052)	-----	-----	-----	+0.00318 (0.00056)	+0.001077 (0.000685)
6B	(3)	ASHR (even) = +0.0075 s.d. = (0.0048)	-----	-----	-----	+0.00310 (0.00056)	+0.001030 (0.000593)

¹ 171 cements; Avg = 0.03000 percent; S.D. = 0.005648.

² 86 cements.

³ 85 cements.

* Coef./s.d. ratio less than 1.

6.3. Expansion of Concretes as a Result of Rewetting

a. Expansion of Series O Concretes made with a water/cement ratio of 0.635. Air-dried and then Rewetted (OEXP)

After 14 days moist-air curing, then drying in laboratory air for 8 weeks, the concrete specimens were placed in water for 28 days. The frequency distribution of the expansion values of the Series O concretes made of the different cements is presented in table 10-11. A comparison of table 10-11 for expansion and table 10-3 for shrinkage of the Series O concretes shows that the range of values for the expansion was not so great as the range of values for the shrinkage. The expansion values of the air-entraining cements, were generally above the average for all cements in table 10-11 and the shrinkage values appeared to be still further above the average shrinkage for all cements in table 10-3. There was an overlapping of the expansion values of the cements of the different types in table 10-11.

Equations for AE + NAE cements are presented in table 10-12 indicating the relationship of the expansion values of Series O concretes (OEXP) to various independent variables. Equation 1 indicates at the 5-percent significance level that there is a probable relationship between the expansion and the percentage weight-gain of the rewetted concrete specimens made of the different cements. The use of commonly determined independent variables in eqs 2 and 4 resulted in highly significant reduction in the S.D. values. (see also table 10-44). By the additional use of the trace

elements, P, Pb, Rb, SrO and Zn in eqs 3 and 5, a further significant reduction in the S.D. value was achieved. It may be noted that in equations for the "odds" and "evens", all of these trace elements except Rb, had coef./s.d. ratios of less than 1 in one or the other of the equations for the smaller groups of cements, see eqs 3A, 3B, 5A and 5B.

A corresponding series of equations for the NAE cements is presented in table 10-13. The relationship of the expansion values to the percentage weight-gain resulting from soaking 28 days in water is presented in eq 1. By reference to table 10-44 it may be noted that the indication of relationship is not significant at the 5-percent probability level. The coef./s.d. ratio for the air content was less than 1 for the NAE cements, and this variable was not included in the equations. The use of commonly determined variables in eqs 2 and 4 resulted in highly significant reductions in the S.D. values. (see table 10-44). The use of trace elements P, Pb, Rb, SrO, and Zn in eqs 3 and 5 resulted in further significant reductions in the S.D. values. In the equations for the "odds" and "evens" (eqs 3A, 3B, 5A and 5B) Pb, and SrO, as well as CaO, had coef./s.d. ratios less than 1 in one or the other of the pairs of equations with the smaller groups of cements.

Calculated contributions of the different variables to the expansion after 28 days rewetting, as well as the estimated ranges of such contributions are presented in table 10-14. It may be noted that increases in fineness, C₃A, C₄AF, Rb, and possibly P were associated with increases in the expansion values. Increases in SO₃ and

age shrinkage after 56 days drying following 14 days moist curing of concretes having a slump of 5 ± 1 in., to various independent variables

Na ₂ O	SO ₃	Loss	APF	Air content	Zr	SrO	Zn	Co	W/C	Percent wt loss	S.D.
									*-0.0099 (0.0177)		0.005659
										+0.00305 (0.00105)	0.005530
-0.00595 (0.00194)	-0.00737 (0.00117)	-0.00392 (0.00066)	+0.00000588 (0.00000081)	+0.000658 (0.000156)							0.004099
-0.00714 (0.00199)	-0.00784 (0.00117)	-0.00381 (0.00067)	+0.00000609 (0.00000079)	+0.000620 (0.000153)	+0.00927 (0.00817)	+0.00828 (0.00420)	-0.0216 (0.0101)				0.004002
-0.00578 (0.00311)	-0.00827 (0.00179)	-0.00359 (0.00123)	+0.00000594 (0.00000131)	+0.000487 (0.000209)	*-0.01896 (0.05798)	*+0.00638 (0.00647)	-0.0277 (0.0130)				0.004044
-0.00858 (0.00276)	-0.00673 (0.00171)	-0.00415 (0.00089)	+0.00000555 (0.00000107)	+0.000848 (0.000242)	+0.00982 (0.00934)	+0.01218 (0.00652)	*-0.0099 (0.0178)				0.004055
-0.00556 (0.00313)	-0.00721 (0.00117)	-0.00393 (0.00117)	+0.00000543 (0.00000076)	+0.000680 (0.000156)							0.004113
-0.00640 (0.00197)	-0.00768 (0.00116)	-0.00377 (0.00066)	+0.00000544 (0.00000075)	+0.000646 (0.000153)		+0.00884 (0.00413)	-0.0145 (0.0102)	-0.3150 (0.2375)			0.004015
-0.00415 (0.00314)	-0.00741 (0.00184)	-0.00379 (0.00124)	+0.00000490 (0.00000133)	+0.000474 (0.000216)		*+0.00386 (0.00665)	-0.0246 (0.0143)	*-0.1099 (0.3058)			0.004178
-0.00756 (0.00267)	-0.00709 (0.00167)	-0.00409 (0.00087)	+0.00000575 (0.00000093)	+0.000826 (0.000231)		+0.01190 (0.00611)	*-0.0015 (0.0163)	-0.7669 (0.4421)			0.003988

TABLE 10-9. Coefficients for equations for NAE cements relating ASHR, the percentage $5\frac{1}{2}$ bags of cement per cubic yard and a slump

Equation	Note	Const.	C ₃ A	C ₂ S	C ₄ AF	Al ₂ O ₃
1-----	(1)	ASHR = -0.0017 s.d. = (0.0149)	-----	-----	-----	-----
2-----	(1)	ASHR = +0.0193 s.d. = (0.0029)	-----	-----	-----	-----
3-----	(1)	ASHR = +0.0144 s.d. = (0.0041)	+0.001319 (0.000134)	-0.0000871 (0.0000540)	+0.000891 (0.000174)	-----
4-----	(1)	ASHR = +0.0152 s.d. = (0.0040)	+0.001205 (0.000145)	-0.0000950 (0.0000528)	+0.000880 (0.000172)	-----
4A-----	(2)	ASHR (odd) = +0.0158 s.d. = (0.0065)	+0.001212 (0.000222)	-0.0001852 (0.0000775)	+0.000916 (0.000275)	-----
4B-----	(3)	ASHR (even) = +0.0140 s.d. = (0.0052)	+0.001074 (0.000202)	*+0.0000337 (0.0000816)	+0.000800 (0.000225)	-----
5-----	(1)	ASHR = +0.0106 s.d. = (0.0034)	-----	-----	-----	+0.00346 (0.00036)
6-----	(1)	ASHR = +0.0120 s.d. = (0.0034)	-----	-----	-----	+0.00310 (0.00038)
6A-----	(2)	ASHR (odd) = +0.0094 s.d. = (0.0055)	-----	-----	-----	+0.00316 (0.00060)
6B-----	(3)	ASHR (even) = +0.0152 s.d. = (0.0044)	-----	-----	-----	+0.00291 (0.00050)

¹ 159 cements; Avg = 0.02964 percent; S.D. = 0.005558.

² 80 cements.

³ 79 cements.

* Coef./s.d. ratio less than 1.

TABLE 10-10. Calculated contributions of independent variables to the percentage shrinkage after 56 days drying following 14 days moist curing of concretes having a nominal $5\frac{1}{2}$ bags of cement per cubic yard and a slump of 5 ± 1 in., and the calculated ranges of such contributions (ASHR)

Independent variables	Range of variables (percent)	Coefficients from eq (4) table 10-9	Calculated contributions to ASHR	Calculated ranges of contributions to ASHR
C ₃ A	0 to 15	+0.001205	Const. = +0.0152	0 to +0.0181
C ₂ S**	20 to 65	-0.0000950	-0.0019 to -0.0062	0.0043
C ₄ AF	4 to 17	+0.00088	+0.0035 to +0.0150	0.0115
Na ₂ O	0 to 0.75	-0.00687	0 to -0.0051	0.0051
SO ₃	1.2 to 3.0	-0.00822	-0.0099 to -0.0247	0.0148
Loss	0.3 to 3.3	-0.00389	-0.0012 to -0.0128	0.0116
APF	*2500 to 5500	+0.00000617	+0.0154 to +0.0339	0.0185
Zr**	0 to 0.5	+0.00866	0 to +0.0043	0.0043
SrO**	0 to 0.4	+0.00681	0 to +0.0027	0.0027
Zn	0 to 0.2	-0.0237	0 to -0.0047	0.0047

*cm²/g.

**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

TABLE 10-11. Frequency distribution of cements with respect to expansion after 28 days moist storage following 56 days drying of nominal $5\frac{1}{2}$ bags of cement per cubic yard concretes made with a water-cement ratio of 0.635 (OEXP)

Type cement	Percentage expansion								Total
	0.011 to 0.014	0.014 to 0.017	0.017 to 0.020	0.020 to 0.023	0.023 to 0.026	0.026 to 0.029	0.029 to 0.032	0.032 to 0.035	
	Number of cements								
I-----	3	6	19	25	22	6	1	-----	82
IA-----	-----	-----	-----	2	3	3	-----	-----	8
II-----	1	13	19	20	11	2	1	1	68
IIA-----	-----	-----	-----	1	1	-----	-----	-----	2
III-----	-----	3	3	9	3	2	-----	-----	20
IIIA-----	-----	-----	1	-----	-----	1	1	-----	3
IV, V-----	1	3	6	2	1	2	-----	-----	15
Total-----	5	25	48	59	41	16	3	1	198

TABLE 10-12. Coefficients for equations for AE+NAE cements relating OEXP, the ex $5\frac{1}{2}$ bags of cement per cubic yard and a water-ce

Equation	Note	Const.	C ₃ A	C ₄ AF	CaO	Al ₂ O ₃
1-----	(1)	OEXP = +0.0164 s.d. = (0.0018)	-----	-----	-----	-----
2-----	(1)	OEXP = +0.0098 s.d. = (0.0023)	+0.000664 (0.000090)	+0.000413 (0.000116)	-----	-----
3-----	(1)	OEXP = +0.0107 s.d. = (0.0022)	+0.000595 (0.000099)	+0.000390 (0.000115)	-----	-----
3A-----	(2)	OEXP (odd) = +0.0101 s.d. = (0.0031)	+0.000595 (0.000136)	+0.000390 (0.000165)	-----	-----
3B-----	(2)	OEXP (even) = +0.0105 s.d. = (0.0035)	+0.000598 (0.000152)	+0.000427 (0.000175)	-----	-----
4-----	(1)	OEXP = +0.0394 s.d. = (0.0123)	-----	-----	-0.000453 (0.000191)	+0.00172 (0.00023)
5-----	(1)	OEXP = +0.0380 s.d. = (0.0121)	-----	-----	-0.000415 (0.000188)	+0.00155 (0.00026)
5A-----	(2)	OEXP (odd) = +0.0310 s.d. = (0.0164)	-----	-----	-0.000316 (0.000255)	+0.00157 (0.00035)
5B-----	(2)	OEXP (even) = +0.0464 s.d. = (0.0191)	-----	-----	-0.000544 (0.000295)	+0.00153 (0.00039)

¹ 176 cements; Avg = 0.02056 percent; S.D. = 0.003593.

² 88 cements.

* Coef./s.d. ratio less than 1.

shrinkage after 56 days drying following 14 days moist curing of concretes having a nominal of 5 ± 1 in., to various independent variables

Fe ₂ O ₃	Na ₂ O	SO ₃	Loss	APF	Zr	SrO	Zn	Co	W/C	Percent wt loss	S.D.
									+0.0489 (0.0232)		0.005499
										+0.00384 (0.00106)	0.005355
	-0.00588 (0.00194)	-0.00776 (0.00117)	-0.00403 (0.00067)	+0.00000594 (0.00000081)							0.004024
	-0.00687 (0.00198)	-0.00822 (0.00117)	-0.00389 (0.00068)	+0.00000617 (0.00000080)	+0.00866 (0.00811)	+0.00681 (0.00432)	-0.0237 (0.0100)				0.003931
	-0.00608 (0.00335)	-0.00876 (0.00209)	-0.00431 (0.00125)	+0.00000765 (0.00000150)	+0.02849 (0.0478)	+0.00263 (0.00750)	-0.0420 (0.0217)				0.004328
	-0.00639 (0.00251)	-0.00812 (0.00152)	-0.00381 (0.00081)	+0.00000492 (0.00000095)	*+0.00301 (0.00822)	+0.01160 (0.00570)	-0.0180 (0.0110)				0.003603
+0.000710 (0.000436)	-0.00543 (0.00192)	-0.00755 (0.00117)	-0.00406 (0.00067)	+0.00000550 (0.00000077)							0.004040
+0.000926 (0.000430)	-0.00601 (0.00195)	-0.00799 (0.00116)	-0.00388 (0.00066)	+0.00000550 (0.00000075)	+0.00697 (0.00424)		-0.0164 (0.0100)	-0.3628 (0.2426)			0.003938
+0.001129 (0.000709)	-0.00483 (0.00345)	-0.00775 (0.00208)	-0.00427 (0.00127)	+0.00000610 (0.00000149)	*+0.00243 (0.00727)		-0.0232 (0.0217)	-0.4419 (0.3592)			0.004459
+0.000590 (0.000559)	-0.00641 (0.00238)	-0.00819 (0.00150)	-0.00376 (0.00078)	+0.00000506 (0.00000085)	+0.01130 (0.00547)		-0.0141 (0.0114)	*-0.2183 (0.3624)			0.003573

pansion after 28 days moist curing following 56 days drying of concretes having a nominal ment ratio of 0.635, to various independent variables

SO ₃	Loss	APF	Air content	Zn	Rb	P	SrO	Pb	Percent wt gain	S.D.
									+0.00163 (0.00070)	0.003549
-0.00394 (0.00078)	-0.00233 (0.00045)	+0.00000294 (0.00000051)	+0.000608 (0.000122)							0.002802
-0.00411 (0.00076)	-0.00220 (0.00044)	+0.00000300 (0.00000050)	+0.000646 (0.000118)	-0.0102 (0.0065)	+0.3234 (0.1150)	+0.00369 (0.00198)	-0.00378 (0.00268)	-0.0518 (0.0345)		0.002711
-0.00380 (0.00103)	-0.00178 (0.00054)	+0.00000290 (0.00000061)	+0.000784 (0.000170)	*-0.0074 (0.0076)	+0.2485 (0.1807)	+0.00587 (0.00290)	-0.00621 (0.00395)	-0.0730 (0.0470)		0.002607
-0.00415 (0.00131)	-0.00271 (0.00080)	+0.00000315 (0.00000091)	+0.000540 (0.000174)	-0.0151 (0.0143)	+0.3601 (0.1707)	*+0.00244 (0.00305)	*-0.00314 (0.00397)	*-0.0248 (0.0537)		0.002920
-0.00435 (0.00079)	-0.00256 (0.00045)	+0.00000326 (0.00000052)	+0.000571 (0.000118)							0.002779
-0.00444 (0.00077)	-0.00243 (0.00044)	+0.00000327 (0.00000051)	+0.000606 (0.000116)	-0.0099 (0.0064)	+0.2853 (0.1150)	+0.00406 (0.00196)	-0.00409 (0.00265)	-0.0499 (0.0341)		0.002677
-0.00402 (0.00105)	-0.00196 (0.00056)	+0.00000310 (0.00000063)	+0.000748 (0.000168)	*-0.0071 (0.0075)	+0.2188 (0.1786)	+0.00593 (0.00283)	-0.00663 (0.00393)	-0.0677 (0.0486)		0.002588
-0.00473 (0.00133)	-0.00298 (0.00078)	+0.00000356 (0.00000092)	+0.000492 (0.000169)	-0.0152 (0.0139)	+0.3308 (0.1673)	+0.00326 (0.00303)	*-0.00329 (0.00389)	*-0.0307 (0.0528)		0.002868

TABLE 10-13. Coefficients for equations for NAE cements relating OEXP, the percentage expansion after 28 days moist storage following 56 days of drying of concretes having a nominal 5½ bags of cement per cubic yard and a water-cement ratio of 0.635, and the calculated ranges of these contributions (OEXP)

Equation	Note	Const.	C ₃ A	C ₄ AF	CaO	Al ₂ O ₃
1.....	(1)	OEXP = +0.0176 s.d. = (0.0019)				
2.....	(1)	OEXP = +0.0106 s.d. = (0.0024)	+0.000688 (0.000093)	+0.000401 (0.000119)		
3.....	(1)	OEXP = +0.0117 s.d. = (0.0023)	+0.000595 (0.000099)	+0.000368 (0.000115)		
3A.....	(2)	OEXP (odd) = +0.0127 s.d. = (0.0030)	+0.000628 (0.000126)	+0.000251 (0.000141)		
3B.....	(2)	OEXP (even) = +0.0108 s.d. = (0.0037)	+0.000565 (0.000159)	+0.000568 (0.000199)		
4.....	(1)	OEXP = +0.0412 s.d. = (0.0126)			-0.000475 (0.000195)	+0.00179 (0.00024)
5.....	(1)	OEXP = +0.0388 s.d. = (0.0122)			-0.000416 (0.000189)	+0.00157 (0.00026)
5A.....	(2)	OEXP (odd) = +0.0101 s.d. = (0.0159)			*+0.000018 (0.000248)	+0.00169 (0.00033)
5B.....	(2)	OEXP (even) = +0.0720 s.d. = (0.0189)			-0.000912 (0.000293)	+0.00140 (0.00040)

1 164 cements; Avg = 0.02031 percent; S.D. = 0.003520.

2 82 cements.

* Coef./s.d. ratio less than 1.

loss on ignition were associated with decreases in the expansion values. Of these variables, differences in fineness, C₃A, and SO₃ had the greatest calculated effect on differences of expansion of the concretes.

TABLE 10-14. Calculated contributions to the percentage expansion after 28 days in water following 56 days air-drying of concretes having a nominal 5½ bags of cement per cubic yard and a water-cement ratio of 0.635, and the calculated ranges of these contributions (OEXP)

Independent variables	Range of variables (percent)	Coefficients from eq (3) table 10-13	Calculated contributions to OEXP	Calculated ranges of contributions to OEXP
			Const. = +0.0117	
C ₃ A.....	0 to 15	+0.000595	0 to +0.0089	0.0089
C ₄ AF.....	4 to 17	+0.000368	+0.0015 to +0.0063	0.0048
SO ₃	1.2 to 3.0	-0.00433	-0.0052 to -0.0130	0.0078
Loss.....	0.3 to 3.3	-0.00214	-0.0006 to -0.0071	0.0065
APF.....	*2500 to 5500	+0.00000315	+0.0079 to +0.0173	0.0094
Zn**.....	0 to 0.2	-0.0110	0 to -0.0022	0.0022
Rb.....	0 to 0.01	+0.3894	0 to +0.0039	0.0039
P.....	0 to 0.5	+0.00438	0 to +0.0022	0.0022
SrO**.....	0 to 0.4	-0.00394	0 to -0.0016	0.0016
Pb**.....	0 to 0.05	+0.0450	0 to +0.0022	0.0022

*cm²/g.

**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

b. Expansion of Series A Concretes made with a Slump of 5 ± 1-in., Air-dried and then Rewetted (AEXP)

The frequency distributions of the different types of cement with respect to the expansion of Series A concretes (AEXP) are presented in table 10-15. There was a fairly broad distribution of values and an overlapping of values for cements of the different types.

Equations relating the expansion values of the concrete specimens made of AE + NAE cements to various independent variables are presented in table 10-16. Equation 1 indicates

TABLE 10-15. Frequency distribution of cements with respect to expansion after 28 days moist storage following 56 days of drying of concretes having a nominal 5½ bags of cement per cubic yard and a 5 ± 1 in. slump (AEXP)

Type cement	Percentage expansion							Total
	0.011 to 0.014	0.014 to 0.017	0.017 to 0.020	0.020 to 0.023	0.023 to 0.026	0.026 to 0.029	0.029 to 0.032	
	Number of cements							
I-----	4	4	12	25	24	7	3	79
IA-----			1	3	4			8
II-----	1	7	26	16	11	5		66
IIA-----				1		1		2
III-----		4	2	5	6	1	2	20
IIIA-----				1		1	1	3
IV, V-----		2	6	4	2	1		15
Total-----	5	17	47	55	47	16	6	193

that there was no apparent relationship between the water/cement ratio required to obtain concretes of 5 ± 1-in slump and the expansion on rewetting the air-dried specimens. There was an indication, significant at the 5-percent probability level, of a relationship between the expansion and the percentage weight-gain of the specimens made of the different cements as indicated in eq 2 (see also table 10-44). Using only the major potential compounds in eq 3 resulted in a significant reduction in the S.D. value, although the C₃S had a coef./s.d. ratio less than 1 and the coefficient for C₄AF was not highly significant. With the use of additional commonly determined variables in eq 4, a further significant reduction in the S.D. value was obtained, (see table 10-44), and the coefficients for C₃S and C₄AF were more highly significant than in eq 3. The additional use of the trace elements Mn, Ni, P, and

SO ₃	Loss	APF	Zn	Rb	P	SrO	Pb	Percent wt gain	S.D.
								+0.00105 (0.00074)	0.003509
-0.00403 (0.00080)	-0.00230 (0.00046)	+0.00000301 (0.00000053)							0.002857
-0.00433 (0.00077)	-0.00214 (0.00044)	+0.00000315 (0.00000051)	-0.0110 (0.0065)	+0.3894 (0.1161)	+0.00438 (0.00199)	-0.00394 (0.00278)	-0.0450 (0.0345)		0.002702
-0.00463 (0.00103)	-0.00164 (0.00051)	+0.00000310 (0.00000058)	-0.0207 (0.0100)	+0.2922 (0.1792)	+0.00592 (0.00278)	*-0.00345 (0.00372)	-0.0867 (0.0483)		0.002475
-0.00325 (0.00128)	-0.00323 (0.00088)	+0.00000284 (0.00000100)	-0.0107 (0.0092)	+0.3526 (0.1702)	+0.00315 (0.00314)	-0.00591 (0.00439)	*-0.0234 (0.0513)		0.002912
-0.00449 (0.00081)	-0.00251 (0.00046)	+0.00000334 (0.00000054)							0.002807
-0.00468 (0.00078)	-0.00235 (0.00045)	+0.00000340 (0.00000052)	-0.0109 (0.0064)	+0.3461 (0.1162)	+0.00473 (0.00196)	-0.00432 (0.00275)	-0.0432 (0.0340)		0.002664
-0.00454 (0.00104)	-0.00157 (0.00051)	+0.00000311 (0.00000060)	-0.0213 (0.0101)	+0.2940 (0.1819)	+0.00629 (0.00276)	*-0.00369 (0.00378)	-0.0837 (0.0487)		0.002490
-0.00424 (0.00127)	-0.00393 (0.00087)	+0.00000362 (0.00000099)	-0.0092 (0.0086)	+0.3070 (0.1631)	+0.00423 (0.00302)	-0.00644 (0.00415)	*-0.0275 (0.0489)		0.002778

V in eq 5 resulted in a further significant reduction in the S.D. value.

The use of major oxides with other commonly determined variables in eq 6 resulted in a highly significant reduction in the S.D. value. It may be noted, however, that the coefficient for Fe₂O₃ was negative whereas that for C₄AF in eqs 4 and 5 were positive. Neither was highly significant⁹. The additional use of the trace elements in eq 7 resulted in a further significant reduction in the S.D. value (see table 10-44).

In equations for the "odds" and "evens" (eqs 5A, 5B, 7A and 7B) there were instances where the coef./s.d. ratios for C₃S, Fe₂O₃, K₂O, MgO, air content, Mn, P, V, and Ni were less than 1 in one or both, of the pairs of equations for the smaller groups of cements.

A corresponding series of equations for NAE cements is presented in table 10-17. The coefficients of the various independent variables and the reduction in variance resulting from the use of additional independent variables were in reasonable agreement with those presented in the previous table 10-16 where the AE cements were included.

Using the coefficients of the independent variables from eq 5 of table 10-17 and the approximate ranges of these variables, calculated contributions to the expansion, and the ranges of such contributions were determined. As indicated in table 10-18, increases in C₃A, C₄AF, MgO, K₂O, fineness, and Mn were associated with increases in the expansion values. Increases in SO₃ and Loss were associated with decreases in the expansion values. Differences in C₃A, fineness, and SO₃ had the largest cal-

culated effect on differences of expansion values obtained.

6.4. Residual Shrinkage or Difference between the Percentage Shrinkage During Drying and the Percentage Expansion with Subsequent Rewetting

a. Residual Shrinkage of Series O Concretes made with a water/cement Ratio of 0.635 (ODSX)

The frequency distributions of the residual shrinkage, defined as the difference between the shrinkage during air drying for 56 days and the expansion during the subsequent water storage for 28 days, of the Series O concretes are presented in table 10-19. There was a fairly broad range of values and an overlapping of the values obtained with the different types of cement.

Equations presenting the variables associated with the residual-shrinkage values for AE + NAE cements are presented in table 10-20. The use of commonly determined variables in eq 1 resulted in a highly significant reduction in the S.D. value, and the additional use of the trace elements SrO, Li, and Rb in eq 2 resulted in a further highly significant reduction in the S.D. value. The S.D. value of eq 2 was higher than that obtained when the shrinkage values were used as independent variables as in eq 3. The use of additional independent variables having coef./s.d. ratios greater than 1 in eq 4 caused a significant reduction in the S.D. value (see table 10-44). The coef./s.d. ratios of C₄AF, APF, and Loss were less than 1 when included with the other variables in eq 4, but the coef./s.d. ratio for MgO was greater than 1. Except for SrO and

⁹ The C₄AF value was computed as a constant times the Fe₂O₃ in accordance with specification requirements.

TABLE 10-16. *Coefficients for equations for AE+NAE cements relating AEXP, the percent a nominal 5½ bags of cement per cubic yard with*

Equation	Note	Const.	C ₃ A	C ₂ S	C ₄ AF	K ₂ O
1-----	(1)	AEXP = +0.0237 s.d. = (0.0076)	-----	-----	-----	-----
2-----	(1)	AEXP = +0.0163 s.d. = (0.0020)	-----	-----	-----	-----
3-----	(1)	AEXP = +0.0166 s.d. = (0.0032)	+0.000485 (0.000105)	*+0.0000389 (0.0000420)	+0.000236 (0.000150)	-----
4-----	(1)	AEXP = +0.0107 s.d. = (0.0030)	+0.000710 (0.000101)	-0.0000542 (0.0000395)	+0.000366 (0.000130)	+0.00174 (0.00130)
5-----	(1)	AEXP = +0.0085 s.d. = (0.0032)	+0.000725 (0.000110)	-0.0000440 (0.0000387)	+0.000344 (0.000131)	+0.00260 (0.00130)
5A-----	(2)	AEXP (odd) = +0.0107 s.d. = (0.0042)	+0.000658 (0.000142)	*-0.0000482 (0.0000489)	+0.000366 (0.000170)	+0.00479 (0.00167)
5B-----	(3)	AEXP (even) = +0.0069 s.d. = (0.0050)	+0.000714 (0.000172)	*-0.0000183 (0.0000662)	+0.000237 (0.000208)	*+0.00055 (0.00217)
			CaO	Al ₂ O ₃	Fe ₂ O ₃	
6-----	(1)	AEXP = +0.0551 s.d. = (0.0167)	-0.000682 (0.000246)	+0.00182 (0.00027)	-0.000451 (0.000258)	+0.00141 (0.00131)
7-----	(1)	AEXP = +0.0483 s.d. = (0.0167)	-0.000606 (0.000248)	+0.00186 (0.00029)	-0.000521 (0.000351)	+0.00237 (0.00132)
7A-----	(2)	AEXP (odd) = +0.0412 s.d. = (0.0226)	-0.000475 (0.000335)	+0.00169 (0.00037)	*-0.000213 (0.000457)	+0.00451 (0.00172)
7B-----	(3)	AEXP (even) = +0.0493 s.d. = (0.0257)	-0.000629 (0.000382)	+0.00185 (0.00045)	-0.000932 (0.000552)	*+0.00077 (0.00209)

¹ 171 cements; Avg=0.02100 percent; S.D.=0.003823.

² 86 cements.

³ 85 cements.

* Coef./s.d. ratio less than 1.

TABLE 10-17. *Coefficients for equations for NAE cements relating AEXP, the percentage nominal 5½ bags of cement per cubic yard with a*

Equation	Note	Const.	C ₃ A	C ₂ S	C ₄ AF	K ₂ O
1-----	(1)	AEXP = +0.0073 s.d. = (0.0101)	-----	-----	-----	-----
2-----	(1)	AEXP = +0.0159 s.d. = (0.0020)	-----	-----	-----	-----
3-----	(1)	AEXP = +0.0165 s.d. = (0.0033)	+0.000498 (0.000107)	-0.0000443 (0.0000435)	+0.000254 (0.000154)	-----
4-----	(1)	AEXP = +0.0122 s.d. = (0.0031)	+0.000722 (0.000102)	-0.0000645 (0.0000401)	+0.000335 (0.000130)	+0.00175 (0.00131)
5-----	(1)	AEXP = +0.0094 s.d. = (0.0032)	+0.000748 (0.000111)	-0.0000504 (0.0000390)	+0.000329 (0.000130)	+0.00270 (0.00130)
5A-----	(2)	AEXP (odd) = +0.0152 s.d. = (0.0040)	+0.000797 (0.000152)	-0.0001181 (0.0000452)	+0.000303 (0.000169)	+0.00363 (0.00186)
5B-----	(3)	AEXP (even) = +0.0024 s.d. = (0.0052)	+0.000738 (0.000705)	*+0.0000484 (0.0000706)	+0.000365 (0.000184)	+0.00254 (0.00197)
			CaO	Al ₂ O ₃	Fe ₂ O ₃	
6-----	(1)	AEXP = +0.0606 s.d. = (0.0168)	-0.000749 (0.000247)	+0.00186 (0.00027)	-0.000578 (0.000358)	*+0.00132 (0.00132)
7-----	(1)	AEXP = +0.0533 s.d. = (0.0168)	-0.000672 (0.000248)	+0.00193 (0.00029)	-0.000626 (0.000348)	+0.00238 (0.00133)
7A-----	(2)	AEXP (odd) = +0.0582 s.d. = (0.0206)	-0.000717 (0.000303)	+0.00209 (0.00040)	-0.000595 (0.000464)	+0.00274 (0.00192)
7B-----	(3)	AEXP (even) = +0.0357 s.d. = (0.0290)	-0.000447 (0.000434)	+0.00195 (0.00045)	-0.000604 (0.000524)	+0.00265 (0.00204)

¹ 159 cements; Avg=0.02085 percent; S.D.=0.003821.

² 80 cements.

³ 79 cements.

* Coef./s.d. ratio less than 1.

age expansion after 28 days moist storage following 56 days of drying of concretes having
a 5 ± 1 in. slump, to various independent variables

SO ₃	MgO	Loss	APF	Air content	Mn	P	V	Ni	W/C	Percent wt gain	S.D.
									*-0.00428 (0.01188)		0.003832
										+0.00184 (0.00075)	0.003767
											0.003611
-0.00494 (0.00091)	+0.000436 (0.000217)	-0.00252 (0.00050)	+0.00000393 (0.00000062)	+0.000214 (0.000117)							0.003011
-0.00548 (0.00090)	+0.000428 (0.000222)	-0.00250 (0.00049)	+0.00000431 (0.00000062)	+0.000208 (0.000114)	+0.00418 (0.00195)	+0.00284 (0.00215)	+0.0237 (0.0154)	+0.1149 (0.0830)			0.002916
-0.00582 (0.00128)	*+0.000257 (0.000304)	-0.00285 (0.00070)	+0.00000386 (0.00000086)	+0.000316 (0.000163)	*+0.00043 (0.00310)	+0.00510 (0.00331)	*+0.0065 (0.0196)	+0.2155 (0.1172)			0.002668
-0.00453 (0.00132)	+0.000555 (0.000342)	-0.00204 (0.00071)	+0.00000429 (0.00000095)	*+0.000114 (0.000170)	+0.00615 (0.00278)	*+0.00051 (0.00303)	+0.0533 (0.0260)	*+0.0043 (0.1353)			0.003125
-0.00522 (0.00091)		-0.00296 (0.00051)	+0.00000398 (0.00000059)	+0.000206 (0.000117)							0.002998
-0.00572 (0.00091)		-0.00290 (0.00050)	+0.00000435 (0.00000060)	+0.000202 (0.000114)	+0.00425 (0.00194)	+0.00278 (0.00213)	+0.0229 (0.0151)	+0.1098 (0.0828)			0.002909
-0.00596 (0.00129)		-0.00316 (0.00074)	+0.00000381 (0.00000083)	+0.000314 (0.000162)	*+0.00031 (0.00307)	+0.00505 (0.00328)	*+0.0071 (0.0185)	+0.2149 (0.1164)			0.002650
-0.00479 (0.00132)		-0.00246 (0.00071)	+0.00000450 (0.00000091)	*+0.000097 (0.000168)	+0.00635 (0.00274)	*+0.00043 (0.00302)	+0.0531 (0.0258)	*-0.0073 (0.1346)			0.003110

expansion after 28 days moist storage following 56 days of drying of concretes having a
 5 ± 1 in. slump, to various independent variables

SO ₃	MgO	Loss	APF	Mn	P	V	Ni	W/C	Percent wt gain	S.D.
								+0.02111 (0.01583)		0.003811
									+0.00190 (0.00077)	0.003760
										0.003597
-0.00523 (0.00091)	+0.000438 (0.000220)	-0.00243 (0.00050)	+0.00000393 (0.00000063)							0.002978
-0.00588 (0.00090)	+0.000446 (0.000222)	-0.00240 (0.00049)	+0.00000434 (0.00000062)	+0.00413 (0.00201)	+0.00272 (0.00212)	+0.0297 (0.0152)	+0.1096 (0.0828)			0.002861
-0.00765 (0.00121)	*+0.000119 (0.000285)	-0.00225 (0.00065)	+0.00000467 (0.00000081)	*-0.00103 (0.00288)	+0.00396 (0.00315)	*-0.0007 (0.0180)	+0.2475 (0.1142)			0.002572
-0.00466 (0.00140)	+0.000597 (0.000356)	-0.00240 (0.00080)	+0.00000407 (0.00000097)	+0.00707 (0.00294)	*-0.00057 (0.00308)	+0.0715 (0.0262)	*-0.0023 (0.1301)			0.003032
-0.00553 (0.00091)		-0.00290 (0.00051)	+0.00000395 (0.00000059)							0.002959
-0.00615 (0.00091)		-0.00282 (0.00050)	+0.00000438 (0.00000060)	+0.00415 (0.00200)	+0.00262 (0.00211)	+0.0293 (0.0149)	+0.1028 (0.0824)			0.002848
-0.00765 (0.00121)		-0.00268 (0.00070)	+0.00000435 (0.00000079)	*-0.00131 (0.0029)	+0.00388 (0.00318)	*+0.0053 (0.0177)	+0.2334 (0.1151)			0.002595
-0.00471 (0.00144)		-0.00269 (0.00078)	+0.00000442 (0.00000095)	+0.00703 (0.00296)	*-0.00054 (0.00310)	+0.0689 (0.0262)	*-0.0258 (0.1302)			0.003050

TABLE 10-18. Calculated contributions to the percentage expansion after 28 days in water following 56 days air-drying of concretes having a nominal $5\frac{1}{2}$ bags of cement per cubic yard and a slump of 5 ± 1 in., and the calculated ranges of such contributions (AEXP)

Independent variables	Range of variables (percent)	Coefficients from eq (5) table 10-17	Calculated contributions to AEXP	Calculated ranges of contributions to AEXP
			Const. = +0.0094	
C ₃ A-----	0 to 15	+0.000748	0 to +0.0112	0.0112
C ₃ S**-----	20 to 65	-0.0000504	-0.0010 to -0.0033	0.0023
C ₄ AF-----	4 to 17	+0.000329	+0.0013 to +0.0056	0.0043
K ₂ O-----	0 to 1.1	+0.00270	0 to +0.0030	0.0030
SO ₃ -----	1.2 to 3.0	-0.00588	-0.0071 to -0.0176	0.0105
MgO-----	0 to 5.0	+0.000446	0 to +0.0022	0.0022
Loss-----	0.3 to 3.3	-0.00240	-0.0007 to -0.0079	0.0072
APF-----	*2500 to 5500	+0.00000434	+0.0108 to +0.0239	0.0131
Mn-----	0 to 1.0	+0.00413	0 to +0.0041	0.0041
P**-----	0 to 0.5	+0.00272	0 to +0.0014	0.0014
V**-----	0 to 0.1	+0.0297	0 to +0.0030	0.0030
Ni**-----	0 to 0.02	+0.1096	0 to +0.0021	0.0021

*cm²/g.

**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

TABLE 10-19. Frequency distribution of cements with respect to difference between the shrinkage and subsequent expansion of concrete of nominal $5\frac{1}{2}$ bags of cement per cubic yard made with a water-cement ratio of 0.635 (ODSX)

Type cement	Percentage shrinkage minus percentage expansion														Total
	-0.002 to 0.000	0.000 to 0.002	0.002 to 0.004	0.004 to 0.006	0.006 to 0.008	0.008 to 0.010	0.010 to 0.012	0.012 to 0.014	0.014 to 0.016	0.016 to 0.018	0.018 to 0.020	0.020 to 0.022	0.022 to 0.024	0.024 to 0.026	
	Number of cements														
I-----	1	1	3	5	14	16	19	13	5	1	1	1	1	82	
IA-----					1	1	1		3	2				8	
II-----		2	4	10	6	14	16	7	5	4				68	
IIA-----								1	1					2	
III-----				3	3	3	4	6	1					20	
IIIA-----				1			1		1					3	
IV, V-----				2	4	3	3		3					15	
Total-----	1	3	7	21	28	37	34	27	19	7	1	1	1	198	

independent variables as in eq 2, a higher S.D. value is obtained.

Calculated contributions and ranges of contributions to the residual shrinkage based on eq 2, table 10-21, are given in table 10-22. Increases in C₄AF, C₃A/SO₃, fineness, and SrO appear to be associated with increases in the residual-shrinkage values. Increases of Na₂O, loss on ignition, and Rb were associated with a decrease in the residual-shrinkage values.

b. Residual Shrinkage of Series A Concretes made having a Slump of 5 ± 1 in. (ADSX)

The frequency distribution of the cements with respect to the residual shrinkage of specimens made of Series A concretes is presented in table 10-23. There was a fairly broad distribution of values, and an overlapping of the values for the cements of the different types.

A series of equations is presented in table 10-24 to indicate the variables associated with the values for the residual shrinkage of the Series A concretes made of AE + NAE cements. A corresponding series of equations for

Rb, the coefficients of none of the added independent variables were highly significant. In eqs 2A and 2B, Na₂O, air content, and Li had coef./s.d. ratios less than 1 in one or the other of the smaller groups of cements. In eqs 4A and 4B, for the "odds" and "evens", Na₂O, C₃A/SO₃, air content, Li, and MgO had coef./s.d. ratios less than 1.

A corresponding series of equations for the NAE cements is presented in table 10-21. As in the previous table, the residual shrinkage appeared highly related to the initial shrinkage (see eq 3). Other additional independent variables such as SrO and Rb appeared to have highly significant effects as indicated in eq 4. If the relationship of the difference between the shrinkage and expansion is computed only with the chemical composition and fineness of the cement and the air content of the concrete as

the NAE cements is presented in table 10-25. The independent variables associated with the shrinkage-expansion differences for the specimens made of Series A concretes and those previously reported in tables 10-20 and 10-21 made of Series O concretes are in reasonable agreement as are the coefficients of these variables. The differences between shrinkage and expansion were significantly associated with the magnitude of the shrinkage as indicated in eq 3 of the two tables (see also table 10-44). The additional use of commonly determined variables in eq 4, or those with trace elements in eq 5, resulted in significantly lower S.D. values. The use of commonly determined variables in eq 1 or those with trace elements in eq 2, but not including the shrinkage as an independent variable, each resulted in significant reductions in the S.D. values, but these values were higher than in eqs 3, 4 and 5 where the shrinkage was also included as an independent variable.

Calculated contributions and ranges of contributions of the variables of eq 2, table 10-25,

TABLE 10-20. Coefficients for equations for AE + NAE cements relating the percentage shrinkage after 56 days drying and the percentage expansion after 28 days water storage ($ODSX = OSHR - OEXP$) of concretes having a nominal $5\frac{1}{2}$ bags of cement per cubic yard and a water-cement ratio of 0.635, to various independent variables

Equation	Note	Const.	C ₄ AF	Na ₂ O	C ₂ A/SO ₃	Air content	APF	Loss	SrO	Li	Rb	MgO	OSHR	S.D.
1A-----	(1) ODSX s.d.	= -0.00608 = (0.00335)	+0.000502 (0.000149)	-0.00459 (0.00169)	+0.00120 (0.00021)	+0.000413 (0.000156)	+0.00000224 (0.00000062)	-0.00179 (0.00057)	-----	-----	-----	-----	-----	0.00387
2A-----	(1) ODSX s.d.	= -0.00416 = (0.00314)	+0.000502 (0.000139)	-0.00485 (0.00183)	+0.00097 (0.00021)	+0.000296 (0.000146)	+0.00000178 (0.00000058)	-0.00164 (0.00053)	+0.0114 (0.0032)	-0.1923 (0.0863)	-0.4920 (0.1381)	-----	-----	0.00340
2A-----	(2) ODSX (odd) s.d.	= -0.00835 = (0.00452)	+0.000513 (0.000185)	-0.00616 (0.00221)	+0.00124 (0.00031)	+0.000465 (0.000196)	+0.00000316 (0.00000094)	-0.00326 (0.00093)	+0.0085 (0.0046)	*-0.0859 (0.1100)	-0.3057 (0.1729)	-----	-----	0.00329
2B-----	(2) ODSX (even) s.d.	= -0.00246 = (0.00460)	+0.000511 (0.000216)	*-0.00235 (0.00327)	+0.00086 (0.00030)	*+0.000136 (0.000224)	+0.00000121 (0.00000078)	-0.00172 (0.00071)	+0.0112 (0.0049)	-0.3084 (0.1424)	-0.7474 (0.2373)	-----	-----	0.00352
3A-----	(1) ODSX s.d.	= -0.00662 = (0.00108)	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	+0.5377 (0.0351)	0.00271
4A-----	(1) ODSX s.d.	= -0.00466 = (0.00115)	-----	-0.00231 (0.00137)	-0.000164 (0.000137)	-0.000158 (0.000112)	-----	-----	+0.00820 (0.00239)	-0.0938 (0.0647)	-0.3570 (0.1009)	-0.000231 (0.000166)	+0.5223 (0.0357)	0.00250
4A-----	(2) ODSX (odd) s.d.	= -0.00435 = (0.00182)	-----	-0.00305 (0.00198)	*-0.000024 (0.000234)	*-0.000016 (0.000170)	-----	-----	+0.00929 (0.00352)	*-0.0222 (0.0931)	-0.3148 (0.1425)	*-0.000168 (0.000270)	+0.4739 (0.0589)	0.00274
4B-----	(2) ODSX (even) s.d.	= -0.00545 = (0.00145)	-----	*-0.00191 (0.00194)	-0.000256 (0.000163)	-0.000375 (0.000150)	-----	-----	+0.00708 (0.00305)	-0.1913 (0.0931)	-0.5051 (0.1496)	*-0.000211 (0.000216)	+0.5737 (0.0509)	0.00226

1 177 cements; Avg = 0.00965; S.D. = 0.00413. 2 89 cements. 3 88 cements. * Coef./s.d. ratio less than 1.

TABLE 10-21. Coefficients for equations for NAE cements relating the percentage shrinkage after 56 days drying and the percentage expansion after 28 days water storage ($ODSX = OSHR - OEXP$) of concretes having a nominal $5\frac{1}{2}$ bags of cement per cubic yard and a water-cement ratio of 0.635, to various independent variables

Equation	Note	Const.	C ₄ AF	Na ₂ O	C ₂ A/SO ₃	Air content	APF	Loss	SrO	Li	Rb	MgO	OSHR	S.D.
1A-----	(1) ODSX s.d.	= -0.00298 = (0.00339)	+0.000430 (0.000149)	-0.00430 (0.00170)	+0.00118 (0.00021)	-0.000830 (0.000388)	+0.00000221 (0.00000062)	-0.00205 (0.00057)	-----	-----	-----	-----	-----	0.00359
2A-----	(1) ODSX s.d.	= -0.00207 = (0.00323)	+0.000440 (0.000141)	-0.00489 (0.00186)	+0.00096 (0.00021)	-0.000559 (0.000380)	+0.00000183 (0.00000060)	-0.00182 (0.00055)	+0.0113 (0.0034)	-0.1529 (0.0877)	-0.3841 (0.1430)	-----	-----	0.00339
2A-----	(2) ODSX (odd) s.d.	= -0.00402 = (0.00538)	+0.000438 (0.000233)	-0.00601 (0.00241)	+0.00100 (0.00034)	-0.000818 (0.000651)	+0.00000271 (0.00000102)	-0.00254 (0.00090)	+0.0121 (0.0051)	*-0.0836 (0.1333)	-0.2931 (0.2064)	-----	-----	0.00339
2B-----	(2) ODSX (even) s.d.	= -0.00119 = (0.00435)	+0.000437 (0.000192)	-0.00342 (0.00326)	+0.00094 (0.00029)	-0.000651 (0.000520)	+0.00000134 (0.00000079)	-0.00139 (0.00075)	+0.0107 (0.0049)	-0.1903 (0.1293)	-0.4976 (0.2207)	-----	-----	0.00353
3A-----	(1) ODSX s.d.	= -0.00670 = (0.00116)	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	+0.5410 (0.0382)	0.00274
4A-----	(1) ODSX s.d.	= -0.00417 = (0.00129)	-----	-0.00253 (0.00141)	-0.000153 (0.000143)	-0.000389 (0.000281)	-----	-----	+0.00860 (0.00255)	-0.0818 (0.0664)	-0.3622 (0.1057)	-0.000232 (0.000172)	+0.5154 (0.0408)	0.00253
4A-----	(2) ODSX (odd) s.d.	= -0.00432 = (0.00198)	-----	-0.00343 (0.00192)	*-0.000144 (0.000219)	*-0.000393 (0.000488)	-----	-----	+0.01287 (0.00385)	*-0.0268 (0.1017)	-0.3957 (0.1532)	-0.000287 (0.000248)	+0.5186 (0.0585)	0.00253
4B-----	(2) ODSX (even) s.d.	= -0.00439 = (0.00187)	-----	*-0.00191 (0.00221)	*-0.000148 (0.000198)	-0.000490 (0.000374)	-----	-----	+0.00457 (0.00372)	-0.1168 (0.0928)	-0.3594 (0.1609)	*-0.000182 (0.000237)	+0.5300 (0.0616)	0.00260

1 165 cements; Avg = 0.00941; S.D. = 0.00408. 2 83 cements. 3 82 cements. * Coef./s.d. ratio less than 1.

TABLE 10-22. Calculated contributions to the percentage shrinkage after 56 days drying minus the percentage expansion after 28 days in water of concretes having a nominal $5\frac{1}{2}$ bags of cement per cubic yard and a water-cement ratio of 0.635, and the calculated ranges of such contributions ($ODSX = OSHR - OEXP$)

Independent variables	Range of variables (percent)	Coefficients from eq (2) table 10-21	Calculated contributions to ODSX	Calculated ranges of contributions to ODSX
C_4AF -----	4 to 17	+0.000440	Const. = -0.00207	0.0056
Na_2O -----	0 to 0.75	-0.00489	+0.0018 to +0.0074	0.0037
C_3A/SO_3 ---	0.4 to 10.1	+0.00096	0 to -0.0037	0.0093
Air content**	0 to 10	-0.000659	+0.0004 to +0.0097	0.0066
APF-----	*2500 to 5500	+0.00000183	0 to -0.0066	0.0066
Loss-----	0.3 to 3.3	-0.00182	+0.0046 to +0.0101	0.0055
SrO -----	0 to 0.4	+0.0113	-0.0005 to -0.0060	0.0055
Li^{**} -----	0 to 0.02	-0.1529	0 to +0.0045	0.0045
Rb -----	0 to 0.01	-0.3841	0 to -0.0030	0.0030
			0 to -0.0038	0.0038

*cm²/g.

**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

to the residual-shrinkage values are presented in table 10-26. Increases in C_4AF , C_3A/SO_3 , fineness, and possibly SrO were associated with greater residual shrinkage. Increases in Na_2O , loss on ignition, and Li were associated with decreases in the residual shrinkage. Of these, differences in C_4AF , C_3A/SO_3 , and fineness had the greatest calculated effect on the ranges of the values for residual shrinkage. The air-content and Rb , though of doubtful significance in eq 2 of table 10-25, may possibly be significant in eq 2 of table 10-24 where the AE cements were included.

TABLE 10-23. Frequency distribution of cements with respect to difference between the shrinkage and subsequent expansion of nominal $5\frac{1}{2}$ bags of cement per cubic yard concretes with a 5 ± 1 in. clump ($ADSX$)

Type cement	Percentage shrinkage minus percentage expansion												Total
	0.000 to 0.002	0.002 to 0.004	0.004 to 0.006	0.006 to 0.008	0.008 to 0.010	0.010 to 0.012	0.012 to 0.014	0.014 to 0.016	0.016 to 0.018	0.018 to 0.020	0.020 to 0.022	0.022 to 0.024	
	Number of cements												
I-----	2	5	10	11	13	17	12	4	1	1	2	1	78
IA-----				1	1		3	1	1				8
II-----	1	6	8	9	18	11	3	5	5	1			66
IIA-----					2								2
III-----			3	3	1	8	5						20
IIIA-----					1		1		1				3
IV, V-----		1	1	4	5	2		2					15
Total-----	3	12	22	28	41	38	24	12	8	1	2	1	192

TABLE 10-24. Coefficients for equations for AE+NAE cements relating the difference between days water storage ($ADSX = ASHR - AEXP$) of concretes having a nominal $5\frac{1}{2}$ bags

Equation	Note	Const.	C_4AF	Na_2O	C_3A/SO_3	SO_3
1-----	(1)	$ADSX = -0.00673$ s.d. = (0.00318)	+0.000539 (0.000142)	-0.00364 (0.00161)	+0.00106 (0.00020)	-----
2-----	(1)	$ADSX = -0.00519$ s.d. = (0.00304)	+0.000552 (0.000136)	-0.00402 (0.00176)	+0.00080 (0.00020)	-----
2A-----	(2)	$ADSX$ (odd) = -0.00423 s.d. = (0.00466)	+0.000498 (0.000199)	-0.00399 (0.00260)	+0.00086 (0.00031)	-----
2B-----	(3)	$ADSX$ (even) = -0.00545 s.d. = (0.00436)	+0.000560 (0.000200)	-0.00412 (0.00253)	-0.00071 (0.00031)	-----
3-----	(1)	$ADSX = -0.00573$ s.d. = (0.00110)	-----	-----	-----	-----
4-----	(1)	$ADSX = -0.00798$ s.d. = (0.00187)	+0.000098 (0.000088)	-----	-----	+0.00112 (0.00057)
5-----	(1)	$ADSX = -0.00729$ s.d. = (0.00184)	+0.000175 (0.000086)	-----	-----	+0.00080 (0.00057)
5A-----	(2)	$ADSX$ (odd) = -0.00708 s.d. = (0.00279)	*+0.000106 (0.000125)	-----	-----	+0.00133 (0.00076)
5B-----	(3)	$ADSX$ (even) = -0.00742 s.d. = (0.00254)	+0.000212 (0.000123)	-----	-----	+0.00254 (0.00088)

¹ 171 cements; Avg = 0.00907; S.D. = 0.00384.

² 86 cements.

³ 85 cements.

* Coef./s.d. ratio less than 1.

6.5. Ratio of Percentage Expansion Divided by the Percentage Shrinkage

a. Ratio of Expansion Divided by the Shrinkage of Series O Concretes Made with a Water/Cement Ratio of 0.635 (OREX = OEXP/OSHR)

The frequency distribution of the cements with respect to the ratio of the expansion divided by the shrinkage of Series O concretes of AE + NAE cements is presented in table 10-27. Although there was a considerable range of values for the ratio, and an overlapping of the values for the cements of the different types, the averages of the ratios of the air-entraining cements were close to the averages of the NAE cements.

Equations relating the expansion-shrinkage ratio of Series O concretes to various independent variables are presented in table 10-28. Equation 1 indicates that this ratio is significantly related to the shrinkage of the specimens when dried in laboratory air for 56 days (see table 10-44). Use of the independent variables associated with the chemical composition of the cements, as in eq 2, produced an equivalent S.D. value. Using the chemical-composition variables and the shrinkage in eq 3 resulted in a significant reduction in the S.D. value (see table 10-44). Equations 4, 6, and 7, calculated with C_3A , Fe_2O_3 , and Al_2O_3 respectively in place of the C_4AF of eq 3, resulted in S.D. values which were close to that obtained in eq 3.

In eqs 2A and 2B for the "odds" and "evens" there were instances where the coef./s.d. ratios of C_4AF , and MgO were less than 1. In equations using the "odds" and "evens" of the equations 3, 4, 5, 6, and 7, not presented in the table, C_4AF , Fe_2O_3 , C_3A , Al_2O_3 , Na_2O , MgO , Li , and Ba , all had coef./s.d. ratios less than one

in one or the other or both of the equations for these smaller groups of cements.

A similar series of equations relating the expansion/shrinkage ratios of concretes made of the NAE cements to various independent variables is presented in table 10-29. The independent variables associated with the differences in the ratios, their probable significance, and the reduction in variance resulting from their use in the multivariable equations were in reasonable agreement with those of the preceding table 10-28 where the AE cements were included.

Using the coefficients of the independent variables of eq 2, table 10-29, and their ranges of values, computations were made of the calculated contributions to the expansion/shrinkage ratio, as well as the calculated ranges of these contributions. These calculated values are presented in table 10-30. Increases in Na_2O , Li , and Rb were associated with increases of the expansion/shrinkage ratio. Increase in SrO associated with a decrease in the ratio, whereas C_4AF , MgO , and Ba had coef./s.d. ratios of doubtful significance.

b. Ratio of Expansion Divided by the Shrinkage of Concretes made having a Slump of 5 ± 1 in (AREX = AEXP/ASHR)

The frequency distribution of the cements with respect to the expansion/shrinkage ratios of Series A concretes is presented in table 10-31. The range of values for the ratio was rather large and there was an overlapping of the values of the cements of the different types.

Equations relating the expansion/shrinkage ratio of the Series A concrete specimens made of AE + NAE are presented in table 10-32. No relationship was found between the ratio and the water/cement ratio used in preparing the

between the percentage shrinkage after 56 days drying and the percentage expansion after 28 of cement per cubic yard and a slump of 5 ± 1 in., to various independent variables

Air content	APF	Loss	MgO	SrO	Li	Mn	Rb	ASHR	S.D.
+0.000338 (0.000129)	+0.00000237 (0.00000059)	-0.00186 (0.00055)							0.00344
+0.000272 (0.000122)	+0.00000165 (0.00000056)	-0.00165 (0.00052)		+0.0108 (0.0031)	-0.1714 (0.0822)	-0.00274 (0.00208)	-0.2812 (0.1337)		0.00323
+0.000299 (0.000165)	+0.00000185 (0.00000094)	-0.00146 (0.00092)		+0.0094 (0.0045)	-0.2033 (0.1179)	-0.00662 (0.00285)	-0.2497 (0.1719)		0.00318
+0.000303 (0.000196)	+0.00000216 (0.00000075)	-0.00203 (0.00073)		+0.0122 (0.0048)	-0.1410 (0.1255)	*+0.00191 (0.00328)	-0.3311 (0.2280)		0.00340
								+0.4933 (0.0360)	0.00265
			-0.000512 (0.000169)					+0.5066 (0.0351)	0.00257
			-0.000370 (0.000169)	+0.0047 (0.0023)	-0.1459 (0.0573)	-0.00305 (0.00162)	-0.2592 (0.1005)	+0.4705 (0.0364)	0.00246
			-0.000329 (0.000250)	+0.0051 (0.0032)	-0.1824 (0.0828)	-0.00640 (0.00224)	-0.2660 (0.1284)	+0.4568 (0.0544)	0.00241
			-0.000352 (0.000245)	+0.0034 (0.0033)	-0.0965 (0.0837)	*+0.00027 (0.00241)	-0.2634 (0.1667)	+0.4947 (0.0505)	0.00251

TABLE 10-25. Coefficients for equations for NAE cements relating the difference between water storage ($ADSX = ASHR - AEXP$) of concretes having a nominal $5\frac{1}{2}$ bags of ce

Equation	Note	Const.	C ₄ AF	Na ₂ O	C ₃ A/SO ₃	SO ₃
1-----	(1)	ADSX = -0.00486 s.d. = (0.00319)	+0.00051 (0.00014)	-0.00365 (0.00159)	+0.00098 (0.00020)	-----
2-----	(1)	ADSX = -0.00424 s.d. = (0.00310)	+0.00054 (0.00014)	-0.00396 (0.00177)	+0.00078 (0.00021)	-----
2A-----	(2)	ADSX (odd) = -0.00514 s.d. = (0.00524)	+0.00058 (0.00023)	-0.00428 (0.00287)	+0.00055 (0.00034)	-----
2B-----	(3)	ADSX (even) = -0.00272 s.d. = (0.00399)	+0.00046 (0.00018)	-0.00346 (0.00239)	+0.00091 (0.00028)	-----
3-----	(1)	ADSX = -0.00547 s.d. = (0.00114)	-----	-----	-----	-----
4-----	(1)	ADSX = -0.00735 s.d. = (0.00199)	+0.000119 (0.000089)	-----	-----	+0.00116 (0.00059)
5-----	(1)	ADSX = -0.00760 s.d. = (0.00193)	+0.000199 (0.000087)	-----	-----	+0.00094 (0.00059)
5A-----	(2)	ADSX (odd) = -0.00969 s.d. = (0.00285)	+0.000318 (0.000146)	-----	-----	+0.00166 (0.00085)
5B-----	(3)	ADSX (even) = -0.00682 s.d. = (0.00282)	+0.000146 (0.000108)	-----	-----	*+0.00084 (0.00084)

¹ 159 cements; Avg=0.00886; S.D.=0.00376.

² 80 cements.

³ 79 cements.

* Coef./s.d. ratio less than 1.

TABLE 10-26. Calculated contributions to the percentage shrinkage after 56 days drying minus the percentage expansion after 28 days in water of concretes having a nominal $5\frac{1}{2}$ bags of cement per cubic yard and a slump of 5 ± 1 in., and the calculated ranges of such contributions ($ADSX = ASHR - AEXP$)

Independent variables	Range of variables (percent)	Coefficients from eq (2) table 10-25	Calculated contributions to ADSX	Calculated ranges of contributions to ADSX
C ₄ AF----	4 to 17	+0.00054	Const. = -0.00424	
Na ₂ O----	0 to 0.75	-0.00396	+0.0022 to +0.0092	0.0070
C ₃ A/SO ₃ ----	0.4 to 10.1	+0.00078	0 to -0.0030	0.0030
Air content.**	0 to 13	-0.000336	+0.0003 to +0.0079	0.0076
APF-----	*2500 to 5500	+0.00000218	0 to -0.0044	0.0044
Loss-----	0.3 to 3.3	-0.00188	+0.0054 to +0.0120	0.0066
SrO-----	0 to 0.4	+0.0089	-0.0006 to -0.0062	0.0056
Li-----	0 to 0.02	-0.1431	0 to +0.0036	0.0036
Mn**-----	0 to 1.0	-0.00294	0 to -0.0029	0.0029
Rb**-----	0 to 0.01	-0.1861	0 to -0.0029	0.0029
			0 to -0.0019	0.0019

*cm²/g.

**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

concretes, see eq 1. Using commonly determined variables in eq 2 resulted in a significant reduction in the S.D. value, but in eq 4 the indication of reduction was significant at the 5-percent probability level (see table 10-44). The additional use of the trace elements Li, SrO, Mn, Rb, Zn, and Ba in eqs 3 and 5 resulted in significant reductions in the S.D. values although none of the individual variables except SrO had high coef./s.d. ratios (see table 10-44). The coef./s.d. ratios of C₃A, C₂S, Wagn, Mn, Zn, and Ba were less than 1 in one or the other of the pairs of equations for the "odds" and "evens", eqs 3A, 3B, or 5A, 5B. Equation 6 gives a significant indication that the expansion/shrinkage ratio was associated with the shrinkage values of the concretes. The use of other independent variables in addition to ASHR in eq 7 resulted in a further significant reduction in the S.D. value (see table 10-44).

the percentage shrinkage after 56 days drying and the percentage expansion after 28 days
ment per cubic yard and a slump of 5 ± 1 in., to various independent variables

Air content	APF	Loss	MgO	SrO	Li	Mn	Rb	ASHR	S.D.
-0.000423 (0.000308)	+0.00000243 (0.00000059)	-0.00214 (0.00055)							0.00333
-0.000336 (0.000309)	+0.00000218 (0.00000057)	-0.00188 (0.00053)		+0.0089 (0.0032)	-0.1431 (0.0834)	-0.00294 (0.00213)	-0.1861 (0.1375)		0.00320
-0.000553 (0.000541)	+0.00000269 (0.00000103)	-0.00211 (0.00094)		+0.0106 (0.0051)	*-0.1217 (0.1292)	-0.00357 (0.00290)	*-0.1142 (0.2071)		0.00352
*-0.000173 (0.000402)	+0.00000173 (0.00000069)	-0.00178 (0.00066)		+0.0067 (0.0044)	-0.1754 (0.1221)	-0.00389 (0.00378)	-0.3084 (0.2034)		0.00301
								+0.4834 (0.0378)	0.00264
-0.000321 (0.000233)			-0.000534 (0.000172)					+0.4925 (0.0368)	0.00254
			-0.000399 (0.000174)	+0.0034 (0.0023)	-0.1473 (0.0573)	-0.00315 (0.00166)	-0.2462 (0.1024)	+0.4715 (0.0382)	0.00245
			*+0.000092 (0.000286)	+0.0051 (0.0036)	-0.1937 (0.0843)	-0.00557 (0.00228)	-0.3112 (0.1477)	+0.4786 (0.0562)	0.00267
			-0.000647 (0.000216)	*+0.0016 (0.0031)	-0.1108 (0.0802)	*+0.00046 (0.00282)	-0.2503 (0.1488)	+0.4767 (0.0543)	0.00223

TABLE 10-27. Frequency distribution of cements with respect to ratio, percentage expansion divided by percentage shrinkage, for concretes having a nominal $5\frac{1}{2}$ bags of cement per cubic yard and a water-cement ratio of 0.635 (OREX=OEXP/OSHR)

Type cement	Ratio														Total
	0.35 to 0.40	0.40 to 0.45	0.45 to 0.50	0.50 to 0.55	0.55 to 0.60	0.60 to 0.65	0.65 to 0.70	0.70 to 0.75	0.75 to 0.80	0.80 to 0.85	0.85 to 0.90	0.90 to 0.95	0.95 to 1.00	1.00 and over	
	Number of cements														
I.....	3			1	6	15	18	14	14	7	1	1		2	82
IA.....					2	3		1	2						8
II.....				2	9	12	18	8	5	7	3	3		1	68
IIA.....						2									2
III.....					1	6	4	4	1	4					20
IIIA.....						1	2				1				3
IV, V.....					2	5	2	3	2	1					15
Total.....	3	0	0	3	20	44	43	30	24	19	5	4	0	3	198

TABLE 10-28. Coefficients for equations for AE+NAE cements, relating the ratio, percentage expansion in moist storage after drying divided by the percentage shrinkage, for concretes of nominal 5½ bags of cement per cubic yard and a water-cement ratio of 0.635, to various independent variables (OREX = OEXP/OSHR)

Equation	Note	Const.	C ₄ AF	Fe ₂ O ₃	C ₃ A	Al ₂ O ₃	Na ₂ O	MgO	Li	SrO	Rb	Ba	OSHR	S.D.
1	(1)	OREX s.d. = +0.9554 = (0.0364)	---	---	---	---	---	---	---	---	---	---	-8.734 (1.183)	0.0913
2	(1)	OREX s.d. = +0.6953 = (0.0380)	-0.00409 (0.00310)	---	---	---	+0.1536 (0.0497)	+0.00858 (0.00612)	+6.458 (2.315)	-0.3628 (0.0840)	+12.77 (3.64)	+0.4314 (0.2146)	---	0.0912
2A	(2)	OREX (odd) s.d. = +0.6810 = (0.0615)	*-0.00199 (0.00479)	---	---	---	+0.2012 (0.0691)	*+0.00663 (0.00961)	+3.563 (3.243)	-0.4433 (0.1337)	+10.14 (5.12)	+0.4280 (0.3696)	---	0.0990
2B	(2)	OREX (even) s.d. = +0.7136 = (0.0483)	-0.00580 (0.00411)	---	---	---	+0.0782 (0.0747)	*+0.00804 (0.00854)	+10.688 (3.576)	-0.2599 (0.1092)	+18.20 (5.52)	+0.3669 (0.2713)	---	0.0844
3	(1)	OREX s.d. = +0.8882 = (0.0490)	**+0.00322 (0.00285)	---	---	---	**+0.1300 (0.0459)	**+0.00908 (0.00563)	**+3.999 (2.174)	-0.2501 (0.0798)	+12.47 (3.35)	**+0.2230 (0.2008)	-6.747 (1.200)	0.0839
4	(1)	OREX s.d. = +0.8390 = (0.0414)	---	---	**+0.00498 (0.00248)	---	**+0.1273 (0.0456)	**+0.00871 (0.00558)	**+4.353 (2.166)	-0.3005 (0.0844)	+11.29 (3.35)	**+0.2764 (0.2015)	-7.294 (1.212)	0.0832
5	(1)	OREX s.d. = +0.6956 = (0.0381)	---	**+0.0126 (0.0065)	---	---	+0.1536 (0.0497)	**+0.00856 (0.00612)	+6.464 (2.315)	-0.3629 (0.0840)	+12.78 (3.64)	+0.4319 (0.2146)	---	0.0912
6	(1)	OREX s.d. = +0.8885 = (0.0490)	---	**+0.0099 (0.0087)	---	---	**+0.1301 (0.0459)	**+0.00907 (0.00563)	**+4.003 (2.174)	-0.2501 (0.0798)	+12.47 (3.35)	**+0.2234 (0.2008)	-6.745 (1.200)	0.0819
7	(1)	OREX s.d. = +0.8087 = (0.0491)	---	---	---	**+0.0143 (0.0082)	**+0.1279 (0.0457)	**+0.00914 (0.00559)	**+4.175 (2.165)	-0.2955 (0.0852)	+11.01 (3.40)	**+0.2728 (0.2024)	-7.403 (1.238)	0.0835

1 176 cements; Avg = 0.6912; S.D. = 0.1043.

2 88 cements.

* Coef./s.d. ratio less than 1.

** Coef./s.d. ratio less than 1 in one or both of the equations for "odds" and "evens".

TABLE 10-29. Coefficients for equations for NAE cements, relating the ratio, percentage expansion in moist storage after drying divided by the percentage shrinkage, for concretes of nominal 5½ bags of cement per cubic yard and a water-cement ratio of 0.635, to various independent variables (OREX = OEXP/OSHR)

Equation	Note	Const.	C ₄ AF	Fe ₂ O ₃	C ₃ A	Al ₂ O ₃	Na ₂ O	MgO	Li	SrO	Rb	Ba	OSHR	S.D.
1	(1)	OREX s.d. = +0.9661 = (0.0391)	---	---	---	---	---	---	---	---	---	---	-9.137 (1.293)	0.0929
2	(1)	OREX s.d. = +0.6935 = (0.0392)	-0.00409 (0.00322)	---	---	---	+0.1665 (0.0520)	+0.00941 (0.00650)	+6.034 (2.399)	-0.3667 (0.0909)	+12.31 (3.78)	+0.4293 (0.2208)	---	0.0931
2A	(2)	OREX (odd) s.d. = +0.6660 = (0.0622)	*-0.00088 (0.00528)	---	---	---	+0.2230 (0.0714)	+0.00496 (0.00917)	+4.523 (3.553)	-0.4620 (0.1370)	+13.10 (5.26)	+0.3925 (0.3604)	---	0.0948
2B	(2)	OREX (even) s.d. = +0.7046 = (0.0530)	-0.00602 (0.00423)	---	---	---	+0.1135 (0.0802)	+0.01539 (0.00968)	+7.412 (3.349)	-0.2817 (0.1247)	+13.25 (5.70)	+0.4904 (0.2923)	---	0.0931
3	(1)	OREX s.d. = +0.8988 = (0.0515)	**+0.00270 (0.00295)	---	---	---	**+0.1340 (0.0480)	**+0.00781 (0.00596)	**+3.864 (2.232)	**+0.2607 (0.0854)	+13.10 (3.47)	**+0.2161 (0.2059)	-7.209 (1.297)	0.0853
4	(1)	OREX s.d. = +0.8546 = (0.0447)	---	---	**+0.00510 (0.00254)	---	**+0.1315 (0.0475)	**+0.00730 (0.00591)	**+4.267 (2.220)	-0.3133 (0.0895)	+11.97 (3.47)	**+0.2739 (0.2063)	-7.786 (1.301)	0.0844
5	(1)	OREX s.d. = +0.6939 = (0.0393)	---	**+0.0126 (0.0098)	---	---	+0.1665 (0.0520)	**+0.00939 (0.00650)	+6.039 (2.400)	-0.3667 (0.0909)	+12.31 (3.78)	+0.4298 (0.2208)	---	0.0931
6	(1)	OREX s.d. = +0.8990 = (0.0516)	---	**+0.0083 (0.0090)	---	---	**+0.1340 (0.0480)	**+0.00779 (0.00596)	**+3.868 (2.233)	-0.3133 (0.0854)	+13.11 (3.47)	**+0.2165 (0.2059)	-7.208 (1.297)	0.0853
7	(1)	OREX s.d. = +0.8207 = (0.0520)	---	---	---	**+0.0160 (0.0084)	**+0.1317 (0.0476)	**+0.00751 (0.00591)	**+4.133 (2.217)	-0.3133 (0.0902)	+11.65 (3.51)	**+0.2755 (0.2070)	-7.997 (1.331)	0.0845

1 164 cements; Avg = 0.6943; S.D. = 0.1060.

2 82 cements.

* Coef./s.d. ratio less than 1.

** Coef./s.d. ratio less than 1 in one or both of the equations for "odds" and "evens".

TABLE 10-30. Calculated contributions to the ratio, percentage expansion after 28 days in water divided by percentage shrinkage after 56 days of drying, for concretes having a nominal 5½ bags of cement per cubic yard and a water-cement ratio of 0.635, and the calculated ranges of such contributions (OREX = OEXP/OSHR)

Independent variables	Range of variables (percent)	Coefficients from eq (2) table 10-29	Calculated contributions to OREX	Calculated ranges of contributions to OREX
C4AF**	4 to 17	-0.00409	Const. = +0.6935 -0.016 to -0.069	0.054
Na2O	0 to 0.75	+0.1665	0 to +0.125	0.125
MgO**	0 to 5.0	+0.00941	0 to +0.047	0.047
Li	0 to 0.02	+6.034	0 to +0.121	0.121
SrO	0 to 0.4	-0.3667	0 to -0.147	0.147
Rb	0 to 0.01	+12.31	0 to +0.123	0.123
Ba**	0 to 0.2	+0.4293	0 to +0.086	0.086

**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

TABLE 10-31. Frequency distribution of cements with respect to the ratio, percentage expansion divided by the percentage shrinkage for concretes having a nominal 5½ bags of cement per cubic yard and a slump of 5 ± 1 in. (AREX = AEXP/ASHR)

Type cement	Ratio												Total
	0.40 to 0.45	0.45 to 0.50	0.50 to 0.55	0.55 to 0.60	0.60 to 0.65	0.65 to 0.70	0.70 to 0.75	0.75 to 0.80	0.80 to 0.85	0.85 to 0.90	0.90 to 0.95	0.95 to 1.00	
	Number of cements												
I	3		2	2	13	15	16	10	8	4	3	2	78
IA			2		3	1		2					8
II			2	4	9	19	13	6	5	3	5		66
IIA							1	1					2
III				1	3	10	2	2	1	1			20
IIIA					1	1	1						3
IV, V					4	4	4	2		1			15
Total	3	0	6	7	33	50	37 *	23	14	9	8	2	192

of the other independent variables all but MgO had coef./s.d. ratios between 2 and 3. MgO had a coef./s.d. ratio of less than 1 in eq 3A for one of the smaller groups of cements (the odds). There is a significant indication then, than an increase of the expansion/shrinkage ratio may be associated with an increase in Na2O, MgO, Li, and Mn. An increase in the expansion/shrinkage ratio may possibly also be associated with decreasing values for C4AF, fineness, and SrO.

6.6. The ratio of the Residual Shrinkage after Rewetting to the Initial Shrinkage (OREX and AREX)

The frequency distributions of the cements with respect to the ratio (OSHR-OEXP)/OSHR and (ASHR-AEXP/ASHR) are presented in tables 10-35 and 10-36 for the Series O and Series A concretes respectively. There was a considerable range of ratios in each of the tables and an overlapping of the values for the cements of different types.

Equations are presented in table 10-37 indicating the independent variables having a

A corresponding series of equations for the NAE cements is presented in table 10-33 indicating the variables which were associated with the expansion/shrinkage ratios of the Series A concretes. The equations presented in table 10-33 are in reasonable agreement with those presented in the previous table 10-32 where the AE cements were included in the calculated multivariable equations. (see also table 10-44).

Using the independent variables of eq 3 of table 10-33 and the ranges of these variables, calculations were made of the calculated contributions and ranges of contributions to the expansion/shrinkage ratios. The results of these calculations are presented in table 10-34. The coef./s.d. ratios of C3A, C2S, Rb, Zn, and Ba were all less than 2 in eq 3 of table 10-33, and

coef./s.d. ratio greater than 1 which may be associated with the (residual shrinkage divided by the shrinkage) ratios (OREX and AREX) for both AE + NAE and the NAE cements. In equations for OEXR, none of the major potential compounds or major oxides had coef./s.d. ratios greater than 1 when used in multivariable equations with other independent variables. None of the other independent variables in eqs 1 and 2 except Na2O and SrO had coef./s.d. ratios greater than 3. Only SrO had a coef./s.d. ratio greater than 3 in the two equations for the Series A concretes, eqs 3 and 4. As indicated in table 10-44, there was a highly significant reduction in variance with each of the four equations of table 10-37.

The calculated contributions to the OEXR ratio, and estimated ranges of these contributions, are presented in table 10-38 for the Series O concretes. Increases in SrO and possibly Pb were associated with increases in the ratio. Increases in Na2O, and possibly SO3, Li, and Rb were associated with decreases in the ratio. The effect of the variables MgO, Ba, V, and P were of doubtful significance.

TABLE 10-32. *Coefficients for equations for AE & NAE cements, relating the ratio, per bags of cement per cubic yard and a slump of 5 ± 1 in.*
Specimens were moist cured for 2 weeks, then air dried

Equation	Note	Const.	C ₃ A	C ₂ S	C ₄ AF	Fe ₂ O ₃
1-----	(1)	AREX = +0.729 s.d. = (0.189)	-----	-----	-----	-----
2-----	(1)	AREX = +1.039 s.d. = (0.109)	-0.0115 (0.0032)	-0.00161 (0.00115)	-0.00924 (0.00368)	-----
3-----	(1)	AREX = +0.924 s.d. = (0.108)	-0.0046 (0.0036)	-0.00180 (0.00109)	-0.00830 (0.00364)	-----
3A-----	(2)	AREX (odd) = +0.860 s.d. = (0.141)	-0.0074 (0.0047)	*-0.00098 (0.00129)	-0.00883 (0.00525)	-----
3B-----	(2)	AREX (even) = +1.017 s.d. = (0.185)	*-0.0049 (0.0064)	-0.00280 (0.00217)	-0.00974 (0.00561)	-----
4-----	(1)	AREX = +0.757 s.d. = (0.067)	-----	-----	-----	*-0.00554 (0.00995)
5-----	(1)	AREX = +0.787 s.d. = (0.065)	-----	-----	-----	-0.0191 (0.0089)
5A-----	(2)	AREX (odd) = +0.700 s.d. = (0.096)	-----	-----	-----	-0.0140 (0.0134)
5B-----	(2)	AREX (even) = +0.855 s.d. = (0.094)	-----	-----	-----	-0.0244 (0.0130)
6-----	(1)	AREX = +0.899 s.d. = (0.036)	-----	-----	-----	-----
7-----	(1)	AREX = +0.861 s.d. = (0.045)	-----	-----	** -0.00467 (0.00277)	-----

¹ 170 cements; Avg = 0.7045; S.D. = 0.0941.

² 85 cements.

* Coef./s.d. ratio less than 1.

** Coef./s.d. ratio less than 1 in equations for "odds" and "evens".

TABLE 10-33. *Coefficients for equations for NAE cements, relating the ratio, percentage cement per cubic yard and a slump of 5 ± 1 in to var*
Specimens were moist cured for 2 weeks, then air dried

Equation	Note	Const.	C ₃ A	C ₂ S	C ₄ AF	Fe ₂ O ₃
1-----	(1)	AREX = +1.051 s.d. = (0.256)	-----	-----	-----	-----
2-----	(1)	AREX = +1.024 s.d. = (0.110)	-0.0101 (0.0033)	-0.00127 (0.00117)	-0.0099 (0.0037)	-----
3-----	(1)	AREX = +0.928 s.d. = (0.110)	-0.0042 (0.0037)	-0.00154 (0.00112)	-0.0090 (0.0037)	-----
3A-----	(2)	AREX (odd) = +1.108 s.d. = (0.170)	-0.0075 (0.0055)	-0.00186 (0.00159)	-0.0140 (0.0061)	-----
3B-----	(2)	AREX (even) = +0.757 s.d. = (0.152)	*-0.0007 (0.0053)	*-0.00102 (0.00177)	-0.0054 (0.0048)	-----
4-----	(1)	AREX = +0.781 s.d. = (0.068)	-----	-----	-----	-0.0104 (0.0095)
5-----	(1)	AREX = +0.809 s.d. = (0.068)	-----	-----	-----	-0.0219 (0.0091)
5A-----	(2)	AREX (odd) = +0.913 s.d. = (0.113)	-----	-----	-----	-0.0288 (0.0155)
5B-----	(2)	AREX (even) = +0.723 s.d. = (0.083)	-----	-----	-----	-0.0174 (0.0111)
6-----	(1)	AREX = +0.895 s.d. = (0.038)	-----	-----	-----	-----
7-----	(1)	AREX = +0.860 s.d. = (0.046)	-----	-----	-0.0054 (0.0028)	-----

¹ 158 cements; Avg = 0.7077; S.D. = 0.0942.

² 79 cements.

* Coef./s.d. ratio less than 1.

** Coef./s.d. ratio less than 1 in equations for "odds" and "evens".

centage expansion divided by the percentage shrinkage, for concretes having a nominal 5½
to various independent variables (AREX = AEXP/ASHR)
for 8 weeks and then placed in water for 4 weeks.

Na ₂ O	MgO	WAGN	Li	SrO	Mn	Rb	Zn	Ba	W/C	ASHR	S.D.
									*-0.0412 (0.2970)		0.0955
+0.0974 (0.0418)	+0.0174 (0.0058)	-0.000092 (0.000032)									0.0885
+0.0946 (0.0461)	+0.0163 (0.0059)	-0.000061 (0.000031)	+6.20 (2.16)	-0.262 (0.088)	+0.088 (0.055)	+7.55 (3.55)	+0.315 (0.214)	+0.260 (0.208)			0.0827
+0.1020 (0.0692)	+0.0156 (0.0082)	*-0.000033 (0.000044)	+6.89 (3.12)	-0.246 (0.122)	+0.201 (0.077)	+10.37 (4.17)	+0.492 (0.273)	+0.486 (0.308)			0.0798
+0.0982 (0.0670)	+0.0150 (0.0098)	-0.000084 (0.000047)	+5.45 (3.19)	-0.237 (0.138)	*+0.008 (0.085)	+7.28 (5.94)	*+0.063 (0.384)	*+0.162 (0.327)			0.0878
+0.0765 (0.0425)	+0.0148 (0.0060)	-0.000047 (0.000029)									0.0914
+0.0844 (0.0459)	+0.0158 (0.0059)	-0.000038 (0.000028)	+6.44 (2.14)	-0.290 (0.079)	+0.075 (0.055)	+6.76 (3.52)	+0.305 (0.211)	+0.295 (0.200)			0.0829
+0.0805 (0.0682)	+0.0162 (0.0082)	*-0.000010 (0.000042)	+7.83 (3.02)	-0.301 (0.117)	+0.175 (0.075)	+9.07 (4.64)	+0.527 (0.273)	+0.547 (0.306)			0.0801
+0.0817 (0.0646)	+0.0113 (0.0093)	-0.000054 (0.000039)	+5.87 (3.16)	-0.255 (0.112)	*-0.011 (0.083)	+6.30 (5.86)	*-0.044 (0.361)	*+0.099 (0.281)			0.0876
										-6.500 (1.183)	0.0869
**+0.0711 (0.0429)	+0.0146 (0.0055)		+5.16 (2.03)	-0.206 (0.077)	**+0.090 (0.052)	+7.42 (3.23)				-5.213 (1.162)	0.0792

expansion divided by the percentage shrinkage, for concretes having a nominal 5½ bags of
ious independent variables (AREX = AEXP/ASHR)
for 8 weeks and then placed in water for 4 weeks.

Na ₂ O	MgO	WAGN	Li	SrO	Mn	Rb	Zn	Ba	W/C	ASHR	S.D.
									-0.539 (0.400)		0.0952
+0.1082 (0.0420)	+0.0200 (0.0059)	-0.000093 (0.000033)									0.0874
+0.1023 (0.0475)	+0.0187 (0.0061)	-0.000068 (0.000032)	+5.18 (2.20)	-0.233 (0.092)	+0.094 (0.057)	+6.07 (3.64)	+0.312 (0.215)	+0.254 (0.210)			0.0828
+0.1519 (0.0814)	*+0.0083 (0.0103)	-0.000110 (0.000053)	+5.05 (3.21)	-0.342 (0.143)	+0.155 (0.079)	+6.65 (5.38)	*+0.075 (0.440)	*+0.313 (0.365)			0.0918
+0.0732 (0.0627)	+0.0253 (0.0083)	*-0.000025 (0.000041)	+6.91 (3.29)	-0.145 (0.125)	*+0.020 (0.095)	+8.35 (5.35)	+0.416 (0.242)	+0.281 (0.278)			0.0757
+0.0934 (0.0424)	+0.0177 (0.0060)	-0.000055 (0.000030)									0.0896
+0.0924 (0.0470)	+0.0182 (0.0061)	-0.000049 (0.000029)	+6.04 (2.17)	-0.258 (0.084)	+0.081 (0.056)	+5.29 (3.60)	+0.307 (0.212)	+0.285 (0.201)			0.0829
+0.1347 (0.0806)	*+0.0084 (0.0103)	-0.000081 (0.000050)	+5.46 (3.16)	-0.394 (0.136)	+0.127 (0.077)	+5.33 (5.30)	*+0.105 (0.434)	+0.408 (0.360)			0.0921
+0.0616 (0.0588)	+0.0239 (0.0079)	*-0.000019 (0.000035)	+7.12 (3.19)	-0.131 (0.108)	*+0.018 (0.094)	+7.93 (5.22)	+0.393 (0.233)	*+0.228 (0.241)			0.0748
										-6.328 (1.259)	0.0877
**+0.0778 (0.0442)	+0.0158 (0.0057)		+5.05 (2.06)	**+0.0181 (0.087)	**+0.095 (0.054)	+6.85 (3.32)				-5.143 (1.220)	0.0796

TABLE 10-34. Calculated contributions to the ratio, percentage expansion after 28 days in water divided by the percentage shrinkage after 56 days in laboratory air, for concretes having a nominal $5\frac{1}{2}$ bags of cement per cubic yard and a slump of 5 ± 1 in., and the calculated ranges of such contributions (AREX = AEXP/ASHR)

Independent variables	Range of variables (percent)	Coefficients from eq (3) table 10-33	Calculated contributions to AREX	Calculated ranges of contributions to AREX
			Const. = +0.928	
C ₃ A**	0 to 15	-0.0042	0 to -0.063	0.063
C ₂ S**	5 to 50	-0.00154	-0.008 to -0.077	0.069
C ₄ AF	4 to 17	-0.0090	-0.036 to -0.153	0.117
Na ₂ O	0 to 0.75	+0.1023	0 to +0.077	0.077
MgO	0 to 5.0	+0.0187	0 to +0.094	0.094
WAGN	*1300 to 2900	-0.000068	-0.088 to -0.197	0.109
Li	0 to 0.02	+5.18	0 to +0.104	0.104
SrO	0 to 0.4	-0.233	0 to -0.093	0.093
Mn	0 to 1.0	+0.094	0 to +0.094	0.094
Rb**	0 to 0.01	+6.07	0 to +0.061	0.061
Zn**	0 to 0.2	+0.312	0 to +0.062	0.062
Ba**	0 to 0.2	+0.254	0 to +0.051	0.051

*cm²/g.

**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

A corresponding series of calculations for the Series A concretes made of NAE cements is presented in table 10-39. Increases in SrO, and possibly Fe₂O₃ and fineness were associated with an increase in the ratio. Increases in Na₂O and Li may possibly be associated with decreases in the ratio.

6.7. Shrinkage of Tops and Bottoms of Concrete Specimens as Cast

All of the data presented in this section on shrinkages and expansions, their differences, and their ratios have been based on averages of the measurements of the tops and bottoms of the specimens as cast. The results of measurements on the top of the concrete specimens and the bottom as cast usually differed. The frequency distribution of these differences obtained on the concretes of constant water/ce-

TABLE 10-35. Frequency distribution of cements with respect to the ratio (OSHR-OEXP)/OSHR of concretes having a nominal $5\frac{1}{2}$ bags of cement per cubic yard and a water-cement ratio of 0.635

Type cement	Ratio														Total
	-0.10 to -0.05	-0.05 to 0	0 to 0.05	0.05 to 0.10	0.10 to 0.15	0.15 to 0.20	0.20 to 0.25	0.25 to 0.30	0.30 to 0.35	0.35 to 0.40	0.40 to 0.45	0.45 to 0.50	0.50 to 0.55	0.55 and over	
	Number of cements														
I-----	1	0	1	1	1	6	14	14	19	15	5	2	-----	3	82
IA-----	-----	-----	1	2	4	5	3	11	18	14	9	-----	1	-----	68
II-----	-----	-----	-----	-----	-----	-----	-----	1	1	1	-----	-----	-----	-----	2
IIA-----	-----	-----	-----	-----	1	2	3	4	4	6	1	-----	-----	-----	20
III-----	-----	-----	-----	-----	-----	-----	-----	1	1	1	-----	-----	-----	-----	3
IIIA-----	-----	-----	-----	-----	-----	1	2	2	3	5	2	-----	-----	-----	15
IV, V-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
Total-----	1	0	2	3	6	14	24	32	46	45	19	2	1	3	198

TABLE 10-36. Frequency distribution of cements with respect to the ratio (ASHR-AEXP)/ASHR of concretes having a nominal $5\frac{1}{2}$ bags of cement per cubic yard and a slump of 5 ± 1 in.

Type cement	Ratio												Total
	0 to 0.05	0.05 to 0.10	0.10 to 0.15	0.15 to 0.20	0.20 to 0.25	0.25 to 0.30	0.30 to 0.35	0.35 to 0.40	0.40 to 0.45	0.45 to 0.50	0.50 to 0.55	0.55 to 0.60	
	Number of cements												
I	1	3	4	8	7	19	16	10	5	2		3	78
IA					1	1	1	2	1	2			8
II		5	2	4	7	12	20	9	5	2			66
IIA					1	1							2
III			1		3	2	10	3	1				20
IIIA						1	1	1					3
IV, V			1		2	3	5	4					15
Total	1	8	8	12	21	39	53	29	12	6	0	3	192

ment ratio are presented in table 10-40. Although there were 12 instances where the bottom of the specimen had greater shrinkage than the top, the tops of the majority of the specimens had greater shrinkage than the bottoms. The equations presented in table 10-41 indicate that this difference was associated with higher average shrinkage (OSHR), as indicated in eqs 1 and 5 (see also table 10-44). The reduction in variance resulting from the use of the additional independent variables in eqs 4 or 8 was significant at the 5-percent probability levels. The use of only commonly determined variables in eq 2, or those with trace elements in eq 3, resulted in a reduction in the S.D. value significant at the 5-percent probability level. In eqs 6 and 7, the reduction in the S.D. was not significant at the 5-percent level (see table 10-44). Of the different independent variables in tables 10-41, only OSHR,

the average shrinkage of the top and bottom as cast, had a coef./s.d. ratio greater than 3.

The frequency distribution of the differences of the shrinkage of tops and bottoms of the specimens as cast of the Series A concrete specimens is presented in table 10-42.

Equations are presented in table 10-43 indicating the relationship of the various independent variables to the difference in the shrinkage of the tops and bottoms of the Series A concrete specimens. The use of commonly determined variables in eqs 1, 3, 5, and 7 resulted in a significant reduction in the S.D. values (see also table 10-44). The additional use of the trace elements, Cr, Rb, Li, and Ni in eqs 2, 4, 6, and 8 respectively, each resulted in a reduction in variance significant at the 1-percent probability level (see table 10-44).

A similar series of equations (not presented in tables) was calculated using the percentage

TABLE 10-37. Coefficients for equations relating the (shrinkage-expansion)/shrinkage ratios
ables; the 6- x 8- by 16-in. concrete specimens were cured in moist air for 14 days, then

Equation	Type cement	Note	Const.	Na ₂ O	MgO
1.....	AE + NAE	(¹)	OEXR = +0.3398 s.d. = (0.0610)	-0.1664 (0.0491)	*-0.0083 (0.0060)
2.....	NAE	(²)	OEXR = +0.3584 s.d. = (0.0654)	-0.1757 (0.0518)	*-0.0088 (0.0063)
3.....	AE + NAE	(³)	AEXR = +0.1939 s.d. = (0.0630)	*-0.0808 (0.0455)	-0.0157 (0.0057)
4.....	NAE	(⁴)	AEXR = +0.1723 s.d. = (0.0652)	-0.0939 (0.0465)	*-0.0181 (0.0059)

¹ 178 cements; Avg = 0.3091; S.D. = 0.1037.

² 166 cements; Avg = 0.3063; S.D. = 0.1054.

³ 172 cements; Avg = 0.2957; S.D. = 0.0937.

⁴ 160 cements; Avg = 0.2926; S.D. = 0.0937.

* Coef./s.d. ratio was less than 1 in equations for "odds" and "evens".

TABLE 10-38. Calculated contributions to the OEXR ratio or (OSHR-OEXP)/OSHR, of concretes of nominal 5½ bags of cement per cubic yard and a water-cement ratio of 0.635, and the calculated ranges of such contributions

Independent variables	Range of variables (percent)	Coefficients from eq (2) table 10-37	Calculated contributions to OEXR	Calculated ranges of contributions to OEXR
			Const. = +0.3584	
Na ₂ O.....	0 to 0.75	-0.1757	0 to -0.132	0.132
MgO**.....	0 to 5.0	-0.0088	0 to -0.044	0.044
SO ₃	1.2 to 3.0	-0.0523	-0.063 to -0.157	0.094
WAGN**.....	*1300 to 2900	+0.000049	+0.064 to +0.142	0.078
Li.....	0 to 0.02	-5.042	0 to -0.101	0.101
Rb.....	0 to 0.01	-8.542	0 to -0.085	0.085
SrO.....	0 to 0.4	+0.4648	0 to +0.186	0.186
Ba**.....	0 to 0.2	-0.2521	0 to -0.050	0.050
V**.....	0 to 0.1	-0.7135	0 to -0.071	0.071
P**.....	0 to 0.5	-0.1011	0 to -0.050	0.050
Pb.....	0 to 0.05	+2.741	0 to +0.137	0.137

*cm²/g.

**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

TABLE 10-39. Calculated contributions to the AEXR ratio or (ASHR-AEXP)/ASHR of concretes of nominal 5½ bags of cement per cubic yard and a slump of $\bar{\delta} \pm 1$ in., and the calculated ranges of such contributions

Independent variables	Range of variables (percent)	Coefficients from eq (4) table 10-37	Calculated contributions to AEXR	Calculated ranges of contributions to AEXR
			Const. = +0.1723	
Na ₂ O.....	0 to 0.75	-0.0939	0 to -0.070	0.070
MgO.....	0 to 5.0	-0.0181	0 to -0.090	0.090
Fe ₂ O ₃	1.0 to 5.0	+0.0184	+0.018 to +0.092	0.074
WAGN.....	*1300 to 2900	+0.000056	+0.073 to +0.162	0.089
Li.....	0 to 0.02	-5.462	0 to -0.109	0.109
Rb**.....	0 to 0.01	-3.889	0 to -0.039	0.039
SrO.....	0 to 0.4	+0.2834	0 to +0.113	0.113
Ba**.....	0 to 0.2	-0.2555	0 to -0.047	0.047

*cm²/g.

**Coefficient of doubtful significance as coef./s.d. ratio was less than 2.

TABLE 10-40. Frequency distribution of cements with respect to the difference of the percentage shrinkage of the top and bottom of specimens as cast of concretes having a nominal 5½ bags of cement per cubic yard and a water-cement ratio of 0.635 (ODSH)

Type cement	Difference in percentage shrinkage of top and bottom												Total
	-0.006 to -0.004	-0.004 to -0.002	-0.002 to 0.000	0.000 to 0.002	0.002 to 0.004	0.004 to 0.006	0.006 to 0.008	0.008 to 0.010	0.010 to 0.012	0.012 to 0.014	0.014 to 0.016	0.016 to 0.018	
	Number of cements												
I.....		3	3	5	16	21	20	10	2		1	1	82
IA.....				1		2	4	1					8
II.....	1	1	2	11	15	20	7	5	2	2	1		67
IIA.....							1						1
III.....				4	3	6	4	2					19
IIIA.....					1	2							3
IV, V.....	1		1	1	4	1	2	1	2	1			14
Total.....	2	4	6	22	39	52	38	19	6	3	2	1	194

of the concretes of nominal 5½ bags of cement per cubic yard to other independent variables dried in laboratory air for 56 days, then placed in water for 28 days

SO ₃	Air content	Fe ₂ O ₃	WAGN	Li	Rb	SrO	Ba	V	P	Pb	S.D.
-0.0515 (0.0204)	*-0.0124 (0.0100)	-----	*+0.000049 (0.000029)	-5.750 (2.432)	-9.889 (3.621)	+0.4542 (0.0867)	*-0.2838 (0.2261)	*-0.7141 (0.4479)	*-0.1190 (0.0638)	*+2.698 (1.142)	0.0874
-0.0523 (0.0215)	-----	-----	*+0.000049 (0.000031)	*-5.042 (2.520)	-8.542 (3.804)	+0.4648 (0.0933)	*-0.2521 (0.2329)	*-0.7135 (0.4614)	*-0.1011 (0.0663)	*+2.741 (1.163)	0.0880
-----	-----	*+0.0156 (0.0088)	+0.000045 (0.000027)	-5.857 (2.112)	-5.434 (3.408)	+0.3182 (0.0778)	*-0.2530 (0.1968)	-----	-----	-----	0.0829
-----	-----	+0.0184 (0.0088)	*+0.000056 (0.000028)	-5.462 (2.148)	*-3.889 (3.480)	+0.2834 (0.0827)	*-0.2355 (0.1984)	-----	-----	-----	0.0829

shrinkage (ASHR) as an additional independent variable. A coef./s.d. ratio slightly greater

than 3 was obtained for ASHR in an equation relating ADSH to ASHR.

7. Discussion

7.1. Comparison of Equations for the Dependent Variables

A summary of equations for shrinkage and expansion characteristics of the Series O concretes using AE + NAE cements is presented in table 10-45. The equations are presented vertically instead of horizontally as was also done in previous Sections. Also presented are the coef./s.d. ratios of each of the independent variables as well as the calculated ranges of the effects of these independent variables. It may be noted that in the first two equations (OSHR and OEXP), columns 1 and 2, increases in SO₃ and loss on ignition were associated with decreases in both shrinkage and expansion. Increased Na₂O was associated with decreased shrinkage. Both the shrinkage and the expansion increased with increased C₃A, C₄AF, fineness of the cement and the air content of the concrete.

It was previously indicated (tables 10-1 and 10-2) that concretes made with the air-entraining cements had lower cement content per unit volume than the others. There was, however, also probably a decrease in water content, and in fine and coarse aggregate per unit volume of concrete with an increase in air content. A decrease in the cement content may be expected to result in a decrease in the shrinkage [1]. L'Hermite [1] quoting Dutron [6] indicated that the shrinkage of concrete,

$\Delta_b = \gamma (e + c + v + f)^n$ where e = volume of water, c = volume of cement, v =

volume of voids, and f = volume of fines. It was not possible in our present series of tests to hold the cement content constant and vary only the air content. The void volume or air content appears to have a major effect when airentraining cements are included. Tables 10-5 and 10-9 indicated that for NAE cements, the air content had a coef./s.d. ratio less than 1 and was not a significant variable. The slump of the concretes made with AE cements was previously reported [3] as being high when a constant water/cement ratio was used. When the water content was adjusted to that required to produce the 5 ± 1 -in-slump concretes, the air content still appeared to be associated with shrinkage, but not with expansion (see table 10-46).

The equations for the differences between the shrinkage on drying and the expansion on rewetting (i.e., the residual shrinkage) are presented in columns 3 and 4 of table 10-45. Column 3 indicates that increases in C₄AF, C₃A/SO₃, fineness, and SrO were associated with increases in the residual-shrinkage values. Increases in Loss, Rb, and possibly Na₂O, and Li were associated with decreases in these values in column 3 where the percentage shrinkage was not used as an independent variable. The greater the initial shrinkage, the greater the residual shrinkage, as indicated in column 4. Except for Rb and SrO, none of the other independent variables had coef./s.d. ratios greater than 2 for the equation presented in column 4.

TABLE 10-41. Coefficients for equations relating ODSH, the difference in percentage shrinkage of the tops and bottoms as cast of the 6-X 8-X 16-in. specimens made of nominal $5\frac{1}{2}$ bags of cement per cubic yard concretes with a water-cement ratio of 0.635, to various independent variables

Equation	Type cement	Note	Const.	C ₁ A	C ₂ S	Na ₂ O	Loss	Mn	Rb	V	Cu	OSHR	APF	S.D.
1	AE+NAE	(1)	ODSH = +0.00022 s.d. = (0.00135)	-----	-----	-----	-----	-----	-----	-----	-----	+0.1407 (0.0436)	-----	0.003282
2	do	(1)	ODSH = +0.00365 s.d. = (0.00176)	+0.000156 (0.000090)	+0.000051 (0.000041)	-0.00357 (0.00153)	-0.000800 (0.000480)	-----	-----	-----	-----	-----	-----	0.003308
3	do	(1)	ODSH = +0.00222 s.d. = (0.00182)	+0.000254 (0.000097)	*+0.000042 (0.000041)	-0.00354 (0.00152)	*-0.000778 (0.000482)	*+0.00336 (0.00207)	*-0.1952 (0.1331)	*+0.0295 (0.0155)	*+0.0441 (0.0297)	-----	-----	0.003249
4	do	(1)	ODSH = +0.00208 s.d. = (0.00227)	-----	-----	*-0.00235 (0.00148)	-----	*+0.00320 (0.00203)	*-0.1303 (0.1287)	*+0.0253 (0.0147)	*+0.0396 (0.0281)	+0.1506 (0.0435)	*-0.00000069 (0.00000048)	0.003208
5	NAE	(2)	ODSH = +0.00013 s.d. = (0.00145)	-----	-----	-----	-----	-----	-----	-----	-----	+0.1438 (0.0476)	-----	0.003343
6	do	(2)	ODSH = +0.00333 s.d. = (0.00188)	+0.000161 (0.000096)	+0.000053 (0.000044)	-0.00372 (0.00159)	-0.000649 (0.000506)	-----	-----	-----	-----	-----	-----	0.003372
7	do	(2)	ODSH = +0.00245 s.d. = (0.00190)	+0.000225 (0.000101)	*+0.000047 (0.000044)	-0.00340 (0.00158)	*-0.000661 (0.000508)	*+0.00317 (0.00220)	*-0.1877 (0.1382)	+0.0296 (0.0160)	-----	-----	-----	0.003327
8	do	(2)	ODSH = +0.00230 s.d. = (0.00239)	-----	-----	*-0.00216 (0.00155)	-----	*+0.00307 (0.00216)	*-0.1426 (0.1330)	*+0.0258 (0.0152)	-----	+0.1428 (0.0477)	*-0.00000061 (0.00000051)	0.003289

1 175 cements; Avg = 0.00551; S.D. = 0.003370.

2 164 cements; Avg = 0.00444; S.D. = 0.003425.

* Coef./s.d. ratio less than 1 in one or both of the smaller groups of "odds" or "evens".

TABLE 10-42. Frequency distribution of cements with respect to the difference of the percentage shrinkage of the top and bottom of specimens as cast of concretes having a nominal $5\frac{1}{2}$ bags of cement per cubic yard and a slump of 5 ± 1 in. (ADSH)

Type cement	Difference in percentage shrinkage of top and bottom											Total
	-0.006 to -0.004	-0.004 to -0.002	-0.002 to 0.000	0.000 to 0.002	0.002 to 0.004	0.004 to 0.006	0.006 to 0.008	0.008 to 0.010	0.010 to 0.012	0.012 to 0.014	0.014 to 0.016	
	Number of cements											
I.....	2	1	2	7	11	20	18	12	1	3	-----	77
IA.....				1	4	1		2				8
II.....	3	1	7	7	11	16	11	6	2		1	65
IIA.....						1	1					2
III.....	1		1	4	8	3		1	2			20
IIIA.....			1			1		1				3
IV, V.....			2	3	2	4	2	1				14
Total.....	6	2	13	22	36	46	32	23	5	3	1	189

The equation for the ratio of the expansion divided by the shrinkage (OREX) (column 5) also indicates that the greater the shrinkage the smaller the expansion/shrinkage ratio. Increases in Rb and possibly Na_2O , Li, and C_3A were associated with an increase in the ratio, and increases in SrO with decreases in the expansion/shrinkage ratios. Other independent variables had coefficients of doubtful significance as related to this ratio.

The equation in column 6 for the ratio of the difference between shrinkage and expansion divided by the shrinkage (OEXR) has only two independent variables, Na_2O and SrO, with coef./s.d. ratios greater than 3. Increases in SrO and possibly Pb were associated with increases of the ratio. Increases in Na_2O and possibly Li, Rb, and SO_3 were associated with decreases of the ratio values.

A series of equations similar to those in table 10-45 is presented in table 10-46 for the Series A concretes made of AE + NAE cements. In general, but with a few exceptions, the same independent variables had coef./s.d. ratios greater than one in equations in this table and in the previous table. The effect of the air content was previously noted and discussed.

7.2. The Effect of Trace Elements

Tables 10-45 and 10-46 show, as indicated in previous subsections where the equations were discussed, that the coef./s.d. ratios of the trace elements were generally less than 3. This effect may have resulted from the fact that the concretes contained a lower percentage of the cements than did the mortars and neat pastes, and the influence of the trace elements is thereby more difficult to determine. It may also be that variations resulting from the use of different lots of sand and gravel in this study may have helped mask the effects of the independent variables used in the equations. Generally, the use of trace elements as independent variables in addition to commonly determined in-

dependent variables in the multivariable equations resulted in a reduction in variance significant at the $\alpha = 0.01$ level although there were many instances where the reduction in variance was significant at the $\alpha = 0.05$ level.

7.3. Agreement of Duplicate Determinations of Percentage Shrinkage and Percentage Expansion

There were 26 pairs of duplicate specimens where the water/cement ratio was the same for the Series O and Series A concretes. The estimated standard deviation of individual measurements from the differences between these pairs was 0.0016 for the shrinkage values and 0.0015 for the expansion values.

A summary of the averages of shrinkage and expansion values obtained with the NAE cements is presented in table 10-47. Also presented are the S.D. values for the individual cements and the equations, and those calculated from duplicate determinations. The $(\text{S.D.})^2$ values for all the cements were from 5 to 12 times the $(\text{S.D.})^2$ values obtained from the duplicates.

It has been indicated in previous tables that, by use of various independent variables, a highly significant reduction in variance was obtained. The S.D. values of the "best" equations were, however, much greater than the S.D. values of the duplicate specimens. Each of these pairs of duplicate specimens was made the same day using a single lot of aggregates. Greater variations could be expected if the test specimens were made over a period of three years with different lots of aggregates and by different operators. This necessary procedure may also have affected the shrinkage and expansion values as measured and may have masked to some extent the effect of the independent variables, such as cement composition, on these properties. No control tests were carried on during the period the specimens of the

TABLE 10-43. Coefficients for equations relating the difference in percentage shrinkage of the tops and bottoms of 6-X-8-X-16-in. specimens as cast made of nominal 5½ bags of cement per cubic yard concretes with a 5 ± 1 in slump to various independent variables (ADSH)

Equation	Type cement	Note	Const.	Air content	C ₃ A	Al ₂ O ₃	Loss	Cr	Rb	Li	Ni	S.D.
1.-----	AE+NAE	(1)	ADSH = +0.00359 s.d. = (0.00092)	-0.000274 (0.000124)	+0.000309 (0.000083)	-----	-0.00122 (0.00048)	-----	-----	-----	-----	0.003361
2.-----	do-----	(1)	ADSH = +0.00452 s.d. = (0.00100)	*-0.000312 (0.000114)	+0.000385 (0.000081)	-----	-0.00166 (0.00044)	*-0.1133 (0.0630)	-0.7113 (0.1298)	*-0.1204 (0.0711)	*+0.1940 (0.0846)	0.003068
3.-----	do-----	(1)	ADSH = +0.00070 s.d. = (0.00146)	-0.000247 (0.000122)	-----	+0.00103 (0.00026)	-0.00117 (0.00026)	-----	-----	-----	-----	0.003342
4.-----	do-----	(1)	ADSH = +0.00077 s.d. = (0.00151)	*-0.000280 (0.000112)	-----	+0.00132 (0.00026)	-0.00160 (0.00044)	*-0.0893 (0.0626)	-0.7746 (0.1295)	*-0.1119 (0.0704)	*+0.1734 (0.0834)	0.003033
5.-----	NAE	(2)	ADSH = +0.00477 s.d. = (0.00101)	-0.00107 (0.00030)	+0.000322 (0.000084)	-----	-0.00135 (0.00049)	-----	-----	-----	-----	0.003307
6.-----	do-----	(2)	ADSH = +0.00494 s.d. = (0.00107)	*-0.00076 (0.00029)	+0.000396 (0.000084)	-----	-0.00170 (0.00046)	*-0.0971 (0.0652)	-0.6535 (0.1339)	*-0.0905 (0.0730)	*+0.1842 (0.0860)	0.003073
7.-----	do-----	(2)	ADSH = +0.00173 s.d. = (0.00154)	-0.00100 (0.00030)	-----	+0.00107 (0.00026)	-0.00129 (0.00048)	-----	-----	-----	-----	0.003291
8.-----	do-----	(2)	ADSH = +0.00093 s.d. = (0.00160)	*-0.00066 (0.00029)	-----	+0.00136 (0.00026)	-0.00162 (0.00045)	*-0.0781 (0.0650)	-0.6990 (0.1341)	*-0.0847 (0.0721)	*+0.1575 (0.0848)	0.003038

1 169 cements; Avg = 0.00412; S.D. = 0.003539.

2 157 cements; Avg = 0.00413; S.D. = 0.003578.

* Coef./s.d. ratio less than 1 in one or both of smaller groups of "odds" and "evens".

TABLE 10-44. "F" Values for significance of reduction of variance due to added variables

Table	Equations	"F" ratio	D.F.	Critical "F" Ratio	
				$\alpha=0.01$	$\alpha=0.05$
10-4-----	*0,1	6.20	2:175	4.72	3.05
	0,2	20.97	9:168	2.51	1.93
	2,3	3.92	3:165	3.88	2.67
	0,4	23.19	8:169	2.61	2.00
	4,5	3.71	3:166	3.88	2.67
10-5-----	**0,1	5.07	2:163	4.73	3.05
	0,2	20.87	8:157	2.62	2.00
	2,3	3.76	3:154	3.90	2.67
	0,4	23.44	7:158	2.75	2.07
	4,5	3.51	3:155	3.90	2.67
10-8-----	*0,2	4.69	2:169	4.72	3.05
	0,3	18.07	9:162	2.52	1.93
	3,4	3.65	3:159	3.89	2.67
	0,5	19.93	8:163	2.62	2.00
	5,6	3.68	3:160	3.89	2.67
10-9-----	**0,1	2.72	2:157	4.74	3.05
	0,2	7.14	2:157	4.74	3.05
	0,3	19.04	8:151	2.63	2.00
	3,4	3.41	3:148	3.90	2.68
	0,5	21.28	7:152	2.76	2.07
10-12-----	5,6	3.66	3:149	3.90	2.68
	*0,1	3.20	2:174	4.72	3.05
	0,2	16.67	7:169	2.74	2.07
	2,3	3.77	5:164	3.13	2.27
	0,4	17.89	7:169	2.74	2.07
10-13-----	4,5	3.62	5:164	3.13	2.27
	**0,1	1.51	2:162	4.73	3.05
	0,2	15.16	6:158	2.93	2.15
	2,3	4.73	5:153	3.14	2.27
	0,4	16.65	6:158	2.93	2.15
10-16-----	4,5	4.48	5:153	3.14	2.27
	*0,2	3.56	2:169	4.72	3.05
	0,3	6.17	4:167	3.42	2.43
	3,4	13.20	6:161	2.93	2.15
	4,5	3.66	4:157	3.43	2.43
10-17-----	0,6	12.90	9:162	2.52	1.93
	6,7	3.52	4:158	3.43	2.43
	**0,2	3.60	2:157	4.74	3.05
	0,3	6.10	4:155	3.43	2.43
	3,4	15.23	5:150	3.14	2.27
10-20-----	4,5	4.13	4:146	3.45	2.44
	0,6	14.27	8:151	2.63	2.00
	6,7	4.00	4:147	3.44	2.44
	*0,1	7.74	7:170	2.74	2.07
	1,2	10.36	3:167	3.88	2.67
10-21-----	0,3	117.05	2:175	4.72	3.05
	3,4	6.11	6:169	2.92	2.15
	**0,1	7.87	7:158	2.75	2.07
	1,2	7.40	3:155	3.89	2.67
	0,3	67.95	3:162	3.89	2.67
10-24-----	3,4	5.00	7:155	2.76	2.07
	*0,1	7.01	7:164	2.75	2.07
	1,2	6.50	4:160	3.43	2.43
	0,3	95.03	2:169	4.72	3.05
	3,4	4.56	3:166	3.88	2.67
10-25-----	4,5	4.79	4:162	3.43	2.43
	**0,1	7.24	7:152	2.76	2.07
	1,2	4.15	4:148	3.44	2.44
	0,3	82.76	2:157	4.74	3.05
	3,4	4.15	4:153	3.44	2.44
10-28-----	*0,1	27.84	2:174	4.71	3.05
	0,2	7.77	8:168	2.61	2.00
	1,3	5.58	7:167	2.74	2.07
	**0,1	25.76	2:162	4.73	3.05
	0,2	7.07	8:156	2.62	2.00
10-29-----	1,3	5.31	7:155	2.75	2.07
	1,4	5.89	7:155	2.75	2.07
	*0,2	4.17	7:163	2.75	2.07
	2,3	4.94	6:157	2.93	2.15
	0,4	3.04	5:165	3.13	2.07
10-32-----	4,5	6.93	6:159	2.93	2.15
	0,6	15.67	2:168	4.73	3.05
	6,7	5.89	7:161	2.75	2.07
	*0,2	4.65	7:151	2.76	2.07
	2,3	3.87	6:145	2.93	2.15
10-33-----	0,4	4.33	5:153	3.14	2.27
	4,5	5.29	6:147	2.93	2.15
	0,6	13.14	2:156	4.74	3.05
	6,7	5.77	7:149	2.76	2.07

TABLE 10-44. "F" values for significance of reduction of variance due to added variables—Continued

Table	Equations	"F" ratio	D.F.	Critical "F" Ratio	
				$\alpha=0.01$	$\alpha=0.05$
10-37-----	*0,1	6.58	13:165	2.31	1.80
	0,2	7.01	12:154	2.32	1.82
	0,3	6.30	9:163	2.52	1.93
	0,4	5.93	9:151	2.53	1.93
	*0,1	5.76	2:173	4.72	3.05
10-41-----	0,2	2.32	5:170	3.12	2.27
	2,3	2.56	4:166	3.42	2.43
	1,4	2.34	6:167	2.92	2.15
	0,5	5.07	2:162	4.73	3.05
	0,6	2.04	5:159	3.13	2.27
10-43-----	6,7	2.44	3:156	3.90	2.67
	5,8	2.07	5:157	3.13	2.27
	**0,1	5.59	4:165	3.43	2.43
	1,2	9.26	4:161	3.43	2.43
	0,3	6.13	4:165	3.43	2.43
	3,4	9.83	4:161	3.43	2.43
	0,5	7.70	4:153	3.44	2.44
	5,6	7.05	4:149	3.44	2.44
	0,7	8.14	4:153	3.44	2.44
	7,9	7.63	4:149	3.44	2.44

*Equation 0 refers to the variance for the values themselves with no fitted equation for AE+NAE cements (See footnote of Table).
**As above, for NAE cements.

different concretes were made, so information on the possible effects of different lots of sand or of manipulative differences are not available.

7.4. Shrinkage and Expansion of Concrete versus Shrinkage Characteristics of Neat Cement

Equations were computed to find the relationship of each of the dependent variables for concrete of this section, i.e., OSHR, ASHR, OEXP, AEXP, ODSX, ADSX, OREX, AREX, OEXR, and AEXR, to the dependent variables for neat cement of the previous section, SH24, SH07, SH28, SH6M, SHBR, TOBR, SB/S7, and SB/S28. The reduction in variance or "F" values were also calculated for each of the equations. There was an indication of a relationship between OEXP and AEXP values for the concretes and SH28 and SH6M values for the neat cements significant at the 5-percent probability level. Neither the shrinkage values of the two series of concretes, the differences between the shrinkage and expansion, nor any of the various ratios showed an indication significant at the 5-percent level of a relationship with any of the shrinkage values obtained for the neat cements. This lack of relationship applied to both the nonrestrained shrinkage of the neat cements and the time of cracking of the annular rings.

There were, possibly, many factors which contributed to this lack of correlation. The neat cement pastes were made with a low water/cement ratio; the concretes with a high water/cement ratio. The neat cement pastes hydrated for only 24 hr prior to drying; the concretes for 14 days. The specimens of neat cement had

TABLE 10-45. Coefficients, coef./s.d. ratios and calculated ranges of contributions of independent variables associated with shrinkage and expansion characteristics of concretes made with a water/cement ratio of 0.635

Column	1	2	3	4	5	6
Equation	3	3	2	4	4	1
Table	10-4	10-12	10-20	10-20	10-28	10-37
Dependent variable	OSHR	OEXP	ODSX	ODSX	OREX	OEXR
	=	=	=	=	=	=
Constant	+0.0139	+0.0107	-0.00416	-0.00466	+0.8390	+0.3398
CaA, coef.	+0.00123	+0.000595			+0.00498	
Coef./s.d.	8.8	6.0			2.0	
Calculated range*	0.0185	0.0089			0.0075	
CaS, coef.	** -0.0000835					
Coef./s.d.	1.6					
Calculated range	0.004					
C4AF, coef.	+0.000898	+0.000390	+0.000502			
Coef./s.d.	5.3	3.4	3.6			
Calculated range	0.0117	0.0051	0.0065			
Na2O, coef.	-0.00822		-0.00485	** -0.00231	+0.1273	-0.1664
Coef./s.d.	4.2		2.7	1.7	2.8	3.4
Calculated range	0.0062		0.0036	0.0017	0.0954	0.1248
SO3, coef.	-0.00894	-0.00411				-0.0515
Coef./s.d.	7.8	5.4				2.5
Calculated range	0.0161	0.0074				0.0927
C3A/SO3, coef.			+0.00097	** -0.000164		
Coef./s.d.			4.6	1.2		
Calculated range			0.0094	0.0016		
MgO, coef.				** -0.000231	** +0.00871	** -0.0083
Coef./s.d.				1.4	1.6	1.3
Calculated range				0.0012	0.0435	0.0415
Loss, coef.	-0.00363	-0.0022	-0.00164			
Coef./s.d.	5.6	5.0	3.1			
Calculated range	0.0109	0.0066	0.0049			
APF, coef.	+0.00000613	+0.0000030	+0.00000178			
Coef./s.d.	7.9	6.0	3.1			
Calculated range	0.0180	0.0090	0.0053			
Wagn., coef.						** +0.000049
Coef./s.d.						1.7
Calculated range						0.0784
Air content, coef.	+0.00100	+0.000646	+0.000296	** -0.000158		** -0.0124
Coef./s.d.	5.6	5.5	2.0	1.4		1.2
Calculated range	0.0100	0.0065	0.0030	0.0016		0.1240
Ba, coef.					** +0.2764	** -0.2838
Coef./s.d.					1.4	1.3
Calculated range					0.0553	0.0568
Li, coef.			-0.1923	** -0.0938	+4.353	-5.750
Coef./s.d.			2.2	1.4	2.0	2.4
Calculated range			0.0038	0.0019	0.0871	0.1150
P, coef.		** +0.00369				** -0.1190
Coef./s.d.		1.9				1.9
Calculated range		0.0018				0.0595
Pb, coef.		** -0.0518				+2.698
Coef./s.d.		1.5				2.4
Calculated range		0.0026				0.1350
Rb, coef.		+0.3234	-0.4920	-0.387	+11.29	-9.889
Coef./s.d.		2.8	3.6	3.8	3.4	2.7
Calculated range		0.0032	0.0049	0.0039	0.1129	0.0989
SrO, coef.	+0.00854	** -0.00378	+0.0114	+0.0082	-0.3005	+0.4542
Coef./s.d.	2.1	1.4	3.6	3.4	3.6	5.2
Calculated range	0.0034	0.0015	0.0046	0.0033	0.1202	0.1817
V, coef.						** -0.7141
Coef./s.d.						1.6
Calculated range						0.0714
Zn, coef.	-0.0198	** -0.0102				
Coef./s.d.	2.0	1.6				
Calculated range	0.0040	0.0020				
Zr, coef.	** +0.0126					
Coef./s.d.	1.6					
Calculated range	0.0063					

See footnotes at end of table.

TABLE 10-45. Coefficients, coef./s.d. ratios and calculated ranges of contributions of independent variables associated with shrinkage and expansion characteristics of concretes made with a water/cement ratio of 0.635—Continued

Column	1	2	3	4	5	6
Equation	3	3	2	4	4	1
Table	10-4	10-12	10-20	10-20	10-28	10-37
Dependent variable	OSHR	OEXP	ODSX	ODSX	OREX	OEXR
	=	=	=	=	=	=
Constant	+0.0139	+0.0107	-0.00416	-0.00466	-0.8390	+0.3398
OSHR, coef.....				+0.5223	-7.294	
Coef./s.d.....				13.5	6.0	
Calculated range.....				0.0157	0.2188	

* The calculated range was computed from the range of values for the cements times the coefficient for each independent variable.

** The independent variable is probably not significant as the coef./s.d. ratio is less than 2.

TABLE 10-46. Coefficients, coef./s.d. ratios, and calculated ranges of contributions of independent variables associated with shrinkage and expansion characteristics of concretes having a slump of 5 ± 1 in.

Column	1	2	3	4	5	6
Equation	4	5	2	5	7	3
Table	10-8	10-16	10-24	10-24	10-32	10-37
Dependent Variable	ASHR	AEXP	ADSX	ADSX	AREX	AEXR
	=	=	=	=	=	=
Constant	+0.0129	+0.0085	-0.00519	-0.00729	+0.861	+0.1939
C ₂ A, coef.....	+0.001221	+0.000725				
Coef./s.d.....	8.4	6.6				
Calculated range *.....	0.0183	0.0108				
C ₃ S, coef.....	-0.0000903	** -0.000044				
Coef./s.d.....	1.7	1.1				
Calculated range.....	0.0004	0.0002				
C ₄ AF, coef.....	+0.000918	+0.000344	+0.000552	+0.000175	** -0.00467	
Coef./s.d.....	5.3	2.6	4.1	2.0	1.7	
Calculated range.....	0.0119	0.0045	0.0072	0.0023	0.0607	
Fe ₂ O ₃ , coef.....						** +0.0156
Coef./s.d.....						1.8
Calculated range.....						0.0702
Na ₂ O, coef.....	-0.00714		+0.00402		+0.0711	** -0.0808
Coef./s.d.....	3.6		2.3		1.7	1.8
Calculated range.....	0.0054		0.0030		0.0533	0.0606
K ₂ O, coef.....		+0.0026				
Coef./s.d.....		2.0				
Calculated range.....		0.0029				
SO ₃ , coef.....	-0.00784	-0.00548		** +0.00080		
Coef./s.d.....	6.7	6.1		1.4		
Calculated range.....	0.0141	0.0099		0.0014		
C ₂ A/SO ₃ , coef.....			+0.00080			
Coef./s.d.....			4.0			
Calculated range.....			0.0078			
MgO, coef.....		** +0.000428		-0.000370	-0.0146	-0.0157
Coef./s.d.....		1.9		2.2	2.7	2.8
Calculated range.....		0.0021		0.0018	0.0730	0.0785
Loss, coef.....	-0.00381	-0.0025	-0.00165			
Coef./s.d.....	5.7	5.1	3.2			
Calculated range.....	0.0114	0.0075	0.0050			
APF, coef.....	+0.00000609	+0.00000431	+0.00000165			
Coef./s.d.....	7.7	7.0	2.9			
Calculated range.....	0.0183	0.0129	0.0049			
Wagn, coef.....						** +0.000045
Coef./s.d.....						1.7
Calculated range.....						0.0720

See footnotes at end of table.

TABLE 10-46. *Coefficients, coef. s.d. ratios, and calculated ranges of contributions of independent variables associated with shrinkage and expansion characteristics of concretes having a slump of 5 ± 1 in.—Continued*

Column	1	2	3	4	5	6
Equation	4	5	2	5	7	3
Table	10-8	10-16	10-24	10-24	10-32	10-37
Dependent Variable	ASHR	AEXP	AD SX	AD SX	AREX	AEXR
	=	=	=	=	=	=
Constant	+0.0129	+0.0085	-0.00519	-0.00729	+0.861	+0.1939
Air content, coef.	+0.000620	**+0.000208	+0.000272			
Coef./s.d.	4.1	1.8	2.2			
Calculated range	0.0062	0.0021	0.0027			
Ba, coef.						** -0.2530
Coef./s.d.						1.3
Calculated range						0.0506
Li, coef.			-0.1714	-0.1459	+5.16	-5.857
Coef./s.d.			2.1	2.5	2.5	2.8
Calculated range			0.0034	0.0029	0.1032	0.1171
Mn, coef.		+0.00418	** -0.00274	** -0.00305	** +0.090	
Coef./s.d.		2.1	1.3	1.9	1.7	
Calculated range		0.0042	0.0027	0.0030	0.0900	
Ni, coef.		** +0.1149				
Coef./s.d.		1.3				
Calculated range		0.0023				
P, coef.		** +0.00284				
Coef./s.d.		1.3				
Calculated range		0.0014				
Rb, coef.			-0.2812	-0.2592	+7.42	** -5.434
Coef./s.d.			2.1	2.6	2.3	1.6
Calculated range			0.0028	0.0026	0.0742	0.0543
SrO, coef.	+0.00828		+0.0108	+0.0047	-0.206	+0.3182
Coef./s.d.	2.0		3.5	2.0	2.7	4.1
Calculated range	0.0033		0.0043	0.0019	0.0824	0.1273
V, coef.		** +0.0237				
Coef./s.d.		1.5				
Calculated range		0.0024				
Zn, coef.	-0.0216					
Coef./s.d.	2.1					
Calculated range	0.0043					
Zr, coef.	** +0.00927					
Coef./s.d.	1.1					
Calculated range	0.0046					
ASHR, coef.				+0.4705	-5.213	
Coef./s.d.				12.9	4.5	
Calculated range				0.0141	0.1564	

* The calculated range was computed from the range of values for the cements times the coefficient for each independent variable.

** The independent variable is probably not significant as the coef./s.d. ratio is less than 2.

TABLE 10-47. *Summary of averages and estimated standard deviations of concretes made of non-air-entraining cements*

	Averages, cements percent	S.D. cements	S.D. equations	S.D. duplicates
OSHR.....	0.0298	0.0056	0.0039	0.0016
ASHR.....	0.0296	0.0056	0.0039	0.0016
OEXP.....	0.0203	0.0035	0.0027	0.0015
AEXP.....	0.0208	0.0038	0.0029	0.0015

a 1-in-square cross-section; the concretes an approximate 6×8 -in cross-section.

A comparison of table 9-35 of the previous section, indicating variables associated with shrinkage of neat cement, with tables 10-45 and 10-46 of this section, indicating summaries

of the variables associated with the shrinkage and expansion of concretes, shows a number of differences. An increase in fineness of the cements was possibly associated with an increase in shrinkage of the neat cements, but the coef./s.d. for fineness was highly significant with the concretes. An increase in the C_3A was associated with a decrease of shrinkage of the neat cements and with an increase in shrinkage of the concretes. Increased Na_2O , K_2O and possibly Rb were associated with increased shrinkage of the neat cements at 7 days and 28 days. For the concretes, an increase in Na_2O was associated with a decrease in shrinkage, and K_2O was apparently not associated with shrinkage.

7.5. Shrinkage and Expansion of Concretes versus the Dynamic Young's Modulus of Elasticity

Equations were calculated to determine the relationship of the shrinkage, and expansion values of the concretes to the dynamic Young's modulus of elasticity of 3 × 4 × 16-in specimens made of the same concretes, cured, and exposed to air, then soaked in water in the same manner as the 6 × 8 × 16-in specimens. The results, as well as the reduction in variance or "F" values, are presented in table 10-48. The nomenclature used, OD14, OD42, OD70, and OD98, refers to the dynamic Young's modulus of elasticity in psi × 10⁻⁶ at 14 days (after moist-air curing), at 42 days (after 4 weeks air-drying), at 70 days (after 8 weeks drying in laboratory air), and at 98 days (after being in water for 4 weeks) respectively. The equations presented in this table are for the concretes made with a water/cement ratio of 0.635. With the number of degrees of freedom involved, 2:180, "F" values of 4.70 and 3.05 must be equaled or exceeded for an indication of a relationship significant at the one or the 5-percent probability levels, respectively.

The equations of table 10-48 indicate that an increase in the dynamic modulus of elasticity was associated with a decrease in the amount of shrinkage, with a decrease in the subsequent expansion when the concretes were rewetted, and with a decrease in the difference between

TABLE 10-48. Relationship of percentage shrinkage (OSHR), percentage expansion, OEXP, and percentage shrinkage minus percentage expansion of concrete (ODSX) versus the dynamic modulus of elasticity of concrete specimens at various ages

Equation				S.D.	"F"
1.....	OSHR = +0.0540 s.d. = (0.0054)	-0.00497 (0.00113)	OD14	0.00552	9.7
2.....	OSHR = +0.0607 s.d. = (0.0051)	-0.00661 (0.00111)	OD42	0.00531	17.7
3.....	OSHR = +0.0619 s.d. = (0.0045)	-0.00709 (0.00100)	OD70	0.00514	25.0
4.....	OSHR = +0.0681 s.d. = (0.0070)	-0.00743 (0.00137)	OD98	0.00539	14.7
5.....	OEXP = +0.0315 s.d. = (0.0037)	-0.00225 (0.00078)	OD14	0.00381	4.2
6.....	OEXP = +0.0363 s.d. = (0.0036)	-0.00337 (0.00078)	OD42	0.00371	9.4
7.....	OEXP = +0.0366 s.d. = (0.0032)	-0.00355 (0.00071)	OD70	0.00365	12.4
8.....	OEXP = +0.0429 s.d. = (0.0048)	-0.00434 (0.00094)	OD98	0.00369	10.7
9.....	ODSX = +0.0224 s.d. = (0.0039)	-0.00272 (0.00082)	OD14	0.00403	5.4
10.....	ODSX = +0.0244 s.d. = (0.0039)	-0.00324 (0.00083)	OD42	0.00399	7.5
11.....	ODSX = +0.0393 s.d. = (0.0034)	-0.00354 (0.00077)	OD70	0.00393	10.7
12.....	ODSX = +0.0252 s.d. = (0.0053)	-0.00309 (0.00103)	OD98	0.00405	4.5

the percentage shrinkage and percentage expansion, ODSX. The indication of relationship in eqs 5 and 12 were significant at the 5-percent level and in the others at the 1-percent level.

In table 9-36, Section 9, a series of equations was presented for the shrinkage of neat cement pastes versus the compressive strengths of 1:2.75 (cement to sand) mortars. Higher strength of the mortars at 1, 3, and 7 days was associated with higher shrinkage of the neat cements at 24 hr (after 24 hr moist curing). There was no significant relationship between the shrinkage of the neat cements at later ages, up to 6 months, and the compressive strength of mortars at 1, 3, 7, or 28 days. An increase in the compressive strength of the mortars was associated with a decrease in the time of the annular specimens made of neat cement of normal consistency.

A series of equations was also calculated for the concretes made with sufficient water to give a 5 ± 1-in slump. These equations and the "F" values, though not presented here, were very similar to the relationships presented in table 10-48 for constant water/cement ratio concretes.

It has been reported that shrinkage increases with an increase of the cement content of the concrete [1]. An increase in cement content would also result in a greater compressive strength and a higher dynamic modulus of elasticity. With any one cement there would be a greater amount of cement gel formed per unit volume in a given period of hydration if there was more cement to start with. The results of the present study indicate that greater shrinkage occurred with lower strength (lower dynamic E), and that concretes with higher strengths had lower shrinkages. The compounds and other independent variables associated with the shrinkage of concretes have been reported in this section; variables associated with dynamic Young's modulus of elasticity will be reported in a later section in this series.

7.6. Top and Bottom Shrinkage and Expansion

The greater shrinkage of the tops of a majority of the specimens as cast is probably the result of settlement of the solids and normal bleeding. The actual water/cement ratio in the upper half of the 8-in depth of concrete at the time of setting may have been greater than in the bottom half, and such a difference could account for the greater shrinkage. It was noted that concretes with greater shrinkage of the top of the specimens when air dried usually had a greater expansion when the specimens were rewetted. Attempts were made (without success) to relate the difference in expansion of the tops and bottoms of specimens to various

independent variables. The differences were usually slight, and the limitations of the measuring device, and the fact that in one case averages of two measurements and in the other case single measurements were used, may have masked any relationship.

7.7. Comparative Test Values on other Hydraulic Cements

Tests were also made of concretes made of two portland-pozzolan cements and seven portland-blast-furnace-slag cements. All but one of

the nine cements had shrinkage values in the "O" series within the range of values of the concretes made of the portland cements. Concretes made of six of the nine cements had greater shrinkage and expansion values than the averages of the NAE portland cements. All the concretes were proportioned on a weight basis rather than a volume basis. The lower specific gravity of these cements resulted in a slightly greater volume of cementitious material in the concrete than would be the case with the portland cements.

8. Summary and Conclusions

(1) Two series of concretes were made with 199 portland cements having a broad range of composition and properties. In Series O the water/cement ratio was 0.635, and in Series A the water/cement ratio was adjusted to give concretes with a 5 ± 1 -in slump. Although designed for a cement content of 5.5 bags per cubic yard, the computed cement contents of concretes made of the air-entraining cements were considerably less than this value.

(2) Specimens, $6 \times 8 \times 16$ in, were made of the two series of concretes with each of the cements. Measurements were made of the shrinkage of the specimens when stored in laboratory air for 8 weeks after an initial 14-day curing period. After the drying period, the specimens were stored in water for 4 weeks and measurements made of the expansion.

(3) Calculations were made of the difference between the percentage shrinkage and the percentage expansion, of the ratio of the expansion to initial shrinkage, and of the ratio of the difference to the initial shrinkage.

(4) The average shrinkage of the concrete specimens was 0.030 percent, with a range of about 0.015 to 0.045 percent. The estimated standard deviation of individual measurements obtained from duplicate specimens was 0.0016.

(5) The average expansion of the air-dried concrete specimens when res soaked for 4 weeks was 0.020 percent, with a range of 0.011 to 0.035 percent. The estimated standard deviation of measurements on duplicate specimens was 0.0015.

(6) The frequency distribution of the shrinkage and expansion for the concretes, as well as the residual shrinkage and ratios of shrinkage and expansion values of the concretes, indicated, in each instance, a fairly wide distribution of values. There was an overlapping of the shrinkage and expansion values obtained with the different types of cement as classified.

(7) Multivariable regression equations were used to determine which of the available independent variables or combinations of variables,

such as cement composition, trace elements, fineness, and the air content of the concrete, were associated with the shrinkage and expansion characteristics. Typical equations are presented in table 10-45 for the concretes made with a 0.635 water/cement ratio.

(7.1) Increases in C_3A , C_4AF , fineness of the cement, and air content of the concrete were all associated with increases in both the shrinkage and the expansion values of the concretes.

(7.2) Increases in Na_2O , SO_3 , and loss on ignition of the cements were associated with decreases in the shrinkage values of the concretes. Increases in SO_3 and loss on ignition were associated with decreases of the expansion values of the concretes.

(7.3) There was no highly significant indication of association of any of the individual trace elements of the cements with either the shrinkage or the expansion of the concrete. An increase in Rb may possibly be associated with an increase in expansion, an increase in SrO possibly with an increase in shrinkage, and an increase in Zn possibly with a decrease in shrinkage values. The trace elements of P, Pb, and Zr may possibly have an effect, but verification of their influence would require further investigation. The additional use of all the trace elements having a coef./s.d. ratio greater than 1 in the equations with the commonly determined variables usually resulted in a highly significant reduction of variance.

(7.4) The difference between the initial drying shrinkage and the subsequent expansion when the concrete specimens were rewetted (i.e. the residual shrinkage) was highly dependent on the values for the initial shrinkage. Equations calculated with only the cement composition and the air content as independent variables indicated that increases in C_4AF , C_3A/SO_3 , fineness, SrO, and possibly air content of the concrete were associated with increases in the residual-shrinkage values. Increases of loss on ignition, Rb, and possibly Na_2O and Li were associated with decreases in the residual-shrinkage values. When the initial

shrinkage was included as an independent variable, only Rb and SrO were related to the residual shrinkage. Sodium oxide, C_3A/SO_3 , MgO, air content and Li had coef./s.d. ratios between one and two and were possibly not associated with residual shrinkage, but further study of their effects may be warranted.

(7.5) The ratio of expansion/shrinkage values was highly dependent on the initial shrinkage value. Increases in Rb, and possibly Na_2O , Li, and C_3A were associated with higher expansion/shrinkage ratios. Increases in SrO as well as the initial shrinkage were associated with decreases in the expansion/shrinkage ratio.

(7.6) The equations for the ratio of residual shrinkage divided by the shrinkage had only two independent variables that were highly significant. Increases in SrO and possibly Pb were associated with increases in the ratio. Increases in Na_2O and possibly SO_3 , Li and Rb were probably associated with decreases in the ratio.

(7.7) Equations calculated for the shrinkage, expansion, differences, and expansion/shrinkage ratios of specimens made of concretes having a 5 ± 1 -in slump were in general agreement with those calculated for the 0.635-water/cement-ratio concretes. Some differences may be noted in comparing tables 10-45 and 10-46.

(7.8) In equations calculated for the non-air-entraining cements, the coef./s.d. ratio for air-

content of the concretes was usually less than 1 and always less than 2.

(7.9) Equations calculated using the principal oxides instead of the potential compounds resulted in approximately the same estimated standard deviations and reduction-in-variance values.

(8) The shrinkage values of the two series of concrete specimens moist cured for 14 days and then air dried did not show, at the 5-percent probability level, a relationship to the shrinkage values of the neat cement bars of normal consistency, moist cured for 24 hr, and then air dried.

(9) The shrinkage and expansion values of the two series of concrete specimens did not show at the 5-percent level a relationship with time of cracking of the restrained annular specimens made of the neat cement, moist cured for 24 hr and then air dried.

(10) Higher shrinkage values and higher expansion values of the concretes were associated with lower dynamic modulus of elasticity values of the concretes.

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