NBS BUILDING SCIENCE SERIES 110

Reliability Basis of Load and Resistance Factors for Reinforced Concrete Design

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ABSTRACT

Engineering decisions must be made in the presence of uncertainties which arise as a consequence of imperfect information and knowledge and inherent randomness in many design parameters. It is on account of these uncertainties and potential risks arising therefrom that safety margins provided by load and resistance factors are required in design. Reliability methods are employed in this study to facilitate the selection of criteria for reinforced concrete design. These methods, which are based on probability theory, provide a logical basis for determining the manner in which uncertainties in resistance and loads affect design safety and how their effects should be controlled in building standards. Following a comprehensive analysis of uncertainty measures, safety indices associated with existing reinforced concrete designs are evaluated. Design criteria commensurate with levels of uncertainty and required reliability are then presented. Simplification of these leads to practical reliability based criteria which retain the relatively simple characteristics of existing criteria and yet have a well established and documented rationale.

KEY WORDS: Buildings (codes); concrete (reinforced); design (criteria); loads; probability theory; reliability; statistical analysis; structural engineering.

NOTATION

In addition to standard American Concrete Institute notation, the following symbols are frequently employed in this report:

A = influence area, equal to $2A_t$ for beams and $4A_t$ for columns;

A_t = tributary loaded area;

B = bias in mathematical model;

b_n, h_n, d_n, A_{sn} = nominal diminsions for width, thickness, effective depth to reinforcement, and reinforcing bar area, respectively;

$E[X]$, $Var[X]$ = mean, variance of random variable X;

F_X, f_X = cumulative distribution function, probability density function for variable X;

$k\ell/r$ = slenderness ratio;

D,L,W = Dead load, live load, wind load, respectively;

L_c, L_I, L_t = code - specified basic live load instantaneous live load, and lifetime maximum live load;

p_f = probability of failure;

p_{ij} = relative likelihood of load ratio combination ij;

Q = maximum variable load which is added to other loads taken at their instantaneous values when combining loads;

R_i, S_j = resistance variable i, load j, respectively;

R_i^*, S_j^*, = the values of resistance variable i and load j which are substituted in the limit state equation for safety checking;

R^* = factored ultimate resistance;

s = spacing of stirrups used to resist shear;

T = time;

U^* = factored ultimate load

$\overline{X}$, σ_X = mean, standard deviation of variable X;

α_i = direction cosine (see Equation 2.8);

β = safety index;

δ_X, Δ_X, Ω_X = basic variability, prediction error, and total uncertainty, respectively, in X;

Δ_b'', Δ_h'', Δ_d'' = tolerances to be subtracted from b_n, h_n, or d_n for safety checking when using the partial factors format;

ρ, ρ_v, ρ_g = reinforcement ratios for flexure, shear and combined bending and compression, respectively; $\rho = A_s/bd$; $\rho_v = A_v/bs_{st}$; $\rho_g = (A_s + A_s')/bh$.

ϕ', γ' = resistance and load factors (i = D,L,W) applied to *nominal* resistance and load;

ϕ, γ_i = resistance and load factors applied to *mean* resistance and load;

Apostrophe (prime) denotes a *nominal* value specified for use in a conventional standard; e.g., nominal compressive strength of concrete, f'_c; nominal live load L' specified by ANSI A58.1 - 1972;

Asterisk denotes a *factored* value of a variable which is used in safety checking.

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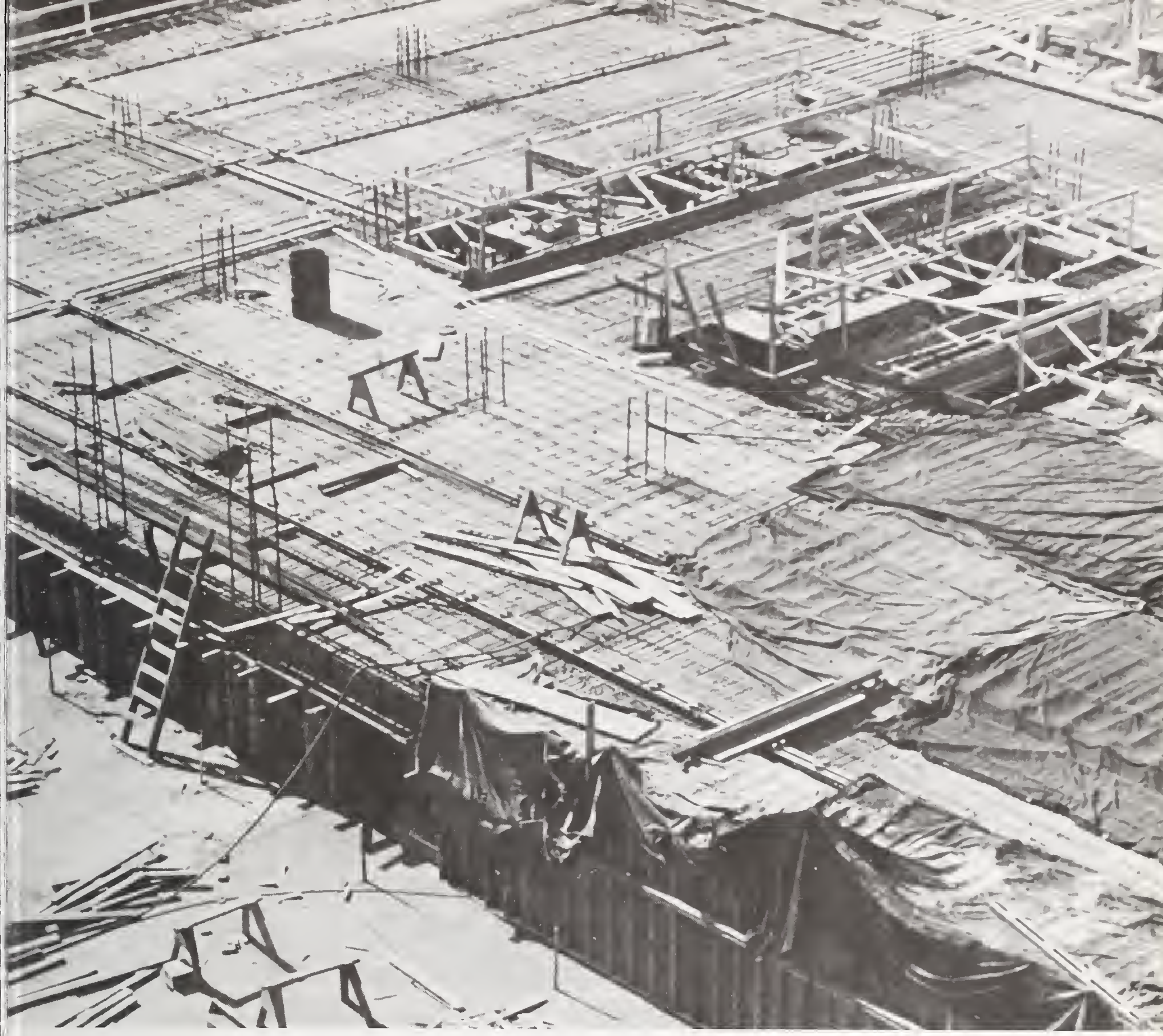

1. INTRODUCTION AND BACKGROUND

The design of safe, functionally reliable and economical structures is of continuing concern to structural engineers. In view of existing and projected shortages in building materials and continuing increases in construction costs, those costs which are attributable to excessive safety margins can and should be reduced. In many situations, material savings may be realized without sacrificing structural performance and safety.

Engineering decisions must be made in the presence of uncertainties which are invariably present in practice [3,4]*. These arise as a consequence of imperfect information and knowledge, lack of experience, and inherent randomness in many design parameters. In fact, it is on account of these uncertainties and the potential risks arising therefrom that the safety margins provided by load and resistance factors, and the like, are required in design. However, the safety factors provided in existing codes and standards have been derived primarily from experience, tradition, and engineering judgment [51]. As a consequence, current codes may differ in the levels of safety they provide for different materials and modes of failure.

In recognition of these factors, the current trend in code development in the United States, as well as Europe, is toward the use of approaches which employ probabilistic concepts as a basis for developing design criteria. This represents an attempt to proportion members and connections so that the probability of their failure is acceptably small. In modern reliability analysis, probability theory furnishes the logical framework for incorporating uncertainty and the consequences of failure which are reflected in the probability of failure. The safety factors obtained are functions of precisely what motivates and necessitates their use in design, and are commensurate with the measure of uncertainty and acceptable failure probability.

An example of reliability based criteria is given by the tentative load and resistance factor design (LRFD) procedure developed by Galambos and Ravindra [28] for steel structures. A preliminary study of such criteria for reinforced concrete structure was conducted by Ellingwood and Ang [21]. In Canada, Allen [1] analyzed the reliability of steel designs implied by using the load factors given in Section 4.1 of the National Building Code of Canada. Siu, Parimi and Lind [58] carried this one step further, in determining an appropriate load and resistance factor set for several materials. In their study, and in the Canadian Standard, it is proposed that one set of load factors be used irrespective of the material.

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*Numbers in brackets denote references at the end of this report.

In Europe, the Comite Europeen du Beton (CEB) has also proposed a material-independent loading criterion for ultimate and serviceability limit states* which would have a probabilistic basis [12,26]. The desirability of using material-independent loading criteria appears to be widely agreed upon. This represents an attempt to avoid the inevitable conflicts which may arise when different material usages are being weighed or combined, as in composite construction, and an attempt to place all materials on an equivalent footing from a design safety point of view.

It should be pointed out that the Canadian and European building codes prescribe live and wind loads that may differ, in some cases considerably, from those presented in American standards such as ANSI A58.1-1972 [9]. Therefore, while the methodologies employed in those studies are useful, the specific criteria and numerical values cannot be assumed to be applicable to American building standards.

In the United States, safety criteria have traditionally been developed separately for different materials and methods of construction by interested groups such as the American Concrete Institute (ACI), the American Institute of Steel Construction (AISC), and similar organizations. Needless to say, this has led to a situation where there are many differences in philosophy and inconsistencies in the levels of safety and performance accorded different structures, depending on the materials used in their design. A stage is rapidly being reached where the implementation of probabilistically derived design criteria is considered both feasible and desirable in the United States. To this end, ANSI A58.1 has designated a subcommittee to investigate the possibility of developing one set of load factors which would be appropriate for use with different materials. In order to do this in a way acceptable to current material groups, appropriate target reliability levels first must be identified for different construction materials and design criteria formats must be decided upon.

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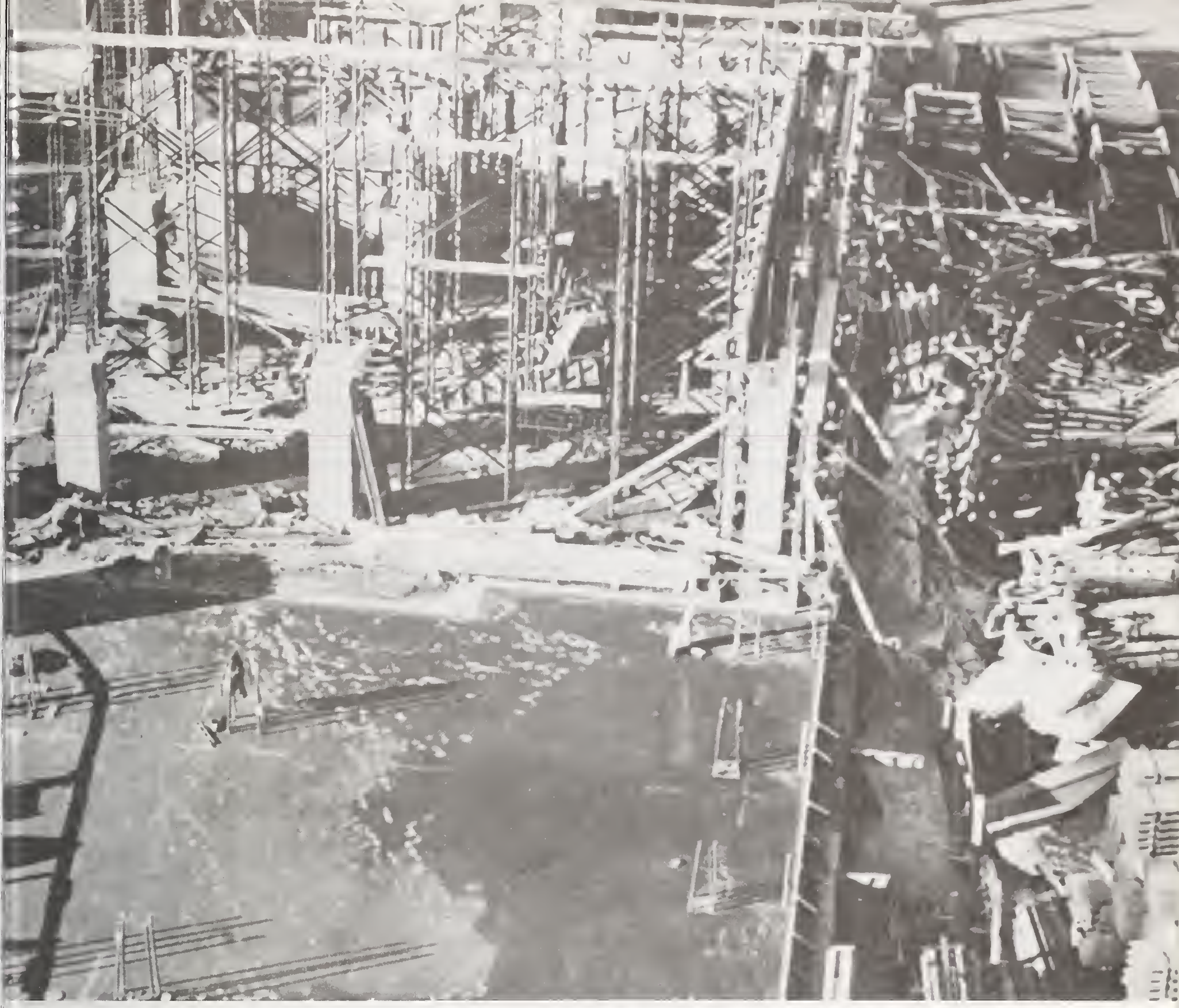
*Limit states describe various conditions where the structure fails in some way to achieve its intended purpose. Ultimate limit states relate to safety under maximum loads, while serviceability limit states relate to loss of function under normal loads.

The objectives of the present study are actually threefold. The first is to provide a convincing demonstration of modern reliability analysis techniques in selecting practical design criteria, taking advantage of recently developed [14,26,30] methods of reliability analysis. For illustrative purposes, the ACI Standard 318-71 [10] which governs much of the concrete construction in the U.S. was selected as a framework for criteria development. A second objective is to determine levels of safety tacitly implied by existing reinforced concrete design standards. This then contributes to the technical data base which would be required prior to developing material-independent loading criteria. Although the reliability analysis methods used in this study differ from those used by Galambos in his study of the AISC specifications, [28,60] the numerical results obtained in these two phases may be considered to complement his work. A final objective is to examine some of the relative advantages and disadvantages of some of the design criteria formats most likely to be proposed for use in first generation probability based standards.

The report is organized to show how a large aggregate of engineering data and judgment can be systematically combined, using probabilistic techniques, to arrive at rational design criteria which need not be overly complex. Section 2 contains a discussion of the recently developed first-order second moment reliability analysis method which serves as the basis for the safety checking and design criteria presented in later sections. The reader familiar with the mathematics may proceed directly to Section 3 without significantly losing continuity. Section 3 contains a brief discussion of how the measures of uncertainty can be estimated and a summary of uncertainties in resistance and load terms. The analysis of individual uncertainty measures is documented in detail in the Appendix. The preceding two sections lay the groundwork for Section 4, in which reliabilities associated with existing reinforced concrete design criteria are evaluated. On this basis, the development of reliability-based design procedures are discussed in Section 5; practical load and resistance factor design criteria are presented in Section 6.



2. RELIABILITY ANALYSIS

The conceptual framework for probability based design is provided by the classical reliability theory described by Freudenthal and others [27,4]. The loads and resistances are assumed to be statistical variables and the necessary statistical information is available.

A mathematical model is first derived which relates resistance to loads by describing the limit state of interest. Express the overall resistance R and overall load effect S as functions of component resistances and loads, i.e.

$$R = g_R \; (R_1, R_2, \ldots R_n) \tag{2.1a}$$

$$S = g_S \; (S_1, S_2, \ldots S_m) \tag{2.1b}$$

where R_i and S_j are (generally random) resistance and load variables. Since a structure is safe as long as its resistance exceeds the effect of loads placed on it, safety is equivalent to $R > S$, or

$$R - S > 0 \tag{2.2}$$

Correspondingly, the limit state is the condition $R = S$. Then safety is assured by assigning a small probability p_f to the event that the limit state will be reached, i.e.,

$$p_f = P(R < S) = \int F_R \; (s) \; f_S \; (s) \; ds \tag{2.3}$$

in which P( ) is the probability of the event enclosed in parentheses, and F_R and f_S are cumulative probability distribution function and density function for R and S, respectively. If the probability distributions are defined by their means and variances (or coefficients of variation, the square root of variance divided by mean), Equation 2.3 may be expressed symbolically as,

$$p_f = p_f \; (\overline{R}, \overline{S}, \Omega_R, \Omega_S) \tag{2.4}$$

where $\overline{R}$ and Ω_R are, respectively, the mean and coefficient of variation of R, and similarly for S. The c.o.v. provide a convenient dimensionless measure of uncertainty.

This provides a theoretical basis for the evaluation of structural safety. It is tacitly assumed that all uncertainties in design are contained in the probability laws for R and S, and these are also assumed to be known. However, in structural safety analyses these probability laws are seldom known precisely, and in fact only the first and second order moments, i.e. mean and variance, may be known with any confidence. While this information usually is sufficient to describe the central behavior of the probability distribution, it may be grossly inadequate to describe the behavior out at the tail regions of the probability distributions which are significant for structural reliability analysis. The problem is compounded by the fact that the probability of failure computed form Eq. 2.3 is sensitive to the choice of probability distributions for R and S at probabilities considered significant for structural applications [5]. Another problem is that while statistical information may be sufficient to define R_i or S_j in Equation 2.1a and 2.1b the convolution required to obtain necessary probability laws for R and S in Equation 2.3 may be impractical.

The difficulties outlined above have motivated the development of first order second moment reliability analysis methods, so called because they rely on a knowledge of the first and second order statistical properties of R and S only rather than the probability distributions as a whole. Thus, this approach is generally consistent with the state of knowledge which exists in structural engineering regarding a random phenomenon. The concepts are summarized here, there being several recent papers on the topic [3,14,26,30].

Let the limit state of interest be given by

$$g = g_R\,(R_1, R_2,\ldots) - g_S(S_1, S_2,\ldots) = 0 \tag{2.5}$$

Failure is considered to occur if $g < 0$. For purposes of design or safety checking, a set of points,

$$R_i^* = \overline{R}_i \exp\,(-\,\alpha_{R_i}\;\beta\,\Omega_{R_i}),\; i = 1, n \tag{2.6a}$$

$$\overline{S}_j^* = \overline{S}_j\,(1 + \alpha_{S_j}\;\beta\,\Omega_{S_j}),\; j = 1, m \tag{2.6b}$$

is determined such that

$$g(R_1^*, R_2^*,\ldots S_1^*, S_2^*,\ldots) = 0 \tag{2.7}$$

An exponential transformation is used on resistance terms to avoid theoretical difficulties that might otherwise arise from negative R^* values [11,26]. Note that the point (R_1^*, R_2^*,... S_1^*, S_2^*,...) lies on the failure surface defined by the limit state. The term β in Equations 2.6 is the safety index and provides a measure of the reliability [30]. The terms

α_{R_i} and α_{S_j} appearing in Equations 2.6 are the direction cosines of the vector normal to the failure surface g = 0 along which β is measured and are given by,

$$\alpha_i = a_i/[\Sigma a_i^2]^{1/2} \qquad (2.8a)$$

in which

$$a_{R_i} = \sigma_{R_i} \exp\left(-\alpha_{R_i} \beta \Omega_{R_i}\right) \frac{\partial g}{\partial R_i}\Big|_* \qquad (2.8b)$$

$$a_{S_j} = -\sigma_{S_j} \frac{\partial g}{\partial S_j}\Big|_* \qquad (2.8c)$$

where σ_{R_i} and σ_{S_j} are standard deviations in R_i and S_j and the derivatives are evaluated at the point on the failure surface given by Equation 2.6. The reliability analysis requires that Equations 2.6 - 2.8 be solved simultaneously using an iterative procedure to search for the α_i which minimize the distance from point $(\overline{R}_1, \overline{R}_2, \ldots, \overline{S}_1, \overline{S}_2, \ldots)$ to the limit state equation.

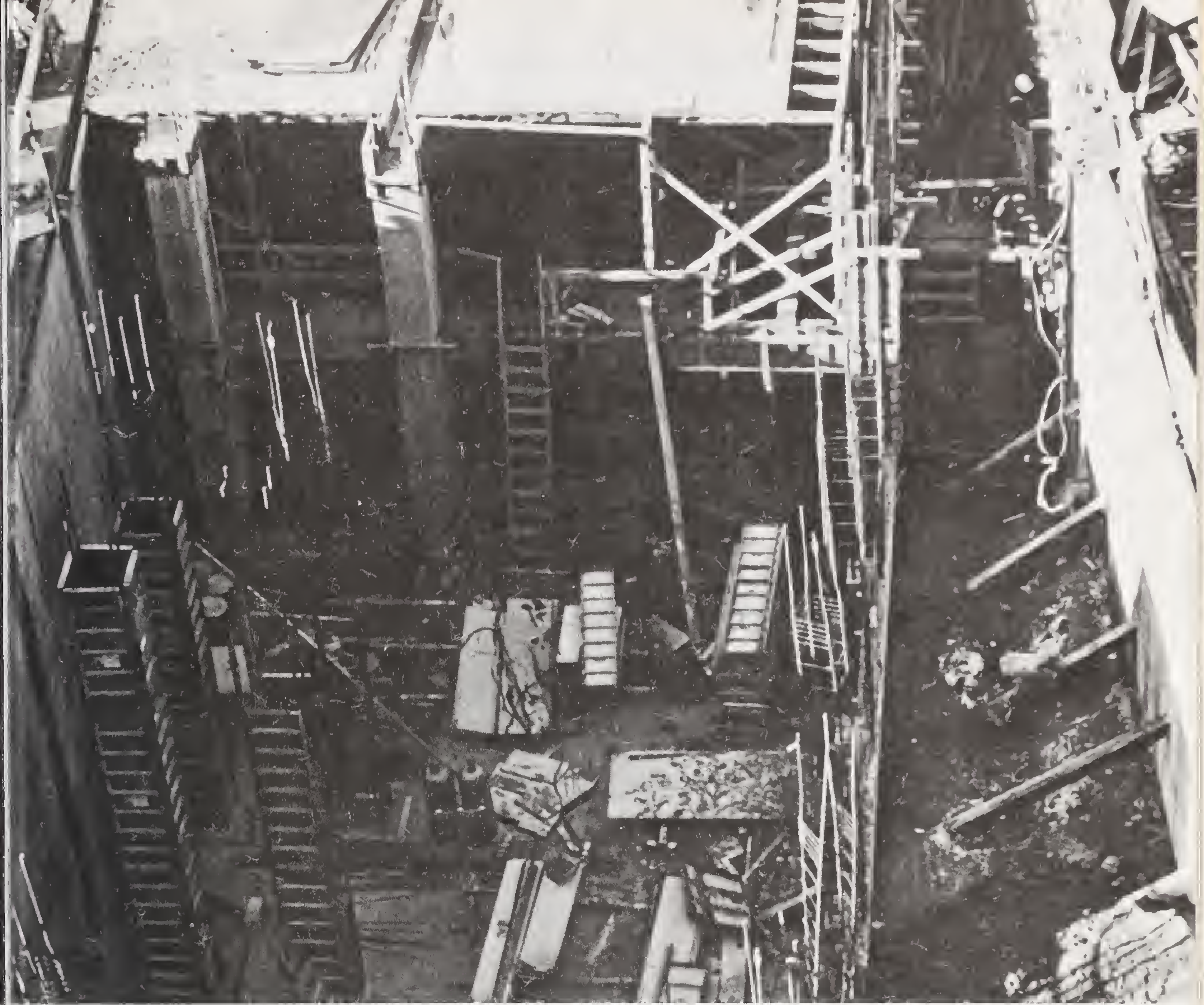
The design problem is to find an appropriate set $(\overline{R}_1, \overline{R}_2, \ldots)$ for a prescribed safety index and load set $(\overline{S}_1, \overline{S}_2 \ldots)$ Conversely, safety analysis or checking requires that a value of β be found corresponding to a prescribed set of resistance and load parameters. In either event, the solution is a function of the reliability β and the measures of uncertainty Ω.

It is important to realize that if the probability distributions for all design variables are known, then the safety index β and p_f in Equations 2.3 and 2.4 are uniquely related, that is, for any prescribed set of means and uncertainties a unique β corresponding to p_f can be computed. For example, if the limit state Equation 2.5 is linear in the R_i and S_j variables and R_i, S_j have normal probability distributions, then $\beta = \Phi^{-1}(1 - p_f)$, where Φ^{-1} is the percent point function of the standard normal variate. For ease of reference, this relation is shown in Table 2.1.

Table 2.1 - Approximate Relation Between p_f and β

p_f	β
10^{-2}	2.33
10^{-3}	3.09
10^{-4}	3.72
10^{-5}	4.26

Even when such distributions and probabilities cannot be obtained, however, β is a useful comparative measure of reliability and can serve to evaluate the relative safety of various design alternatives, provided that the uncertainties are handled consistently. This will be illustrated in detail subsequently.



3. MEASURES OF UNCERTAINTY IN RESISTANCE AND LOADS

The discussion in Section 2 has shown that the assessment of the various measures of uncertainty plays a central role in reliability-based design. As noted previously, the uncertainty associated with a particular variable is measured conveniently in a non-dimensional form by its coefficient of variation abbreviated c.o.v. The uncertainties used in the reliability analysis should include all imponderables which may affect design safety, and would include not only inherent statistical variability but also sources due to errors in prediction and in imperfect or incomplete information [3,15]. The key test in differentiating between these two sources of uncertainty is whether the acquisition of additional information would materially reduce their estimated value [15]. If the variability is intrinsic to the problem, additional sampling is not likely to reduce its estimate (although the confidence interval on the estimate will contract). In contrast, uncertainties due to modeling and prediction should decrease as improved models and additional data become available.

In this section general methods for analyzing uncertainties are discussed and results of this analysis are summarized for reinforced concrete member capacities and structural loads. A detailed examination of individual measures of uncertainty is deferred to the Appendix to this report.

3.1 Methods of Analysis

Let X denote a basic resistance or load design variable. The true statistical characteristics of X should be employed when evaluating p_f or β and when deriving appropriate factors of safety for design. However, the random characteristics of X usually are not known precisely in structural engineering problems owing to insufficient data and imperfect information. Generally available instead are estimates $\bar{X}$ and δ_X of the mean and c.o.v. of X. These are usually computed from data gathered under ideal or carefully controlled conditions. Therefore, while δ_X reflects basic statistical variability, it fails to encompass all sources of uncertainty that contribute to the total variability in X, Ω_X. Portions of Ω_X may arise from factors which are inherently random, but which were not practical or feasible to include in the experiment or its analysis. For example, material strengths which are commonly determined for design purposes from standard laboratory tests may differ significantly from their strengths in situ. Load idealizations whose effect cannot be evaluated objectively are an additional source of uncertainty. Moreover, despite a comprehensive effort to identify sources and magnitudes of design uncertainty, there will invariably be some that are neglected or unknown.

These various omissions constitute prediction and modeling errors. If the uncertainty measure attributed to these factors is given by the c.o.v. Δ_X, then according to procedures described in detail by Ang and Cornell, [3,4,15] Ω_X is evaluated as,

$$\Omega_X = [\delta_X^2 + \Delta_X^2]^{1/2} \tag{3.1}$$

Frequently, Δ_X can be broken down into several parts, in which case, $\Delta_X^2 = \Delta_1^2 + \Delta_2^2 + \ldots$ It is implicit in this formulation that Δ_X measures primarily the uncertainty in predicting the true mean of X by $\overline{X}$. Often, Δ_X must be estimated on the basis of engineering experience, including the engineer's feel for the completeness and adequacy of his assessment of the design problem. Thus, the capability for the exercise and incorporation of engineering judgment is not entirely removed. This is attractive in engineering reliability analyses, where experience is often as valuable an indication of field peformance as data taken under artificially controlled test conditions that may not be representative of actual practice.

When data are available, $\overline{X}$ and δ_X can be estimated from the data samples using classical statistical analysis techniques. However, it is frequently the case, at least in structural reliability analyses, that very little data are available to supplement the engineer's judgement. Occasionally, he may only be able to estimate the range over which he feels, from past experience, the data should lie. In this case, the c.o.v. may be estimated from the range if some assumption is made regarding the shape of the probability distribution as well. For example, if it is assumed that values between x_1 and x_2 are equally likely (X is uniformly distributed), then

$$\delta_X \simeq \frac{1}{\sqrt{3}} \frac{x_2 - x_1}{x_2 + x_1} \tag{3.2a}$$

If it is assumed instead that values in the midrange are more likely than those near the extremes (X has a bell-shaped probability density) and that roughly 95 percent of the values fall within x_1 and x_2, then

$$\delta_X \simeq \frac{1}{2} \frac{x_2 - x_1}{x_2 + x_1} \tag{3.2b}$$

Similar techniques may be employed to estimate Δ_X, provided that information on the range of the <u>means</u> is available.

The resistance and load terms are usually functions of other basic random variables and in order to apply Equations 2.5 through 2.8, their means and variances (or c.o.v.) must be determined from the known statistics of the component basic variables. This may be accomplished using linear statistical analysis, which involves expanding the function in a Taylor series about some convenient value, truncating the higher order terms, and taking mathematical expectations of the linearized result. If the expansion is taken about the means of the component resistance variables, for example, this leads to [4],

$$E[R] \simeq g_R\ (\bar{R}_1, \bar{R}_2, \ldots) \tag{3.3a}$$

$$\sigma_R^2 \simeq \Sigma\left(\frac{\partial f}{\partial R_i}\right)^2 \sigma_{R_i}^2 + \sum_i \sum_j P_{ij}\left(\frac{\partial f}{\partial R_j}\right)\left(\frac{\partial f}{\partial R_j}\right)\sigma_{R_i}\sigma_{R_j} \tag{3.3b}$$

in which the derivatives are evaluated at the point of expansion and P_{ij} is the correlation coefficient between R_i and R_j. Usually, the basic variables are selected so as to be mutually uncorrelated, so that the second term in Equation 3.3b is zero.

There may be certain instances where it may be appropriate to expand about some other point than the means. For example, the reliability analysis described in Section 2 involves expanding the limit state function about a point on the failure surface.

In addition to variability in the basic variables, the mathematical model used to describe resistance R (or load S) may be biased and may contribute to the overall variability in resistance or load. This is because analyses of structural resistance are invariably based on assumptions and approximations which may not be strictly valid. For instance, let R be the actual resistance and R_u be the modeled, or predicted, resistance. Using a simple linear model, R and R_u are related by

$$R = BR_u + \varepsilon \tag{3.4}$$

in which B is model bias and ε is a zero-mean error term. The mean and variance of R_u are given by,

$$\bar{R} = \bar{B}\ \bar{R}_u \tag{3.5a}$$

$$\sigma_R^2 = \bar{B}^2\ \sigma_{R_u}^2 + \sigma_\varepsilon^2 \tag{3.5b}$$

in which σ_ε^2 is a measure of the observed variability in R_u on the condition that all parameters in R_u are known and where $\overline{R}_u$ and $\sigma^2_{R_u}$ are mean and variance in R_u, which may be found using Equation 3.3. If $\overline{B} = 1$, the model is unbiased. As with Equation 3.1, Equation 3.5 emphasizes the contribution of two separate components to the variance in the actual resistance R. If $\sigma^2_\varepsilon = \overline{R}^2_u \sigma^2$, conforming to the commonly observed tendency for scatter in R to increase as R_u increases, $\overline{B}$ and σ^2 may be estimated as the mean and variance of the ratio R/R_u, which is a common way of comparing test data with theoretical predictions. More precisely, σ^2 is mathematically equivalent to the conditional variance Var $[R/R_u | R_u = x]$. Defining $\Omega_R = \sigma_R/\overline{R}$ and $\Omega_{R_u} = \sigma_{R_u}/\overline{R}_u$, we have

$$\Omega^2_R = \Omega_{R_u}^2 + (\sigma/\overline{B})^2 \tag{3.6}$$

from Equations 3.5a and 3.5b. The second term in Eq. 3.6 is seen to be equivalent to the Δ - term in Eq. 3.1, discussed by Ang and Cornell [3,4] when applied to the strength prediction equation itself.

3.2 Summary of Resistance and Load Uncertainties

An analysis of individual measures of uncertainty in the capacity of reinforced concrete members and in structural loads was performed using the concepts described in Section 3.1. This analysis is described in detail in the Appendix to this report. The results are merely summarized in this section, so as to better maintain continuity in the overall presentation. Along with the c.o.v. in resistances and loads, the ratios between mean and nominal values $\overline{R}/R'$, $\overline{S}/S'$ are also presented, as this information is required to perform the calibration in Section 4. Finally, statistics used in other recent reliability studies are also given; the similarity of the results obtained may be noted, despite the sometimes different methods used in their derivation.

Resistance and load statistics are summarized in Tables 3.1 and 3.2, respectively.

Table 3.1 - Summary of Statistics for Resistance

Source	Flexure		Shear		Compression and Bending	
	$\overline{R}/R'$	Ω_R	$\overline{R}/R'$	Ω_R	$\overline{R}/R'$	Ω_R
This study	1.12	0.13-0.16	1.18	0.21-0.23	0.97-1.13	0.14-0.21
Ellingwood, Ang [21]	1.16	0.16-0.17	-	0.18-0.24	-	0.14-0.22
Siu, et. al [58]	1.14	0.15	1.10	0.21	1.14	0.16

Table 3.2 - Summary of Statistics for Load**

Source	Dead		Maximum Live		Instantaneous Live		Maximum Wind	
	$\overline{S}/S'$	Ω_S	$\overline{S}/S'$	Ω_S	$\overline{S}/S'$	Ω_S	$\overline{S}/S'$	Ω_S
This study	1.0	0.10	Fig. A.6	0.26	Varies	Varies	1.2	0.31
Ellingwood, Ang [21]	1.0	0.13	Varies	0.33	Varies	Varies	1.0	0.37
*Siu, et. al. [58]	1.0	0.07	0.7	0.30	-	-	0.80	0.25
Galambos [28]	1.0	0.06	Varies	0.24	Varies	Varies	1.0	0.26
*Allen [1]	1.0	0.07	0.7	0.30	Varies	Varies	0.8	0.25

*Siu, et. al [58] and Allen [1] based their estimates of $\overline{S}/S'$ on the Canadian Standard [43].

**The Ω_S terms include a component to account for the analysis used to transform the load to a load effect (moment, shear, etc.), as discussed in the Appendix.

4. CALIBRATION WITH EXISTING DESIGNS

Target reliabilities or safety indices, along with the measures of uncertainty summarized in Section 3, provide the basis for reliability-based criteria. These may be established either by prescribing *a priori* values which are consistent with overall societal objectives or by calibration with existing building codes. The calibration approach appears appropriate in the short term because it references the reliability analysis to existing designs, relying on the consensus that provisions in current design codes are generally adequate from a safety standpoint, and thus avoids the need to specify an acceptable risk explicitly. It allows the usefulness of the reliability analysis techniques to be demonstrated in a familiar context and enables certain inconsistencies in current criteria to be eliminated.

The calibrations in this section determine the β values which are tacitly associated with design criteria in Chapter 9 of ACI 318-71 [10] for the ultimate limit states. These criteria are given by,

$$\phi' R' \geq \begin{cases} 1.4D + 1.7L'; & \text{[ACI 318-71 Eq. 9-1]} \\ 0.75\ (1.4D + 1.7W' + 1.7L'); & \text{[ACI 318-71 Eq. 9-2]} \\ 0.9D + 1.3W'; & \text{[ACI 318-71 Eq. 9-3]} \end{cases} \tag{4.1}$$

where the resistance factor ϕ' depends on the particular ultimate limit state (flexure, shear, etc.) under consideration. Other loads may be substituted for W'. Calibrations are performed on an individual member basis, as this is the approach normally taken in proportioning members.

Calibration according to Equations 2.5 through 2.8 requires that the mean resistance obtained using existing design criteria be computed for various load combinations. The dead plus maximum live load case is considered to be of fundamental importance. It is virtually impossible to calibrate against all possible design situations or load combinations, since not only would the number be prohibitive, but in many cases the information available may be of insufficient quality for a meaningful calibration to result. Instead, it appears appropriate to thoroughly analyze design cases which have demonstrated satisfactory past behavior and where the information is sufficient that the β values obtained from calibration can be viewed with some confidence. The gravity load case is the most obvious of these, and indeed controls in many practical design situations. Calibration against the gravity plus wind load combination will also be briefly considered for comparison.

4.1 Gravity Loads

Safety indices associated with structural designs which conform to Equation 9-1 of ACI

318-71 are shown in Figures 4.1a, b, c and d for flexure, shear and axial compression combined with bending.

The live load specified by ANSI A58.1-1972 [9] has been selected for these calibrations. The ANSI design live load L' consists of a basic uniform load L_c (e.g., L_c = 50 psf (2394 N/m^2) in offices) which may be reduced when L_c is less than 100 psf (4788 N/m^2) and tributary area A_t exceeds 150 ft^2 (13.9m^2). Thus,

$$L' = L_c \left[1 - \min \left[0.0008A_t, 0.6, 0.23 (1 + D/L_c)\right] \right] \qquad (4.2)$$

This relation is illustrated in Figure A.6 of the Appendix for the case where L_c = 50 psf (2394 N/m^2). If $L_c/D < 0.62$, the maximum live load reduction of 60 percent may be applied. This would be the case for many reinforced concrete structures in which the dead load is a major portion of the total load. However, the allowable reduction is limited where L_c/D is large. An examination of L_c/D for typical occupancies in reinforced concrete buildings including offices, corridors, schools, hotels, and retail establishments showed that L_c/D may vary from 0.25 to about 1.6, with a substantial percentage being less than 0.8. On the other hand, the live load of interest for analyzing the reliability is the maximum value which occurs during the useable life of the structure. Its statistics are given in the Appendix; although these are based on an analysis of office live loads, the limited data on c.o.v. in live loads for other occupancies appears similar. The mean value is compared to the ANSI live load in Figure A.6. The ratio between the nominal live load and mean live load is a function of the tributary loaded area and L_c/D.

This discussion shows that β for existing designs will depend on L_c/D and A_t. However, the reduction in live load permitted by ANSI A 58.1-1972 is not consistent, in a statistical sense, with the actual reduction in the mean live load. This is reflected in the dip in the safety index (Figure 4.1) which occurs e.g., at A_t = 750 ft^2 (69.7 m^2) when L_c/D = 0.5. Similar results were obtained by Galambos and Ravindra [28] in their calibration work with AISC specificiations [60] which govern steel design. In contrast, no such irregularities occur in calibrations performed on the Canadian Building Code [1,43,58] because the live load reduction formula therein has a form similar to Equation A.13 which is consistent with the statistical analysis of the data.

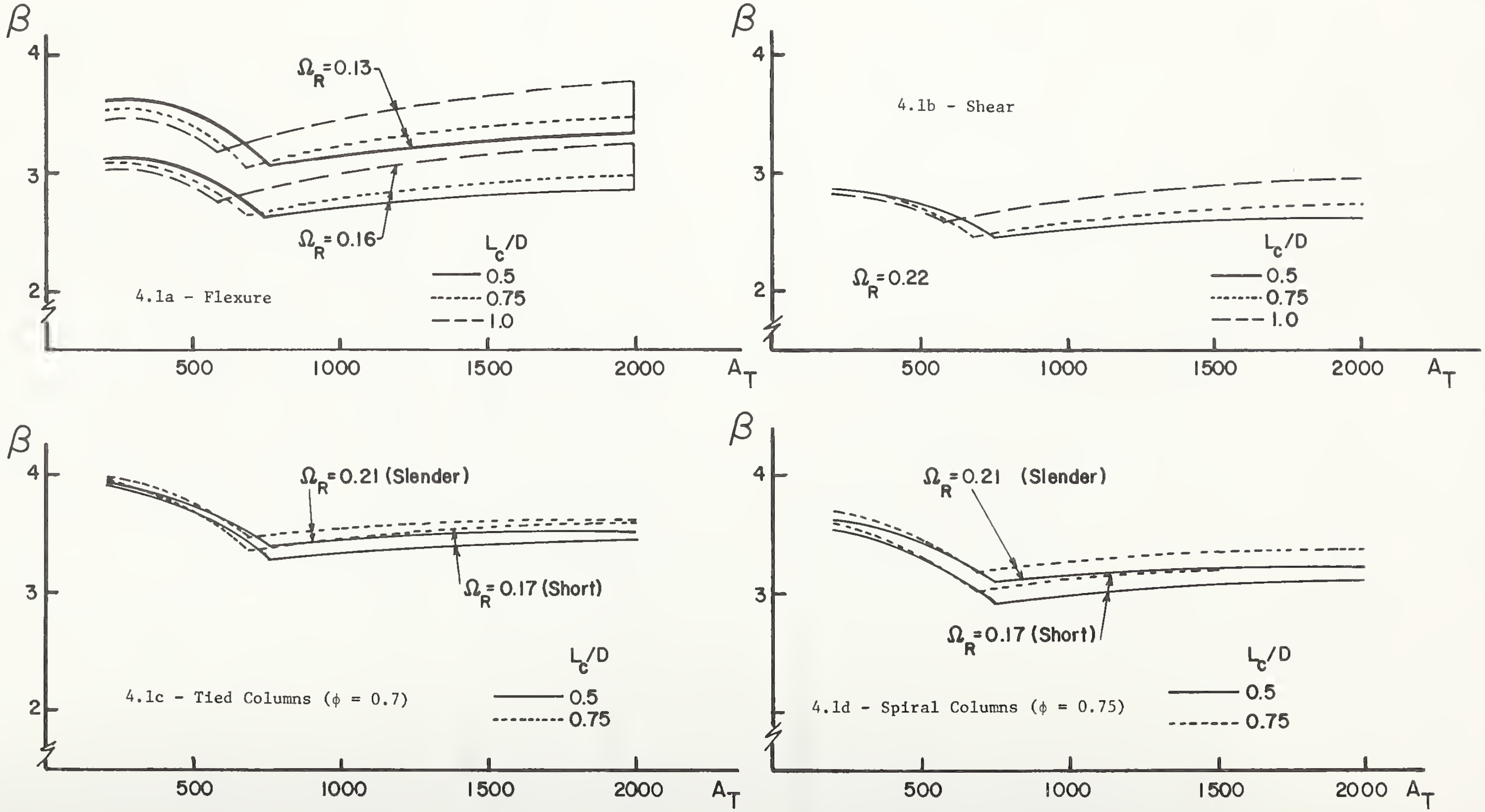


Figure 4.1 - Safety Index for Designs Conforming to ACI Standard 318-71 (Gravity Loads)

When $L_c/D > 0.62$ the point of minimum β shifts toward smaller tributary areas. This is caused by the limit $0.23(1 + D/L_c)$ on the maximum allowable live load reduction, which shifts the point of minimum $L'/\bar{L}$ to smaller tributary areas, as shown in Figure A.6. While the higher live load and its large attendant variability causes the reliability to be less at small A_t where the above limit is not effective, at large areas the limit on load reduction actually causes the β to increase as L_c/D increases.

The sensitivity of β to variations in Ω_R is also shown in Figure 4.1. As a coarse rule of thumb, an increase of 0.01 in Ω_R causes a decrease of 0.10 in β. It may be recalled from Section 3.2 and the Appendix that Ω_R in flexural resistance decreases from 0.16 as the depth of the member increases because the variability in bar placement d decreases for deep members.

It may be observed from Figure 4.1b that in all cases the β value for shear is less than for flexure, despite the desirability for beam and slab failures to occur in the ductile flexural rather than the relatively brittle shear mode. This conclusion has also been reached in other studies [21,58]. The reason for this should be clearly understood. While the c.o.v. in flexural resistance is of the order 0.13 - 0.16, the c.o.v. in shear capacity is of the order 0.21 - 0.23. On the other hand, the present ACI code accounts for this increase in variability and the undesirability of a shear failure by subjectively reducing the resistance factor from 0.9 for flexure to 0.85 for shear. This small reduction is inadequate to account for the increased variability and desired reliability in shear. This is an excellent example of the type of inconsistency which may be common in current building codes because of their failure to consider uncertainties explicitly.

Figure 4.1c shows that the safety for tied columns is greater than for beams, which is in accordance with ACI objectives; the corresponding difference in β implies over an order of magnitude difference in risk. As shown by comparing Figures 4.1c and d, the increase in ϕ' to 0.75 for columns with spiral reinforcement results in a decrease in β of roughly 0.3 from that for tied columns at all tributary areas. The spirally reinforced columns behave in a more ductile fashion than tied columns and it is primarily on account of this ductility that ϕ' is increased to 0.75. Thus, the difference in β is apparently what additional ductility is currently worth to ACI, in terms of reliability. The ACI code also currently permits ϕ' for axial compression combined with bending to be increased toward the ϕ' for bending when the axial load is small. This increase in ϕ' appears justifiable primarily on the grounds of the associated increase in ductility (see Appendix).

For purposes of comparing reinforced concrete and steel designs, the same analysis was performed for steel beams designed according to Section 2 of the AISC specifications [60]. Therein, $\phi' = 1.0$ while $\gamma_D' = \gamma_L' = 1.7$. According to Galambos and Ravindra [28], $R'/\bar{R} = 0.93$ and $\Omega_R = 0.13$. The corresponding safety indices are shown in Figure 4.2 for various L_c/D, which tend to be higher for steel construction than for reinforced concrete. It appears from Figure 4.2 that an appropriate β would be about 3.25 for the design of steel beams. Galambos used a lognormal type of reliability analysis suggested by several researchers [3,4,52] which differs from the one described in Section 2. A portion of his results are also shown in Figure 4.2 for comparison.

Several conclusions may be drawn from this analysis of concrete and steel flexural designs. First of all, a comparison of Figures 4.1a and 4.2 shows that current gravity load design criteria for reinforced concrete and steel beams appear to result in similar levels of reliability. Secondly, there does not appear to be a practically significant difference, at least in this case, in the results obtained using the two different reliability analyses, as may be seen in Figure 4.2 Finally, the use of one overall factor of safety, such as the 1.7 prescribed by AISC, Section 2 (or, for that matter, as implied by any working stress provision where the factor of safety is, effectively, the reciprocal of the ratio of allowable to ultimate stress) leads to more scatter in the resulting value of β than the use of the split load factors followed by ACI 318-71. This may be seen quite clearly in Table 4.1.

Table 4.1 - Comparison of β for Steel and Concrete Designs when $A_t = 500\ ft^2 (46.5\ m^2)$

	L_c/D				
	0.25	0.50	0.75	1.0	1.5
Steel ($\Omega_R = 0.13$)	3.731	3.551	3.357	3.182	2.984
Concrete ($\Omega_R = 0.16$)	3.028	3.004	2.946	2.881	2.839

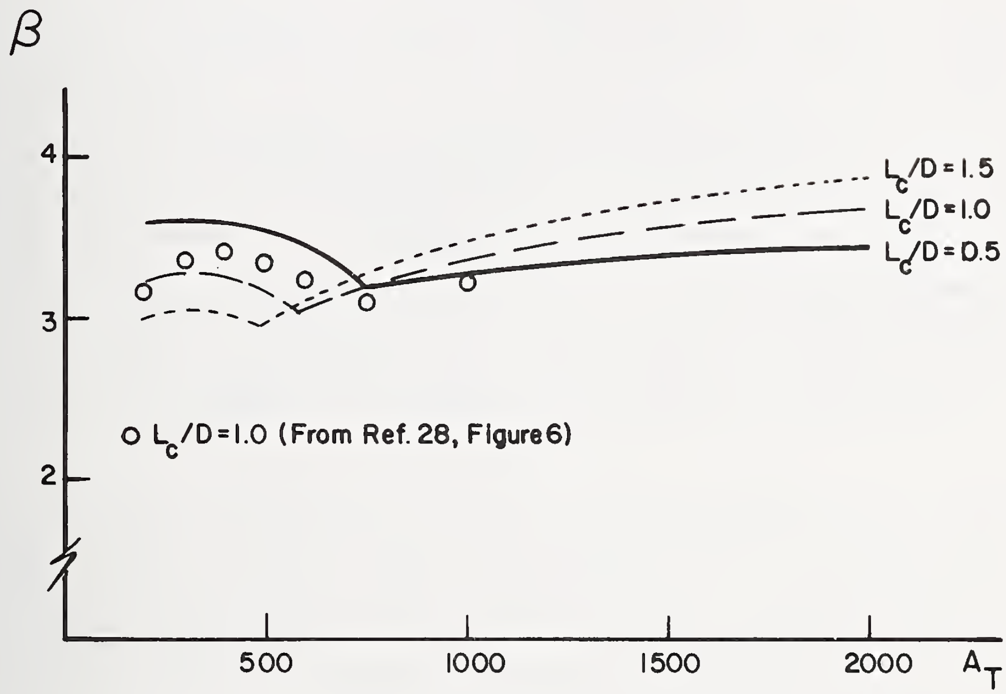


Figure 4.2 - Safety Index for Steel Beams Conforming to AISC Specifications, Section 2 (Gravity Loads)

As the L_c/D increases, the larger variability in L becomes increasingly more important for reliability and must be accounted for by a larger load factor. It may be inferred that the split load factor approach is more desirable than the single load factor approach in situations where one overall load criteria (independent of load ratio) must be given.

4.2 Gravity Loads Combined with Wind

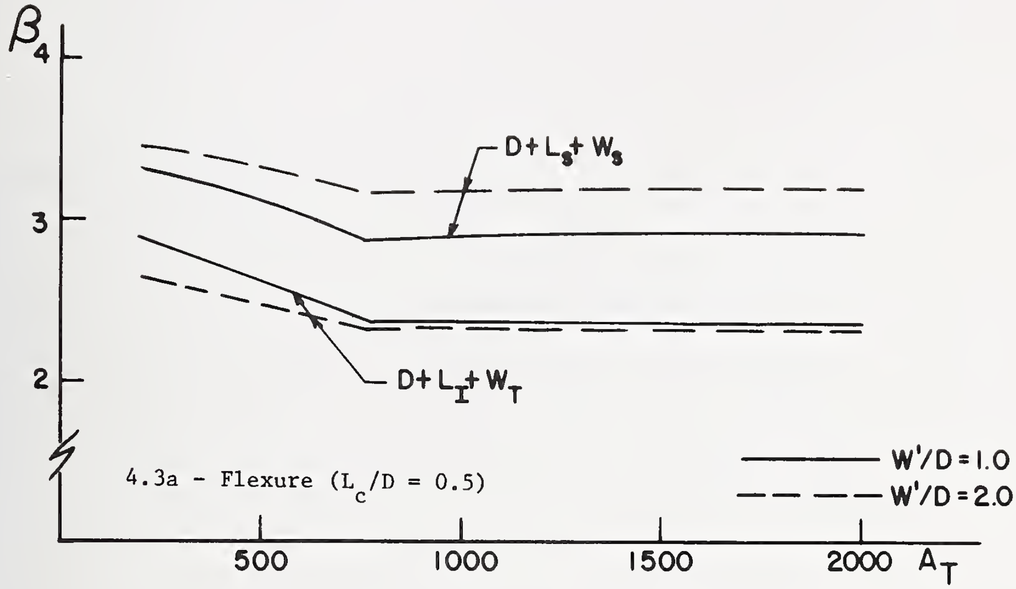
Calibration performed on structural designs conforming to Equation 9-2 and resistance factors of Section 9.2 of ACI 318-71 are shown in Figures 4.3a and 4.3b for flexure and thrust in tied columns. β is shown as a function of A_t and is computed for both load combinations in Equations A.17a and A.17b. It may be seen that Equation A.17b represents the critical combination for the case illustrated ($L_c/D=0.5$). β is virtually constant when A_t is large because the ratio $L'/\overline{L}_I$ is constant at large areas for short term live load and the contribution of the large variability Ω_{L_I} is dampened by the typically low values of $\overline{L}_I/\overline{D}$. (As seen from Figure 4.1, this is not the case for maximum live load).

An example of safety indices achieved by existing column designs is given in Table 4.2. The column supports a tributary loaded area of 750 ft.^{2} (69.7m^2). This corresponds to the "worst" case of Figure 4.2 in that the ratio $L'/\overline{L}_I$ is smallest at this area. The slight tendency for the reliability to increase with increasing L_c/D may be noted, as well as its relative constancy with W/D at moderate values of L_c/D.

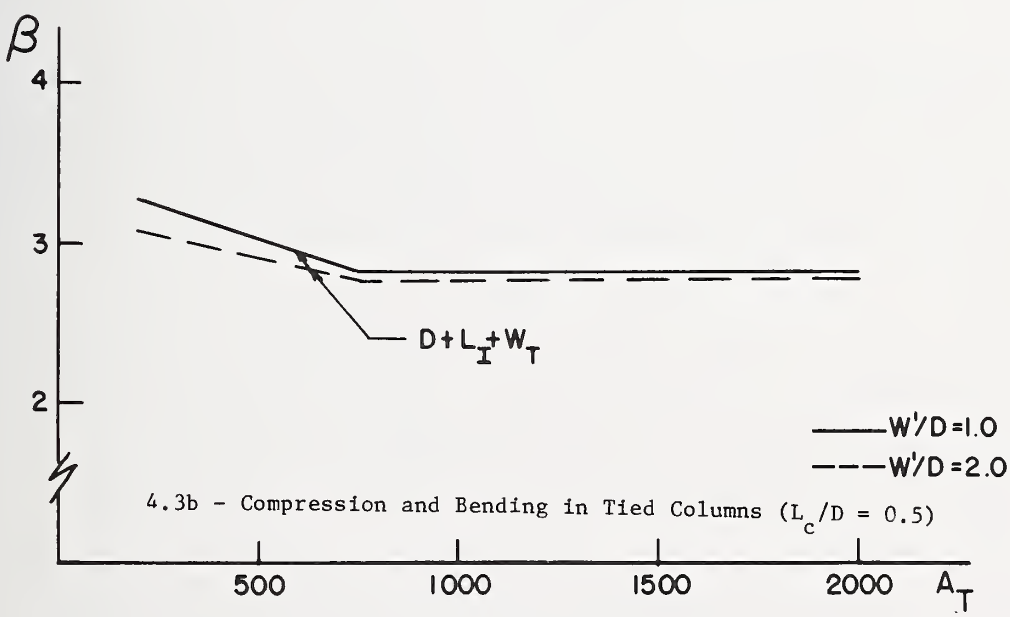
Table 4.2 - Safety Index for Existing Tied Column Designs

A_t = 750 ft^2 (69.7m^2)

	L_c/D				
W'/D	0.25	0.50	0.75	1.0	1.5
0.5	2.680	2.825	3.084	3.417	3.907
1.0	2.727	2.839	3.041	3.309	3.741
2.0	2.698	2.773	2.911	3.100	3.431
3.0	2.667	2.724	2.827	2.971	3.233
$\phi' = 0.7$; $\gamma_D' = 1.05$; $\gamma_L' = \gamma_W' = 1.275$					



4.3a - Flexure ($L_c/D = 0.5$)



4.3b - Compression and Bending in Tied Columns ($L_c/D = 0.5$)

Figure 4.3 - Safety Index for Designs Conforming to ACI Standard 318-71 (Gravity plus Wind Loads)

For consistent reliability, it would be desirable for the β values to be the same for both (D + L) and (D + L + W) loading combinations. However, comparison of Figures 4.1 and 4.3 indicates that the probability of failure under the (D + L + W) combination is approximately one order of magnitude higher than for (D + L). There are a number of reasons why the actual reliability under wind load may exceed the apparent values in the analysis above. Among these are the wind directionality effect alluded to earlier, the fact that the extreme wind may not act in the direction most unfavorable to structural response. In addition, elements which are not designed as load bearing may, nevertheless, carry a certain portion of load and thus reduce the wind load resisted by the structural system. Lateral drift limitations for tall buildings may also require that the lateral load carrying system be stiffened above that required by an ultimate strength analysis alone. There remains, however, the very real possibility that current reinforced concrete designs may not have as high an implied margin of safety for combined loads as for gravity loads alone.

When lateral and gravity loads counteract one another, as e.g., the case for columns on the windward side of a building, designs must conform to Equation 9 - 3 of ACI 318-71. Corresponding β values obtained for columns with tributary areas exceeding 750 ft^2 (69.7m^2) are shown in Table 4.3.

Table 4.3 - Safety Index for Tied Columns with Counteracting Loads

A_t = 750 ft^2 (69.7m^2)

	L_c/D				
W'/D	0.25	0.50	0.75	1.0	1.5
1.0	2.680	2.846	2.993	3.126	3.359
2.0	2.754	2.840	2.920	2.995	3.134
3.0	2.727	2.784	2.839	2.891	2.990
$\phi' = 0.7$; $\gamma'_D = 0.9$; $\gamma'_W = 1.3$					

Safety indices for smaller W'/D are not given, as one of the two preceding load combinations would normally govern design in that case. Neglecting the beneficial contribution of live load when gravity and wind loads effects counteract one another is obviously conservative, as shown by the increase in β with L_c/D. Comparison of the results in Tables 4.2 and 4.3 shows that Equations 9-2 and 9-3 of ACI 318-71 are basically risk-consistent for moderate values of L_c/D.

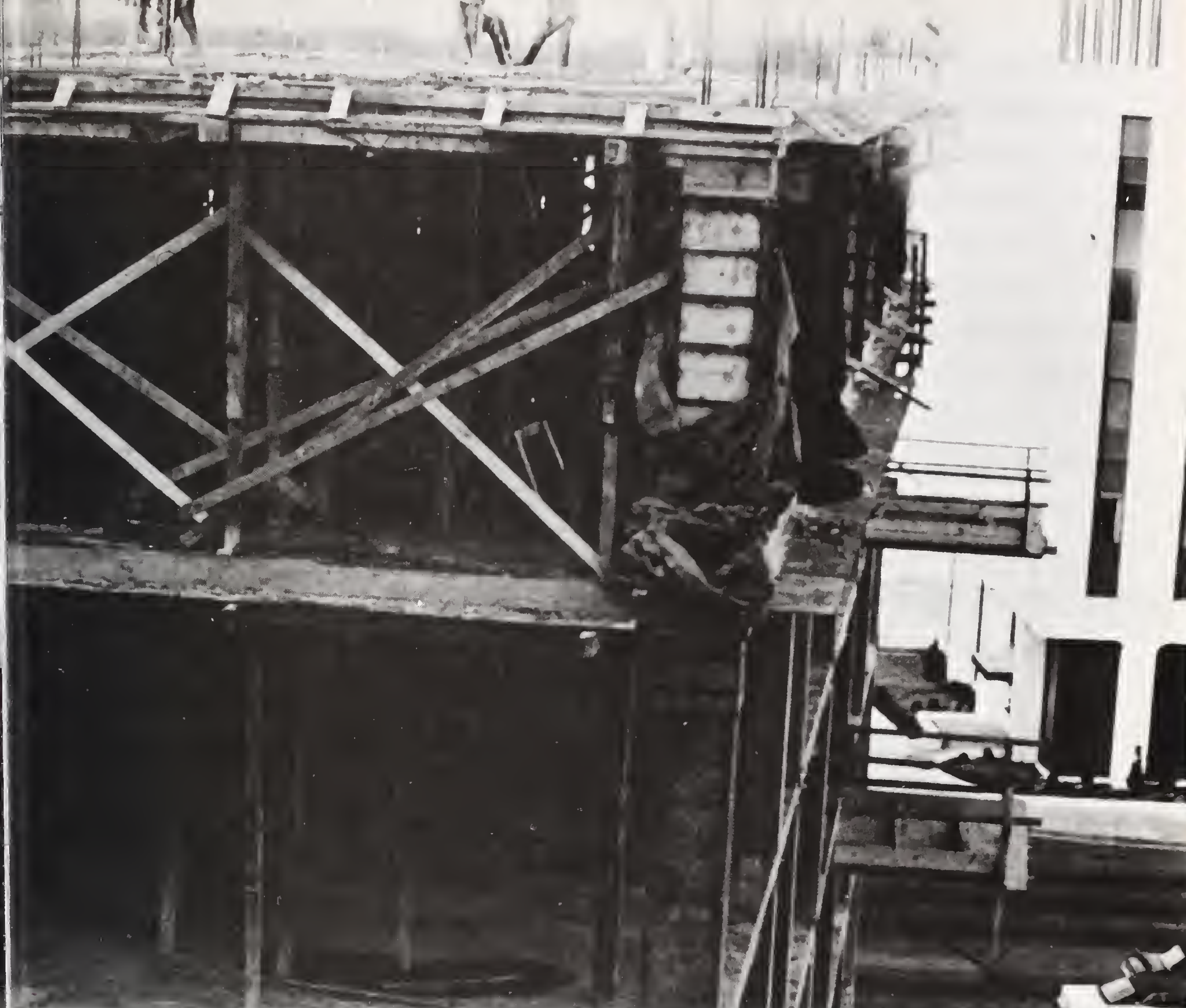
4.3 Target Reliabilities for Design

Target safety indices for reinforced concrete design were selected considering all of the calibrations performed in the previous sections. In the study by Siu, et. al. [58] on the Canadian code, a weighted average of all the β - values was taken, weighted by the probabilities of occurrence of the various mean load ratios ($\overline{L}/\overline{D}$, $\overline{W}/\overline{D}$). However, the live load reduction employed in the Canadian Standard is basically consistent with statistical analysis while the ANSI reduction formula is not. As a consequence, Siu's values of β do not depend on the tributary or influence area while the ones in this study do. Thus, appropriate target β values were necessarily determined more subjectively in this study.

For flexure, the value of β selected was 3.0, for tied columns, 3.5, and for spirally reinforced columns, 3.2. These values will tend to provide slightly more safety than existing designs at large tributary areas and slightly less at small areas. Considering consequences of failure, additional safety would probably be desirable at large areas. The difference in β implies that columns will be designed with a probability of failure which is at about one order of magnitude less than for beams. While this may not be deemed to be a sufficient additional margin of safety in view of the relative consequences to the structure of the beam and column failure, it appears justifiable on the basis of current practice. This points out one disadvantage in relying too heavily on calibration to set design safety standards. On the other hand, an increase in β to 4 or more for columns would result in designs which are more conservative than currently acceptable for *all* A_t and would probably be rejected by the profession on economic grounds.

The value of β for shear presents a problem in that current values obviously are too low and are incompatible with the desired objective of having members fail in flexure rather than shear. In this study, β was increased to 3.0 for shear, this being the minimum adjustment necessary to make the reliability in shear no less than that in flexure. While it may well be desirable that β for shear exceed β for flexure, further discussion of this point by standards committees appears advisable because of the impact that such a recommendation would have on current design. It should also be noted that β for columns ranges from 3.2 to 3.5. In view of the relative consequences of beam and column failure, it certainly does not appear necessary to increase β for shear above 3.2.

In contrast, Galambos [28] prescribed β = 3.0 for all structural members in his development of load and resistance factor criteria for steel design. While simplifying criteria development, the relative consequences of failure in beams and columns, the latter of which is typically more severe, thus is not considered. It should be observed from the above discussion that the use of calibration as the sole basis for developing reliability based design criteria implies that the current code provisions are internally consistent. Thus, inconsistencies of the type found in shear would tend to carry over into the reliability based code if the results of the calibration are not treated judiciously.



5. DEVELOPMENT OF RELIABILITY BASED DESIGN CRITERIA

Specific design criteria are developed on the basis of target reliabilities furnished by the calibrations discussed in Section 4. Although many different formats are possible, it is felt that simplicity, consistency or continuity with existing formats, and ease of enforcibility initially should guide the selection of these criteria. Such practical considerations are necessary in order for the approach to be accepted by the profession at large.

Three different levels of sophistication for reliability-based design can be identified. At all three levels, statistical variabilities in resistance and loads are taken into account, however. At the highest level, termed Level III, the probability distributions of all variables and necessary convolutions must be known or computed, and the design is based on an acceptable small probability of failure previously agreed upon by the profession. As noted before, this information generally is not available at present. Level II methods employ safety checks at discrete points of the limit state equation (e.g., at selected values of $\overline{L}/\overline{D}$, $\overline{W}/\overline{D}$, etc.), the basic design variables in the limit state equation being specified in advance. Probability distributions are not required for Level II methods in which reliability is measured by the safety index β. Level I methods involve the selection of one set of load and resistance factors to be applied to all designs, regardless of $\overline{L}/\overline{D}$, $\overline{W}/\overline{D}$, etc.; current design criteria are of the Level I type. Levels I and II can be made equivalent if the load and resistance factors are allowed to vary in accordance with the load ratios.

In this section, Level II design procedures for flexure, shear and thrust are presented. The criteria in this study are presented in terms of <u>mean</u> resistance and load values; to convert to the <u>nominal</u> resistance and load values that designers may be more familiar with, simply multiply the factors by the ratio of the mean to nominal value. (The nominal value is the code-specified minimum value used in design; for example, for Grade 40 reinforcement, the nominal yield stress is 40 ksi, while the <u>mean</u> may be about 47 ksi, depending on the bar diameter.)

Specific design criteria generally take the form of an equation describing some limit state in which certain "resistance" and "load" factors are appended to terms on the resistance and load sides of the equation, respectively. The safety check for ultimate limit states requires that the factored ultimate resistance R* exceed the factored load U*, or

$$R^* > U^* \tag{5.1}$$

There are at least two ways of specifying the factored resistance R*. In the first, followed by the current ACI provisions, one overall resistance factor (RF) ϕ is appended to the resistance side of the design equation for each limit state. Using this approach, the factored ultimate resistance, R* for a particular limit state, thus becomes,

$$R^* = \phi \, \overline{R} \tag{5.2}$$

Here the basic resistance variable is R itself, and the resistance function g_R in Equation 2.5 is simply R. This is also the format selected by Galambos [28], for steel design. It has the virtues of simplicity and familiarity to American designers, as well as an ability to incorporate the consequences of failure in a particular limit state in the resistance factor for that limit state.

A second approach employs partial factors (PF) which are applied to the means of the individual variables in the resistance equation. For example, let $f_y^* = \phi_y \overline{f}_y$, $f_c^* = \phi_c \overline{f}_c$, $h^* = h_n - \Delta_h''$ and $d^* = d_n - \Delta_d''$, where Δ_h'' and Δ_d'' are small dimensions (on the order of 1/4 - 1/2 inch (6.4 - 12.7 mm)) subtracted from nominal dimensions for purposes of safety checking. The factored ultimate resistance is then,

$$R^* = R\,(f_y^*, f_c^*, h^*, d^*) \tag{5.3}$$

In this approach, the partial factors ϕ_y, ϕ_c, etc. are specified only once in the design standard and the same factors would be used for all ultimate limit states. This format was discussed by ACI Committee 318 over a decade ago [48], and has become popular in Europe [12]. The application of safety factors directly to the parameters which are the source of variability is an attractive feature. Despite certain conceptual advantages, this approach was not acceptable to many American engineers and was rejected by ACI membership when it was originally proposed, as there was a feeling that it might penalize concrete construction in relation to steel. This point will be discussed further after specific reliability-based partial factors are derived.

Similarly, the load side of the equation may be treated several ways. The simplest is to apply one load factor to the sum of whatever loads the structure's limit states must be checked against; the factored ultimate load would then be,

$$U^* = \gamma\ (D + L + \cdots) \tag{5.4}$$

This is equivalent to AISC Section 2 [60]. This format recognizes that it is the sum of the loads acting on the structure at any time instant which is important. However, there has been a tendency to attach separate load factors to each load. Among the reasons for this are that the variability in permanent and transient loads are often of different orders of magnitude and that individual design loads, such as live load, wind load, and earthquake load, are usually determined by different technical committees. The factored ultimate load in this format is

$$U^* = \gamma_D D + \gamma_L L + \gamma_W W \tag{5.5}$$

or some variant of this equation.

In this study, criteria of the form

$$\phi\overline{R} > \gamma_D\overline{D} + \gamma_L\overline{L} + \gamma_W\overline{W} \tag{5.6}$$

will be emphasized because it is easiest to tie these to existing ACI criteria and the criteria proposed by Galambos [28] for steel structures. However, the other criteria will also be examined. It is not the intent here to advocate any particular one of these alternatives; indeed, each has certain advantages and disadvantages. Regardless of which format is selected, however, the safety factors determined by the reliability methods are correctly apportioned to the resistance and load sides of the design equations.

It is assumed in this study that the total load effect is a linear combination of the individual loads. The limit state equation may then be expressed in terms of the mean load ratios by dividing Equation 2.7 by $\overline{D}$; thus, the importance of $\overline{L}/\overline{D}$, $\overline{W}/\overline{D}$, etc. in Sections 5 and 6. The load factors in Equations 5.4 through 5.6 should, in general, be applied to the load prior to performing the analysis which transforms the load to a load effect. This is also required by ACI 318-71, Section 9.1.1. Provided that the relation between load and load effect is linear, or nearly so, it makes no difference when the load factors are applied. However, for certain non-linear problems, such as stability checks, it is unconservative to factor the load effect.

5.1 Overall Resistance Factors

For consistency with Galambos' study, criteria taking the form of Equation 5.6 shall be referred to as load and resistance factor design (LRFD). In this regard, it might be noted that the coefficients appended to terms $\overline{R}_i$ and $\overline{S}_j$ in Equations 2.6a and 2.6b are tantamount to resistance and load factors when the criterion takes the form of Equation 5.6. Thus, the reliability-based design problem is to find an appropriate mean resistance and load and resistance factors for a prescribed set of Ω_R, Ω_{S_i}, β and load ratios. The same c.o.v. that were specified for calibration in the previous section are also used here.

The resistance and load factors for the gravity load combination (D + L) are shown as functions of $\overline{L}/\overline{D}$ in Figure 5.1 for flexure, Figure 5.2 for shear, and Figure 5.3 for thrust and bending. Since the calibration showed that β was not constant for existing designs, the sensitivity of ϕ, γ_D and γ_L to β is also examined by showing their variation for two different but reasonable values of β for the flexure case.

Several points are worth noting from Figures 5.1 through 5.3. The most important is that the load factors γ_D and γ_L are quite insensitive to Ω_R, as may be seen by comparing the flexure and shear cases. A certain amount of coupling between the resistance and load factors exists because the structural safety condition is described by one inequality out of which (at most) one independent piece of information (overall safety factor) can be extracted. The fact that this coupling appears relatively weak will have some important implications in developing Level I design criteria to be discussed subsequently. The increase in γ_L with increasing $\overline{L}/\overline{D}$ occurs because as $\overline{L}/\overline{D}$ increases the live load with its associated higher variability comprises a larger portion of the total design load. The

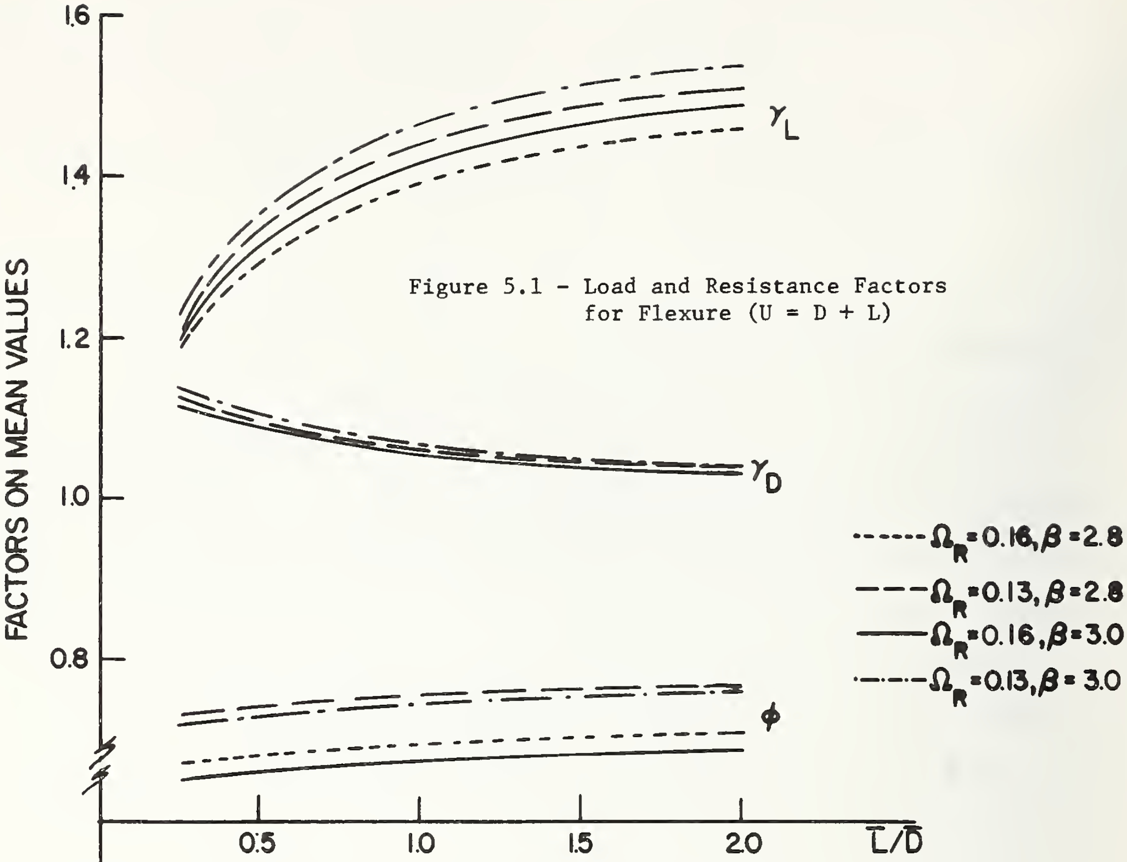


Figure 5.1 - Load and Resistance Factors for Flexure (U = D + L)

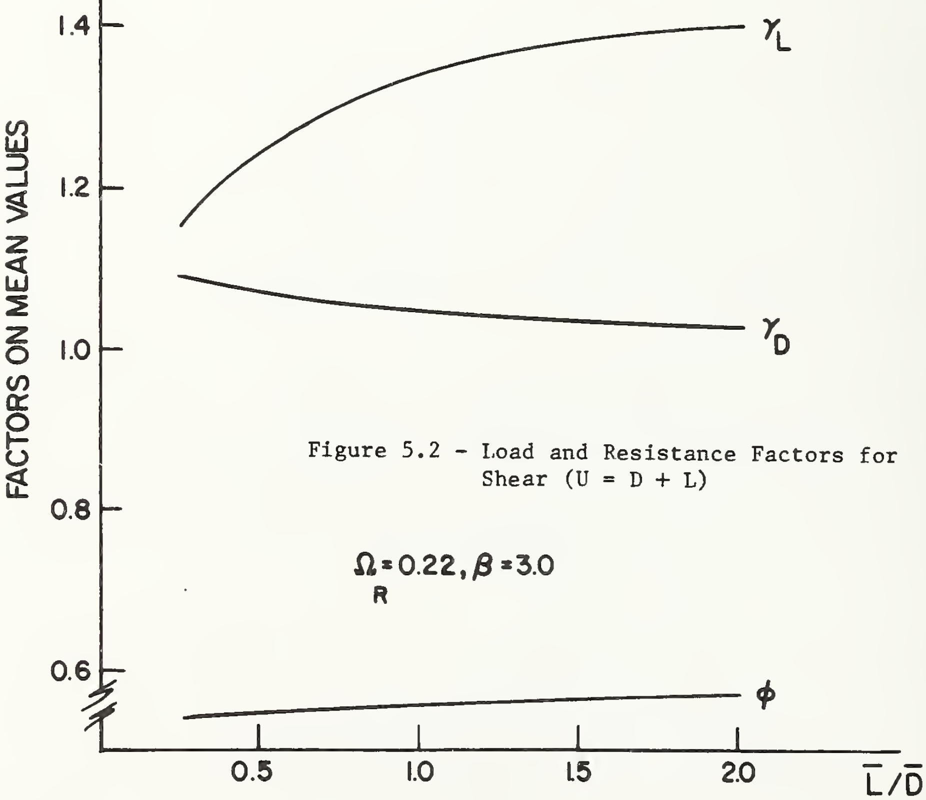


Figure 5.2 - Load and Resistance Factors for Shear (U = D + L)

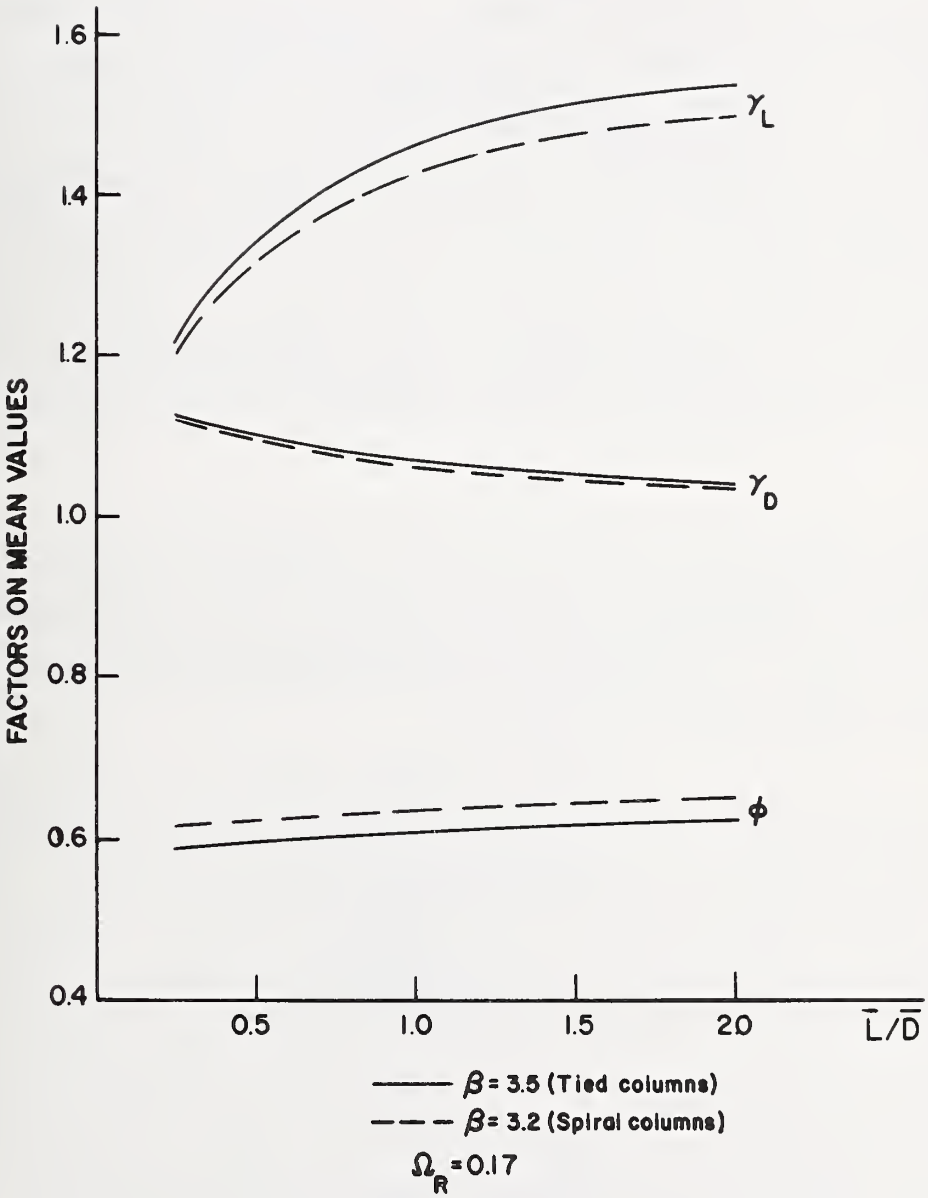


Figure 5.3 - Load and Resistance Factors for Compression and Bending (U = D + L)

fact that γ_L depends on $\overline{L}/\overline{D}$ when β is prescribed also means that in existing design criteria as well as Level I methods, there will be some violation of the reliability requirement for certain values of $\overline{L}/\overline{D}$ (unless, of course, γ_L is chosen as the asymptotic value at large $\overline{L}/\overline{D}$; in this case, however, the designs at other $\overline{L}/\overline{D}$ would be grossly conservative).

It might be observed from Figures 5.1 through 5.3 that γ_D and ϕ appear to be virtually independent of $\overline{L}/\overline{D}$. Moreover, the appropriate dead load factor ranges from about 1.04 to 1.12, and since $D' \approx \overline{D}$, the load factor against nominal dead load should be about 1.1. This is because Ω_D is quite small compared to other variabilities in the design problem. However, existing dead load factors range from 1.4 to 1.7, which may result in design penalties when $\overline{L}/\overline{D}$ is small. It is also apparent that the resistance factors are considerably lower than those in existing standards. This is true for nominal ϕ-factors as well, as may be seen by multiplying the ϕ-values in Figures 5.1 - 5.3 by the appropriate $\overline{R}/R'$.

This analysis suggests that current load factors and resistance factors tend to be too high. Although the combination of the two factors may give results that are about right, the current tendency to lump most of the effect of design uncertainty on the load side of the design equation clearly is unjustified.

Resistance and load factors for the gravity plus wind load case ($D + L_I + W$) are shown in Figure 5.4 for various $\overline{L}_I/\overline{D}$ and $\overline{W}/\overline{D}$ for the flexure case; the discussion which follows also applies if shear or compression and bending are considered instead. The behavior of ϕ and γ_D is virtually the same as when only gravity loads act on the structure. The very rapid increase in γ_{L_I} in Figure 5.4a occurs because Ω_{L_I} is quite large ($\Omega_{L_I} \approx 0.54$ in this case, corresponding to beams supporting tributary areas of greater than 750 ft.^{2} (69.7m^2)). As expected, γ_W also increases with $\overline{W}/\overline{D}$; this is also shown in Figure 5.4b, where γ_W is plotted for $\overline{L}_I/\overline{D}$ = 0.25 and 0.50, which values, according to the load ratio analysis discussed in Section 4; bracket most values of $\overline{L}_I/\overline{D}$. A comparison of Figures 5.4b and 5.1 shows that γ_W is almost the same as γ_L in the gravity load case; the reason for this is that Ω_L and Ω_W are fairly close in value. This property will be taken advantage of when Level I load and resistance factor design criteria are developed.

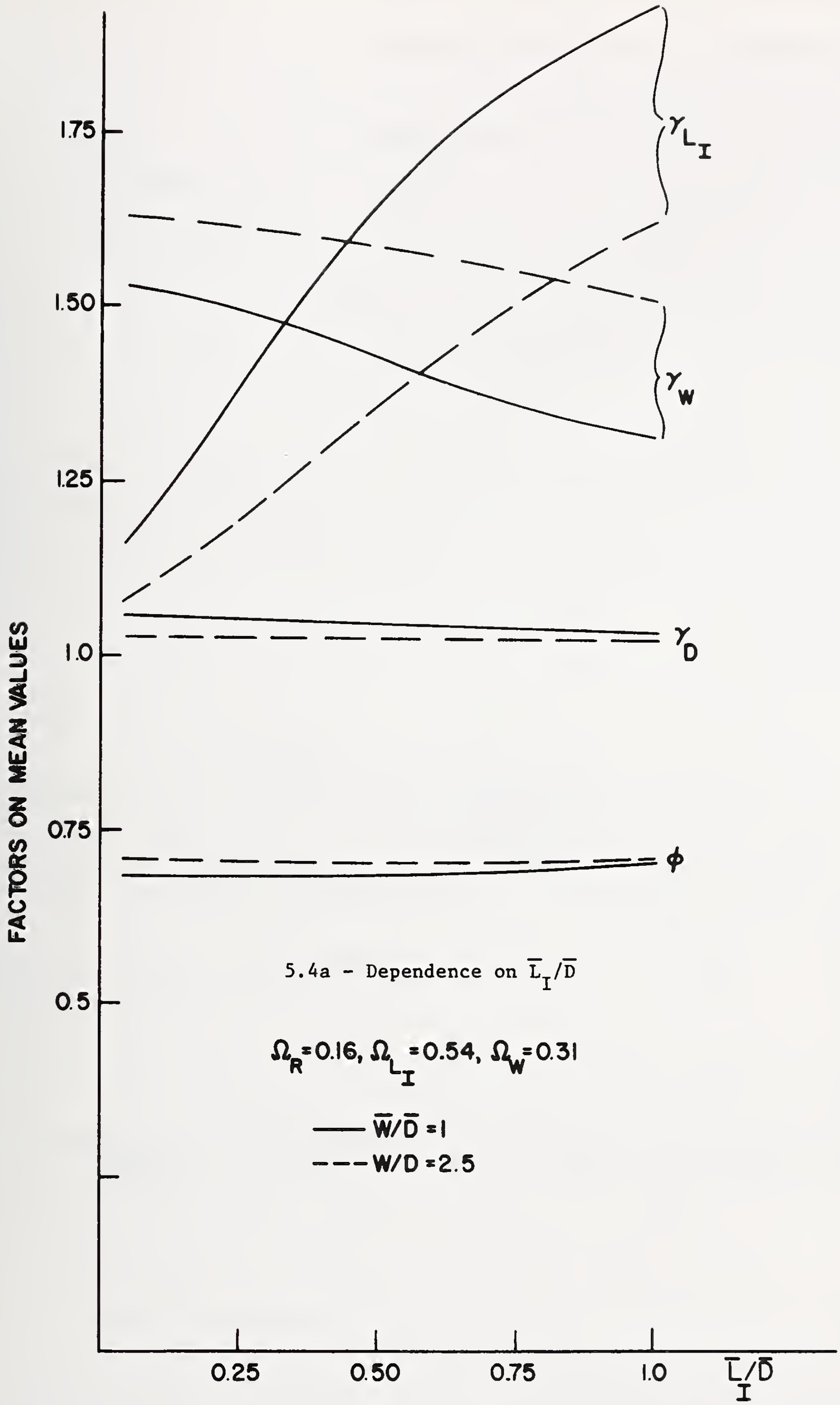
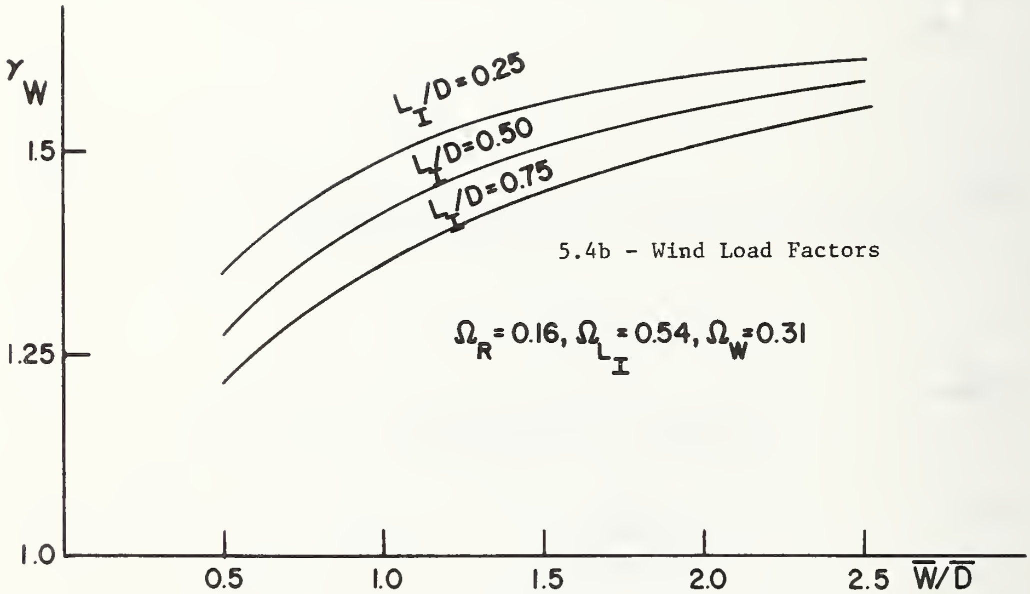


Figure 5.4 - Load and Resistance Factors for Flexure (U = D + L + W)



5.4b - Wind Load Factors

Finally, resistance and load factors for the case where wind and gravity loads counteract one another, W - (D + L), are shown in Figure 5.5 for compression and bending. The behavior of the load factors is quite different than before, both γ_D and γ_{L_I} being less than unity, while γ_W is significantly higher (cf. Figures 5.3 and 5.5). Figure 5.5 shows that in contrast with present practice, a portion of the live load should be subtracted along with the dead load from $\gamma_W \bar{W}$ when computing U*. However, the decrease in γ_{L_I} with increasing $\bar{L}_I/\bar{D}$ indicates that as the live load becomes a larger portion of the total gravity load, $\gamma_{L_I} \bar{L}_I$ becomes smaller and less is subtracted from $\gamma_W \bar{W}$.

It may also be observed that the ϕ-factors are considerably higher than in the two previous cases examined, and appear to be more sensitive to $\bar{W}/\bar{D}$ (cf. Figures 5.5 and 5.3). For example, here ϕ appears to be about 0.75 for compression as opposed to the values of about 0.6 previously observed. This is a potential source of difficulty, assuming that it is desirable for ϕ values to be the same for all load combinations in Level I design.

It is apparent from these three cases that the reliability criteria $\beta = \beta_o$ is impossible to satisfy for all load ratios ($\bar{L}_I/\bar{D}$, $\bar{W}/\bar{D}$) with only one set of load factors. Thus, Level I formats necessarily result in some local violation of the reliability criterion.

5.2 Partial Resistance Factors

In the partial factors approach, the factored ultimate resistance is given by Equation 5.3. It is assumed that capacities derived from ultimate strength principles define the resistance function required by Equations 2.5 and 2.8, when corrected for equation bias. Thus, for example, R* for flexure would be,

$$R^* = B^* \, (A_s f_y)^* d^* \, (1 - 0.59q^*) \tag{5.7}$$

and similarly for shear. Difficulties arise in applying Equations 2.5 through 2.8 to the combined thrust and bending case, however, because the expression for resistance does not exist in closed form. The derivatives in Equation 2.8 must then be evaluated numerically,

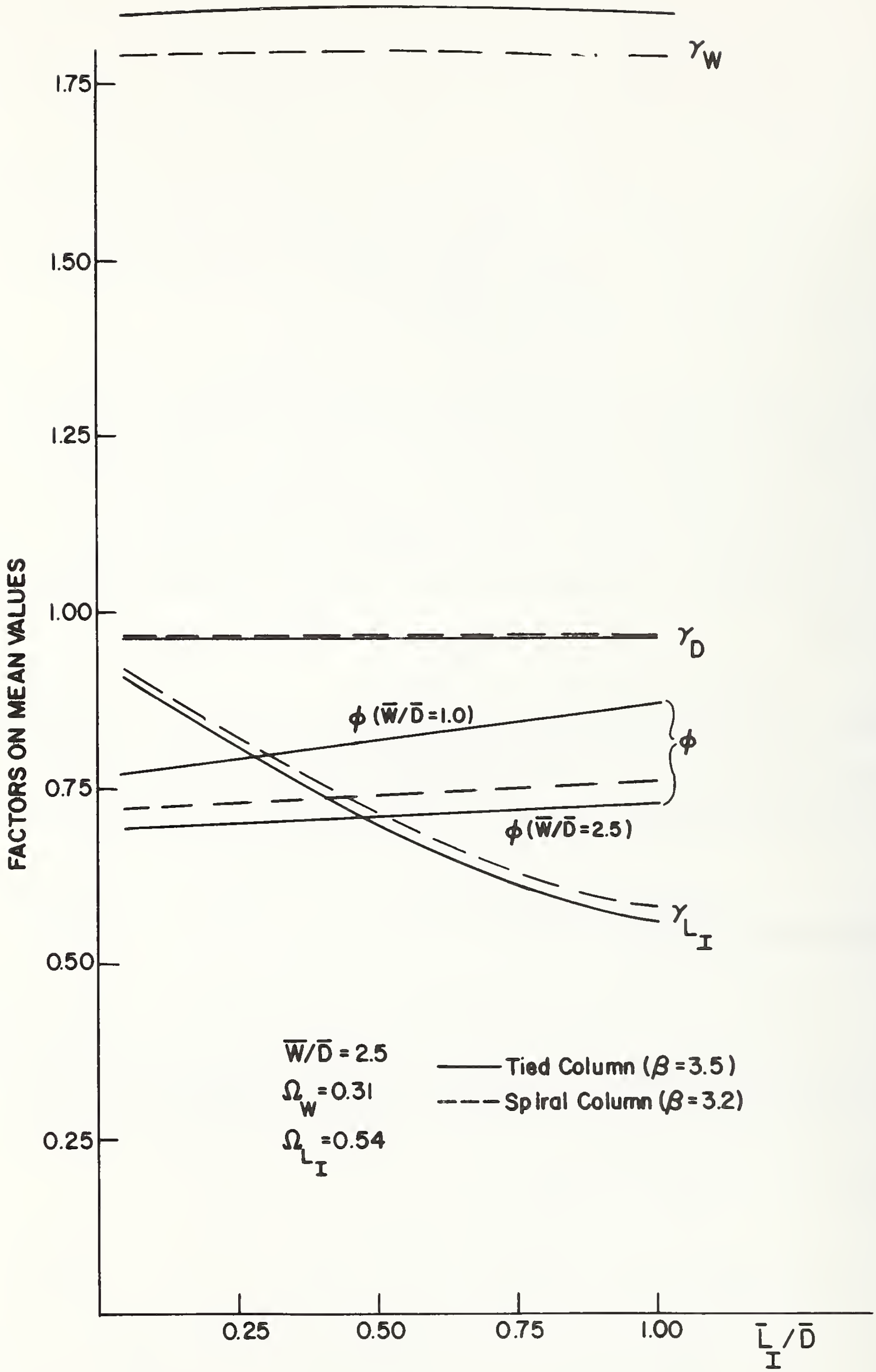


Figure 5.5 - Load and Resistance Factors for Compression and Bending (U = W - D - L)

and while this presents no conceptual problem, in practice it is very awkward. For computing partial factors for columns, then, the case of pure thrust will be considered, where a simple closed form expression for ultimate capacity is available. As the load eccentricity increases, these PF should approach those for flexure, provided that β remains constant.

The inclusion of the bias term B*, is important. It is essential that the total variability associated with resistance in the limit-state, given by Equations 2.1a and 2.5, be the same regardless of whether the RF or PF approach is selected for evaluating R*. $\overline{B}$, and Ω_B are incorporated in the $\overline{R}$ and Ω_R used to derive ϕ when the RF approach is used. The most straightforward way to incorporate their effect when using the PF method is to include B as one of the basic design variables. While this may appear unusual at first, it should not create any difficulties in practice, as all that would be required would be to multiply the calculated ultimate strength, e.g., M*, by the numerical constant B*.

Partial factors are developed in the following sections considering that the factored ultimate load has the form $U^* = \gamma_D D + \gamma_L L$. The PF values and their behavior are virtually the same when wind load is included. As before, $\beta = 3.0$ for flexure, $\beta = 3.0$ for shear and $\beta = 3.5$ for thrust in tied columns and it is emphasized that the individual c.o.v. are the same as those used to evaluate Ω_R. This should permit a direct comparison later of required resistances obtained with the RF and PF methods.

Specific partial factors derived by considering the bending limit state are shown in Figure 5.6 as functions of $\overline{L}/\overline{D}$ and in Figure 5.7 as functions of $\rho\overline{f}_y/\overline{f}_c$ when $\overline{L}/\overline{D} = 0.75$. While ϕ_B, ϕ_y and ϕ_c appear relatively insenstive to $\overline{L}/\overline{D}$, d_n and $\rho f_y/f_c$, Δ''_d is extremely sensitive to d_n. This occurs because Ω_d decreases significantly as d_n increases. Thus, for an 8 in (203 mm) thick beam or slab where $d_n \simeq 6.3$ in (160 mm) $\Omega_d \simeq 0.09$ and $\Delta''_d \simeq$ 7/8" (Figure 5.7), and for safety checking a value $d^* = d_n$ - 7/8" (d_n - 22 mm) should be used. On the other hand, if $b_n \simeq 20$ in (508 mm) and $d_n \simeq 16$ in (406 mm) $\Omega_d \simeq 0.04$ and d^* = d_n - 3/8" (d_n - 9.5 mm) would be appropriate. The value Δ''_d does not appear to be especially sensitive to $\rho\overline{f}_y/f_c$.

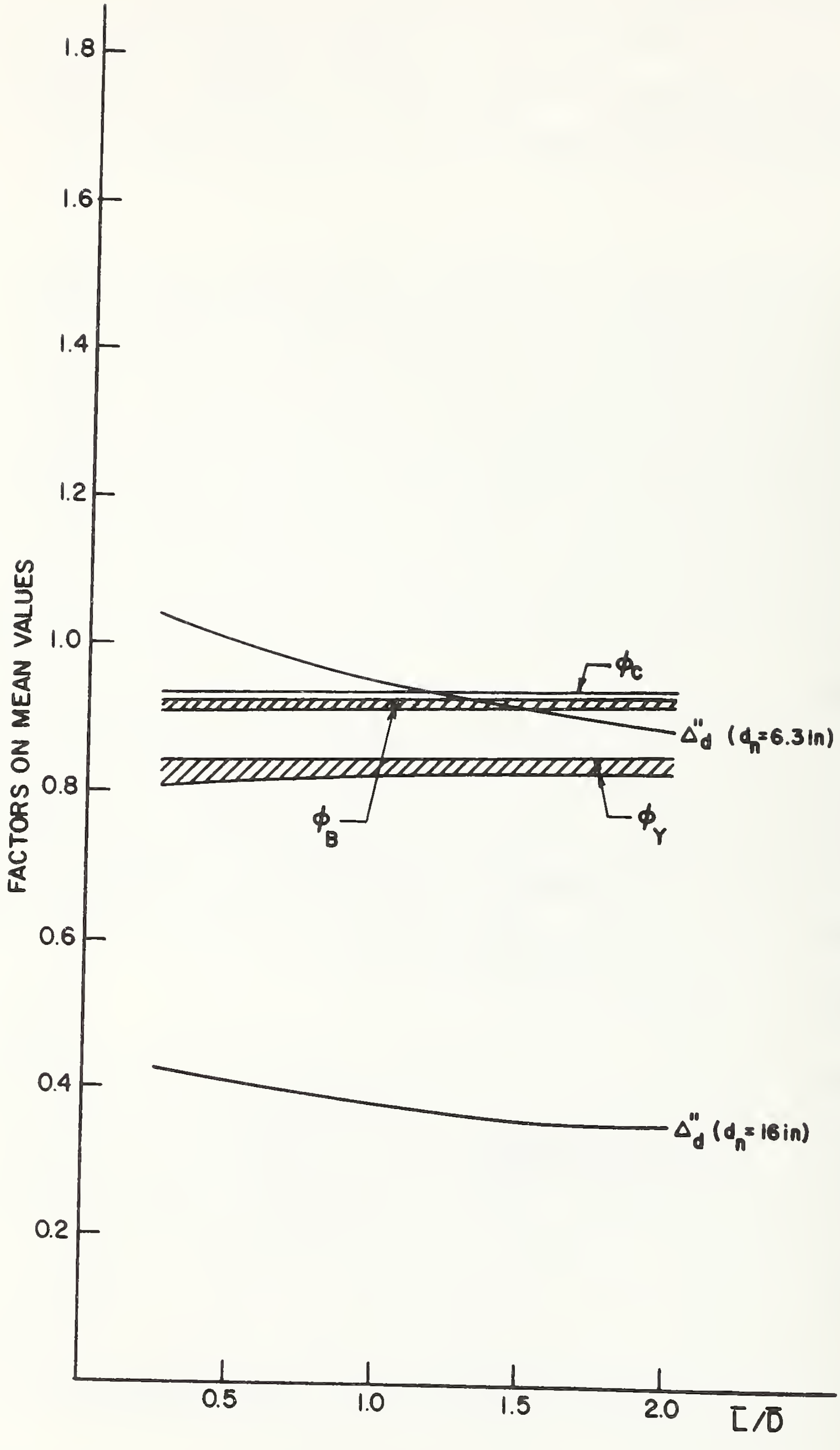


Figure 5.6 - Variation in Partial Factors with $\overline{L}/\overline{D}$ ($\rho\overline{f}_y/\overline{f}_c$ = 0.15; 6.3 in < d_n < 16 in)

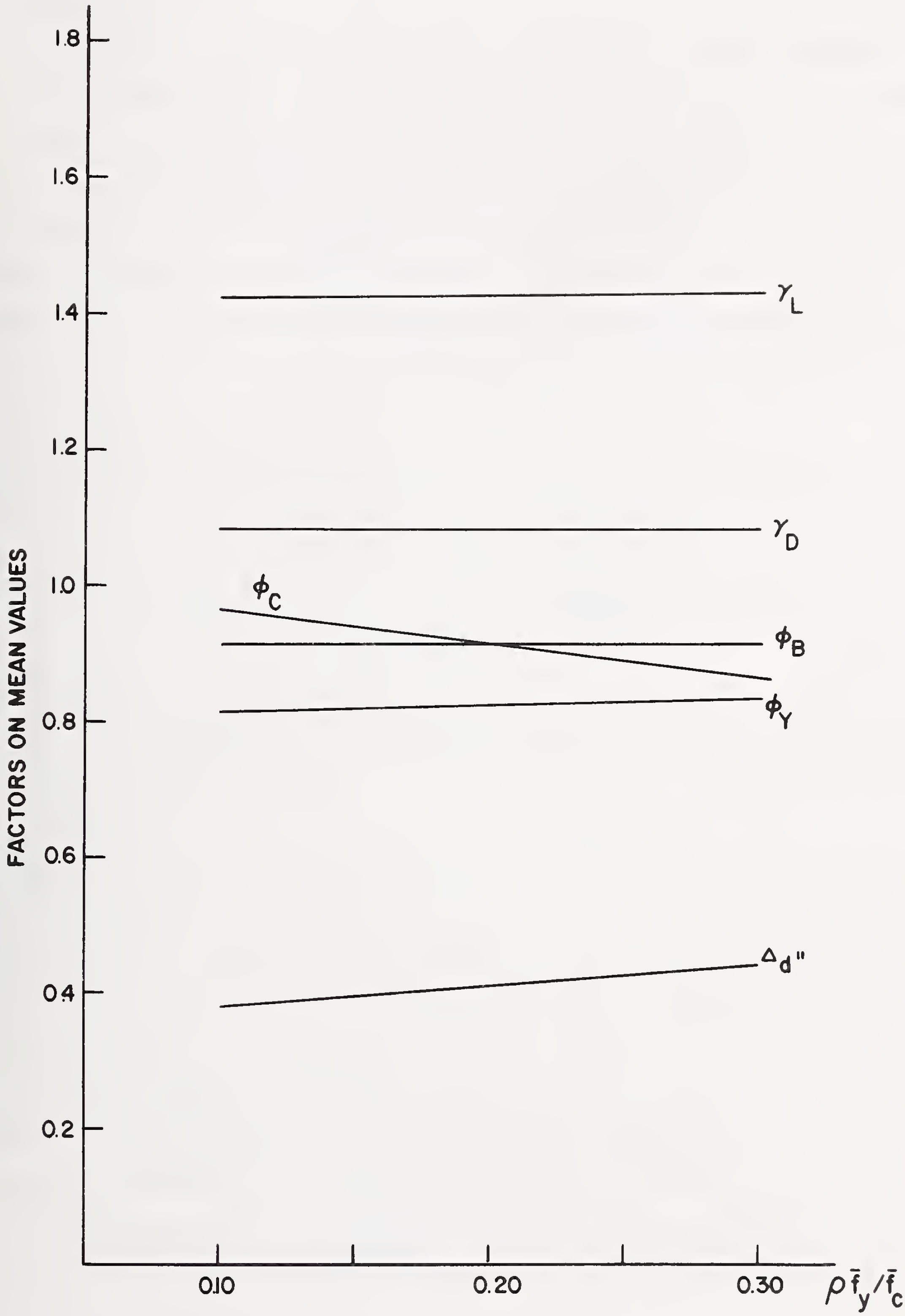


Figure 5.7 - Partial Factors for Flexure (d_n = 16 in; $\overline{L}/\overline{D}$ = 0.75)

The slight decrease in ϕ_c observed in Figure 5.7 reflects the increased importance of f_c in determining flexural capacity for heavily reinforced sections. In other words, as the importance of a particular variable increases, its partial factor tends to become more conservative. This may also be observed with d. The reason that ϕ_c is greater than ϕ_y is that in a properly designed underreinforced flexural member, f'_c is of secondary importance to $A_s f_y$ and d in determining ultimate strength, as a few simple calculations show. On the other hand, small variations in effective depth are more important in the safety of thin members than deep ones, and the importance of good workmanship in placing reinforcement in shallow members can hardly be overemphasized.

Partial factors derived from the shear limit state equation are shown in Figure 5.8 as functions of the parameter $\overline{\rho}_v \overline{f}_y / \overline{v}_c$. With the exception of Δ''_d, the PF were found to be insensitive to $\overline{L}/\overline{D}$ and to d_n. As with flexure, Δ''_d is dependent on d_n for the reasons noted earlier. The increase in ϕ_c and decrease in ϕ_y observed when $\overline{\rho}_v \overline{f}_y / \overline{v}_c$ increases reflects the added importance of steel yield stress and lessening contribution of v_c when the web is heavily reinforced. The fact that ϕ_c generally is lower than for flexure while ϕ_y generally is higher reflects the fact that, relatively speaking, concrete strength is more important in shear than in flexure.

Finally, partial factors appropriate for thrust are shown in Figure 5.9 for different $\overline{\rho}_g \overline{f}_y / 0.85\overline{f}_c$, where $\overline{\rho}_g = (A'_s + A'_S)\overline{bh}$. Variability in b and h are incorporated here because these tend to be more important for thrust than in flexure and shear. The PF for thrust are insensitive to $\overline{L}/\overline{D}$ and to nominal column dimensions. The tolerance Δ''_h depends to a certain extent on h, as Ω_h is related to the size of the column. Since few columns would be less than 12 in. (305 mm) in size, it would be a simplifying assumption to set h* = h_n - 1/4" (h_n - 6.4 mm) for safety checking columns. ϕ_c increases as $\overline{\rho}_g \overline{f}_y / 0.85\overline{f}_c$ becomes large and the relative amount of the axial load carried by the reinforcement increases. In all cases, ϕ_c is much less than for either flexure or shear, being in the range 0.6 - 0.7, because of the relative importance of f_c in determining axial capacity.

A comparison of γ_L and γ_D computed using RF and PF is shown in Figures 5.10a and b for flexure and thrust, respectively. While small differences naturally exist (due in part to

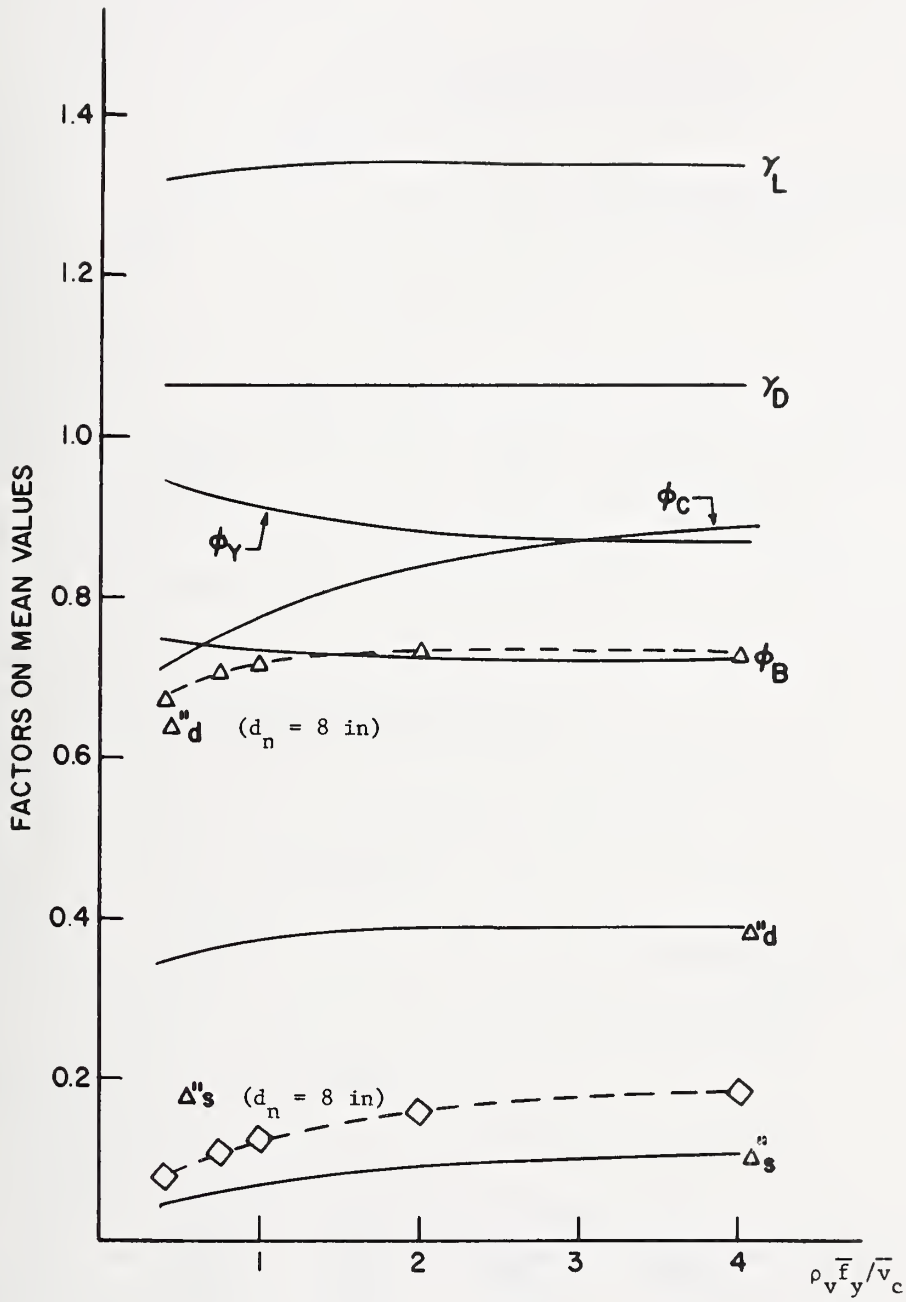


Figure 5.8 - Partial Factors for Shear (d_n = 16 in, $\bar{L}/\bar{D}$ = 0.75)

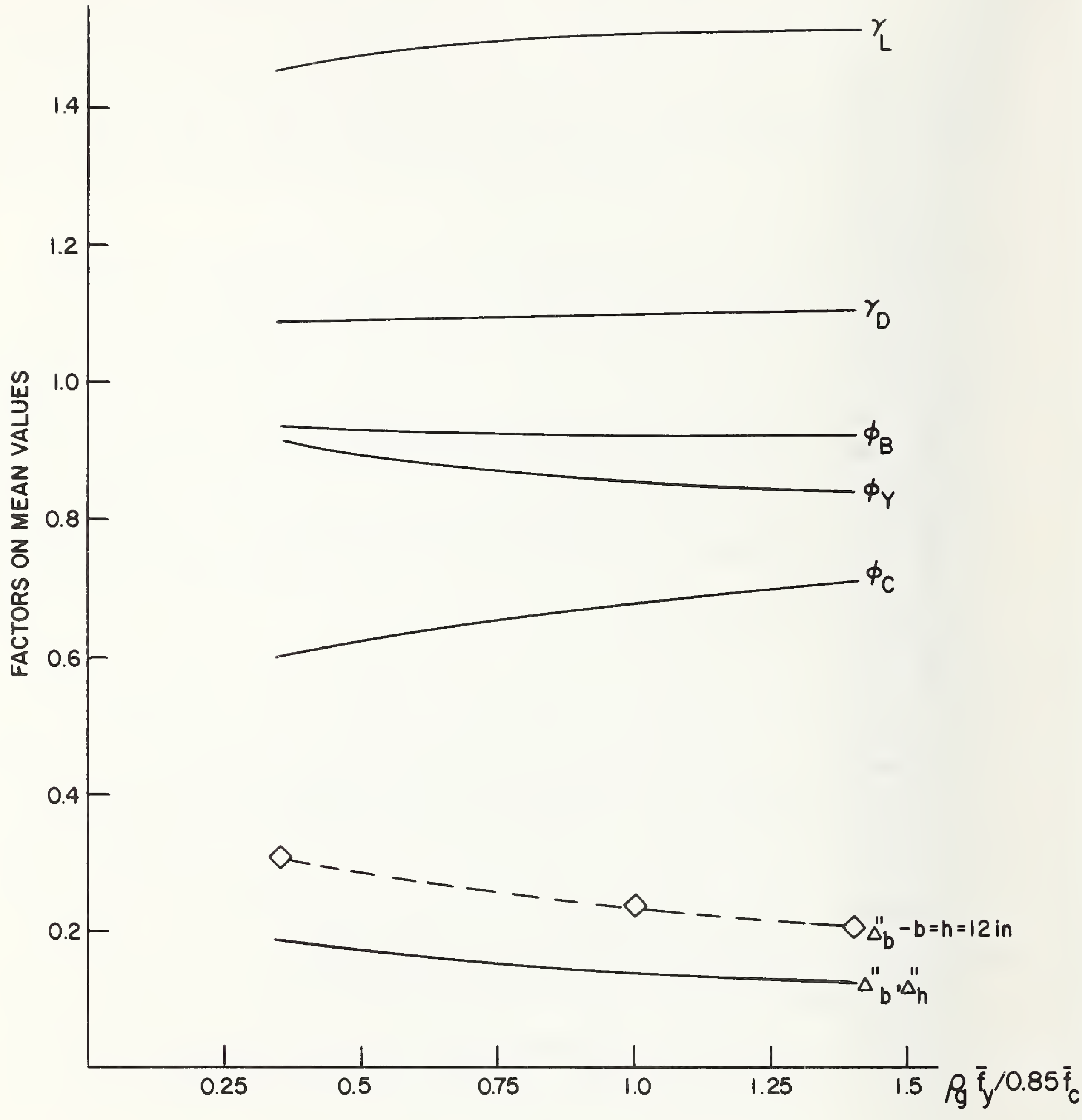


Figure 5.9 - Partial Factors for Thrust ($b_n = h_n = 20$ in, $\overline{L}/\overline{D} = 0.75$)

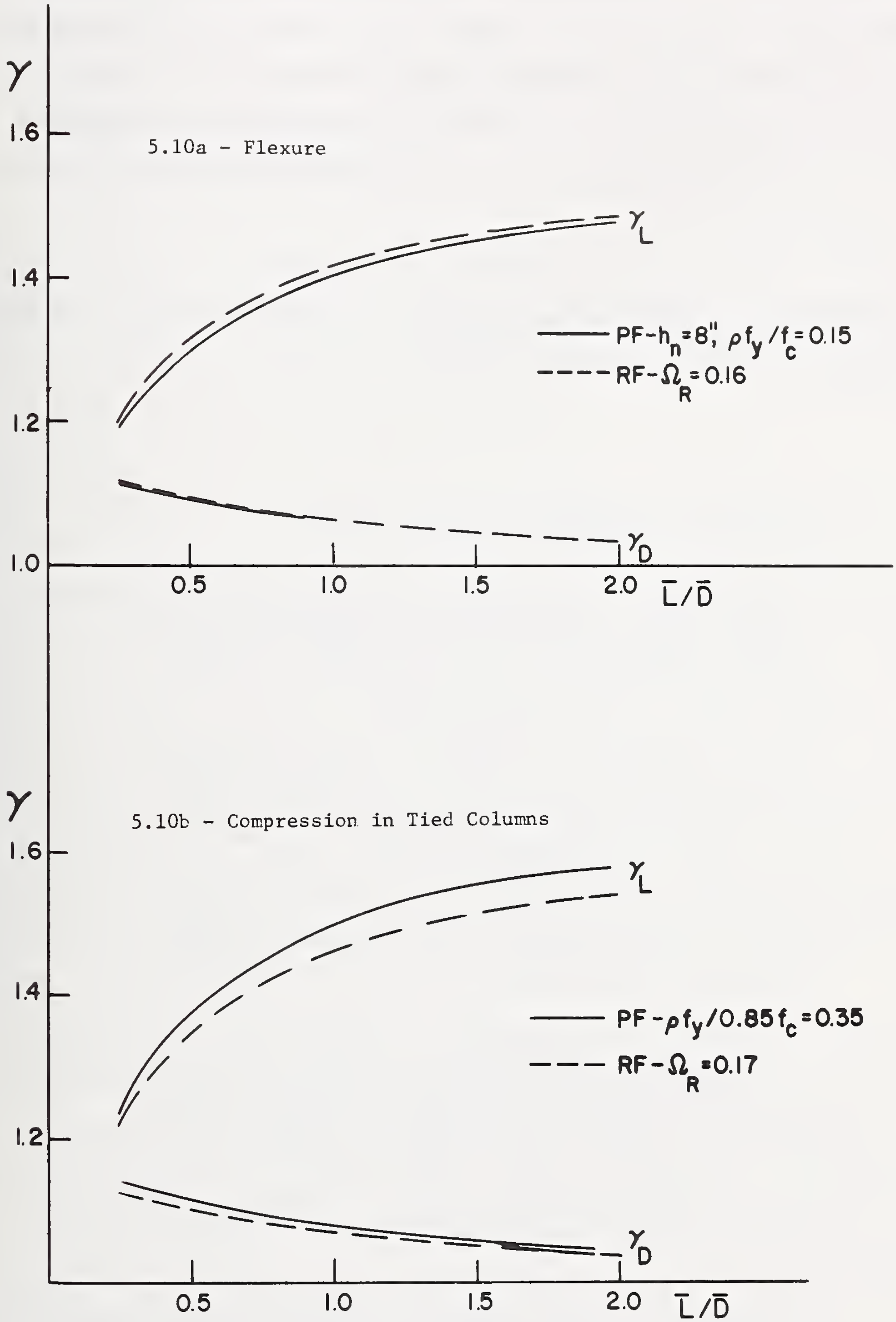


Figure 5.10 - Comparison of Load Factors Computed Using Partial and Total Resistance Factors in Limit State Equation

rounding of uncertainties to two significant figures in RF) the load factors computed using the two approaches are virtually the same. (This is also true for γ_W, as in Equation 5.4). It may therefore be concluded that the choice of whether to use RF or PF in design codes may be deferred, while the load factors derived from either method may be implemented without delay. This is encouraging in view of the current lively discussion on whether to use the RF or PF approach.

Designs obtained using the PF and RF are compared in Tables 5.1a, b and c for flexure, shear, and thrust.

Table 5.1 - Comparison of RF and PF Designs.

(5.1a) Flexure: Design Parameter $\bar{B}\ \bar{A}_s\bar{f}_y\bar{d}_n/\bar{D}$ when $\rho\bar{f}_y/\bar{f}_c = 0.15$

L/D	0.25	0.50	0.75	1.00	2.00
		cov(R) = 0.13			
RF	2.199	2.691	3.214	3.753	5.961
PF	2.195	2.687	3.209	3.747	5.951
		cov(R) = 0.16			
RF	2.376	2.903	3.459	4.031	6.372
PF	2.394	2.922	3.479	4.052	6.399

(5.1b) Shear: Design Parameter $\bar{B}\ \bar{v}_c\overline{bd}_n/D$ when $\rho_v\bar{f}_y/\bar{v}_c = 1.0$

L/D	0.25	0.50	0.75	1.00	2.00
RF(Ω_R = 0.22)	1.276	1.554	1.845	2.143	3.362
PF	1.177	1.435	1.706	1.985	3.126

(5.1c) Thrust: Design Parameter B $0.85\bar{f}_c\overline{bh}/\bar{D}$ when $\rho_g\bar{f}_y/0.85\bar{f}_c = 0.35$

L/D	0.25	0.50	0.75	1.0	2.0
RF (Ω_R=0.17)	1.812	2.216	2.568	3.088	4.903
PF	1.739	2.131	2.547	2.977	4.740

The small variation in Ω_R for each limit state discussed in Section 3.2 is sufficient to cause the differences between the RF and PF methods observed in Table 5.1. However, since these differences seldom exceed a few percent, it may be concluded that the two methods would result in essentially the same designs under gravity loading and that the PF format would not penalize reinforced concrete designs.

A primary advantage to the PF approach, at least in a conceptual sense, is that the safety factors are applied directly to the individual design variables whose randomness is the source of variability in resistance. Presumably, if one variable was particularly important to safety its PF would reflect that importance.

It has been proposed [12,26] that this method be implemented in design by prescribing one set of factors, $\phi_y, \phi_c, \Delta''_d, \Delta''_h$, etc., that would be used for all ultimate limit states, and that this would simplify design. Although the importance of including equation bias and variability arising therefrom has not been specifically mentioned, its inclusion presents no particular problem as long as B is accepted as one of the basic design variables. As was the case with RF design, however, PF consistent with a specified reliability differ for the various limit states. This is due to differences in the relative importance of steel and concrete strength and member geometry in each limit state, as well as the different β required in each case. The use of the same β for all limit states reduces, but does not entirely eliminate, this difference. Thus, the use of one set of PF for flexure, shear and thrust would result in local violations of the reliability constraint. For example, if $\phi_c = 0.85 - 0.9$, as suggested by the flexure and shear solutions, the corresponding safety index for thrust would be about 3.0 rather than the 3.5 required. Similarly, the use of one bar placement tolerance, say $\Delta''_d = 3/8''$ (9.5 mm) would be unconservative for shallow beams and their slabs. On the other hand, the effect of small changes in individual variables on the total variability in resistance tends to get damped out in the RF approach. This is particularly true for shear, where variability associated with the prediction equation itself is a significant component of the total variability.

Although the partial factors procedure will not be considered further for the above reasons, with the exception of shallow members (h < 8", say) the following PF on mean values appear appropriate: $\phi_y = 0.84$, $\phi_c = 0.65$, $\Delta''_d = 1/2''$ (13 mm), $\Delta''_h = 1/4''$ (6 mm); $\Delta_s = 1/8''$ (3 mm). The factored resistance R* should include the term B^*_i as noted previously. The load factors should be the same as for LRFD.

5.3 Single Load Factors

As a final design format example, we consider the equation,

$$\phi\bar{R} > \gamma\,(\bar{D} + \bar{L}) \qquad (5.8)$$

which is the simplest format in which it is possible to treat the load and resistance sides of the design equation separately. For illustration, flexural resistance will be considered, wherein $\Omega_R = 0.16$ and $\beta = 3.0$; similar results are obtained with shear or thrust. The c.o.v. in the total load is,

$$\Omega_{D+L} = \frac{1}{1+\bar{L}/\bar{D}}\,[\Omega_D^2 + (\bar{L}/\bar{D})^2\Omega_L^2]^{1/2} \qquad (5.9)$$

Ω_D and Ω_L being the same as before.

The behavior of ϕ and γ is shown in Figure 5.11, along with the γ_D and γ_L determined as before. As would be expected, γ falls between γ_D and γ_L, approaching γ at small $\bar{L}/\bar{D}$. The resistance factor ϕ is exactly the same as the LRFD value. This indicates again that the functional form of the resistance or load side of the design equation may be altered without significantly affecting the safety factors on variables on the opposite side. It may also be recalled from the previous section that γ_D and γ_L were the same for both PF and overall ϕ-factor approaches. This suggests that standard-writing groups concerned with resistances and those concerned with loads could proceed independently in developing criteria without agreeing precisely on an overall format for design (e.g. PF vs. RF). It is only necessary that they agree on the underlying measures of uncertainty Ω_i and the reliability analysis methods through which Ω_i and β are incorporated to derive design criteria.

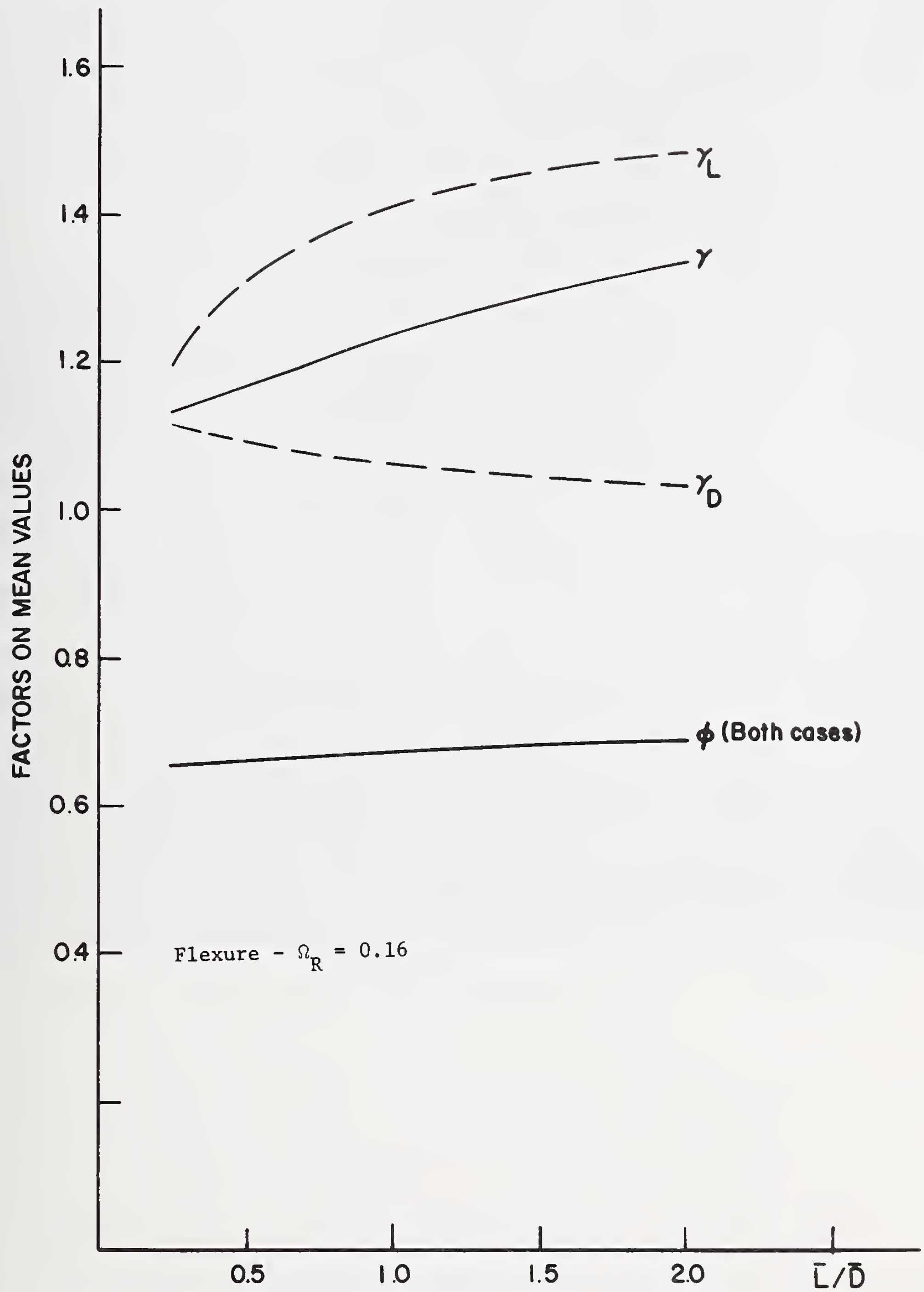
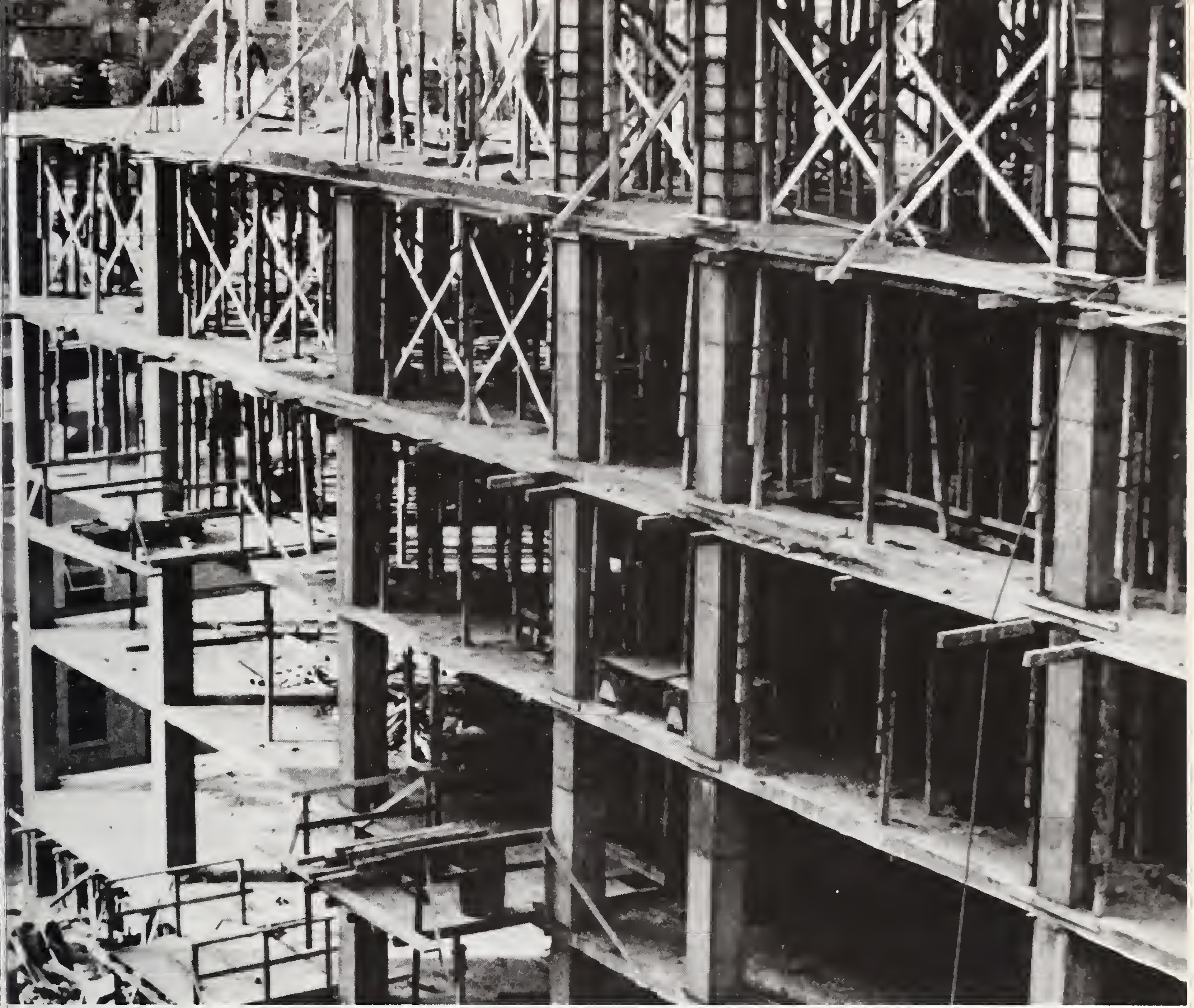


Figure 5.11 - Design Criteria of the Form $\phi\bar{R} = \gamma(\bar{D} + \bar{L})$

It was observed in Table 4.1, that when only one (constant) load factor is applied to the *sum* of load effects in a Level I procedure, as in Equation 5.4, the associated value of β varies more with $\overline{L}/\overline{D}$ than if the load factors are separated, as in Equation 5.5. The Level I procedure, to be discussed subsequently, requires that a set of load and resistance factors be chosen that minimize the reliability constraint violation, and this may be better achieved with the separate load factor procedure. Additional practical difficulties with Equation 5.4 arise when lateral forces counteract gravity loads, as the c.o.v. in (W - D - L) may become very large in this case. If one overall γ is desired, it appears, on the basis of Figure 5.11 and supplementary calculations considering D + L + W, that γ should be in the range 1.25 - 1.3. It seems worthwhile to re-emphasize the conceptual advantages, discussed at the beginning of Section 5, that this approach has for Level II design procedures, wherein variation of ϕ and γ with ($\overline{L}/\overline{D}$, $\overline{W}/\overline{D}$, $\cdots$) would be an acceptable feature.



6. PRACTICAL LOAD AND RESISTANCE FACTOR DESIGN

At present, Level II methods are useful primarily to technical committees charged with developing design standards. However, for simplicity and continuity with existing formats, it is necessary to eliminate the dependence of the safety factors on the load ratios and to prescribe a minimal number of such factors suitable for use in everyday design. In the short term, the LRFD approach appears most appropriate for Level I formats in the United

States because of its consistency with the existing ACI [10] and the proposed AISC provisions [60].

The maximum total load on a structure or component during its life may result from any one of many different possible load combinations. It has been found [63] that since the probability of a joint occurrence of two or more extreme loads is negligible, the maximum total load effect results when one variable load assumes its maximum lifetime value while others are at their instantaneous values. The factored ultimate load U* thus becomes,

$$U^* = \gamma_D \bar{D} + \gamma_Q \bar{Q} + \sum_i^{n_s} \gamma_{s_i} \bar{S}_i \tag{6.1}$$

where D is mean permanent (dead) load, $\bar{Q}$ is the mean maximum variable load during some suitable period, e.g., the life of the structure, and $\bar{S}_i$ is the mean instantaneous or short-term load which acts simultaneously with $\bar{Q}$. In the general case, the individual loads under consideration must be rotated in the load equation, each load taking the position of the maximum mean variable load $\bar{Q}$, while the remaining loads are assigned the value $\bar{S}_i$. So that a direct comparison can be made with the requirements of Sections 9.2 and 9.3 of ACI 318-71, it will be assumed that $\bar{Q}$ equals either $\bar{L}$ or $\bar{W}$, and that $\bar{S}_i = \bar{L}_I$. This by no means encompasses all possible load combinations, but will enable the concepts required for Level I criteria development to be demonstrated. The factored ultimate resistance R* against which U* is compared is simply $R^* = \phi\bar{R}$, in accordance with Equation 5.2.

The variation in (ϕ, γ_D, γ_Q, γ_{L_I}) with the load ratios in Section 5, shows that if a constant set of these values are prescribed, the reliability constraint will be violated locally. While such violations are unavoidable in Level I design, it nevertheless should be possible to select one set of LRF that minimizes the extent of this violation when considered over all likely load ratio combinations. One way to do this is to select a set of LRF that minimize the quantity $\Sigma\Sigma p_{ij}(\beta_{ij}-\beta_o)^2$, where β_{ij} is the safety index obtained using a prescribed LRF set for load combination ij occurring with probability p_{ij}, and β_o is the target reliability [58]. However, this approach is numerically difficult to apply with the reliability procedure given by Eqs. 2.5-2.8 because their solutions are not available in closed form.

As an alternative procedure, a function of the form

$$I = \Sigma\Sigma p_{ij}[\bar{R}_{ij}(\beta_o) - \bar{R}\text{ (Safety factors; load ratios}_{ij})]^2 \qquad (6.2)$$

is minimized. In Eq. 6.2, $\bar{R}_{ij}(\beta_o)$ is the mean resistance required according to Eqs. 2.5-2.8 for a target reliability β_o and the load combination ij. The function $\bar{R}$ is the mean resistance furnished by the Level I design procedure, e.g., if LRFD is used, $\bar{R} = (\gamma_D\bar{D} + \gamma_L\bar{L} + \gamma_W\bar{W})/\phi$. Note that this minimization procedure, as well as the one described in the preceding paragraph, requires the selection, *a priori*, of a Level I design format. Indeed, the specific safety factors extracted will depend on the functional form of $\bar{R}$ and constraints placed upon it. Compared to the procedure outlined in the previous paragraph, however, the minimization of Eq. 6.2 is relatively simple, since $\bar{R}_{ij}(\beta_o)$ are tabulated during the Level II development discussed in Section 5. Moreover, it was found by computing $\Sigma\Sigma p_{ij}(\beta_{ij} - \beta_o)^2$ for small perturbations in the LRF values obtained from Equation 6.2 that the LRF that minimize both expressions are within a few percent of one another.

The determination of an appropriate set of mean load ratios for this minimization and the assignment of relative likelihoods p_{ij} to them cannot be done objectively given the present state of knowledge. In a recent study [58], a set for all limit states and materials was selected subjectively. However, since the present study is concerned with reinforced concrete structures, the $\bar{L}/\bar{D}$ and $\bar{W}/\bar{D}$ ratios would be smaller than those previously reported. Accordingly, the values listed in Table 6.1a were assigned for the (D + L) case;

Table 6.1a - Mean Load Ratios and Weights for D + L

$\bar{L}/\bar{D}$	0.25	0.50	0.75	1.0	2.0
p_i	0.22	0.31	0.25	0.14	0.08

and when live load is combined with wind, as in Table 6.1b;

Table 6.1b - Mean Load Ratios and Weights for D + L_I + W

$\bar{L}_I/\bar{D}$	0.10	0.25	0.50	0.75	1.00
p_i	0.21	0.44	0.29	0.04	0.02

For each $\overline{L}_I/\overline{D}$, these p_i must be apportioned to possible $\overline{W}/\overline{D}$ values also. On the basis of an analysis of a multistory moment resisting reinforced concrete frame, using loads defined in Appendix A, equal weights p_{ij} were assigned all $\overline{W}/\overline{D}$ values between 0.5 and 2.5. A study was also conducted to determine the sensitivity of the Level I criteria to the load ratio and probability assignment. It was found that although the load and resistance factors depend on the *range* of $\overline{L}/\overline{D}$, $\overline{W}/\overline{D}$ assumed, surprisingly enough they do not appear to be especially sensitive to the assignment of p_{ij} *within* that range.

Using the LRFD format for load combination k, Eq. 6.2 has the form (some factors may be zero),

$$I_k(\phi,\gamma_D,\gamma_Q,\gamma_{L_I}) = \Sigma\Sigma p_{ij}\ [\overline{R}_{ij}/\overline{D} - (\gamma_D+\gamma_Q\overline{Q}/\overline{D}_i+\gamma_{L_I}\overline{L}_I/\overline{D}_j)]/\phi \qquad (6.3a)$$

Considering more than one load combination simultaneously, the function to be minimized is,

$$I(\phi,\gamma_D,\gamma_Q,\gamma_{L_I}) = \Sigma_k\ I_k\ (\phi,\gamma_D,\gamma_Q,\gamma_{L_I}) \qquad (6.3b)$$

in which $(\phi,\gamma_D,\gamma_Q,\gamma_{L_I})$ are the required Level I load and resistance factor set.

Certain additional restrictions are placed on the minimization problem. First of all, it was apparent from the results in Section 5 that γ_D = 1.10 in all cases where gravity and lateral loads are additive. Secondly, it was observed (cf. Figures 5.1 and 5.4b) that the load factors applied to the maximum live load and wind loads were quite similar because Ω_L and Ω_W are close in magnitude. Thus, it appears reasonable to set γ_Q (= γ_L = γ_W) constant in Equation 6.1. In other words, the load factor applied to the *maximum* variable load in the load equation should be the same, regardless of the load combination. Intuitively, this appears reasonable because the c.o.v. associated with most maximum variable loads (with the exception of earthquake loads) is generally about 0.30. However, this represents a slight departure from current ACI practice, wherein according to Eq. 9-1 and 9-2, γ_L = 1.7 and γ_W = 1.275, respectively.

Minimization of Equations 6.3 is first performed separately for flexure, shear, and axial compression and bending, considering the two load combinations (D+L, $D+W+L_I$) where load effects are additive. The results of this analysis are presented in the first four columns of Table 6.2.

Table 6.2 - Optimal Load and Resistance Factors

Limit State	ϕ	γ_D	γ_Q	γ_{L_I}	ϕ (γ_Q = 1.55; γ_{L_I} = 1.6)
Flexure					
(Ω_M=0.13)	0.77	1.10	1.59	1.63	0.76
(Ω_M=0.16)	0.71	1.10	1.54	1.59	0.71
Shear	0.59	1.10	1.47	1.50	0.61
Compression & Bending					
Tied Columns	0.63	1.10	1.59	1.63	0.62
Spiral Columns	0.67	1.10	1.55	1.59	0.66

Selecting γ_Q = 1.55 and γ_{L_I} = 1.6, thus, the values of ϕ which minimize Equations 6.3 are listed in the last column of Table 6.1.

In order to determine an appropriate set of load factors for the third load combination, when gravity and wind forces counteract one another ($W - D - L_I$), we require only that the ϕ-values be identical to those listed in Table 6.1. No other constraints are the values of γ_Q,γ_D^- and $\gamma_{L_I}^-$ are placed (the "minus" superscript means that the associated loads are to be subtracted in the load combination). Minimizing Equation 6.3 loads to γ_Q = 1.60, γ_D^- = 0.8 and γ_L^- = 0.60. It would be conservative to neglect the latter term. The load criteria analogous to Equations 9-1, 9-2 and 9-3 in ACI 318-71 would thus be,

$$U^* \; 1.1\,\overline{D} + 1.55\,\overline{L} \tag{6.4a}$$

$$U^* = 1.1\,\overline{D} + 1.6\,\overline{L}_I + 1.55\overline{W} \tag{6.4b}$$

$$U^* = 1.60\,\overline{W} - (0.8\,\overline{D} + 0.6\,\overline{L}_I) \tag{6.4c}$$

The ϕ values in the last column of Table 6.1 and γ in Equations 6.4 are associated with *mean* resistance and loads. To obtain appropriate values to be applied to *nominal* quantities, simply multiply ϕ by $\overline{R}/R'$ and γ_i by $\overline{S}_i/S_i'$.

The typical violation of the reliability constraint arising from the selecting of a Level I criteria is illustrated in Table 6.3, where β_{ij} for the shear criterion $0.61\bar{R} = 1.1\bar{D} + 1.55\bar{W} + 1.6\bar{L}_I$ are given. The target value β_o was 3.0.

Table 6.3 - β Achieved by Level I Criterion for Shear

$\bar{W}/\bar{D}$	$\bar{L}_I/\bar{D}$				
	0.10	0.25	0.50	0.75	1.0
0.5	2.98	3.02	2.98	2.90	2.82
1.0	3.01	3.06	3.07	3.03	2.98
1.5	3.00	3.05	3.09	3.08	3.05
2.5	2.97	3.01	3.06	3.08	3.09

Using the p_{ij} in Table 6.1, the mean and variance of the safety index achieved are listed in Table 6.4.

Table 6.4 - Variability in β Caused by Level I Criteria

Limit State	β_o	Load Combination	$E[\beta]=\bar{\beta}$	$E[(\beta-\bar{\beta})^2]$
Flexure Ω_M=0.16(0.13)	3.0	D + L	2.99(3.03)	0.00975(0.00974)
		D + L + W	3.01(3.01)	0.00322(0.00536)
Shear	3.0	D + L	2.96	0.00876
		D + L + W	3.03	0.00174
Compression and Bending (Tied Columns)	3.5	D + L	3.55	0.00698
		D + L + W	3.54	0.00408
Compression and Bending (Spiral Columns)	3.2	D + L	3.23	0.00838
		D + L + W	3.24	0.00327

It may be observed that $\bar{\beta}$ is within 0.1 of β_o in all cases. This may be contrasted with the scatter in β values associated with existing design criteria, discussed in Chapter 4.

It is worth reemphasizing that, in accordance with Equation 6.2, the Level I load and resistance factors depend on the format that is assumed, *a priori*, for the design equation.

In principle, this could range from a single equation with one overall safety factor to the complex formats currently under consideration by CEB [12] and other European code-writing groups. As with multiple regression analysis, the more independent factors that are assigned, the less the reliability constraint will be violated, i.e., $\bar{\beta}$ in Table 6.4 will approach β_o and $E[(\beta-\bar{\beta})^2]$ will approach zero. This militates against the use of one overall safety factor or load factor in Level I criteria development, and would suggest that as many factors be assigned as would be practical and easy to handle for everyday design usage.

7. CONCLUSIONS

Reliability analysis methods have been employed in this study to facilitate the selection of criteria for reinforced concrete design. These methods, which are based on probability theory, provide a logical basis for determining the manner in which uncertainties in resistance and loads affect design safety and how their effects should be controlled in building codes. Using data obtained from a comprehensive analysis of uncertainty measures (described in Appendix A), safety indices associated with existing reinforced concrete designs were evaluated. This provided target reliabilities for subsequent criteria development. Finally, Level II design criteria commensurate with levels of uncertainty and required reliability were presented. Simplification of these led to practical reliability based criteria which retain the relatively simple characteristics of existing criteria and yet have a well established and documented rationale.

This study has presented a rather broad treatment of the use of reliability methods in developing criteria for structural design against various ultimate limit states. By examining the sensitivity of the solutions to certain assumptious or simplifications a decision can be made as to whether further examination is worthwhile. Additional studies on the treatment of load combinations appears desirable in the short term, for example. Additional data are also needed to determine appropriate load ratios and relative frequencies as in Table 6.1. However, since standards development is evolutionary in nature, many of the concepts and results discussed herein can be implemented concurrent to supplementary research.

The following specific conclusions can be drawn:

1. Analysis of uncertainties in resistance for reinforced concrete members shows that, with the exception of shear, they are about the same as for steel members [60]. c.o.v. in resistance may approach, and occasionally exceed, those for loads.

2. Levels of safety for reinforced concrete designs, measured by β, average from 2.5 to 4, depending on the limit state and tributary loaded area. This is the same order as found by Galambos [28] for steel designs. It is less than found by Siu, et.al [58] because the ANSI live load is less than the Canadian [43] live load.

3. Reliabilities associated with reinforced concrete beams in flexure and columns appear appropriate in a relative sense. Current shear provisions appear inadequate; the nominal $\phi' = 0.85$ is insufficient to account for the large (0.22) c.o.v. in shear resistance.

4. Reliabilities associated with the current loading provisions contained in Chapter 9 of ACI 318-71 generally appear risk-consistent, although there is evidence that

the reliability against wind may be slightly less than against gravity loads.

5. Load and resistance factors for a limit state which are consistent with a prescribed reliability depend on mean load ratios. The γ factors do not appear to be strongly dependent on Ω_R, however, suggesting that the coupling between resistance and load sides of the design equation is weak.

6. Safety factors in existing standards tend to be allocated improperly to resistance and load sides of the design equation because of failure to account for relative magnitudes of uncertainty in resistance and load. Calibration assures that existing and reliability-based designs will be about equal; although the resistance and load factors in the latter will both tend to be lower, their net effect will be about the same.

7. The load and overall resistance factor (LRFD) format

$$\phi\overline{R} > \Sigma \quad (\gamma_i \overline{S})$$

appears more attractive for American designers than either the partial resistance factor or single load factor approaches. However, since the factors obtained on one side of the design equation are not sensitive to the format chosen for the other side, provided that measures of uncertainty and β are agreed upon, the question of whether to use overall or partial resistance factors need not be agreed upon prior to load criteria development.

8. Level I criteria depend on the format selected and constraints imposed. Practical criteria can be developed from reliability bases which are no more complex than design procedures currently in use. As an example, for reinforced concrete design, the following load criteria and resistance factors could replace those in ACI 318-71. Note that the load and resistance factors are applied to *mean* values.

Load:

$$U^* = 1.1\ \overline{D} + 1.55\ \overline{L}$$

$$U^* = 1.1\ \overline{D} + 1.55\ \overline{W} + 1.6\ \overline{L}_I$$

$$U^* = 1.6\ \overline{W} - (0.8\ D + 0.6\ \overline{L}_I)$$

Resistance

Flexure	$\phi = 0.73$
Shear	$\phi = 0.61$
Compression and Bending (Tied Columns)	$\phi = 0.62$
Compression and Bending (Spiral Columns)	$\phi = 0.66$

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REFERENCES

1. Allen, D.E., "Limit States Design - A Probabilistic Study," Canadian Journal of Civil Engineering, Vol. 2, No. 1, 1976, pp. 36-49

2. Allen, D.E., "Statistical Study of the Mechanical Properties of Reinforcing Bars," Building Research Note 85, Division of Building Research, National Research Council of Canada, Ottawa, 1972.

3. Ang, A. H.-S., "Structural Risk Analysis and Reliability-Based Design," Journal of the Structural Division, ASCE, Vol. 99, No. ST9, Proc. Paper 10011, September 1973, pp. 1891-1910.

4. Ang. A.H.-S., Cornell, C.A., "Reliability Bases of Structural Safety and Design," Journal of the Structural Division, ASCE, Vol. 100, No. ST9, Proc. Paper 10777, September 1974, pp. 1755-1769.

5. Ang, A.H.-S., Ellingwood, B.R., "Critical Analysis of Reliability Principles Relative to Design," in Statistics and Probability in Structural Engineering, Hong Kong University Press, 1972, pp. 1-16.

6. Baker, M.J., "The Evaluation of Safety Factors in Structures," CIRIA Research Report 72, Department of Civil Engineering, Imperial College, London, 1970.

7. Birkeland, P.W., Westhoff, L.J., "Dimensional Tolerance - Concrete," Proceedings of International Conference on Planning and Design of Tall Buildings, Vol. 1b, American Society of Civil Engineers, New York, 1972.

8. Bloem, D., "Concrete Strength in Structures," Journal of the American Concrete Institute, Vol. 65, No. 3, March 1968, pp. 176-187.

9. "Building Code Requirements for Minimum Design Loads in Buildings and other Structures," ANSI A58.1-1972, American National Standards Institute, New York, 1972.

10. "Building Code Requirements for Reinforced Concrete," ACI Standard 318-71, American Concrete Institute, Detroit Mich., 1971.

11. "Code of Basic Data for the Design of Buildings: Chapter V - Loading," British Standards Institution, CP3:Chapter V: Part 2 Wind Loads, September 1972.

12. "Common Unified Rules for Different Types of Construction and Material, "Bulletin D' Information No. 116E, Comite Euro-International du Beton (CEB), Paris, November 1976.

13. Corley, W.G., "Rotational Capacity of Reinforced Concrete Beams," Journal of the Structural Division, ASCE, Vol. 92, No. ST5, Proc. Paper 4939, October 1966, pp. 121 - 146.

14. Cornell, C.A., "Summary Report on Structural Design Parameters," Proceedings of 2nd International Conference on Applications of Statistics and Probability to Soil and Structural Engineering, Technische Hochschule Aachen, West Germany, September 1975.

15. Cornell,C.A., "Requirements for Probabilistic Approach," in Probabilistic Approaches to Structural Safety, Journal of the American Concrete Institute, Vol. 73 No. 1, January 1976, pp. 37-48.

16. Culver, C. C., "Survey Results for Fire Loads and Live Loads in Office Buildings," National Bureau of Standards Building Science Series No. 85, Washington, D.C., May 1976.

17. Davenport, A.G., "Gust Loading Factors, "Journal of the Structural Division, ASCE Vol. 93, No. ST3, Proc. Paper 5255, June 1967, pp. 11 - 34.

18. Drysdale, R.C., "Placement Errors for Reinforcing in Concrete Columns," Journal of the American Concrete Institute, Vol. 72, No. 1, January 1975, pp. 9-15.

19. Drysdale, R.G., "Variation of Concrete Strength in Existing Buildings," Magazine of Concrete Research, Vol. 25, No. 85, December 1973, pp. 201-207.

20. Eaton, K.J., Mayne, J.R., Cook, N.F., "Wind Loads on Low-Rise Buildings--Effects of Roof Geometry, "Building Research Establishment Current Paper CP 1/76, Garston, Watford UK, January 1976.

21. Ellingwood, B., Ang, A.H.-S.,"Probabilistic Study of Safety Criteria for Design," Structural Research Series No. 387, Department of Civil Engineering, University of Illinois, Urbana, June 1972.

22. Ellingwood, B., "Statistical Analysis of RC Beam-Column Interaction," Journal of the Structural Division, ASCE, Vol. 103, No. ST7, Proc. Paper 13061, July 1977, pp. 1377-1388.

23. Ellingwood, B., Culver, C., "Analysis of Live Loads in Office Buildings," Journal of the Structural Division, ASCE, Vol. 103, No. ST8, Proc. Paper (3109, August 1977, pp. 1551-1560.

24. Ferry Borges, J., Castanheta, M., Structural Safety, 2nd Edition, Laboratorio Nacional de Engenharia Civil, Lisbon, Portugal, March 1971.

25. Fiorato, A.E., "Geometric Imperfections in Concrete Structures," National Swedish Building Research Report D5:1973, Stockholm, 1973.

26. "First Order Reliability Concepts for Design Codes, "Bulletin D' Information No. 112, Comite Europeen du Beton, Munich, July 1976.

27. Freudenthal, A.M., Garrelts, J., Shinozuka, M., "The Analysis of Structural Safety," Journal of the Structural Division, ASCE, Vol. 92, No. ST1, Proc. Paper 4682, February 1966, pp. 267-325.

28. Galambos, T.V., Ravindra, M.K., "Tentative Load and Resistance Factor Design Criteria for Steel Buildings," Structural Division Research Report No. 18, Washington University, St. Louis, September 1973.

29. Hald, A., Statistical Theory with Engineering Applications, John Wiley and Sons, Inc., New York, 1952.

30. Hasofer, A., Lind, N.C., "Exact and Invariant Second-Moment Code Format," Journal of the Engineering Mechanics Division, ASCE, Vol. 100, No. EM1., Proc. Paper 10376, February 1974, pp. 111-121.

31. Hognestad, E., "A Study of Combined Bending and Axial Load in Reinforced Concrete Members," University of Illinois Engineering Experiment Station Bulletin No. 399, Urbana, Ill., November 1951.

32. Johansson, A., Warris, B., "Deviations in the Location of Reinforcement," Proceedings No. 40, Swedish Cement and Concrete Research Institute, Royal Institute of Technology, Stockholm, 1969.

33. Johnson, A., "Strength, Safety, and Economical Dimensions of Structures," Bulletin of the Division of Building Statics and Structural Engineering, Royal Institute of Technology, Stockholm, 1953.

34. Jones, P., Reichart, F.E., "The Effect of Testing Speed on Strength and Elastic Properties of Concrete," Proceedings, American Society for Testing and Materials, Vol. 36, 1936, pp. 380-391.

35. Julian, O.G., "Synopsis of First Progress Report of Committee on Factors of Safety," Journal of the Structural Division, ASCE, Vol. 83, No. ST4, Proc. Paper 1316, July 1957, pp. 1316-1322.

36. Kani, G.N.J., "How Safe are Our Large Reinforced Concrete Beams?" Journal of the American Concrete Institute, Vol. 64, No. 3, March 1967.

37. Lind, N.C., "Approximate Analysis and Economics of Structures," Journal of the Structural Division, ASCE, Vol. 102, No. ST6, Proc. Paper 12201, June 1976, pp. 1177-1196.

38. MacGregor, J., Breen, J., Pfrang, E.O., "Design of Slender Concrete Columns," Journal of the American Concrete Institute, Vol. 67, No. 1, June 1970, pp. 6-28.

39. Mattock, A., Kriz, L., Hognestad, E., "Rectangular Concrete Stress Distribution in Ultimate Strength Design," Journal of the American Concrete Institute, Vol. 57, No. 8, February 1961, pp. 875-928.

40. McGuire, R., Cornell C.A., "Live Load Effects in Office Buildings," Department of Civil Engineering Report R73-28, Massachusetts Institute of Technology, Cambridge, Mass., 1973.

41. Mirza, S.A., MacGregor, J.G., "A Statistical Study of Variables Affecting The Strength of Reinforced Normal Weight Concrete Members," Structural Engineering Report No. 58, Department of Civil Engineering, University of Alberta, Edmonton, Alberta, Canada, December 1976.

42. Mitchell, G.R., Woodgate, R.W., "Floor Loadings in Office Buildings--The Results of a Survey, "Current Paper 3/71, Building Research Station, Garston, Watford, UK, 1971.

43. "National Building Code of Canada," National Research Council, Ottawa, Canada, 1970.

44. Peir, J.C., Cornell, C.A., "Spatial and Temporal Variability of Live Loads," Journal of the Structural Division, ASCE, Vol. 99, No. ST5, Proc. Paper 9747, May 1973, pp. 903-922.

45. Pfrang, E.O., Sozen, M.A., Siess, C.P., "Load-Moment-Curvature Characteristics of Reinforced Concrete Cross Sections," Journal of the American Concrete Institute, Vol. 61, No. 7, July 1964, pp. 763-778.

46. Placas, A., Regan. P.E., "Shear Failure of Reinforced Concrete Beams," Journal of the American Concrete Institute, Vol. 68, October 1971. pp. 763-778.

47. Popovics, S., "Relations Between Various Strengths of Concrete," in Highway Research Record No. 210, Symposium on Concrete Strength, National Academy of Engineering, 1967.

48. "Proposed Revision of Building Code Requirements for Reinforced Concrete (ACI 318-56), Journal of the American Concrete Institute, Vol. 59, February 1962, pp. 145-276 (Published discussion in Vol. 59, September 1962 - November 1962).

49. Rajagopolan, K.S., Ferguson, P.N., "Exploratory Shear Tests Emphasizing Percentage of Longitudinal Steel," Journal of the American Concrete Institute, Vol. 65, August 1968, pp. 634-638.

50. "Recommended Practice for Evaluation of Compression Test Results of Field Concrete," Journal of the American Concrete Institute, Vol. 61, September 1964, pp. 1057-1072.

51. Reese, R., "Safety Requirements in Structural Design and ACI 318-71," Journal of the American Concrete Institute, Vol. 70, March 1973, pp. 190-198.

52. Rosenblueth, E., Esteva, L., "Reliability Basis for Some Mexican Codes," in Probabilistic Design of Reinforced Concrete Buildings, American Concrete Institute Publication SP-31, Detroit, Mich., 1972, pp. 1-42.

53. Rusch, N., "Researches Toward a General Flexural Theory for Structural Concrete," Journal of the American Concrete Institute, Vol. 32, July 1960.

54. "Shear and Diagonal Tension," ACI-ASCE Joint Committee 326, Journal of the American Concrete Institute, Vol. 59, January, February, March 1962.

55. Sexsmith, R.G., "Reliability Analysis of Concrete Members," Journal of the American Concrete Institute, Vol. 66, May 1969, pp. 413-420.

56. Simiu, E., Filliben, J., "Statistical Analysis of Extreme Winds," National Bureau of Standards Technical Note No. 868, Washington, D.C., June 1975.

57. Simiu, E., "Equivalent Static Wind Loads for Tall Building Design," in Journal of the Structural Division, ASCE, Vol. 102, No. ST4, Proc. Paper 12057, April 1976, pp. 719-737.

58. Siu, W.C., Parimi, S., Lind, N.C., "Practical Approach to Code Calibration," Journal of the Structural Division, ASCE, Vol. 101, No. ST7, Proc. Paper 11404, July 1975, pp. 1469-1480.

59. Sozen, M.A., Zwoyer, E.M., Siess, C.P., "Strength in Shear of Beams Without Web Reinforcement," University of Illinois Engineering Experiment Station Bulletin No. 452, Urbana, Illinois, April 1959.

60. "Specification for the Design, Fabrication and Erection of Structural Steel for Buildings," in Manual of Steel Construction, American Institute of Steel Construction, New York, 1970.

61. "The Shear Strength of Reinforced Concrete Members, "Task Committee on Masonry and Reinforced Concrete of the Structural Division, James G. MacGregor, Chairman, Journal of the Structural Division, ASCE, Vol. 99, No. ST6, Proc. Paper 9791, June 1973, pp. 1091-1187.

62. Tso, W.K., Zelman, I.M., "Concrete Strength Variations in Actual Structures," Journal of the American Concrete Institute, Vol. 67, No. 12, December 1970, pp. 981-988.

63. Turkstra, C.J., "Theory of Structural Design Decisions," Solid Mechanics Study No. 2, University of Waterloo, Waterloo, Ontario, Canada, 1972.

64. Vickery, B.J., "On the Reliability of Gust Loading Factors," in Wind Loads on Buildings and Structures, R.D. Marshall and H.C.S. Thom, editors, National Bureau of Standards Building Science Series No. 30, November 1970, pp. 93-106.

65. Wen, Y.K., "Statistical Combination of Extreme Loads," Journal of the Structural Division, ASCE, Vol. 103, No. ST6, Proc. Paper 12930 May 1977. pp. 1079-1095.

APPENDIX - ANALYSIS OF UNCERTAINTIES

The analysis of uncertainties, which was summarized in Tables 3.1 and 3.2 in Section 3 is described in detail in this Appendix. The estimation of measures of uncertainty in such basic resistance variables as f_y, f_c, etc. is discussed in section A.1. These are combined to determine total variability Ω and mean-to-nominal ratio $\overline{R}/R'$ in A.2 for flexure, shear, and thrust combined with bending. A similar evaluation of load statistics is presented in A.3. ACI 318-71 standard notation is used whenever possible.

A.1 Estimation of Individual Measures of Uncertainty

Dispersion in the basic variables such as concrete strength, reinforcement yield stress, placement of bars, and member dimensions all contribute to the variability in resistance of reinforced concrete members.

Dispersion in concrete strength is caused by variability in material properties, mix quality control, and in transporting, placing and field curing the concrete. The basic variability δ_{f_c} is determined from standard cylinder test data which relate quality control to δ_{f_c} by [50]

$0.10 < \delta_{f_c} < 0.15$	-	Good quality
$0.15 < \delta_{f_c} < 0.20$	-	Average quality
$0.20 < \delta_{f_c}$	-	Poor quality

Additional uncertainty in f_c arises from unavoidable variations in quality control at the batch plant in preparing the concrete. The dispersion of the observed sample mean strengths $\overline{f}_c$ of concrete delivered during 12 successive monthly periods is about 0.07 [50]. The design compressive strength f_c' is intended to be a conservative value, so that the probability of actually achieving a lesser strength is small. The mean cylinder strength for a batch of concrete $\overline{f}_{c,\ cyl}$ therefore must be somewhat higher than f_c' in order to satisfy section 4.2.2 of ACI 318.71. For a nominal 4000 psi mix with average quality control ($\delta_{f_c} = 0.15$ or $\sigma_{f_c} = 600$ psi) $\overline{f}_{c,\ cyl} \simeq 1.2\ f_c'$ to satisfy this requirement.

It is widely recognized that the strength of concrete in situ, which would be the relevant value to use for reliability calculations, is different from the standard cylinder test value. Data presented by Bloem [8] show the relation between in situ strength (in slabs), as determined from drilled cores, and the standard cylinder test strength. Data given in Table 2 of Ref 8 are plotted in Figure A.1. Regression analyses of the "actual" drilled core strength $f_{c,\ core}$ on standard cylinder strength f'_c yielded,

$$f_{c,\ core} = \begin{cases} 0.67\ f_{c,\ cyl} - \text{Poor cure} \\ 0.84\ f_{c,\ cyl} - \text{Good cure} \end{cases} \tag{A.1a}$$

with σ (see Equation 3.6) = 0.098 and 0.071 for poor and good field curing, respectively. According to Equation 3.6, then, the contribution of field curing to concrete strength variability ranges from 0.08 to 0.15, with an average value of about 0.12. An alternate regression analysis with two undetermined constants yielded

$$f_{c,core} = \begin{cases} 662 + 0.49\ f_{c,cyl} - \text{Poor cure} \\ 596 + 0.67\ f_{c,\ cyl} - \text{Good cure} \end{cases} \tag{A.1b}$$

Within the range $3000 < f'_c < 4000$ psi, the results are practically the same.

Other factors influence the variability in f_c. For example, moisture migration upward and settlement of aggregate cause vertical or deep members to be stronger at the bottom than at the top. At the bottom and top of columns, the concrete strength was 8 percent higher and 4 percent lower, respectively, than the average column strength [19]. Using either Equations 3.2a or 3.2b, the contribution of this segregation to total variability in resistance is 0.03. Other factors are more difficult to assess; the sustained load strength of concrete is about 85 percent of its short term strength for example, and low strain rates may only yield about 85 percent of the strength obtained using ASTM test methods [34,50,53]. On the other hand, concrete gains strength beyond its 28-day design strength; after one year, Bloem [8] found this increase could be in excess of 30 percent. Moreover, confining the concrete elevates its strength also. These factors tend to offset one another.

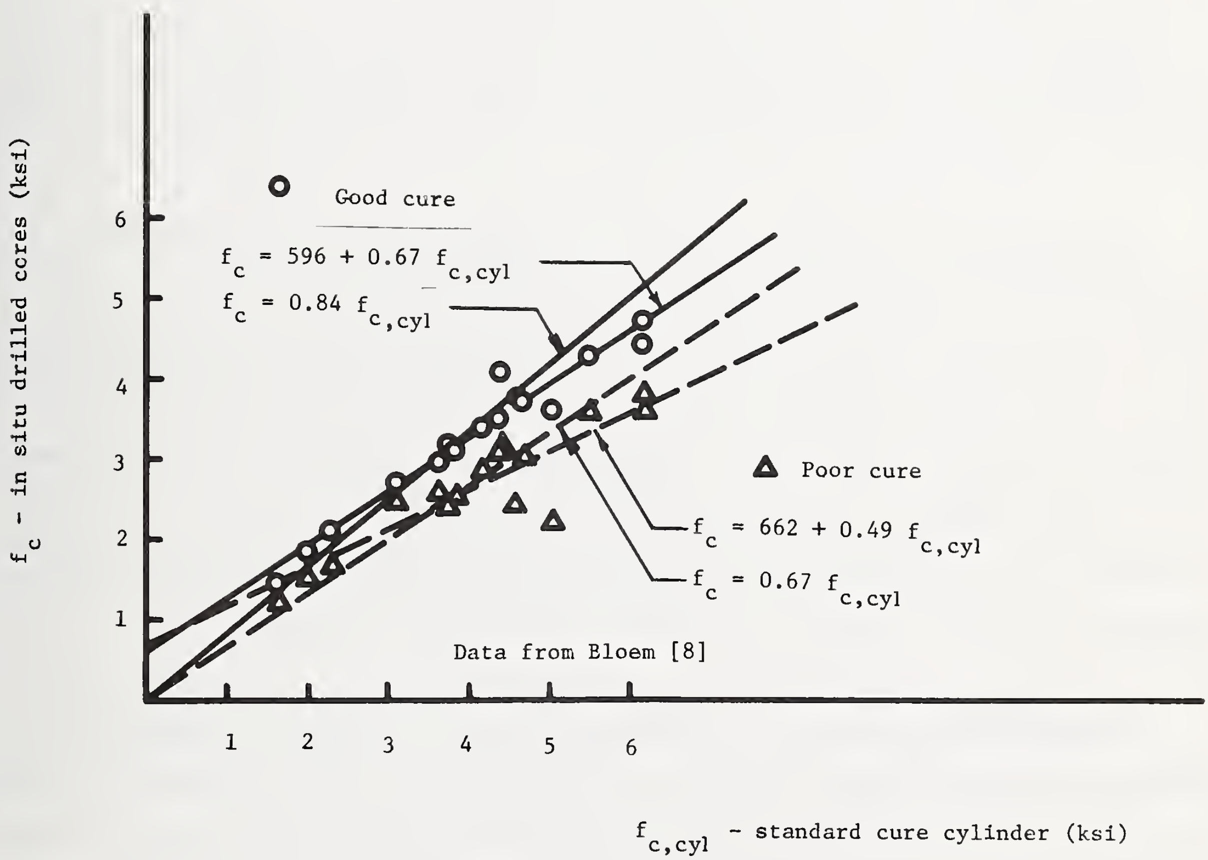

Figure A.1 - Variability in Concrete Strength

The strength of concrete in situ f_c is thus,

$$f_c = f'_c \frac{f_{c,\,cyl}}{f'_c} \frac{f_{c,\,core}}{f_{c,\,cyl}} \text{ (other factors)} \tag{A.2a}$$

Using Eq. A.1 and recalling that $f_{c,\,cyl} \simeq 1.2\, f'_c$, the mean strength $\bar{f}_c$ is

$$\bar{f}_c \simeq \begin{cases} 0.8\, f'_c \text{ for poor field cure} \\ 1.01\, f'_c \text{ for good field cure} \end{cases} \tag{A.2b}$$

The total in situ variability is,

$$\Omega_{f_c} \simeq [\delta^2_{f_c} + (0.07)^2 + (0.12)^2 + (0.03)^2]^{1/2} \tag{A.2c}$$

This leads to values of Ω_{f_c} which range from 0.174 to 0.207 as δ_{f_c} increases from 0.10 to 0.15.

The variability in concrete strength in existing buildings was determined by Drysdale [19] using ultrasonic techniques. Variability in pulse velocity appears to correlate quite well to that in f_c. For 1145 columns in 14 buildings Drysdale found that Ω_{f_c} varied from 0.07 to 0.30. In four buildings, Ω_{f_c} was in the range 0.05 to 0.10; in eight 0.10 to 0.20; and in two, Ω_{f_c} exceeded 0.20. Accompanying standard cylinder test data were unavailable. In an earlier study using similar techniques, Tso and Zelman [62] found that test cylinder strengths did not correlate well with in situ strength, undoubtedly due in part to the additional sources of variability discussed in the preceding paragraphs.

Tensile strength in concrete is commonly measured by the modulus of rupture or the cylinder splitting strength. In one set of tests [59], the c.o.v. in the modulus of rupture ranged from 0.13 - 0.18; in another [47] the c.o.v. in the splitting strength varied from 0.06 to 0.14. These appear to be roughly equivalent to the δ_{f_c} found from standard cylinder compression tests; additional sources of uncertainty tend to increase the variability in tensile strength as in compression. Thus, variabilities in tensile strength and compressive strength do not appear to be distinguishable and will be assumed to be equal in this study.

Variability in reinforcement yield strength is determined primarily from mill test data. Data reported by Julian [35] for Grade 40 reinforcement, No. 3 through No. 10 bars, showed that $\bar{f}_y$ = 47.7 ksi [329 MN/m^2] and δ_{f_y} = 0.12. Recent data reported by Mirza and MacGregor [41] showed that for Grade 40 bars (No. 3 through No. 11) $\bar{f}_y$ = 48.8 ksi (336 MN/m^2) and δ_{f_y} = 0.107 while for Grade 60 bars (Nos. 5 through 11) $\bar{f}_y$ = 71.1 ksi (490 MN/m^2) and δ_{f_y} = 0.093. These data include an uncertainty due to size effects. Earlier data reported by Baker [6] showed that $\bar{f}_y$ decreased from 50.4 ksi (348 MN/m^2) for No. 3 bars to 44.1 ksi (304 MN/m^2) for No. 8 bars; δ_{f_y} ranged from 0.07 to 0.11 but this variation was not related to bar size. On the basis of these data, it appears that $\delta_{f_y} \simeq 0.09$ and the variability in the mean $\bar{f}_y$ due to the size effect is about 0.04 [21].

It should be noted that this δ_{f_y} may also actually include the effect of variability in the bar area A_s, since it is not clear in the data sources cited above whether f_y was determined on the basis of nominal area A_n or actual bar area. If $T = A_s f_y$, and

$$f_{y,\,n} = \frac{A_s f_y}{A_n} \qquad \text{(A.3a)}$$

then the observed $\delta^2_{f_{y,\,n}}$ is actually

$$\delta^2_{f_{y,\,n}} \simeq \delta^2_{A_S} + \delta^2_{f_y} \simeq \delta^2_{f_y} \qquad \text{(A.3b)}$$

Since δ_{A_s} is about 0.02 [21] this source of error should be negligible.

Mill tests upon which the above assessments of variability are made indicate a bar yield strength which is higher than would probably be observed in practice because of the elevated strain rates at which tests are made. Julian [35] has indicated that this elevation may be on the order of 5 - 10 percent. This was borne out by Allen [2], who found that the elevation in mean value was about 3 ksi (20.7 MN/m^2) for Grade 40 reinforcement, with a c.o.v. of about 0.13. If the yield strength in the structure is given as

$$f_y = x_1 f_{y,\,mill} - x_2 \qquad \text{(A.4a)}$$

where x_1 and x_2 are size effect and strength elevation, respectively, then

$$\Omega_{f_y}^2 = \frac{\Delta_{x_1}^2 + \delta_{f_y}^2 + (\overline{x}_2/\overline{x}_1\ \overline{f}_{y,\ mill})^2\ \Delta_{x_2}^2}{(1 - \overline{x}_2/\overline{x}_1\ \overline{f}_{y,\ mill})^2} \tag{A.4b}$$

Substituting $\Delta_{x_1} = 0.04$, $\delta_{f_y} = 0.09$ and $\Delta_{x_2} = 0.13$, and assuming that x_2 is 10 percent of $x_1\ \overline{f}_{y,\ mill}$ leads to $\Omega_{f_y} = 0.11$.

Recent data [25] on variability in reinforced concrete members dimensions and bar placement has shown that these variabilities have a definite tendency to decrease as the member size increases. This is illustrated in Figure A.2 for member thickness, h. The δ_h reported by Johnson [33] and Johansson and Warris [32] was obtained from measurements taken on in situ slabs, all of which were less than 8 in (203 mm) thick. The data reported by Tso and Zelman [62] for columns ranging from 12 to 30 inches (304 to 762 mm) shows this decrease more clearly. It appears from Figure A.2 that $\delta_h \simeq 0.4/h_n$, where h_n is the nominal minimum member dimension. Additional confirmation for this is provided by Birkeland and Westhoff [7], who estimate (on the basis of extensive field experience) that member dimensions vary by $\pm$ 7/8 inch (22 mm). Using Equation 3.2b as previously discussed leads to $\delta_h \simeq 0.44/h_n$. Finally, Mirza and MacGregor [41] suggest that $\delta_h \simeq 0.46/h_n$ for slabs, $0.214/h_n$ for beams, and $0.262/h_n$ for columns. The latter study also indicates that the variability in beam width is quite small.

Additional uncertainty arises from variation in mean dimensions from those nominally specified. The slab data suggest that this variability is about 0.04 [32,33]. Data in Ref. 62, Table 2 for thicker members reveals that the <u>mean</u> tolerance varies between -3 and +5 percent; using Equation 3.2b, this implies the variability due to uncertainty in mean value prediction is about 0.02; thus, Δ_h also depends on the size of the member, and appears to be roughly $0.25/h_n$. Combining this with the previous estimate for δ_h, the total variability is $\Omega_h \simeq 0.45/h_n$. Variability in the width of beams is quite small [41].

Basic variability δ_d in the placing of reinforcement is illustrated in Figure A.3. As with member thickness, δ_d is seen to decrease with member size. Measurements of effective depth to reinforcement in slabs [32,33] were taken directly. In Drysdale's study [18] of

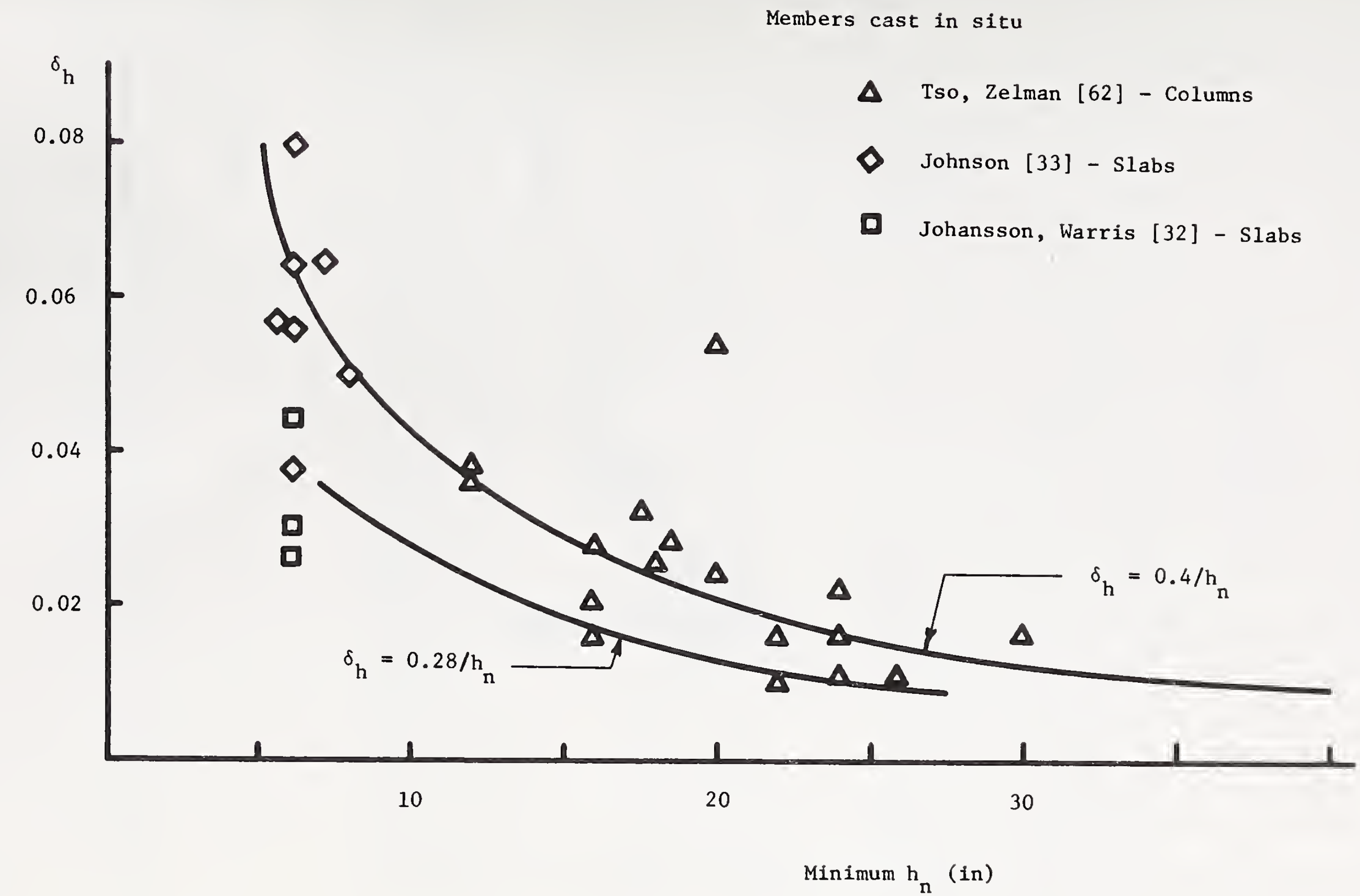

Figure A.2 - Variability in Member Dimensions

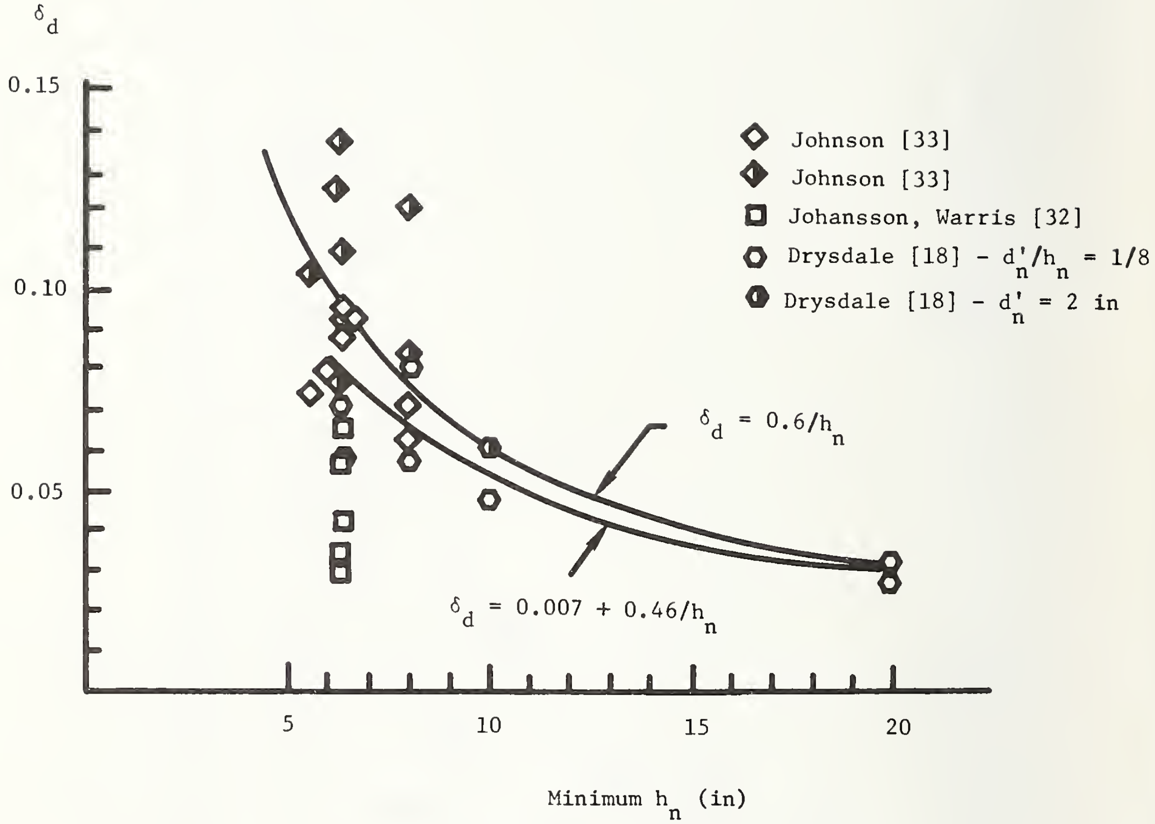

Figure A.3 - Variability in Effective Depth to Reinforcement columns, however, statistics for the ratio $r = d'/d'_n$, the ratio of the actual to nominal concrete cover, were reported instead for column sizes ranging from 12 in (305 mm) to 42 in (1067 mm) in size. In order to extract δ_d from the statistics of r, note that $d = h - d'_n r$, assuming symmetrical reinforcement in the column, and thus

$$\delta_d \approx \frac{[\sigma_h^2 + (d_n')^2\sigma_r^2]^{1/2}}{\overline{h}_n - d_n' \overline{r}} \qquad (A.5)$$

In order to use this relationship, it was assumed that d_n'/h_n was 1/8 or $d_n' = 2$ in (51 mm) and that $\delta_n \approx 0.4/h_n$ as previously determined; the resulting δ_d are plotted in Figure A.3. It appears that $\delta_d \approx 0.6/h_n$, which is somewhat higher than for member dimensions. Birkland and Westhoff [7] suggest that variations in placement ranging from + 1" to -1 3/8" (+25 to -35 mm) may be common in practice; using Equation 3.2b, this leads to an estimate of δ_h which is even slightly higher. The study by Mirza and MacGregor [41] suggests that $\delta_d \approx 0.594/d_n$ in slabs.

Additional variability is induced by the deviation of mean effective depth from d_n. The ratio d/d_n appears to be consistently biased, varying between 0.97 and 1.0. While for slabs, this $\Delta_d \approx 0.05$ [32,33] it appears to be about 0.02 for columns [18]. This implies that $\Delta_d \approx 0.32/h_n$; the total variability is then $\Omega_d = 0.68/h_n$.

The data discussed above have been derived for concrete members which are cast in situ. Fiorato's recent study [25] also contains data on variabilities in precast members. These data indicate that the variability in precast member geometry is less than one-half that for members cast in situ.

A.2 Structural Resistance

A.2.1 Flexure

The calculated ultimate flexural capacity of a properly designed underreinforced concrete beam according to ultimate strength provisions of ACI 318-71 is,

$$M_u = A_s f_y d \left(1 - 0.59 \frac{A_s}{bd} \frac{f_y}{f_c}\right) \qquad (A.6)$$

The mean and c.o.v. may be calculated as,

$$\overline{M}_u \simeq \overline{A}_s \overline{f}_y \overline{d} \left(1 - 0.59 \frac{\overline{A}_s}{\overline{b}\overline{d}} \frac{\overline{f}_y}{\overline{f}_c}\right) \tag{A.7a}$$

$$\Omega^2_{M_u} \simeq [(1 - 1.18\,\overline{q})^2\, \Omega^2_{f_y}$$

$$+ (0.59\,\overline{q})^2\, \Omega^2_{f_c} + \Omega^2_d]/(1 - 0.59\,\overline{q})^2 \tag{A.7b}$$

using Equation 3.3 in which $\overline{q} = (\overline{A}_s \overline{f}_y/\overline{b}\overline{d}\ \overline{f}_c) = \rho\ \overline{f}_y/\overline{f}_c$. Contributions of variability A_s and b have been neglected, as they and the coefficients by which they are multiplied in Equation A.7b are quite small.

Additional uncertainties arise from the inability of Equation A.6 to predict the failure load precisely when all its parameters are known (modeling errors). This contribution of bias and variability is analyzed using data provided by Corley [13] and Sexsmith [55] and Equations 3.5. The values $\overline{B} = 1.11$ and $\sigma/\overline{B} = 0.07$ are based on laboratory test data, wherein all parameters presumably are known with some confidence, and thus represent bias and variability due to the equation itself.

The total variability Ω_M in flexural resistance is thus evaluated as $\Omega_M = [\Omega^2_{M_u} + (0.07)^2]^{1/2}$. With Grade 60 reinforcement, 4000 psi (2758 KN/m^2) concrete, and $\rho = 0.02$, parameter $\overline{q} = 0.3$. With these values, Ω_M is found to decrease from 0.159 when h = 8 in (203 mm) to 0.126 when h = 24 in (610 mm). In Ref. 21 Ω_M was found to vary slightly with ρ, but this variation is small and can be ignored.

The ratio between the mean in-situ resistance and the nominal calculated resistance (capacity computed using nominal material strengths and dimensions) is also required for calibration. Using the data presented in Section A.1, $\overline{M}/M'$ is found to average about 1.12. Any reduction in concrete strength under long-term load has little effect on flexural strength, since this is determined primarily by $A_s f_y$ and d.

A.2.2 Shear

The analysis of uncertainties in shear resistance is considerably more complex than for flexure. The truss analogy model used to compute the capacity is approximate in nature because the mechanism of shear in beam and slabs is not completely understood. Recent studies [54,61] have shown the c.o.v. in shear resistance to be in the range 0.18 to 0.24. Because of the implications that such high variabilities have for structural safety calculations, the analysis of uncertainties in shear is discussed at some length in the following.

According to the truss analogy equation, the ultimate shear capacity of reinforced concrete beams is given by

$$V_u = V_c + V_s = v_c \, bd + \frac{d}{s} A_v f_y \qquad \text{(A.8a)}$$

or

$$v_u = v_c + \rho_v f_y \qquad \text{(A.8b)}$$

in which v_c is the nominal shear stress at which inclined cracking occurs;

$$v_c = 1.9 \sqrt{f'_c} + 2500 \, \rho \, V \frac{d}{M} \leq 3.5 \sqrt{f'_c} \qquad \text{(A.9a)}$$

and $\rho_v = A_v/bs$, A_v and s are area and spacing of the stirrups.

A great number of tests of reinforced concrete beams in shear conducted during the past twenty years are summarized in reports of the joint ACI-ASCE Committee 426 [53,61]. The variability in these tests is illustrated in Figure A.4, where test values of v_u are plotted against the v_u calculated from Equation A.8b. As these tests were conducted under laboratory conditions, the variability may be interpreted as that which would exist when all parameters in the prediction model are known. Inspection of this large body of test data reveals that Equation A.8b is conservatively biased.

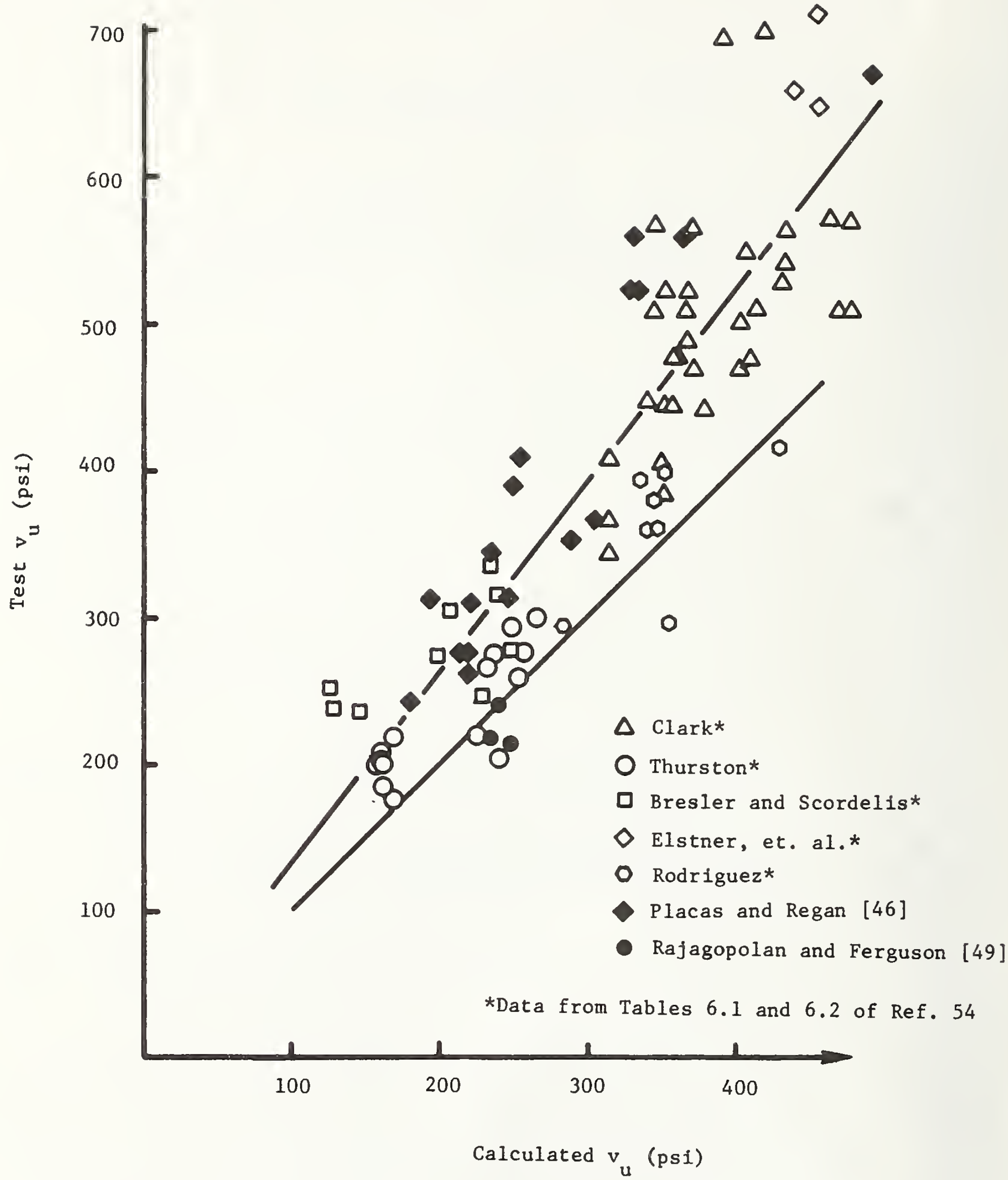

Figure A.4 - Bias and Variability in Shear Equation

A portion of this bias and variability is caused by Equation A.9a, which is also empirical. Table 5.20 of ref. 53 summarizes the mean and c.o.v. of the ratio $v_{c,test}/v_{c,calc}$ for 430 beams without web reinforcement. In that analysis, data with varying characteristics were lumped together. In an effort to obtain a more homogeneous sample, these data were first separated according to source and to whether $v_c/bd\sqrt{f'_c}$ was less than or equal to 3.5. The mean and standard deviation of the ratio of test to calculated v_c were then recomputed. The results are summarized in Table A.1. The mean and variance viewed on this basis both tend to be lower than those values reported in Table 5.20 of ref. 53. Statistically unbiased estimates of this mean and variance are $\bar{B} = 1.105$ and $\sigma^2 = (0.116)^2$. Inclusion of the more recent data of Rajagopolan [49] and Kani [36] does not change these estimates significantly.

Additional bias and uncertainty arise from the truss analogy model itself, including its failure to consider shear friction and doweling, and the assumption that the stirrups have yielded. However, it is difficult to sort out these various sources of bias from available data since the actual portions of shear carried by the concrete and steel in a beam with web reinforcement are unknown. Much of the data presented in Tables 6.1 and 6.2 of ref. 54 are not helpful in answering these questions, because in many cases the limiting values of $8\sqrt{f'_c}$ or $10\sqrt{f'_c}$ were used to calculate the capacity and the contributions of the steel and concrete cannot be correctly apportioned. Moreover, it was apparent that the equation bias is not constant for all beam geometries, but tends to increase as the parameter $\rho_v f_y/v_c$ increases. Finally, the bias tends to be higher for restrained beams and for T-beams.

Data on beams with web reinforcement tested in shear [46,54] were separated according to source and whether $v_u < 8\sqrt{f'_c}$ in an attempt to obtain more homogeneous samples than those analyzed in Tables 6.1 and 6.2 of ref. 54. Beams with diagonal stirrups, those which failed by splitting, and T-beams were omitted. The analysis of these data is summarized in Table A.2. The c.o.v. in the ratio of test to calculated strength tends to be lower than the values 0.2 and 0.22 presented previously [54].

The simplest approach for incorporating bias and variability arising from the truss analogy equation is to apply an overall factor B to V_u. The quantities $\bar{V}_u$ and $\sigma^2_{V_u}$ in

Table A.1 - Analysis of Inclined Cracking Equation

	Source	n	β	σ
$v_c/bd\sqrt{f_c} < 3.5$	Moody, et. al (S)*	28	1.164	0.125
	Moody, et. al (R)*	15	1.082	0.140
	Bower (S)*	5	1.004	0.090
	Bower (R)*	28	1.004	0.090
	Rodriquez (c)*	13	1.001	0.109
	Thurston (S)*	58	1.241	0.129
	PCA - anson*	5	1.196	0.063
	Rajagopolan (S)[49]	8	1.144	0.208
	Kani (S) [36]	13	1.107	0.141
$v_c/nd\sqrt{f_c} = 3.5$	Moody, et. al.* (S)	12	0.884	0.094
	Moody, et. al.* (R)	43	1.052	0.112

R = restrained; S = simple beam

*Data from Chapter 5, ACI Committee 326 Report [54].

Table A.2 - Analysis of Truss Analogy Equation

	Source	n	β	σ
$v_u < 8\sqrt{f'_c}$	Clark (S)*	37	1.325	0.229
	Bresler (S)*	9	1.475	0.306
	Thurston (S)*	19	1.165	0.121
	Placas (S) [46]	17	1.421	0.159
	Elstner (R)*	4	1.549	0.122
	Rodriguez (R)*	8	1.045	0.106
$v_u = 8\sqrt{f_c}$	Clark (S)*	9	1.198	0.106
	Elstner (R)*	23	1.601	0.227
	Rodriguez (R)*	12	1.523	0.304

*Data from Chapter 6, ACI Committee 326 Report [54].

R = Restrained beam; S = simple beam

this approach would be determined from linear statistical analysis of Equation A.8a, while σ_ε^2 would equal $V_u^2\sigma^2$, conforming to the increase in scatter observed in Figure A.4 as v_u increases. Statistically unbiased estimates for $\overline{B}$ and σ^2, obtained from the data in Table A.2, are $\overline{B} = 1.31$, $\sigma^2 = (0.197)^2$ and $\sigma/\overline{B} = 0.150$ if $v_u < 8\sqrt{f'_c}$. This simple model can include terms where $v_u = 8\sqrt{f'_c}$ (or $10\sqrt{f'_c}$) in the equation uncertainty analysis, as we are looking simply at overall bias. However, when $\rho_v f_y$ becomes small, approaching the case of a beam without web reinforcement, the equation bias should be about 1.11 in accordance with the previous analysis of the inclined cracking load equation. This approach thus is incompatible with the data which show that the bias increases as $\rho_v f_y/v_c$ increases.

To eliminate this incompatibility, individual bias terms may be applied instead to the portion of shear carried by the concrete and web steel. The mean and variance would be

$$\overline{V} = \overline{B}_1 \overline{v}_c \overline{b}\overline{d} + \overline{B}_2 \frac{\overline{d}}{\overline{s}} \overline{A}_v f_y \tag{A.10a}$$

$$\sigma_V^2 = \overline{B}_1^2 \sigma_{V_c}^2 + \overline{B}_2^2 \sigma_{V_s}^2 + \sigma_\varepsilon^2 \tag{A.10b}$$

If $\sigma_\varepsilon^2 = (v_c + \rho_v f_y)^2\sigma^2$, σ^2 may still be interpreted as the variance in the ratio of test to calculated strength. So that Equation A.10 is valid when $\rho_v f_y/v_c \simeq 0$, $\overline{B}_1$, is first set equal to 1.11; $\overline{B}_2 = 1.38$ is then determined by least squares fitting of that data (Table A.2) where $v_u < 8\sqrt{f'_c}$ (or $10\sqrt{f'_c}$).

The total variability in resistance to shear is $\Omega_V = \sigma_V/\overline{V}$, were $\overline{V}$ and σ_V are obtained from Equations A.10 and σ_{V_c} and σ_{V_s} are determined from linear statistical analysis of Equation A.8. The appearance of the term $\sqrt{f'_c}$ in Equation A.9a is intended to reflect the tensile strength f_t of the concrete. Thus, by replacing $\sqrt{f'_c}$ with $x\, f_t$, where x is a constant, it is found that $\Omega_{v_c} \simeq \Omega_{f_t} = \Omega_{f_c}$. This result also can easily be shown to obtain in those situations where $v_c = 3.5\sqrt{f'_c}$, or at very small reinforcement ratios where it has been suggested [61] that the inclined cracking equation should be

$$v_c = (0.8 + 120\rho)\sqrt{f'_c} \leq 2.3\sqrt{f'_c} \tag{A.9b}$$

While Ω_V tends to be slightly higher for very lightly reinforced webs, for practical purposes it may be treated as constant at $\Omega_V = 0.22$. The variation in Ω_d and Ω_b caused by member size has little effect in Ω_V, because of the magnitude of the other variabilities, particularly that due to the equation.

Calculations using Equations A.10 and the data in section A.1 show that $\overline{V}/V'$ ranges from about 1.14 when the minimum web reinforcement permitted by ACI 318-71 is used ($A_v = 50\ bs/f_y$) up to about 1.30 for very heavily reinforced webs. For purposes of calibration to existing designs, a value of 1.18 will be used, recognizing that the results may be slightly conservative for heavily reinforced webs.

A.2.3 Axial Thrust and Bending

The analysis of uncertainty in the resistance of reinforced concrete sections subjected to a combination of axial thrust and bending presents special problems. This resistance, denoted R_u, is described by an interaction diagram, a locus of thrust and moment combinations (P, M) that cause failure. General expressions [45] for resistance derived directly from ultimate strength principles are unavailable in closed form. Therefore, Equation 3.4 cannot be applied in evaluating the mean and variance in resistance.

Monte Carlo simulation provides a useful alternate method for evaluating the statistics of R_u. Values of f_y, f_c, etc. are sampled numerically from probability distributions having means and c.o.v. described in section A.1 and a series of interaction diagrams is generated. The mean and c.o.v. of R_u can then be evaluated as a function of the eccentricity ratio $e/h = M/Ph$ or at particular mean thrust or moment levels, depending on the character of the applied load [24,52]. If thrust and moment increase in proportion to failure, the statistics at fixed eccentricity are most appropriate.

This analysis has been described in some detail in a recent publication [22]. Therein, δ_{R_u} and Δ_{R_u} were analyzed separately; according to Equation 3.1, $\Omega_{R_u} = [\delta^2_{R_u} + \Delta^2_{R_u}]^{1/2}$. Ω_{R_u} may also be obtained directly using the estimates of total variability (Ω_b,Ω_h, etc.) presented in Section A.1 in the simulation. Ω_{R_u} is shown as a fuction of ρ_g and e/h in

Figure A.5 for a short beam-column constructed of 4000-psi concrete and Grade 60 reinforcement with average quality control. Simulations on sections of different size yielded essentially the same results. When ρ_g is small, Ω_{R_u} is elevated at small e/h; however, it decreases as the thrust component lessens and the compressive strength of the concrete becomes less significant to the overall resistance. It appears that Ω_{R_u} may be taken as sensibly constant provided that $\rho_g > 0.02$ (22). Thus, except for lightly reinforced columns, the increase in ϕ' with e/h permitted in ACI Standard 318-71 must be justified on the basis of the accompanying additional ductility prior to failure rather than any significant decrease in Ω_R. As a point of interest, the Monte-Carlo simulation should yield approximately the same results as Equation A.7b as $e/h \to \infty$. With $\rho = 0.03$, the latter equation yields $\Omega_{M_u} = 0.125$, which may be seen to compare quite well with the simulated values in Figure A.5.

The bias and variability in the strength prediction are assessed from analyzing laboratory tests of short columns [31]. These data show that $\overline{B} = 0.99$ and $\sigma^2 = (0.0603)^2$, making the contribution of equation variability to Ω_R about 0.06. That this is close to the flexural equation variability is explained by the similarity in the assumptions underlying the flexural and combined thrust and bending analyses [39,45]. Using the results in Figure A.5, most columns would fall in the range $0.14 < \Omega_R < 0.18$. The value of 0.17 used in the calibration studies corresponds to a moderately reinforced column constructed with average quality control.

The ratio between the mean in situ and nominally calculated resistance, calculated using methods similar to those for flexure and shear, was found to depend on e/h. A best overall value was 0.965. This is lower than for flexure, since the difference between in situ and mean concrete discussed earlier becomes more important for sections which must resist significant axial loads.

The data used above to analyze the equation bias and variability were obtained from tests on tied columns. The results should not differ significantly for columns with spiral reinforcement, since the primary advantage in using spiral reinforcement is in the greatly improved column ductility that it affords [38,39].

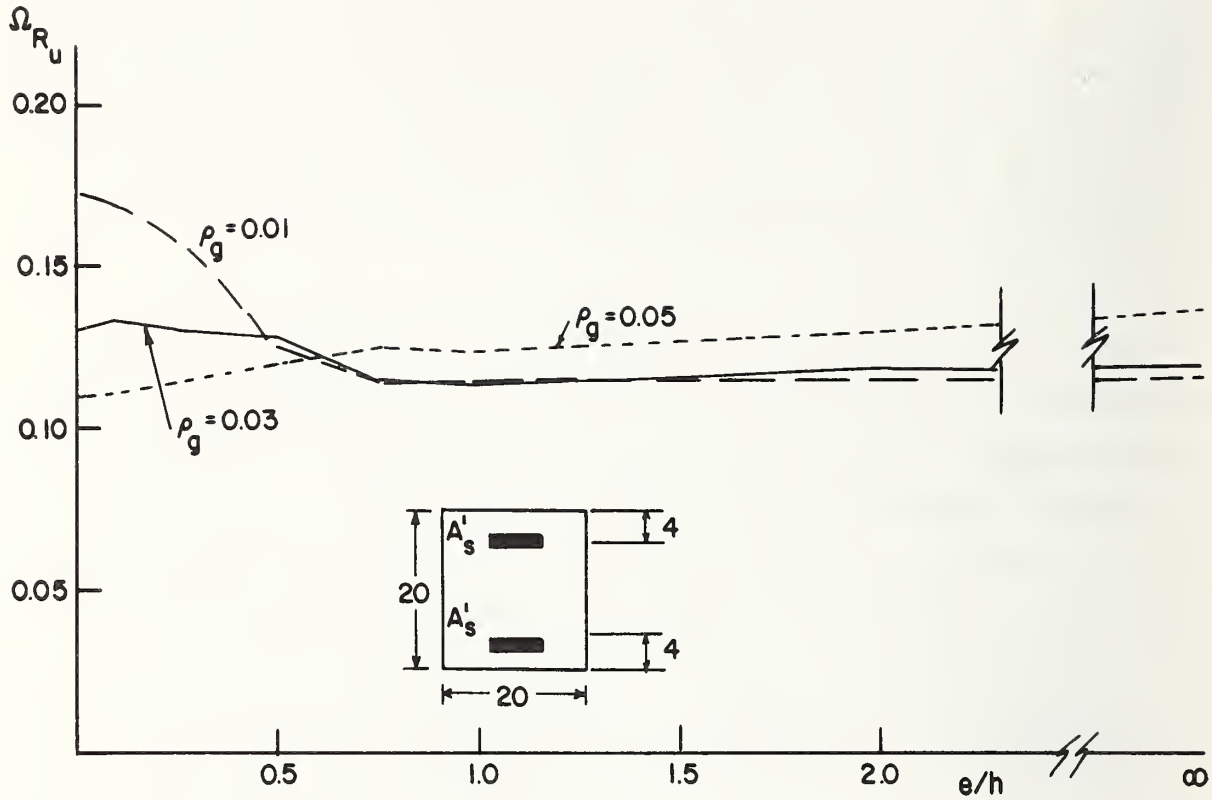

Figure A.5 - Coefficient of Variation in Beam Column Resistance Due to Coefficients of Variation in Individual Variables (f'_y = 60000 psi, f'_c = 4000 psi)

Slenderness effects in columns are accounted for in existing designs using the moment magnifier method which amplifies the design moment on the column. This may affect the bias and variability in R. In an extensive comparison of test values to predictions for a $k\ell/r \leq 100$ it was found [38] that E(Test/Calc) = 1.13, with a c.o.v. of 0.169. These data are stratified according to $k\ell/r$. Analysis of those data where $k\ell/r = 46 - 57$, for example, leads to $\sigma = 0.152$. Removing that component of variability inherent to the short column analysis (0.06), the c.o.v. attributable to the moment magnifier method is about 0.13 for columns where $k\ell/r < 70$. Studies have shown [38] that $k\ell/r$ usually is less than 70 in reinforced concrete columns; in fact, in most braced frame structures, $k\ell/r < 40$. Thus, for slender columns, Ω_R would increase from 0.17 to about 0.21. In terms of safety, however, this is partially offset by the effect of the more conservative equation bias ($\overline{B} \simeq 1.13$) on $\overline{R}/R'$.

A.3 - Structural Loads and Load Effects

Variabilities in load effects which arise from gravity and wind are emphasized herein since for most structures they are the loads often checked in preliminary designing. The basic approach taken is to relate the load A to its load effect S_A using the influence coefficient c_A:

$$S_A = c_A A \qquad \text{(A.11a)}$$

assuming the transformation from load to load effect is a linear one. The total variability is defined by

$$\Omega_{S_A} = \Omega_{c_A}^2 + \Omega_A^2 \qquad \text{(A.11b)}$$

in which uncertainties in c_A arise from the analysis transforming the load to a load effect and Ω_A would include basic variability in A, as well as uncertainty in the load model itself. It may be noted that the term Ω_{c_A} in Equation A.11b serves the same purpose as the "analysis" term proposed by Cornell [15] and used by Galambos[28].

A.3.1 Gravity loads

The dead or permanent load results from the weight of elements comprising the structure and also includes permanent equipment, partitions and installations. This weight is quite predictable when the geometry of the structure is specified and equipment specifications are known. The ratio of mean to nominal value is assumed to be unity. Previous studies have employed values of Ω_D ranging from 0.06 to 0.20 [21,24,28,37], most being between 0.06 and 0.13; herein, a value of 0.10 is used.

The specification of live loads for structural design consists of a basic uniformly distributed unit load (load per unit area), which depends on a particular occupancy type, and a reduction factor by which the basic load may be multiplied to account for the decrease in unit load observed as the loaded or tributary area increases. The load reduction factor currently recommended by ANSI A58.1-1972 and other standards can be traced back thirty years where it was determined, somewhat subjectively, from load survey data gathered in two federal office buildings.

In recent years, a significant amount of load survey data have become available for office buildings in the United Kingdom (UK) [42] and the United States (US), the latter data being gathered in a recent NBS load survey [16]. Concurrently, analytical procedures to model live loads on structures probabilistically [40,44] have been developed. The methods enable an equivalent uniformly distributed load (EUDL) to be determined which will produce (statistically) the same load effect on a structural member (beam moment, column thrust, etc.) as the actual random set of loads. This methodology has been applied successfully to the UK load survey data [40], and to the US data [23], as summarized here.

On the basis of a statistical analysis of NBS data from all rooms which were randomly selected, the total mean unit live load was found to be 11.6 psf (555 N/m^2); this is also equal to the mean EUDL. In comparison, the mean unit load obtained using the UK data [40] was 11.8 psf (565 N/m^2). The variance in L was found to be Var [L] = 26.2 + 14300/A, in which A is the influence area.

The EUDL L obtained from load survey data L denotes the point-in-time sustained live load. The probability distribution of the maximum sustained load L_s seen during the lifetime T of the building is determined by assuming that occupancy changes occur as a Poisson random process with rate ν_L; the distribution of L_s is then given by,

$$F_{L_s}(x) = \exp\left[-\nu_L T(1 - F_L(x))\right] \tag{A.12}$$

in which F_L is the distribution function for L. NBS survey data show an average occupancy duration of 8 years; with T about 64 years, $\nu_L T = 8$. Loads may also occur from infrequent clustering of people above and beyond normal personnel loads and other activities such as remodeling (extraordinary loads). For the purposes of structural design, the mean lifetime maximum total EUDL L_t is of interest, as it is this EUDL which is analogous to the design live load specified in building codes and standards. The upper fractiles (0.9 - 0.99) of L_t may be estimated as the corresponding upper fractiles of the maximum sustained load L_s plus the mean of the extraordinary load for the period over which L_s acts [40]. Approximate formulas for the mean and variance of L_t or L_s may be derived by fitting a Type I Extreme Value distribution to the upper load fractiles and calculating the mean and variance of the fitted Type I distribution.

A comparison of expressions for $E[L_s]$ and $E[L_t]$ with the live loads currently permitted by ANSI A58.1-1972 is presented in Figure A.6. As the loads are presented as functions of influence area rather than tributary area, separate curves are required for ANSI floor and column loads. The agreement between values of $E[L_s]$ based on the UK and NBS data is very close. It may also be observed that the maximum sustained load is much less than the ANSI load where areas are small, but is almost identical to it at larger areas.

For the maximum total EUDL L_t the analysis described above gives, approximately,

$$E[L_t] = 18.7 + 520/\sqrt{A} \tag{A.13a}$$

$$Var[L_t] = 14.2 + 18900/A \tag{A.13b}$$

The analysis using the UK data resulted in $E[L_t] = 14.9 + 763/\sqrt{A}$ and $Var\ [L_t] = 11.3 + 15000/A$. As seen in Figure A.6 there is a significant difference between the two mean

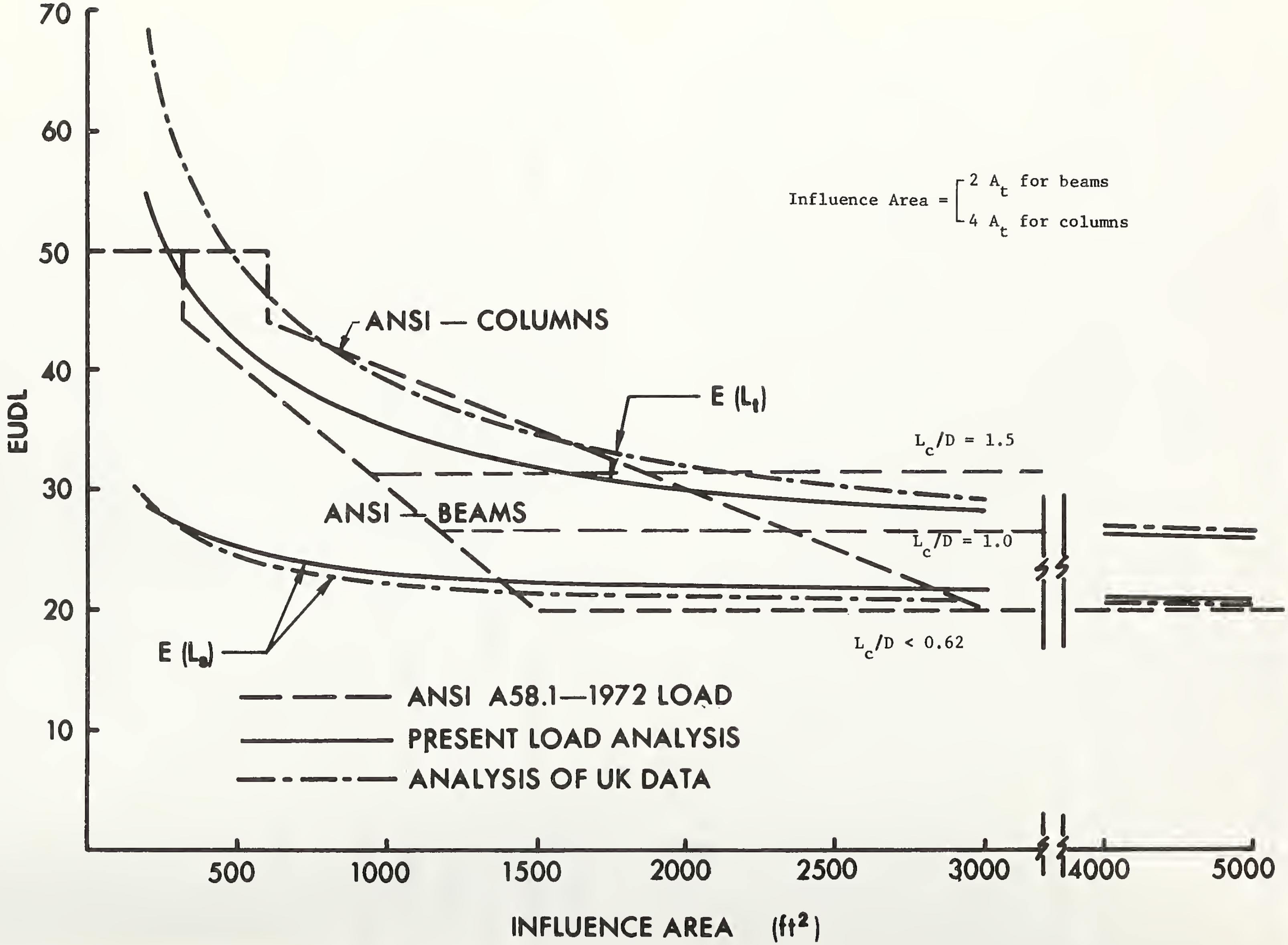

Figure A.6 - Comparison of Live Loads

loads at small influence areas. This difference is attributed to differences in the parameters used in the extraordinary load model. This is somewhat disturbing, as unfortunately there is little objective basis for selecting the extraordinary load parameters. Additional research is needed so that the parameters of the extraordinary live load model can be estimated with confidence.

At large areas, $E[L_t]$ exceeds the ANSI live load regardless of which extraordinary load model is chosen. It seems clear that the maximum load reduction of 60 percent allowed by ANSI is too large; according to this analysis, the maximum load reduction should be something less than 50 percent. Moreover, the current load reduction rate of 0.08 percent per square foot of area over 150 ft^2 (13.94 m^2) permits too rapid a reduction in load for beam loads, although it appears to be about right for column loads.

The total variability Ω_L in maximum live load effect is obtained by augmenting the data-based variability, obtained as $Var[L_t]/E[L_t]$ from Equations A.13, with uncertainty measures attributable to the load model and the structural analysis relating load and load effect. These are taken as 0.20 and 0.05, respectively [15,21,28]. Ω_L thus ranges from about 0.28 for $A = 200\ ft^2$ to about 0.26 for $A = 2000\ ft^2$, and for purposes of this study will be taken as constant at 0.26.

Statistics for the instantaneous live load are required when combining loads from several sources [63]. As above, this uncertainty is obtained by adding $(0.20)^2 + (0.05)^2$ to the data-based variability which is given by $[(26.2 + 14300/A)/11.6]^{1/2}$. Ω_{L_I} ranges from 0.71 when $A = 400\ ft^2$ $(37.2m)^2$ to 0.51 when $A = 4000\ ft^2$ $(372m^2)$, a variation that must be considered in the calibration.

A.3.3 Wind Loads

The wind pressure on a structure is given by an expression of the type

$$W = GF[1/2\rho\ C_p\ V^2\ (\tfrac{z}{10})^{2\alpha}] \qquad (A.14)$$

using the power law for wind velocity profile with height z, in which ρ, C_p and V are mass density of air, pressure coefficient, and basic wind speed at a height of 10 m (30 ft) respectively, GF is the gust factor which reflects the dynamic characteristics of the structure and which enables the wind forces to be used in a static structural analysis [17,64]. The wind force W may be written as,

$$W = W(C_p, V, \kappa, \alpha, f_o, \beta) \tag{A.15}$$

in which κ and α are exposure terms while f_o and β are fundamental natural frequency and damping, respectively. Inspection of Equation A.14 reveals that the major component of variability in W arises from uncertainty in the wind speed, V.

When gravity and wind loads are combined, account must be taken of the reduced probability that extreme live and wind loads occur simultaneously. For reliability analyses, the load combination of interest is,

$$Z = \max_T [L(t) + W(t)] \tag{A.16}$$

where T is the life of the structure. The duration of structurally significant winds is almost instantaneous compared to that for live loads. Under these conditions, McGuire and Cornell [40] found by numerical simulation that the upper percentiles of the maximum of the sum of two random processes, one with duration and one without, was well approximated by the same percentiles of one of the following:

$$Z \simeq \begin{cases} L_s + W_s & \text{(A.17a)} \\ L_I + W_T & \text{(A.17b)} \end{cases}$$

where L_I and L_s are as defined in A.3.2, W_s is the maximum of W(t) during the period in which L_s acts (here about 8 yr) and W_T is the lifetime maximum of W(t). Analyses by Ferry Borges [24] and Wen [65] lead to similar conclusions. Accordingly, the analysis of reliability under gravity plus wind loads requires that these two load cases both be considered. However, studies using the more exact procedure suggested by Wen [65] show that the mean of Z is well approximated in many cases by the mean of Equation A.17b.

Recent statistical analysis of wind speed data conducted by the National Bureau of Standards [56] shows that for extratropical winds the Type I Extreme Value distribution best models the annual maximum wind speeds. With this distribution, the mean and c.o.v. of the maximum V of n identically distributed and statistically independent variables V_o are related by

$$\bar{V} = \bar{V}_o \left(1 - \frac{\sqrt{6}}{\pi} \delta_{V_o} \ln \frac{1}{n}\right) \qquad \text{(A.18a)}$$

$$\frac{1}{\delta_V} = \frac{1}{\delta_{V_o}} - \frac{\sqrt{6}}{\pi} \ln \frac{1}{n} \qquad \text{(A.18b)}$$

Using data reported by Simiu [56] on 37 year records of wind speeds excluding tropical sites, it was found that $0.07 < \delta_{V_o} < 0,24$. The corresponding range in c.o.v. for a period of 64 years (equivalent, on the average, to 8 tenancies; see previous section) is $0.06 < \delta_V < 0.14$, with 0.11 as a representative value; for 8 years, $0.06 < \delta_V < 0.17$.

The mean lifetime wind speed may be related to the wind speeds currently specified in codes, e.g. ANSI A58.1-1972, which are based on mean recurrence intervals. Corresponding to a mean recurrence interval $\bar{N}$, the wind speed v_f is (for Type I distribution for V_o) is

$$v_f = \bar{V}_o \left[1 - \delta_{V_o} \left(0.45 + 0.7797 \ln \ln \frac{1}{1 - 1/\bar{N}}\right)\right] \qquad \text{(A.19)}$$

Using this v_f and $\bar{V}$ defined by Equation A.18a, it may be seen that for most δ_{V_o} and $\bar{N}$, n of interest, $\bar{V}/v_f$ is typically 1.05 - 1.07; in particular, if $n = \bar{N}$, $\bar{V}/v_f$ is very close to 1.0.

Additional uncertainty in the wind force W arises from variability in the pressure coefficient C_p. For low-rise structures, Ω_{C_p} falls in the range of 0.10 - 0.20 [20] and may be even higher for buildings with unusual shapes. For more conventional buildings, this uncertainty is somewhat less, and a value of 0.15 is used in this study.

The total variability in W is evaluated as

$$\Omega_W^2 = \frac{1}{\overline{W}^2} \quad \Sigma \left(\frac{\partial W}{\partial X_i}\right)^2 \overline{X}_i^2 \, \Omega_{X_i}^2 \tag{A.20}$$

where X_i are the parameters in Equation A.14. These derivatives have been evaluated in Appendix D of ref. 21 using Davenport's [17] gust factor. The gust factor analysis presented by Simiu [57] was also used as a check. For example, corresponding to the wind speed term,

$$\frac{1}{\overline{W}} \frac{\partial W}{\partial V} \overline{V} = 2 + a_V$$

in which $a_v = \frac{1}{\overline{GF}} \frac{\partial GF}{\partial V} \overline{V}$. Using Simiu's analysis, a_V is of the order 0.1 - 0.3. In contrast, ref. 21 showed that $0.1 < a_V \leq 0.4$ for buildings ranging from 100 - 600 ft (30.5 - 183 m) in height. Thus, the contribution of Ω_V to Ω_W may be obtained by multiplying Ω_V by roughly 2.3.

The terms κ and α are related to the building exposure. While $\Omega_\kappa \simeq 0.20$ and $\Omega_\alpha \simeq 0.10$ [64], the coefficients associated with them in Equation A.18 are roughly 0.32 and 0.4, respectively [21]. The contribution of exposure to overall wind variability thus is small. Finally, the natural frequency and structural damping contribute uncertainty but because the derivative terms are so small their effect can be neglected in a practical sense. The total variability in lifetime maximum wind load is thus,

$$\Omega_{W_T} \simeq [\underbrace{(2.3 \times 0.11)^2}_{\text{velocity}} + \underbrace{(1 \times 0.15)^2}_{\substack{\text{Pressure} \\ \text{Coeff.}}} + \underbrace{[(.32 \times .2)^2 + (.4 \times .1)^2]}_{\text{Exposure}} + \underbrace{(0.05)^2}_{\text{Modeling}}]^{1/2}$$

$$\Omega_{W_T} \simeq 0.31$$

Of this, 0.29 is due simply to the velocity and pressure coefficient terms which are basically structure-independent. Similarly, $\Omega_{W_S} \simeq 0.35$.

The ratio of nominal to mean wind loads, required for the calibration, depends on the ratio $(v_f/\overline{V})$, the ratio between the ANSI pressure coefficient and the mean pressure coefficient, and the orientation of the building to the direction of the extreme wind. As previously noted, $v_f/\overline{V}$ is about 0.93 - 0.95. The pressure coefficients used in ANSI A58.1-1972 tend to be conservative for the leeward side of a building; rather than C_ℓ = -0.5, the mean would be more like -0.3. This would make the total mean drag coefficient $\overline{C}_p \simeq 1.1$ rather than 1.3. Data contained in BSI CP3: Chapter V: Part 2: 1972 [11] tend to support this position. Finally, there is a reduced probability that the wind will act on the structure in its most unfavorable direction. The effect on $\overline{W}$ may be estimated by assuming the orientation of the force vector to be uniformly distributed between $-\pi/4$ and $\pi/4$ with respect to the axis of the building, leading to a mean force which is from 85 - 90 percent of the most unfavorable. Combining all these factors leads to

$$W'/\overline{W}_T = (0.94)^2 \times (1.3/1.1) \times (1/0.85) \simeq 1.2$$

Similarly, $W'/\overline{W}_s \simeq 1.75$.

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16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

Engineering decisions must be made in the presence of uncertainties which arise as a consequence of imperfect information and knowledge and inherent randomness in many design parameters. It is on account of these uncertainties and potential risks arising therefrom that safety margins provided by load and resistance factors are required in design. Reliability methods are employed in this study to facilitate the selection of criteria for reinforced concrete design. These methods, which are based on probability theory, provide a logical basis for determining the manner in which uncertainties in resistance and loads affect design safety and how their effects should be controlled in building standards. Following a comprehensive analysis of uncertainty measures, safety indices associated with existing reinforced concrete design are evaluated. Design criteria commensurate with levels of uncertainty and required reliability are then presented. Simplification of these leads to practical reliability based criteria which retain the relatively simple characteristics of existing criteria and yet have a well established and documented rationale.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)*

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