NBS BUILDING SCIENCE SERIES 129

Cost Estimation and Cost Variability in Residential Rehabilitation

U.S. DEPARTMENT OF COMMERCE • NATIONAL BUREAU OF STANDARDS

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Cost Estimation and Cost Variability in Residential Rehabilitation

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PREFACE

This research was conducted under the sponsorship of the Rehabilitation Technology Group, Building Economics and Regulatory Technology Division, Center for Building Technology, National Engineering Laboratory, National Bureau of Standards. This paper analyzes four methods for estimating the costs of residential rehabilitation. A theoretical procedure based on an approach which integrates the performance concept with established engineering economics techniques is also developed.

The author would like to extend appreciation to those individuals who generously provided information on all aspects of the residential rehabilitation process. Special appreciation is extended to Dr. Harold E. Marshall, Applied Economics Group, Building Economics and Regulatory Technology Division, who provided many useful comments at the formative stages of this study while the author was a member of the Applied Economics Group. (The author is currently with the Operations Research Division, Center for Applied Mathematics, National Engineering Laboratory, National Bureau of Standards.) Special appreciation is also extended to Dr. Joseph G. Kowalski, formerly with the Building Economics and Regulatory Technology Division, who prepared a working draft of several portions of chapter 3, and to Mr. William G. Hall, Operations Research Division, whose programming efforts in the area of fire safety in health care facilities has greatly affected the formulation of an approach for applying the performance concept to the residential rehabilitation problem.

Cover: *Residential rehabilitation activities have increased dramatically in recent years. In some cases these activities have resulted in substantial gains to investors in inner city housing. In other cases the uncertainty surrounding renovation costs have had a significant and adverse effect on the demand for houses in older neighborhoods.*

ABSTRACT

This study analyzes four methods of estimating the costs of residential rehabilitation. Each method is critiqued with regard to its treatment of changes in the size of the renovation project, the productivity of labor, and the contractor's markup for overhead and profit. Cost comparisons and a discussion of the way in which the inherent riskiness of renovation activities may be asessed are also presented. A theoretical approach for dealing with cost variability which integrates the performance concept with established engineering economics techniques is also developed.

Key words: Applied economics; building codes; building economics; cost estimation; economic analysis; engineering economics; housing; mathematical programming; optimization; rehabilitation; renovation.

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Most Common SI Units and their
Equivalent Values in
Customary Units

QUANTITY	INTERNATIONAL (SI) UNIT	U.S. CUSTOMARY UNIT	APPROXIMATE CONVERSION	
LENGTH	meter (m)	foot (ft)	1 m	= 3.2808 ft
	millimeter (mm)	inch (in)	1 mm	= 0.0394 in
AREA	square meter (m^2)	square yard (yd^2)	1 m^2	= 1.1960 yd^2
		square foot (ft^2)	1 m^2	= 10.764 ft^2
	square millimeter (mm^2)	square inch (in^2)	1 mm^2	= 1.5500 x $10^{-3} in^2$
VOLUME	cubic meter (m^2)	cubic yard (yd^3)	1 m^3	= 1.3080 yd^3
		cubic foot (ft^3)	1 m^3	= 25.315 ft^3
	cubic millimeter (mm^3)	cubic inch (in^3)	1 mm^3	= 61.024 x $10^{-6} in^3$
CAPACITY	liter (L)	gallon (gal)	1 L	= 0.2642 gsl
	milliliter (mL)	fluid ounce (fl oz)	1 mL	= 0.0338 fl oz
VELOCITY, SPEED	meter per second (m/s)	foot per second (ft/s or f.p.s.)	1 m/s	= 3.2808 ft/s
	kilometer per hour (km/h)	mile per hour (mile/h or m.p.h.)	1 km/h	= 0.6214 mile/h
ACCELERATION	meter per second squared (m/s^2)	foot per second squared (ft/s^2)	1 m/s^2	= 3.2808 ft/s^2
MASS	metric ton (t) [1000 kg]	short ton [2000 lb]	1 t	= 1.1023 ton
	kilogram (kg)	pound (lb)	1 kg	= 2.2046 lb
	gram (g)	ounce (oz)	1 g	= 0.0353 oz
DENSITY	metric ton per cubic meter (t/m^3)	ton per cubic yard (ton/yd^3)	1 t/m^3	= 0.8428 ton/yd^3
	kilogram per cubic meter (kg/m^3)	pound per cubic foot (lb/ft^3)	1 kg/m^3	= 0.0624 lb/ft^3
FORCE	kilonewton (kN)	ton-force (tonf)	1 kN	= 0.1124 tonf
		kip [1000 lbf]	1 kN	= 0.2248 kip
	newton (N)	pound-force (lbf)	1 N	= 0.2248 lbf
MOMENT OF FORCE, TORQUE	kilonewton meter (kN•m)	ton-force foot (tonf•ft)	1 kN•m	= 0.3688 tonf•ft
	newton meter (N•m)	pound-force inch (lbf•in)	1 N•m	= 8.8508 lbf•in
PRESSURE, STRESS	megapascal (MPa)	ton-force per square inch ($tonf/in^2$)	1 MPa	= 0.0725 $tonf/in^2$
		ton-force per square foot ($tonf/ft^2$)	1 MPa	= 10.443 $tonf/ft^2$
	kilopascal (kPa)	pound-force per square inch (lbf/in^2)	1 kPa	= 0.1450 lbf/in^2
		pound-force per square foot (lbf/ft^2)	1 kPa	= 20.885 blf/ft^2
WORK, ENERGY, QUANTITY OF HEAT	megajoule (MJ)	kilowatthour (kWh)	1 MJ	= 0.2778 kWh
	kilojoule (kJ)	British thermal unit (Btu)	1 kJ	= 0.9478 Btu
	joule (J)	foot pound-force (ft•lbf)	1 J	= 0.7376 ft•lbf
POWER, HEAT FLOW RATE	kilowatt (kW)	horsepower (hp)	1 kW	= 1.3410 hp
	watt (W)	British thermal unit per hour (Btu/h)	1 W	= 3.4121 Btu/h
		foot pound-force per second (ft•lbf/s)	1 W	= 0.7376 ft•lbf/s
COEFFICIENT OF HEAT TRANSFER [U-value]	watt per square meter kelvin (W/m^2•K) [=(W/m^2•°C)]	Btu per square foot hour degree Fahrenheit (Btu/ft^2•h•°F)	1 W/m^2•K	= 0.1761 Btu/ft^2•h•°F
THERMAL CONDUCTIVITY [k-value]	watt per meter kelvin (W/m•K) [=(W/m•°C)]	Btu per square foot degree Fahrenheit (Btu/ft^2•°F)	1 W/m•K	= 0.5778 Btu/ft^2•°F

NOTES: (1) The above conversion factors are shown to three or four places of decimals.

(2) Unprefixed SI units are underlined. (The kilogram, although prefixed, is an SI base unit).

REFERENCES: NBS Guidelines for the Use of the Metric System, LC1056, Revised August 1977;
The Metric System of Measurement, Federsl Register Notice of October 26, 1977, LC 1078, Revised November 1977;
NBS Special Publication 330, "The Internationsl System of Units (SI)," 1977 Edition;
NBS Technical NOte 938, "Recommended Practice for the use of Metric (SI) Units in Building Design and Construction," Revised edition June 1977;
ASTM Standard E621-78, "Standard Practice for the Use of Metric (SI) Units in Building Design and Construction," (based on NBS TN 938), March 1978;
ANSI Z210 .1-1976, "American National Standard for Metric Practice;" also issued as ASTM E380-76$^{\epsilon}$, or IEEE Std. 268-1976.

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The rapidly rising costs of new housing coupled with the locational advantages of the central city have stimulated rehabilitation activities in many urban areas. Unfortunately this trend is mitigated by the extreme variability of residential rehabilitation costs.

1. INTRODUCTION

1.1 BACKGROUND

Between 1972 and 1976 the median cost of a new single-family housing unit increased at an annual rate of 12.5 percent.[1] Such rapidly rising prices have forced many prospective homeowners to look elsewhere in order to satisfy their housing demands. This trend is likely to continue over the next 5 to 10 years. Current forecasts indicate that between 22 and 30 million additional housing units will be needed by 1988.[2] A significant increase in the number of housing units being renovated is therefore likely

[1] U.S. Department of Housing and Urban Development, Final Report of the Task Force on Housing Costs, May 1978.

[2] "Millions of Housing Units Needed by 1988," Engineering News Record, April 27, 1978.

if the nation's housing needs are to be satisfied. Since costs play a vital role in all investment decisions, any significant change in current housing investment trends calls for a reduction in the uncertainty associated with the extreme cost variability of residential rehabilitation activities.

The housing renovation industry has received little research attention in the past. Due to its rapid growth, however, many research questions are now being raised. One important area where knowledge is lacking relates to the nature of the cost structure of the housing renovation industry. Focusing more narrowly, how reliable is cost estimation in the area of housing renovation? What methods are available for homeowners and/or investors, contractors, subcontractors, and public officials for estimating renovation costs? How reliable are such methods? Need they be improved?

Accurate cost estimation is important to all the major participants in the building process. Reasonably accurate cost information is essential to the housing owner as an input into the investment decision. Unreliable cost information renders housing renovation investments riskier than they would be if accurate cost estimates existed. To contractors and subcontractors the reliability of cost estimation procedures also affects the riskiness of doing business. Finally, accurate cost estimation is needed by Federal, State, and local program managers for numerous reasons. For example, a local public housing official may be faced with a decision about choosing a contractor to upgrade publicly owned housing units. Accurate cost estimates would be a major concern to the HUD official engaged in program budgeting for an Urban Homesteading Demonstration Program.

The primary sources of cost data for housing renovation are cost estimation guides or manuals, which subdivide tasks into fine levels of detail. These guides can be put into two categories. The first includes the guides whose primary market is for new building construction contractors, subcontractors, architects, engineers, and cost estimators. These guides are often used to prepare budgetary estimates of construction costs. They provide reference information on unit costs. More detailed or final estimates in new construction, however, usually rely directly on a construction firm's cost estimator. These estimates would be based on detailed quantity takeoffs coupled with unit prices drawn from the firm's past experiences with similar construction projects and current prices in the local market for labor and materials. The second category of guides are those directly aimed at the housing renovation contractor or subcontractor. These guides can also be used to prepare preliminary or budgetary estimates for housing renovation costs.

1.2 PURPOSE

The purpose of this report is to present an analysis of four basic methods for estimating housing renovation costs. Particular emphasis is placed on how these methods permit (or prohibit) the introduction of the inherent riskiness of the renovation process into the housing investment decision. This report also serves to develop a conceptual procedure, based on engineering economics principles, which will permit risk to be explicitly incorporated into the preparation of a pre-bid cost estimate. Guidelines are given to show how this procedure could eventually be used by participants in the renovation process. This report is intended to serve as a reference document for researchers; architectural and engineering consultants; and Federal, State, and local program managers concerned with the problems of estimating the costs of housing renovation.

1.3 SCOPE AND APPROACH

The focus of this study is on analyzing the ways in which cost calculations are performed and cost variability is treated in several alternative procedures for estimating the costs of housing renovation. For the most part, the emphasis of this study is on theoretical considerations. Actual cost data are introduced in several instances, however, to illustrate how these considerations could be treated in practice. It is not the intent of this study to downgrade a particular method of estimation. Each method has its strengths and weaknesses. However, if a particular method is deficient in its treament of risk, it would seem advisable, given the importance of this factor in the renovation process, that this deficiency be called to the attention of potential users.

The basic format of this study consists of a description of the four primary methods for estimating housing renovation costs, a critique of each method, the development of a "theoretical" approach for dealing with cost variability, and the implications that the use of such an approach would have on the purchases of renovation services.

Specifically, chapter 2 describes each of the four basic methods for estimating renovation costs, what their data requirements are, how they may be applied to the problem, and a measure of their flexibility in dealing with varying technical and economic conditions.

Chapter 3 focuses on a critique of the per unit cost method of estimation with regard to such crucial factors as the impact of changing job size; the treatment of productivity and contractor markup; and the treatment of risk.

Chapter 4 develops a theoretical approach for dealing with cost variability based on the economic theory of cost functions. A framework for introducing the performance concept is also developed. The cost saving potential of this approach (based on past studies in the area of fire safety) is then discussed. The chapter concludes with a discussion of how such an approach could be of benefit to individual homeowners and/or investors, financial institutions, and governmental program managers.

Chapter 5 contains a short summary of the research findings and recommendations for future research, including an econometric analysis of actual housing renovation projects.

The report also includes a technical appendix which shows how, given an underlying set of technical relationships, a cost function suitable for estimating residential rehabilitation costs may be derived.

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The extreme variability of residential rehabilitation costs is a reflection of constraints on the construction process due to the condition of the building as well as economic factors determined in the market place. Accounting for the interactions among technical and economic variables can only be achieved by increasing the complexity of the cost estimating procedure.

2. ALTERNATIVE METHODS OF ESTIMATING REHABILITATION COSTS

2.1 AVERAGE COSTS AND COSTS INDICES

Cost estimation is usually based on past experiences projected to include anticipated increases in wage and material costs. A premium is, thus, placed on historical data as a basis for estimating future costs. Guidebooks and manuals are frequently the only source of cost information for prospective homeowners and/or investors. They also form an important supplement to a contractor's own past cost experience. In general, cost guides or manuals rely upon cost reports submitted, under a variety of conditions, by contractors who report their own historical experience. Thus, subtask cost figures reported in a cost manual are based on the average cost experiences of the reporting contractors. Furthermore, the guidebooks rely on the judgment of construction experts to modify, if necessary, their collected cost figures to ensure that the reported figures are representative of cost behavior under typical or average conditions.

The first three publications listed in table 2.1 are primarily directed to the estimation of new construction costs.[1] The last two publications focus on the estimation of home repair, remodeling and renovation costs. There are, of course, other manuals available which address slightly different aspects of the building construction process. Table 2.1 should therefore be looked at as a representative sample of the manuals available rather than an exhaustive listing.

In every case the particular figures that the guidebooks report are average figures per unit, e.g., per square foot or linear foot. The common approach taken is to report average costs per unit as equal to the sum of average material cost per unit and average labor cost per unit where each of the latter have been factored upwards by the contractor's markup for overhead and profit.

More specifically the average total cost of task i may be expressed mathematically by either equation 2.1, 2.2 or 2.3.

$$ATC_i = AMC_i + ALC_i \qquad 2.1$$

$$ATC_i = k(ADMC_i) + k(ADLC_i) = k(ADMC_i + ADLC_i) \qquad 2.2$$

$$ATC_i = k(P_m + W_i/PROD_i) \qquad 2.3$$

where

ATC_i = Average Total Cost of the i^{th} subtask
AMC_i = Average Material Cost of the i^{th} subtask
ALC_i = Average Labor Cost of the i^{th} subtask
$ADMC_i$ = Average Direct Material Cost (excludes markup) of the i^{th} subtask
$ADLC_i$ = Average Direct Labor Cost (excludes markup) of the i^{th} subtask
k = Markup factor for contractor's (or subcontractor's) overhead and profit,
P_m = Average gross material price per unit,

[1]All three guides referred to above contain provisions which would allow the cost of certain types of renovation processes to be estimated.

Table 2.1 A Description of Selected Cost Manuals

Title Publisher	Approximate Number of Subitems Listed	Data Recorded for Each Suhitem	Nature and Source of Data	Overhead and Profit	Regional Variation	Statistical Assessibility
Dodge Manual for Building Construction Pricing and Scheduling McGraw-Hill Information Systems Company	over 10,000	1. Description of task 2. Crew composition 3. Output per day 4. Unit definition 5. Per unit material cost 6. Per unit labor costs 7. Total unit costs	Average figures; compiled from records of Wood & Tower Incorporated.	Data includes subcontractor overhead and profit but not general contractor's overhead & profit.	Adjustment indices for 152 cities are reported. 24 separate trade & subtrade indicies are reported for each city.	No standard errors published. Reliability not determinable.
Building Construction Cost Data Robert Snow Means, Company	over 16,000	1. Description of task 2. Crew composition 3. Output per day 4. Unit definition 5. Per unit material cost 6. Per unit labor costs 7. Total unit costs	Average figures compiled from actual joh costs reported on industrial and commercial buildings costing $150,000 and up or large housing projects	Overhead profit included in cost figures which can be subcontracted; bare costs reported for items that are not suhcontracted.	Over 108 city cost indices can be used to adjust per unit material, labor, and total cost in 16 categories for each city.	No standard errors published. Reliability not determinable.
Building Cost File Construction Publishing Company	over 7,000	1. Specification and description of task 2. Unit definition 3. Per unit material cost 4. Per unit labor cost 5. Per unit total cost	Data based on cost records of McKee, Berger, and Mansueto.	Overhead and profit included and specified markups are obtainable.	Regional editions and city cost indices are reported.	No standard errors published. Reliability not determinable.
Home-Tech Estimator Vol. II Manager's Manual	over 9,000	1. Specification and description of task 2. Per unit cost of material 3. Per unit cost of labor 4. Total per unit cost (no markup included) 5. Per unit price (cost times markup)	Data based on "actual time and material studies."	Overhead and profit not included in cost figures.	Local area cost modification index available for over 100 cities.	No standard errors published. Reliability not determinable.
National Repair and Remodeling Estimator Craftsman Book Company	over 2,000	1. Description of task 2. Crew composition 3. Ouput per day 4. Unit definition 5. Per unit material 6. Per unit labor costs 7. Total unit costs	Developed by author through consultation with experts. Derivation of figures explicitly described in many cases.	Overhead and profit not included in cost figures.	Geographical wage modification factors for 16 crafts and 17 cities are reported.	No standard errors published. Reliability not determinable.

W_i = Average wage rate per hour of the crew required to perform task i, and

$PROD_i$ = Number of units of output per hour of the crew required to do task i.

Equations 2.1 through 2.3 are very useful because they explicitly identify the major items which must be verified by the cost estimator to ensure that a cost figure from a guide will be applicable to the specific job under consideration. First, is the markup factor appropriate? This is because the figure in equations 2.2 and 2.3 will vary from subcontractor to subcontractor and depends upon market conditions as well as those factors which affect general overhead and project specific overhead costs. Second, are the material prices assumed in equations 2.1 through 2.3 appropriate to that location at that particular time? Third, are local wage rates consistent with the average hourly rate assumed in the guidebook? Fourth, is the crew mix required to accomplish a specific task consistent with a given firm's practices? Finally, and probably most critically, is the productivity figure appropriate to the situation at hand?

The above questions indicate the potential sources of error which may render estimates based on the figures reported in a guide inaccurate in any particular construction application. The authors and editors of these manuals are fully aware of such variations, and for this reason, they include methods for adjusting or modifying estimates for major sources of variations. Assumptions for making such adjustments are usually given in the preface.

> "The unit costs presented in the DODGE MANUAL represent average prices. Users of construction costs recognize that such costs are not exact for a broad range of building projects. Implicit in the development of these unit costs are a number of assumptions which include purchase of materials in quantities normal for most building projects, no volume or special discounts, no labor cost premiums due to trade shortages, no unusual weather conditions, standard "good workmanship" and standard grade materials...Prices contained in the DODGE MANUAL are those that would be incurred by a general contractor who subcontracted for all Items of Work, with the exception of the mechanical and electrical items (Divisions 15 & 16) which do not include the subcontractor's overhead and profit. The general contractor's overhead and markup are not included in any of the prices. No two projects are identical nor are the Items of Work that make up two different projects. When using unit costs, the user must consider special project conditions such as weather, site, etc., and must reflect the effect of simplicity or complexity on the Items of Work."[1]

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[1] 1979 Dodge Manual for Building Construction Pricing and Scheduling, McGraw-Hill Information Systems Company, New York, 1978, p. vi.

2.2 PARAMETRIC COST ESTIMATING PROCEDURES

The second category of cost estimating procedures is parametric or regression based. These procedures make use of historical cost data just as do the average cost procedures. The major difference is that parametric procedures use a set of key factors (parameters) which are weighted to reflect their importance in order to estimate a response variable (usually the cost of a particular subtask). Parametric procedures are superior to average cost procedures because they permit specific factors to (differentially) register their impact on average total cost (or on total cost for that matter). Thus, they may remove to some extent the reliance on judgmental decisions which can neither be confirmed nor denied. Their application, however, is more complicated. In addition, they require more data inputs in order to develop a satisfactory procedure.

The equation used to estimate the response in a parametric procedure is frequently referred to as a cost model. The process of developing such a cost model may be long and involved. It always begins with the postulation of a theoretical model. Ideally, information on the renovation process would be used to postulate the theoretical model. One should be cautioned against postulating a model without knowledge of the process. Using a trial and error approach to "discover" the key factors may produce a model which requires information as input which is not available before the fact.

One commonly used cost model, where cost is in dollars, is of the form

$$ATC = \beta_o + \sum_{j=1}^{m} \beta_j X_j + e \qquad 2.4$$

where ATC = average total cost, the response variable,
β_o = the intercept term,[1]
β_j = the coefficients of the explanatory variables (weighting factors),
X_j = the explanatory variables (key factors), and
e = the error term.

Models having a structure[2] similar to equation 2.4 are referred to as linear. This may be seen by noting that for the case where m is equal to 1 the above model reduces to:

$$ATC = \beta_o + \beta_1 X_1 + e$$

---

[1]The intercept term is included since the assumption that the response is zero when all explanatory variables are zero is a rather strong and usually unjustified assumption.

[2]The term structure is used to denote the basic relationship between the variables which affect average total cost.

which is the equation of a straight line. In higher dimensions (m greater than or equal to 2) the equation defines a plane or hyperplane in m-dimensional space.

Models of a nonlinear form may also be included under the category of parametric cost estimating procedures. An example of a nonlinear cost model would be:

$$ATC = \beta_o (T)^{\alpha_o} (L)^{\alpha_1} (M)^{\alpha_2} (E)^{\alpha_3} e \qquad 2.5$$

where T = the construction technology factor,

L = the labor inputs,
M = the materials inputs,
E = the equipment inputs,
β_o = the intercept term,
α_i = the weighting factors, and
e = the error term.

As one might suspect the estimation of the β and α terms (weighting factors) for nonlinear models is much more complicated than for the general linear case. The flexibility of the nonlinear model is, however, greater than that of the general linear model. Cost models having a structure similar to equation 2.5 are frequently referred to as cost estimating relationships or cost functions. Since the emphasis of this section is on the more commonly used linear cost models, a thorough discussion of cost functions will be postponed until section 2.4.

The methodology through which the underlying structure and determinants of average total cost may be analyzed involves a four stage iterative process of (1) selecting a model, (2) estimating the coefficients, β_j, (3) testing the validity of the underlying assumptions, and (4) testing the adequacy of the model. Once estimated, the finalized model can then be used to predict average total costs under a wide variety of conditions. If desired, a similar approach may also be used to estimate the markup ratio.

The actual estimation of the β's in equation 2.4 would normally be accomplished through application of the econometric technique known as ordinary least squares. For example, suppose we have n observations on average total cost, where n is greater than m + 1. Then the i^{th} observation may be expressed as:

$$ATC_i = \beta_o + \sum_{j=1}^{m} \beta_j X_{ij} + e_i$$

All n observations may be expressed in matrix form by:

$$\begin{bmatrix} ATC_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ ATC_n \end{bmatrix} = \begin{bmatrix} 1 X_{11} & X_{1m} \\ \cdot\ \cdot & \cdot \\ \cdot\ \cdot & \cdot \\ \cdot\ \cdot & \cdot \\ \cdot\ \cdot & \cdot \\ \cdot\ \cdot & \cdot \\ 1X_{n1} & X_{nm} \end{bmatrix} \begin{bmatrix} \beta_o \\ \beta_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \beta_m \end{bmatrix} + \begin{bmatrix} e_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ e_n \end{bmatrix}$$

or more compactly

$$\underline{ATC} = \underline{X}\ \underline{\beta} + \underline{e} \qquad 2.6$$

where the line under ATC, X, β and e indicates that they are matrices. In order to apply ordinary least squares to equation 2.6, the following assumptions should be satisfied:

(1) the explanatory variables, X_{ij}, are fixed,
(2) the expected value of the error term, $E(e_i)$, is zero,
(3) the variance of the error term, $E(e_ie_i)$, is constant, and
(4) the covariance among the i^{th} and k^{th} error terms, $E(e_ie_k)$, is zero.

Under these conditions ordinary least squares will produce best linear unbiased estimators of the β's. The estimates, β, are unbiased because the expected values of $\hat{\beta}_j$ is equal to β_j for each j (i.e., $E(\hat{\beta}_j) = \beta_j$). The estimates are linear because they are linear combinations of the experimental observations. That is, using the matrix notation of equation 2.6, $\underline{\hat{\beta}}$ may be expressed as:

$$\underline{\hat{\beta}} = (\underline{X}'\underline{X})^{-1}\ \underline{X}'\ \underline{ATC}$$

The ordinary least squares estimates are best because no other linear estimator of the β's has a smaller variance. This attribute is very useful because it permits more precise statements to be made about the impact of changes in the value of one or more of the key factors on average total cost.

As will be shown in chapter 3, economists and engineers generally agree that average total cost may be expressed as a function of the number of units of output (say square feet of floor area renovated). This usually implies that average total cost falls over a certain range and then remains constant over a fairly wide range. (Eventually, for very large projects, average total costs will begin to rise again.) Suppose now that only the square feet of floor area renovated is allowed to vary (i.e., all other factors are held constant). Figure 2.1 illustrates how the ordinary least squares procedure operates. In this case average total cost, ATC, is shown along the vertical axis whereas the number of units of output, Q, is shown along the horizontal axis. (The dots shown on the figure represent (ATC,Q) combinations.) The straight line shown in the figure represents the "best fit" for the given data. Note that the straight line is declining, indicating that the average total cost decreases as the number of square feet of floor area increases.

It is important to point out that the straight line is not extended beyond the range of observation. This is because ordinary least squares estimates are rather "data specific" and extrapolating beyond the range of observation may lead to gross inaccuracies in the predicted value of the response.

In order to better illustrate the ordinary least squares approach, we shall refer to the particular combination (ATC_i, Q_i) in figure 2.1. In this case ATC_i is the response and Q_i is the number of square feet of floor area renovated. The predicted response, $\hat{ATC}_i$, lies on the straight line. The difference between $\hat{ATC}_i$ and ATC_i, d_i, is denoted by a darkened line connecting the two points.

The rationale behind any estimation procedure, including ordinary least squares, is to keep as small as possible the distance between the response and the predicted response. An examination of figure 2.1 would reveal that some of the (ATC,Q) combinations lie above the line and some lie below it. Consequently, any summation of just the d_i would probably end up being zero. In order to get around this problem ordinary least squares sums the squared deviations (i.e., d_i^2). Since the square of a number is always greater than zero unless the number is zero (i.e., $\hat{ATC}_i = ATC_i$), the sum of their squares will have to be greater than or equal to zero. The concept of "closeness" that ordinary least squares uses is to minimize the sum of the squared deviations. Although it is conceptually "nice", ordinary least squares may give too much weight to outliers (points way above or way below the line). Consequently, the stability of the estimates of the β's may be very sensitive to the exclusion of a particular observation.[1]

As mentioned earlier, a cost model combines one or more explanatory variables in order that an estimate of a response variable (direct cost, material cost, output per hour, or markup) is produced. Although these

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[1]Another more theoretical issue relates to the stability of the estimates over time. For an authorative source on this problem see R.L. Brown, J. Durbin, and J. M. Evans, "Techniques for Testing and Constancy of Regression Relationships Over Time," Journal of the Royal Statistical Society, Series B, Vol. 37, pp. 149-192, 1975.

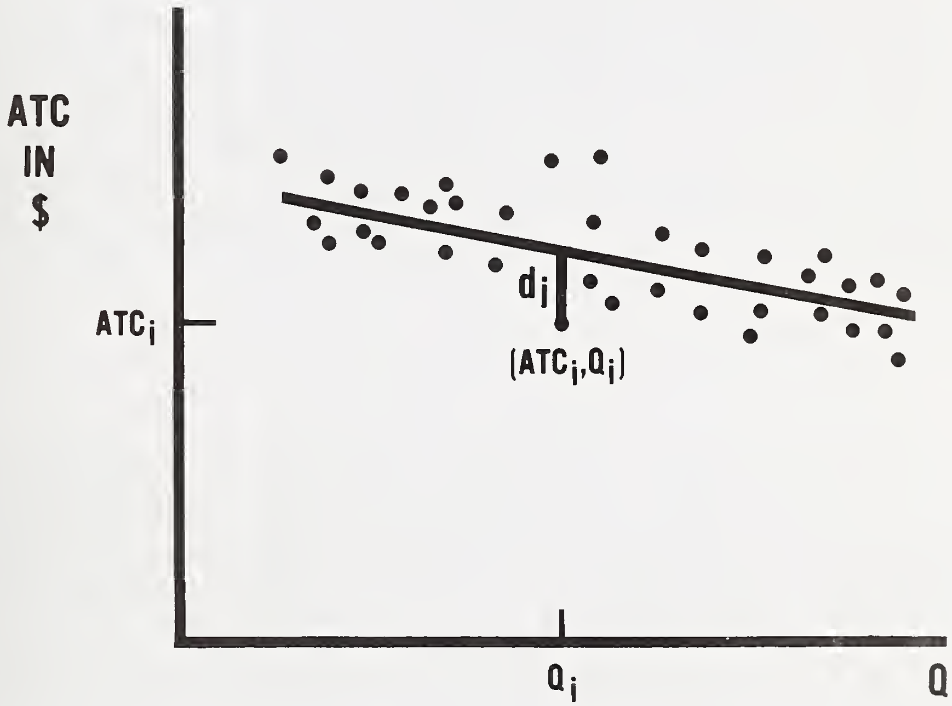

Figure 2.1 Fitting a Model to Data

models can be shown to be best (minimum variance) linear unbiased estimators of the desired response variable, they represent point estimates rather than interval estimates. Thus, there may be situations in which other technical approaches might have costs very near to the one revealed as the least costly. This would imply that the least costly approach can not be unambiguously identified.

Even though in most cases there will be a wide enough variation in costs between technical approaches to identify the one which is least costly, there may be instances where confidence intervals about each estimated response are desired. In addition, confidence intervals tell us how good or how bad our cost model is for that particular combination of inputs. Other things being equal small confidence intervals are preferred to large confidence intervals. The purpose of this discussion is to illustrate how a confidence interval may be fitted about the response. First, we must define precisely what we mean by a confidence interval. The 100(1 - ε) percent confidence interval associated with a particular cost model is defined as:

$$\underline{C}'\ \beta \pm (t_{n-k,1-\varepsilon/2})\ (\underline{C}'(\underline{X}'\underline{X})^{-1}\underline{C})$$

where

$\hat{\underline{\beta}}$ = the vector of estimated coefficients (weighting factors);

$\underline{C}$ = the vector of explanatory variables (key factors);

$s(\underline{C}'(\underline{X}'\underline{X})C^{-1})^{\frac{1}{2}}$ = an estimator of the standard deviation of the predicted value, $\underline{C}'\hat{\underline{\beta}}$ and

$t_{n-k,1-\varepsilon/2}$ = the 1 - ε/2 value of the t statistic with n - k degrees of freedom.[1]

It is important to point out that the output of the cost model is the point estimate $\underline{C}'\hat{\underline{\beta}}$

Suppose we wish to fit a 90 percent confidence interval about the estimate resulting from covering over 510 square feet of wall area in a dwelling unit with plywood paneling. Prior information on wage rates and prices for a four by eight foot sheet of plywood paneling indicate that the average wage rate is $10.00 per hour and the cost per square foot of plywood paneling is $0.45.

The predicted value, $\underline{C}'\hat{\underline{\beta}}$, which results is $1.20. That is, the average cost of installing plywood paneling is $1.20 per square foot in this case.

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[1]The number of degrees of freedom is based on information used in estimating the cost model; in particular, n is the number of observations and k is the number of explanatory variables.

We now wish to compute the width of the confidence interval, w, where:

$$w = t_{8,\ 0.95}\ (\underline{C}'s^2(\underline{X}'\underline{X})^{-1}\underline{C})^{\frac{1}{2}}$$

A statistical table may be used to show that the appropriate value for the t distribution with eight degrees of freedom is 1.86. This permits the above statement to be reduced to:

$$w = 1.86\ (\underline{C}'s^2(\underline{X}'\underline{X})^{-1}\underline{C})^{\frac{1}{2}}$$

Performing the indicated matrix multiplication,[1] $\underline{C}'s^2\ (\underline{X}'\underline{X})^{-1}\underline{C}$, and taking the square root of the resulting scalar yields:

$$(\underline{C}'s^2(\underline{X}'\underline{X})^{-1}\underline{C})^{\frac{1}{2}} = 0.06$$

Thus, the width of the confidence interval is 11 cents (w = (0.06)x (1.86)). The 90 percent confidence interval, $I_{0.90}$, about the predicted value for plywood paneling in this case is thus:

$$I_{0.90} = (1.09,\ 1.31) \text{ or}$$

$$I_{0.90} = 1.20\ \pm 0.11 = \underline{C}'\hat{\underline{\beta}} \pm w.$$

The above discussion has shown how a confidence interval would be fitted about the estimate for a particular set of inputs. If a whole set of inputs were considered, one would get a confidence band about the best fit straight line. An example of such a confidence band is shown in figure 2.2. Note that the band becomes wider at the ends. This is because the cost model is most accurate in the area where the largest number of observations lie (i.e., in the central region).

It was mentioned earlier that nonlinear cost models tended to be more flexible than linear cost models. This stems from the fact that real world considerations are more frequently nonlinear than linear.[2] Figure 2.3 serves to illustrate this point. As in the earlier cases, average total cost is shown along the vertical axis and Q along the horizontal axis. The theoretical average total cost curve is labeled ATC in the figure. By contrast the linear approximation of the desired portion

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[1]The $s^2(\underline{X}'\underline{X})^{-1}$ matrix is usually printed as a part of the standard output of the ordinary least squares package. The matrix actually used in this exercise is available from the author upon written request.

[2]There are many applications where the assumption of linearity is quite appropriate (see, Robert E. Chapman and Joseph G. Kowalski, Lead Paint Abatement Costs: Some Technical and Theoretical Considerations, National Bureau of Standards, Technical Note 979, February 1979). In these cases, however, it is often true that the key factors affecting cost are only varying over a small range. In such cases, even if the process were nonlinear, a linear cost model could be extremely accurate.

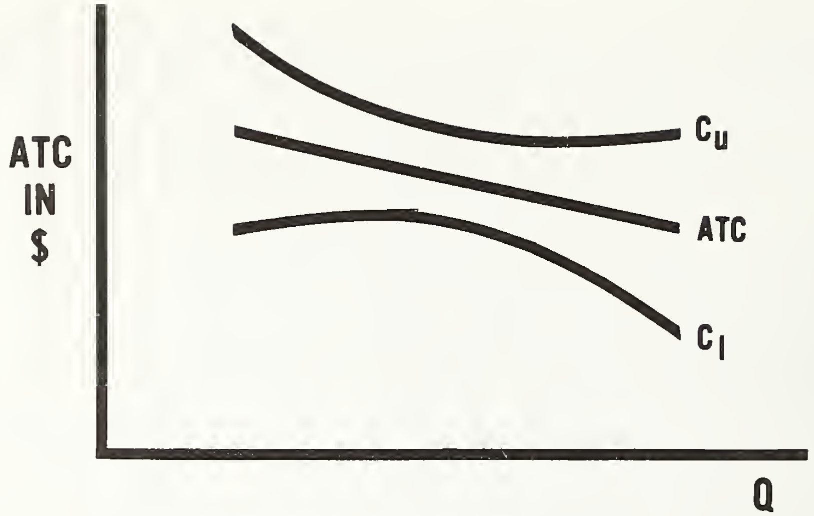

Figure 2.2 Fitting a Confidence Band About a Parametric Cost Estimate

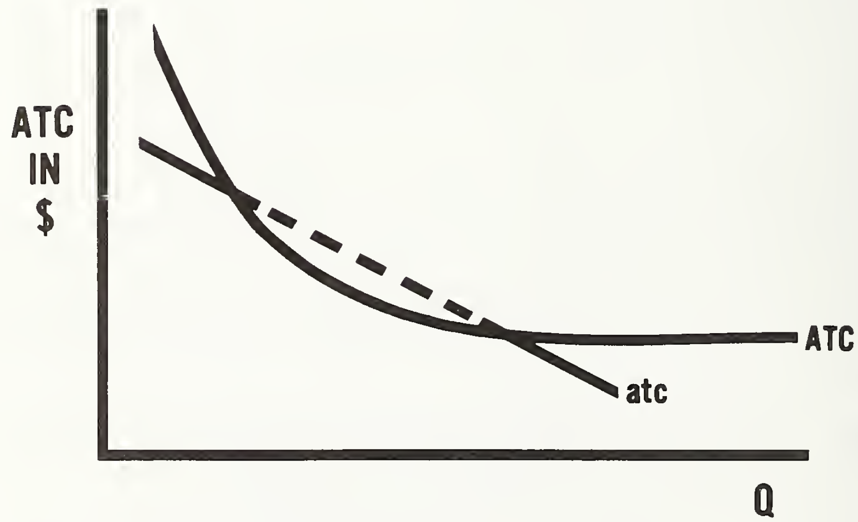

Figure 2.3 Implications of Extrapolation With a Linear Cost Model

of the theoretical average total cost curve based on actual observations is labeled atc. Note that atc consists of three distinct segments. The two segments which appear at the ends are drawn as solid lines.

In these regions the predicted response for average total costs, atc, would be too low. Furthermore, if the line atc were extended in either direction the magnitude of the error would increase. This illustrates why one should not extrapolate beyond the range of observation. The segment in the middle of the line atc is dashed. In this region the predicted response would be too high. It is worth noting, however, that even though the cost estimates lie above the ATC curve, they may be quite accurate. This is due to the fact that the ATC curve is the theoretical average total cost curve. Consequently it is not possible for actual average total costs to lie below it whereas due to inefficient (less than optimal) utilization of labor, material, and equipment, it is possible to lie above it.[1]

2.3 PROBABILISTIC COST ESTIMATING PROCEDURES

One form of construction cost estimation which has grown rapidly as low-cost computer software packages have become available is probabilistic cost estimation. The term probabilistic is a reflection of the fact that a probability distribution can be associated with each key factor in the renovation process. Once these probability distributions have been specified for each factor, it becomes possible to perform a monte carlo simulation of the cost estimation process. The usual output of such a simulation

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[1]The above statement implies that the theoretical average total cost curve will never be observed in its entirety. To illustrate why this is so, a discussion on technical and allocative inefficiency similar to that given by Farrell (M. J. Farrell, "The Measurement of Productive Efficiency," Journal of the Royal Statistical Society, 1957) will be presented. For a two factor constant returns to scale production technology, the two types of inefficiency may be defined in a straightforward manner. Technical inefficiency (i.e., producing using inputs in the right proportions but wrong quantities) is measured by the ratio of the inputs required to produce a given output as a percent of those actually used. Allocative inefficiency (i.e., producing using inputs in the wrong proportions but using quantities that are technically correct) is measured, for a given input price ratio, by constructing a line tangent to the desired isoquant (a curve giving all the technically feasible ways of producing a given level of output) and extending it until it intersects (denoted as point A) the line connecting the origin and the point on the desired isoquant corresponding to the combination of inputs actually used (denoted as point B). The measure of allocative inefficiency is then defined as the distance OA divided by OB. In both cases, the total cost of production is higher due to inefficiency. Since firms may not be flexible enough to always use the right quantities (technical efficiency) or right proportions (allocative efficiency) observed cost will lie above the envelope curve defined by the theoretical average total cost function.

is what is known as a cost profile. A cost profile may be defined as a graphical or tabular portrayal, for the given values of the data input, of the probability of overrunning any given budget estimate. Probabilistic cost estimating procedures are quite attractive because they can be applied to either an average cost method or a parametric cost estimating procedure. Since the application of probabilistic procedures to average costs is simpler than for a whole series of parameters, the discussion in this section will focus upon average cost methods.

Although no theoretical limitation exists on which probability distribution can be used in the simulation of the cost estimating process, most actual applications rely on four basic distributions. These distributions are: (1) the uniform; (2) the normal; (3) the log normal; and (4) the triangular.

These four probability distributions are plotted graphically in figure 2.4. In each case the value of the random variable (average total cost for a particular subtask) is plotted along the horizontal (x) axis. The value taken on by the density function, f(x), is shown along the vertical axis on figure 2.4. The mathematical form of each density function is given in table 2.2. A nonmathematical interpretation of the density function is that it provides a measure of the frequency with which a certain event will take place for a given "small" interval along the x axis. Note that in figure 2.4 the uniform and the triangular distribution both have well defined starting and stopping points. These distributions might be appropriate if average total cost was known to be at least $a but no more than $c. It is also important to point out that the log normal (of necessity) and the triangular (by construction) distributions can be skewed. That is, the tails of the distribution are of unequal length. For the distributions as drawn, it reflects the possibility that an extremely high value of the random variable (average total cost) can occur with non-zero probability. Such cost patterns are a common occurrence in the construction industry, where the most likely cost (the point where f(x) is a maximum) for a particular subtask may be rather low but due to the riskiness of the process, costs may take on a very large value with non-zero probability.

The crucial step which must be taken prior to the application of any probabilistic cost estimation procedure is to determine which distribution is appropriate for the case at hand. The first step is to collect cost data from similar projects or contact local building material suppliers and contractors. The actual cost of each subitem should then be recorded. Once all data has been collected it will then be necessary to group them into intervals. (A general rule of thumb states that at least five data points should be in each interval.)

The number of times average total cost occurred within the interval may then be used to construct a histogram. The histogram may then be compared to sample standard histograms such as appear in the text by Hastings and Peacock.[1] Based on these comparisons, a class of distributions which

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[1] N. Hastings and S.B. Peacock, Statistical Distributions, John Wiley and Sons, New York, 1975.

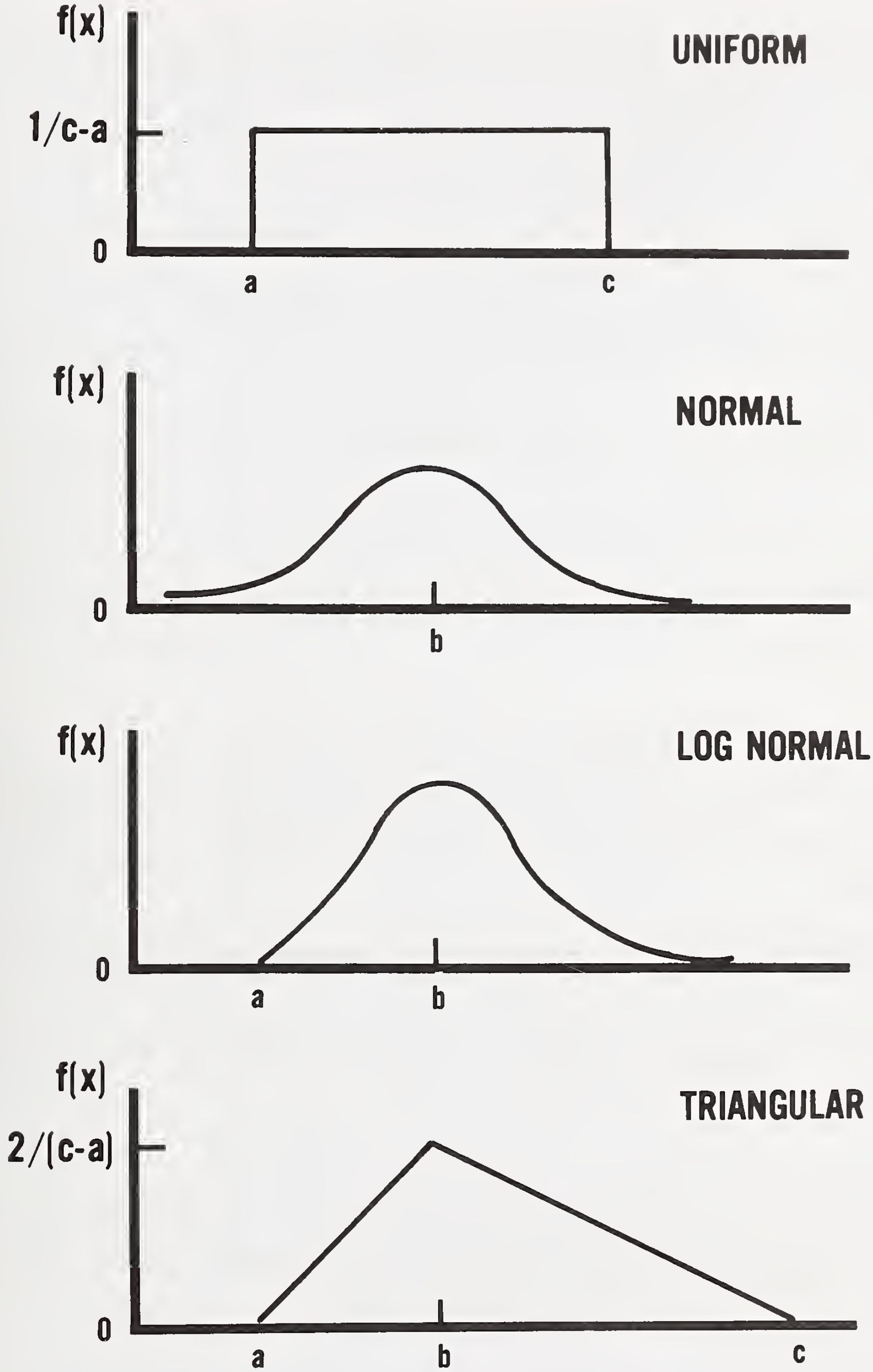

Figure 2.4 Commonly Used Probability Distributions for Key Construction Factors

Table 2.2 Density Functions for Commonly Used Probability Distributions[a]

UNIFORM		
	$f(x) = \frac{1}{c - a}$	in the interval [a, c]
	0	otherwise
NORMAL		
	$f(x) = \frac{1}{\sigma(2\Pi)} \exp\left(-(x-m)^2/2\sigma^2\right)$	$-\infty < x < \infty$
LOG NORMAL		
	$f(x) = 0$	$x \leq a$
	$\frac{1}{(x - a)\sigma(2^{\Pi})} \exp\left(-(\ell n(x-a)-m)^2/2\sigma^2\right)$	$x > a$
TRIANGULAR		
	$f(x) = \frac{2}{(c - a)} \cdot \frac{x}{(b - a)}$	in the interval [a, b]
	$\frac{2}{(c - a)} \cdot \frac{x}{(c - b)}$	in the interval [b, c]
	0	otherwise

[a]Source: Marek Fisz, Probability Theory and Mathematical Statistics, Third Edition, John Wiley and Sons, New York, 1963.

might fit the data can be hypothesized. The next step is to develop maximum likelihood estimators for the parameters of the distribution (e.g., the mean and variance) under the assumption that the hypothesized distribution is correct.[1] The final step is to perform a goodness-of-fit test on the data. If the fit is unacceptable it will be necessary to hypothesize a new class of distributions and reestimate the parameters of the distribution. Several points along these lines are worth noting. First, there are two goodness-of-fit tests which may be used. They are: (1) the Chi-square test; and (2) the Kolmogorov-Smirnov test.[2] Second, it is possible to have the goodness-of-fit test accept more than one distribution as appropriate. This unfortunate circumstance is rather common with the Chi-square test. However, since the Chi-square test is by far the easiest to apply, such a potential problem may not be deemed serious. Should the situation result where two distributions are "accepted" by the goodness-of-fit test, it is advisable to carefully examine the tails of the two distributions. If one tail is significantly more important than the other, then choose the distribution which better fits that tail. Finally, it is important to recognize that the Chi-square test is an asymptotic result whereas the Kolmogorov-Smirnov test is exact for any number of data points. Thus, if the sample size is small the emphasis should be on the result of the Kolomogorov-Smirnov test rather than the Chi-square test.

The focus of the previous paragraph on fitting distributions to data was not to overpower the reader with a whole set of statistical procedures which must be followed prior to any meaningful analysis. The focus was rather on recognizing that if probability distributions are used to estimate renovation costs, the underlying assumptions implicit in that application must be understood. It is crucial to recognize that the distributions are ideal theoretical constructions whereas the data are from the "real world" and hence can not be "ideal." In some cases it may in fact be impossible to obtain a sample of actual average total cost figures for a particular subtask. In this case expert judgment can be used to choose the distribution. In such cases past researchers have recommended the use of the triangular distribution because it explicitly allows for low-probability high-cost events. Furthermore, it may be completely defined by only three points: (1) the minimum point; (2) the maximum point; and (3) the mode or most likely point. (If one believed that costs were equally likely to be distributed throughout the interval then the uniform distribution would be appropriate. The uniform distribution may be completely defined by only two points: (1) the minimum point; and (2) the maximum point.)

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[1]The likelihood function of a sample, given a parameter, is the product of the density function with respect to the parameter at each sample point. For an excellent discussion see John Freund, Mathematical Statistics, Second Edition, McGraw-Hill, New York, 1970.

[2]Both tests are discussed in L. Breiman, Statistics: With a View Toward Applications, Haughton Mifflin, Boston, 1973.

The previous discussion has touched upon how the distributions would be applied in an actual monte carlo simulation to estimate the costs of residential rehabilitation. Although only two of the four distributions were discussed it can be easily shown that the normal and log normal distributions may be applied almost as simply in actual cost studies. The basic difference between the normal--log normal and uniform--triangular distributions is that the upper tail of the normal and log normal distributions extends to infinity. This is not true for both lower tails since the log normal distribution has a minimum point at a (a is greater than or equal to zero). The lower tail on the normal distribution has no minimum point however. Thus, in applying probabilistic cost estimating techniques it is useful to specify three sets of numbers when a normal or log normal distribution enters the process. Each set of three numbers may be either estimated from actual data or based on the opinions of construction experts. The first number needed is the "optimistic" estimate of average total cost. This estimate is called optimistic because there is only a 10 percent (subjective) probability that average total cost will fall below it. Mathematically, the optimistic estimate is the 10th percentile point of the distribution. The second number needed is the "middle ground" estimate of average total cost. The term middle ground is used because in 50 percent of the cases average total cost will fall below it and in 50 percent of the cases average total cost will fall above it. Mathematically, the middle ground estimate is the median of the distribution. The third number needed is the "pessimistic" estimate of average total cost. This estimate is called pessimistic because there is only a 10 percent (subjective) probability that average total cost will fall above it. Mathematically, the pessimistic estimate is the 90th percentile point of the distribution. In essence, once the distribution is specified the user need enter (at most) three numbers for each subtask into the computer software package. These numbers are summarized in table 2.3.

Assuming that the user has correctly specified the probability distribution and correctly input the information identified in table 2.3, the software package will perform a monte carlo simulation. The term monte carlo is used to indicate that the process is patterned after several popular games of chance. Basically what the computer program does is estimate the cost of each subtask sequentially. This is done by generating a random number. Each random number will correspond to a value between 0 and 1. The random number is then associated with the parent probability distribution (i.e., the average total cost distribution for that subtask). Since each random number is between 0 and 1 it can be interpreted as the probability that the value of the random variable will be less than or equal to a specified amount. If we denote the random number as R, the random variable as χ and the specified amount as X then the expression may be written formally as

$$\Pr(\chi \leq X) = R.$$

Table 2.3 Input Requirements for a Standard Probabilistic Cost Estimating Procedure

DISTRIBUTION	NUMERICAL INPUTS
UNIFORM	1. MINIMUM COST 2. MAXIMUM COST
TRIANGULAR	1. MINIMUM COST 2. MAXIMUM COST 3. MODAL OR MOST LIKELY COST
NORMAL AND LOG NORMAL	1. OPTIMISTIC: 10^{th} PERCENTILE 2. MIDDLE GROUND: 50^{th} PERCENTILE 3. PESSIMISTIC: 90^{th} PERCENTILE

Since we are concerned with the average total cost of the subtask, the relevant cost is X. Thus, if there are n (say 100) subtasks we will get n random numbers and n separate X's. Suppose there are N (say 1000) iterations. Then for the first iteration we get

$$\Pr(\chi \leq X_{1j}) = R_{1j} \qquad j = 1, \ldots n$$

The estimated total cost for the overall job on the first iteration, TC_1, is thus

$$TC_1 = \sum_{j=1}^{n} X_{1j}$$

Similarly, the estimated total cost for the overall job on the second iteration, TC_2, would be

$$TC_2 = \sum_{j=1}^{n} X_{2j}$$

and for the i^{th} iteration

$$TC_i = \sum_{j=1}^{n} X_{ij}$$

The computer will generate N estimates of total cost for the overall job. These estimates are then ranked from least costly to most costly. For example, the least costly estimate (i.e., the first order statistic) is denoted as $TC_{(1)}$. The parentheses are used to distinguish the first order statistic from the estimated cost on the first iteration. Since the total costs are ranked from least to most costly, it is possible to compute the probability that total cost will be less than or equal to a specified dollar amount. For example,

$$\Pr(tc \leq TC_{(k)}) = k/N,$$

where tc = the random variable total cost

$TC_{(k)}$ = the k^{th} order statistic for total cost; and

N = the number of iterations.

Conversely, the probability of exceeding $TC_{(k)}$, the projected budget, may be expressed as

$$\Pr(tc \geq TC_{(k)}) = 1 - k/N$$

The process described above is most easily understood through reference to a cost profile. An example of a cost profile is shown in figure 2.5.

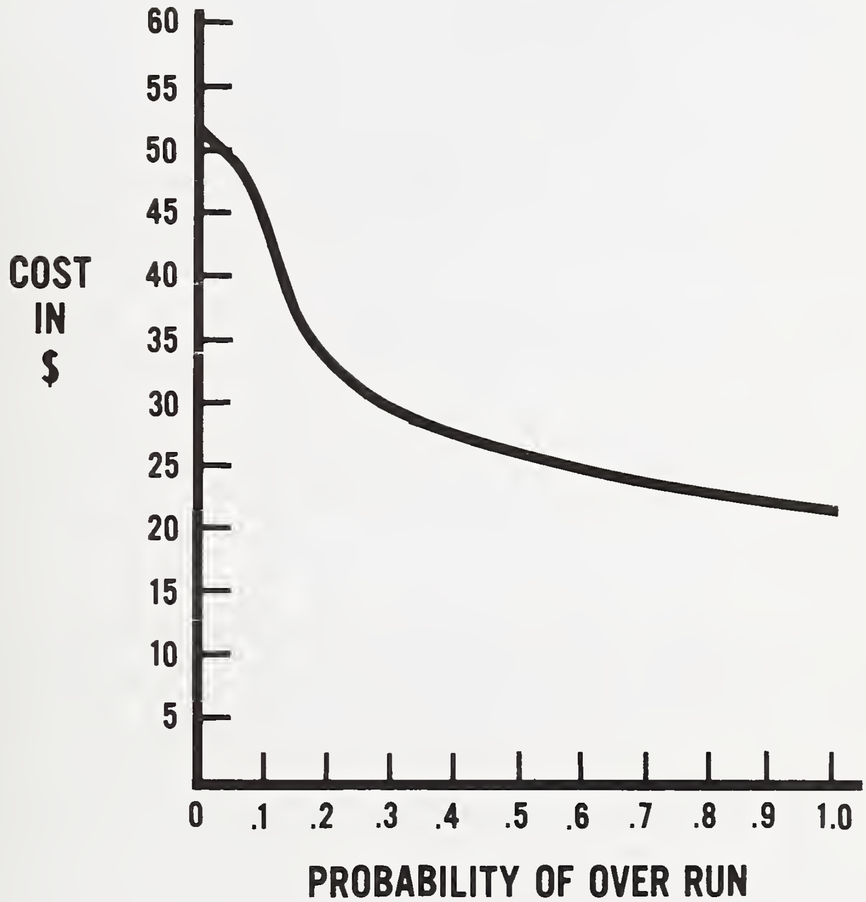

Figure 2.5 Cost Profile for a Renovation Project

Note that the total cost of the job is shown along the vertical axis. In this case, the probability of overrunning a specified budget is shown along the horizontal axis. For example, the probability of overrunning a $35,000 budget is only 15 percent whereas the probability of overrunning a $30,000 budget is 30 percent. Thus, the perspective investor can specify a given level of risk (in terms of probability of overrun) and then choose a budget which will satisfy this constraint. In addition to dealing with risk, the use of probabilistic cost estimation permits the investor to more effectively manage any funds held for contingencies. That is, the investor may proceed with a basic contract which permits contingencies in terms of better quality products (say floor coverings) to be installed should the cost of the job fall below some agreed upon figure.

It is important to point out that the term monte carlo is quite appropriate since there is still some element of chance remaining. In essence there are no absolutes. Total costs will either exceed or fall below the projected budget. The measure of risk is only approximate. In particular, different sequences of random numbers can yield different estimates. Thus, one should be cautioned against demanding a one percent risk. On one simulation the figure might be $45,000 and on another $55,000. This is because each simulation (N iterations) yields a single estimate of the cost profile. Therefore, in order to get a more meaningful measure of the true risk being assumed, one should replicate the simulation using a different starting random number (seed). Should the investor be using the simulation to choose between two alternative methods of renovation, replications should certainly be performed. For an excellent discussion of this topic the interested reader is referred to the article by Law.[1]

2.4 COST FUNCTIONS

The discussion so far has focused on engineering based procedures for estimating rehabilitation costs. The purpose of this section is to show how the requirements of having a sound cost engineering approach can be integrated with the economic theory of cost functions. Combining both aspects of the problem results in a cost estimating procedure which is sensitive to the technical considerations of the renovation process as well as local market conditions. Furthermore, the development of such a procedure facilitates the treatment of risk through the use of probabilistic methods. (A theoretical structure which allows performance concepts to be introduced into the rehabilitation investment decision will be developed in chapter 4.)

The cost function approach is highly desirable due to the duality relationship between the cost function and the production function associated with the physical process. (The term production function as used in this study refers to an explicit relationship between a set of inputs (i.e., labor, materials, and capital) and technological factors which taken together produce a given output, e.g., square feet of floor area renovated.)

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[1]Averill M. Law, "Confidence Intervals in Discrete Event Simulation: A Comparison of Replication and Batch Means," Naval Research Logistics Quarterly, Vol. 24, 1977.

This attribute has been documented in numerous economic and engineering economics articles.[1,2] Through reference to the duality relationship, it is possible to assert that the cost function tells us the least-cost way of renovating Q square feet of floor area. In an actual empirical study, the cost function associated with the underlying process would be derived from the production function by solving a constrained optimization problem. In order to illustrate this concept more fully, a mathematical discussion followed by a numerical example will be presented.

The derivation of the cost function will proceed on the assumption that the production function underlying the construction process is Cobb-Douglas. This assumption is made both because it is consistent with previous studies of the construction industry[3] and for ease of exposition. In the mathematical discussion which follows, it is assumed that the prices of all inputs are independent. Under this assumption the quantity of output, Q, the square feet of floor area renovated, and the cost of output, C, may be expressed as:

$$Q = \left(\tau_o \prod_{i=1}^{m} \tau_i(k_i)\right)\left(\prod_{j=1}^{n} (X_j)^{\alpha_j}\right) \qquad 2.7$$

$$C = \sum_{j=1}^{n} X_j P_j \qquad 2.8$$

where

Q = total square feet of floor area to be renovated;

τ_o = basic construction technology factor;

τ_i = construction technology factor associated with i^{th} building system or subsystem (e.g., plumbing, mechanical, electrical);

k_i = measure of the condition of the i^{th} system or subsystem (e.g., 1 = excellent; 2 = sound; 3 = some deterioration . . .);

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[1]Eugene Silberberg, The Structure Of Economics: A Mathematical Analysis, McGraw-Hill Publishing Company, New York, 1978.

[2]Gerald L. Musgrave and Robert H. Rasche, "Estimation of Cost Functions," The Engineering Economist, Vol. 22, No. 3, 1973.

[3]For an authoritative source on this subject see John S. McConnaughey, Production Functions in Contract Construction for the United States, 1972 (unpublished), Ph.D. dissertation, Michigan State University, 1976.

m = number of systems or subsystems considered;

X_j = quantity of the j^{th} input (materials, labor, equipment) required to perform the job;

α_j = the percentage change in output associated with a one percent change in the utilization of the j^{th} input;[1]

n = number of inputs considered;

C = the total cost (expected bid price) of the job; and

P_j = the unit cost of the j^{th} input.

Note also that the summation sign, Σ, and the product sign, Π, are used in equations 2.7 and 2.8. They are defined as:

$$\sum_{j=1}^{n} X_j P_j = X_1 P_1 + X_2 P_2 + \cdots + X_n P_n$$

and

$$\prod_{j=1}^{n} X_j^{\alpha_j} = (X_1)^{\alpha_1} (X_2)^{\alpha_2} \cdots (X_n)^{\alpha_n}$$

Letting n equal 2 would thus imply that

$$\sum_{j=1}^{n} X_j P_j = \sum_{j=1}^{2} X_j P_j = X_1 P_1 + X_2 P_2$$

and

$$\prod_{j=1}^{n} (X_j)^{\alpha_j} = \prod_{j=1}^{2} (X_j)^{\alpha_j} = ((X_1)^{\alpha_1})((X_2)^{\alpha_2})$$

The constrained optimization problem may then be solved through application of the method of La Grange multipliers.[2]

Based on the constrained optimization problem stated above, it is possible to assert that the cost minimizing demand curve for each input, X^*_ℓ, reduces to:

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[1]In economics α_j is referred to as an output elasticity.

[2]In order to avoid a rather mathematical discussion at this point the derivation of the cost function will not be presented here. Those readers wishing an in-depth discussion are encouraged to turn directly to appendix A.

$$X^*_\ell = (Q)^{1/R} \left(\tau_o \prod_{i=1}^{m} \tau_i(k_i)\right)^{-1/R} \left(\prod_{j=1}^{n} (\alpha_j/\alpha_\ell)^{-\alpha_j/R} (P_\ell/PP_j)^{-\alpha_j/R} \right) \qquad 2.9$$

where $R = \sum_{j=1}^{n} \alpha_j$; and

$\ell = 1, \ldots, n.$

The minimum cost solution, C*, for renovating Q square feet of floor area is then obtained by substituting equation 2.9 into equation 2.8. This substitution yields:

$$C^* = \sum_{j=1}^{n} X^*_j P_j,$$

or equivalently:

$$C^* = (Q)^{1/R} \left(\tau_o \prod_{i=1}^{m} \tau_i(k_i)\right)^{-1/R} \left(\prod_{j=1}^{n} (P_j)^{\alpha_j/R} \right) (R) \left(\prod_{j=1}^{n} (\alpha_j)^{-\alpha_j/R} \right) \qquad 2.10$$

A closer examination of equation 2.10 reveals that it consists of three distinct factors: (1) a technology factor; (2) a size factor; and (3) a market factor. The technology factor is defined by the underlying construction/renovation process. That is, certain basic construction techniques (technologies) interact with the condition of the building's system and subsystems in defining an approach which is feasible in the engineering sense. The technology factor, TF, in equation 2.10 is given as:

$$TF = \left(\tau_o \prod_{i=1}^{m} \tau_i(k_i)\right)^{-1/R} .$$

The size factor may be expressed as the product of the number of structures being renovated, N, and the average number of square feet per structure renovated, q. This division is important because it permits the existence or lack of existence, of economies of scale to be tested. The size factor, SF, in equation 2.10 is given as:

$$SF = Q^{1/R} = (N^{1/R})\,(q^{1/R}).$$

The market factor reflects the influence that supply and demand conditions in the local construction market for key labor, material, and equipment inputs have on the overall cost of the job. The market factor, MF, in equation 2.10 is given as:

$$MF = \left(\prod_{j=1}^{n} (P_j)^{\alpha_j/R} \right) (R) \left(\prod_{j=1}^{n} (\alpha_j)^{\alpha_j/R} \right)..$$

Equation 2.10 may thus be written as:

$$C^* = (TF)x(SF)x(MF). \qquad 2.11$$

Thus far no mention has been made about how such a cost function would be estimated. In its present state neither equation 2.10 or 2.11 is in a form which is directly estimable. Taking the natural logarithm of either equation 2.10 or 2.11 would, however, produce an equation which was directly estimable. Stated in its most simple form the natural logarithm of C* would be:

$$\ell n(C^*) = b_o + b_1 \ell n(TF_a) \ (b_2 \ell n(SF_a) + b_3 \ \ell n(MF_a) + \varepsilon_a$$

where a, a=1, ... A, denotes the contract package of one or more dwelling units and ε_a is a normally distributed random variable, error term, with mean 0 ($E(\varepsilon_a) = 0$ for all a), variance σ^2 ($E(\varepsilon_a, \varepsilon_a) = \sigma^2$ for all a) and covariance 0 ($E(\varepsilon_i, \varepsilon_i) = 0$ for all $i \neq j$). Taking the natural logarithm of equation 2.10, letting $t_i\ (k_i) = \delta_i k_i^{\beta_i}$, and collecting all constant terms into b_o yields

$$\ell n(C_a^*) = \beta_o + \sum_{j=1}^{n} \beta_j \ell n(P_{ja}) + \sum_{i=1}^{m} (\beta_{n+1}) \ell n(k_{ia})$$

$$+ (\beta_{m+n+1}) \ell n(Q_a) + \varepsilon_a. \qquad 2.12$$

The method of ordinary least squares may now be applied to equation 2.12. Equation 2.12 in its estimated form would be given as

$$\ell n(C_a^*) = \beta_o + \sum_{j=1}^{n} \beta_j \ell n(P_{ja}) + \sum_{i=1}^{m} (\beta_{n+i}) \ell n(k_{ia})$$

$$+ (\beta_{m+n+1}) \ell n(Q_a) \qquad 2.13$$

In equation 2.13 the β_χ, $\chi=0, \ . \ . \ . \ , m+n+1$, are the ordinary least squares estimates of the β_χ.

It is important to point out that the left hand side of equation 2.13 is an estimate of the natural logarithm of C_a^*. Therefore, in order to get an estimate of C_a^*, it is necessary to take the antilogarithm of $\ell n(C_a^*)$. This may be accomplished simply by making use of the relationship $C_a = \exp(\ell n(C_a))$.

Example

In order to illustrate how the actual estimation of a cost function would be accomplished, the methodology developed earlier will be applied to the problem of estimating the costs of surface refinishing work. Examples of surface refinishing include covering over walls on which the paint may be in a cracked or peeling state, may be of an undersirable color or texture, or may contain lead pigments which are potentially dangerous to small children.

One method of surface refinishing which is currently used is the application of a flexible wall covering. Flexible wall coverings include cement coated fiberglass and gypsum impregnated jute fabric. Both products are applied in the same manner as wall paper, usually with a water base adhesive. Their strength and durability, however, greatly exceed that of most commercially available wallpaper. Due to their similarity, it is possible to use one cost function for both products. The discussion given here is patterned after results presented in a report by Chapman and Kowalski on methods for eliminating the lead-based paint hazard from housing. The reader interested in further details on surface refinishing techniques is referred to the NBS report Lead Paint Abatement Costs: Some Technical and Theoretical Considerations.[1]

The products under study, cement-coated fiberglass and gypsum impregnated jute fabric, consist of a woven glass or jute fabric impregnated with portland cement. After application, the portland cement absorbs moisture from the environment and hardens becoming a fairly rigid and penetration-resistant material. Whenever the material is to be used in a wet area, such as a bathroom or a kitchen, a protective coating should also be applied. Based on a previous study, the method of application is known to be affected by the surface condition and the occupancy status. The primary labor cost is the wage of the painter/wallpaper hanger who installs the product. The only material cost which is significant is the delivered cost per square yard of the product itself.

The production function for cement-coated fiberglass and gypsum impregnated jute fabric may thus be hypothesized as:

$$Q = \tau_0 (k_1)^{\beta_1} (k_2)^{\beta_2} (L)^{\alpha_1} (M)^{\alpha_2}$$

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[1] Robert E. Chapman and Joseph G. Kowalski, Lead Paint Abatement Costs: Some Technical and Theoretical Considerations, National Bureau of Standards, Technical Note 979, February 1979.

where:

Q = the total number of square feet of wall area covered;

τ_o = the basic technology factor;

k_1 = the occupancy status (1 = unoccupied; 2 = occupied);

k_2 = the condition of the wall surface;

β_1, β_2 = the "complexity" factors;

L = the amount of painter/wallpaper hanger labor required to do the job;

M = the square feet of cement-coated fiberglass or gypsum impregnated jute fabric required; and

α_1, α_2 = the output elasticities.

Using equation 2.10 as a guide, it is possible to write the cost function as:

$$C^* = (Q)^{1/R} \, (\tau_o)^{-1/R} \left(\prod_{i=1}^{2} (k_i)^{-\beta_i/R} \right) \left(\prod_{j=1}^{2} (P_j)^{\alpha_j/R} \right) (D_g)^{\alpha} , \qquad 2.14$$

where:

$R = \alpha_1 + \alpha_2$;

P_1 = the hourly wage rate of a painter/wallpaper hanger;

P_2 = the delivered price per square yard for cement-coated fiberglass or gypsum impregnated jute fabric; and

Dg = a dummy variable associated with the use of gypsum jute (1 = gypsum jute not used; 2 = gypsum jute used).

Using the general form given in equation 2.12 to put equation 2.14 into a form which is readily estimable yields:

$$\begin{aligned} \ell n(C_i^*) = {} & \beta_o + \beta_1 \ell n(Q_i) + \beta_2 \ell n(k_{1i}) + \beta_3 \ell n(k_{2i}) \\ & + \beta_4 \ell n(P_{Li}) + \beta_5 \ell n(P_{Mi}) + \beta_6 \ell n(D_{gi}) + \varepsilon_i \end{aligned} \qquad 2.15$$

Twenty three separate observations were then used to estimate β_o through β_6 in equation 2.15. The estimates for β_o through β_6 are given in table 2.4.

Table 2.4 Estimated Coefficients for the Cement-Coated Fiberglass and Gypsum Impregnated Jute Fabric Cost Model

FACTOR	COEFFICIENT	ESTIMATE
Intercept	o	-1.63
Size	1	0.97
Occupancy	2	-0.24
Condition	3	0.16
Wage Rate	4	0.52
Material Price	5	0.46
Jute Dummy	6	-0.42

An examination of table 2.4 reveals the following. First, the coefficient of the size factor is slightly less than one, indicating that (other things being equal) the size of the job can be doubled without doubling the cost. Second, the coefficient of the occupancy factor is negative, indicating that occupied units are relatively less expensive to treat. This rather surprising result stems from the fact that although the presence of furniture and people in the dwelling does affect the installation of the product, these added costs are more than compensated for by being able to store the materials in the dwelling until the job is completed.[1] Third, the coefficients associated with condition, the average wage rate and the material price per square yard were all positive, indicating that a rise in the value of those factors would result in higher installation cost. Finally, the coefficient associated with the gypsum jute dummy variable is negative indicating that (others things being equal) it is relatively less expensive to install this product than cement-coated fiberglass. This difference is due to less material wastage and a greater ease in cutting the product as well as minor differences in the method of applying the material.

An illustration of the model is now in order. Suppose we wished to estimate the direct cost of using cement-coated fiberglass to refinish 800 square feet of wall area in an unoccupied dwelling unit. An inspection of the unit revealed the walls to be in a sound but peeling condition. The average wage rate and delivered material price in the locale are \$10.00 per hour and \$3.60 per square yard, respectively. The estimated cost thus becomes

$$\begin{aligned}\hat{\ln}\, C^* = -1.63 &+ 0.97\ \ln(800) - 0.24\ \ln(1) \\ &+ 0.16\ \ln(2) + 0.51\ \ln(10.00) \\ &+ 0.45\ \ln(3.60) - 0.42\ \ln(1),\end{aligned}$$

which reduces to

$$\hat{\ln}\, C^* = -1.63 + 6.49 - 0 + 0.11 + 1.18 + 0.58 - 0,$$

---

[1]Theft and vandalism were prevalent in the neighborhoods containing the dwellings making up the sample.

or

$$\hat{\ell n}\ \hat{C}* = 6.73.$$

Recall that in order to get an estimate of $\hat{C}*$, the direct cost of the job, it was necessary to make use of the formula $\hat{C}* = \exp(\hat{\ell n}C*)$, which implies, in this case, that

$$\hat{C}* = e^{6.73},$$

or

$$\hat{C}* \doteq \$840.$$

Chapter Summary

The purpose of this chapter was to provide an introduction to each of four basic methods for estimating rehabilitation costs. The presentation of the methods stressed both theoretical and empirical considerations with particular emphasis being placed on statistical techniques. The development of the theoretical discussion was arranged to highlight the increasing order of complexity of the cost models in terms of data requirements and ease of application. The reason for this approach was to demonstrate why increased flexibility in dealing with varying technical and economic conditions could only be achieved through a greater emphasis on detail in the data inputs or sophistication on the part of model builders and/or users.

Facing page:

Although many aspects of residential rehabilitation are similar to those of new construction, differences in job size, productivity, and contractor markup cause the application of average cost methods to systematically underestimate the true costs of building renovation.

3. CRITIQUE OF ALTERNATIVE METHODS OF ESTIMATING REHABILITATION COSTS

The data reported in cost guides on per unit costs is essentially of two kinds. First the data can be based on averages. Average figures are computed by compiling cost reports submitted by many independent reporting units. Second, the data reported in these manuals may be nonstatistical, not derived from samples, and thus to some extent, judgmental. In this case, per unit cost figures are based on opinions of experts who understand the nature of the subtask, how it is best done, and how long it would take to do it. Such figures represent the expert's judgment as to what is typical (or average) behavior in completing a specific subtask. The repair and alteration cost guides appear to lean to expert opinion as their sources for their reported subtask figures.

None of the cost guides, either of the new construction genre or of the repair and remodeling genre, describe the degree of variability in their data. Such information would be useful in order to assess the reliability of estimates based on such data.

In this chapter we shall discuss in more detail some of the difficulties associated with judgmental or expert estimates. The focus will be on the notion of the "reliability" of the estimates and how to determine it.

Most guidebooks also contain detailed cost adjustment indices which differ by region and labor categories. They are usually applied to the average figures in order to attain figures which are more appropriate to local construction market conditions. Although cost indices serve to control for systematic differences in the structure of construction costs, they may promote a false sense of security.[1] This stems from three factors: (1) the construction project mix; (2) local construction practices; and (3) local building code requirements. Understanding the reasoning behind the previous statement may be seen by recognizing that cost indices are really little more than a market basket of construction labor, materials, and sometimes equipment. The weights used in computing the index are based on specific types of construction. Consequently, should the type of construction being put in place for a particular region differ from that used to compute the weights for the index, it is quite likely that the index would introduce a bias into any cost to which it was applied. Unfortunately, there is no prior information which can be used to identify the direction and size of such a bias. This claim is supported by recent empirical studies which have shown that the use of construction cost indices for military construction projects is likely to introduce systematic biases into the decisionmaking process.[2]

Local construction practices relating to crew size and composition and local building code requirements may also introduce biases which will render meaningless the weights used in a construction cost index.[3]

In addition to these factors, the impact of changing job size, the treatment of productivity, and variations in contractor's markup ratios, will all affect the reliability of the cost estimates. Each of these topics will be examined in turn prior to an assessment of the reliability of the most commonly used techniques for introducing risk into the process of estimating

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[1]Lawrence Jaquith, "The Cost Index: Working Tool or Trap," Architectural Record, February 1969.

[2]James M. Johannes, Paul D. Koch, and Robert H. Rasche, An Investigation of Factors Affecting Geograpic Cost Differentials on Military Construction Projects, National Bureau of Standards, NBS-GCR-80-197, February 1980.

[3]For an authorative source on assessing the cost implications of a building code or code provision, see John S. McConnaughey, Jr., An Economic Analysis of Building Code Impacts: A Suggested Approach, National Bureau of Standards, NBSIR 78-1528, October 1978.

rehabilitation costs. The chapter will conclude with a discussion of how and why the calculation of risk can be rigorously introduced into the cost estimation process.

3.1 IMPACT OF CHANGING JOB SIZE

The term job size really has two meanings, only one of which is purely economic. Throughout this paper the meaning, unless otherwise stated, will be the one which is purely economic in nature. The economic definition of job size may be thought of as an aggregate measure of the gross square feet of floor/wall area treated, the number of building systems renovated, and the number of items installed. This economic concept of job size is closely related to a more pragmatic concept of job size, namely the dollar value of a particular construction contract or subcontract. The economic meaning will be used because it makes more sense to talk about costs varying as a function of scale rather than as a function of costs.

A cost estimation procedure is basically a summation approach. A construction project is subdivided into hundreds of subtasks. Ideally the cost estimation effort would therefore also be subdivided into hundreds of subtasks. Separate cost estimates are then made for each subtask and summed to obtain overall cost figures. Usually the estimation process follows, as closely as possible, the order of the construction process. It may start with items involving excavation or earth work and end with fixture installation.

Job size affects the estimation process along a horizontal dimension, where the "horizontal dimension" refers to the relationship between job size and how many square feet, linear feet or item counts are specified in any one of the particular subtasks. Job size also affects the "vertical dimension" of a cost estimate. The number of subtasks included in the summation process is the vertical dimension of a cost estimate. Both dimensions will be positively related to the total size and price of the job. Table 3.1 illustrates the two dimensions of cost estimation. We shall consider each dimension in turn.

The "Horizontal" Impact of Job Size

The data base in the major cost estimating guides is drawn from cost reports of contractors whose new construction projects had total project costs in excess of $150,000. (This minimum figure varies from manual to manual but in all cases it is large.) Average housing renovation costs would most likely fall in the $20,000 to $30,000 range, however.[1,2]

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[1] Bureau of Building Marketing Research, Homeowners Remodeling/Modernization Study, November 1975.

[2] "HUD Launches a New Rehab Program," Engineering News Record, November 25, 1976.

Table 3.1 An Illustration of the "Horizontal" and "Vertical" Dimensions of Cost Estimation

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Item #	Item Description	Cost Per Unit Lab.	Cost Per Unit Mat.	Cost Per Unit Total	Markup Factor	Total Unit Costs (4 x 5)	Total No. Units	Total Costs (6 x 7)
V E	1	____________	xx	xx	xx	xx	xx	xx	xx
R T	2	.	.	.	.	.	.	.	.
I C	3	.	.	.	.	.	.	.	.
A L	.	.	.	.	.	.	.	.	.
D	.	.	.	.	.	.	.	.	.
I M	.	.	.	.	.	.	.	.	.
E N	.	.	.	.	.	.	.	.	.
S I	.	H O R I Z O N T A L D I M E N S I O N							
O N	.	____________	xx	xx	xx	xx	xx	xx	xx
	Total								xxx

Because the descriptive subtask of the work to be done in housing renovation can be made to coincide with the descriptive subtasks reported in new construction manuals, however, cost estimates for residential rehabilitation can be gleaned from these manuals. For example, both a new construction and renovation subtask may require painting of an interior wall (one primer coat and one finish coat). Thus, since the per square foot cost from a manual "can" be specified in terms compatible with the guidebooks' descriptions, they have often been used in putting together a renovation cost estimate. That this may lead to substantial errors may be seen by examining an economic construct known as a cost function. For purposes of illustration in this section we shall focus on the average cost curve (function).

Economists generally think of average total costs as depending on units of output. Average costs fall as the output increases, reach a minimum at some size of job, and begin to rise as output level continues to increase.[1] An illustration of the typical behavior of average total costs as a function of job size is shown in figure 3.1.[2] Note that for any particular output level, whether it be Q_1 or Q_2, (see figure 3.1), it will always be true that the accounting definitions of equations 2.1 through 2.3 will be met. Thus, the guidebooks could be thought as reporting average cost figures from one such point along a given cost curve. Most economists and engineers agree, however, that average total cost remains constant over some fairly wide range. This concept of average total cost is illustrated in figure 3.2. If costs do in fact behave in this way, little error will be contained in a cost figure which ignores the impact of job size. The above being true as long as the number of units involved (e.g., gross square feet) exceeds some critical minimum. Therefore in order to ensure that average total costs are approximately constant, most major guides give a minimum total cost figure in their preface.

Given that there may be two orders of magnitude in the difference in job size between housing renovations and "major" new construction projects, the applicability of the constant cost assumption is probably not valid. The practical meaning, or source, of such differences in cost as a function of job size is illustrated by the following quote from R. L. Peurifoys book. Estimating Construction Costs:

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[1]The economic principles behind this claim are discussed in detail in Walter Nicholson, Microeconomic Theory: Basic Principles and Extensions, The Dryden Press, Inc., Hinsdale, Illinois, 1972.

[2]Recall that average total cost is equal to total cost divided by Q, where Q is output (the economic measure of job size). Although average total cost may in fact be falling, it does not follow that total cost will also be falling.

> "All experienced construction men know that the productivity of labor is usually low during the early stages of construction. As the organization becomes more efficient, its productivity rates will improve, then as the construction job enters the final stages, there will usually be a reduction in productivity rates. This is important to an estimator. For a small job it is possible that labor will never reach its most efficient rate of production because there will not be sufficient time. If a job is of such a type that laborers must frequently be transferred from one operation to another or if there are frequent interruptions, the productivity rates will be lower than when the laborers remain on one operation for long periods of time without interruption."[1]

The importance of the role of job size on cost behavior for renovation activities is supported by empirical work in the area of lead-based paint hazard abatement.[2] In the lead paint report aluded to above, cost data were collected and analyzed for using barrier materials to cover over the lead-based paint hazard (e.g., gypsum wallboard, and plywood paneling) on existing walls. These tasks would easily be subitems in a room remodeling or renovation contract. In every case where barrier materials were applied, it was found that average direct costs[3] were significantly and negatively related to job size (square feet of wall area covered).[4] This suggests that using the average cost from new construction activities in housing renovaton might lead to persistent errors in (per square foot) average direct cost estimates which would lead to significant errors in the estimate of the total cost of the job. The differences in job size between major new construction projects and housing renovation are so great that it is reasonable to assume that the new construction average costs will be significantly lower than those experienced in renovation activities (see figure 3.2).

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[1]R.L. Peurifoy, Estimating Construction Costs, McGraw-Hill Book Co., New York, 1958.

[2]Robert E. Chapman and Joseph G. Kowalski, Guidelines for Cost-Effective Lead Paint Abatement, National Bureau of Standards, Technical Note 971, January 1979.

[3]Recall that direct costs do not include a markup for the contractor's overhead and profit.

[4]A negative relationship between job size and average direct cost means that as the job size increased average direct cost decreased.

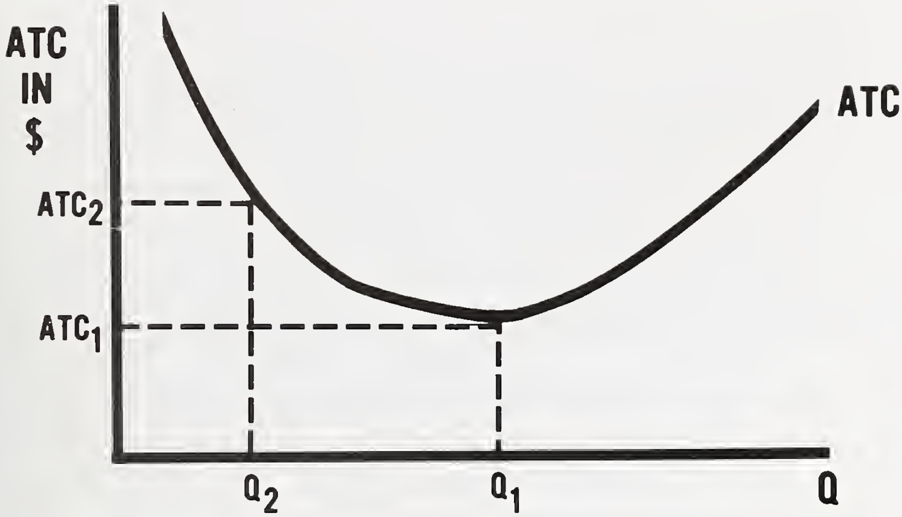

Figure 3.1 Average Total Cost as a Function of Job Size

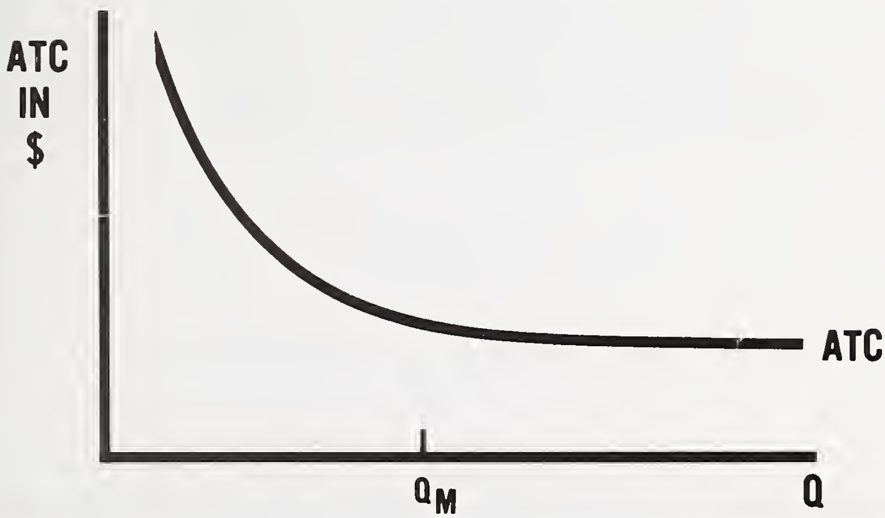

Figure 3.2 An Average Total Cost Curve With a Constant Cost Per Unit Segment

The "Vertical" Impact of Job Size

Although errors may occur at the horizontal level of detail in a construction cost estimate, some cancellation may in fact occur (i.e., positive errors may be compensated for by negative errors). If for any particular task the direction and size of error is random, and if each subitem comprises a small proportion of total project costs, then violations of the constant cost assumptions will likely be canceling in effect because of the cumulative nature of cost estimation.[1] Since a cost estimate for a new construction job is quite detailed in the vertical direction (hundreds of subtasks may be involved), the statistical laws involved in the summation approach will tend to promote a minimization of the aggregated error. Thus, there is a built-in safeguard against large errors of estimation for the job as a whole which is due to the additive nature of cost estimation. Again job size is important, but now in the vertical sense.[2] Here increasing job size will be associated with an increasing number of estimates for the required subtasks. Since job sizes are smaller in housing renovation activities, the potential number of subitems will also be smaller. This means that the chances for the estimation error of any one subtask to cancel an error in another subtask is reduced. This implies that estimating the costs of housing renovation may be inherently less accurate than for new construction activities. Unfortunately, it is not clear how many subtasks are needed in order to achieve a satisfactory, much less optimal level, of protection against estimation errors.

3.2 TREATMENT OF PRODUCTIVITY

The data reported in cost guides can be adjusted relatively easily for differences in wage rates and material prices (see equation 2.3). As we have seen in the earlier parts of this paper differences in average costs because of differences in job size may be less amenable to judgmental modification in transfering new construction cost experiences to housing renovation applications.

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[1]The statistical laws of large numbers provide a theoretical rationale for this claim.

[2]Harrison D. Weed in his report, A Study of Variability of Construction Cost Estimates, U.S. Army Armament Command, Rock Island, Illinois, July 1976, found that as job size increases, the coefficient of variation of bids submitted fell. (See page 11 of the above cited report.) His finding is consistent with the discussion above.

Inherent in the arithmetic of cost accounting equations (equations 2.1 through 2.3) was that job size will be reflected in changing values of the productivity figures associated with the average labor costs. Job size, however, is not the only factor which affects productivity.[1]

Repair, remodeling, and renovation contracting is conducted in a very different construction environment than new building construction. Productivity figures for renovation activities (beyond the modification due to job size differences) may differ from productivity in new construction because of factors completely foreign to typical new construction activity. Several factors readily suggest themselves.

First, productivity of labor will undoubtedly be affected by the occupancy status of the dwelling unit. The productivity of labor will be affected by the accessability of building elements requiring treatment. Such access may be significantly reduced due to interference from the occupants.

It seems likely that productivity will be affected by the size and composition of the crew carrying out the work; the construction practices which govern how these workers carry out their tasks; and the relative skill levels of the labor force engaged in housing renovation. Whether labor is union or non-union may have a consistent impact on average productivity. It shouldn't be assumed that union labor is more productive in the renovation environment. The contrary may be true because renovation activities may best be served by labor with breadth of skills rather than depth of skills. For example, one writer argues that:

> "More skill is required for remodeling work than for new construction. Most people working on new homes are highly specialized; one carpenter will do nothing but hang doors, while another installs trim and another lays floors. This kind of specialization is not practicable in remodeling, and each worker must have a variety of skills."[2]

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[1]Past empirical studies indicate that technical as well as economic considerations are of crucial importance in estimating the level of productivity. See Robert E. Chapman and Joseph G. Kowalski; Guidelines for Cost-Effective Lead Paint Abatement, National Bureau of Standards, Technical Note 971, January 1979.

[2]R.M. Burns, How to Buy and Fix Up an Old House, Home-Tech Publication, Bethesda, Maryland, 1976.

If higher paid union workers are proportionately more productive than non-union workers, the total wage bill may be lower under union than non-union contracts. The fact that union workers are accepting reduced scales in order to get major rehab jobs in several large urban areas, however, seems to indicate that the total wage bill is higher for union contracts.[1,2]

Another factor affecting labor productivity will be reflected in differences, if any exist, in the amount of mechanical equipment and specialized tools that workers have available to them to work with. Since housing renovation contractors tend to be small relative to new construction contractors and subcontractors, it seems reasonable to assume that less "capital" equipment will be available to their workers.

One other important factor bearing on productivity is managerial skill. Given that new construction contractors often have fairly sophisticated management systems and techniques, it implies that their average productivities will be higher than in the area of housing renovation where the physical process is more complicated and less expertise is available.

Although none of the above factors have been explicitly quantified in terms of their impacts on the levels of productivity in housing renovation, they do suggest that differences in the working environment are large enough to cast doubt on the applicability of new construction productivity figures for renovation activity.

Another question of crucial importance is whether the production function for new (housing) construction is the same as for housing renovation. Since the technical options open to a renovation contractor are much more constrained than for a new housing contractor, it is likely that the physical process will exhibit systematic differences in the utlization of labor and materials.[3] The levels of productivity are therefore likely to differ significantly. Past empirical studies[4] have shown that production functions for different classes of construction activities may be significantly

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[1]"Unions and Contractors Try to Get More Rehab Work," Engineering News Record, July 22, 1976.

[2]"HUD Launches a New Rehab Program," Engineering News Record, November 25, 1976.

[3]The mathematical concepts underlying this statement are based on the Le Chatelier principle. For an indepth discussion of the Le Chatelier principle the interested reader is referred to Eugene Silberberg, The Structure of Economics: A Mathematical Analysis, McGraw-Hill Book Company, New York, 1978.

[4]For an authortative source, see John S. McConnaughey, Jr., Production Functions in Contract Construction For the United States, 1972 (unpublished) Ph.D. dissertation, Michigan State University, 1976.

different. This would imply that marked differences in the marginal physical product of the factors of production (labor, materials and equipment) would quite likely exist.

3.3 MARKUP AND ITS DETERMINANTS

The average material cost and average labor cost for each subtask must be multiplied by a markup factor to arrive at an average total cost estimate for that subtask. By definition, the markup figure is equal to the sum of the total costs for a job divided by the variable costs for that job. By definition, average total cost is equal to average variable cost plus average fixed cost. Fixed costs are composed of general overhead, project specific overhead,[1] and a target profit figure. General overhead represents those costs which represent payments for general office expense, rental for space, payments for such services as telephones and advertising, and the portion of the salaries of management which is not assignable to any specific job. Project specific overhead is a cost which would not be incurred if the project were not undertaken but which can not be assigned to, or tied to, any particular subtask's output level. Profit, although included for cost estimation purposes as a targeted dollar amount, is in reality the residual which remains after total costs have been subtracted from the gross revenue of the project.

In practice, the estimating manuals and guidebooks treat the markup factor as a constant across the "horizontal" items of the cost estimate. Such a practice will be generally accurate when a firm is involved in a project which is large enough so that it is producing in the constant cost range (see figure 3.2) of its average total cost curve. If a particular job is small, i.e., if the firm is operating at output levels which do not demonstrate constant costs, the markup factor will be inversely related to the job size. This is because it is easier to lose money on a small job than a big one due both to unanticipated technical problems and relatively higher average costs (see figure 3.2 where it is assumed that renovation activities are carried out at a level below Q_M). In addition, markup factors will be higher for smaller sized jobs because fixed costs are spread over a smaller base. Both of these effects suggest that housing renovation markup rates should be higher than new (housing) construction rates. One further point is that we would expect markup rates for the successful firm to demonstrate greater variability across jobs. This is due both to variations in job size and in the contractor's assessment of the building's condition.

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[1] In an accounting sense, project specific overhead is often a horizontal or line item in a cost estimate.

The above, although quite speculative and requiring empirical confirmation, suggests that new (housing) construction markup factors are not transferable to housing renovation. And that doing so could lead to gross underestimates of the true costs of renovating the unit.

3.4 ASSESSMENT OF RELIABILITY

The primary difficulty with (subtask) cost per unit estimates based on "expert" opinion is that the user of such estimates, if not an "expert," may improperly apply those estimates. Experts in subfields of construction will have some shared scenarios in mind when referring to a subtask like removing and replacing a sink in the same location of a 40 year old house. The steps involved in such a task will be well understood by such experts. Thus, variations between the actual case in hand and a reported case in a cost guide may be clear to an expert cost estimator in housing renovation construction. However, the cost guides cannot detail the scenarios which lie behind any particular subtask figure. Space considerations alone make such detailing impossible.

The irony in all this is that cost figures reported in cost guides are most usable, from the viewpoint of accuracy, by those who need to refer to them the least. This implies that data in cost guides become less and less reliable as the user becomes less and less expert. (Incorrect estimates based on guidebook data can be explained away by asserting that the guidebook user did not correctly modify the reported figures.) Thus, a homeowner (the potential guide user most in need of assistance) attempting to use a repair and alteration guide will be least capable of judgmentally modifying the reported figures.

If the data were statistically derived it would be possible to define an interval estimate which could be believed with a specified degree of confidence. Data of this type, if reported, would enable a user to estimate a range which would probably contain the expected costs. However, data of this kind is considerably more expensive to collect. For example, the repair and alteration manuals break out figures for over 500 separate items. If the data were to be statistically based, samples of at least 20 observations would probably have to be collected for each subitem for each major region of rehab activity. This means that at least 10,000 separate figures would have to be collected and processed for each region. Such data collecting, processing and evaluating requirements are probably beyond the present size of the market for this kind of information.

Cost estimation is a form of prediction. Reliability refers to the accuracy of predictions; more intuitively, the difference between predicted costs and actual costs. Esoteric discussions exist on how to evaluate predictive methods. Unfortunately, there is no basis for assessing, in these terms, the reliability of the data contained in cost manuals.[1]

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[1]The above statement applies with equal force to both the new construction manuals and the repair and alteration manuals.

In order to test the reliability of the figures in any manual, a systematic matching or comparison of subtask predictions and actual experiences would have to be undertaken. Also total job estimates and actual figures would have to be compared. Errors of prediction could be measured, and explicitly quantified statements could then be made about the accuracy of the cost estimates based on guidebook data.

A mechanism does not exist for such an undertaking for a variety of reasons. Subcontractors and contractors usually do not rely on guides for their final estimates. They use them primarily as references. Architects and engineers may use such reference data in preliminary stages of design work but have little incentive to compare preliminary estimates with actual figures. Government contracting officers can only compare their predicted bids (when based on guidebook data) with actual bids. Such a comparison is not between predicted costs and actual costs, but between predicted costs and predicted costs (to the extent that a bid price represents a contractor's own prediction).

The above discussion implies that given currently available information it is impossible to assess the reliability of cost figures reported in such manuals. Statistically derived data at least is conceptually assessable. Based on existing sample data, standard errors for each factor in the cost estimation "equation" could be computed as well as standard errors of prediction and confidence intervals.[1] Judgmentally derived estimates may be of such a nature that in theory there may be no way of demonstrating that they could be wrong.

In order to illustrate the notion of reliability we can compare cost data given in an NBS report[2] on the actual cost of refinishing interior walls to cost estimates derived from repair and alteration manuals. Per unit material and labor costs and their sum, per unit direct costs, are reported in the NBS report for gypsum wallboard and plaster and metal lath. It is important to point out that these costs were actually experienced by contractors for surface refinishing activities in a number of separate dwelling units. Twelve separate observations are reported in the NBS report on the costs of installing and painting gypsum wallboard on interior walls. Nine separate observations of the costs of plaster and lath are also observed. Three series of estimates are derived from three different cost manuals. They are the per unit direct costs of: (1) labor; (2) materials; and (3)

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[1]Those readers who are interested in the mechanics of "fitting" confidence intervals about the estimated cost for a particular subtask are referred to Robert E. Chapman and Joseph G. Kowalski, Lead Paint Abatement Costs: Some Technical and Theoretical Considerations, National Bureau of Standards, Technical Note 979, February 1979.

[2]Robert E. Chapman, Economic Analysis of Experiment Lead Paint Abatement Methods: Phase I, National Bureau of Standards, Technical Note 922, September 1976.

the total. A comparison of the observed average direct cost experiences with the cost estimates permits us to compute a percentage error of prediction for each series of manual estimates. Table 3.2 contains the results of this exercise.

Upon examination of table 3.2 it can be seen that the manuals underestimate both labor and material costs. In general, however, the underestimates for labor costs are more serious than those for material costs. The dollar errors, the difference between actual and estimated costs, range from six cents to 42 cents per square foot. For even a modest job, say refinishing a single room (approximately 500 square feet of wall and ceiling area) these errors could exceed $200. In terms of percentage errors, the amount actual costs exceeded estimated costs, the manuals ranged from 4.8 percent to 80.3 percent. It may come as a surprise that the widely used technique of installing gypsum wallboard had relatively higher percentage errors than for plaster with metal lath.

Data similar to that illustrated in table 3.2 would be needed on a subtask by subtask basis in order to assess the overall reliability of the figures reported in the cost manuals. To do this adequately would require a comprehensive research project. In the absence of such a research project, a quantitative assessment of the reliability of the data in the cost manuals cannot be made.

Two comments on the data in table 3.2 are worth reiterating. First, the direct cost estimates are in all cases underestimates of the actual average experiences reported in table 3.2. Although it would not be appropriate to generalize from two small subtasks to the population of subtasks as a whole, it is of interest that the examples chosen do underestimate housing renovation subtask costs. This is consistent with the observations in sections 3.1 and 3.2 of this paper. Second, note that the repair and alteration manual's estimates are not appreciably better than the new construction manual's estimates.

3.5 Treatment of Risk

Although it was possible to give only a qualitative assessment of the reliability of the data in the cost manuals, it is possible to discuss in more detail the means through which risk can be treated in these manuals. As in the previous section, the discussion will be qualitative rather than quantitative.

There are three basic methods of treating risk in construction cost estimation. These three methods are: (1) judgmental modifications; (2) defining a confidence interval about the estimate; and (3) probabilistic (monte carlo simulation) concepts. Each method will be treated in turn. In the discussion which follows, unless explicitly stated, it will be assumed that the cost estimating technique being discussed is based on average total costs.

Table 3.2 A Comparison of Estimated and Actual Direct Costs for Two Surface Refinishing Tasks[a]

MANUAL Interior Wall Covering	Estimated Direct Cost Per Square Foot			Actual[b] Direct Cost Per Square Foot			$ ERROR	PERCENTAGE ERROR
	Labor	Materials	Total	Labor	Materials	Total		
Building Construction Cost Data								
1. Gypsum Wallboard	0.38	0.17	0.55	0.72	0.20	0.92	0.37	+67.3
2. Gypsum Plaster and Metal Lath	0.82	0.24	1.06	0.92	0.40	1.31	0.25	+23.5
National Repair and Remodeling Estimator								
1. Gypsum Wallboard	0.34	0.17	0.51	0.72	0.20	0.92	0.41	+80.3
2. Gypsum Plaster and Metal Lath	0.45	0.33	0.78	0.92	0.40	1.31	0.42	+67.9
Home Tech Estimator								
1. Gypsum Wallboard	0.36	0.15	0.51	0.72	0.20	0.92	0.41	+80.3
2. Gypsum Plaster and Metal Lath[c]	0.87	0.38	1.25	0.92	0.40	1.31	0.06	+ 4.8

[a]All estimates were based on figures from the 1975 editions of the respective guidebooks. This is because the renovation activities presented in the NBS report were carried out in 1975.

[b]Source: Robert E. Chapman, Economic Analysis of Experimental Lead Paint Abatement Methods: Phase I, National Bureau of Standards Technical Note 922, September 1976.

[c]This manual only reports an estimate for three coats (two brown and one finish). The actual data is based on two coats (one brown and one finish). The two other estimates are for two coats.

Judgmental Mofifications

By far the most popular method of risk asessment is to make use of judgmental modifications. Unfortunately, the use of such a method can only be viewed as a placebo since no a priori grounds can be established which would enable the user to associate percentage changes in risk due to a particular judgmental modification. In reality, judgmental modifications are more a means for adjusting the cost estimate to reflect a change in some technical attribute rather than a measure of the interaction of technical attributes and economic forces in the market place. Thus, experts applying judgmental modification methods are, in a sense, superimposing a parametric cost estimating procedure on top of an average total cost procedure. Although this concept may seem appropriate, it is important to point out that parametric or regression based techniques obey certain rules of the real numbers. This is because most cost estimating relationships are fitted using a continuum of data. An expert who modifies the average total cost procedure, however, is usually only capable of establishing a relative order or rank among different scenarios. Such an approach makes use of what is known as ordinal level data.[1]

Unfortunately, ordinal level data need not obey all the rules of the real number system which a regression based procedure obeys by construction. Since the estimator of the rehab project is concerned with the absolute rather than the relative cost of the project, it does not appear that any level of significance can be attached to the "reduction in risk" associated with the judgmental modification method.

Defining a Confidence Interval About the Estimate

The second method for treating risk involves fitting a confidence interval about the estimate. In section 2.2 where the mechanics of fitting a confidence interval were discussed, it was shown that an estimate of the standard deviation of the predicted value must be known in order to define the upper and lower limits on the confidence interval. Since none of the guidebooks cited in table 2.1 provide measures of variability associated with their average total cost estimates, it is not possible to fit a

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[1] Statisticians have defined four levels of measurement in data. The first level, nominal, makes no assumption about the values being assigned to the data. The second level, ordinal, assumes that it is possible to rank-order all categories but that any numeric values assigned to the categories does not imply that any other properties of the real numbers follow. The third level, interval, implies that the distance between categories are defined in terms of fixed and equal units. The fourth level, ratio, implies that a zero point is inherently defined by the measurement scheme. In the context of the previous discussion, parametric procedures are usually estimated at the interval or ratio levels. For an excellent discussion on the levels of measurement see Norman H. Nie, C. Hadlai Hull, Jean G. Jenkins, Karin Steinbrenner, and Dale Bent, SPSS: Statistical Package for the Social Sciences, Second Edition, McGraw-Hill Book Company, New York, 1975.

confidence interval about the estimate. Even if the guidebooks published such information, there is some question about the effects of aggregation on the variance of the sample. This criticism stems from the fact that observations are drawn from cities across the nation. Consequently, the structure of costs in St. Louis might so differ from Boston that the variances about the means would not be homogeneous. In this case, pooling information about the variance of the sample would not be justified. The thrust of this criticism is that in order to fit confidence intervals about the average total cost figures in the guidebooks, it would be necessary to have sample variances for each city.

Although the use of confidence intervals helps in assessing the impacts of risk on the rehab decision, they are lacking in some respects. For example, a 95 percent confidence interval implies that if samples were taken over and over under identical circumstances from the same population, then 95 out of 100 intervals would contain the true mean of the population. If the sample is not random or the desired estimate is for an atypical case, then the estimate of the mean may be biased. This implies that the concept of a confidence interval loses some of its meaning. The previous statement is reinforced by the fact that no discussions of sampling and nonsampling errors are given in the guidebooks. Consequently, it is not possible to quantify the bias, if any, associated with these average total cost estimates. Under these circumstances fitting a confidence interval must be viewed as a rather academic exercise.

Closely related to the concept of a confidence interval is a technique known as sensitivity analysis. This technique derives its name from measuring the cost sensitivity to a change in one or more factors involved in the process. That is, it permits one to determine how "sensitive" the average total cost estimate is to a change in one or more of the key factors. Sensitivity analysis is extremely difficult to apply to average total cost procedures due to the aggregation of all key factors affecting average total cost. Parametric procedures are quite easily adapted to sensitivity analysis, however. More succinctly, the sensitivity of the estimate to a change in a factor may be defined by simply differentiating the cost estimation equation. In the simplest case, where the relationship is linear and cost depends on only two factors, we see that

$$C = \beta_o + \beta_1 X_1 + \beta_2 X_2$$

which upon differentiation (partially) with respect to X_1 reduces to

$$\partial C = \beta_1 (\partial X_1)$$

That is, the change in cost, ΔC, associated with a change in X_1, ΔX_1, is given as

$$\Delta C = \beta_1 \; (\Delta X_1)$$

If both factors were likely to change, it would be necessary to compute the total derivative, which is given as (using the Δ notation)

$$\Delta C = \beta_1(\Delta X_1) + \beta_2(\Delta X_2)$$

The change in C would then be computed by plugging in the ΔX's. Delta C would then be added to the estimate of C. A similar equation can be written for a linear relationship containing n factors. Now if due to the riskiness of the process, one felt ΔX_1, would be -1 with a probability of 0.1 and +5 with a probability of 0.9, the expected value of ΔX_1, $\overline{\Delta X}_1$, would be

$$\overline{\Delta X}_1 = (0.1)(-1) + (0.9)(5) = -0.1 + 4.5 = 4.4$$

If probabilities (either objective or subjective) for the other X's could be computed, then the expected change in average total cost due to the riskiness of the process would be

$$\Delta C = \beta_1(\overline{\Delta X}_1) + \beta_2(\overline{\Delta X}_2)$$

or in general terms

$$\Delta C = \sum_{i=1}^{n} \beta_i(\overline{\Delta X}_i) \qquad 3.1$$

As in the previous case, ΔC would then be added to the estimate of C to determine if a revision of the cost estimate is needed.

In the previous example where the relationship was linear, the rate of change of the cost estimate was independent of the level of any given factor. Furthermore, by virtue of linearity, a change in one factor had no effect on any other factor. That is, all factors are assumed to be independent and to cause no interaction. In some cases these assumptions may not be justified. Based on the information presented in section 2.4, it can readily be seen that cost functions do not suffer from these problems.[1] This is one of the many reasons why the cost function approach will be stressed in chapter 4. Another reason is the relative ease with which probabilistic methods can be introduced into its framework.

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[1]Following the notation of equation 3.1, C may be expressed as

$$\Delta C = \sum_{i=1}^{n} (\partial C/\partial X_i)(\overline{\Delta X}_i)$$

Probalilistic (Monte Carlo Simulation) Concepts[1]

The third and most comprehensive method for treating risk draws upon probabilistic concepts. Probabilistic concepts are the most appropriate for treating risk because they allow the estimator to "fit" a distribution about each factor and then run through a whole series of "what if" questions. Although it is possible to manually fit a confidence interval about an average total cost estimate, most probabilistic methods rely on computer software for support. The major advantages of probabilistic methods are: (1) they do explicitly treat risk, and (2) they do not suffer from some of the criticisms voiced about confidence intervals. In particular, if one has strong *a priori* beliefs about the structure of rehab costs, it is still possible to apply probabilistic methods even if the average total cost estimate in the guide is biased. More pragmatically, it is possible to adjust for the bias through a judicious choice of a distribution so that the behavior of the distribution parallels the critical steps in the process.

As was shown in section 2.3, probabilistic methods can be applied to both average cost and parametric procedures. If one wishes to use probabilistic methods in conjunction with a parametric procedure, however, it would be necessary to fit a probability distribution about each factor rather than just about the average total cost estimate. It is fairly easy to see that this approach is superior to the average total cost approach since costs are "sensitive" to a change in any factor.[2] The "degree" of sensitivity is, however, dependent upon the relative weight of that factor in determining the renovation cost for that task. In the case where the relationship is linear, the application of probabilistic methods is rather simple and straightforward. The computer software package will first perform a monte carlo simulation on the cost estimating equation (relationship). As mentioned in section 2.3, this process will be repeated until a cost profile for the task results (see figure 2.5 in section 2.3). The user can then choose a particular level of risk which he is willing to accept. In order to compute a confidence interval about the desired level, the user must replicate the simulation a certain number of times (the number of replications is dependent on the desired "tightness" of the confidence interval).

Probabilistic methods may also be applied to cost functions. To properly apply these methods to a cost function is relatively more difficult, however. This can be seen by noting that the basic form of the cost function, as given by equation 2.10, was shown to be nonlinear. From the discussion in

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[1]Many of the major advantages associated with the use of probabilistic methods have already been discussed in section 2.3. Consequently, this discussion will focus on how these methods would be used in conjunction with procedures based on average total costs, parametric relationships, and cost functions.

[2]If costs are not sensitive to a particular factor, then the estimate of its coefficient is probably not statistically significant. In such an event, that factor could probably be eliminated from the cost estimation equation.

section 2.4, equation 2.10 is known to be log linear. That is, if one takes the natural logarithm of equation 2.10, the resulting equation is linear in the logarithms of the factors. In order to determine the appropriate distribution, it will be necessary for the user to fit a probability distribution to the logarithms of each key factor. Guidelines for doing this were provided in section 2.3. Monte carlo simulations can then be performed just as in the previous cases. Probabilistic methods can also be used to perform a type of sensitivity analysis. The type of sensitivity analysis may be either qualitative, in the sense that it is based solely on judgmental modification, or quantitative. For example, one might not be sure about the true distribution of a particular factor. (Perhaps the Chi-square test accepted two distributions.) The monte carlo simulation could then be repeated under the assumption that the factor was distributed differently. By the same token, uncertainty about the condition of the building can be incorporated by either shifting the entire distribution upward or by requiring the distribution to be more skewed. Such an approach could thus complement expert judgment about the physical process. In any event by using the baseline estimate as a reference point, it will be possible to attach a percentage change in the risk being borne by the investor due to a change in a particular factor or group of factors.

Chapter Summary

The emphasis of this chapter has been on the assessment of per unit cost estimation techniques relating to the impact of changing job size, the treatment of productivity and contractor markup. Two examples were also presented which introduced the importance of assessing the reliability of the model and its ability to explicitly deal with risk. Although each of the four techniques is capable of generating per unit cost estimates, the focus was on the cost index/average total cost technique used in most costruction cost manuals. The inflexibility of this technique was shown to exist across the horizontal measure of job size, the treatment of productivity and contractor markup. Since the vertical measure of job size depends on the number of subtasks, no one technique can be claimed to be best. However, in the event that a technique systematically under or overestimates the cost of the job, increasing the vertical dimension will increase the overall error due to the additive nature of the error term.

Theoretical and empirical evidence was also presented which suggested the technical constraints imposed on rehabilitation activities would lead to systematic differences in the utilization of construction labor and materials which could profoundly affect the productivity of the factor inputs. Qualitative evidence was advanced that showed why the use of markup factors had to be more closely studied and that transferring new construction markup rates to rehabilitation projects was probably not justified. An example, based on actual cost data was presented to show that the reliability of the cost estimates from guidebooks specializing in repair and alteration are probably no better than those for new construction. Both types of guidebooks resulted in substantial underestimates of the actual costs experienced.

Of the three methods for dealing with risk, judgmental modifications, confidence intervals, and probablistic concepts, only the probabilistic approach is capable of assessing the risk being borne by the investor as a function of changes in one or more factors. In addition, this method was revealed to be less sensitive to any biases in the average total cost figures presented in the guidebooks or in technical reports. Probalistic methods were also shown to complement expert judgment since they are flexible enough to allow for judgmental modifications in cost figures due to *a priori* beliefs about the system or process.

Facing page:

The application of the performance concept is a key ingredient in reducing the costs of residential rehabilitation. If this approach were combined with a technically sound cost estimation procedure, the risks of undertaking a particular residential rehabilitation job could be reduced significantly.

4. AN APPROACH FOR DEALING WITH COST VARIABILITY

The focus of this chapter is on combining the advantages of the use of cost functions with the cost reducing potential of the performance concept in developing a means for dealing comprehensively with cost variability. Although the discussion in this chapter will be of a theoretical nature, empirical results will be presented which support the claim that this method of solution will permit substantial cost reductions in some types of rehab work.

4.1 COST FUNCTIONS AND THE PERFORMANCE CONCEPT: AN OVERVIEW

In section 2.4 it was shown that the use of cost functions permitted the requirements for a sound cost engineering approach to be integrated with traditional economic theory. More specifically, the duality relationship between cost and production functions permitted:

(1) local market conditions; and

(2) technical considerations of the renovation process

to register their impact on residential rehabilitation costs.

In section 3.5 it was shown that the use of probabilistic methods permitted the effects of risks to be explicitly introduced into the estimation of rehab costs. When these advantages are combined with the cost reducing potential of the performance concept, an approach which reduces both cost and cost variability begins to emerge.

There are two major advantages of the performance concept which make it highly desirable for application to the rehab process:

(1) its cost reducing potential; and

(2) the fact that it provides decisionmakers with greater latitude in making choices.

The major disadvantage is that performance is difficult to measure unless it can be tied to some prescriptive solution. In order to get around this obstacle, previous researchers have focused on the development of a hybrid system known as an equivalence methodology.[1] This system separates the levels of performance into a series of discrete steps, or states, each of which can be tied to a prescriptive solution. Futhermore, the equivalency methodology approach is sufficiently general that it is possible to define a distinct level of performance (or performance score) associated with each code or code attribute under consideration. Thus, the use of an equivalent methodology:

(1) provides a mechanism for demonstrating code compliance; and

(2) promotes greater latitude in making choices by permitting substitutions among building components.

It is the second reason which lends support to the claim that this approach will reduce cost variability. (A detailed discussion of the cost reducing potential of this system is given in section 4.2.) This relationship may be seen more clearly by noting that greater freedom in making retrofit

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[1]Harold E. Nelson and A. J. Shibe, A System for Fire Safety Evaluation of Health Care Facilities, National Bureau of Standards, NBSIR 78-1555, November 1978.

choices will permit the investor to avoid some of those situations which are more risk prone. This would imply that the variance about any estimate where a substitution was made could be reduced. Consequently, the end result would be a reduction in the variance of the budget estimate.

The discussion which follows will be divided into four parts. The first deals with the structure of the equivalence methodology. The second outlines how cost functions and probabilistic methods can be used to develop a cost structure for the problem. The third provides a general mathematical formulation of the problem as well as a graphical means for selecting the least-cost rehabilitation (retrofit) strategy and for defining an alternative class of retrofit strategies. The final part provides guidelines for selecting the best retrofit strategy from those provided by a computerized version of the "equivalency methodology."

STRUCTURE OF THE EQUIVALENCE METHODOLOGY

This subsection will develop a series of tables (matrices) which reveal the basic structure for a generalized equivalency methodology. The discussion is of necessity abstract in order to highlight the flexibility of such a generalized methodology.

Prior to the development of the generalized methodology, however, it is necessary to define several terms and state the assumptions upon which the analysis rests. The following terms will be used throughout the discussion.

(1) Building Component - any portion of the building or a building system for which a prescriptive solution is or can be defined in the code(s) under consideration.

(2) State - a discrete level of performance for a particular building component.

(3) State Value - a numeric score associated with the level of performance of that state.

(4) State Variable - a variable which takes on a value of 1 if the building component is in that state and a value of 0 if it is not.

(5) Code Requirement - a numeric score associated with the level of performance required by each code under consideration.

(6) Score Assessment - a numeric score associated with the level of performance provided within the building for each code under consideration.

(7) Retrofit Cost - the cost of moving from any given or initial state to any other state.

The following assumptions will be made in the discussion which follows:[1]

(1) All code requirements must be satisfied simultaneously in order for the building to be deemed in compliance.

(2) The score assessment is the sum of all state values for those building components which affect the code under consideration.

(3) Each building component can be in one and only one state.

(4) The state of a building component is defined by the worst case condition of that component.

(5) The number of building components is much larger than the number of codes considered.

(6) It is possible to stay in a given or initial state at no cost.

(7) Moving to a lower state is impossible.

As one might suspect, the structure of the generalized equivalency methodology will be rather complex. Fortunately, previous efforts in the area of fire safety have demonstrated that such a methodology can be easily adapted to computer optimization. The specific method which will be used in this exposition is the mathematical programming technique known as linear programming. In its usual context, linear programming deals with the problem of allocating limited resources among competing activities in an optimal way. At the foundation of any linear programming problem is a mathematical model which describes the problem of concern. The term "linear" refers to the requirement that all mathematical functions in the model are linear.[2] The term "program" is used in the general sense in that it refers to a plan rather than a computer program per se. (The basic reason why all mathematical functions involved in the program are linear may be explained through reference to tables 4.1 and 4.2.)

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[1]Assumptions 4 and 7 are not necessary from a computational viewpoint. They are included here to promote a more consistent approach to the engineering problem.

[2]A linear function is defined as a function of the form

$$f(X) = a_o + a_1x_1 + . . . + a_jx_j + . . . + a_nx_n$$

where a_j are coefficients not all zero and the x_j are variables. The geometrical representation of a linear function is a straight line, a plane, or a hyperplane. For example, $f(x) = a + bx$, a straight line, is a linear function where $g(x) = c + dx^2$, a parabola, is not.

Table 4.1 Building Component/State Score Matrix

Building Component	Level of Performance State and Value				
	1	2	3	. . .	m
1	V_{11}	V_{12}	V_{13}	. . .	V_{1m}
2	V_{21}	V_{22}	V_{23}	. . .	V_{2m}
3	V_{31}	V_{32}	V_{33}	. . .	V_{3m}
⋮	⋮	⋮	⋮	⋮	⋮
n	V_{n1}	V_{n2}	V_{n3}	. . .	V_{nm}

Table 4.2 Score Assessment Matrix

Building Component	Code				
	1	2	3	. . .	p
1				. . .	
2				. . .	
3				. . .	
:				. . .	
n				. . .	
Total	$P_1 =$	$P_2 =$	$P_3 =$	. . .	$P_p =$

The foundation of the linear programming model is shown in table 4.1, the Component/State Score Matrix. An examination of table 4.1 reveals that there are n building components and m states associated with each building component.[1] The state value associated with the i^{th} component and j^{th} state is denoted V_{ij}. The state values, V_{ij}, ascend in score as one moves from left to right. That is, $V_{11} < V_{12} < \ldots < V_{1m}$. As a general point of departure, it will be assumed that negative state values represent undesirable circumstances whereas positive state values represent desirable circumstances. A state value of 0 is assumed to be neutral.

Based on assumption 2 and the previous discussion it can be seen that a movement to a higher state represents a potential retrofit.[2]

Thus far, the mechanics of actually calculating the level of performance provided within the building for a particular code have not been discussed. Table 4.2 provides the format for performing these calculations. Notice that table 4.2 is a matrix with n rows, one for each building component, and p columns, one for each code or code attribute under consideration. An examination of table 4.2 would reveal that some cells of the matrix are shaded. This is due to the fact that some building components have no effect on the level of performance required by the code. Thus, to include them in the score assessment for that code would be redundant.[3] In order to perform the score assessment calculation, one would take the state values from table 4.1 and enter them in the appropriate spaces in table 4.2. Once this task has been completed, it is only necessary to sum each column and enter the resultant score in the appropriate space in the row labeled "Total" in table 4.2. Each of these scores are labeled P_1, P_2, . . . P_p, indicating the level of performance provided within the building for each of the p codes under consideration.

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[1] In reality there need not be an equal number of states for each building component. This stronger assumption is used only for purposes of expository clarity. It will be shown in the subsequent discussion that an unequal number of states can easily be accommodated in the mathematical formulation of the problem.

[2] It is of course possible that a movement to a higher state in one building component will affect the performance of another building component. This "row interdependence" can easily be handled through the use of dummy state variables representing dummy states. See exhibit 4.2 in section 4.2 for an example of a "row interdependence."

[3] Rigorously speaking, each cell shaded in table 4.2 is assigned a weight of 0 and hence contributes 0 points to the score assessment for that code, whereas each cell not shaded in table 4.2 is assigned a weight of 1. It is of course possible to have any weight w_{ij}, but for purposes of expository clarity, w_{ij} is assumed to be either 0 or 1.

The code requirements are given in table 4.3. These numeric values are determined by identifying the state in table 4.1 which includes or is the prescriptive solution defined in the code. These values are then entered in the appropriate spaces in table 4.2 and summed in order to get the code requirements, R_1, R_2, ... , R_p. Note that the equivalency methodology is sufficiently general to accomodate variations in the level of performance required by different code jurisdictions.

The final step, determining if the building is in compliance with each code, is outlined in table 4.4. This step is taken by transferring the first set of scores P_1, P_2, ... P_p, in table 4.4, from table 4.2 to the boxes labeled P_1, P_2, ... P_p, in table 4.4. The second set of scores R_1, R_2, ... R_p, is then transferred from table 4.3 to the boxes labeled R_1, R_2, ... R_p. Code equivalence is tested by determining if the differences between the first set of numbers, P_1, P_2, ... P_p, and the second set of numbers, R_1, ... R_p, are <u>all</u> greater than or equal to zero.

In the event that one or more of the differences is negative, the building is deemed not to be in compliance with the code. At this point it becomes necessary to define a plan of correction or, more simply, a retrofit strategy. Although retrofits can be defined based on score improvement alone, such an approach overlooks the potential of the linear programming procedure alluded to earlier.

COST STRUCTURE

Given the basic framework of the equivalency methodology discussed in the previous subsection, it becomes possible to use cost functions and probabilistic methods to develop a cost structure for the problem. The structure of costs is, however, more complicated than that discussed in section 2.4. The nature of these complications is twofold. First, through reference to table 4.1 it can be seen that there are nm (n rows by m columns) possible states.[1] The likelihood of one cost function being able to treat all of these cases is quite remote. Thus, in the discussion which follows, it will be assumed that there are N distinct cost functions, $1 < N \leq nm$. The second complication is of a more subtle nature.

From the discussion in section 2.4 we know that each cost function contains a set of key factors. Since we now have N cost functions, it is likely that some of them will contain factors in common. This implies that greater care must be exercised in applying probabilistic methods. For example, several building components might be associated with certain categories of plumbing system retrofits all of which require a skilled plumber. We shall assume that the same plumbing contractor would perform whichever tasks are selected by the investor and would use the same staff in carrying

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[1]The number of possible retrofits is at most nm-n and, in general, will depend upon the initial state of each factor.

Table 4.3 Code Requirements Matrix

	Code				
	1	2	3	. . .	p
Score	R_1	R_2	R_3	. . .	R_p

Table 4.4 Equivalency Test Matrix

Code	P − R = ■	≥ 0	
		Yes	No
1	$P_1 - R_1 = \square$ □ □		
2	$P_2 - R_2 = \square$ □ □		
:	: : :		
p	$P_p - R_p = \square$ □ □		

them out. Now if each retrofit were treated independently using probabilistic methods, it would be possible to get differing wages from the same plumber doing different tasks. Such a state of affairs does not accurately reflect the way in which construction services are contracted. Fortunately, the solution to the problem is rather simple. To see this, denote the key factors contained in the first cost function as K_1, in the second cost function K_2, and so on. The universe of key factors, $\mathcal{K}$, is thus the union of all key factors in the N cost functions. This relationship may be expressed mathematically as

$$\mathcal{K} = \bigcup_{a=1}^{n} K_a$$

Now if there are $\mathcal{L}$ distinct key factors in $\mathcal{K}$, then we may express K mathematically as

$$\mathcal{K} = \left\{ k_1, k_2, \ldots k_{\mathcal{L}} \right\}$$

As in the discussion in section 2.4, each element of $\mathcal{K}$ has associated with it an estimated mean $\hat{k}$ and a probability distribution.[1]

Once all elements of $\mathcal{K}$ have been identified and the relevant information about their probability distributions have been put into the software package, it becomes possible to generate a meaningful set of cost estimates. The cost estimates desired for this problem are shown in table 4.5. From the table it can be seen that a cost estimate is needed for each building component/state pair. Based on assumption 6, however, we know that it is possible to stay in the initial or given state at zero cost. This implies that costs of staying in the n input states of the nm possible states are 0. From assumption 7 we know that it is impossible to move to a lower state. This may be assured by making the cost of a lower state arbitrarily high. For example, n times the highest single retrofit cost. Thus, although costs for all nm entries of table 4.5 will eventually have to be computed, the only costs which are of importance at this stage are the costs associated with the potential retrofits. Since the goal of this operation is to reduce both cost and variability, probabilistic methods must be used in conjunction with the linear programming procedure.

In order to apply probabilistic methods to the problem at hand, it is first necessary to select a random number for each factor, refer to the probability distribution of that factor, and select the appropriate value of the random variable. Once all $\mathcal{L}$ random variables have been determined, it is necessary to plug them into the appropriate cost function in order to get an estimated cost for each potential retrofit. Just as in the discussion in section 2.4, this process is reiterated until a cost profile for each potential retrofit is defined (see figure 2.5). The next step is for the user to

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[1]In the event that k is a fixed constant, say square feet of floor area, then the probability distribution associated with k is the point distribution.

Table 4.5 Cost Matrix Used in Linear Programming Approach

	RETROFIT COST				
	STATE				
BUILDING COMPONENT	1	2	3	. . .	m
1	C_{11}	C_{12}	C_{13}	. . .	C_{1m}
2	C_{21}	C_{22}	C_{23}	. . .	C_{2m}
3	C_{31}	C_{32}	C_{33}	. . .	C_{3m}
⋮	⋮	⋮	⋮		⋮
n	C_{n1}	C_{n2}	C_{n3}	. . .	C_{nm}

define a risk level, say 20 percent. The software package will then survey each potential retrofit and pick off the retrofit cost which has a 20 percent chance of being overrun. The program would then automatically select the maximum value, multiply it by some constant, say n, and use it as the "arbitrarily high" cost for all retrofits corresponding to a lower state than the initial or given state. The result of this exercise is that table 4.5 is now filled with all the appropriate cost figures. These values can now be used as inputs for the linear programming procedure.

What is particularly attractive about this is that the linear programming algorithm will produce the least-cost solution for achieving compliance to all p codes. However, by construction this solution has only a 20 percent chance of being overrun. Thus, we have obtained the least-cost solution for a given level of risk.

PROBLEM FORMULATION[1]

The subject of the next two subsections is the formulation of the linear programming problem and a discussion of how such a computer algorithm would select the optimal solution and generate a set of alternatives. Certain portions of the discussion which follows are more technical than previous sections. Therefore, the reader who is primarily interested in obtaining a conceptual understanding of linear programming and how it may be applied to the problem of residential rehabilitation is directed to the subsection entitled "Graphical Formulation".

MATHEMATICAL FORMULATION

To formulate the mathematical model (linear program) for this problem, we shall refer to information in table 4.1. Table 4.1 may be thought of as a matrix with n rows, one for each of the n building components, and m columns. The variable X_{ij} is used to identify the state of each building component. By definition the state variable X_{ij} is equal to one if the i^{th} factor is in the j^{th} state and is zero otherwise. (Recall that the score for each factor is determined by the worst case condition.) The variable V_{ij} is the score, state value, associated with the i^{th} building component and j^{th} state (entry in table 4.1). These values are taken directly from table 4.1. For example, if the i^{th} building component is in the j^{th} state the value, or score, associated with the i^{th} row is $V_{ij}X_{ij}$, since X_{ij} is equal to zero for j' unequal to j. Score improvements are thus possible as a result of retrofits. If the i^{th} building component is currently in the j^{th} state, X_{ijc} would be the pre retrofit or current state, jc, and X_{ij} would be the post retrofit or new state. It is of course required

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[1]The discussion of the mathematical and graphical formulation of the problem is a modified and abbreviated version of a writeup prepared by Mr. William G. Hall and which appears in Robert E. Chapman, William G. Hall, and Phillip T. Chen, A Computerized Approach for Identifying Cost-Effective Fire Safety Retrofits in Health Care Facilities, National Bureau of Standards, NBSIR 80-1926, January 1980.

that $j > j_c$ otherwise no improvement has been made. In actual practice (see section 4.2) the score associated with X_{ij}, V_{ij}, would be greater than the score associates with X_{ijc}. It is now possible to show how costs may be introduced in the mathematical model. The variable C_{ij} is used to record the cost of upgrading the i^{th} building component from the input state, j_c, to the j^{th} state. For each possible retrofit being considered, C_{ij} is calculated through use of a cost function.

Keeping in mind that the objective is to match or exceed the scores for each of the p code requirements in the least costly manner, it is now possible to formulate the problem mathematically. The problem is to choose X_{ij}, $j_c^i \leq j \leq j_m^i$, where j_c^i is the current state of the i^{th} factor and j_m^i is the maximal state of factor i^1 so as the minimize total retrofit costs

$$\sum_{i=1}^{n} \sum_{j=j_c^i}^{j_m^i} C_{ij}X_{ij} \qquad X_{ij} = 0 \text{ or } 1$$

subject to

(1) parameter value constraints for each building component given as:

$$\sum_{j=j_c^i}^{j_m^i} X_{ij} = 1 \qquad \text{for } i = 1, \ldots , n$$

(2) total value requirements

$$\sum_{i\varepsilon S_1} \sum_{j=j_c^i}^{j_m^i} V_{ij}X_{ij} - Y_1 = R_1$$

$$\sum_{i\varepsilon S_2} \sum_{j=j_c^i}^{j_m^i} V_{ij}X_{ij} - Y_2 = R_2$$

$$\vdots$$

$$\sum_{i\varepsilon S_p} \sum_{j=j_c^i}^{j_m^i} V_{ij}X_{ij} - Y_p = R_p$$

[1] The superscript on j, j^i, indicates that the maximal state, j_m^i, may vary and hence the total number of states, as a function of the building component under consideration.

where

j = the state index (column of table 4.1),

V_{ij} = the score associated with the i^{th} building component and j^{th} state (entry in table 4.1),

C_{ij} = the total retrofit cost of going from the j_c^{th} state to the j^{th} state, for the i^{th} building component,

$S_1, S_2, \ldots, S_p$ = the set of building components not shaded in table 4.2,

$R_1, R_2, \ldots, R_p$ = the mandatory safety requirements from table 4.3, and

$Y_1, Y_2, \ldots, Y_p$ = nonnegative surplus variables representing a requirement excess.

GRAPHICAL FORMULATION

The mathematical formulation of the problem presented in the previous discussion is of necessity rather terse. It is also possible to show how an optimal combination of retrofit options can be selected via a graphical procedure.

In order to illustrate graphically how such a computerized procedure would operate, it would be necessary to use an example in which it is assumed that the code level is a function of the performance of two techniques, "A" and "B." A graphical solution is the least-cost means of achieving compliance to the code is shown in figure 4.1. In the figure, any movement up the vertical axis indicates higher levels of performance for technique "A." Similarly, any movement along the horizontal axis indicates higher levels of performance for technique "B." Consequently, if one were to construct a straight line which passed through the origin, any movement along that line (i.e., a move in the north-easterly direction) would indicate a movement to higher code levels.

In part A of figure 4.1 the lightly shaded region represents all the combinations of the performance of techniques "A" and "B" which match or exceed the requirements of the code. The lightly shaded region in part A of figure 4.1 is referred to as the feasible region since all points which lie along its boundary or within are technically feasible. The boundary is all those combinations of the two techniques which exactly satisfy the requirements of the code.

In part B of figure 4.1 a series of equal cost lines are shown. An equal cost line shows all the combinations of technique "A" and technique "B" which cost the same. It is drawn based on the assumption that the unit prices for technique "A" and technique "B" are constant. Higher equal cost lines imply greater costs are being incurred. By referring to part B of figure 4.1, it can be seen that the first equal cost line does not touch the shaded region, implying that not enough funds are being allocated

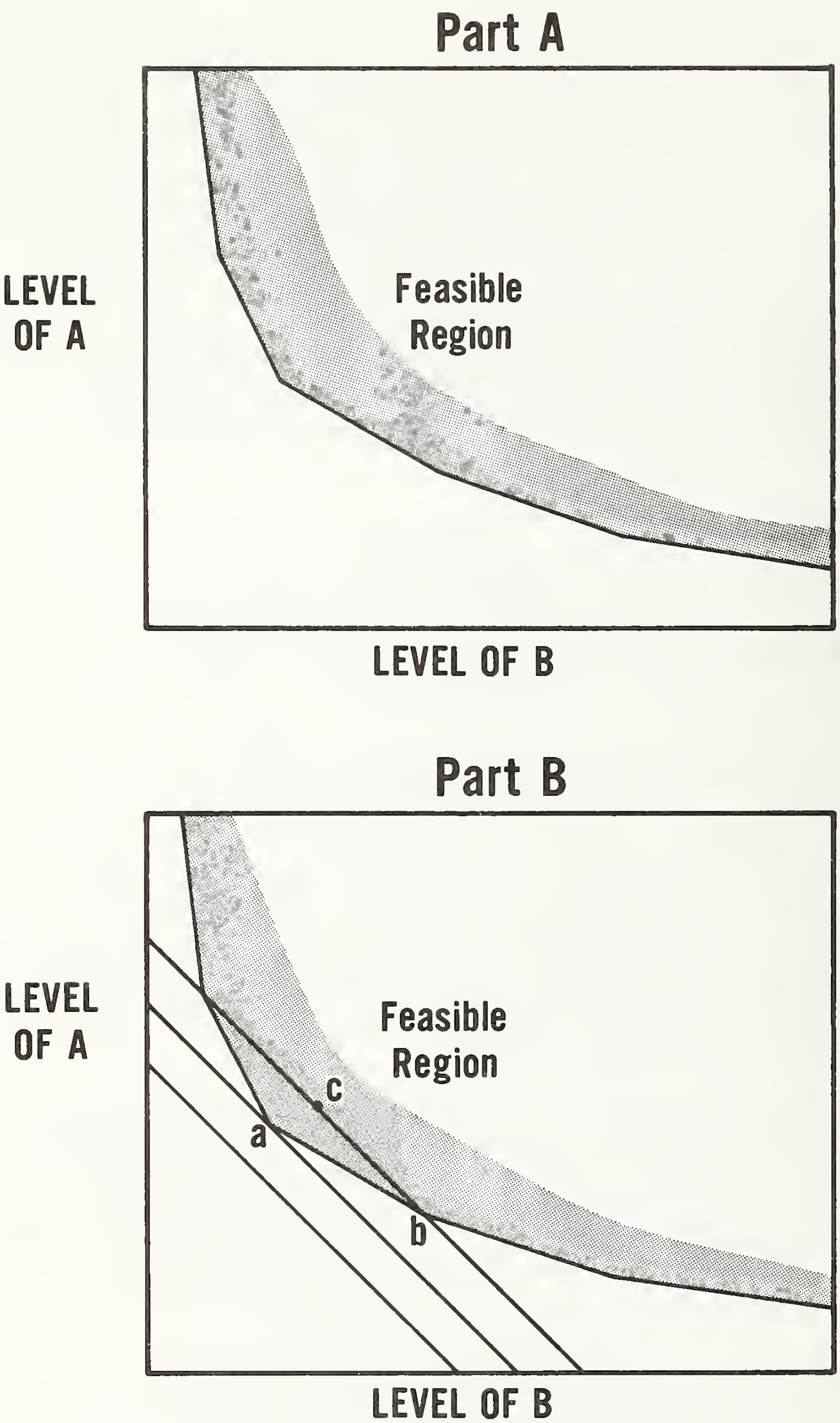

Figure 4.1 Optimal Solution: Graphical Method

to attain the level of "performance" required by the code. The second equal cost curve just touches a vertex of the shaded feasible region. Since no other equal cost line which is lower also touches the feasible region, it can be asserted that the point of contact is the least-cost combination of the two factors satisfying the code. Figure 4.1 also illustrates that if more money were spent it would still be possible to achieve the same level of performance as shown by the vertex of the feasible region labeled "b." However, from the graph it can be seen that it would be more cost effective to use those dollars to achieve a higher level of performance than required by the code. Such a strategy would result in a point lying entirely within the feasible region such as "c."

One way in which alternative solutions may be generated by such a computerized procedure is shown in figure 4.2.

In real world applications, the procedure actually used would probably generate two classes of alternative solutions. Only the first class is illustrated in figure 4.2. The optimal solution derived earlier is denoted by "a" in figure 4.2.

For example, suppose X_{ij} appears in the optimal solution as a retrofit (i.e., $X_{ij} = 1$), then the cost, C_{ij}, of going from X_{ij_c}, the pre retrofit state, to X_{ij}, the post retrofit state is made arbitrarily high. This step guarantees that X_{ij} can no longer be in the optimal solution. Graphically this would correspond to a rotation of the equal cost line. The new solution would thus be the point at which the lowest of the new class of equal cost curves just touches the feasible region. Such a point is designated by "b" in figure 4.2. Exactly analagous is the case where X_{ij} did not appear in the optimal solution (i.e., $X_{ij} = 0$). In this case, the cost of going from X_{ij_c} to X_{ij}, C_{ij}, is held fixed and all C_{ik}, $k \neq j$, are made arbitrarily high. This step guarantees that X_{ij} will appear in the optimal solution as a retrofit. The number of solutions generated in this class is equal to the number of variables in the original linear programming problem minus the number of retrofit states which cannot be attained (those within a row having a lesser value than the input). In the simple case illustrated graphically, up to six alternative solutions could be generated. The second class of alternatives is determined by the rows of the table. For example, suppose X_{ij} appears in the optimal solution as a retrofit, then the cost of going from X_{ij_c} to $X_{ij'}$ for all j', $j_c < j' \leq j^i_m$, where j^i_m is the maximal state in row i, is made arbitrarily high. This step guarantees that no potential retrofits associated with that building component can appear in the optimal solution. The number of solutions generated is equal to the number of building components (rows in table 4.5) which moved to a higher state (were retrofitted) in the optimal solution.

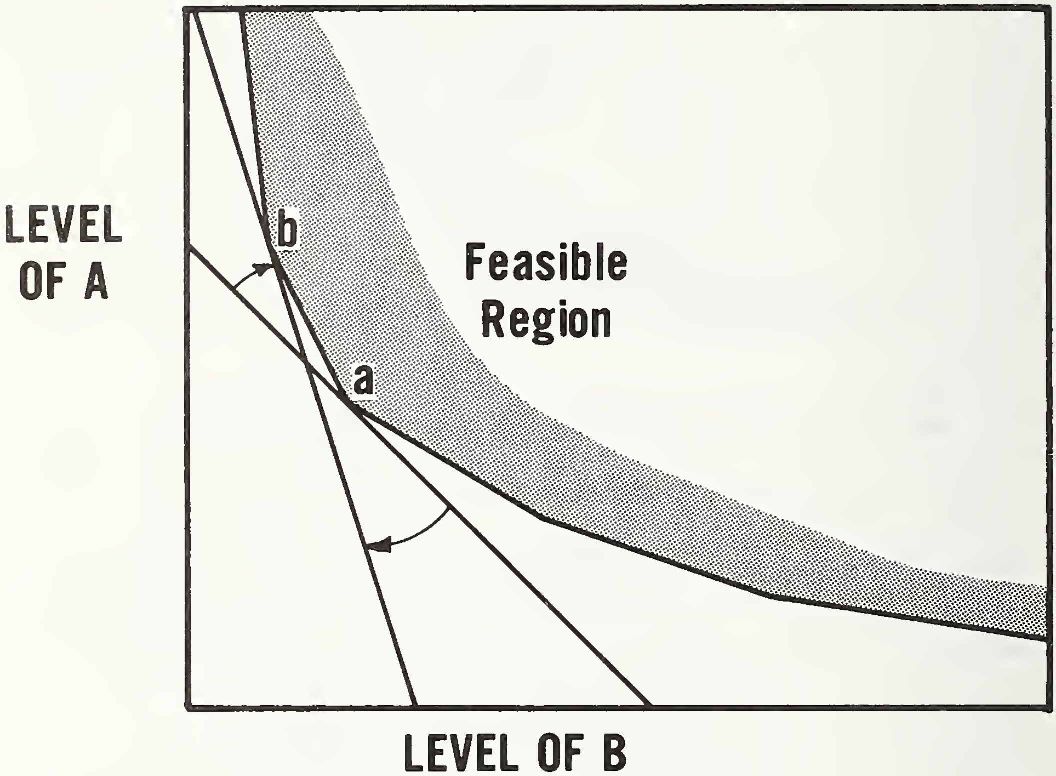

Figure 4.2 Alternative Solution: Graphical Method

The two classes of alternative solutions described above are very interesting because they are "close" to the optimal solution. Close is put in quotation marks because through reference to figure 4.2 it can be seen that the alternative solutions are adjacent vertices of the boundary of the feasible region.[1] Although these solutions will be more costly than the optimal solution, some of them will probably be close (in dollar terms) to the least-cost combination of retrofits.

All of the discussion has proceeded as if the continuous solution to the optimization were actually the integer solution. In general, this is not true; however, there are several strong arguments for the approach used. The first is the "near integer" property of the solutions; the second, the judicious selection of the family of alternative solutions; and the third, the computational cost, efficiency, and reliability of the algorithm. Each of these is discussed in turn.

The near integer property occurs because of the structure of the constraint matrix. A solution will have exactly n + m variables, where n was assumed to be much greater than m, since n constraints corresponding to the building component in table 4.1 have no variables in common and each of the constraints has a right hand side of 1. Therefore, there are n mutually exclusive subsets of the n + m variables which must sum to 1. Since it requires at least two fractions to sum to 1, there must be at least n-m variables which are exactly one; n-m or more integers among the n variables of interest is defined as near integer. Furthermore, this is the worst case since some of the variables may be zero and/or some of the surplus variables (the Y's) may be nonzero. Should either condition occur, there are fewer than n + m variables to be included in the n subsets and more than n-m of the variables may be integer.

Regardless of the integer characteristics of the optimal solution, it represents a lower bound on the total retrofit cost. Should this approach be adopted a heuristic integerization procedure could be applied to each solution generated. This would produce an integer solution by making a minimal change in the basic continuous solution.

An additional reason for advocating the design of the alternate solution family discussed earlier is that any variable appearing as a fraction in the optimal solution must appear in at least one of the alternates as one and be supressed in at least one of the alternates. Another, more pedestrian, criterion is that the alternates be amenable to generation via a systematic but comprehensive specification, that they meet user needs, and that the number of alternates be adequate but not burdensome.

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[1]In n dimensional space, the alternative solutions correspond to the vertices of the polyhedron (feasible region) in the neighborhood of the optimal basis (optimal combination of retrofits).

SELECTING THE "BEST" RETROFIT STRATEGY

The output of the computer program discussed in the previous subsection will include:

(1) the optimal retrofit strategy; and

(2) a series of alternative retrofit strategies.

It is important to point out that all retrofit strategies are ranked on the basis of construction cost only. As such, they do not include any recurring or nonrecurring costs which will take place in the future. Differences in the length of the renovation period are only treated through their effect on construction cost. In reality, longer renovation periods will result in lost revenues or increased housing expenses if the dwelling is to be occupied by the investor. By the same token, computer printouts would not include a measure of aesthetic quality or professional design judgment, unless these attributes were reflected in the retrofit cost of going to a particular state. For these reasons, it is important that the user of the linear programming model carefully review the complete set of retrofit strategies with respect to any of the additional objectives outlined above in order to select the one which is optimal for the case at hand. For example, if one of the attributes under consideration was related to energy performance, one might opt for a higher initial cost in order to increase savings on future energy bills. Similar statements can be made about building aesthetics.

The previous discussion was not intended to leave the reader with a feeling that the final selection was an arbitrary one. On the contrary, different investors have different objectives which are not always reflected in renovation costs. In some cases, the additional constraints placed on the problem by the potential investor are purely subjective, and hence, not amenable to mathematical optimization. In other cases the costs (and benefits) of a particular retrofit strategy can be quantified. It is these cases where a clear-cut economic rationale can be applied. Stated more simply, if future costs (e.g., fire insurance, energy, water) are unlikely to vary across retrofit strategies, then other things being equal, the potential investor would be advised to select the least-cost solution. Any differences in amenities provided by one retrofit strategy over any other can be evaluated by the investor's willingness to pay for that amenity. On the other hand, if future costs are likely to vary across retrofit strategies, then a well proven building investment tool such a life-cycle costing should be used. In this case the user would assess all costs occurring over the investment horizon and select that retrofit strategy which minimizes life-cycle costs. Furthermore, if any differences in amenities exist between the retrofit strategy which minimizes life-cycle costs and any other, the economic viability of that strategy can be assessed by the user's willingness to pay for that amenity or set of amenities.

4.2 ACHIEVING CODE COMPLIANCE IN THE LEAST COSTLY MANNER: A CASE STUDY

This section provides a case illustration of how the performance approach, developed in the previous section, may permit the costs of code compliance to be drastically reduced. It is important to point out that although this case application is concerned with the problem of fire safety in health care facilities, it exhibits many of those attributes discussed in the previous section. More precisely, the methodology presented in this section includes:

(1) the concept of code equivalence; and

(2) the cost reduction potential of the performance approach over that of strict compliance to the code.

Although the methodology is concerned with compliance to only one code, it explicitly identifies four code attributes all of which must be satisfied simultaneously in order for the facility to be deemed "up to code." If each of these attributes is thought of as a code, then the problem becomes quite similar to the one discussed in the previous section.

THE FIRE SAFETY EVALUATION SYSTEM

The Fire Safety Evaluation System is a quantitative evaluation system for grading fire safety in health care facilities. The primary use of the system is to determine how combinations of widely accepted fire safety equipment and building construction features may provide a level of safety equivalent to that required by the 1973 Life Safety Code[1]. Three major concepts basic to the Fire Safety Evaluation System are:[2]

(1) Occupancy Risk: the number of people affected by a given fire, the level of fire they are likely to encounter, and their ability to protect themselves.

(2) Building Safety Features: the ability of the building and its fire protection systems to provide measures of safety commensurate with the risk.

(3) Safety Redundancy: in-depth protection, through the simultaneous use of alternative safety methodologies such as Containment, Extinguishment, and People Movement. The design of the complete fire safety system is intended to ensure that the failure of a single protection device or method will not result in a major failure of the entire system.

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[1] Code for Safety to Life from Fire in Buildings and Structures, National Fire Protection Association, NFPA 101-1973 (Chapter 10: Institutional Occupancies)

[2] Definitions are taken from the report by H.E. Nelson, and A. J. Shibe, A System for Fire Safety Evaluation of Health Care Facilities, National Bureau of Standards NBSIR 78-1555, November 1978.

The concept of safety redundancy in the Fire Safety Evaluation System is directly related to Provision 2-1111 of the 1973 Life Safety Code.

The task of ensuring that the Fire Safety Evaluation System satisfied Provision 2-1111 of the Life Safety Code was the responsibility of a panel of fire safety experts. The goal of the panel was to reach consensus on all relevant fire safety issues. To facilitate this process, a management tool known as the Delphi Method was used. The Delphi Method, as used in developing the Fire Safety Evaluation System, consisted of four steps. These steps are illustrated in figure 4.3. Note that the steps illustrated in the figure form a closed loop. This is because a certain amount of recycling of ideas was needed in order to achieve consensus. The first step in the process was to select a set of key safety factors. These factors were related to the concepts of Occupancy Risk and Building Safety Features mentioned earlier. (For example, Occupancy Risk includes as a factor Patient Mobility whereas Building Safety Features includes as a factor Interior Finishes in the Corridor and Exits.) The second step focused upon the identification of a set of parameters associated with each factor. (For example, Patient Mobility includes as parameters Mobile, Limited Mobility, Not Mobile, and Not Movable. Interior Finishes in the Corridor and Exits includes Class A, Class B, and Class C flame spread ratings as parameters.) The third and most critical step was to assign a weight to each parameter which best reflected the relative degree of risk or safety associated with that parameter. More specifically, negative values reflected greater risks whereas positive values contributed toward a higher level of safety within the fire zone. (The term fire zone is defined as a space separated from all other spaces by floors, horizontal exists, or smoke barriers.) The system treats a value of zero as "safety" neutral. (The values for the three parameters associated with the safety factor Interior Finishes in the Corridor and Exits are: Class C, -5 points; Class B, 0 points; and Class A, 3 points.) In the fourth step, the value of each safety parameter assigned by the panel was reassessed for adequacy and consistency. In the event that the system which emerged from the panel was shown to be inadequate or inconsistent, the entire four-stage process was repeated.

The end result of the panel's work was a series of worksheets which permitted the relative merits of each fire safety parameter to be carefully assessed. These worksheets are presented as exhibits 4.1 through 4.4. In addition, since each of the 13 Building Safety Features has a unique parameter which corresponds to strict compliance, it was possible to compute the score, or level of safety, provided by the Life Safety Code for Extinguishment, Containment and People Movement Safety. These values were then used as a base which any alternative to strict compliance to the Life Safety Code had to match or exceed.

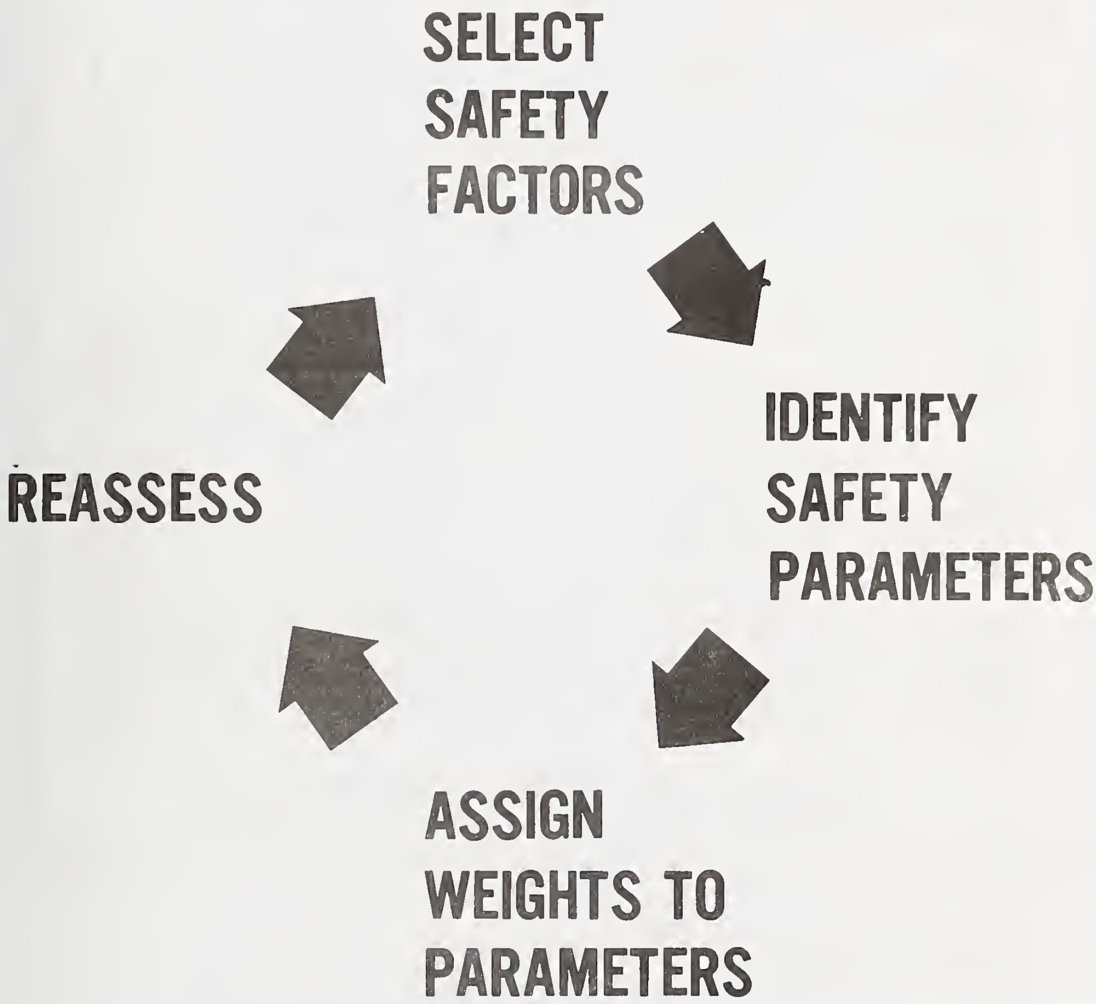

Figure 4.3 Delphi Method

Exhibit 4.1 Occupancy Risk Factor Calculation Worksheet

Table 1. OCCUPANCY RISK PARAMETER FACTORS

RISK PARAMETERS — RISK FACTOR VALUES

1. PATIENT MOBILITY (M)

MOBILITY STATUS	MOBILE	LIMITED MOBILITY	NOT MOBILE	NOT MOVABLE
RISK FACTOR	1.0	1.6	3.2	4.5

2. PATIENT DENSITY (D)

PATIENT	1-5	6-10	11-30	>30
RISK FACTOR	1.0	1.2	1.5	2.0

3. ZONE LOCATION (L)

FLOOR	1ST	2ND OR 3RD	4TH TO 6TH	7TH AND ABOVE	BASE-MENTS
RISK FACTOR	1.1	1.2	1.4	1.6	1.6

4. RATIO OF PATIENTS TO ATTENDANTS (T)

PATIENTS / ATTENDANT	1-2 / 1	3-5 / 1	6-10 / 1	>11 / 1	ONE OR* MORE / NONE
RISK FACTOR	1.0	1.1	1.2	1.5	4.0

5. PATIENT AVERAGE AGE (A)

AGE	UNDER 65 YEARS AND OVER 1 YEAR	65 YEARS & OVER 1 YEAR & YOUNGER
RISK FACTOR	1.0	1.2

* RISK FACTOR OF 4.0 IS CHARGED TO ANY ZONE THAT HOUSES PATIENTS WITHOUT ANY STAFF IN IMMEDIATE ATTENDANCE

Table 2. OCCUPANCY RISK FACTOR CALCULATION

OCCUPANCY RISK M [ ] × D [ ] × L [ ] × T [ ] × A [ ] = F [ ]

Table 3A. (NEW BUILDINGS)

1.0^{F} [ ] = R [ ]

Table 3B. (EXISTING BUILDINGS)

0.5^{F} [ ] = R [ ]

Exhibit 4.2 Worksheet for Determining the Value of the Safety Parameters

Table 4. SAFETY PARAMETERS VALUES

PARAMETERS		PARAMETERS VALUES						
1. CONSTRUCTION		COMBUSTIBLE				NON-COMBUSTIBLE		
		WOOD FRAME		ORDINARY				
FLOOR OF ZONE		UNPROTECTED	PROTECTED	UNPROTECTED	PROTECTED	UNPROTECTED	PROTECTED	FIRE RESIST.
	FIRST	−2	0	−2	0	0	1	2
	SECOND	−7	−2	−7	−2	−7	2	4
	THIRD	−9	−2	−9	−7	−7	2	4
	4TH & ABOVE	−13	−7	−13	−7	−9	−7	4

PARAMETERS					
2. INTERIOR FINISH (Corr. & Exit)	CLASS C	CLASS B	CLASS A		
	−5	0	3		
3. INTERIOR FINISH (Rooms)	CLASS C	CLASS B	CLASS B		
	−3	3	3		
4. CORRIDOR PARTITIONS/WALLS	NONE OR INCOMPLETE	<1/3 HR	>1/3 <1.0 HR	≥1.0 HR.	
	−10 (0)*	0	1 (0)*	2 (0)*	
5. DOORS TO CORRIDOR	NO DOOR	<20 MIN.FR	>20 MIN. FR	>20 MIN. FR & AUTO CLOS.	
	−10	0	1 (0)***	2 (0)***	
6. ZONE DIMENSIONS	DEAD END MORE THAN 100'	DEAD END 30'−100'	NO DEAD ENDS>30' & ZONE LENGTH IS:		
			>150'	100'−150'	<100'
	−6 (0)**	−4 (0)**	−2	0	1
7. VERTICAL OPENINGS	OPEN 4 OR MORE FLOORS	OPEN 2 OR 3 FLOORS	ENCLOSED WITH INDICATED FIRE RESIST.		
			<1 HR.	≥1HR.<2 HR.	≥2 HR.
	−14	−10	0	2 (0)*	3 (0)*
8. HAZARDOUS AREAS	DOUBLE DEFICIENCY		SINGLE DEFICIENCY		NO DEFICIENCIES
	IN ZONE	OUTSIDE ZONE	IN ZONE	IN ADJACENT ZONE	
	−11	−5	−6	−2	0
9. SMOKE CONTROL	NO CONTROL	SMOKE PARTITION	MECH. ASSISTED SYSTEMS		
			BY ZONE	BY CORRIDOR	
	−2 (0)***	0	3	4	
10. EMERGENCY MOVEMENT ROUTES	<2 ROUTES	MULTIPLE ROUTES			
		DEFICIENT CAPACITY	W/O HORIZONTAL EXIT(s)	HORIZONTAL EXIT(s)	DIRECT EXIT(s)
	−8	−2	3	3	5
11. MANUAL FIRE ALARM	NO MANUAL FIRE ALARM		MANUAL FIRE ALARM		
			W/O F.D. CONN.	W/F.D. CONN.	
	−4		1	2	
12. SMOKE DETECTION & ALARM	NONE	CORRIDOR ONLY	ROOMS ONLY	CORRIDOR & HABIT. SPACE	TOTAL SPACE
	0	2	3	4	5
13. AUTOMATIC SPRINKLERS	NONE	CORRIDOR	CORRIDOR & HABIT. SPACE	TOTAL SPACE	
	0	2 (0)**	8	10	

NOTE: *Use (0) when item 5 is −10.
**Use (0) when item 10 is −8.
***Use (0) in zone with less than 31 patients in existing buildings.

*Use (0) when item 1 is based on first floor zone or on an unprotected type of construction.
**Use (0) when item 1 is based on an unprotected type of construction.
***Use (0) when item 4 is −10.

Exhibit 4.3 Worksheet for Calculating Containment Safety, Extinguishment Safety, People Movement Safety, and General Safety

Table 5. **INDIVIDUAL SAFETY EVALUATIONS**

SAFETY PARAMETERS	CONTAINMENT SAFETY (S_1)	EXTINGUISHMENT SAFETY (S_2)	PEOPLE MOVEMENT SAFETY (S_3)	GENERAL SAFETY (S_G)
1. CONSTRUCTION				
2. INTERIOR FINISH (Corr. & Exit)				
3. INTERIOR FINISH (Rooms)				
4. CORRIDOR PARTITIONS/WALLS				
5. DOORS TO CORRIDOR				
6. ZONE DIMENSIONS				
7. VERTICAL OPENINGS				
8. HAZARDOUS AREAS				
9. SMOKE CONTROL				
10. EMERGENCY MOVEMENT ROUTES				
11. MANUAL FIRE ALARM				
12. SMOKE DETECTION & ALARM				
13. AUTOMATIC SPRINKLERS			÷ 2 =	
TOTAL VALUE	S_1 =	S_2 =	S_3 =	S_G =

Exhibit 4.4 Worksheet for Evaluating Fire Zone Safety Equivalency

Table 6. **MANDATORY SAFETY REQUIREMENTS**

	CONTAINMENT S_a		EXTINGUISHMENT S_b		PEOPLE MOVEMENT S_c	
ZONE LOCATION	New	Exist.	New	Exist.	New	Exist.
FIRST FLOOR	9.0	4.0	6.0	3.0	6.0	1.0
ABOVE FIRST FLOOR	14.0	8.0	8.0	5.0	9.0	3.0

Table 7. **ZONE SAFETY EQUIVALENCY EVALUATION**

					YES	NO
CONTAINMENT SAFETY (S_1)	less	MANDATORY CONTAINMENT (S_a)	≥0	S_1 ☐ – S_a ☐ = C ☐		
EXTINGUISHMENT SAFETY (S_2)	less	MANDATORY EXTINGUISHMENT (S_b)	≥0	S_2 ☐ – S_b ☐ = E ☐		
PEOPLE MOVEMENT SAFETY (S_3)	less	MANDATORY PEOPLE MOVEMENT (S_c)	≥0	S_3 ☐ – S_c ☐ = P ☐		
GENERAL SAFETY (S_G)	less	OCCUPANCY RISK (R)	≥0	S_G ☐ – R ☐ = G ☐		

The first worksheet, exhibit 4.1, consists of a brief description of the fire zone and the means for calculating occupancy risk. Occupancy risk is calculated by selecting the appropriate value for each of the five factors shown in table 1 and multiplied together to get an "unadjusted" occupancy risk factor. The resultant is then entered either in table 3A if the building is new, or in table 3B if the building was in existence at the time of the adoption of the 1973 Life Safety Code. The occupancy risk factor for the fire zone is then calculated by taking the product of the weighting factor and the "unadjusted" occupancy risk factor. The occupancy risk factor is then used to establish the minimum level of General Safety which must be provided by the 13 Building Safety Features in order to be deemed in compliance to the Life Safety Code.

The second worksheet, table 4 shown as exhibit 4.2, provides the foundation for the computerized procedure. Table 4 gives the scores associated with the parameters for each of the 13 Building Safety Features. These scores are used in evaluating the fire zone. To evaluate the fire zone it is first necessary to identify the appropriate parameter and score associated with each of the 13 Building Safety Features. The existing state of the fire zone is then defined by recording (circling, checking or marking in some other manner) all of these parameters and their associated scores in table 4. In performing the evaluation, it is important to point out that for each Building Safety Feature which has parameters which have a higher score, those parameters represent potential retrofits. For example, if in the current state the flame spread rating on Interior Finishes in the Corridor and Exits was Class C, then both Class B and Class A flame spread ratings would be potential retrofits. More importantly, by systematically combining improvements in score with anticipated retrofit costs, it becomes possible to establish a means for upgrading the level of fire safety within the fire zone in the most cost-effective manner.

The third worksheet, table 5, is shown in exhibit 4.3. Table 5 provides the means for calculating the score associated with each of the four safety redundancy requirements. (The four safety redundancy requirements are: (1) Extinguishment Safety, (2) Containment Safety, (3) People Movement Safety, and (4) General Safety.) In order to calculate the score for each of the safety redundancy requirements, it is necessary to enter the score associated with the parameter identified in table 4 as corresponding to the existing state of the Building Safety Feature into the appropriate spaces in the coded rows of table 5. (No values are entered in the shaded spaces of table 5.) Each of the four columns is then summed to get an overall score. These scores are labeled S_1, S_2, S_3, and S_G in table 5.

The fourth worksheet provides the means for determining if the fire zone possesses a level of fire safety equivalent to that of the 1973 Life Safety Code. Basically this is done by taking the four scores calculated in table 5 and entering them in the boxes labeled S_1, S_2, S_3, and S_G in table 7. The user then selects the values from table 6 for Containment Safety, Extinguishment Safety, and People Movement Safety for the appropriate building type and fire zone location. These values are entered in the boxes labeled S_a, S_b, and S_c in table 7. The occupancy risk factor calculated on the first worksheet is then entered in the box labeled R. Based on these two sets of numbers it is possible to test if the fire zone provides a level of safety equivalent to the 1973 Life Safety Code. This test is performed by determining if the differences between the first set of numbers, S_1, S_2, S_3, and S_G, and the second set of numbers, S_a, S_b, S_c, and R, in table 7 are greater than or equal to zero.

A COMPUTERIZED VERSION OF THE FIRE SAFETY EVALUATION SYSTEM

In the event that the fire zone fails to pass the equivalency test, it will be necessary to select a retrofit strategy which will ensure that the Building Safety Features produce scores which match or exceed each of the four redundancy requirements. A systematic means for doing this which explicitly introduces relative costs into the retrofit decision is the computerized version of the Fire Safety Evaluation System developed jointly by the Center for Building Technology and the Center for Applied Mathematics at the National Bureau of Standards.[1]

This computerized procedure is particularly attractive since it uses information collected during the fire safety evaluation as its primary input. This information is used not only to define the current state of fire safety in the fire zone, but also to calculate the expected retrofit costs[2] for that fire zone.

Using linear programming, the computerized procedure utilizes information on the current state of the fire zone, the minimum passing "score" needed to achieve compliance, and the anticipated cost of each retrofit measure to identify the least-cost or optimal combination of retrofits. The computerized procedure then systematically analyzes other retrofit combinations to see if alternatives might exist which are close to the one identified as least costly. The least costly combination of retrofits and any alteratives which the program produces, usually between 10 and 20, are then summarized in tabular form so that they can be ranked from least costly to most costly. By using this approach, health care facility decisionmakers have greater flexibility in choosing among retrofit combinations. In particular, by providing alternatives, the decisionmaker has the opportunity to assess the relative costs of the alternative retrofit strategies, to assess the effects of nonconstruction costs, and to match

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[1]Robert E. Chapman, Phillip T. Chen and William G. Hall, Economic Aspects of Fire Safety in Health Care Facilities: Guidelines for Cost-Effective Retrofits, National Bureau of Standards, NBSIR 79-1902, November 1979.

[2]Unless otherwise noted, the term retrofit cost refers to initial cost of a given retrofit measure.

common retrofit packages across the fire zones. The computerized procedure also contains a series of user options which make it possible to alter the cost of any retrofit, preclude a retrofit, force a retrofit to be included, or demand a higher level of safety than required by the Life Safety Code.[1]

CASE APPLICATION

The focus of this section is on the results of the application of the computerized procedure to a "typical" general hospital. This approach was taken in order to introduce as many engineering and architectural design considerations as possible into the analysis. The formulation of the typical hospital design was based on an extensive review of actual hospital layouts and working drawings. The review was also useful in defining an expected level for each of the 13 Building Safety Features.

The design used in the case application was a seven story "T" shaped structure capable of housing approximately 300 patients. The facility also houses an outpatient clinic and an emergency room. The facility, assumed to have been built around 1960, is constructed with structural steel framing protected by a fire resistive concrete covering, reinforced concrete floors, fixed windows and masonry exterior walls. In all cases, but one, the fire zone under examination was the entire floor.[2]

Through reference to floor plans for the hospital, it was possible to establish fire hazard scenarios for each fire zone. All fire hazard scenarios showed the expected level of each Building Safety Feature as well as potential retrofit courses of action should the fire zone be deficient. Figure 4.4 shows the plan for the patient room floors. (The patient room floors are floors three through seven.) Through reference to the figure, one can identify such important factors as the location of the means of egress, hazardous areas and a dead end corridor as well as information on zone dimensions and the expected number of patients being housed. Additional information was then used to establish a series of critical element counts which uniquely defined the scope of the retrofit. They include information on such topics as the number of "No Door" charges which must be removed or the square feet of floor space requiring sprinkler protection in order to move to a higher state. Recall that the level of each Building Safety Feature is determined by the worst case condition within the fire zone. Consequently, there may be several retrofit options possible for a particular Building Safety Feature. (This may be seen by examining exhibit 4.2.)

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[1]Those readers interested in a detailed description of the computerized procedure are referred to the report by R. E. Chapman, W. G. Hall and P. T. Chen, A Computer Approach for Identifying Cost-Effective Retrofits in Health Care Facilities, National Bureau of Standards, NBSIR 80-1929, January 1980.

[2]The first floor, which houses the emergency room, consists of two fire zones.

FLOOR PLAN OF THE PROTOTYPICAL HOSPITAL: THIRD THROUGH SEVENTH FLOORS

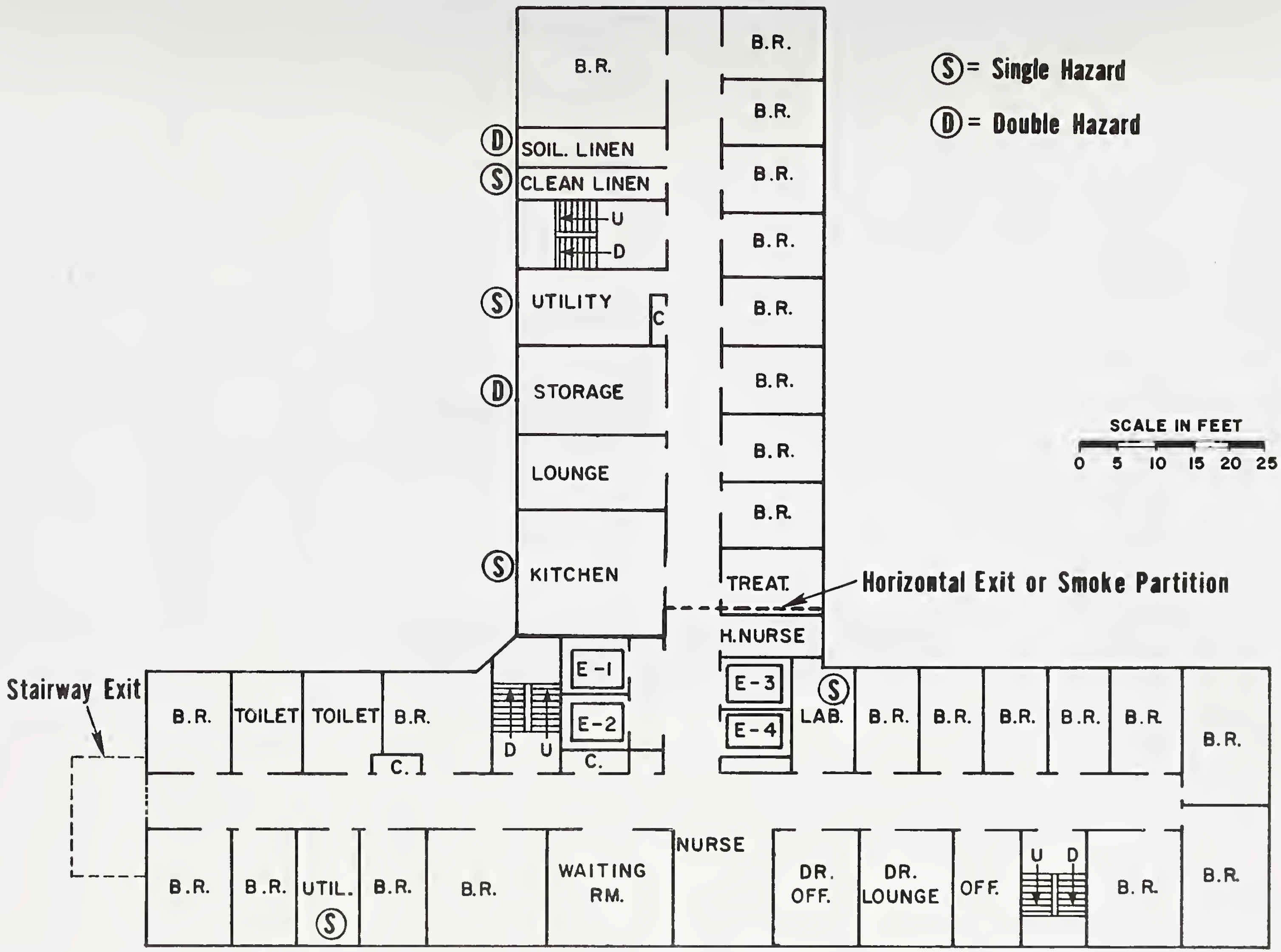

Figure 4.4 Floorplan for the Patient Room Floors

Application of the Fire Safety Evaluation System to the selected hospital design indicated that all fire zones in areas of patient use, with the exception of the fire zone containing the emergency room, would require some degree of renovation in order to meet the requirements of the 1973 Life Safety Code. Computer analyses were then conducted for each floor to identify retrofit options which would satisfy Provision 1-3118 of the Life Safety Code. These analyses produced anywhere from seven to 21 retrofit strategies which were less costly than strict compliance to the prescriptive provisions of the Life Safety Code. In most cases, the cost of the least-cost solution was about half the cost of the prescriptive solution. Since the retrofit strategies mentioned earlier were for a particular floor, it was necessary to use information on all fire zones to synthesize a comprehensive retrofit strategy. In reality, synthesizing a comprehensive retrofit strategy could be a major problem. Due to the built in procedure for generating alternative retrofits, however, this problem is reduced to a minimum. More succinctly, a large number of retrofit strategies for each fire zone is desirable from the viewpoint of design flexibility because they provide an opportunity to match common retrofits across fire zones. In the case application discussed here, the retrofit strategy identified as the least costly on an individual fire zone basis exhibited a marked similarity across fire zones. Thus, the least-cost solution for the entire building is composed of the least-cost solutions for all fire zones within the building. As in actual practice, the synthesis of comprehensive retrofit strategies was based on technical considerations as well as anticipated retrofit costs.

The cost saving potential of the Fire Safety Evaluation System is shown in figure 4.5. It displays graphically the anticipated costs of retrofitting the entire hospital under two courses of action. The first course of action involves the selection of the least-cost solution based on the Fire Safety Evaluation System. This case is labeled "Performance" in figure 4.5. The second course of action implies that strict compliance to the prescriptive provisions of the Life Safety Code is adhered to. This case is labeled "Prescriptive" in figure 4.3. Even a brief examination of figure 4.5 would reveal that the performance based Fire Safety Evaluation System reduces overall retrofit costs by more than 50 percent. The estimated retrofit costs for the two courses of action are $115,000 and $250,000, respectively.

Even though the cost saving potential of the Fire Safety Evaluation System is substantial in this case, it is possible that the overall least-cost solution may vary considerably from one fire zone to another. The use of this solution may, thus, be unwarranted from a design viewpoint. In order to address this point, engineering judgment was used to establish a series of alternative retrofit strategies which matched retrofit packages across fire zones. The retrofit costs associated with the least-cost solution,

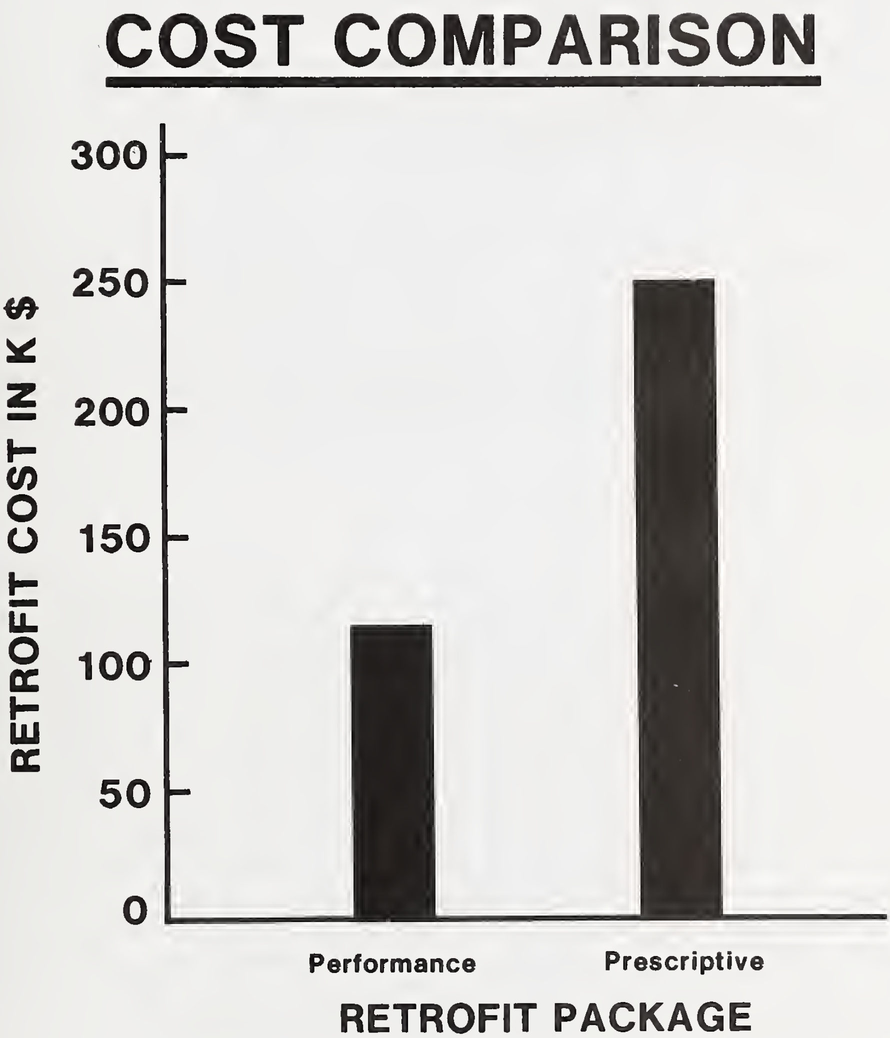

Figure 4.5 Cost Comparison

five alternative retrofit strategies and the prescriptive solution are shown graphically in figure 4.4. A brief description of each package is given in table 4.6. A more detailed description of each retrofit package is given in the NBS report by Chapman et al.[1]

A close examination of table 4.6 and figure 4.4 reveals that at least three alternative retrofit strategies can be defined which are close in cost to the least-cost solution. Additional retrofit strategies could also be defined which both require a specific retrofit package on a particular floor and are fairly close to the least-cost solution. Through the use of this procedure, health care facility decisionmakers will have a great deal of freedom in tailoring the retrofit strategy to their own particular needs and objectives.

Two additional retrofit strategies based on the Fire Safety Evaluation System are also shown in figure 4.4. These are total sprinklering of the entire facility (package E) and the removal of the dead end corridor through the installation of an exit stairwell (package F). They are included to show how the Fire Safety Evaluation System can promote substantial cost savings even if a relatively expensive retrofit strategy, such as totally sprinklering each fire zone, is required. In particular, it was possible to totally sprinkler the entire facility for only two thirds the cost of strict compliance.

The inherent flexibility of the Fire Safety Evaluation System is more evident, however, when retrofit package F is compared to the prescriptive package G. This stems from the fact that a significant portion of the cost of strict compliance to the Life Safety Code was due to the installation of an exit stairwell to remove the dead end corridor. Consequently, any requirement that the exit stairwell be included in the retrofit package would provide a critical test of the Fire Safety Evaluation System's cost-saving capability. Since the computer program generates two classes of alternative solutions, one of which includes the installation of the exit stairwell as a retrofit, it is possible to formulate a comprehensive retrofit package from the standard computer output.[2] Based on this information, it was possible to develop a retrofit strategy which met the requirements of the Life Safety Code at a cost saving of over 25 percent. This is due to the fact that the Building Safety Features which are not constrained are upgraded in the most cost-effective manner.

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[1]R. E. Chapman, P. T. Chen, and W. G. Hall, Economic Aspects of Fire Safety In Health Care Facilities: Guidelines for Cost-Effective Retrofits, National Bureau of Standards, NBSIR 79-1902, November 1979.

[2]Users of the computer program could also utilize the option CHANGE in putting together a comprehensive retrofit package. For a programmer-oriented description, see R. E. Chapman, W. G. Hall, and P. T. Chen, A Computerized Approach for Identifying Cost-Effective Retrofits in Health Care Facilities, National Bureau of Standards, NBSIR 80-1929, January 1980.

Table 4.6 Retrofit Packages for the Case Application

RETROFIT PACKAGE	DESCRIPTOR	APPROXIMATE RETROFIT COST
A	Least-Cost	$115,000
B	Fire Department Connection on Manual Fire Alarms	$121,000
C	Sprinklers in the Corridor	$131,000
D	Smoke Detection and Alarm System in the Corridor	$133,000
E	Total Sprinkler	$158,000
F	Removal of the Dead End Corridor	$184,000
G	Prescriptive	$250,000

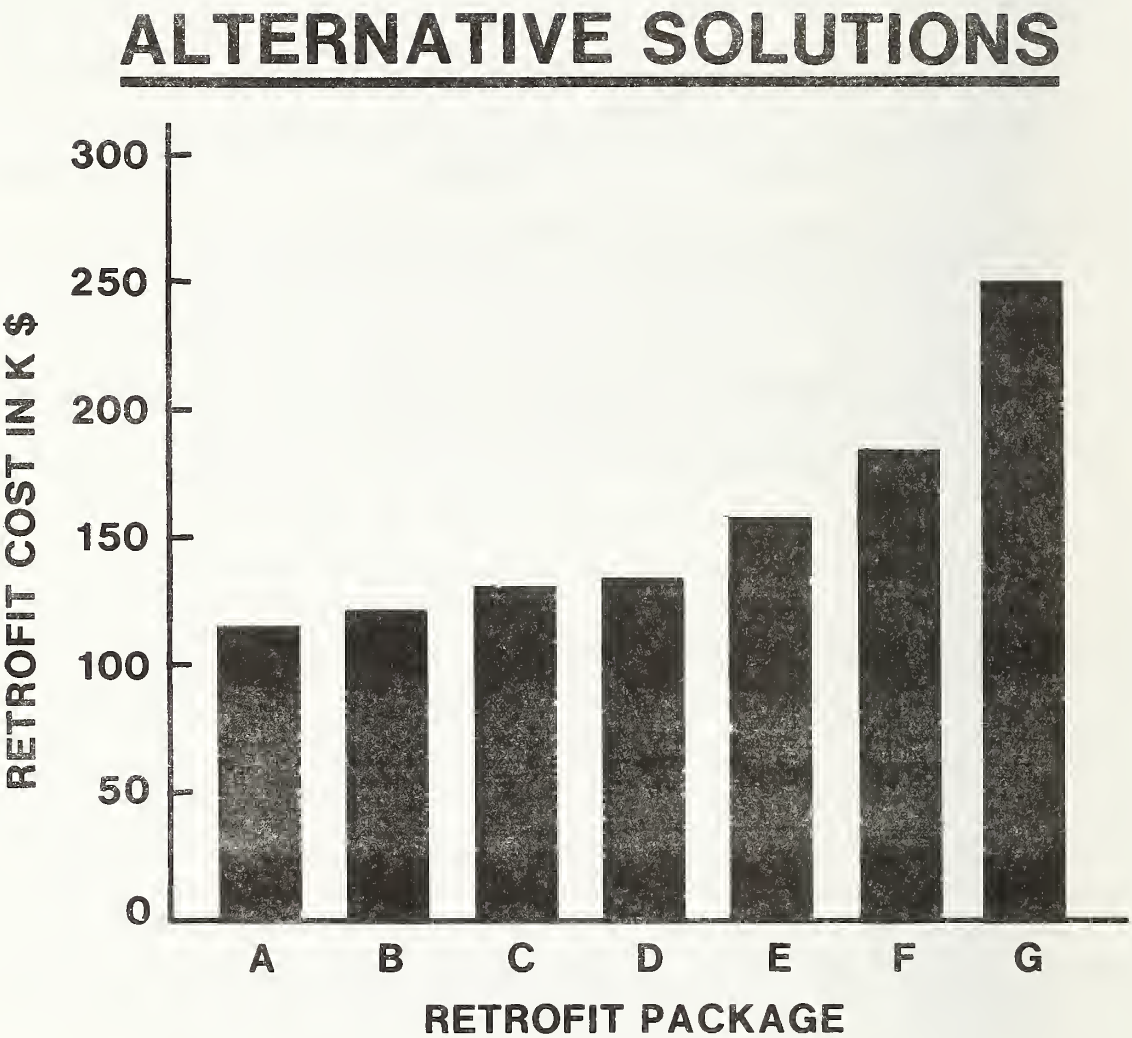

Figure 4.6 Alternative Retrofit Packages for the Case Application

In addition to making it easier to formulate a comprehensive retrofit strategy, the use of alternative retrofits has a second advantage in that it allows health care facility decisionmakers to introduce explicitly nonconstruction costs into the process of retrofit selection and design development. Nonconstruction costs include not only the potential for lost revenues and a reduction in the level of services being provided, but insurance differentials and changes in operating and maintenance costs as well. Since these costs may be significant, the use of alternatives permits the decisionmaker to choose an option which may have slightly higher construction costs but which is less costly when other cost considerations are introduced. Furthermore, some retrofit decisions may affect the costs of owning, operating and maintaining the facility in future periods of time. Should this be the case, it would be worthwhile to base the retrofit decision on an established building investment technique such as life-cycle costing which explicitly weighs all costs and savings over a set period of time.

The previous discussion has shown how the Fire Safety Evaluation System can be used to achieve significant savings in the costs of compliance to fire safety codes without compromising the safety and well being of the persons being housed in health care facilities. Although the Fire Safety Evaluation System is the only comprehensive equivalence methodology in use at this time and its application is clearly nonresidential, the basic concept shows much promise for applications in other code areas and for other occupancy classifications. At this time, the nonfire related code areas where a full fledged equivalence methodology seems appropriate, relate to health and sanitation and security and egress, these areas offer a definite potential because alternatives do not exist and the interdependence among potential alternatives does not appear to be as serious as in other areas such as strength and stability.

4.3 POTENTIAL APPLICATIONS OF AN APPROACH INTEGRATING COST FUNCTIONS AND THE PERFORMANCE CONCEPT

The discussion, thus far, has focused primarily on the mechanics of a combined cost function/performance concept approach. That such an approach could both reduce costs and cost variability was examined in section 4.1. Empirical evidence which documents the cost saving capability of the performance concept was presented in the previous section. In terms of the real world application of this procedure, however, a natural question is "who really benefits from it?". The answer to this question is quite basic. The primary benefactors of such an approach are the purchasers of rehabilitation services, because this approach permits them to purchase a known quality of housing services (mandated by the code(s)) at the least cost for a known level of risk acceptance. It can also be argued that this approach will benefit contractors preparing bids for residential rehabilitation projects.[1] However, since the major burden of the costs of residential rehabilitation must be borne by the purchaser a contractor's services, the emphasis in this section will be on how this approach helps these individuals.

There are three types of "purchasers" who will benefit from the approach outlined in section 4.1. They are:

(1) individual homeowners and/or investors;
(2) financial institutions; and
(3) government

To some extent, all three groups of "purchasers" are interrelated, however, the group which would be most affected by this approach is the first.

At the present time, the individual homeowner or investor shoulders the greatest burden in undertaking a residential rehabilitation project. To be sure the use of the approach proposed in this report will not shift the responsibility from the owner to someone else, it must be assumed that people undertake rehabilitation because, in the end, they believe it will be profitable. Current rehabilitation efforts, however, tend to be overly risky and may promote speculation which often adversely affects the inhabitants of the area being "developed." As rehabilitation activities become less risk prone, they will attract a larger segment of society. In some cases it may even be possible for low-income homeowners to undertake remodeling efforts which were previously beyond their means. By the same token, renters may suffer lower rent increases due to rehabilitation efforts because the absolute cost of renovation is lower.[2]

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[1]For an excellent discussion of this topic the interested reader is referred to the following articles: Marvin Gates, "Bidding Strategies and Probabilities," Journal of the Construction Division, American Society of Civil Engineers, Vol. 93, March 1967; and Michael Curran, "A Scientific Approach to Bidding: Range Estimating," Constructor, January 1975.

[2]As Anthony Downs points out (Anthony Downs, "Investing in Rehabilitation Can Be Successful," Real Estate Review, Vol. 6, No. 2, Summer 1976.) any renovation activity must be accompanied with an increase in the rental rate for the property. Reducing the cost of renovation will, however, reduce the absolute increase in the rental rate. It may also promote a more rapid upgrading of lower quality housing. For an extensive discussion of this topic which includes a model of housing supply, the interested reader is referred to the study by Valenza (Joseph J. Valenza, Residential Rehabilitation, unpublished Ph.D. dissertation, George Washington University, 1978).

The crucial issue for either the homeowner or the investor is of course the profitability of residential rehabilitation. In terms of profitability, one must differentiate between expected return and expected risk. Most people are to some extent risk averse. Thus, maximizing the expected return on an investment regardless of the risk is an unlikely investment strategy for most people. Economic studies have shown that individuals tend to prefer a balance between risk and rate of return.[1] It is uncertain what effect the widespread use of the approach discussed in section 4.1 would have on the rate of return of a rehabilitation project.[2]

It is certain, however, that the inherent riskiness of residential rehabilitation will be reduced somewhat by better cost estimates and known levels of risk acceptance.

The second group of "purchasers," financial institutions, would also be affected by a reduction in the costs and riskiness of rehabilitation activities. In certain instances, financial institutions practice a subtle form of redlining, thus precluding any investor, no matter how qualified, from receiving a loan for undertaking a rehabilitation effort. Many of these activities are defended on the basis of "poor past performance," or "high risks." As the trend toward rehabilitation has grown, many financial institutions have formed consortiums or other risk-sharing arrangements.[3,4]

Even the widespread use of an approach which provides accurate cost estimate that can be tied to a measure of risk will not remove the need for financial risk-sharing arrangements. In all cases one would expect residential rehabilitation to be more risky than new housing construction. What this approach would do, which is not done now, is provide an objective means for assessing the merits of a particular rehabilitation loan application.

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[1] J. Tobin, "Liquidity Preference as Behavior Towards Risk," The Review of Economic Studies, Vol. XXVI, No. 1, February 1958.

[2] This may be seen by noting that more people will probably be drawn into rehabilitation activities if the costs begin to fall. As more people enter and the market interactions become more frequent, the likelihood of individuals behaving as "free riders" and hence appropriating a portion of the investors profit for themselves will be increased. For an excellent discussion of this topic see the report by Colwell (Peter F. Colwell, An Economic Analysis of Residential Abandonment and Rehabilitation, National Bureau of Standards, NBSIR 76-1043, May 1976).

[3] David Gressel, Financing Techniques for Local Rehabilitation Programs, National Association of Housing and Redevelopment Officials, 1976.

[4] J. Thomas Black, Allan Bornt, and Robert Dubinsky, Private Market Housing Renovation in Older Urban Areas, The Urban Land Institute, 1977.

Furthermore, if financial institutions are aware that the likelihood of a given budget being overrun is very small, they may be more willing to make loans in neighborhoods where little or no rehabilitation is being carried out.

The last "purchaser" of rehabilitation services to benefit is government. As in many public policy situations, the government recognizes that private market rehabilitation may, if left to itself, create serious problems. Thus, the government may find itself in a position of advocating rehabilitation in order to get houses into a higher tax paying status while coping with the problem of providing adequate shelter for those families displaced by upward filtering of the housing stock due to rehabilitation. The problem is further complicated since government agents (Federal, State, and local) all operate under a budget constraint. The fact that the emphasis in the past has been on urban renewal reflects this claim (throughout most of the 50's and 60's renewal efforts were thought to be less expensive than rehabilitation efforts). Unfortunately, renewal activities generated many of the same problems of dislocation that widespread private market rehabilitation activities generate. The current government emphasis on rehabilitation seems to reflect a desire to save what we already have. Housing conservation in the future, however, can be more than just saving. It may even be possible to pursue a moderately aggressive program with lower budgets. More precisely reducing the costs of renovation may be a valuable tool for government planners in providing adequate housing for low-income families. It may make code enforcement activities more effective and promote the goal of providing standard (higher quality) housing for lower income households.[1] If costs can be reduced substantially, government sponsored rehabilitation may be used in conjunction with private market rehabilitation so that dislocation problems are minimized while assuring that increased tax revenues will be forthcoming due to private market activities.

Unfortunately, the rehabilitation of inner city housing is not a panacea. There are many problems facing the nation's cities. The fact that rehabilitation activities cannot be divorced from these other problems is a direct result of the economic infrastructure of the modern urban community. On the positive side, improving the housing stock may make other problems seem somewhat more tractable. On the other side, the uncertainty associated with the efficacy of treating some of the major problems facing the nation's cities is so grave that it may prevent the inner city renaiassance so desired from coming to fruition.

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[1]Daniel R. Mandelker and Roger Montgomery, "Housing Codes and Housing Code Enforcement--Saving What We Have," Housing in America: Problems and Perspectives, Bobbs-Merrill Company, Inc., Indianapolis, Indiana, 1973.

Chapter Summary

The purpose of this chapter was to develop a theoretical approach for dealing with cost variability. This approach combined the technical and economic advantages of cost functions with the cost-reducing potential of the performance concept. It was shown that through the use of monte carlo simulation this approach could identify the least-cost solution for achieving compliance to a given set of codes for a specified level of risk acceptance. Consequently, the generalized equivalence methodology outlined in this chapter provides a means for dealing comprehensively with cost variability.

As a means of illustrating the cost-saving potential of the generalized methodology, an example from the area of fire safety in health care facilities was presented. Although the method presented was concerned with only one code, the Life Safety Code, it contained four code attributes which had to be satisfied simultaneously in order to be deemed in compliance. If each of these attributes is thought of as a code, the analogy to the residential rehabilitation problem comes into sharp focus. The equivalence methodology was explained through a series of worksheets and applied to a prototypical hospital. The results of this application revealed a cost-saving potential of over 50 percent when compared to the costs of prescriptive compliance. In addition, it was shown that several retrofit strategies could be identified which were significantly less expensive than prescriptive compliance. These alternatives were a means through which nonconstruction costs could be introduced into the design selection process.

The chapter also included a discussion of how the generalized methodology could benefit individual homeowners and/or investors, financial institutions, and government. Basic concepts which highlighted the interdependence among the three purchasers of rehabilitation services were also presented.

Facing page:

The gains from residential rehabilitation are both financial and aesthetic. Work outlined in this study has shown that a potential for reducing the extreme variability of rehabilitation costs may both promote private market activities and permit the upgrading of lower-income housing units by government authorities.

5. SUMMARY AND RECOMMENDATIONS FOR FURTHER RESEARCH

5.1 SUMMARY

Accurate cost estimation is important to all major participants in the rehabilitation process. Reasonably accurate cost information is essential to the housing owner as an input to the investment decision and to financial institutions in assessing the merits of each investment decision. Unreliable cost information renders residential rehabilitation investments riskier than they would be if accurate cost estimates existed. To contractors and subcontractors, the reliability of cost estimation procedures has a substantial effect on the riskiness of doing business. Finally, accurate cost estimation is needed by Federal, State and local government agents in developing a coordinated approach which promotes private market activity without escalating the housing problems faced by lower income residents.

Four methods for estimating the costs of residential rehabilitation were analyzed in this study. The first method, average total cost, is the most widely used and least flexible of the four methods studied. The second method, parametric procedures, is more flexible but suffers from the assumed linear relationships between the key factors in the procedure. The third method, probabilistic cost estimation, is extremely flexible; it may be used with either average total cost or parametric procedures. The fourth method, cost functions, is the most consistent from an engineering economics viewpoint. In addition, probabilistic concepts are easily incorporated into the cost function approach.

Due to the extreme variability of residential rehabilitation costs, it is necessary that any totally acceptable procedure be able to explicitly treat risk in developing a cost estimate. Three means of risk assessment were studied for each of the four methods. Based on this analysis, judgmental modifications were shown to be inappropriate, since it is qualitative rather than quantitative, and the use of confidence intervals might be misleading if the estimated value for either average total cost or sample variance was biased. On the other hand, probabilistic concepts were shown to be highly desirable since they permitted one to define percentage changes in the level of risk being borne by the investor as a function of changes in one or more factors in the rehabilitation process.

The engineering economics construct known as the cost function was then adapted to a general framework based on the performance concept. The performance concept was then used to define a generalized equivalence methodology for residential rehabilitation. This theoretical methodology permitted both compliance to the code(s) under consideration and a mathematical structure suitable for optimization techniques to be identified. Probabilistic methods were then developed which would permit a potential investor to identify an optimal retrofit strategy for a given level of risk acceptance. Past empirical studies in the area of fire safety in health care facilities were used to illustrate how such an approach might reduce retrofit costs by as much as 50 percent.

5.2 RECOMMENDATIONS FOR FURTHER RESEARCH

In order to expand the concepts developed in this study, future research in several areas is needed.

This study has concluded that the engineering economics construct known as a cost function is the most consistent technique for estimating residential rehabilitation costs. Unfortunately, the development of a series of cost functions for residential rehabilitation can only be accomplished through the application of econometric techniques to actual housing renovation cost data. Past empirical studies have defined the type and nature of such cost data. It would be helpful if these guidelines could be followed in collecting a complete set of residential rehabilitation cost data.

The use of probabilistic methods is an important tool for assessing the riskiness of a potential residential rehabilitation project. At the present time, little empirical evidence exists on the likely candidates for the probability distributions associated with a given factor in the rehabilitation process. Further work in this area has two basic merits. First, it explicitly ties the problem of risk in the physical process to the cost estimation process. Second, since cost functions are more complicated than the other methods of estimating rehabilitation costs, defining the most likely probability distributions for each factor would simplify their use in the field.

This study has presented empirical evidence which shows that cost reductions of 50 percent were possible in other areas of building renovation. If these savings can be achieved in residential rehabilitation activities, the level of rehabilitation may increase substantially. The theoretical model developed in this paper is of necessity abstract. However, previous experience in other areas has facilitated the development of the generalized methodology presented in this paper. The actual process of selecting each building component, defining each state and assigning each state value will be extremely difficult. If this effort is balanced against the cost reducing capability of such a procedure it may seem less formidable, however. The fact that the generalized equivalence methodology can actually identify the least-cost solution for a given level of risk implies that a comprehensive treatment of costs in this area is theoretically feasible.

The productivity of labor in residential rehabilitation activities is one of the major topics which affects rehabilitation costs. At this time there is a widespread belief that productivity levels in all sectors of the construction industry are declining. If this is true and can be measured, one could show directly how rehabilitation costs will be changing as a result of the decline in productivity. The sparsity of empirical data on this crucial topic, however, prevents one from giving a definitive answer as to the existence or severity of the decline in productivity. Along similar lines, it would be beneficial to know how the level of productivity in residential rehabilitation compares to that in new construction. If answers to this question could be obtained, one might be able to identify the circumstances under which rehabilitation would have a distinct cost advantage over new construction. Guidelines for making these decisions would be of value to potential investors whether they contract for renovation services or do the job themselves.

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APPENDIX A

DERIVATION OF A COST FUNCTION FOR RESIDENTIAL REHABILITATION

This appendix developes a form of economic model known as a cost function. The derivation of the cost function will proceed on the assumption that the production function underlying the construction process is Cobb-Douglas. This assumption is made both because it is consistent with previous studies of the construction industry[1] and for ease of exposition. The term production function as used in this study refers to an explicit relationship between a set of inputs (labor, materials, and capital) and technological factors which taken together produce a given output (square feet of floor area renovated). The cost function associated with the underlying process may then be derived from the production function by solving a constrained optimization problem. A non-mathematical statement of the problem is to minimize the cost of renovation subject to the requirement that Q square feet of floor area are renovated. Since portions of the discussion which follows are of a rather technical nature, readers interested primarily in the implications of such an approach are encouraged to go directly to sections 2.4 and 4.1.

In the mathematical discussion which follows, it is assumed that the prices of all inputs are independent. Under this assumption the quantity of output, Q, the square feet of floor area renovated, and the cost of output, C, may be expressed as

$$Q = \tau_o \prod_{i=1}^{m} \tau_i(k_i) \prod_{j=1}^{n} (X_j)^{\alpha_j} \qquad \text{A.1}$$

$$C = \sum_{j=1}^{n} X_j P_j \qquad \text{A.2}$$

where

Q = total square feet of floor area to be renovated;

τ_o = basic construction technolory factor;

τ_i = construction technology factor associated with the i^{th} building system or subsystem (e.g., plumbing, structural, electrical);

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[1]For an authoritative source on this subject see John S. McConnaughey, Jr. Production Functions in Contract Construction for the United States, 1972 (unpublished), Ph.D. dissertation, Michigan State University, 1976.

k_i = measure of the condition of the i^{th} system or subsystem (e.g., 1 = excellent, 2 = sound, 3 = some deterioration, ...);[1]

m = the number of building systems or subsystems;

X_j = quantity of the j^{th} input (materials, labor, equipment) required to perform the job;

α_j = the percentage change in output associated with a one percent change in the utilization of the j^{th} input;[2]

n = the number of inputs;

C = the total cost (expected bid price) of the job; and

P_j = the unit cost of the j^{th} input.

Note also that the summation sign, Σ, and the product sign, Π, are used in equations A.1 and A.2. They are defined as

$$\sum_{j=1}^{n} X_j P_j = X_1P_1 + X_2P_2 + \ . \ . \ . + X_nP_n$$

and

$$\prod_{j=1}^{n} (X_j)^{\alpha_j} = (X_1)^{\alpha_1} (X_2)^{\alpha_2} \ . \ . \ . (X_n)^{\alpha_n}$$

Letting n equal 2 would thus imply that

$$\sum_{j=1}^{n} X_j P_j = \sum_{j=1}^{2} X_j P_j = X_1P_1 + X_2P_2$$

and

$$\sum_{j=1}^{n} (X_j)^{\alpha_j} = \sum_{j=1}^{2} (X_j)^{\alpha_j} = (X_1)^{\alpha_1} (X_2)^{\alpha_2}$$

The constrained optimization problem may then be solved through application of the method of La Grange multipliers. This method consists of the formation of an auxiliary function subject to a constraint condition $\phi(X_1, ..., X_n) = 0$.

---

[1]In terms of the four levels of measurement discussed in section 3.5, the numerical values assigned to the condition variable should be defined over the interval or ratio levels.

[2]In economics α_j is referred to as an output elasticity.

Since the problem may be written as

$$L = C + \lambda\phi\ (X_1,\ \ldots,\ X_n)$$

and $\phi\ (X_1, \ldots, X_n)$ is equal to zero, minimizing minimizes C (equation A.2) subject to the constraint that Q square feet of floor area are renovated (equation A.1). More succinctly, the auxiliary function may be written as

$$L = \sum_{j=1}^{n} X_j P_j + \lambda\ (Q - \tau_o \prod_{i=1}^{m} \tau_i(k_i) \prod_{j=1}^{n} (X_j)^{\alpha_j})$$

Differentiating partially equation A.3 with repect to each input and equating the resultant to zero, produces the following system of equations

$$\frac{\partial L}{\partial X_1} = P_1 - \lambda \frac{\alpha_1}{X_1} (\tau_o \prod_{i=1}^{m} \tau_i(k_i) \prod_{j=1}^{n} (X_j)^{\alpha_j}) = 0$$

$$\vdots$$

$$\frac{\partial L}{\partial X_h} = P_h - \lambda \frac{\alpha_h}{X_h} (\tau_o \prod_{i=1}^{m} \tau_i(k_i) \prod_{j=1}^{n} (X_j)^{\alpha_j}) = 0$$

$$\vdots$$

$$\frac{\partial L}{\partial X_\ell} = P_\ell - \lambda \frac{\alpha_\ell}{X_\ell} (\tau_o \prod_{i=1}^{m} \tau_i(k_i) \prod_{j=1}^{n} (X_j)^{\alpha_j}) = 0$$

$$\vdots$$

$$\frac{\partial L}{\partial X} = P_n - \lambda \frac{\alpha_n}{X_n} (\tau_o \prod_{i=1}^{m} \tau_i(k_i) \prod_{j=1}^{n} (X_j)^{\alpha_j}) = 0$$

This system consists of n equations in n + 1 unknowns, $X_1, \ldots, X_n$ and . Thus it is only possible to express the level of each input, say X_ℓ, as a function of one of the n inputs, say X_h. For example

$$\frac{P_\ell\ P_\ell}{\alpha_\ell\ (\tau_o \prod_{i=1}^{m} \tau_i(k_i) \prod_{j=1}^{n} (X_j)^{\alpha_j})} = \quad = \frac{P_h\ P_h}{\alpha_h (\tau_o \prod_{i=1}^{m} \tau_i(k_i) \prod_{j=1}^{n} (X_j)^{\alpha_j})}$$

which implies

$$X_\ell = \frac{\alpha_\ell}{\alpha_h} \frac{P_h}{P_\ell} X_h \qquad \ell = 1, \ldots, n \qquad \text{A.4}$$

The cost minimizing demand curve for the h^{th} input[1] may then be obtained by substituting equation A.4 in equation A.1 and solving for X_h. This is expressed mathematically as

$$Q = \tau_o \prod_{i=1}^{m} (\tau_i)(k_i) \left[\prod_{\substack{j=1 \\ j \neq h}}^{n} ((\alpha_j)/(\alpha_h))^{\alpha_j} \right] \left[\prod_{\substack{j=1 \\ j \neq h}}^{n} ((P_h)/(P_j))^{\alpha_j} \right] \left[\prod_{j=1}^{n} (X_j)^{\alpha_j} \right]$$

or equivalently

$$\prod_{j=1}^{n} (X_h)^{\alpha_j} = X_h^R = Q \left\{ \tau_o \prod_{i=1}^{m} \tau_i(k_i) \prod_{\substack{j=1 \\ j \neq h}}^{n} ((\alpha_j)/(\alpha_h))^{\alpha_j} \prod_{j=1}^{n} ((P_h)/(P_j))^{\alpha_j} \right\}^{-1} \qquad \text{A.5}$$

where

$$R = \sum_{j=1}^{n} \alpha_j .$$

The cost minimizing demand curve for the h^{th} input, X_h^*, is then obtained by taking the one over R^{th} root of equation A.5. The cost minimizing demand curve for the h^{th} input, X_h^*, is therefore

$$X_h^* = Q^{\frac{1}{R}} \left(\tau_o \prod_{i=1}^{m} \tau_i(k_i) \right)^{\frac{-1}{R}} \left(\prod_{\substack{j=1 \\ j \neq h}}^{n} ((\alpha_j)/(\alpha_h))^{\alpha_j} \right)^{\frac{-1}{R}} \left(\prod_{\substack{j=1 \\ j \neq h}}^{n} ((P_h)/(P_j))^{\alpha_j} \right)^{\frac{-1}{R}} \qquad \text{A.6}$$

Substituting equation A.6 into equation A.4 permits the cost minimizing demand curves for the remaining n-1 factor inputs to be derived. These demand curves are given as

$$Q^{\frac{1}{R}} \left(\tau_o \prod_{i-1}^{m} \tau_i(k_i) \right)^{\frac{-1}{R}} \prod_{\substack{j=1 \\ j \neq h}}^{n} ((\alpha_j)/(\alpha_h))^{-\alpha_j/R} \prod_{\substack{j=1 \\ j \neq h}}^{n} ((P_h)/(P_j))^{-\alpha_j/R} ((\alpha_\ell)/(\alpha_h))((P_h)/(P_\ell))$$

$$\ell = 1, \ldots, n, \neq h.$$

---

[1]The cost minimizing demand curve for the h^{th} input gives the level of the h^{th} input needed in order to minimize equation A.2.

The above equations may be simplified by noting that

$$\frac{\alpha_\ell}{\alpha_h} = (\alpha_\ell)(\alpha_h)^{-1}$$

and

$$\prod_{\substack{j=1\\ j\neq h}}^{n} ((\alpha_j)/(\alpha_h))^{-\alpha_j/R} = (\alpha_h)^{(1-(\alpha_h/R))} \prod_{\substack{j=1\\ j\neq h}}^{n} (\alpha_j)^{-((\alpha_j)/R)}$$

which implies

$$\frac{\alpha_\ell}{\alpha_h} \prod_{\substack{j=1\\ j\neq h}}^{n} ((\alpha_j)/(\alpha_h))^{-\alpha_j/R} = \alpha_\ell \prod_{j=1}^{n} (\alpha_j)^{-((\alpha_j)/R)}$$

Applying the same logic one gets

$$\alpha_\ell \prod_{j=1}^{n} (\alpha_j)^{-((\alpha_j)/R)} = \prod_{j=1}^{n} ((\alpha_j)/(\alpha_\ell))^{-((\alpha_j)/R)}$$

Through a similar set of manipulations it is possible to show that

$$\frac{P_h}{P_\ell} \prod_{\substack{j=1\\ j\neq h}}^{n} ((P_h)/(P_j))^{-((\alpha_j)/R)} = \prod_{j=1}^{n} ((P_\ell)/(P_j))^{-((\alpha_j)/R)}$$

The cost minimizing demand curve for each input is thus

$$X_\ell^* = Q^{1/R}(\tau_o \prod_{i=1}^{m} \tau_i(k_i))^{-1/R} \prod_{j=1}^{n} ((\alpha_j)/(\alpha_\ell))^{-((\alpha_j)/R)} ((P_\ell)/P_j))^{-((\alpha_j)/R)} \qquad \text{A.7}$$

where $\ell = 1, \ldots, n$.

The minimum cost solution for renovating Q square feet of floor area if then gotten by substituting equation A.4 into equation A.2. This substitution yields

$$C^* = \sum_{\ell=1}^{n} X^*_{\ell} P_{\ell}$$

or equivalently

$$C^* = Q^{1/R} (\tau_o \prod_{i=1}^{m} \tau_i(k_i))^{-1/R} \sum_{\ell=1}^{n} P_{\ell} \prod_{j=1}^{n} ((\alpha_{\ell}))^{-((\alpha_j)/R)} ((P_{\ell})/P_j))^{-((\alpha_j)/R)} \qquad A.8$$

Fortunately, equation A.8 can be simplified by noting that

$$P_{\ell} \prod_{j=1}^{n} ((P_{\ell})/(P_j))^{-((\alpha_j)/R)} = \prod_{j=1}^{n} (P_j)^{((\alpha_j)/R)}$$

which implies

$$C^* = Q^{1/R} (\tau_o \prod_{i=1}^{m} \tau_i(k_i))^{-1/R} \prod_{j=1}^{n} P_j^{((\alpha_j)/R)} \sum_{\ell=1}^{n} \prod_{j=1}^{n} ((\alpha_j)/(\alpha_{\ell}))^{-((\alpha_j)/R}$$

The last term in brackets in the previous equation can also be simplified by noting that

$$\prod_{j=1}^{n} ((\alpha_j)/(\alpha_{\ell}))^{-((\alpha_j)/R)} = \alpha_{\ell} \prod_{j=1}^{n} (\alpha_j)^{-((\alpha_j)/R)}$$

which implies

$$\sum_{\ell=1}^{n} \prod_{j=1}^{n} ((\alpha_j)/(\alpha_{\ell}))^{-((\alpha_j)/R)} = \prod_{j=1}^{n} (\alpha_j)^{-((\alpha_j)/R)} \sum_{\ell=1}^{n} \alpha_{\ell}$$

where $R = \sum_{\ell=1}^{n} \alpha_{\ell}$

Equation A.8 may thus be rewritten as

$$C^* = Q^{1/R} \left(\tau_o \prod_{i=1}^{m} \tau_i(k_i) \right)^{-1/R} \prod_{j=1}^{n} P_j^{((\alpha_j)/R)} \quad (R) \quad \prod_{j=1}^{n} ((\alpha_j)^{-((\alpha_j)/R)} \qquad \text{A.9}$$

A comparison of equation A.9 to equation 2.10 in section 2.4 reveals the two to be identical in form. The empirical estimation of the theoretical cost function (either equation A.9 or 2.10) could then proceed in a manner similar to that outlined in section 2.4.

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This study analyzes four methods of estimating the costs of residential rehabilitation. Each method is critiqued with regard to its treatment of changes in the size of the renovation project, the productivity of labor, and the contractor's markup for overhead and profit. Cost comparisons and a discussion of the way in which the inherent riskiness of renovation activities may be assessed are also presented. A theoretical approach for dealing with cost variability which integrates the performance concept with established engineering economics techniques is also developed.

12. KEY WORDS *(Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)*

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