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# DENSITY AND THERMAL EXPANSION OF AMMERICAN PETROLEUM OILS 

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## I. INTRODUCTION

## 1. OBJECT OF INVESTIGATION

The work presented in this paper was undertaken for the purpose of securing data from which to calculate standard density and volumetric tables for American petroleum oils. The data have been secured and the tables prepared and published as Circular No. 57 of this Bureau. These tables are intended to be applicable to all petroleum oils, both crude and refined, produced in the United States. They cover a density range of 0.620 to 0.950 , and a temperature range of $30^{\circ} \mathrm{F}$ to $120^{\circ} \mathrm{F}$. In addition to these, a special table for heavy lubricating and fuel oils has been prepared with a temperature range up to $210^{\circ} \mathrm{F}$.

## II. MATERIAL USED

The material used in the investigation here reported was for the most part supplied by producers and refiners of oil in various parts of the United States. Oil samples were received from the States of Pennsylvania, New York, Ohio, Louisiana, Texas, Oklahoma, Kansas, Indiana, and California.

A part of the samples of lubricating oil were of unknown origin, having been submitted to the Bureau for test in connection with the fulfilment of contracts with the Government for lubricating oils.

It is assumed that the samples examined fairly represent the commercial petroleum oils produced in the United States. As the object of the investigation was to determine the rate of expansion of commercial petroleum oils, no special precautions were taken to insure more than ordinary purity in the samples collected.

## III. METHODS OF MEASUREMENT EMPLOYED

In making the density determinations on the oil samples two methods were employed: (a) The method of hydrostatic weighing; (b) the picnometer method.

In making use of the first method a sinker or plummet of known mass and volume is weighed in the sample of oil whose density is to be measured, and the density is calculated from the known volume and the difference between the weight of the sinker in vacuo and when immersed in the oil.

By the second method the weight of a known volume of the oil in question is determined, and the density calculated in the usual way. Details of the calculation will be given at a later point in this paper.

## IV. APPARATUS USED

The greater part of the apparatus used in this investigation has been previously described in publications ${ }^{1}$ of the Bureau and need not be described in great detail here. The essential features of the apparatus used in the method of hydrostatic weighing are shown in Figs. I and 2. The picnometer is shown in Fig. 3.

The sinker employed has a mass of 99.9630 g , and the following volumes at the temperatures indicated:

TABLE 1
Volume of Sinker No. 7

| Tempera- <br> ture in <br> degrees <br> centigrade | Volume <br> in milli- <br> liters |
| :---: | :---: |
| 0 | 47.6882 |
| 10 | 47.6998 |
| 20 | 47.7113 |
| 25 | 47.7170 |
| 30 | 47.7227 |
| 40 | 47.7339 |
| 50 | 47.7450 |

The picnometers used have the following internal volumes at the temperatures indicated:

TABLE 2

| Tempera- <br> ture in <br> degrees <br> centigrade | Internal volume in <br> milliliters |  |
| :---: | :---: | :---: |
|  | No.1 | No.2 |
| 0 | 108.3803 | 109.0834 |
| 25 | 108.4543 | 109.1545 |
| 50 | 108.5378 | 109.2335 |
| 75 | 108.6283 | 109.3216 |
| 95 | 108.6906 | 109.3954 |

The picnometers have the following external volumes at $20^{\circ}$ C: No. I, 156.04 Iml ; No. 2 , 155.592 ml . The external volume is used only in calculating the correction for air buoyancy and need not be known with great accuracy.

The temperature control bath (Fig. r) is so arranged that either the densimeter tube $H$ (Fig. 2) or the picnometer (Fig. 3) may be used. By means of an electric heating coil and a refrigerating brine coil any desired temperature between $0^{\circ}$ and $50^{\circ} \mathrm{C}$ may be secured and automatically maintained within the bath.

The temperature of the bath is observed by means of mercury thermometers suspended in the bath parallel to the picnometer or densimeter tube, as shown in Fig. I. The thermometers are subdivided to 0.1 C , and by means of a long-focus microscope are read to o.or C.

The thermometers used are well aged and have been frequently calibrated, and when used repeatedly over the same temperature
range in the same regular order and with occasional determinations of the ice point, the temperature observations are very consistent and are believed to be reliable to oor or 0.02 C . The temperatures were nearly always read with a stationary or slowly rising meniscus, as a falling meniscus is known to be unsteady and unreliable.

## V. CALIBRATION OF APPARATUS

The density sinker and the picnometers were calibrated by the use of pure, air-free, twice-distilled water, assuming Chappuis's ${ }^{2}$ values for the density of water to be correct. Calibrations were made at each temperature at which densities were to be determined.

Throughout this paper all densities are expressed in grams per milliliter and all weights are reduced to vacuo. The densities are, therefore, in all cases numerically the same as true specific gravities at the various temperatures referred to water at $4^{\circ} \mathrm{C}$ as unity.

## VI. TEMPERATURE RANGE OF DENSITY DETERMINATIONS

Density determinations were made on most of the samples at the following temperatures: $0^{\circ}, 10^{\circ}, 20^{\circ}, 25^{\circ}, 30^{\circ}, 40^{\circ}$, and $50^{\circ} \mathrm{C}$. On a few samples determinations were not made at the lower temperatures, while on others the temperatures were carried up to $75^{\circ}, 85^{\circ}$, and $95^{\circ} \mathrm{C}$.

## VII. METHOD OF PROCEDURE

## 1. BY THE METHOD OF HYDROSTATIC WEIGHING

The oil sample whose density is to be measured is placed in the densimeter tube $H$ with the sinker $E$ immersed in it (Fig. I) and the tube secured in position in the temperature-control bath. The temperature of the bath is then brought to the point at which the first density determination is to be made and is allowed to remain constant until the apparatus reaches a condition of temperature equilibrium. After about 20 minutes at the constant temperature observations are begun. First, a weighing is made with the sinker $E$ immersed in the oil sample and suspended from the arm of a balance. The temperature is then read on each of two thermometers suspended in the tube $L$, which is immersed in the same bath and close to the densimeter tube $H$. Next, a weighing is made with the sinker $E$ detached from the suspension and resting on the botton of the tube $H$. Then, a second weighing is

[^0]

Fig. 1.-Densimeter tube, sinker and thermometers in temperature-control bath

Fig. 2.-Densimeter tube and sinker
made with the sinker $E$ suspended; and, finally, the temperature is again read on the two thermometers.

By means of the small sinker $F$ the suspension wire is kept in position and passing through the surface of the oil at all times, both when the large sinker $E$ is suspended and when it is detached. In this way the effect of surface tension on the suspension wire is eliminated. The observations at each temperature, as outlined above, consist of two weighings with the sinker attached, one weighing with it detached, and two readings on each of two thermometers. The reason for making two weighings with the sinker attached and only one with it detached is because in the former case a slight change in the temperature of the oil makes an appreciable change in the apparent weight of the sinker on account of its large volume, while in the latter case the change is not appreciable.

After completing the observations at one point the temperature of the bath is changed to the next in the series and the process repeated in the same order.

## 2. BY THE PICNOMETER METHOD

The method of hydrostatic weighing above described is applicable only to such oils as are of sufficient fluidity to allow the sinker to readily take up a position of static equilibrium when suspended in the oil. With the more viscous oils the sensibility of the balance is greatly reduced and the weighings become more difficult to make, and of doubtful accuracy. For such oils it is therefore necessary, or at least desirable, to use some other method. The method usually resorted to is that of the picnometer or specific-gravity bottle.

For the work here described, special picnometers were designed and constructed somewhat similar to those previously used in alcoholometric determinations. ${ }^{3}$ The essential features are the tube $E$ (Fig. 3) extending nearly to the lower end of the picnometer, the tube $D$ extending up through the bottom of the reservoir $I$, the funnel $G$ and the attachment $F$ provided with a stopcock.

In filling the picnometer the oil is placed in the funnel $G$ and drawn in through $E$ by exhausting the air through $F$. By this means much time is saved in filling the picnometer and the method is equally efficacious in emptying and cleaning the picnometer when it is desired to introduce a new sample.

When the filling is completed $F$ and $G$ are replaced by the caps $A$ and $B$, all parts being provided with well-fitting ground joints. The picnometer having been filled, it is placed in the
temperature-control bath in place of the densimeter tube and the temperature brought to the desired point as before. The quantity of oil in the picnometer is so adjusted that when temperature equilibrium has been established the oil surface is just flush with the tip of the capillary tubes $C$ and $D$. The excess oil in the reservoir $I$ is removed and the interior of the reservoir


Fig. 3.-Special picnometer
carefully cleaned by means of a pipette, filter paper, and gasoline. The temperature is then observed and the picnometer removed from the bath, dried on the outside, allowed to come to room temperature, and then weighed.

The density of the oil at each temperature is calculated by dividing the mass by the volume; that is, by the internal volume of the picnometer at that temperature.

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VIII. SAMPLE RECORDS OF OBSERVATIONS AND CALCULATION OF DENSITY
[Sample, Texaco spirits from Oklahoma crude. Date, Mar. 16, 1914. Observer, E. L. P.]

| Time | Observed temperature |  | Corrections to thermometers |  | Corrected temperature |  | Mean temperature $\mathbf{t}^{\prime}$ | Balance reading | Apparent weight of sinker in liquid | Buoyancy correction on weights | True weight of sinker in liquid | Weight of displaced liquid | Volume of sinker at tem- ature $t^{\prime}$ | Density of liquid at $\mathrm{t}^{\prime} / 4^{\circ} \mathrm{C}$ | Correction to integral degrees | Density of liquid at $\mathrm{t} / 4^{\circ} \mathrm{C}$ | Integral temperature $t$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10.15 a.m.. | $\begin{array}{r} \text { Haak } \\ \text { No. } 2040 \\ 24.93 \\ 24.93 \end{array}$ | $\begin{array}{r} \text { Homann } \\ \text { No. } 9552 \\ 24.86 \\ 24.86 \end{array}$ | $\begin{gathered} { }^{\circ} \mathrm{C} \\ -0.01 \end{gathered}$ | $\begin{gathered} { }^{\circ} \mathrm{C} \\ +0.05 \end{gathered}$ | $\begin{aligned} & { }^{\circ} \mathrm{C} \\ & 24.92 \\ & 24.92 \end{aligned}$ | $\begin{aligned} & { }^{\circ} \mathbf{C} \\ & 24.91 \\ & 24.91 \end{aligned}$ | ${ }^{\circ} \mathrm{C}$ | $\begin{gathered} \mathrm{g} \\ 20.1770 \\ 83.1187 \\ 20.1773 \end{gathered}$ | $\begin{gathered} \mathbf{g} \\ 62.9417 \\ 62.9414 \end{gathered}$ | g | g | g | m1 | $\mathrm{g} / \mathrm{ml}$ |  | $\mathrm{g} / \mathrm{ml}$ | ${ }^{\circ} \mathrm{C}$ |
|  |  |  |  |  |  |  | 24.92 |  | 62.9416 | $-0.0089$ | 62.9327 | 37.0303 | 47.71690 | 0.77604 | -0.00006 | 0.77598 | 25 |
| $11.40 \mathrm{a} . \mathrm{mi}.$. | $\begin{aligned} & .08 \\ & .10 \end{aligned}$ | $\begin{aligned} & .00 \\ & .02 \end{aligned}$ | $-.10$ | $-.01$ | $-.02$ | $\begin{aligned} & -.01 \\ & +.01 \end{aligned}$ |  | 21. 0888 83.1508 | $\begin{aligned} & 62.0620 \\ & 62.0628 \end{aligned}$ |  |  |  |  |  |  |  |  |
|  |  |  |  |  | . 00 |  | . 00 |  | 62.0624 | $-.0087$ | 62.0537 | 37.9093 | 47.68816 | . 79494 | . 00000 | . 79494 | 0 |
| 12.30 p.m.. | 10.08 | $\begin{aligned} & 10.03 \\ & 10.07 \end{aligned}$ | $-.01$ | $+.04$ | $\begin{aligned} & 10.07 \\ & 10.11 \end{aligned}$ | $\begin{aligned} & 10.07 \\ & 10.11 \end{aligned}$ |  | $\begin{array}{r} 20.7200 \\ 83.1379 \end{array}$ | $\begin{array}{r}62.4179 \\ 62.4191 \\ \hline\end{array}$ |  |  |  |  |  |  |  |  |
|  | 10.12 |  |  |  |  |  | 10.09 |  | 62.4185 | $-.0088$ | 62.4097 | 37.5533 | 47.69990 | . 78728. | $+.00007$ | . 78735 | 10 |
| $1.30 \mathrm{p} . \mathrm{m} . .$. | $\begin{aligned} & 20.01 \\ & 20,02 \end{aligned}$ | $\begin{aligned} & 19.95 \\ & 19.95 \end{aligned}$ | . 00 | +. 06 | $\begin{aligned} & 20.01 \\ & 20.02 \end{aligned}$ | $\begin{aligned} & 20.01 \\ & 20.01 \end{aligned}$ |  | $\begin{aligned} & 20.3571 \\ & 83.1249 \end{aligned}$ | $\begin{aligned} & 62.7678 \\ & 62.7680 \end{aligned}$ |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | 20.01 |  | 62.7679 | $-.0088$ | 62.7591 | 37. 2039 | 47.71130 | . 77977 | +. 00001 | . 77978 | 20 |
| 2.15 p. m... | $\begin{aligned} & 24.84 \\ & 24.85 \end{aligned}$ | $\begin{aligned} & 24.78 \\ & 24.78 \end{aligned}$ | $-.01$ | +. 05 | $\begin{aligned} & 24.83 \\ & 24.84 \end{aligned}$ | $\begin{aligned} & 24.83 \\ & 24.83 \end{aligned}$ |  | 20. 1808 <br> 83.1190 <br> 20. 1810 | $\begin{aligned} & 62.9382 \\ & 62.9380 \end{aligned}$ |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | 24.83 |  | 62.9381 | $-.0089$ | 62.9292 | 37.0338 | 47.71680 | . 77612 | -. 00013 | . 77599 | 25 |
| 2.55 p. m... | 29.98 | $\begin{aligned} & 29.94 \\ & 29.95 \end{aligned}$ | . 00 | $+.01$ | $\begin{aligned} & 29.98 \\ & 29.99 \end{aligned}$ | $29.95$ |  | 19. 9914 83. 1124 <br> 19. 9914 | $\begin{aligned} & \text { 63. } 1210 \\ & 63.1210 \end{aligned}$ |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  | 29.96 | 29.97 |  | 63.1210 | $-.0089$ | 63.1121 | 36.8509 | 47.72264 | . 77219 | $-.00002$ | . 77217 | 30 |
| 3.40 p. m... | $\begin{array}{r} \text { No. } 264 \\ 39.94 \\ 39.94 \end{array}$ | $\begin{aligned} & 39.90 \\ & 39.89 \end{aligned}$ | -. 03 | -. 01 | $\begin{aligned} & 39.91 \\ & 39.91 \end{aligned}$ | $\begin{aligned} & 39.89 \\ & 39.88 \end{aligned}$ |  | 19.6252 83.0997 19 | $\begin{aligned} & \text { 63. } 4745 \\ & 63.4743 \end{aligned}$ |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | 39.90 | 1.6254 | 63.4744 | -. 0089 | 63.4655 | 36. 4975 | 47.73380 | . 76461 | -. 00008 | . 76453 | 40 |


Buoyancy constant at $25^{\circ}$ and $50^{\circ}=1043$; air density at $25^{\circ}$ and $50^{\circ}=0.001183$.
Buoyancy constant at $75^{\circ}$ and $95^{\circ}=1055$; air density at $75^{\circ}$ and $95^{\circ}=0.001197$.

## IX. CALCULATION OF RESULTS

Having made the weighings and observed the temperatures at each of the several points at which the density is to be determined, the calculation of density is carried out by means of the following equations:
(I) By the method of hydrostatic weighing:

$$
D_{t}=\frac{S-w\left(\mathrm{I}-\frac{\rho}{8.4}\right)}{V_{\mathrm{t}}}=\frac{M}{\overline{V_{t}}}
$$

$D_{\mathrm{t}}=$ density of oil at the temperature $t$
$S=$ mass of sinker
$w=$ apparent mass of sinker in oil at temperature $t$
$\rho=$ density of air
$8.4=$ density of brass weights
$V_{t}=$ volume of sinker at temperature $t$
$M=$ mass of oil displaced by sinker
(2) By picnometer:

$$
D_{\mathrm{t}}=\frac{w\left(\mathrm{I}-\frac{\rho}{8.4}\right)+\rho v-P}{V_{\mathrm{t}}}=\frac{M}{V_{\mathrm{t}}}
$$

$D_{t}=$ density of oil at the temperature $t$
$w=$ apparent mass of picnometer filled with oil at temperature $t$
$\rho=$ density of air
$8.4=$ density of brass weights
$v=$ external volume of picnometer
$P=$ mass of empty picnometer
$V_{\mathrm{t}}=$ internal volume of picnometer at temperature $t$
$M=$ mass of oil contained in picnometer.

## X. REDUCTION OF OBSERVATIONS

Having determined the density of the individual samples at the several temperatures, the rate of change of density with change of temperature was calculated by the application of the method of least squares to the density determinations made on each sample.

It is assumed that the expansion of any sample may be represented by an equation having the form,

$$
D_{\mathrm{t}}=D_{\mathrm{T}}+\alpha(t-T)+\beta(t-T)^{2}
$$

in which,
$D_{\mathrm{t}}=$ density at any temperature $t$ $D_{\mathrm{T}}=$ density at the standard temperature $T$
$\alpha$ and $\beta$ are constant coefficients to be determined for each sample.

The various steps in the operation of working out the values of a, $\beta, D_{\mathrm{T}}$, and the most probable values of $D_{\mathrm{t}}$ for an average sample of oil are shown in Tables 5 and 6 . From the closeness of the agreement between the observed and the calculated values of $D_{t}$, it is seen that the assumed equation can not be much in error.

A similar reduction of the observations has been made for each sample; this work, however, will not be given here in detail.

TABLE 5
Sample of Reduction of Observations

| t | $\mathrm{C}_{1}$ | $\mathrm{C}_{1}{ }^{2}$ | $\mathrm{C}_{2}$ | $\mathrm{C}_{1} \mathrm{C}_{2}$ | $\mathrm{C}^{2}$ | D. | N | $\mathrm{C}_{1} \mathrm{~N}$ | $\mathrm{C}_{2} \mathrm{~N}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | -25 | 625 | 375 | -9375 | 140625 | 0.79494 | +0.01900 | -0.47500 | +7.12500 |
| 10 | -15 | 225 | - 25 | + 375 | 625 | . 78735 | +. 01141 | -. 17115 | -. 28525 |
| 20 | - 5 | 25 | -225 | +1125 | 50625 | . 77978 | +. 00384 | - . 01920 | -. 86400 |
| 25 | 0 | 0 | -250 | 0 | 62500 | . 77599 | +. 00005 | . 00000 | -. 01250 |
| 30 | 5 | 25 | -225 | -1125 | 50625 | . 77217 | -. 00377 | -. 01885 | +. 84825 |
| 40 | 15 | 225 | - 25 | - 375 | 625 | . 76453 | -. 01141 | -. 17115 | $+.28525$ |
| 50 | 25 | 625 | 375 | +9375 | 140625 | . 75683 | -. 01911 | -. 47775 | -7.16625 |
|  |  | 7)1750 |  | 0 | 446250 | . 77594 |  | $-1.33310$ | -. 06950 |

The normal equations are:

$$
\begin{aligned}
& \Sigma C_{1}{ }^{2} a+\Sigma C_{1} C_{2} \beta=\Sigma C_{1} N \\
& \Sigma C_{1} C_{2} a+\Sigma C_{2}{ }^{2} \beta=\Sigma C_{2} N \\
& \left.X_{\mathrm{m}}+\frac{\Sigma C_{1}{ }^{2} \beta}{n} \quad D_{\mathrm{t}}\right]_{\mathrm{m}}
\end{aligned}
$$

in which

$$
\begin{array}{rlrl}
C_{1} & =t-t_{\mathrm{m}} & & \left(t_{\mathrm{m}}=\text { mean temperature }\right) \\
C_{2} & =C_{1}{ }^{2}-\left[C_{1}{ }^{2}\right]_{\mathrm{m}} & & \left(\left[C_{1}{ }^{2}\right]_{\mathrm{m}}=\text { mean } C_{1}{ }^{2}\right) \\
N & =D_{\mathrm{t}}-\left[D_{\mathrm{t}}\right]_{\mathrm{m}} & & \left(\left[D_{\mathrm{t}}\right]_{\mathrm{m}}=\text { mean } D_{\mathrm{t}}\right) \\
n & =\text { number of measured temperatures in series }=7 \\
X_{\mathrm{m}} & =\text { density at mean temperature }=D_{25} \\
D_{\mathrm{t}} & =\text { density at temperature } t
\end{array}
$$

By solving the above normal equations, the following values of $a, \beta$, and $D_{25}$ are obtained:

$$
\begin{aligned}
a & =-0.000762 \\
\beta & =-0.00000016 \\
D_{25} & =0.77598
\end{aligned}
$$

These values when substituted in the general equation give the density values shown in Table 6:

$$
\begin{aligned}
D_{\mathrm{t}} & =D_{25}+\alpha(t-25)+\beta(t-25)^{2} \\
& =0.77598-0.000762(t-25)-0.00000016(t-25)^{2}
\end{aligned}
$$

TABLE 6

| t | t-25 | $(t-25)^{2}$ | $\alpha(t-25)$ | $\beta(t-25)^{2}$ | $\underset{\text { (calculated) }}{\mathrm{D}_{t}}$ | $\underset{\text { (observed) }}{D_{t}}$ | Difference (obs.-cal.) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | -25 | 625 | +0.01904 | -0.00010 | 0.79492 | 0.79494 | 2 |
| 10 | -15 | 225 | +.01143 | -. 00004 | . 78737 | . 78735 | -2 |
| 20 | - 5 | 25 | + .00381 | . 00000 | . 77979 | . 77978 | -1 |
| 25 | 0 | 0 | . 00000 | . 00000 | . 77598 | . 77599 | 1 |
| 30 | 5 | 25 | -. 00381 | . 00000 | . 77217 | . 77217 | 0 |
| 40 | 15 | 225 | -. 01143 | -. 00004 | . 76451 | . 76453 | 2 |
| 50 | 25 | 625 | -. 01904 | -. 00010 | . 75684 | . 75683 | -1 |

The above reductions are for a sample of "Texaco spirits" from Oklahoma crude.

The density determination made on the individual samples, and the calculated thermal density coefficient for each sample are shown in the following pages. The samples in each group are arranged in increasing order of their densities at $25^{\circ} \mathrm{C}$.

## XI. RESULTS IN DETAIL

## TABLE 7

Refined Oils

| Locality produced | Nature of oil | Density (g/ml) at- |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $0^{\circ}$ | $10^{\circ}$ | $20^{\circ}$ | $25^{\circ}$ | $30^{\circ}$ | $40^{\circ}$ | $50^{\circ}$ | $\alpha \times 10^{5}$ | $\beta \times 10^{7}$ |
|  | Pentane...... | 0.6388 | ...... | .... | 0.6131 |  |  |  | 103 | ........ |
|  | . .do. | . 6410 |  | ..... | . 6164 | ..... |  |  | 98 | ........ |
|  | .do. | . 6564 |  |  | . 6321 |  |  |  | 97 | ........ |
|  | . do. | . 6562 |  | ...... | . 6324 |  |  |  | 95 | ........ |
|  | Naphtha. | . 7002 | 0. 6914 | 0.6825 | . 6781 | 0. 6736 | 0.6647 | 0.6558 | 89 | - 1 |
|  | Gasoline. | . 7193 | . 7107 | . 7020 | . 6975 | . 6933 | . 6845 | . 6757 | 87 | - 2 |
|  | Naphtha. | . 7237 | . 7150 | . 7062 | . 7019 | . 6975 | . 6887 | . 6798 | 88 | -2 |
| Tezas. | Gasoline. | . 7506 | . 7422 | . 7339 | . 7296 | . 7256 | . 7172 | . 7087 | 84 | 1 |
| Pennsylvania. | ...do. | . 7515 | . 7433 | . 7351 | . 7310 | . 7270 | . 7188 | . 7105 | 82 | - 1 |

TABLE 7-Continued
Refined Oils-Continued

| Locality produced | Nature of oil | Density ( $\mathrm{g} / \mathrm{ml}$ ) at- |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $0^{\circ}$ | $10^{\circ}$ | $20^{\circ}$ | $25^{\circ}$ | $30^{\circ}$ | $40^{\circ}$ | $50^{\circ}$ | $\alpha \times 10^{5}$ | $\beta \times 10^{7}$ |
| California. | Gasoline (treated) | . 7532 | . 7447 | . 7362 | . 7319 | . 7277 | . 7190 | . 7104 | 86 | 3 |
| Do. | Benzine (treated) | . 7641 | . 7556 | . 7473 | . 7431 | . 7389 | . 7305 | . 7220 | 84 |  |
| Indiana. | Naphtha. | . 7657 | . 7575 | . 7493 | . 7453 | . 7411 | . 7328 | . 7244 | 82 |  |
| California | Engine distillate.. | . 7879 | . 7799 | . 7719 | . 7679 | . 7639 | . 7559 | . 7478 | 80 |  |
| Oklahorna | Gasoline. | . 7949 | . 7874 | . 7798 | . 7760 | . 7722 | . 7645 | . 7568 | 76 | 2 |
| Pennsylvania. | Kerosene. | . 7981 | . 7908 | . 7835 | . 7799 | . 7762 | . 7689 | . 7615 | 73 |  |
| Do. | .do. | . 7990 | . 7918 | . 7845 | . 7809 | . 7772 | . 7700 | . 7627 | 73 |  |
| Do | do | . 7994 | . 7921 | . 7848 | . 7812 | . 7775 | . 7702 | . 7629 | 73 |  |
| Louisiana | Lighthouse | . 8017 | . 7944 | . 7872 | . 7835 | . 7799 | . 7726 | . 7650 | 73 |  |
| Pennsylvania. | Kerosene. | . 8040 | . 7968 | . 7896 | . 7860 | . 7824 | . 7753 | . 7681 | 72 |  |
| Do. | do | . 8054 | . 7982 | . 7910 | . 7874 | . 7838 | . 7766 | . 7694 | 72 |  |
| Ohio. | . do | . 8096 | . 8023 | . 7949 | . 7913 | . 7876 | . 7802 | . 7729 | 73 |  |
| Indiana | Refined | . 8128 | . 8054 | . 7979 | . 7942 | . 7905 | . 7830 | . 7756 | 74 |  |
| Oklahoma | Kerosene | . 8136 | . 8062 | . 7989 | . 7953 | . 7916 | . 7842 | . 7769 | 73 |  |
| Mid-continen | . ${ }^{\text {do. }}$ | . 8177 | . 8104 | . 8030 | . 7994 | . 7957 | . 7883 | . 7809 | 74 |  |
| California. | .do. | . 8249 | . 8175 | . 8101 | . 8064 | . 8027 | . 7953 | . 7878 | 74 |  |
| Do. | do. | . 8301 | . 8228 | . 8155 | . 8119 | . 8082 | . 8009 | . 7935 | 73 |  |
|  | Burning oil (high F.T.) | . 8389 |  |  | . 8220 |  |  | . 8053 | 67 |  |
| Louisiana. | Mineral Seal.... | . 8390 | . 8322 | . 8254 | . 8221 | . 8187 | . 8119 | . 8052 | 68 |  |
| Pennsylvania. | Refined. | . 8573 | . 8507 | . 8440 | . 8407 | . 8374 | . 8308 | . 8243 | 66 | + |
| Indiana.......... | . do | . 8614 | . 8546 | . 8478 | . 8444 | . 8410 | . 8343 | . 8275 | 68 | + |
|  | Neutral | . 8646 |  |  | . 8481 |  |  | . 8317 | 66 |  |
|  | .....do. | . 8714 |  |  | . 8551 |  |  | . 8388 | 65 |  |
| California. | Stove oil (treated). | . 8763 | . 8692 | . 8620 | . 8585 | . 8550 | . 8479 | . 8408 | 71 | + |
| Pennsylvania. | Refined. |  | . 8703 | . 8538 | . 8606 | . 8574 | . 8510 | . 8447 | 64 | + |
| Do. | Dynamo oi | . 8800 | . 8736 | . 8672 | . 8639 | . 8607 | . 8542 | . 8478 | 64 |  |
| Do. | Refined. | . 8802 | . 8735 | . 8671 | . 8639 | . 8607 | . 8544 | . 8481 | 64 | + 3 |
| Louisiana. | ....do. | . 8832 |  |  | . 8661 |  |  | . 8498 | 67 |  |
|  | Neutral. | . 8834 |  |  | . 8672 |  |  | . 8512 | 65 |  |
| Pennsylvania..... | Refined. | . 8841 | . 8782 | . 8718 | . 8688 | . 8657 | . 8594 | . 8531 | 62 | -3 |
|  | Marine engine... | . 8938 |  |  | . 8766 |  |  | . 8609 | 66 |  |
|  | Gas engine. | . 8974 |  |  | . 8810 |  |  | . 8651 | 65 |  |
| Louisiana........ | Refined | . 9007 |  |  | . 8838 |  |  | . 8678 | 66 |  |
|  | Cylinder oil. | . 9108 |  |  | . 8910 |  |  | . 8730 | ${ }^{\text {a }} 76$ |  |
|  | Paraffin oil. | . 9118 |  |  | . 8947 |  |  | . 8786 | 66 |  |
| Indiana........... | Refined | . 9111 | . 9045 | . 8980 | . 8948 | . 8915 | . 8850 | . 8786 | 65 | +1 |
|  | Engine oil. | . 9124 | . 9053 | . 8988 | . 8956 | . 8924 | . 8859 | . 8794 | a 66 | +14 |
|  | Cylinder oil. | . 9153 |  |  | . 8971 |  |  | . 8803 | 70 |  |
|  | Paraffin oil. | . 9205 |  |  | . 9036 |  |  | . 8876 | 66 |  |
|  | Cylinder oil. | . 9285 |  |  | . 9086 |  |  | . 8912 | a 75 |  |
| Texas. | Refined | . 9385 | . 9319 | . 9252 | . 9219 | . 9186 | . 9119 | . 9054 | 66 | -1 |
| Indiana. | ..do | . 9421 |  |  | . 9221 |  |  | . 9052 | a 74 |  |
|  | Gas engine oil. | . 9470 |  |  | . 9306 |  |  | . 9145 | 65 |  |
| Texas. | Refined | . 9497 |  |  | . 9333 |  |  | . 9173 | 65 |  |
| Do. | . do | . 9551 |  |  | . 9389 |  |  | . 9230 | 64 |  |

$a$ These samples probably contained solid particles at the low temperatures.

## TABLE 8

## Crude Oils

| Locality produced | $\underset{\mathrm{g} / \mathrm{ml}}{\mathrm{D}_{0}}$ | $\begin{gathered} \alpha^{1} \\ 0^{\circ}-25^{\circ} \end{gathered}$ | $\underset{\mathrm{g} / \mathrm{ml}}{\mathrm{D}_{25}}$ | $25^{\alpha^{1}}-50^{\circ}$ | $\underset{\mathrm{g} / \mathrm{ml}}{\mathrm{D}_{50}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Pennsylvania... | 0.8253 | 0.00074 | 0.8067 | 0.00070 | 0.7892 |
| Louisiana. | . 8309 | . 00076 | . 8118 | . 00071 | . 7940 |
| Texas. | . 8425 | . 00074 | . 8241 | . 00072 | . 8061 |
| Pennsylvania.. | . 8432 | . 00072 | . 8251 | . 00069 | . 8078 |
| Louisiana.. | . 8526 | . 00065 | . 8363 | . 00065 | . 8200 |
| Ohio. | . 8648 | . 00076 | . 8459 | . 00068 | . 8288 |
| Mid-continent. | . 8726 | . 00067 | . 8558 |  |  |
| Oklahoma | . 8815 | . 00074 | . 8629 | . 00058 | . 8460 |
| California. | . 9082 | . 00070 | . 8908 | . 00070 | . 8734 |
| Do. | . 9162 | . 00067 | . 8995 | . 00067 | . 8828 |
| Louisiana.. | . 9193 | . 00069 | . 9021 | . 00068 | . 8850 |
| Texas. | . 9232 | . 00068 | . 9062 | . 00066 | . 8897 |
| California | . 9361 | . 00071 | . 9183 | . 00068 | . 9012 |
|  |  |  | $\underset{\mathbf{g} / \mathrm{ml}}{\mathbf{D}_{30}}$ | $\begin{gathered} \alpha^{1} \\ 30^{\circ}-40^{\circ} \end{gathered}$ | $\underset{\mathbf{g} / \mathrm{ml}}{\mathbf{D}_{40}}$ |
| Texas. |  |  | . 9180 | . 00071 | . 9109 |
| Do. |  |  | . 9194 | . 00066 | . 9128 |
| Do. |  |  | . 9296 | . 00067 | . 9229 |
| Do. |  |  | . 9396 | . 00068 | . 9328 |
| California. |  |  | . 9564 | . 00066 | . 9498 |

## TABLE 9

Fuel Oils and Heavy Lubricating Oils

| Nature of oil | $\begin{aligned} & \mathbf{D}_{25} \\ & \mathrm{~g} / \mathrm{ml} \end{aligned}$ | $25^{\alpha^{1}}-50^{\circ}$ | $\begin{aligned} & \mathbf{D}_{50} \\ & \mathrm{~g} / \mathrm{ml} \end{aligned}$ | $\begin{gathered} \alpha^{1} \\ 50^{\circ}-75^{\circ} \end{gathered}$ | $\begin{gathered} \mathbf{D}_{75} \\ \mathrm{~g} / \mathrm{ml} \end{gathered}$ | $75^{\alpha^{1}}-95^{\circ}$ | $\begin{gathered} \mathbf{D}_{9 \mathrm{~g}} \\ \mathrm{~g} / \mathrm{ml} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Autocylinder | 0.8620 | 0.00063 | 0.8462 | 0.00064 | 0. 8303 | 0.00063 | 0.8177 |
| Fuel. | . 8641 | . 00068 | . 8472 | . 00067 | . 8304 | . 00068 | a. 8168 |
| Autocylinder | . 8651 | . 00064 | . 8492 | . 00063 | . 8335 | . 00063 | . 8209 |
| Fuel. | . 8713 | . 00067 | . 8546 | . 00066 | . 8380 | . 00068 | a. 8144 |
| Gas engine | . 8809 | . 00063 | . 8652 | . 00062 | . 8496 | . 00062 | . 8373 |
| Locomotive | . 9003 | . 00068 | . 8833 | . 00062 | . 8678 | . 00062 | . 8555 |
| Noncondensing cylinde | . 9010 | . 00069 | . 8838 | . 00061 | . 8686 | . 00062 | . 8563 |
| Locomotive. | . 9140 | . 00068 | . 8971 | . 00063 | . 8814 | . 00064 | . 8687 |
| Marine engine | . 9171 | . 00065 | . 9009 | . 00064 | . 8848 | . 00064 | . 8720 |
| Gas engine. | . 9202 | . 00064 | . 9043 | . 00064 | . 8884 | . 00064 | . 8757 |
| Do | . 9204 | . 00064 | . 9044 | . 00064 | . 8885 | . 00064 | . 8758 |
| Stationary engine | . 9285 | . 00064 | . 9126 | . 00063 | . 8968 | . 00063 | . 8842 |
| Marine engine. | . 9386 | . 00064 | . 9225 | . 00064 | . 9065 | . 00064 | . 8936 |
| Fuel. | . 9526 | . 00065 | . 9363 | . 00066 | . 9199 | . 00066 | a. 9066 |
| Do | . 9537 | . 00065 | . 9374 | . 00065 | . 9212 | . 00065 | . 9082 |

$a$ Calculated from the density at $85^{\circ} \mathrm{C} . \alpha^{1}$ is the change of density per degree centigrade.

## XII. PLOT OF $\alpha$ AND $\beta$ AGAINST DENSITY AT $25^{\circ} \mathrm{C}$

Following the detailed results the values of $a$ and $\beta$ for each sample are shown graphically, these values having been plotted against the density of the sample at $25^{\circ} \mathrm{C}$.


Fig. 4.-Plot of $\alpha$ and $\beta$ against density
A smooth curve was drawn through the points so plotted and the average values of $\alpha$ and $\beta$ read from this curve (Fig. 4). These values, when transformed to the basis of specific gravity at $60^{\circ} / 60^{\circ} \mathrm{F}$ were used in calculating the expansion tables published in Circular No. 57 of this Bureau.

# XIII. TABULATED VALUES OF $\mathrm{D}_{25}, \alpha$, AND $\beta$ 

TABLE 10
Average Values of $\alpha$ and $\beta$ from Curves (Fig 4)

| $\underset{25^{\circ} \mathrm{C}}{\mathrm{C}}$ | $\alpha$ | $\beta$ | $\underset{25^{\circ}}{\text { D }}$ | $\alpha$ | $\beta$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.62.. | -0.00099 | -0.0000004 | 0.79.. | -0.00073 | 0.0000000 |
| 0.63.. | . 00097 | . 0000003 | 0.80 | . 00072 | . 0000000 |
| 0.64. | . 00095 | . 0000003 | 0.81 | . 00071 | . 0000000 |
| 0.65 | . 00094 | . 0000003 | 0.82 | . 00070 | . 0000000 |
| 0.66.. | . 00092 | . 0000003 | 0.83 | . 00070 | . 0000000 |
| 0.67 | . 00091 | . 0000003 | 0.84 | . 00069 | . 0000000 |
| 0.68 . | . 00090 | . 0000002 | 0.85. | . 00068 | +. 0000001 |
| 0.69.. | . 00088 | . 0000002 | 0.86 | . 00068 | . 0000001 |
| 0.70. | . 00087 | . 0000002 | 0.87 | . 00068 | . 0000001 |
| 0.71 | . 00086 | . 0000002 | 0.88 . | . 00067 | . 0000001 |
| 0.72.. | . 00084 | . 0000002 | 0.89 | . 00067 | . 0000002 |
| 0.73 | . 00083 | . 0000001 | 0.90 | . 00067 | . 0000002 |
| 0.74 | . 00081 | . 0000001 | 0.91 | . 00066 | . 0000002 |
| 0.75 | . 00080 | . 0000001 | 0.92 | . 00066 | . 0000002 |
| 0.76.. | . 00078 | . 0000001 | 0.93 | . 00066 | . 0000002 |
| 0.77.. | . 00077 | . 0000001 | 0.94 | . 00066 | . 0000002 |
| 0.78.. | . 00075 | . 0000000 | 0.95 | . 00066 | . 0000003 |

## XIV. CALCULATION OF STANDARD DENSITY AND VOLUMETRIC TABLES

Measurements of petroleum oils in the United States are usually made on the basis of $60^{\circ} \mathrm{F}$ as the standard temperature, and instead of density either the specific gravity at $60^{\circ} \mathrm{F}$, referred to water at $60^{\circ} \mathrm{F}$ as unity, or the degrees Baumé is ordinarily employed. The volume of the oil is also usually corrected to $60^{\circ} \mathrm{F}$. For that reason it is necessary to transform the general equation representing the expansion of petroleum oils in such a way that by its use the specific gravity at any temperature can be calculated from the specific gravity at $60^{\circ} / 60^{\circ} \mathrm{F}$ ( $15 \% 56$ / $15{ }^{\circ}{ }_{56} \mathrm{C}$ ).

When so transformed, the equation $D_{\mathrm{t}}=D_{\mathrm{T}}+a_{\mathrm{T}}(t-T)+\beta_{\mathrm{T}}(t-T)^{2}$ becomes:
Specific gravity $t / 15 \div 56=$ specific gravity $15{ }^{\circ} \cdot 56 / 15{ }^{\circ} \cdot 56+a_{t}(t-$

$$
15.56)+\beta_{t}(t-15.56)^{2}
$$

in which,

$$
a_{\mathrm{t}}=\alpha_{\mathrm{T}}+2 \beta(t-T) \text {, and } \beta_{\mathrm{t}}=\beta_{\mathrm{T}}
$$

$\alpha_{\mathrm{T}}=$ change of density per degree centigrade at $25^{\circ} \mathrm{C}$
$a_{t}=$ change of specific gravity per degree centigrade at $15^{\circ} \cdot 56 \mathrm{C}$

For the purpose of calculating the volume of oil at any temperature from the volume at $60^{\circ} \mathrm{F}\left(15^{\circ} .56 \mathrm{C}\right)$, the equation is put in the form:

$$
V_{\mathrm{t}}=V_{\mathrm{T} 5956}\left[\mathrm{I}+A(t-\mathrm{I} 5.56)+B(t-\mathrm{I} 5.56)^{2}\right]
$$

in which

$$
A=\frac{a_{\mathrm{t}}}{D_{25}} \text { and } B=A^{2}-\frac{\beta_{\mathrm{t}}}{D_{25}}
$$

The volume coefficients $A$ and $B$ for oils of different specific gravities are shown in the following table:

TABLE 11

| Specific gravity $\frac{15956}{159.56} \mathrm{C}$ | A | B | Specific gravity $\frac{15956}{15956} \mathrm{C}$ | A | B |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.630. | 0.00156 | 0.0000030 | 0.800.. | 0.00091 | 0.0000009 |
| 0.640.. | . 00150 | . 0000028 | 0.810.. | . 00089 | . 0000008 |
| 0.650 | . 00145 | . 0000026 | 0.820. | . 00087 | . 0000007 |
| 0.660. | . 00140 | . 0000024 | 0.830 | . 00084 | . 0000007 |
| 0.670. | . 00136 | . 0000023 | 0.840 | . 00082 | . 0000006 |
| 0.680. | . 00132 | . 0000022 | 0.850. | . 00081 | . 0000006 |
| 0.690 | . 00128 | . 0000020 | 0.860.. | . 00079 | . 0000005 |
| 0.700. | . 00125 | . 0000019 | 0.870. | . 00077 | . 0000005 |
| 0.710. | . 00121 | . 0000018 | 0.880. | . 00076 | . 0000004 |
| 0.720 | . 00118 | . 0000016 | 0.890. | . 00075 | . 0000004 |
| 0.730 | . 00114 | . 0000015 | 0.900 | . 00074 | . 0000004 |
| 0.740. | . 00111 | . 0000014 | 0.910.. | . 00073 | . 0000003 |
| 0.750. | . 00107 | . 0000013 | 0.920. | . 00072 | . 0000003 |
| 0.760. | . 00104 | . 0000012 | 0.930. | . 00071 | . 0000002 |
| 0.770. | . 00100 | . 0000011 | 0.940. | . 00070 | . 0000002 |
| 0.780. | . 00097 | . 0000010 | 0.950 | . 00069 | . 0000002 |
| 0.790. | . 00094 | . 0000009 | 0.960.. | . 00069 | . 0000002 |

## XV. APPLICABILITY AND ACCURACY OF THE EXPANSION TABLES OF CIRCULAR NO. 57

The expansion tables contained in Circular No. 57 are applicable to all petroleum oils, both crude and refined, produced in the United States, that are of sufficient fluidity at ordinary temperatures to allow their specific gravities to be determined by means of the hydrometer.

The accuracy with which the tables give the change of specific gravity or volume of any particular oil is dependent directly upon the closeness with which the rate of expansion of that particular oil agrees with the average rate of expansion on which the tables are based. Examination of the average curve and the closeness
with which the individual determinations agree with it indicate that it is very unlikely that the change of specific gravity per degree centigrade of any sample of oil will differ from the average change for oils of that specific gravity by more than two units of the fifth decimal place ( 0.00002 ). For example, the average change of specific gravity per degree centigrade at $25^{\circ} \mathrm{C}$ for oils having a specific gravity of 0.8000 is 0.00072 , and it is very unlikely that any sample of American petroleum oil having a specific gravity of 0.8000 at $\frac{25^{\circ}}{4^{\circ}} \mathrm{C}$ will have a rate of change less than 0.00070 or more than 0.00074 per degree at $25^{\circ} \mathrm{C}$.

Let it be supposed that the rate of expansion of some particular oil differs from the average rate by this maximum amount, then the specific-gravity value calculated from the average rate of expansion will be in error for this particular oil by 0.00002 per degree centigrade, and if the reduction is made over a temperature range of $10^{\circ} \mathrm{C}$ the error in the reduced specific gravity caused by the error in the assumed rate of expansion will amount to two units in the fourth decimal place.

On account of the variation in the rate of expansion of different oils of the same density it has been deemed advisable to carry the expansion tables only to the nearest five units of the fourth decimal place over a temperature range of about $10^{\circ} \mathrm{C}$ on each side of the standard temperature, and to the nearest unit in the third decimal place outside of this range. By thus arbitrarily limiting the implied accuracy of the tables, it is believed that the slight variations that occur between different samples of the same density need not be considered. In commercial measurements of petroleum oils density or specific-gravity determinations are seldom made with greater accuracy than one unit of the third decimal place (o.oor), and it is therefore unnecessary to carry commercial oil tables beyond that point.

It is quite possible that further work on American petroleums, with greater uniformity in the methods of examination, and especially in the time of examination after the collection of the samples, may make it possible to classify the oils from different localities and to take into account the slight differences that have not been considered in the calculation of the tables of Circular 57. It may then be possible and desirable to separate the oils into groups, and for each group to construct a table that will be more exact for that group than are the general tables of Circular 57.

It may, for example, be found desirable to have different tables for Pennsylvania and for California oils, as it is generally believed that California oils have a much higher rate of expansion than do central or eastern oils. The present investigation has not shown any great difference, though the California oils have shown a slightly higher rate of expansion.

## 1. SOURCES OF ERROR

The errors entering into density determinations made by the methods described in this paper are of two kinds, namely, (a) errors in weighing; (b) errors in temperature measurement.

In the work herein reported the magnitude of these errors is, in general, such as to produce errors of not more than from two to four units in the fifth decimal place of the determined densities of the oil samples. It is very unlikely that any of the density determinations are in error by more than five units in the fifth decimal place.

The density determinations were usually made over a temperature interval of $10^{\circ} \mathrm{C}$, and it is therefore apparent that the resulting error in the rate of change of density with change of temperature can not well be more than o.oooor per degree centigrade, even if both density determinations are in error by the maximum amount and in opposite directions. Since the errors in the observed densities are, on the average, as likely to occur in one direction as in the other, and further, since the observed densities were subjected to an adjustment by the method of least squares, to determine the most probable density at each temperature, it is evident that the final errors in the densities and in the rate of change of density with change of temperature can not be very great.

It has already been pointed out that the differences occurring in the rate of expansion of two oils of the same density may be of the order of two units of the fifth decimal place per degree centigrade, and these unavoidable differences are sufficient to render insignificant the experimental errors in the density determinations.

It is evident, therefore, that expansion tables based on the results contained in this paper are susceptible of an accuracy for all petroleum oils well within the demands of observations that are themselves reliable to one unit in the third decimal place.

## XVI. RATE OF EXPANSION OF FUEL OILS AND LUBRICATING OILS OF HIGH TEMPERATURES

The Bureau has several times been requested to furnish information in regard to the rate of change of density and volume of petroleum oils at high temperatures, especially in connection with the calculation of volume of fuel oil at the standard temperature of $60^{\circ} \mathrm{F}$, from its volume measured at relatively high temperatures when fresh from the topping plant. The information is also desired for use in the stillroom, where specific-gravity determinations of the different cuts must be made with as little delay as possible. It was therefore thought advisable to make density determinations on certain of the oil samples at temperatures considerably higher than $50^{\circ} \mathrm{C}$, the upper temperature limit of the greater part of the investigation. Accordingly, determinations were made on several samples at temperatures up to $95^{\circ} \mathrm{C}$.

From the results shown in Table 9, it will be seen that on certain samples the rate of change of density is practically the same at all temperatures between $25^{\circ}$ and $95^{\circ} \mathrm{C}$, while on other samples there is a marked falling off in the rate of expansion at the higher temperatures. This falling off is usually attributed to the melting of particles of paraffin, petrolatum, or other material that is solid at the lower temperatures and which gradually becomes liquid at the high temperatures.

The arrangement of the molecules of the liquid and the solid particles appears to be such as to prevent the same closeness of packing at the low temperatures that exists at the higher temperatures at which the solid particles have themselves become liquid. This arrangement has the effect of giving such a mixture an abnormally high rate of expansion at temperatures below the point of solidification of the particles.

In order to try the effect of dissolved paraffin on the rate of expansion of oil, density measurements were made on a sample of automobile cylinder oil not containing paraffin, and then a known amount of paraffin was dissolved in the oil and the rate of change of density again measured. The results are shown below:

TABLE 12

|  | Change of density per degree centigrade |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $19^{\circ}$ to $25^{\circ}$ | $25^{\circ}$ to $50^{\circ}$ | $50^{\circ}$ to $75^{\circ}$ | $75^{\circ}$ to $95^{\circ}$ |
| Automobile cylinder oil... | 0.00063 | 0.00063 | 0.00063 | 0.00063 |
| Same oil with 4.5 per cent paraffin added. | . 00072 | . 00063 | . 00064 | . 00063 |

It is seen that the dissolved paraffin caused a marked increase in the rate of expansion between $19^{\circ}$ and $25^{\circ} \mathrm{C}$, while at the higher temperatures the rate was not materially changed.
It is probable that if measurements had been made at lower temperatures a still further increase in the rate of expansion would have been found, but conditions were such that at that time the measurements could not conveniently be carried lower. The sample was, however, placed in a glass tube and packed in an ice bath and its appearance noted as its temperature was lowered. At temperatures above $25^{\circ} \mathrm{C}$ the oil was as clear as before the paraffin was added; at about $20^{\circ} \mathrm{C}$ it became somewhat cloudy; and at $15^{\circ} \mathrm{C}$ it was very cloudy or opaque, with a characteristic flaky appearance. At still lower temperatures it became practically a solid vaseline-like mass with very pronounced irregular fractures and transverse fissures.

The behavior of the above sample was very similar to that of certain other samples previously examined, in which abnormally high rates of expansion were found at the lower temperatures, and this would seem to indicate that their high rate of expansion was also due to the presence of dissolved substances that became solid at low temperature.

## XVII. COMPARISON OF RESULTS WITH PREVIOUS WORK

The results presented in this paper, so far as comparison can be made, are in substantial agreement with those given by D . Holde in his book entitled "Examination of hydrocarbon oils," and that of other experimenters; for example, Hans Höfer and Augustus H. Gill.

Since the direct object of this investigation was to obtain data from which to calculate expansion tables for petroleum oils, the work itself can perhaps best be judged by a consideration of these tables. A comparison of the tables (Circular No 57, this Bureau) with those published by the Kaiserlichen Normal Eichungs Kommission (Germany) in 1892 and republished in 1906 shows that when reduced to the same basis they are in excellent agreement throughout their entire range. Only in rare instances do the reduced specific gravities differ by more than one unit in the third decimal place. In most cases the two tables are in perfect agreement or differ by not more than five units in the fourth decimal place.

Another table that is used to some extent in this country is the Baumé table published by C. J. Tagliabue in his Manual for Inspectors of Coal Oil. It is interesting to compare this with the new table prepared by this Bureau. (Table 2, Circular No: 57.) Such a comparison shows that for the heavier grades of oil the agreement between the two tables is all that could be wished. For example, with oils having observed values of $20^{\circ}$, $30^{\circ}$, and $40^{\circ}$ Baumé at various temperatures, the two tables give the following values for the degrees Baumé at $60^{\circ} \mathrm{F}$.:

TABLE 13
[A=Data in this column from Bureau of Standards Circular No. 57, Table 2. B=Data in this column from Tagliabue's Manual for Inspectors of Coal Oil, 8th ed.]

| Observed temperature, ${ }^{\circ} \mathrm{F}$ | Observed values, $20^{\circ}$ Bé. |  | Observed values, $30^{\circ}$ Bé. |  | Observed values, $40^{\circ}$ Bé. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Degrees Baumé at $60^{\circ} \mathrm{F}$ |  | Degrees Baumé at$60^{\circ} \mathrm{F}$ |  | Degrees Baumé at $60^{\circ} \mathrm{F}$ |  |
|  | A | B | A | B | A | B |
| 30. | 21.7 | 21.8 | 32.0 | 32.2 | 42.4 | 42.5 |
| 40. | 21.2 | 21.1 | 31.4 | 31.4 | 41.6 | 41.6 |
| 50. | 20.6 | 20.5 | 30.7 | 30.7 | 40.8 | 40.8 |
| 60. | 20.0 | 20.0 | 30.0 | 30.0 | 40.0 | 40.0 |
| 70. | 19.4 | 19.4 | 29.3 | 29.3 | 39.2 | 39.2 |
| 80. | 18.9 | 18.9 | 28.7 | 28.6 | 38.5 | 38.4 |
| 90. | 18.3 | 18.4 | 28.0 | 28.0 | 37.7 | 37.6 |
| 100. | 17.8 | 17.8 | 27.4 | 27.3 | 37.0 | 36.8 |
| 110.. | 17.2 | a 17.3 | 26.8 | a 26.7 | 36.3 | a 36.1 |

$a$ Extrapolated from $10{ }^{\circ} \mathrm{F}$.
For the lighter oils, however, the agreement between the two tables is by no means as good as for the heavier oils. A comparison of $70^{\circ}, 80^{\circ}$, and $90^{\circ}$ Baumé is given in Table 14 .


[^0]:    ${ }^{2}$ P. Chappuis, Bureau International des Poids et Mesures, Travaux et Memoires, XIII; 190\%.

