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1 **FIPS 203 (Draft)**

2 **Federal Information Processing Standards Publication**
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4 **Module-Lattice-based** 5 **Key-Encapsulation** 6 **Mechanism Standard**

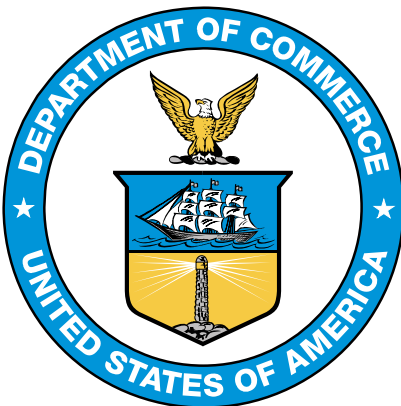
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19

Foreword

20 The Federal Information Processing Standards Publication (FIPS) Series of the National Institute of
21 Standards and Technology is the official series of publications relating to standards and guidelines developed
22 under 15 U.S.C. 278g-3, and issued by the Secretary of Commerce under 40 U.S.C. 11331.

23 Comments concerning this Federal Information Processing Standard publication are welcomed and should
24 be submitted using the contact information in the “Inquiries and comments” clause of the announcement
25 section.

26

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27

Abstract

28 A key-encapsulation mechanism (or KEM) is a set of algorithms that, under certain conditions,
29 can be used by two parties to establish a shared secret key over a public channel. A shared
30 secret key that is securely established using a KEM can then be used with symmetric-key
31 cryptographic algorithms to perform basic tasks in secure communications, such as encryption
32 and authentication.

33 This standard specifies a key-encapsulation mechanism called ML-KEM. The security of
34 ML-KEM is related to the computational difficulty of the so-called Module Learning with Errors
35 problem. At present, ML-KEM is believed to be secure even against adversaries who possess a
36 quantum computer.

37 This standard specifies three parameter sets for ML-KEM. In order of increasing security strength
38 (and decreasing performance), these parameter sets are ML-KEM-512, ML-KEM-768, and
39 ML-KEM-1024.

40 **Keywords:** computer security; cryptography; encryption; Federal Information Processing
41 Standards; lattice-based cryptography; key-encapsulation; post-quantum; public-key cryptography

Federal Information Processing Standards Publication 203

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Announcing the Module-Lattice-based Key-Encapsulation Mechanism Standard

Federal Information Processing Standards Publications (FIPS) are issued by the National Institute of Standards and Technology (NIST) under 15 U.S.C. 278g-3 and issued by the Secretary of Commerce under 40 U.S.C. 11331.

1. **Name of Standard.** Module-Lattice-based Key-Encapsulation Mechanism Standard (ML-KEM) (FIPS PUB 203).
2. **Category of Standard.** Computer Security. **Subcategory.** Cryptography.
3. **Explanation.** This standard specifies a set of algorithms for applications that require a secret cryptographic key that is shared by two parties who can only communicate over a public channel. A cryptographic key (or simply "key") is represented in a computer as a string of bits. A *shared secret key* is computed jointly by two parties (e.g., Party A and Party B) using a set of rules and parameters. Under certain conditions, these rules and parameters ensure that both parties will produce the same key and that this shared key is secret from adversaries. Such a shared secret key can then be used with symmetric-key cryptographic algorithms (specified in other NIST standards) to perform tasks, such as encryption and authentication of digital information.

While there are many methods for establishing a shared secret key, the particular method described in this specification is a key-encapsulation mechanism (KEM). In a KEM, the computation of the shared secret key begins with Party A generating a *decapsulation key* and an *encapsulation key*. Party A keeps the decapsulation key private and makes the encapsulation key available to Party B. Party B then uses Party A's encapsulation key to generate one copy of a shared secret key along with an associated *ciphertext*. Party B then sends the ciphertext to Party A over the same channel. Finally, Party A uses the ciphertext from Party B along with Party A's private decapsulation key to compute another copy of the shared secret key.

The security of the particular KEM specified here is related to the computational difficulty of solving certain systems of noisy linear equations, specifically the so-called *Module Learning With Errors* (MLWE) problem. At present, it is believed that this particular method of establishing a shared secret key is secure even against adversaries who possess a quantum computer. In the future, additional KEMs may be specified and approved in FIPS publications or in NIST Special Publications.

4. **Approving Authority.** Secretary of Commerce.
5. **Maintenance Agency.** Department of Commerce, National Institute of Standards and Technology, Information Technology Laboratory (ITL).

79 6. **Applicability.** Federal Information Processing Standards apply to information systems used
80 or operated by federal agencies or by a contractor of an agency or other organization on behalf
81 of an agency. They do not apply to national security systems as defined in 44 U.S.C. 3552.

82 This standard must be implemented wherever the establishment of a shared secret key is
83 required for federal applications, including the use of such a key with symmetric-key cryp-
84 tographic algorithms, in accordance with applicable Office of Management and Budget and
85 agency policies. Federal agencies may also use alternative methods that NIST has indicated
86 are appropriate for this purpose.

87 The adoption and use of this standard are available to private and commercial organizations.

88 7. **Implementations.** A key-encapsulation mechanism may be implemented in software,
89 firmware, hardware, or any combination thereof. A conforming implementation may replace
90 the given sequence of steps in the top-level algorithms of ML-KEM (i.e., [ML-KEM.KeyGen](#),
91 [ML-KEM.Encaps](#), and [ML-KEM.Decaps](#)) with any equivalent process. In other words, dif-
92 ferent procedures that produce the correct output for every input are permitted. In particular,
93 conforming implementations are not required to use the same subroutines (of the aforemen-
94 tioned main algorithms) as are used in this specification.

95 NIST will develop a validation program to test implementations for conformance to the
96 algorithms in this standard. Information about validation programs is available at <https://csrc.nist.gov/projects/cmvp>. Example values for cryptographic algorithms are available at
97 <https://csrc.nist.gov/projects/cryptographic-standards-and-guidelines/example-values>.
98

99 8. **Other Approved Security Functions.** Implementations that comply with this standard **shall**
100 employ cryptographic algorithms that have been **approved** for protecting Federal Government-
101 sensitive information. **Approved** cryptographic algorithms and techniques include those that
102 are either:

103 (a) Specified in a Federal Information Processing Standards (FIPS) publication,

104 (b) Adopted in a FIPS or NIST recommendation, or

105 (c) Specified in the list of approved security functions for FIPS 140-3.

106 9. **Export Control.** Certain cryptographic devices and technical data regarding them are subject
107 to federal export controls. Exports of cryptographic modules that implement this standard
108 and technical data regarding them must comply with all federal laws and regulations and
109 be licensed by the Bureau of Industry and Security of the U.S. Department of Commerce.
110 Information about export regulations is available at <https://www.bis.doc.gov>.

111 10. **Patents.** NIST has entered into two patent license agreements to facilitate the adoption of
112 NIST's announced selection of public-key encryption PQC algorithm CRYSTALS-KYBER.
113 NIST and the licensing parties share a desire, in the public interest, the licensed patents be
114 freely available to be practiced by any implementer of the ML-KEM algorithm as published by
115 NIST. ML-KEM is the name given to the algorithm in this standard derived from CRYSTALS-
116 KYBER. For a summary and extracts from the license, please see [https://csrc.nist.gov/csrf/m
117 edia/Projects/post-quantum-cryptography/documents/selected-algos-2022/nist-pqc-license-
118 summary-and-excerpts.pdf](https://csrc.nist.gov/csrf/media/Projects/post-quantum-cryptography/documents/selected-algos-2022/nist-pqc-license-summary-and-excerpts.pdf). Implementation of the algorithm specified in the standard may be
119 covered by U.S. and foreign patents of which NIST is not aware.

- 120 11. **Implementation Schedule.** This standard becomes effective immediately upon final publica-
121 tion.
- 122 12. **Specifications.** Federal Information Processing Standards (FIPS) 203, Module-Lattice-based
123 Key-Encapsulation Mechanism Standard (affixed).
- 124 13. **Qualifications.** In applications, the security guarantees of a KEM only hold under certain
125 conditions (see NIST SP 800-227 [1]). One such condition is the secrecy of several values,
126 including the randomness used by the two parties, the decapsulation key, and the shared secret
127 key itself. Users **shall**, therefore, guard against the disclosure of these values.
- 128 While it is the intent of this standard to specify general requirements for implementing
129 ML-KEM algorithms, conformance to this standard does not ensure that a particular imple-
130 mentation is secure. It is the responsibility of the implementer to ensure that any module that
131 implements a key establishment capability is designed and built in a secure manner.
- 132 Similarly, the use of a product containing an implementation that conforms to this standard
133 does not guarantee the security of the overall system in which the product is used. The
134 responsible authority in each agency or department **shall** ensure that an overall implementation
135 provides an acceptable level of security.
- 136 NIST will continue to follow developments in the analysis of the ML-KEM algorithm. As
137 with its other cryptographic algorithm standards, NIST will formally reevaluate this standard
138 every five years.
- 139 Both this standard and possible threats that reduce the security provided through the use of
140 this standard will undergo review by NIST as appropriate, taking into account newly available
141 analysis and technology. In addition, the awareness of any breakthrough in technology or
142 any mathematical weakness of the algorithm will cause NIST to reevaluate this standard and
143 provide necessary revisions.
- 144 14. **Waiver Procedure.** The Federal Information Security Management Act (FISMA) does
145 not allow for waivers to Federal Information Processing Standards (FIPS) that are made
146 mandatory by the Secretary of Commerce.
- 147 15. **Where to Obtain Copies of the Standard.** This publication is available by accessing
148 <https://csrc.nist.gov/publications>. Other computer security publications are available at the
149 same website.
- 150 16. **How to Cite this Publication.** NIST has assigned **NIST FIPS 203 ipd** as the publication
151 identifier for this FIPS, per the [NIST Technical Series Publication Identifier Syntax](#). NIST
152 recommends that it be cited as follows:
- 153 National Institute of Standards and Technology (2023) Module-Lattice-based Key-
154 Encapsulation Mechanism Standard. (Department of Commerce, Washington,
155 D.C.), Federal Information Processing Standards Publication (FIPS) NIST FIPS
156 203 ipd. <https://doi.org/10.6028/NIST.FIPS.203.ipd>
- 157 17. **Inquiries and Comments.** Inquiries and comments about this FIPS may be submitted to
158 fips-203-comments@nist.gov.

159 Call for Patent Claims

160 This public review includes a call for information on essential patent claims (claims whose
161 use would be required for compliance with the guidance or requirements in this Information
162 Technology Laboratory (ITL) draft publication). Such guidance and/or requirements may be
163 directly stated in this ITL Publication or by reference to another publication. This call also
164 includes disclosure, where known, of the existence of pending U.S. or foreign patent applications
165 relating to this ITL draft publication and of any relevant unexpired U.S. or foreign patents.

166 ITL may require from the patent holder, or a party authorized to make assurances on its behalf, in
167 written or electronic form, either:

168 a) assurance in the form of a general disclaimer to the effect that such party does not hold and
169 does not currently intend holding any essential patent claim(s); or

170 b) assurance that a license to such essential patent claim(s) will be made available to appli-
171 cants desiring to utilize the license for the purpose of complying with the guidance or
172 requirements in this ITL draft publication either:

173 (i) under reasonable terms and conditions that are demonstrably free of any unfair
174 discrimination; or

175 (ii) without compensation and under reasonable terms and conditions that are demonstra-
176 bly free of any unfair discrimination.

177 Such assurance shall indicate that the patent holder (or third party authorized to make assurances
178 on its behalf) will include in any documents transferring ownership of patents subject to the
179 assurance, provisions sufficient to ensure that the commitments in the assurance are binding on
180 the transferee, and that the transferee will similarly include appropriate provisions in the event of
181 future transfers with the goal of binding each successor-in-interest.

182 The assurance shall also indicate that it is intended to be binding on successors-in-interest
183 regardless of whether such provisions are included in the relevant transfer documents.

184 Such statements should be addressed to: fips-203-comments@nist.gov.

185 **Federal Information Processing Standards Publication 203**

186 **Specification for the**

187 **Module-Lattice-based Key-Encapsulation**

188 **Mechanism Standard**

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248 1. Introduction

249 1.1 Purpose and Scope

250 This standard specifies the *Module-Lattice-based Key-Encapsulation Mechanism*, or ML-KEM.
251 A key-encapsulation mechanism (or KEM) is a set of algorithms that can be used to establish
252 a shared secret key between two parties communicating over a public channel. A KEM is a
253 particular type of key establishment scheme. Current NIST-**approved** key establishment schemes
254 are specified in NIST SP-800-56A, *Recommendation for Pair-Wise Key-Establishment Schemes*
255 *Using Discrete Logarithm-Based Cryptography* [2], and NIST SP-800-56B, *Recommendation for*
256 *Pair-Wise Key Establishment Schemes Using Integer Factorization Cryptography* [3].

257 It is well-known that the key establishment schemes specified in NIST SP-800-56A and NIST
258 SP-800-56B are vulnerable to attacks using sufficiently capable quantum computers. ML-KEM
259 is an **approved** alternative that is presently believed to be secure even against adversaries
260 in possession of a quantum computer. ML-KEM is derived from the round-three version
261 of the CRYSTALS-KYBER KEM [4], a submission in the NIST post-quantum cryptography
262 standardization project. For the differences between ML-KEM and CRYSTALS-KYBER, see
263 Section 1.3.

264 This standard specifies the algorithms and parameter sets of the ML-KEM scheme. It aims to
265 provide sufficient information for implementing ML-KEM in a manner that can pass validation
266 (see <https://csrc.nist.gov/projects/cryptographic-module-validation-program>). For general
267 definitions and properties of KEMs, including requirements for the secure use of KEMs in
268 applications, see NIST SP 800-227 [1].

269 This standard specifies three parameter sets for ML-KEM. These parameter sets offer differ-
270 ent trade-offs in security strength versus performance. All three parameter sets of ML-KEM
271 are **approved** to protect sensitive, non-classified communication systems of the U.S. Federal
272 Government.

273 1.2 Context

274 Over the past several years, there has been steady progress toward building quantum computers.
275 If large-scale quantum computers are realized, the security of many commonly used public-key
276 cryptosystems will be at risk. This would include key-establishment schemes and digital signature
277 schemes that are based on integer factorization and discrete logarithms (both over finite fields and
278 elliptic curves). As a result, in 2016, the National Institute of Standards and Technology (NIST)
279 initiated a public process to select quantum-resistant public-key cryptographic algorithms for
280 standardization. A total of 82 candidate algorithms were submitted to NIST for consideration for
281 standardization.

282 After three rounds of evaluation and analysis, NIST selected the first four algorithms to stan-
283 dardize as a result of the Post-Quantum Cryptography (PQC) Standardization process. These
284 algorithms are intended to protect sensitive U.S. Government information well into the foresee-
285 able future, including after the advent of quantum computers. This standard specifies a variant
286 of the selected algorithm CRYSTALS-KYBER, a lattice-based key-encapsulation mechanism
287 (KEM) [4]. Throughout this standard, the KEM specified here will be referred to as ML-KEM,

288 as it is based on the so-called Module Learning With Errors assumption.

289 **1.3 Differences From the CRYSTALS-KYBER Submission**

290 Below is a list of all scheme differences between CRYSTALS-KYBER (as described in [4]) and
291 the ML-KEM scheme specified in this document. The list consists only of those differences that
292 result in differing input-output behavior of the main algorithms (i.e., KeyGen, Encaps, Decaps) of
293 CRYSTALS-KYBER and ML-KEM. Recall that a conforming implementation need only match
294 the input-output behavior of these three algorithms (see “Implementations” above, and Section 3.3
295 below). Consequently, the list below does not include any of the numerous differences in how
296 the main algorithms actually produce outputs from inputs (e.g., via different computational steps
297 or different subroutines). The list below also does not include any differences in presentation
298 between this standard and [4].

- 299 • In the third-round specification [4], the shared secret key was treated as a variable-length
300 value whose length depends on how this key would be used in the relevant application. In
301 this specification, the length of the shared secret key is fixed to 256 bits. In this specification,
302 this key can be used directly in applications as a symmetric key; alternatively, symmetric
303 keys can be derived from this key, as specified in Section 3.3.
- 304 • The [ML-KEM.Encaps](#) and [ML-KEM.Decaps](#) algorithms in this specification use a dif-
305 ferent variant of the Fujisaki-Okamoto transform (see [5, 6]) than the third-round specifi-
306 cation [4]. Specifically, [ML-KEM.Encaps](#) no longer includes a hash of the ciphertext in
307 the derivation of the shared secret, and [ML-KEM.Decaps](#) has been adjusted to match this
308 change.
- 309 • In the third-round specification [4], the initial randomness m in the [ML-KEM.Encaps](#)
310 algorithm was first hashed before being used. Specifically, between lines 1 and 2 in
311 Algorithm 16, there was an additional step that performed the operation $m \leftarrow H(m)$. The
312 purpose of this step was to safeguard against the use of flawed randomness generation
313 processes. As this standard requires the use of NIST-approved randomness generation, this
314 step is unnecessary and is not performed in ML-KEM.
- 315 • This specification includes explicit input validation steps that were not part of the third-
316 round specification [4]. For example, [ML-KEM.Encaps](#) requires that the byte array
317 containing the encapsulation key correctly decodes to an array of integers modulo q without
318 any modular reductions.

319 2. Glossary of Terms, Acronyms, and Mathematical Symbols 320

321 2.1 Terms and Definitions

322	approved	FIPS-approved and/or NIST-recommended. An algorithm or technique
323		that is either 1) specified in a FIPS or NIST recommendation, 2) adopted
324		in a FIPS or NIST recommendation, or 3) specified in a list of NIST-
325		approved security functions.
326	decapsulation	The process of applying the Decaps algorithm of a KEM. This algo-
327		rithm accepts a KEM ciphertext and the decapsulation key as input and
328		produces a shared secret key as output.
329	decapsulation key	A cryptographic key produced by a KEM during key generation and
330		used during the decapsulation process. The decapsulation key must be
331		kept private, and must be destroyed after it is no longer needed.
332	decryption key	A cryptographic key that is used with a PKE in order to decrypt cipher-
333		texts into plaintexts. The decryption key must be kept private, and must
334		be destroyed after it is no longer needed.
335	destroy	An action applied to a key or other piece of secret data. After a piece of
336		secret data is destroyed, no information about its value can be recovered.
337	encapsulation	The process of applying the Encaps algorithm of a KEM. This algorithm
338		accepts private randomness and the encapsulation key as input and
339		produces a shared secret key and an associated ciphertext as output.
340	encapsulation key	A cryptographic key produced by a KEM during key generation and
341		used during the encapsulation process. The encapsulation key can be
342		made public.
343	encryption key	A cryptographic key that is used with a PKE in order to encrypt plain-
344		texts into ciphertexts. The encryption key can be made public.
345	equivalent process	Two processes are equivalent if the same output is produced when the
346		same values are input to each process (either as input parameters, as
347		values made available during the process, or both).
348	hash function	A function on bit strings in which the length of the output is fixed.
349		Approved hash functions relevant to this standard are specified in FIPS
350		202 [7].
351	KEM ciphertext	A bit string that is produced by encapsulation and used as an input to
352		decapsulation.
353	key	A bit string that is used in conjunction with a cryptographic algorithm.
354		Examples applicable to this standard include: the encapsulation and
355		decapsulation keys (of a KEM), the shared secret key (produced by a
356		KEM), and the encryption and decryption keys (of a PKE).

357	key-encapsulation	A set of three cryptographic algorithms (KeyGen, Encaps, and Decaps)
358	mechanism (KEM)	that can be used by two parties to establish a shared secret key over a
359		public channel.
360	key pair	A set of two keys with the property that one key can be made public
361		while the other key must be kept private. In this standard, this could
362		refer to either the (encapsulation key, decapsulation key) key pair of a
363		KEM, or the (encryption key, decryption key) key pair of a PKE.
364	party	An individual (person), organization, device, or process. In this specifi-
365		cation, there are two parties (Party A and Party B, or Alice and Bob),
366		and they jointly perform the key establishment process using a KEM.
367	pseudorandom	A process (or data produced by a process) is said to be pseudorandom
368		when the outcome is deterministic yet also appears random as long
369		as the internal action of the process is hidden from observation. For
370		cryptographic purposes, “effectively random” means “computationally
371		indistinguishable from random within the limits of the intended security
372		strength.”
373	public channel	A communication channel between two parties; such a channel can be
374		observed and possibly also corrupted by third parties.
375	public-key	A set of three cryptographic algorithms (KeyGen, Encrypt, and Decrypt)
376	encryption scheme	that can be used by two parties to send secret data over a public channel.
377	(PKE)	Also known as an asymmetric encryption scheme.
378	shared secret key	The final result of a KEM key establishment process. It is a crypto-
379		graphic key that can be used for symmetric-key cryptography. It must
380		be kept private, and it must be destroyed when no longer needed.
381	security category	A number associated with the security strength of a post-quantum cryp-
382		tographic algorithm as specified by NIST (see Appendix A, Table 4).
383	security strength	A number associated with the amount of work that is required to break
384		a cryptographic algorithm or system.
385	shall	Used to indicate a requirement of this standard.
386	should	Used to indicate a strong recommendation but not a requirement of
387		this standard. Ignoring the recommendation could lead to undesirable
388		results.
389		

390 2.2 Acronyms

391	AES	Advanced Encryption Standard
392	CBD	Centered Binomial Distribution
393	FIPS	Federal Information Processing Standard

394	KEM	Key-encapsulation Mechanism
395	LWE	Learning With Errors
396	MLWE	Module Learning with Errors
397	NIST	National Institute of Standards and Technology
398	NISTIR	NIST Interagency or Internal Report
399	NTT	Number-Theoretic Transform
400	PKE	Public-Key Encryption
401	PQC	Post-Quantum Cryptography
402	PRF	Pseudorandom Function
403	RBG	Random Bit Generator
404	SHA	Secure Hash Algorithm
405	SHAKE	Secure Hash Algorithm KECCAK
406	SP	Special Publication
407	XOF	Extendable-Output Function
408		

409 2.3 Mathematical Symbols

410	S^*	If S is a set, this denotes the set of finite-length tuples (or arrays) of elements from the set S , including the empty tuple (or empty array).
411		
412	S^k	If S is a set, this denotes the set of k -tuples (or length- k arrays) of elements from the set S .
413		
414	$\text{BitRev}_7(r)$	Bit reversal of a seven-bit integer r . Specifically, if $r = r_0 + 2r_1 + 4r_2 + \dots + 64r_6$ with $r_i \in \{0, 1\}$, then $\text{BitRev}_7(r) = r_6 + 2r_5 + 4r_4 + \dots + 64r_0$.
415		
416	\hat{f}	The element of T_q that is equal to the NTT representation of a polynomial $f \in R_q$ (see Section 4.3).
417		
418	\mathbb{Q}	The set of rational numbers.
419	\mathbb{Z}_m	The ring of integers modulo m , i.e., the set $\{0, 1, \dots, m-1\}$ equipped with the operations of addition and multiplication modulo m .
420		
421	\mathbb{Z}	The set of integers.
422	$\mathbf{v}^T, \mathbf{A}^T$	The transpose of a row or column \mathbf{v} ; also, the transpose of a matrix \mathbf{A} .
423	f_j	The coefficient of X^j of a polynomial $f = f_0 + f_1X + \dots + f_{255}X^{255} \in R_q$.
424	$r \bmod m$	The unique integer r' in $\{0, 1, \dots, m-1\}$ such that m divides $r - r'$.

425	$r \bmod^{\pm} m$	For m even (respectively, odd), this denotes the unique integer r' such that
426		$-m/2 < r' \leq m/2$ (respectively, $-(m-1)/2 \leq r' \leq (m-1)/2$) and m divides
427		$r - r'$.
428	$ B $	If B is a number, this denotes the absolute value of B . If B is an array, this
429		denotes its length.
430	$\lceil x \rceil$	The ceiling of x , i.e., the smallest integer greater than or equal to x .
431	$\lfloor x \rfloor$	The rounding of x to the nearest integer; if $x = y + 1/2$ for some $y \in \mathbb{Z}$, then
432		$\lfloor x \rfloor = y + 1$.
433	$\lfloor x \rfloor$	The floor of x , i.e., the largest integer less than or equal to x .
434	\mathbb{B}	The set $\{0, 1, \dots, 255\}$ of unsigned 8-bit integers (bytes).
435	$A B$	The concatenation of two arrays or bit strings A and B .
436	$B[i]$	The entry at index i in the array B . All arrays have indices that begin at zero.
437	$B[k : m]$	The subarray $(B[k], B[k + 1], \dots, B[m - 1])$ of the array B .
438	n	Denotes the integer 256 throughout this document.
439	q	Denotes the prime integer $3329 = 2^8 \cdot 13 + 1$ throughout this document.
440	R_q	The ring $\mathbb{Z}_q[X]/(X^n + 1)$ consisting of polynomials of the form $f = f_0 +$
441		$f_1X + \dots + f_{255}X^{255}$ where $f_j \in \mathbb{Z}_q$ for all j , equipped with addition and
442		multiplication modulo $X^n + 1$.
443	$s \leftarrow x$	In pseudocode, this notation means that the variable s is assigned the value
444		of the expression x .
445	$s \xleftarrow{\$} \mathbb{B}^\ell$	In pseudocode, this notation means that the variable s is assigned the value of
446		an array of ℓ random bytes. The bytes must be generated using randomness
447		from an approved RBG (see Section 3.3).
448	T_q	The image of R_q under the number-theoretic transform. Its elements are
449		called “NTT representations” of polynomials in R_q (see Section 4.3).

450 2.4 Interpreting the Pseudocode

451 This section outlines the conventions of the pseudocode used to describe the algorithms in this
 452 standard. All algorithms are understood to have access to two global integer constants: $n = 256$
 453 and $q = 3329$. There are also five global integer variables: k , η_1 , η_2 , d_u and d_v . All other variables
 454 are local. The five global variables are set to particular values when a parameter set is selected
 455 (see Section 7).

456 When algorithms in this specification invoke other algorithms as subroutines, all arguments
 457 (inputs) are passed by value. In other words, a copy of the inputs is created, and the subroutine is
 458 invoked with the copie. There is no “passing by reference.”

459

460 **Data types.** For variables that represent the input or output of an algorithm, the data type (e.g.,

461 bit, byte, array of bits) will be explicitly described at the start of the algorithm. For most local
 462 variables in the pseudocode, the data type is easily deduced from context. For all other variables,
 463 the data type will be declared in a comment. In a single algorithm, the data type of a variable is
 464 determined the first time that the variable is used and will not be changed. Variable names can
 465 and will be reused across different algorithms, including with different data types.

466 In addition to standard atomic data types (e.g., bits, bytes) and data structures (e.g., arrays),
 467 integers modulo m (i.e., elements of \mathbb{Z}_m) will also be used as an abstract data type. It is implicit
 468 that reduction modulo m takes place whenever an assignment is made to a variable in \mathbb{Z}_m . For
 469 example, for $z \in \mathbb{Z}_m$ and any integers x, y , the statement

$$z \leftarrow x + y \tag{2.1}$$

470 means that z is assigned the value $x + y \bmod m$. The pseudocode is agnostic regarding how an
 471 integer modulo m is represented in actual implementations or how modular reduction is computed.

472

473 **Loop syntax.** The pseudocode will make use of both “while” and “for” loops. The “while” syntax
 474 is self-explanatory. In the case of “for” loops, the syntax will be in the style of the programming
 475 language C. Two simple examples are given in Algorithm 1.

Algorithm 1 ForExample

Performs two simple “for” loops.

```

1: for ( $i \leftarrow 0; i < 10; i++$ )
2:    $A[i] \leftarrow i$                                 ▷  $A$  is an integer array of length 10
3: end for                                          ▷  $A$  now has the value (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
4:  $j \leftarrow 0$ 
5: for ( $k \leftarrow 256; k > 1; k \leftarrow k/2$ )
6:    $B[j] \leftarrow k$                                 ▷  $B$  is an integer array of length 8
7:    $j \leftarrow j + 1$ 
8: end for                                          ▷  $B$  now has the value (256, 128, 64, 32, 16, 8, 4, 2)

```

Arithmetic with arrays of integers. This standard makes extensive use of arrays of integers modulo m (i.e., elements of \mathbb{Z}_m^ℓ). In a typical case, the relevant values are $m = q$ and $\ell = n = 256$. Arithmetic with arrays in \mathbb{Z}_m^ℓ will be done as follows. Let $a \in \mathbb{Z}_m$ and $X, Y \in \mathbb{Z}_m^\ell$. The statements

$$\begin{aligned} Z &\leftarrow a \cdot X \\ W &\leftarrow X + Y \end{aligned}$$

476 will result in two arrays $Z, W \in \mathbb{Z}_m^\ell$, with the property that $Z[i] = a \cdot X[i]$ and $W[i] = X[i] + Y[i]$
 477 for all i . Multiplication of arrays in \mathbb{Z}_m^ℓ will only be meaningful when $m = q$ and $\ell = n = 256$, in
 478 which case it corresponds to multiplication in a particular ring. This operation will be described
 479 in (2.2) below.

480

481 **Representations of algebraic objects.** An essential operation in ML-KEM is the number-

482 theoretic transform (NTT), which maps a polynomial f in a certain ring R_q to its “NTT repre-
 483 sentation” \hat{f} in a different ring T_q . The rings R_q and T_q and the NTT are discussed in detail in
 484 Section 4.3. This standard will represent elements of R_q and elements of T_q in pseudocode using
 485 arrays of integers modulo q , as follows.

An element f of R_q is a polynomial of the form

$$f = f_0 + f_1X + \cdots + f_{255}X^{255} \in R_q$$

and will be represented in pseudocode by the array

$$(f_0, f_1, \dots, f_{255}) \in \mathbb{Z}_q^{256}$$

486 whose entries contain the coefficients of f . Abusing notation somewhat, this array will also be
 487 denoted by f . The i -th entry of the array f will thus contain the i -th coefficient of the polynomial
 488 f (i.e., $f[i] = f_i$).

An element (sometimes called “NTT representation”) \hat{g} of T_q is a tuple of 128 polynomials, each
 of degree at most one. Specifically,

$$\hat{g} = (\hat{g}_{0,0} + \hat{g}_{0,1}X, \hat{g}_{1,0} + \hat{g}_{1,1}X, \dots, \hat{g}_{127,0} + \hat{g}_{127,1}X) \in T_q.$$

Such an algebraic object will be represented in pseudocode by the array

$$(\hat{g}_{0,0}, \hat{g}_{0,1}, \hat{g}_{1,0}, \hat{g}_{1,1}, \dots, \hat{g}_{127,0}, \hat{g}_{127,1}) \in \mathbb{Z}_q^{256}.$$

489 Abusing notation somewhat, this array will also be denoted by \hat{g} . In this case, the mapping
 490 between array entries and coefficients is $\hat{g}[2i] = \hat{g}_{i,0}$ and $\hat{g}[2i+1] = \hat{g}_{i,1}$ for $i \in \{0, 1, \dots, 127\}$.

491 Converting between a polynomial $f \in R_q$ and its NTT representation $\hat{f} \in T_q$ will be done via the
 492 algorithms [NTT](#) (Algorithm 8) and [NTT⁻¹](#) (Algorithm 9). These algorithms act on arrays of
 493 coefficients, as described above, and satisfy $\hat{f} = \text{NTT}(f)$ and $f = \text{NTT}^{-1}(\hat{f})$.

494

495 **Arithmetic with polynomials and NTT representations.** The algebraic operations of addition
 496 and scalar multiplication in R_q and T_q are done coordinate-wise. For example, if $a \in \mathbb{Z}_q$ and $f \in R_q$,
 497 the i -th coefficient of the polynomial $a \cdot f \in R_q$ is equal to $a \cdot f_i \bmod q$. In pseudocode, elements
 498 of both R_q and T_q are represented by coefficient arrays (i.e., elements of \mathbb{Z}_q^{256}), as described above.
 499 The algebraic operations of addition and scalar multiplication are thus performed by addition and
 500 scalar multiplication of the corresponding coefficient arrays. For example, the addition of two
 501 NTT representations in pseudocode is performed by a statement of the form $\hat{h} \leftarrow \hat{f} + \hat{g}$, where
 502 $\hat{h}, \hat{f}, \hat{g} \in \mathbb{Z}_q^{256}$ are coefficient arrays.

503 The algebraic operations of multiplication in R_q and T_q are treated as follows. For efficiency
 504 reasons, multiplication in R_q will not be used. The algebraic meaning of multiplication in T_q is
 505 discussed in Section 4.3.1. In pseudocode, it will be performed by the algorithm [MultiplyNTTs](#)
 506 (Algorithm 10). Specifically, if $\hat{f}, \hat{g} \in \mathbb{Z}_q^{256}$ are a pair of arrays (each representing the NTT of

507 some polynomial), then

$$\hat{h} \leftarrow \hat{f} \times_{T_q} \hat{g} \quad \text{means} \quad \hat{h} \leftarrow \text{MultiplyNTTs}(\hat{f}, \hat{g}). \quad (2.2)$$

508 The result is an array $\hat{h} \in \mathbb{Z}_q^{256}$.

509

510 **Matrices and vectors.** In addition to arrays of integers modulo q , the pseudocode will also make
 511 use of arrays whose entries are themselves elements of \mathbb{Z}_q^{256} . For example, an element $\mathbf{v} \in (\mathbb{Z}_q^{256})^3$
 512 will be a length-three array whose entries $\mathbf{v}[0]$, $\mathbf{v}[1]$ and $\mathbf{v}[2]$ are themselves elements of \mathbb{Z}_q^{256} (i.e.,
 513 arrays). One can think of each of these entries as representing a polynomial in R_q , so that \mathbf{v} itself
 514 represents an element of the module R_q^3 .

515 When arrays are used to represent matrices and vectors whose entries are elements of R_q , they
 516 will be denoted with bold letters (e.g., \mathbf{v} for vectors and \mathbf{A} for matrices). When arrays are used
 517 to represent matrices and vectors whose entries are elements of T_q , they will be denoted with a
 518 “hat” (e.g., $\hat{\mathbf{v}}$ and $\hat{\mathbf{A}}$). Unless an explicit transpose operation is performed, it is understood that
 519 vectors are column vectors. One can then view vectors as the special case of matrices with only
 520 one column.

521 Converting between matrices over R_q and matrices over T_q will be done coordinate-wise. Specifi-
 522 cally, if $\mathbf{A} \in (\mathbb{Z}_q^{256})^{k \times \ell}$, then the statement

$$\hat{\mathbf{A}} \leftarrow \text{NTT}(\mathbf{A})$$

523 will result in $\hat{\mathbf{A}} \in (\mathbb{Z}_q^{256})^{k \times \ell}$ such that $\hat{\mathbf{A}}[i, j] = \text{NTT}(\mathbf{A}[i, j])$ for all i, j . This involves running
 524 **NTT** a total of $k \cdot \ell$ times. Note that the case of vectors corresponds to $\ell = 1$.

525

526 **Arithmetic with matrices and vectors.** The following describes how to perform arithmetic with
 527 matrices while continuing to view vectors as a special case of matrices.

Addition and scalar multiplication is performed coordinate-wise. Addition of matrices over R_q
 and T_q is then straightforward. In the case of T_q , scalar multiplication is done via (2.2). For
 example, if $\hat{f} \in \mathbb{Z}_q^{256}$ and $\hat{\mathbf{u}}, \hat{\mathbf{v}} \in (\mathbb{Z}_q^{256})^k$, then

$$\begin{aligned} \hat{\mathbf{w}} &\leftarrow \hat{f} \cdot \hat{\mathbf{u}} \\ \hat{\mathbf{z}} &\leftarrow \hat{\mathbf{u}} + \hat{\mathbf{v}} \end{aligned}$$

528 will result in $\hat{\mathbf{w}}, \hat{\mathbf{z}} \in (\mathbb{Z}_q^{256})^k$ satisfying $\hat{\mathbf{w}}[i] = \hat{f} \times_{T_q} \hat{\mathbf{u}}[i]$ and $\hat{\mathbf{z}}[i] = \hat{\mathbf{u}}[i] + \hat{\mathbf{v}}[i]$ for all i . Note that
 529 the multiplication and addition of individual entries here is performed using the appropriate
 530 arithmetic for coefficient arrays of elements of T_q .

It will also be necessary to multiply matrices with entries in T_q . This is done using standard
 matrix multiplication with the base-case multiplication (i.e., multiplication of individual entries)
 being multiplication in T_q . If $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ are two matrices with entries in T_q , their matrix product
 will be denoted $\hat{\mathbf{A}} \circ \hat{\mathbf{B}}$. Some example pseudocode statements involving matrix multiplication are
 given below. In these examples, $\hat{\mathbf{A}}$ is a $k \times k$ matrix, while $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$ are vectors of length k . All

three of these objects are represented in pseudocode by arrays: a $k \times k$ array for $\hat{\mathbf{A}}$ and length- k arrays for $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$. The entries of $\hat{\mathbf{A}}$, $\hat{\mathbf{u}}$, and $\hat{\mathbf{v}}$ are elements of \mathbb{Z}_q^{256} . The first two pseudocode statements below produce a new length- k vector whose entries are specified on the right-hand side. The third pseudocode statement computes a dot product; the result is therefore in the base ring (i.e., T_q), and is represented by an element \hat{z} of \mathbb{Z}_q^{256} .

$$\begin{aligned} \hat{\mathbf{w}} &\leftarrow \hat{\mathbf{A}} \circ \hat{\mathbf{u}} & \hat{\mathbf{w}}[i] &= \sum_{j=0}^{k-1} \hat{\mathbf{A}}[i, j] \times_{T_q} \hat{\mathbf{u}}[j] \\ \hat{\mathbf{y}} &\leftarrow \hat{\mathbf{A}}^T \circ \hat{\mathbf{u}} & \hat{\mathbf{y}}[i] &= \sum_{j=0}^{k-1} \hat{\mathbf{A}}[j, i] \times_{T_q} \hat{\mathbf{u}}[j] \\ \hat{z} &\leftarrow \hat{\mathbf{u}}^T \circ \hat{\mathbf{v}} & \hat{z} &= \sum_{j=0}^{k-1} \hat{\mathbf{u}}[j] \times_{T_q} \hat{\mathbf{v}}[j] \end{aligned}$$

531 The multiplication \times_{T_q} of individual entries above is performed using [MultiplyNTTs](#), as described
 532 in (2.2) above.

533

534 **Applying algorithms to arrays.** The conventions of coordinate-wise arithmetic described above
 535 will also be extended to algorithms that act on (and/or produce) an atomic data type. When
 536 the pseudocode invokes such an algorithm on an array input, it is implied that the algorithm is
 537 invoked repeatedly for each entry of the array. For example, the function $\text{Compress}_d : \mathbb{Z}_q \rightarrow \mathbb{Z}_{2^d}$
 538 defined in Section 4 can be invoked on an array input $F \in \mathbb{Z}_q^{256}$ with the statement

$$K \leftarrow \text{Compress}_d(F). \tag{2.3}$$

539 The result will be that $K \in \mathbb{Z}_{2^d}^{256}$ and $K[i] = \text{Compress}_d(F[i])$ for every i . This computation
 540 involves running the [Compress](#) algorithm 256 times.

541 3. Overview of the ML-KEM Scheme

542 This section gives a high-level overview of the ML-KEM scheme.

543 3.1 Key-Encapsulation Mechanisms

544 The following is a brief and informal overview of key-encapsulation mechanisms (or KEMs). For
545 more details, see NIST SP 800-227 [1].

546 A key-encapsulation mechanism (or KEM) is a set of algorithms that can be used, under certain
547 conditions, to establish a shared secret key between two communicating parties. This shared
548 secret key can then be used for symmetric-key cryptography.

549 A KEM consists of three algorithms and a collection of parameter sets. The three algorithms are:

- 550 • a key generation algorithm denoted by KeyGen;
- 551 • an "encapsulation" algorithm denoted by Encaps;
- 552 • a "decapsulation" algorithm denoted by Decaps.

553 The collection of parameter sets is used to select a trade-off between security and efficiency.
554 Each parameter set in the collection is a list of specific numerical values, one for each parameter
555 required by the above algorithms.

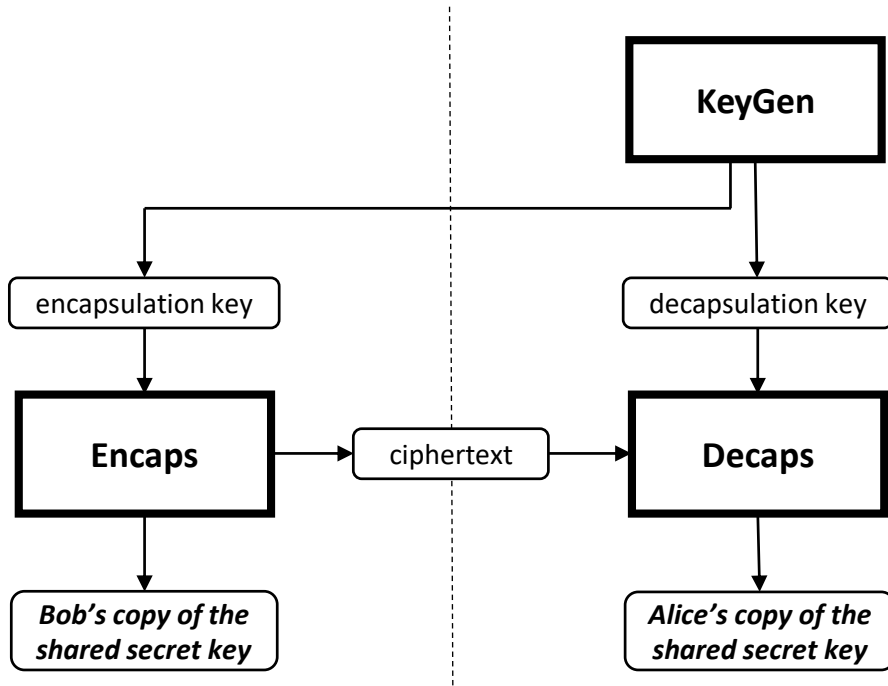


Figure 1. A simple view of key establishment using a KEM

556 A KEM can be used to establish a shared secret key between two parties (see Figure 1) referred
557 to here as Alice and Bob. Alice begins by running KeyGen in order to generate a (public)
558 encapsulation key and a (private) decapsulation key. Upon obtaining Alice's encapsulation key,

559 Bob runs the Encaps algorithm; this produces Bob’s copy K_B of the shared secret key along with
 560 an associated ciphertext. Bob sends the ciphertext to Alice, and Alice completes the process by
 561 running the Decaps algorithm using her decapsulation key and the ciphertext; this step produces
 562 Alice’s copy K_A of the shared secret key.

563 After completing the process above, Alice and Bob would like to conclude that their individual
 564 outputs satisfy $K_A = K_B$ and that this value is a secure, random, shared secret key. However, these
 565 properties only hold under certain important assumptions, as discussed in NIST SP 800-227 [1].

566 3.2 The ML-KEM Scheme

567 ML-KEM is a key-encapsulation mechanism based on CRYSTALS-KYBER [4], a scheme
 568 that was initially described in [8]. The following is a brief and informal description of the
 569 computational assumption underlying ML-KEM, and how the ML-KEM scheme is constructed.

570

571 **The computational assumption.** The security of ML-KEM is based on the presumed difficulty
 572 of solving the so-called Module Learning with Errors (MLWE) problem [9], a generalization of
 573 the Learning with Errors (LWE) problem introduced by Regev in 2005 [10]. The hardness of the
 574 MLWE problem is itself based on the presumed hardness of certain computational problems in
 575 module lattices [9]. This motivates the name of the scheme ML-KEM.

576 In the LWE problem, the input is a set of random “noisy” linear equations in some secret
 577 variables $x \in \mathbb{Z}_q^n$, and the task is to recover x . The noise in the equations is such that standard
 578 algorithms (e.g., Gaussian elimination) are intractable. The LWE problem lends itself naturally to
 579 cryptographic applications. For example, if x is interpreted as a secret key, then one can encrypt a
 580 one-bit value by sampling either an approximately correct linear equation (if the bit value is zero)
 581 or a far-from-correct linear equation (if the bit value is one). Plausibly, only a party in possession
 582 of x can then distinguish these two cases. Encryption can then be delegated to another party by
 583 publishing a large collection of noisy linear equations, which can be combined appropriately by
 584 the encrypting party. The result is an asymmetric encryption scheme.

585 At a high level, the MLWE problem poses the same task as LWE but with \mathbb{Z}_q^n replaced with the
 586 module R_q^k constructed by taking the k -fold Cartesian product of a certain polynomial ring R_q for
 587 some integer $k > 1$. In particular, the secret is now an element \mathbf{x} of the module R_q^k .

588

589 **The ML-KEM construction.** At a high level, the ML-KEM construction proceeds in two
 590 steps. First, the idea mentioned above is used to construct a public-key encryption scheme
 591 from the MLWE problem. Second, this public-key encryption scheme is converted into a
 592 key-encapsulation mechanism using the so-called Fujisaki-Okamoto (FO) transform [11, 12].
 593 In addition to producing a KEM, the FO transform is also intended to provide security in a
 594 significantly more general adversarial attack model. As a result, ML-KEM is believed to satisfy
 595 so-called IND-CCA security [1, 4, 13].

596 The specification of the ML-KEM algorithms in this standard will follow the above pattern.
 597 Specifically, this standard will first describe a public-key encryption scheme called K-PKE and
 598 then use the algorithms of K-PKE as subroutines when describing the algorithms of ML-KEM.
 599 The cryptographic transformation from K-PKE to ML-KEM is crucial for achieving full security.

600 The scheme K-PKE is not sufficiently secure and **shall not** be used as a stand-alone scheme (see
601 Section 3.3).

602 A notable feature of ML-KEM is the use of the *number-theoretic transform* (NTT). The NTT
603 converts a polynomial $f \in R_q$ to an alternative representation as a vector \hat{f} of linear polynomials.
604 Although NTT representations enable fast multiplication, other operations such as rounding and
605 sampling must be applied to standard polynomial representations.

606 ML-KEM satisfies the key properties of KEM correctness, and a proof of asymptotic theoretical
607 security (in a certain heuristic model) is known [4]. Each of the parameter sets of ML-KEM
608 comes with an associated security strength, which was estimated based on current cryptanalysis
609 (see Section 7 for details).

610

611 **Parameter sets and algorithms.** Recall that a KEM consists of algorithms KeyGen, Encaps,
612 and Decaps, together with a collection of parameter sets. In the case of ML-KEM, the three
613 aforementioned algorithms are:

- 614 • [ML-KEM.KeyGen](#) (Algorithm 15);
- 615 • [ML-KEM.Encaps](#) (Algorithm 16);
- 616 • [ML-KEM.Decaps](#) (Algorithm 17).

617 These algorithms are described and discussed in detail in Section 6.

618 ML-KEM comes equipped with three parameter sets:

- 619 • ML-KEM-512 (security category 1);
- 620 • ML-KEM-768 (security category 3);
- 621 • ML-KEM-1024 (security category 5).

622 These parameter sets are described and discussed in detail in Section 7; the security categories
623 1-5 are defined in Appendix A. Each parameter set assigns a particular numerical value to five
624 integer variables: k , η_1 , η_2 , d_u , and d_v . The values of these variables in each parameter set are
625 given in Table 2 of Section 7. In addition to these five variable parameters, there are also two
626 constants: $n = 256$ and $q = 3329$.

627

628 **Decapsulation failures.** Provided all inputs are well-formed, the key establishment procedure of
629 ML-KEM will never explicitly fail. Specifically, the [ML-KEM.Encaps](#) and [ML-KEM.Decaps](#)
630 algorithms will always output a value with the same data type as a shared secret key, and will never
631 output an error or failure symbol. However, it is possible (though extremely unlikely) that the
632 process will fail in the sense that Alice (via [ML-KEM.Decaps](#)) and Bob (via [ML-KEM.Encaps](#))
633 will produce different outputs, even though both of them are behaving honestly and no adversarial
634 interference is present. In this case, Alice and Bob clearly did not succeed in producing a shared
635 secret key. This event is called a decapsulation failure. The decapsulation failure probability is
636 defined to be the probability that the process

- 637 1. $(ek, dk) \leftarrow \text{ML-KEM.KeyGen}()$

- 638 2. $(c, K) \leftarrow \text{ML-KEM.Encaps}(ek)$
- 639 3. $K' \leftarrow \text{ML-KEM.Decaps}(c, dk)$

640 results in $K \neq K'$ (i.e., the encapsulated key is different from the decapsulated key). Estimates for
 641 the decapsulation failure probability (or rate) for each of the ML-KEM parameter sets are given
 642 in Table 1 (see [4]).

Table 1. Decapsulation failure rates for ML-KEM

Parameter set	Decapsulation failure rate
ML-KEM-512	2^{-139}
ML-KEM-768	2^{-164}
ML-KEM-1024	2^{-174}

643
 644 **A note on terminology for keys.** A KEM involves three different types of keys: encapsulation
 645 keys, decapsulation keys, and shared secret keys. ML-KEM is built on top of the component
 646 public-key encryption scheme K-PKE, and K-PKE has two additional key types: encryption
 647 keys and decryption keys. In the literature, encapsulation keys and encryption keys are sometimes
 648 referred to as “public keys,” while decapsulation keys and decryption keys can sometimes be
 649 referred to as “private keys.” In order to reduce confusion, this standard will not use the terms
 650 “public key” and “private key.” Instead, keys will be referred to using the more specific terms
 651 above (i.e., encapsulation key, decapsulation key, encryption key, decryption key, or shared secret
 652 key).

653 3.3 Requirements for ML-KEM Implementations

654 This section describes several requirements for implementing the algorithms of ML-KEM.
 655 Requirements for using ML-KEM in specific applications are given in NIST SP 800-227 [1].

656
 657 **K-PKE is only a component.** The public-key encryption scheme K-PKE described in Section
 658 5 shall not be used as a stand-alone cryptographic scheme. Instead, the algorithms that comprise
 659 K-PKE may only be used as subroutines in the algorithms of ML-KEM. In particular, the algo-
 660 rithms `K-PKE.KeyGen` (Algorithm 12), `K-PKE.Encrypt` (Algorithm 13), and `K-PKE.Decrypt`
 661 (Algorithm 14) are **not approved** for use as a public-key encryption scheme.

662
 663 **Equivalent implementations.** Each of the three top-level algorithms (i.e., `ML-KEM.KeyGen`,
 664 `ML-KEM.Encaps`, and `ML-KEM.Decaps`) defines a particular mathematical operation, mapping
 665 any given input to a corresponding output. For example, the operation defined by the algorithm
 666 `ML-KEM.Encaps` takes one byte array as input and produces two byte arrays as output.

667 In this standard, the three operations defined by `ML-KEM.KeyGen`, `ML-KEM.Encaps`, and
 668 `ML-KEM.Decaps` are described using particular sequences of computational steps. A conform-
 669 ing implementation can replace each of these sequences with a different sequence of steps,
 670 provided that the resulting operation is an equivalent process to the one specified in this standard.

671 For example, a conforming implementation of the encapsulation operation must have the property
672 that, for any parameter set and any input byte array ek , the distribution of output byte arrays is
673 identical to the distribution $ML\text{-}KEM.Encaps(ek)$ as specified in this standard.

674

675 **Approved usage of the shared secret key.** The output of the encapsulation and decapsulation
676 algorithms of ML-KEM is always a 256-bit value. Under appropriate conditions (see above; see
677 also NIST SP 800-227 [1]), this output is a shared secret key K . This shared secret key K can
678 be used directly as a key for symmetric cryptography. When key derivation is needed, the final
679 symmetric key(s) **shall** be derived from this 256-bit shared secret key K in an **approved** manner,
680 as specified in NIST SP 800-108 [14].

681

682 **Randomness generation.** Three algorithms in this standard require the generation of randomness:
683 $K\text{-}PKE.KeyGen$, $ML\text{-}KEM.KeyGen$, and $ML\text{-}KEM.Encaps$. In pseudocode, the step in which
684 this randomness is generated is denoted by a pseudocode statement of the form $m \xleftarrow{\$} \mathbb{B}^{32}$. A fresh
685 string of random bytes must be generated for every such invocation. These random bytes **shall**
686 be generated using an **approved** RBG, as prescribed in NIST SP 800-90A, NIST SP 800-90B,
687 and NIST SP 800-90C [15, 16, 17]. Moreover, the RBG used **shall** have a security strength of at
688 least 128 bits for ML-KEM-512, at least 192 bits for ML-KEM-768, and at least 256 bits for
689 ML-KEM-1024.

690

691 **Input validation.** The algorithms $ML\text{-}KEM.Encaps$ and $ML\text{-}KEM.Decaps$ require input val-
692 idation. Implementers **shall** ensure that $ML\text{-}KEM.Encaps$ and $ML\text{-}KEM.Decaps$ are only
693 executed on validated inputs, as described in Section 6. As discussed above, implementers can
694 choose to implement the top-level algorithms (i.e., $ML\text{-}KEM.Encaps$, $ML\text{-}KEM.Decaps$, or
695 $ML\text{-}KEM.KeyGen$) using any equivalent process; the validation of inputs is considered part of
696 this process. A conforming implementation **shall** be equivalent to first validating the input, and
697 then running the appropriate algorithm.

698

699 **Destruction of intermediate values.** Data used internally by KEM algorithms in intermediate
700 computation steps could be used by an adversary to compromise security. Implementers **shall**,
701 therefore, ensure that such intermediate data is destroyed as soon as it is no longer needed.

702

703 **No floating-point arithmetic.** Implementations of ML-KEM **should not** use floating-point
704 arithmetic. All division and rounding steps in the algorithms of ML-KEM can be performed
705 within the set of rational numbers.

706 4. Auxiliary Algorithms

707 4.1 Cryptographic Functions

708 The algorithms specified in this publication require the use of several cryptographic functions.
 709 Each function **shall** be instantiated by means of an **approved** hash function or an **approved**
 710 eXtendable-Output function (XOF), as prescribed below. The relevant hash functions and XOFs
 711 are described in detail in FIPS 202 [7]. They will be used as follows.

712 SHA3-256 and SHA3-512 are hash functions with variable-length input and fixed-length output.
 713 In this standard, invocations of these functions on an input M will be denoted by $\text{SHA3-256}(M)$
 714 and $\text{SHA3-512}(M)$, respectively.

715 SHAKE128 and SHAKE256 are XOFs with variable-length input and variable-length output.
 716 Invocations of these functions on an input M will be denoted in two different ways, depending
 717 on whether the desired output length ℓ (in bytes) is known at invocation time. If ℓ is known at
 718 invocation time, the invocation will be denoted by $\text{SHAKE128}(M, \ell)$ or $\text{SHAKE256}(M, \ell)$. For
 719 SHAKE128, the output length will sometimes not be known at invocation time; in those cases,
 720 the invocation will be denoted by $\text{SHAKE128}(M)$ and the hashing routine will behave like a byte
 721 stream that provides pseudorandom bytes (by performing additional “squeezing” rounds [7]) until
 722 no more bytes are needed.

723 The above functions will play several different roles in the algorithms specified in this standard.
 724 It will be convenient to assign a specific notation to each of these roles, as follows.

725

726 **Pseudorandom function (PRF)**. The function PRF takes a parameter $\eta \in \{2, 3\}$, one 32-byte
 727 input, and one 1-byte input. It produces one $(64 \cdot \eta)$ -byte output. It will be denoted by $\text{PRF} :$
 728 $\{2, 3\} \times \mathbb{B}^{32} \times \mathbb{B} \rightarrow \mathbb{B}^{64\eta}$, and it **shall** be instantiated as

$$\text{PRF}_\eta(s, b) := \text{SHAKE256}(s \| b, 64 \cdot \eta), \quad (4.1)$$

729 where $\eta \in \{2, 3\}$, $s \in \mathbb{B}^{32}$, and $b \in \mathbb{B}$. Here, η is only used to specify the desired output length
 730 and not to perform domain separation. Note that the output length parameter for SHAKE256 is
 731 specified in bytes.

732

733 **eXtendable-output function (XOF)**. The function XOF takes one 32-byte input and two 1-
 734 byte inputs. It produces a variable-length output. This function will be denoted by $\text{XOF} :$
 735 $\mathbb{B}^{32} \times \mathbb{B} \times \mathbb{B} \rightarrow \mathbb{B}^*$, and it **shall** be instantiated as

$$\text{XOF}(\rho, i, j) := \text{SHAKE128}(\rho \| i \| j), \quad (4.2)$$

736 where $\rho \in \mathbb{B}^{32}$, $i \in \mathbb{B}$, and $j \in \mathbb{B}$. The function XOF will only be invoked to provide a stream
 737 of pseudorandom bytes for the sampling algorithm [SampleNTT](#) (Algorithm 6). As [SampleNTT](#)
 738 performs rejection sampling, the total number of needed bytes will not be known at the time that
 739 XOF is invoked.

740

741 **Three hash functions.** The specification will also make use of three hash function instantiations
 742 H , J , and G , as follows.

743 The functions H and J each take one variable-length input and produce one 32-byte output. They
 744 will be denoted by $H : \mathbb{B}^* \rightarrow \mathbb{B}^{32}$ and $J : \mathbb{B}^* \rightarrow \mathbb{B}^{32}$, respectively, and **shall** be instantiated as

$$H(s) := \text{SHA3-256}(s) \quad \text{and} \quad J(s) := \text{SHAKE256}(s, 32) \quad (4.3)$$

745 where $s \in \mathbb{B}^*$.

746 The function G takes a variable-length input and produces two 32-byte outputs. It will be denoted
 747 by $G : \mathbb{B}^* \rightarrow \mathbb{B}^{32} \times \mathbb{B}^{32}$. The two outputs of G will be denoted by, e.g., $(a, b) \leftarrow G(c)$, where
 748 $a, b \in \mathbb{B}^{32}$, $c \in \mathbb{B}^*$, and $G(c) = a \| b$. The function G **shall** be instantiated as

$$G(c) := \text{SHA3-512}(c). \quad (4.4)$$

749

750

751 4.2 General Algorithms

752 This section specifies a number of algorithms that will be used as subroutines in the main
 753 ML-KEM algorithms. For a discussion of how to interpret the pseudocode of these algorithms,
 754 see Section 2.4.

755 4.2.1 Conversion and Compression Algorithms

756 This section specifies several algorithms for converting between bit arrays, byte arrays, and arrays
 757 of integers modulo m . It also specifies a certain compression operation for integers modulo q , as
 758 well as the corresponding decompression operation.

759

760 **Converting between bits and bytes.** Algorithms 2 and 3 convert between bit arrays and byte
 761 arrays. The inputs to `BitsToBytes` and the outputs of `BytesToBits` are bit arrays, with each
 762 segment of 8 bits representing a byte in little-endian order.

Algorithm 2 `BitsToBytes`(b)

Converts a bit string (of length a multiple of eight) into an array of bytes.

Input: bit array $b \in \{0, 1\}^{8 \cdot \ell}$.

Output: byte array $B \in \mathbb{B}^\ell$.

- 1: $B \leftarrow (0, \dots, 0)$
 - 2: **for** ($i \leftarrow 0$; $i < 8\ell$; $i++$)
 - 3: $B[\lfloor i/8 \rfloor] \leftarrow B[\lfloor i/8 \rfloor] + b[i] \cdot 2^{i \bmod 8}$
 - 4: **end for**
 - 5: **return** B
-

Algorithm 3 BytesToBits(B)

Performs the inverse of BitsToBytes, converting a byte array into a bit array.

Input: byte array $B \in \mathbb{B}^\ell$.

Output: bit array $b \in \{0, 1\}^{8 \cdot \ell}$.

```

1: for ( $i \leftarrow 0; i < \ell; i++$ )
2:   for ( $j \leftarrow 0; j < 8; j++$ )
3:      $b[8i + j] \leftarrow B[i] \bmod 2$ 
4:      $B[i] \leftarrow \lfloor B[i]/2 \rfloor$ 
5:   end for
6: end for
7: return  $b$ 

```

Compression and decompression. Recall that $q = 3329$, and note that the bit length of q is 12. For $d < 12$, define

$$\text{Compress}_d : \mathbb{Z}_q \longrightarrow \mathbb{Z}_{2^d} \quad (4.5)$$

$$x \longmapsto \lceil (2^d/q) \cdot x \rceil.$$

$$\text{Decompress}_d : \mathbb{Z}_{2^d} \longrightarrow \mathbb{Z}_q \quad (4.6)$$

$$y \longmapsto \lceil (q/2^d) \cdot y \rceil.$$

763 Note that the input and output types of these functions are integers modulo m (see discussion
764 of types in Section 2.4). Division and rounding in the computation of the above functions are
765 performed in the set of rational numbers. Floating-point computations **should not** be used.

766 Informally, **Compress** discards low-order bits of the input, and **Decompress** adds low-order bits
767 set to zero. These algorithms satisfy two important properties. First, decompression followed
768 by compression preserves the input, that is, $\text{Compress}_d(\text{Decompress}_d(y)) = y$ for all $y \in \mathbb{Z}_q$ and
769 all $d < 12$. Second, if d is large (i.e., close to 12) — meaning that the number of discarded
770 bits is small — compression followed by decompression does not significantly alter the value.
771 Specifically,

$$\lceil \text{Decompress}_d(\text{Compress}_d(x)) - x \rceil \bmod^\pm q \leq \lceil q/2^{d+1} \rceil \quad (4.7)$$

772 for all $x \in \mathbb{Z}_q$ and all $d < 12$.

773

774 **Encoding and decoding.** The algorithms **ByteEncode** (Algorithm 4) and **ByteDecode** (Algorithm
775 5) will be used for serialization and deserialization of arrays of integers modulo m . All serialized
776 arrays will be of length $n = 256$. ByteEncode_d serializes an array of d -bit integers into an array
777 of $32 \cdot d$ bytes. ByteDecode_d performs the corresponding deserialization operation, converting an
778 array of $32 \cdot d$ bytes into an array of d -bit integers.

779 For the following discussion, it is convenient to view **ByteDecode** and **ByteEncode** as converting
780 between integers and bits. (The conversion between bits and bytes is straightforward and done
781 using **BitsToBytes** and **BytesToBits**.)

782 The valid range of values for the parameter d is $1 \leq d \leq 12$. Bit arrays are divided into d -bit
 783 segments. In the case where $1 \leq d \leq 11$, `ByteDecoded` converts each d -bit segment of the input
 784 into one integer modulo 2^d , and `ByteEncoded` performs the inverse operation. In this case, the
 785 conversion is one-to-one.

786 The case $d = 12$ is treated differently. In this case, `ByteEncode12` receives integers modulo q
 787 as input, and `ByteDecode12` produces integers modulo q as output. `ByteDecode12` converts each
 788 12-bit segment of the input into an integer modulo $2^{12} = 4096$, and then reduces the result modulo
 789 q . This is no longer a one-to-one operation. Indeed, some 12-bit segments could correspond to an
 790 integer greater than $q = 3329$ but less than 4096; however, this cannot occur for arrays produced
 791 by `ByteEncode12`. These aspects of `ByteDecode12` and `ByteEncode12` will be important when
 792 considering validation of the ML-KEM encapsulation key in Section 6.

Algorithm 4 `ByteEncoded(F)`

Encodes an array of d -bit integers into a byte array, for $1 \leq d \leq 12$.

Input: integer array $F \in \mathbb{Z}_m^{256}$, where $m = 2^d$ if $d < 12$ and $m = q$ if $d = 12$.

Output: byte array $B \in \mathbb{B}^{32d}$.

```

1: for ( $i \leftarrow 0$ ;  $i < 256$ ;  $i++$ )
2:    $a \leftarrow F[i]$   $\triangleright a \in \mathbb{Z}_{2^d}$ 
3:   for ( $j \leftarrow 0$ ;  $j < d$ ;  $j++$ )
4:      $b[i \cdot d + j] \leftarrow a \bmod 2$   $\triangleright b \in \{0, 1\}^{256 \cdot d}$ 
5:      $a \leftarrow (a - b[i \cdot d + j]) / 2$   $\triangleright$  note  $a - b[i \cdot d + j]$  is always even.
6:   end for
7: end for
8:  $B \leftarrow \text{BitsToBytes}(b)$ 
9: return  $B$ 

```

Algorithm 5 `ByteDecoded(B)`

Decodes a byte array into an array of d -bit integers, for $1 \leq d \leq 12$.

Input: byte array $B \in \mathbb{B}^{32d}$.

Output: integer array $F \in \mathbb{Z}_m^{256}$, where $m = 2^d$ if $d < 12$ and $m = q$ if $d = 12$.

```

1:  $b \leftarrow \text{BytesToBits}(B)$ 
2: for ( $i \leftarrow 0$ ;  $i < 256$ ;  $i++$ )
3:    $F[i] \leftarrow \sum_{j=0}^{d-1} b[i \cdot d + j] \cdot 2^j \bmod m$ 
4: end for
5: return  $F$ 

```

793 4.2.2 Sampling Algorithms

794 The algorithms of ML-KEM require two sampling subroutines that are specified in Algorithms 6
 795 and 7. Both of these algorithms can be used to convert a stream of uniformly random bytes into a
 796 sample from some desired distribution. In this standard, these algorithms will be invoked with a
 797 stream of pseudorandom bytes as the input. It follows that the output will then be a sample from
 798 a distribution that is computationally indistinguishable from the desired distribution.

799

800 **Uniform sampling of NTT representations.** The algorithm `SampleNTT` (Algorithm 6) converts
 801 a stream of bytes into a polynomial in the NTT domain. If the input stream consists of uniformly
 802 random bytes, then the result will be drawn uniformly at random from T_q . The output is an array
 803 in \mathbb{Z}_q^{256} that contains the coefficients of the sampled element of T_q (see Section 2.4).

Algorithm 6 `SampleNTT`(B)

If the input is a stream of uniformly random bytes, the output is a uniformly random element of T_q .

Input: byte stream $B \in \mathbb{B}^*$.

Output: array $\hat{a} \in \mathbb{Z}_q^{256}$. ▷ the coefficients of the NTT of a polynomial

```

1:  $i \leftarrow 0$ 
2:  $j \leftarrow 0$ 
3: while  $j < 256$  do
4:    $d_1 \leftarrow B[i] + 256 \cdot (B[i+1] \bmod 16)$ 
5:    $d_2 \leftarrow \lfloor B[i+1]/16 \rfloor + 16 \cdot B[i+2]$ 
6:   if  $d_1 < q$  then
7:      $\hat{a}[j] \leftarrow d_1$  ▷  $\hat{a} \in \mathbb{Z}_q^{256}$ 
8:      $j \leftarrow j + 1$ 
9:   end if
10:  if  $d_2 < q$  and  $j < 256$  then
11:     $\hat{a}[j] \leftarrow d_2$ 
12:     $j \leftarrow j + 1$ 
13:  end if
14:   $i \leftarrow i + 3$ 
15: end while
16: return  $\hat{a}$ 

```

Algorithm 7 `SamplePolyCBD` $_{\eta}$ (B)

If the input is a stream of uniformly random bytes, outputs a sample from the distribution $\mathcal{D}_{\eta}(R_q)$.

Input: byte array $B \in \mathbb{B}^{64\eta}$.

Output: array $f \in \mathbb{Z}_q^{256}$. ▷ the coefficients of the sampled polynomial

```

1:  $b \leftarrow \text{BytesToBits}(B)$ 
2: for ( $i \leftarrow 0$ ;  $i < 256$ ;  $i++$ )
3:    $x \leftarrow \sum_{j=0}^{\eta-1} b[2i\eta + j]$ 
4:    $y \leftarrow \sum_{j=0}^{\eta-1} b[2i\eta + \eta + j]$ 
5:    $f[i] \leftarrow x - y \bmod q$  ▷  $f \in \mathbb{Z}_q^{256}$ 
6: end for
7: return  $f$ 

```

804

805 **Sampling from the centered binomial distribution.** ML-KEM makes use of a special distri-
 806 bution $\mathcal{D}_{\eta}(R_q)$ of polynomials in R_q with small coefficients. Such polynomials will sometimes

807 be referred to as “errors” or “noise.” The distribution is parameterized by an integer $\eta \in \{2, 3\}$.
 808 To sample a polynomial from $\mathcal{D}_\eta(R_q)$, each of its coefficients is independently sampled from a
 809 certain centered binomial distribution (CBD) on \mathbb{Z}_q . The algorithm [SamplePolyCBD](#) (Algorithm
 810 7) samples the coefficient array of a polynomial $f \in R_q$ according to the distribution $\mathcal{D}_\eta(R_q)$,
 811 provided that its input is a stream of uniformly random bytes.

812 4.3 The Number-Theoretic Transform

813 The number-theoretic transform (or NTT) can be viewed as a specialized, exact version of the
 814 discrete Fourier transform. In the case of ML-KEM, the NTT is used to improve the efficiency of
 815 multiplication in the ring R_q . Recall that R_q is the ring $\mathbb{Z}_q[X]/(X^n + 1)$ consisting of polynomials
 816 of the form $f = f_0 + f_1X + \dots + f_{255}X^{255}$ where $f_j \in \mathbb{Z}_q$ for all j , equipped with arithmetic
 817 modulo $X^n + 1$.

818 The ring R_q is naturally isomorphic to another ring, denoted T_q , which is a direct sum of 128
 819 quadratic extensions of \mathbb{Z}_q . The NTT is a computationally efficient isomorphism between these
 820 two rings. On input a polynomial $f \in R_q$, the NTT outputs an element $\hat{f} := \text{NTT}(f)$ of the ring
 821 T_q , where \hat{f} is called the “NTT representation” of f . The isomorphism property implies that

$$f \times_{R_q} g = \text{NTT}^{-1}(\hat{f} \times_{T_q} \hat{g}), \quad (4.8)$$

822 where \times_{R_q} and \times_{T_q} denote multiplication in R_q and T_q , respectively. Moreover, since T_q is a
 823 product of 128 rings, each consisting of degree-one polynomials, the operation \times_{T_q} is much more
 824 efficient than the operation \times_{R_q} . For these reasons, the NTT is considered to be an integral part
 825 of ML-KEM and not merely an optimization.

826 As the rings R_q and T_q have a vector space structure over \mathbb{Z}_q , the most natural abstract data type
 827 to represent elements from either of these rings is \mathbb{Z}_q^n . For this reason, the choice of data structure
 828 for the inputs and outputs of [NTT](#) and [NTT⁻¹](#) are length- n arrays of integers modulo q ; these
 829 arrays are understood to represent elements of T_q or R_q , respectively (see Section 2.4). Both [NTT](#)
 830 and [NTT⁻¹](#) can be computed in-place. In fact, Algorithms 8 and 9 demonstrate an efficient means
 831 of computing [NTT](#) and [NTT⁻¹](#) in-place. However, for clarity in understanding the distinction
 832 of the algebraic objects before and after the conversion, the algorithms are written with explicit
 833 inputs and outputs.

834

835 **The mathematical structure of a simple NTT.** Recall that, in ML-KEM, q is the prime
 836 $3329 = 2^8 \cdot 13 + 1$ and $n = 256$. There are 128 primitive 256-th roots of unity and no primitive
 837 512-th roots of unity in \mathbb{Z}_q . Note that $\zeta = 17 \in \mathbb{Z}_q$ is a primitive 256-th root of unity modulo q .
 838 Thus $\zeta^{128} \equiv -1$.

839 Define [BitRev₇](#)(i) to be the integer represented by bit-reversing the unsigned 7-bit value that
 840 corresponds to the input integer $i \in \{0, \dots, 127\}$.

841 The polynomial $X^{256} + 1$ factors into 128 polynomials of degree 2 modulo q as follows:

$$X^{256} + 1 = \prod_{k=0}^{127} \left(X^2 - \zeta^{2\text{BitRev}_7(k)+1} \right). \quad (4.9)$$

842 Therefore, $R_q := \mathbb{Z}_q[X]/(X^{256} + 1)$ is isomorphic to a direct sum of 128 quadratic extension fields
 843 of \mathbb{Z}_q , denoted T_q . Specifically, this ring has the structure

$$T_q := \bigoplus_{k=0}^{127} \mathbb{Z}_q[X]/\left(X^2 - \zeta^{2\text{BitRev}_7(k)+1}\right). \quad (4.10)$$

844 Thus, the NTT representation $\hat{f} \in T_q$ of a polynomial $f \in R_q$ is the vector that consists of the
 845 corresponding degree one residues:

$$\hat{f} := \left(f \bmod (X^2 - \zeta^{2\text{BitRev}_7(0)+1}), \dots, f \bmod (X^2 - \zeta^{2\text{BitRev}_7(127)+1})\right). \quad (4.11)$$

846 In the algorithms below, \hat{f} is stored as an array of 256 integers modulo q . Specifically,

$$f \bmod (X^2 - \zeta^{2\text{BitRev}_7(i)+1}) = \hat{f}[2i] + \hat{f}[2i+1]X.$$

847 for i from 0 to 127.

Algorithm 8 NTT(f)

Computes the NTT representation \hat{f} of the given polynomial $f \in R_q$.

Input: array $f \in \mathbb{Z}_q^{256}$. ▷ the coefficients of the input polynomial

Output: array $\hat{f} \in \mathbb{Z}_q^{256}$. ▷ the coefficients of the NTT of the input polynomial

```

1:  $\hat{f} \leftarrow f$  ▷ will compute NTT in-place on a copy of input array
2:  $k \leftarrow 1$ 
3: for ( $len \leftarrow 128$ ;  $len \geq 2$ ;  $len \leftarrow len/2$ )
4:   for ( $start \leftarrow 0$ ;  $start < 256$ ;  $start \leftarrow start + 2 \cdot len$ )
5:      $zeta \leftarrow \zeta^{\text{BitRev}_7(k)} \bmod q$ 
6:      $k \leftarrow k + 1$ 
7:     for ( $j \leftarrow start$ ;  $j < start + len$ ;  $j++$ )
8:        $t \leftarrow zeta \cdot \hat{f}[j + len]$  ▷ steps 8-10 done modulo  $q$ 
9:        $\hat{f}[j + len] \leftarrow \hat{f}[j] - t$ 
10:       $\hat{f}[j] \leftarrow \hat{f}[j] + t$ 
11:     end for
12:   end for
13: end for
14: return  $\hat{f}$ 

```

848

849 **The ML-KEM NTT algorithms.** An algorithm for the NTT is described in Algorithm 8. An
 850 algorithm for the Inverse-NTT is described in Algorithm 9. These two algorithms are overloaded
 851 in this standard. First, they represent the transformation used to map elements of R_q to elements
 852 of T_q (using NTT) and vice versa (using NTT^{-1}). Second, they represent the coordinate-wise
 853 transformation of structures over those rings; specifically, they map matrices/vectors with entries
 854 in R_q to matrices/vectors with entries in T_q (using NTT) and vice versa (using NTT^{-1}).

Algorithm 9 $\text{NTT}^{-1}(\hat{f})$

Computes the polynomial $f \in R_q$ corresponding to the given NTT representation $\hat{f} \in T_q$.

Input: array $\hat{f} \in \mathbb{Z}_q^{256}$. ▷ the coefficients of input NTT representation
Output: array $f \in \mathbb{Z}_q^{256}$. ▷ the coefficients of the inverse-NTT of the input

```

1:  $f \leftarrow \hat{f}$  ▷ will compute in-place on a copy of input array
2:  $k \leftarrow 127$ 
3: for ( $len \leftarrow 2$ ;  $len \leq 128$ ;  $len \leftarrow 2 \cdot len$ )
4:   for ( $start \leftarrow 0$ ;  $start < 256$ ;  $start \leftarrow start + 2 \cdot len$ )
5:      $zeta \leftarrow \zeta^{\text{BitRev}_7(k)} \bmod q$ 
6:      $k \leftarrow k - 1$ 
7:     for ( $j \leftarrow start$ ;  $j < start + len$ ;  $j++$ )
8:        $t \leftarrow f[j]$ 
9:        $f[j] \leftarrow t + f[j + len]$  ▷ steps 9-10 done modulo  $q$ 
10:       $f[j + len] \leftarrow zeta \cdot (f[j + len] - t)$ 
11:     end for
12:   end for
13: end for
14:  $f \leftarrow f \cdot 3303 \bmod q$  ▷ multiply every entry by  $3303 \equiv 128^{-1} \bmod q$ 
15: return  $f$ 

```

855 4.3.1 Multiplication in the NTT Domain

856 As discussed in Section 2.4, addition and scalar multiplication of elements of T_q is straightforward:
857 it can be done using the corresponding coordinate-wise arithmetic operations on the coefficient
858 arrays. This section describes how to do the remaining ring operation (i.e., multiplication in T_q).

859 Recall from (4.11) that $\hat{f} \in T_q$ is a vector of 128 degree one residues modulo quadratic polynomials.
860 Algebraically, multiplication in the ring T_q consists of independent multiplication in each of the
861 128 coordinates with respect to the quadratic modulus of that coordinate. Specifically, the i -th
862 coordinate in T_q of the product $\hat{h} = \hat{f} \times_{T_q} \hat{g}$ is determined by the calculation

$$\hat{h}[2i] + \hat{h}[2i + 1]X = (\hat{f}[2i] + \hat{f}[2i + 1]X)(\hat{g}[2i] + \hat{g}[2i + 1]X) \bmod (X^2 - \zeta^{2\text{BitRev}_7(i+1)}). \quad (4.12)$$

863 Thus, one can compute the product of two elements of T_q using the algorithm [MultiplyNTTs](#)
864 (Algorithm 10). Note that [MultiplyNTTs](#) uses [BaseCaseMultiply](#) (Algorithm 11) as a subroutine.
865 As discussed in Section 2.4, [MultiplyNTTs](#) enables one to perform linear-algebraic arithmetic
866 operations with matrices and vectors with entries in T_q .

Algorithm 10 `MultiplyNTTs`(\hat{f}, \hat{g})

Computes the product (in the ring T_q) of two NTT representations.

Input: Two arrays $\hat{f} \in \mathbb{Z}_q^{256}$ and $\hat{g} \in \mathbb{Z}_q^{256}$. ▷ the coefficients of two NTT representations

Output: An array $\hat{h} \in \mathbb{Z}_q^{256}$. ▷ the coefficients of the product of the inputs

- 1: **for** ($i \leftarrow 0; i < 128; i++$)
 - 2: $(\hat{h}[2i], \hat{h}[2i+1]) \leftarrow \text{BaseCaseMultiply}(\hat{f}[2i], \hat{f}[2i+1], \hat{g}[2i], \hat{g}[2i+1], \zeta^{2\text{BitRev}_7(i)+1})$
 - 3: **end for**
 - 4: **return** \hat{h}
-

Algorithm 11 `BaseCaseMultiply`($a_0, a_1, b_0, b_1, \gamma$)

Computes the product of two degree-one polynomials with respect to a quadratic modulus.

Input: $a_0, a_1, b_0, b_1 \in \mathbb{Z}_q$. ▷ the coefficients of $a_0 + a_1X$ and $b_0 + b_1X$

Input: $\gamma \in \mathbb{Z}_q$. ▷ the modulus is $X^2 - \gamma$

Output: $c_0, c_1 \in \mathbb{Z}_q$. ▷ the coefficients of the product of the two polynomials

- 1: $c_0 \leftarrow a_0 \cdot b_0 + a_1 \cdot b_1 \cdot \gamma$ ▷ steps 1-2 done modulo q
 - 2: $c_1 \leftarrow a_0 \cdot b_1 + a_1 \cdot b_0$
 - 3: **return** c_0, c_1
-

867 5. The K-PKE Component Scheme

868 This section describes the component scheme K-PKE. As discussed in Section 3.3, K-PKE is
 869 **not approved** for use in a stand-alone fashion. It serves only as a collection of subroutines for
 870 use in the algorithms of the **approved** scheme ML-KEM, as described in Section 6.

871 K-PKE consists of three algorithms:

- 872 1. Key generation ([K-PKE.KeyGen](#));
- 873 2. Encryption ([K-PKE.Encrypt](#));
- 874 3. Decryption ([K-PKE.Decrypt](#)).

875 When K-PKE is instantiated as part of ML-KEM, K-PKE inherits the parameter set selected
 876 for ML-KEM. Each parameter set specifies numerical values for each parameter. While n is
 877 always 256 and q is always 3329, the values of the remaining parameters k , η_1 , η_2 , d_u , and d_v vary
 878 among the three parameter sets. The individual parameters and the parameter sets are described
 879 in Section 7.

880 The algorithms in this section do not perform any input validation. This is because they are
 881 only invoked as subroutines of the main ML-KEM algorithms. The algorithms of ML-KEM
 882 do perform input validation as needed; they also ensure that all inputs to K-PKE algorithms
 883 (invoked as subroutines) will be valid.

884 Each of the algorithms of K-PKE below is accompanied by a brief, informal description in text.
 885 For simplicity, this description is written in terms of vectors and matrices whose entries are
 886 elements of R_q . In the actual algorithm, most of the computations occur in the NTT domain in
 887 order to improve the efficiency of multiplication. The relevant vectors and matrices will then
 888 have entries in T_q . Linear-algebraic arithmetic with such vectors and matrices (see, e.g., line 19
 889 of [K-PKE.KeyGen](#)) is performed as described in Sections 2.4 and 4.3.1. The encryption and
 890 decryption key of K-PKE are also stored in the NTT form.

891 5.1 K-PKE Key Generation

892 The key generation algorithm [K-PKE.KeyGen](#) of K-PKE (Algorithm 12) takes no input, requires
 893 randomness, and outputs an encryption key ek_{PKE} and a decryption key dk_{PKE} . From the typical
 894 point of view of public-key encryption, the encryption key can be made public, while the
 895 decryption key and the randomness must remain private. This will be the case in the context
 896 of this standard as well. Indeed, the encryption key of K-PKE will serve as the encapsulation
 897 key of ML-KEM (see [ML-KEM.KeyGen](#) below) and can thus be made public; meanwhile, the
 898 decryption key and randomness of [K-PKE.KeyGen](#) must remain private as they can be used to
 899 perform decapsulation in ML-KEM.

900

901 **Informal description.** The decryption key of [K-PKE.KeyGen](#) is a length- k vector \mathbf{s} of elements
 902 of R_q , i.e., $\mathbf{s} \in R_q^k$. Roughly speaking, \mathbf{s} is a set of secret variables, while the encryption key is a
 903 collection of “noisy” linear equations ($\mathbf{A}\mathbf{s} + \mathbf{e}$) in the secret variables \mathbf{s} . The rows of the matrix
 904 \mathbf{A} form the equation coefficients. This matrix is generated pseudorandomly using XOF, with
 905 only the seed stored in the encryption key. The secret \mathbf{s} and the “noise” \mathbf{e} are sampled from the

Algorithm 12 $K\text{-PKE.KeyGen}()$

Generates an encryption key and a corresponding decryption key.

Output: encryption key $ek_{\text{PKE}} \in \mathbb{B}^{384k+32}$.

Output: decryption key $dk_{\text{PKE}} \in \mathbb{B}^{384k}$.

```

1:  $d \xleftarrow{\$} \mathbb{B}^{32}$                                 ▷  $d$  is 32 random bytes (see Section 3.3)
2:  $(\rho, \sigma) \leftarrow G(d)$                   ▷ expand to two pseudorandom 32-byte seeds
3:  $N \leftarrow 0$ 
4: for ( $i \leftarrow 0; i < k; i++$ )                ▷ generate matrix  $\hat{\mathbf{A}} \in (\mathbb{Z}_q^{256})^{k \times k}$ 
5:   for ( $j \leftarrow 0; j < k; j++$ )
6:      $\hat{\mathbf{A}}[i, j] \leftarrow \text{SampleNTT}(\text{XOF}(\rho, i, j))$   ▷ each entry of  $\hat{\mathbf{A}}$  uniform in NTT domain
7:   end for
8: end for
9: for ( $i \leftarrow 0; i < k; i++$ )                ▷ generate  $\mathbf{s} \in (\mathbb{Z}_q^{256})^k$ 
10:   $\mathbf{s}[i] \leftarrow \text{SamplePolyCBD}_{\eta_1}(\text{PRF}_{\eta_1}(\sigma, N))$   ▷  $\mathbf{s}[i] \in \mathbb{Z}_q^{256}$  sampled from CBD
11:   $N \leftarrow N + 1$ 
12: end for
13: for ( $i \leftarrow 0; i < k; i++$ )                ▷ generate  $\mathbf{e} \in (\mathbb{Z}_q^{256})^k$ 
14:   $\mathbf{e}[i] \leftarrow \text{SamplePolyCBD}_{\eta_1}(\text{PRF}_{\eta_1}(\sigma, N))$   ▷  $\mathbf{e}[i] \in \mathbb{Z}_q^{256}$  sampled from CBD
15:   $N \leftarrow N + 1$ 
16: end for
17:  $\hat{\mathbf{s}} \leftarrow \text{NTT}(\mathbf{s})$                     ▷ NTT is run  $k$  times (once for each coordinate of  $\mathbf{s}$ )
18:  $\hat{\mathbf{e}} \leftarrow \text{NTT}(\mathbf{e})$                     ▷ NTT is run  $k$  times
19:  $\hat{\mathbf{t}} \leftarrow \hat{\mathbf{A}} \circ \hat{\mathbf{s}} + \hat{\mathbf{e}}$                 ▷ noisy linear system in NTT domain
20:  $ek_{\text{PKE}} \leftarrow \text{ByteEncode}_{12}(\hat{\mathbf{t}} \parallel \rho)$   ▷  $\text{ByteEncode}_{12}$  is run  $k$  times; include seed for  $\hat{\mathbf{A}}$ 
21:  $dk_{\text{PKE}} \leftarrow \text{ByteEncode}_{12}(\hat{\mathbf{s}})$         ▷  $\text{ByteEncode}_{12}$  is run  $k$  times
22: return ( $ek_{\text{PKE}}, dk_{\text{PKE}}$ )

```

906 centered binomial distribution using randomness expanded from a seed via PRF. Once \mathbf{A} and \mathbf{s}
 907 and \mathbf{e} are generated, the computation of the final part $\mathbf{t} = \mathbf{A}\mathbf{s} + \mathbf{e}$ of the encryption key takes place.

908 In $K\text{-PKE.KeyGen}$, the choice of parameter set affects the length of the secret \mathbf{s} (via the parameter
 909 k) and, as a consequence, the sizes of the noise vector \mathbf{e} and the pseudorandom matrix \mathbf{A} . The
 910 choice of parameter set also affects the noise distribution (via the parameter η_1) used to sample
 911 the entries of \mathbf{s} and \mathbf{e} .

912 5.2 K-PKE Encryption

913 The encryption algorithm $K\text{-PKE.Encrypt}$ of K-PKE (Algorithm 13) takes an encryption key
 914 ek_{PKE} and a plaintext m as input, requires randomness r , and outputs a ciphertext c . While many al-
 915 gorithms specified in this document require randomness, only the description of $K\text{-PKE.Encrypt}$
 916 interprets this randomness as part of the input. This is because ML-KEM will need to invoke
 917 $K\text{-PKE.Encrypt}$ with a specific choice of randomness (see Algorithm 16 for details).

918

919 **Informal description.** The algorithm `K-PKE.Encrypt` begins by extracting the vector \mathbf{t} and
 920 the seed from the encryption key. The seed is then expanded to re-generate the matrix \mathbf{A} , in
 921 the same manner as was done in `K-PKE.KeyGen`. If \mathbf{t} and \mathbf{A} are derived correctly from an
 922 encryption key produced by `K-PKE.KeyGen`, then they are equal to their corresponding values
 923 in `K-PKE.KeyGen`.

Algorithm 13 `K-PKE.Encrypt`($\text{ek}_{\text{PKE}}, m, r$)

Uses the encryption key to encrypt a plaintext message using the randomness r .

Input: encryption key $\text{ek}_{\text{PKE}} \in \mathbb{B}^{384k+32}$.

Input: message $m \in \mathbb{B}^{32}$.

Input: encryption randomness $r \in \mathbb{B}^{32}$.

Output: ciphertext $c \in \mathbb{B}^{32(d_u k + d_v)}$.

```

1:  $N \leftarrow 0$ 
2:  $\hat{\mathbf{t}} \leftarrow \text{ByteDecode}_{12}(\text{ek}_{\text{PKE}}[0 : 384k])$ 
3:  $\rho \leftarrow \text{ek}_{\text{PKE}}[384k : 384k + 32]$ 
4: for ( $i \leftarrow 0; i < k; i++$ )
5:   for ( $j \leftarrow 0; j < k; j++$ )
6:      $\hat{\mathbf{A}}[i, j] \leftarrow \text{SampleNTT}(\text{XOF}(\rho, i, j))$ 
7:   end for
8: end for
9: for ( $i \leftarrow 0; i < k; i++$ )
10:   $\mathbf{r}[i] \leftarrow \text{SamplePolyCBD}_{\eta_1}(\text{PRF}_{\eta_1}(r, N))$ 
11:   $N \leftarrow N + 1$ 
12: end for
13: for ( $i \leftarrow 0; i < k; i++$ )
14:   $\mathbf{e}_1[i] \leftarrow \text{SamplePolyCBD}_{\eta_2}(\text{PRF}_{\eta_2}(r, N))$ 
15:   $N \leftarrow N + 1$ 
16: end for
17:  $e_2 \leftarrow \text{SamplePolyCBD}_{\eta_2}(\text{PRF}_{\eta_2}(r, N))$ 
18:  $\hat{\mathbf{r}} \leftarrow \text{NTT}(\mathbf{r})$ 
19:  $\mathbf{u} \leftarrow \text{NTT}^{-1}(\hat{\mathbf{A}}^\top \circ \hat{\mathbf{r}}) + \mathbf{e}_1$ 
20:  $\mu \leftarrow \text{Decompress}_1(\text{ByteDecode}_1(m))$ 
21:  $v \leftarrow \text{NTT}^{-1}(\hat{\mathbf{t}}^\top \circ \hat{\mathbf{r}}) + e_2 + \mu$ 
22:  $c_1 \leftarrow \text{ByteEncode}_{d_u}(\text{Compress}_{d_u}(\mathbf{u}))$ 
23:  $c_2 \leftarrow \text{ByteEncode}_{d_v}(\text{Compress}_{d_v}(v))$ 
24: return  $c \leftarrow (c_1 \| c_2)$ 

```

\triangleright extract 32-byte seed from ek_{PKE}
 \triangleright re-generate matrix $\hat{\mathbf{A}} \in (\mathbb{Z}_q^{256})^{k \times k}$
 \triangleright generate $\mathbf{r} \in (\mathbb{Z}_q^{256})^k$
 $\triangleright \mathbf{r}[i] \in \mathbb{Z}_q^{256}$ sampled from CBD
 \triangleright generate $\mathbf{e}_1 \in (\mathbb{Z}_q^{256})^k$
 $\triangleright \mathbf{e}_1[i] \in \mathbb{Z}_q^{256}$ sampled from CBD
 \triangleright sample $e_2 \in \mathbb{Z}_q^{256}$ from CBD
 \triangleright NTT is run k times
 \triangleright NTT⁻¹ is run k times
 \triangleright encode plaintext m into polynomial v .
 \triangleright ByteEncode _{d_u} is run k times

924 Recall from the description of key generation that the pair $(\mathbf{A}, \mathbf{t} = \mathbf{A}\mathbf{s} + \mathbf{e})$ can be thought of as a
 925 system of noisy linear equations in the secret variables \mathbf{s} . One can generate an additional noisy
 926 linear equation in the same secret variables — without knowing \mathbf{s} — by picking a random linear
 927 combination of the noisy equations in the system (\mathbf{A}, \mathbf{t}) . One can then encode information in the
 928 “constant term” (i.e., the entry which is a linear combination of entries of \mathbf{t}) of such a combined
 929 equation. This information can then be deciphered by a party in possession of \mathbf{s} . For example,
 930 one could encode a single bit by deciding whether or not to significantly alter the constant term,

931 thus making either a nearly correct equation (corresponding to the decrypted bit value of 0) or a
 932 far-from-correct equation (corresponding to the decrypted bit value of 1). In the case of K-PKE,
 933 the constant term is a polynomial with 256 coefficients, so one can encode more information: one
 934 bit in each coefficient.

935 To this end, **K-PKE.Encrypt** proceeds by generating a vector $\mathbf{r} \in R_q^k$ and noise terms $\mathbf{e}_1 \in R_q^k$ and
 936 $e_2 \in R_q$, all of which are sampled from the centered binomial distribution using pseudorandomness
 937 expanded (via PRF) from the input randomness r . One then computes the “new noisy equation”
 938 which is (up to some details) computed by $(\mathbf{u}, v) \leftarrow (\mathbf{A}^\top \mathbf{r} + \mathbf{e}_1, \mathbf{t}^\top \mathbf{r} + e_2)$. An appropriate encoding
 939 μ of the input message m is then added to the term $\mathbf{t}^\top \mathbf{r} + e_2$. Finally, the pair (\mathbf{u}, v) is compressed,
 940 serialized into a byte array, and output as the ciphertext.

941 5.3 K-PKE Decryption

942 The decryption algorithm **K-PKE.Decrypt** of K-PKE (Algorithm 14) takes a decryption key
 943 dk_{PKE} and a ciphertext c as input, requires no randomness, and outputs a plaintext m .

944

945 **Informal description.** The algorithm **K-PKE.Decrypt** begins by computing the “noisy equation”
 946 (\mathbf{u}, v) underlying the ciphertext c , as discussed in the description of **K-PKE.Encrypt**. Here, one
 947 can think of \mathbf{u} as the coefficients of the equation and v as the constant term. Recall that the
 948 decryption key dk_{PKE} contains the vector of secret variables \mathbf{s} . The decryption algorithm can thus
 949 use the decryption key to compute the true constant term $v' = \mathbf{s}^\top \mathbf{u}$ and then calculate $v - v'$. The
 950 decryption algorithm ends by decoding the plaintext message m from $v - v'$ and outputting m .

Algorithm 14 **K-PKE.Decrypt**($\text{dk}_{\text{PKE}}, c$)

Uses the decryption key to decrypt a ciphertext.

Input: decryption key $\text{dk}_{\text{PKE}} \in \mathbb{B}^{384k}$.

Input: ciphertext $c \in \mathbb{B}^{32(d_u k + d_v)}$.

Output: message $m \in \mathbb{B}^{32}$.

- 1: $c_1 \leftarrow c[0 : 32d_u k]$
 - 2: $c_2 \leftarrow c[32d_u k : 32(d_u k + d_v)]$
 - 3: $\mathbf{u} \leftarrow \text{Decompress}_{d_u}(\text{ByteDecode}_{d_u}(c_1))$ ▷ ByteDecode_{d_u} invoked k times
 - 4: $v \leftarrow \text{Decompress}_{d_v}(\text{ByteDecode}_{d_v}(c_2))$
 - 5: $\hat{\mathbf{s}} \leftarrow \text{ByteDecode}_{12}(\text{dk}_{\text{PKE}})$
 - 6: $w \leftarrow v - \text{NTT}^{-1}(\hat{\mathbf{s}}^\top \circ \text{NTT}(\mathbf{u}))$ ▷ NTT^{-1} and NTT invoked k times
 - 7: $m \leftarrow \text{ByteEncode}_1(\text{Compress}_1(w))$ ▷ decode plaintext m from polynomial v
 - 8: **return** m
-

951 **6. The ML-KEM Key-Encapsulation Mechanism**

952 The ML-KEM scheme consists of three algorithms:

- 953 1. Key generation ([ML-KEM.KeyGen](#))
- 954 2. Encapsulation ([ML-KEM.Encaps](#))
- 955 3. Decapsulation ([ML-KEM.Decaps](#))

956 To instantiate ML-KEM, one must select a parameter set, each of which is associated with a
 957 particular trade-off between security and performance. The three possible parameter sets are called
 958 ML-KEM-512, ML-KEM-768, and ML-KEM-1024 and are described in detail in Table 2 of
 959 Section 7. Each parameter set assigns specific numerical values to the individual parameters n ,
 960 q , k , η_1 , η_2 , d_u , and d_v . While n is always 256 and q is always 3329, the remaining parameters
 961 vary among the three parameter sets. Implementers **shall** ensure that the three algorithms of
 962 ML-KEM listed above are only invoked with a valid parameter set, and that this parameter set is
 963 selected appropriately for the desired application. In addition, the algorithms [ML-KEM.Encaps](#)
 964 and [ML-KEM.Decaps](#) require validation of inputs, as discussed below.

965 **6.1 ML-KEM Key Generation**

966 The key generation algorithm [ML-KEM.KeyGen](#) for ML-KEM (Algorithm 15) accepts no input,
 967 requires randomness, and produces an encapsulation key and a decapsulation key. While the
 968 encapsulation key can be made public, the decapsulation key must remain private.

969 **Informal description.** The core subroutine of [ML-KEM.KeyGen](#) is the key generation algorithm
 970 of K-PKE (Algorithm 12). The ML-KEM encapsulation key is simply the encryption key of
 971 K-PKE. The ML-KEM decapsulation key is comprised of the decryption key of K-PKE, the
 972 encapsulation key, a hash of the encapsulation key, and a pseudorandom 32-byte value. This
 973 random value will be used in the "implicit rejection" mechanism of the decapsulation algorithm
 974 (Algorithm 17).

Algorithm 15 [ML-KEM.KeyGen\(\)](#)

Generates an encapsulation key and a corresponding decapsulation key.

Output: Encapsulation key $ek \in \mathbb{B}^{384k+32}$.

Output: Decapsulation key $dk \in \mathbb{B}^{768k+96}$.

- 1: $z \xleftarrow{\$} \mathbb{B}^{32}$ ▷ z is 32 random bytes (see Section 3.3)
 - 2: $(ek_{\text{PKE}}, dk_{\text{PKE}}) \leftarrow \text{K-PKE.KeyGen}()$ ▷ run key generation for K-PKE
 - 3: $ek \leftarrow ek_{\text{PKE}}$ ▷ KEM encaps key is just the PKE encryption key
 - 4: $dk \leftarrow (dk_{\text{PKE}} \| ek \| H(ek) \| z)$ ▷ KEM decaps key includes PKE decryption key
 - 5: **return** (ek, dk)
-

976 **6.2 ML-KEM Encapsulation**

977 The encapsulation algorithm `ML-KEM.Encaps` of ML-KEM (Algorithm 16) accepts an encapsulation key as input, requires randomness, and outputs a ciphertext and a shared key.

979

980 **Input validation.** To validate a given input¹ \tilde{ek} to `ML-KEM.Encaps`, perform the following checks.

- 982 1. (*Type check.*) If \tilde{ek} is not an array of bytes of length $384k + 32$ for the value of k specified
 983 by the relevant parameter set, the input is invalid.
- 984 2. (*Modulus check.*) Perform the computation $ek \leftarrow \text{ByteEncode}_{12}(\text{ByteDecode}_{12}(\tilde{ek}))$. If
 985 $ek \neq \tilde{ek}$, the input is invalid. (See Section 4.2.1.)

986 If either of the above checks declare the input to be invalid, then `ML-KEM.Encaps` **shall not** be
 987 performed with input \tilde{ek} . Instead, application-appropriate steps **shall** be taken to abort. If both
 988 of the above checks pass (i.e., none of them declare the input to be invalid), then the input is
 989 considered valid and `ML-KEM.Encaps` can be performed with input $ek = \tilde{ek}$.

990 It is important to note that the above input validation process does not ensure that \tilde{ek} is an actual
 991 output of `ML-KEM.KeyGen`. In fact, the ability to ensure that (without using the decapsulation
 992 key) would violate the security assumption.

993 Recall that, as discussed in Section 3.3, implementations are only required to correctly reproduce
 994 the input-output behavior of the top-level algorithms. In the case of `ML-KEM.Encaps`, this
 995 means that an implementation can perform any process that is equivalent to executing checks 1
 996 and 2 above and then running Algorithm 16. (For example, the second check could be performed
 997 during the execution of `ByteDecode`₁₂ in line 2 of `K-PKE.Encrypt`.)

Algorithm 16 `ML-KEM.Encaps`(ek)

Uses the encapsulation key to generate a shared key and an associated ciphertext.

Validated input: encapsulation key $ek \in \mathbb{B}^{384k+32}$.

Output: shared key $K \in \mathbb{B}^{32}$.

Output: ciphertext $c \in \mathbb{B}^{32(d_u k + d_v)}$.

- 1: $m \xleftarrow{\$} \mathbb{B}^{32}$ ▷ m is 32 random bytes (see Section 3.3)
- 2: $(K, r) \leftarrow G(m || H(ek))$ ▷ derive shared secret key K and randomness r
- 3: $c \leftarrow \text{K-PKE.Encrypt}(ek, m, r)$ ▷ encrypt m using K-PKE with randomness r
- 4: **return** (K, c)
-

998

999 **Informal description.** The core subroutine of `ML-KEM.Encaps` is the encryption algorithm of
 1000 K-PKE, which is used to encrypt a random value m into a ciphertext c . A copy of the shared
 1001 secret K and the randomness used during encryption are derived from m and the encapsulation

¹In discussions of input validation, the tilde in the notation indicates that the input might not be properly formed, e.g., \tilde{ek} for a candidate encapsulation key input, as opposed to ek for a valid input.

1002 key ek via hashing. Specifically, H is applied to ek , and the result is concatenated with m and then
 1003 hashed using G . The algorithm completes by outputting the ciphertext c and the shared secret K .

1004 6.3 ML-KEM Decapsulation

1005 The decapsulation algorithm [ML-KEM.Decaps](#) of ML-KEM (Algorithm 16) accepts a decap-
 1006 sulation key and a ML-KEM ciphertext as input, does not use any randomness, and outputs a
 1007 shared secret.

1008

1009 **Input validation.** To validate a given pair of inputs \tilde{c} (candidate ciphertext) and \tilde{dk} (candidate
 1010 decapsulation key) to [ML-KEM.Decaps](#), perform the following checks.

1011 1. (*Ciphertext type check.*) If \tilde{c} is not a byte array of length $32(d_u k + d_v)$ for the values of d_u ,
 1012 d_v , and k specified by the relevant parameter set, the input is invalid.

1013 2. (*Decapsulation key type check.*) If \tilde{dk} is not a byte array of length $768k + 96$ for the value
 1014 of k specified by the relevant parameter set, the input is invalid.

1015 If either of the above checks declares the input to be invalid, then [ML-KEM.Decaps](#) **shall not**
 1016 be performed with input (\tilde{c}, \tilde{dk}) . Instead, application-appropriate steps **shall** be taken to abort.
 1017 If both of the checks pass (i.e., neither one declares the input to be invalid), then the input is
 1018 considered valid and [ML-KEM.Decaps](#) can be performed with input $(c, dk) = (\tilde{c}, \tilde{dk})$.

1019 For some applications, further validation of the decapsulation key \tilde{dk} may be appropriate. For
 1020 instance, in cases where \tilde{dk} was generated by a third party, users may want to ensure that the four
 1021 components of \tilde{dk} have the correct relationship with each other, as in line 4 of [ML-KEM.KeyGen](#).
 1022 In all cases, implementers **shall** validate the inputs to [ML-KEM.Decaps](#) in a manner that is
 1023 appropriate for their application.

1024

1025 **Informal description.** The algorithm [ML-KEM.Decaps](#) begins by parsing out the components
 1026 of the decapsulation key dk of ML-KEM. These components are an (encryption key, decryption
 1027 key) pair for K-PKE, a hash value h , and a random value z . The decryption key of K-PKE is
 1028 then used to decrypt the input ciphertext c to get a plaintext m' . The decapsulation algorithm then
 1029 re-encrypts m' and computes a candidate shared secret key K' in the same manner as should have
 1030 been done in encapsulation. Specifically, K' and the encryption randomness r' are computed by
 1031 hashing m' and the encryption key of K-PKE, and a ciphertext c' is generated by encrypting m'
 1032 using randomness r' . Finally, decapsulation checks whether the resulting ciphertext c' matches
 1033 the provided ciphertext c . If it does not, the algorithm performs an “implicit rejection”: the value
 1034 of K' is changed to a hash of c together with the random value z stored in the ML-KEM secret
 1035 key (see the discussion on decapsulation failures in Section 3.2). In either case, decapsulation
 1036 outputs the resulting shared secret key K' .

1037

1038

Algorithm 17 $\text{ML-KEM.Decaps}(c, \text{dk})$

Uses the decapsulation key to produce a shared key from a ciphertext.

Validated input: ciphertext $c \in \mathbb{B}^{32(d_u k + d_v)}$.

Validated input: decapsulation key $\text{dk} \in \mathbb{B}^{768k + 96}$.

Output: shared key $K \in \mathbb{B}^{32}$.

```

1:  $\text{dk}_{\text{PKE}} \leftarrow \text{dk}[0 : 384k]$            ▷ extract (from KEM decaps key) the PKE decryption key
2:  $\text{ek}_{\text{PKE}} \leftarrow \text{dk}[384k : 768k + 32]$            ▷ extract PKE encryption key
3:  $h \leftarrow \text{dk}[768k + 32 : 768k + 64]$            ▷ extract hash of PKE encryption key
4:  $z \leftarrow \text{dk}[768k + 64 : 768k + 96]$            ▷ extract implicit rejection value
5:  $m' \leftarrow \text{K-PKE.Decrypt}(\text{dk}_{\text{PKE}}, c)$            ▷ decrypt ciphertext
6:  $(K', r') \leftarrow G(m' || h)$ 
7:  $\bar{K} \leftarrow J(z || c, 32)$ 
8:  $c' \leftarrow \text{K-PKE.Encrypt}(\text{ek}_{\text{PKE}}, m', r')$            ▷ re-encrypt using the derived randomness  $r'$ 
9: if  $c \neq c'$  then
10:    $K' \leftarrow \bar{K}$            ▷ if ciphertexts do not match, “implicitly reject”
11: end if
12: return  $K'$ 

```

1039

1040

1041 **7. Parameter Sets**

1042 ML-KEM is equipped with three parameter sets. Each of the three parameter sets is comprised
 1043 of five individual parameters: k , η_1 , η_2 , d_u , and d_v . There are also two constants: $n = 256$ and
 1044 $q = 3329$. The following is a brief and informal description of the roles played by the variable
 1045 parameters in the algorithms of K-PKE (and hence in ML-KEM). See Section 5 for details.

- 1046 • The parameter k determines the dimensions of the vectors \mathbf{s} and \mathbf{e} in [K-PKE.KeyGen](#), as
 1047 well as the dimensions of the matrix $\hat{\mathbf{A}}$ and the vectors \mathbf{r} , \mathbf{e}_1 , and \mathbf{e}_2 in [K-PKE.Encrypt](#).
- 1048 • The parameter η_1 is required for specifying the distribution for generating the vectors \mathbf{s} and
 1049 \mathbf{e} in [K-PKE.KeyGen](#) and the vector \mathbf{r} in [K-PKE.Encrypt](#).
- 1050 • The parameter η_2 is required for specifying the distribution for generating the vectors \mathbf{e}_1
 1051 and \mathbf{e}_2 in [K-PKE.Encrypt](#).
- 1052 • The parameters d_u and d_v serve as parameters and inputs for the functions [Compress](#),
 1053 [Decompress](#), [ByteEncode](#), and [ByteDecode](#) in [K-PKE.Encrypt](#) and [K-PKE.Decrypt](#).

1054 This standard approves the parameter sets given in Table 2. Each parameter set is associated
 1055 with a required security strength for randomness generation (see Section 3.3). The sizes of the
 1056 ML-KEM keys and ciphertexts for each parameter set are summarized in Table 3.

	n	q	k	η_1	η_2	d_u	d_v	required RBG strength (bits)
ML-KEM-512	256	3329	2	3	2	10	4	128
ML-KEM-768	256	3329	3	2	2	10	4	192
ML-KEM-1024	256	3329	4	2	2	11	5	256

Table 2. Approved parameter sets for ML-KEM

	encapsulation key	decapsulation key	ciphertext	shared secret key
ML-KEM-512	800	1632	768	32
ML-KEM-768	1184	2400	1088	32
ML-KEM-1024	1568	3168	1568	32

Table 3. Sizes (in bytes) of keys and ciphertexts of ML-KEM

1057 A parameter set name can also be said to denote a (parameter-free) KEM. Specifically, ML-KEM- x
 1058 can be used to denote the parameter-free KEM that results from instantiating the scheme
 1059 ML-KEM with the parameter set ML-KEM- x .

1060 The three parameter sets included in Table 2 were designed to meet certain security strength
 1061 categories defined by NIST in its original Call for Proposals [4, 18]. These security strength
 1062 categories are explained further in Appendix A.

1063 Using this approach, security strength is not described by a single number, such as “128 bits of
 1064 security.” Instead, each ML-KEM parameter set is claimed to be at least as secure as a generic

1065 block cipher with a prescribed key size or a generic hash function with a prescribed output
1066 length. More precisely, it is claimed that the computational resources needed to break ML-KEM
1067 are greater than or equal to the computational resources needed to break the block cipher or
1068 hash function, when these computational resources are estimated using any realistic model of
1069 computation. Different models of computation can be more or less realistic and, accordingly,
1070 lead to more or less accurate estimates of security strength. Some commonly studied models are
1071 discussed in [19].

1072 Concretely, ML-KEM-512 is claimed to be in security category 1, ML-KEM-768 is claimed
1073 to be in security category 3, and ML-KEM-1024 is claimed to be in security category 5. For
1074 additional discussion of the security strength of MLWE-based cryptosystems, see [4].

1075

1076 **Selecting an appropriate parameter set.** When initially establishing cryptographic protections
1077 for data, the strongest possible parameter set **should** be used. This has a number of advantages,
1078 including reducing the likelihood of costly transitions to higher-security parameter sets in the
1079 future. At the same time, it should be noted that some parameter sets might have adverse
1080 performance effects for the relevant application (e.g., the algorithm may be unacceptably slow).

1081 NIST recommends using ML-KEM-768 as the default parameter set, as it provides a large
1082 security margin at a reasonable performance cost. In cases where this is impractical or where
1083 even higher security is required, other parameter sets may be used.

1084 **References**

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1159 **Appendix A — Security Strength Categories**

1160 NIST understands that there are significant uncertainties in estimating the security strengths of
1161 post-quantum cryptosystems. These uncertainties come from two sources: first, the possibility
1162 that new quantum algorithms will be discovered, leading to new cryptanalytic attacks; and second,
1163 our limited ability to predict the performance characteristics of future quantum computers, such
1164 as their cost, speed, and memory size.

1165 In order to address these uncertainties, NIST proposed the following approach in its original Call
1166 for Proposals [18]. Instead of defining the strength of an algorithm using precise estimates of
1167 the number of “bits of security,” NIST defined a collection of broad security strength categories.
1168 Each category is defined by a comparatively easy-to-analyze reference primitive, whose security
1169 will serve as a floor for a wide variety of metrics that NIST deems potentially relevant to practical
1170 security. A given cryptosystem may be instantiated using different parameter sets in order to fit
1171 into different categories. The goals of this classification are:

- 1172 • To facilitate meaningful performance comparisons between various post-quantum algo-
1173 rithms by ensuring – insofar as possible – that the parameter sets being compared provide
1174 comparable security
- 1175 • To allow NIST to make prudent future decisions regarding when to transition to longer keys
- 1176 • To help submitters make consistent and sensible choices regarding what symmetric prim-
1177 itives to use in padding mechanisms or other components of their schemes that require
1178 symmetric cryptography
- 1179 • To better understand the security/performance trade-offs involved in a given design approach

1180 In accordance with the second and third goals above, NIST based its classification on the range
1181 of security strengths offered by the existing NIST standards in symmetric cryptography, which
1182 NIST expects to offer significant resistance to quantum cryptanalysis. In particular, NIST defined
1183 a separate category for each of the following security requirements (listed in order of increasing
1184 strength):

- 1185 1. Any attack that breaks the relevant security definition must require computational resources
1186 comparable to or greater than those required for key search on a block cipher with a 128-bit
1187 key (e.g., AES-128) .
- 1188 2. Any attack that breaks the relevant security definition must require computational resources
1189 comparable to or greater than those required for collision search on a 256-bit hash function
1190 (e.g., SHA-256/ SHA3-256).
- 1191 3. Any attack that breaks the relevant security definition must require computational resources
1192 comparable to or greater than those required for key search on a block cipher with a 192-bit
1193 key (e.g., AES-192).
- 1194 4. Any attack that breaks the relevant security definition must require computational resources
1195 comparable to or greater than those required for collision search on a 384-bit hash function
1196 (e.g., SHA-384/ SHA3-384).
- 1197 5. Any attack that breaks the relevant security definition must require computational resources

1198 comparable to or greater than those required for key search on a block cipher with a 256-bit
 1199 key (e.g., AES-256).

Table 4. NIST Security Strength Categories

Security Category	Corresponding Attack Type	Example
1	Key search on block cipher with 128-bit key	AES-128
2	Collision search on 256-bit hash function	SHA3-256
3	Key search on block cipher with 192-bit key	AES-192
4	Collision search on 384-bit hash function	SHA3-384
5	Key search on block cipher with 256-bit key	AES-256

1200 Here, computational resources may be measured using a variety of different metrics (e.g., number
 1201 of classical elementary operations, quantum circuit size). In order for a cryptosystem to satisfy one
 1202 of the above security requirements, any attack must require computational resources comparable
 1203 to or greater than the stated threshold, with respect to all metrics that NIST deems to be potentially
 1204 relevant to practical security.

1205 NIST intends to consider a variety of possible metrics, reflecting different predictions about the
 1206 future development of quantum and classical computing technology, and the cost of different
 1207 computing resources (such as the cost of accessing extremely large amounts of memory).² NIST
 1208 will also consider input from the cryptographic community regarding this question.

1209 In an example metric provided to submitters, NIST suggested an approach where quantum attacks
 1210 are restricted to a fixed running time or circuit depth. Call this parameter MAXDEPTH. This
 1211 restriction is motivated by the difficulty of running extremely long serial computations. Plausible
 1212 values for MAXDEPTH range from 2^{40} logical gates (the approximate number of gates that
 1213 presently envisioned quantum computing architectures are expected to serially perform in a year)
 1214 through 2^{64} logical gates (the approximate number of gates that current classical computing
 1215 architectures can perform serially in a decade), to no more than 2^{96} logical gates (the approximate
 1216 number of gates that atomic scale qubits with speed of light propagation times could perform in a
 1217 millennium). The most basic version of this cost metric ignores costs associated with physically
 1218 moving bits or qubits so they are physically close enough to perform gate operations. This
 1219 simplification may result in an underestimate of the cost of implementing memory-intensive
 1220 computations on real hardware.

1221 The complexity of quantum attacks can then be measured in terms of circuit size. These numbers
 1222 can be compared to the resources required to break AES and SHA-3. During the post-quantum
 1223 standardization process, NIST gave the following estimates for the classical and quantum gate
 1224 counts³ for the optimal key recovery and collision attacks on AES and SHA-3, respectively, where

²See the discussion in [19, Appendix B].

³Quantum circuit sizes are based on the work in [20].

1225 circuit depth is limited to MAXDEPTH]⁴.

Table 5. Estimates for classical and quantum gate counts for the optimal key recovery and collision attacks on AES and SHA-3

AES-128	$2^{157}/\text{MAXDEPTH}$ quantum gates or 2^{143} classical gates
SHA3-256	2^{146} classical gates
AES-192	$2^{221}/\text{MAXDEPTH}$ quantum gates or 2^{207} classical gates
SHA3-384	2^{210} classical gates
AES-256	$2^{285}/\text{MAXDEPTH}$ quantum gates or 2^{272} classical gates
SHA3-512	2^{274} classical gates

1226 It is worth noting that the security categories based on these reference primitives provide substan-
 1227 tially more quantum security than a naïve analysis might suggest. For example, categories 1, 3,
 1228 and 5 are defined in terms of block ciphers, which can be broken using Grover’s algorithm [21],
 1229 with a quadratic quantum speedup. However, Grover’s algorithm requires a long-running serial
 1230 computation, which is difficult to implement in practice. In a realistic attack, one has to run many
 1231 smaller instances of the algorithm in parallel, which makes the quantum speedup less dramatic.

1232 Finally, for attacks that use a combination of classical and quantum computation, one may
 1233 use a cost metric that rates logical quantum gates as being several orders of magnitude more
 1234 expensive than classical gates. Presently envisioned quantum computing architectures typically
 1235 indicate that the cost per quantum gate could be billions or trillions of times the cost per classical
 1236 gate. However, especially when considering algorithms claiming a high security strength (e.g.,
 1237 equivalent to AES-256 or SHA-384), it is likely prudent to consider the possibility that this
 1238 disparity will narrow significantly or even be eliminated.

⁴NIST believes the above estimates are accurate for the majority of values of MAXDEPTH that are relevant to its security analysis, but the above estimates may understate the security of SHA for very small values of MAXDEPTH and may understate the quantum security of AES for very large values of MAXDEPTH.