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DATA ON ELECTRIC RAILWAY TRACK LEAKAGE

BY

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CONTENTS

	Page.
I. Introduction	3
A. Definition of track leakage	3
B. Principles involved	4
II. Discussion	4
A. General method	4
B. Presentation of data	5
1. The Chevy Chase line, Washington, D. C.	5
(a) General condition of line and test	5
(b) Current-leakage data	7
(c) Calculation of track and roadbed resistance	10
2. The second line	12
(a) General condition of line and test	12
(b) Current-leakage data	14
(c) Calculation of track and roadbed resistance	16
3. The third line	17
(a) General condition of line and test	17
(b) Current-leakage data	18
(c) Calculation of track and roadbed resistance	19
III. Summary	21

I. INTRODUCTION

A. DEFINITION OF TRACK LEAKAGE

The stray currents which create the hazard from electrolytic corrosion and kindred dangers in our cities usually escape from the street railway tracks and return to the same circuit after having in many cases done more or less damage to the structures in their path. The amount of damage evidently depends not only on the amount of leakage from the tracks, but the per cent of this leakage that gets to the structures, the value of the latter, and their susceptibility to corrosion.

B. PRINCIPLES INVOLVED

There are two principal factors which govern the magnitude of the leakage current from any section of track. The first is the potential impressed on the leakage path. This potential depends on the magnitude of the return current flowing in the rails and earth and the resistance of this divided circuit. This resistance, in turn, depends on the conductance of the rails and connections and the condition of the bonding and cross-bonding and the resistance of the roadbed. The second factor is the resistance of the leakage path. This resistance is influenced by the area of conductors in contact with the ground, the resistance of this contact, the conductance of the ties, and the length and conductivity of the earth path. Thus it is evident that the roadbed resistance is involved as a first and second order effect, influencing the potential impressed, as well as offering resistance to current flow from the track. This is made clear in equations shown under the individual tests. McCollum and Logan [1] have discussed in considerable detail the theory of track leakage and shown the effects on the magnitude of the leakage currents, of the length of feed, the track resistance, and the leakage conductance. It is the purpose of the following report to show in several actual cases the amounts of current that leak away from the track with load concentrated at the end of the line from which can be derived the constants of the track and roadbed. These constants can then be used in the equations given by McCollum and Logan for further analyzing electrolysis conditions. The data here given were obtained in connection with the Bureau's general investigations.

II. DISCUSSION

A. GENERAL METHOD

In order to determine the amount of leakage which actually occurs from trolley-line tracks, several lines of simple geometrical form were selected having no complication of track networks throughout the greater part of their length and having different physical characteristics as far as roadbed and soil conditions were concerned. It was arranged to impress a constant current on these lines during the night when the regular car service was off. The current was then measured at regular intervals along the tracks, the difference between this current flowing in the rails

[1] McCollum and Logan, Technologic Paper No. 63, Earth Resistance and its Relation to Electrolysis.

and the impressed current being the leakage current. Since the current flow was calculated from the potential drop on a solid length of rail and the resistance of this length, which resistance may vary, due to difference in composition, heat treatment, or rolling of rails in manufacture or wear after being laid, the accuracy can not be greater than that of the potential measurement and consistency of rail resistance. Since only a short length of rail (10 feet or 4 feet) was used, a low reading, low-resistance millivoltmeter was necessary and the accuracy of the readings was affected somewhat by the resistance of the contacts on the rail, readings being taken so rapidly that no great amount of time was spent in perfecting each contact. Two rail spanners having saw-blade contact points 4 feet apart were employed. These aided in expediting the tests, but even these contacts showed considerable variation in resistance and, in general, the results given below may have individual errors as great as 10 per cent, but the general tendency is shown and the average results are much better.

B. PRESENTATION OF DATA

1. THE CHEVY CHASE LINE, WASHINGTON, D. C.—(*a*) *General Condition of Line and Test.*—Although most of the trolley lines in the District of Columbia are not grounded, there are several outlying or interurban sections which use the track as return. Among these is the Chevy Chase line, running through the extreme northwest portion of the District of Columbia from Calvert Street Bridge to the District line at Chevy Chase, a distance of 3½ miles, and from there to Chevy Chase Lake, about 2 miles farther. The country is rather rolling and the elevation is between 200 and 300 feet above sea level. The soil is generally well-drained and there is considerable rock near the surface. Eighty-pound, T-rail, double-track construction is used throughout except on the first one-half mile where 83-pound girder rails are employed. The roadbed is of varied character, the first one-half mile with girder rails being concrete with asphalt street surface (Fig. 1). The next 3.5 miles is rock ballast with ties and rails embedded, the surface being finished with "Tarvia" (Fig. 2). From the end of this section to the lake, 1.5 miles, the rails are raised above the ballast, as shown in Fig. 3. At the extreme end, where a network of car-barn tracks is embedded in soil, the power house is located,

and the negative bus is connected to these tracks as well as the main-line tracks, and to a small ground plate in the lake.

The test was made on this line on April 30, 1911, from 2 a. m. to 5 a. m., when the regular car service was suspended. The ground was rather dry, no rain having fallen for some time.

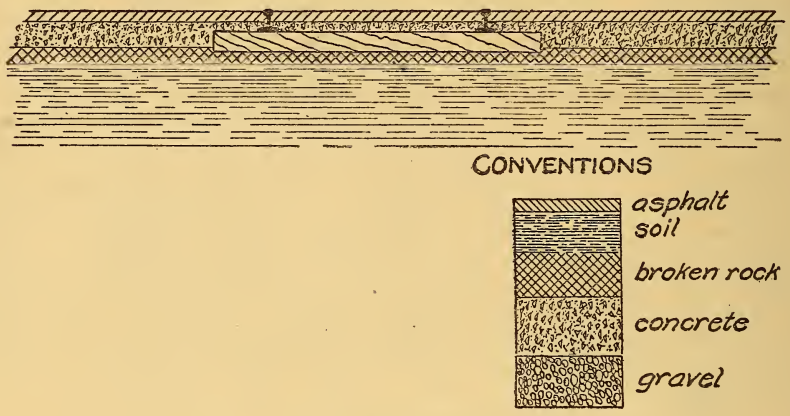

FIG. 1.—*Roadbed construction of first portion of Chevy Chase line*

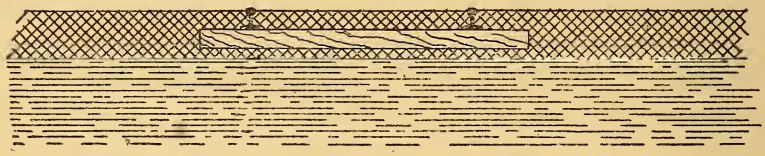

FIG. 2.—*Roadbed construction of second portion of Chevy Chase line*

FIG. 3.—*Roadbed construction of third portion of Chevy Chase line*

After the cars stopped running the power was cut off and the trolley connected to the rails by a No. 0000 bond at Rock Creek Bridge. The generator voltage was then reduced and a current of 300 amperes was supplied to the track. This current was maintained throughout the test. A record of the generator current and voltage is given in Table 1.

TABLE 1

Power-House Load

[Apr. 30, 1911, Chevy Chase line]

Time	Generator voltage	Generator current	Time	Generator voltage	Generator current
		Amperes			Amperes
2.10 a. m.	110	300	4.00 a. m.	103	300
2.20 a. m.	110	300	4.10 a. m.	104	300
2.30 a. m.	110	300	4.20 a. m.	103	300
2.40 a. m.	105	290	4.30 a. m.	103	300
2.50 a. m.	103	300	4.40 a. m.	103	300
3.00 a. m.	100	290	4.50 a. m.	103	300
3.10 a. m.	103	300	5.00 a. m.	105	310
3.20 a. m.	103	300	5.10 a. m.	103	300
3.30 a. m.	103	300	5.20 a. m.	103	300
3.40 a. m.	103	300	5.30 a. m.	103	300
3.50 a. m.	103	300			

(*b*) *Current-Leakage Data.*—The current in the rails was determined by measuring the drop on a 4-foot length on each of the four rails at regular intervals. These data are shown in Table 2 and Fig. 4. In the table the first column gives the time and the second the distance from the power house. The next four columns contain the millivolt drop on each of the four rails, counting the north or east one as number one; the sixth column totals these four columns, the values having been corrected for lead resistance, etc. The last column shows the total current in the four rails, based on a specific rail resistance of 0.000295 ohm per pound-foot and a weight of 80 pounds per yard.

The current is quite evenly distributed among the rails, thereby indicating reasonably good bonding and cross-bonding on this line. The first set of readings was taken beyond the point where the trolley was connected to the rails and indicates that 50.5 amperes were flowing into the city network of tracks and thence through the earth back to grounded connections of the negative bus.

TABLE 2

Current Leakage

[Apr. 30, 1911, Chevy Chase line]

Time	Distance from power house	Millivolt drop on 4 feet of rails					Current
		1st rail	2d rail	3d rail	4th rail	Corrected total	
	Feet						Amperes
2.40 a. m.	28 600	0.0	0.8	2.3	2.4	6.1	50.5
2.50 a. m.	28 400	4.0	9.2	7.4	6.5	30.1	249.0
3.00 a. m.	27 900	6.6	6.8	6.2	5.8	28.2	233.0
3.04 a. m.	27 400	6.0	6.4	5.5	6.3	26.8	222.0
3.08 a. m.	26 900	8.0	5.0	7.0	5.0	27.8	230.0
3.14 a. m.	26 400	8.0	8.0	7.0	6.0	32.2	266.0
3.16 a. m.	25 900	6.0	5.0	5.0	6.0	24.4	202.0
3.20 a. m.	25 400	6.0	5.0	4.0	5.0	22.2	189.0
3.24 a. m.	24 900	6.0	6.0	4.0	5.0	23.3	193.0
3.28 a. m.	24 400	7.0	7.0	7.5	6.0	30.5	252.0
3.32 a. m.	23 900	7.0	6.0	6.0	7.0	28.9	239.0
3.35 a. m.	23 400	6.5	5.5	7.0	6.0	27.8	230.0
3.40 a. m.	22 900	6.0	8.0	6.5	6.0	29.4	243.0
3.43 a. m.	22 400	6.0	6.0	6.0	7.0	27.8	230.0
3.45 a. m.	21 900	7.0	6.5	7.0	6.5	30.0	248.0
3.48 a. m.	21 400	7.0	6.5	6.0	7.0	29.4	243.0
3.51 a. m.	20 900	7.0	5.0	7.5	6.5	28.9	239.0
3.55 a. m.	20 400	8.5	5.5	7.0	7.5	31.6	261.0
4.04 a. m.	20 100	6.0	6.5	7.0	5.0	27.2	225.0
4.13 a. m.	19 600	6.5	6.0	7.0	7.5	30.0	248.0
4.15 a. m.	19 100	7.5	6.0	6.0	6.5	28.9	239.0
4.18 a. m.	18 600	6.5	6.0	6.5	6.0	27.8	230.0
4.20 a. m.	18 100	6.5	7.0	6.5	6.0	28.9	239.0
4.22 a. m.	17 600	7.5	7.0	7.5	6.5	31.6	261.0
4.26 a. m.	17 100	7.0	6.0	7.5	7.0	30.5	252.0
4.27 a. m.	16 600	6.0	6.5	6.5	7.0	28.9	239.0
4.31 a. m.	16 100	6.5	7.0	7.0	7.0	30.5	252.0
4.34 a. m.	15 600	5.5	7.0	6.0	6.0	27.2	225.0
4.38 a. m.	15 100	7.0	6.5	6.0	7.0	29.4	243.0
4.41 a. m.	14 600	7.0	6.0	7.0	6.0	28.9	239.0
4.43 a. m.	14 100	6.0	7.0	7.0	5.0	27.8	230.0
4.46 a. m.	13 600	6.0	5.5	5.0	7.0	26.1	216.0
4.48 a. m.	13 100	5.8	5.5	6.5	6.5	27.0	223.0
4.50 a. m.	12 600	6.0	6.0	7.0	7.5	29.4	243.0
4.53 a. m.	12 100	7.5	6.5	7.5	7.0	31.6	262.0
5.02 a. m.	8 500	7.0	5.5	6.0	7.0	28.3	234.0
5.12 a. m.	5 900	7.0	6.0	7.5	6.5	30.0	248.0
5.27 a. m.	2 600	8.0	7.0	7.5	7.0	32.2	266.0

It is very evident from the curve in Fig. 4 that the current values are quite irregular, not decreasing smoothly toward the center as they theoretically should do with uniform track resistance. This can be explained at some locations by a change in resistance of the return circuit, and this is emphasized by the potential gradient curve shown in Table 3, also plotted in Fig. 4. A small current and also low gradients are noted in the neighborhood of Kindle Run Bridge, where the rails are in contact with the steel structure which is carrying part of the current, returning some of it to the tracks and discharging the rest to the earth. The last

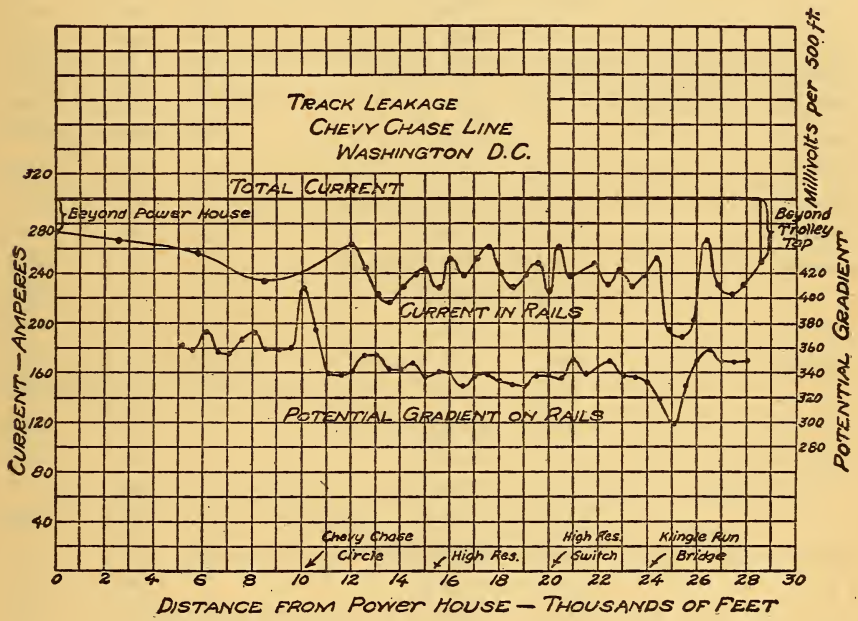

Fig. 4.—*Track leakage, Chevy Chase line*

reading shows a difference of about 35 amperes between the total current and the current on the rails at a distance of 2100 feet from the power house. Most of this current must find its way to the bus through a ground in the lake, the switching and barn tracks, and the Kensington line tracks beyond the power house. With the exception of the place where the bridge structure parallels the track, the point of smallest current is near the center of the line (14 000 feet from the power house), the current here being 216 amperes. The loss of current to connecting tracks at the outer end of the line is 50 amperes, and returning to connecting tracks on the other, 30 amperes.

TABLE 3

Potential Gradient in Rails, 500-foot spans

[Apr. 30, 1911, Chevy Chase line]

Time	Volts for 500 feet	Distance to power house	Time	Volts for 500 feet	Distance to power house
		Feet			Feet
3.04 a. m.	0.31	28 500	4.28 a. m.	0.33	16 500
3.07 a. m.	.35	28 000		.342	16 000
3.15 a. m.	.35	27 500		.34	15 500
3.20 a. m.	.35	27 000		.338	15 000
3.24 a. m.	.36	26 500		.35	14 500
3.27 a. m.	.354	26 000	4.42 a. m.	.344	14 000
3.30 a. m.	.33	25 500		.344	13 500
3.33 a. m.	.30	25 000		.356	13 000
3.36 a. m.	.32	24 500		.356	12 500
3.38 a. m.	.334	24 000		.342	12 000
3.41 a. m.	.336	23 500	4.54 a. m.	.34	11 500
3.43 a. m.	.328	23 000		.34	11 000
3.46 a. m.	.35	22 500	5.00 a. m.	.376	10 500
	.32	22 000		.41	10 000
	.34	21 500		.36	9 500
3.55 a. m.	.353	21 000	5.12 a. m.	.36	9 000
	.336	20 500		.36	8 500
	.404	20 000		.376	8 000
4.10 a. m.	.34	19 500		.37	7 500
	.33	19 000		.358	7 000
	.334	18 500		.358	6 500
4.19 a. m.	.334	18 000	5.25 a. m.	.376	6 000
	.34	17 500		.362	5 500
	.34	17 000		.364	5 000

(*c*) *Calculation of Track and Roadbed Resistance.*—The over-all potential drop was measured, using telephone lines for leads, and was found to be 21 volts.

The following equations may be used to determine the track resistance and also the leakage resistance to earth, where:

I_A = current at sending or receiving end

E_A = potential to ground at sending or receiving end

$2L$ = total length of track

r = track resistance per unit length

g = track leakage conductance per unit length

I_B = current at center of track

The equations governing the relations between these quantities with fixed load delivered to the outer end of the line are given by Kennelly [2] and are:

(1) $$I_A = \frac{E_A}{r_o} \coth \theta \text{ where } r_o = \sqrt{\frac{r}{g}}, \text{ and } \theta = L\sqrt{rg}$$

(2) $$\frac{I_B = E_A}{r_o \sinh \theta}$$

Dividing (1) by (2)

$$\therefore \frac{I_A}{I_B} = \coth \theta \sinh \theta = \cosh \theta = \cosh L\sqrt{rg}.$$

Taking as a particular case the Chevy Chase line, it is necessary to consider one half of the line as a unit. Considering first the power-house end,

$$I_A = 270 \text{ amperes}$$
$$I_B = 216 \text{ amperes}$$
$$L = 14\,300 \text{ feet}$$
$$E_A = 10.5 \text{ volts}$$
$$\therefore \frac{I_A}{I_B} = 1.25 = \cosh \theta$$

Referring to hyperbolic tables,

$$\theta = 0.693 = L\sqrt{rg}$$

The equation involving the drop in potential on the line is,

$$E_A = I_B \sqrt{\frac{r}{g} \sinh \theta}$$

in which all values except $\sqrt{\frac{r}{g}}$ are known, sinh θ being obtained from cosh θ in the above equations.

Then substituting known values, we have

$$\sqrt{\frac{r}{g}} = 0.0648$$

We now have two equations and two unknown quantities. Solving we get:

$$r = 0.00314 \text{ ohm per 1000 feet}$$
$$g = 0.748 \text{ ohm per 1000 feet}$$

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[2] A. E. Kennelly, The Application of Hyperbolic Functions to Electrical Engineering Problems.

Hence the resistance of 1000 feet of roadbed is $1/g = 1.34$ ohms. The value of track resistance, 0.00314 ohm per 1000 feet, is about 15 per cent greater than the calculated resistance of the continuous rail. This can be accounted for by an average rail-joint resistance equivalent to 10 feet of rail on the basis of 60-foot rails, which is very low, especially since there were a number of high-resistance joints. We will find that a much greater effect due to bad joints exists on the other lines discussed below. The resistance of the roadbed, 1.34 ohms for 1000 feet, is a reasonable value as checked by other measurements.

Taking the other half of the Chevy Chase line,

$$I_A = 250 \text{ amperes}$$
$$I_B = 216 \text{ amperes}$$
$$E_B = 10.5 \text{ volts}$$
$$L = 14\ 300 \text{ feet}$$
$$r \text{ and } g \text{ unknown}$$

$$\frac{I_A}{I_B} = \cosh L\sqrt{rg} = 1.155$$

$$\sqrt{rg} = \frac{0.550}{14.3}$$

and

$$E_A = I_B\sqrt{r/g} \sinh \theta \text{ or } \sqrt{r/g} = 0.0841$$

Solving for r and g,

$$r = 0.00323 \text{ ohm per 1000 feet.}$$
$$g = 0.458 \text{ mho per 1000 feet.}$$

or the resistance of 1000 feet of roadbed is 2.18 ohms. Both the rail resistance and roadbed resistance are higher than for the half nearer the power house, the rail resistance being 3 per cent higher and that of the roadbed 63 per cent. The averages for the entire length are 0.00318 ohm per 1000 feet and 1.76 ohms for each 1000 feet of double track. These average values are probably the better since two assumptions were made in stating the conditions of each half of the line; that is, that the minimum current occurs exactly at the center of the line and that the potential is also exactly divided between the two halves. Any error due to these approximations is practically eliminated when the average of the two is taken.

2. THE SECOND LINE—(*a*) *General Condition of Line and Test.*—The second trolley line runs a very straight course a distance of 11

miles. The track is single with 48-pound rails and has two or three switches or turnouts. The base of the rails is only occasionally in contact with the soil, the type of construction being shown in Fig. 5.

The soil is chiefly New Jersey gravel, well drained, and of rather high resistance. The elevation of the line above sea level is 123 feet at the highest point and 15 feet at the lowest, with an average of 75 feet. The power house is located at the western end, and the track at the eastern end is connected to a ground plate in the river near which the line terminates. Data as to trolley and feeders were obtained so that the potential difference of the different parts of the circuit could be calculated.

The trolley wire was connected to the track after the cars stopped and the current adjusted to 150 amperes (Table 4), about

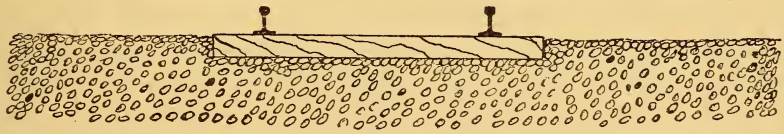

FIG. 5.—*Roadbed construction of second line*

210 volts being required to maintain this current. Current readings were taken about every one-quarter mile by observing the potential drop in both rails with the 4-foot rail spanners or contact points.

TABLE 4

Power-Station Readings

[Nov. 7, 1915, second line]

Time	Generator current	Generator voltage	Time	Generator current	Generator voltage
	Amperes			Amperes	
1 a. m.	150	215	2.30 a. m.	145	205
1.15 a. m.	150	215	2.45 a. m.	150	215
1.30 a. m.	150	215	3.00 a. m.	150	215
1.45 a. m.	150	215	3.15 a. m.	150	215
2.00 a. m.	145	205	3.30 a. m.	145	210
2.15 a. m.	150	215	3.45 a. m.	150	210

(*b*) *Current-Leakage Data.*—The time, distance from power house, and potential drops on the rails are shown in Table 5, the calculations of current flow being given in the last column. These current values are plotted in Fig. 6. It will be noted that about 12 amperes are flowing into the ground plate at the extreme end of the line and that the city tracks at the power-house end are taking up about 50 amperes. The leakage curve is considerably

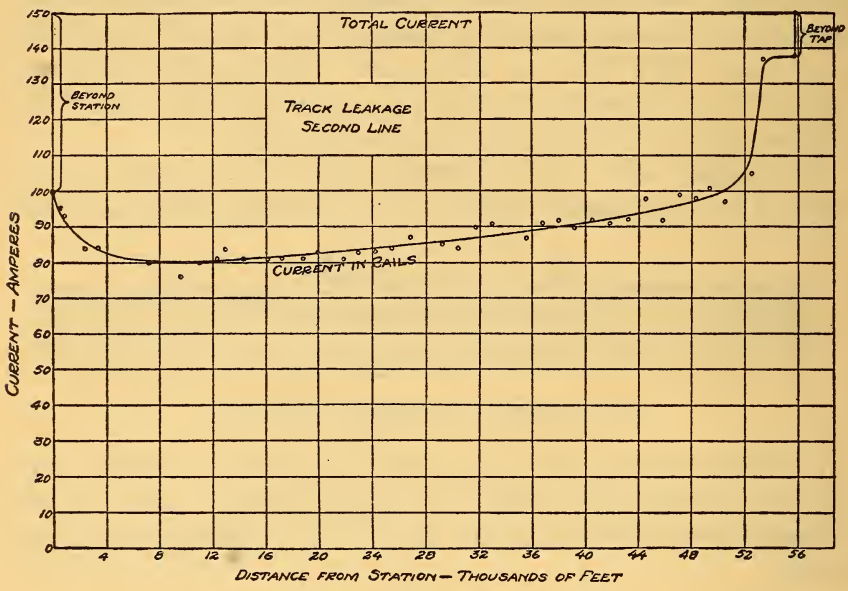

FIG. 6.—*Track leakage, second line*

smoother than that on the Chevy Chase line, but the percentage of loss is considerably greater, the rails at 1.5 miles from the power house are carrying about 80 amperes or 58 per cent of the total delivered to the track, and the remainder, 42 per cent, is leaking into the earth. As pointed out above, only a portion of the current escapes from or returns to the straight track, 58 amperes escaping from the extreme half and 20 amperes returning to the other portion before it connects with the city networks.

TABLE 5

Current Leakage

[Nov. 7, 1913, second line]

Time	Distance east of power house	Millivolt drop on 4 feet of rail			Weight of rail	Current
		North rail	South rail	Total		
	Feet				Lbs./yd.	Amperes
1.07 a. m.	56 160	0.2	0.4	0.6	65	[b] 11.5
1.12 a. m.	55 720	.6	7.3	7.9	65	150.0
3.50 a. m.	53 530	10.0	.0	10.0	48	149.0
1.22 a. m.	51 400	6.3	1.2	7.5	48	105.0
1.27 a. m.	50 590	.0	6.9	6.9	48	97.0
1.37 a. m.	49 400	3.8	3.4	7.2	48	101.0
1.42 a. m.	48 240	5.8	1.2	7.0	48	98.0
1.45 a. m.	47 000	5.7	1.4	7.1	48	99.0
1.49 a. m.	45 745	3.6	3.0	6.6	48	92.0
1.50 a. m.	44 500	3.5	3.5	7.0	48	98.0
1.52 a. m.	43 270	3.4	3.2	6.6	48	92.0
1.54 a. m.	41 800	3.4	3.1	6.5	48	91.0
1.56 a. m.	40 550	3.6	3.0	6.6	48	92.0
1.57 a. m.	39 100	3.3	3.1	6.4	48	90.0
1.59 a. m.	37 900	3.5	3.1	6.6	48	92.0
2.02 a. m.	36 700	3.5	3.0	6.5	48	91.0
2.04 a. m.	35 560	3.0	2.8	5.8	48	81.0
2.06 a. m.	34 300	3.8	2.4	6.2	48	87.0
2.09 a. m.	33 030	4.0	2.5	6.5	48	91.0
2.12 a. m.	31 800	3.0	3.4	6.4	48	90.0
2.14 a. m.	30 530	3.6	2.4	6.0	48	84.0
2.15 a. m.	29 300	6.1	.0	6.1	48	85.0
2.17 a. m.	28 030	3.0	2.8	5.8	48	81.0
2.19 a. m.	26 800	3.1	3.1	6.2	48	87.0
2.21 a. m.	25 540	.1	5.9	6.0	48	84.0
2.23 a. m.	24 300	3.3	2.6	5.9	48	83.0
2.25 a. m.	22 930	2.8	3.1	5.9	48	87.0
2.27 a. m.	21 700	3.6	2.2	5.8	48	81.0
2.30 a. m.	20 020	2.9	3.0	5.9	48	83.0
2.33 a. m.	18 800	4.0	1.8	5.8	48	81.0
2.35 a. m.	17 330	3.0	2.8	5.8	48	81.0
2.37 a. m.	16 100	.0	5.8	5.8	48	81.0
2.40 a. m.	14 230	3.1	2.7	5.8	48	81.0
2.42 a. m.	13 000	3.0	3.0	6.0	48	84.0
2.44 a. m.	12 220	3.1	2.7	5.8	48	81.0
2.46 a. m.	11 000	2.7	3.0	5.7	48	80.0
2.48 a. m.	9 720	2.8	2.6	5.4	48	76.0
2.50 a. m.	8 500	5.7	.0	5.7	48	80.0
2.52 a. m.	7 220	5.7	.0	5.7	48	80.0
2.55 a. m.	6 000	5.5	.0	5.5	48	77.0
2.57 a. m.	4 717	3.2	2.8	6.0	48	84.0
3.00 a. m.	2 582	6.0	.0	6.0	48	84.0
3.02 a. m.	788	2.6	2.6	5.2	60	91.0
3.06 a. m.	[a] 200	.1	.0	.1	60	1.7

[a] Beyond power house. [b] Flowing to ground plate at end of line.

(c) *Calculation of Track and Roadbed Resistance.*—From the current and feeder data it is calculated that a pressure of 66.7 volts was impressed on the track during this test. Referring to the table and curve it is seen that for the half of the track nearest the power house

$$E_A = 33.4 \text{ volts}$$
$$I_A = 100 \text{ amperes}$$
$$I_B = 80 \text{ amperes}$$
$$L = 28\ 000 \text{ feet.}$$

Referring to the preceding equations Nos. 1 and 2 and substituting numerical values,

$$\cosh \theta = \frac{100}{80} = 1.25$$
$$\therefore \theta = L\sqrt{rg} = 0.693 \qquad (3)$$
$$E_A = I_B\sqrt{\frac{r}{g}}\sinh \theta.$$

Substituting known values,

$$\sqrt{\frac{r}{g}} = 0.556 \qquad (4)$$

solving for r and g in (3) and (4)

$$g = 0.0445 \text{ mho per 1000 feet}$$

or the resistance of 1000 feet of roadbed is 22.5 ohms.

$$r = 0.01378 \text{ ohm per 1000 feet.}$$

For the outer half of the line

$$E_A = 33.4 \text{ volts}$$
$$I_A = 143 \text{ amperes}$$
$$I_B = 80 \text{ amperes}$$
$$L = 28\ 000 \text{ feet}$$

as above $\cosh \theta = \frac{I_A}{I_B} = \frac{143}{80} = 1.788$

$$\theta = 1.185 = L\ \sqrt{rg} \qquad (5)$$

and

$$E_A = I_B\sqrt{\frac{r}{g}}\sinh\ \theta$$
$$\sqrt{\frac{r}{g}} = 0.2817 \qquad (6)$$

solving for r and g in (5) and (6)

$$g = 0.1503 \text{ mho per 1000 feet}$$

or resistance to ground = 6.65 ohms for 1000 feet and

$$r = 0.01182 \text{ ohm per 1000 feet.}$$

The averages for the whole line are then:

Resistance to ground $1/g = 14.57$ ohms per each 1000 feet of single track

$$r = 0.0128 \text{ ohm per 1000 feet.}$$

The apparent difference in the roadbed resistance between the two halves of the lines is probably much greater than actually exists, but the average value is reasonable when the type of roadbed is considered, the dry, sandy soil being in contact only with the ties. The resistance of the rails is increased about 40 per cent, or each joint has an average resistance of 12 feet of solid rail on a basis of 30-foot rails.

3. The Third Line—(*a*) *General Condition of Line and Test.*—This trolley line is a single track with 60-pound rails, having a length of about 6.5 miles. The track has a short spur or Y about 1 mile from the extreme end, but it has no complicated network of tracks at either end, and the total current is easily measured. The roadbed is clay and New Jersey gravel, at many points in contact with or covering the base of the rail. The elevation, compared to sea level, is the most striking feature of this line, the highest point being 15 feet above, the lowest 2 feet below, and the average 10 feet above. Salt marshes lie close to the track for the greater part of the distance, and the conductivity of this soil must be very high. There is probably a ground plate attached to the outer end of the track, but this is too small to materially affect the general leakage conditions.

The rotary converter in the substation carried the load for this test, the machine voltage being reduced to 275 volts and the current being maintained at 205 amperes throughout the night, as shown by Table 6. The readings were taken every one-quarter mile on both rails, using the 4-foot spanners mentioned in the preceding test.

TABLE 6

Substation Load

[Nov. 8, 1913, third line]

Time	Amperes	Volts	Time	Amperes	Volts
1.30 a. m.	210	275	2.45 a. m.	205	275
1.45 a. m.	205	275	3.00 a. m.	205	275
2.00 a. m.	205	275	3.15 a. m.	205	275
2.15 a. m.	205	275	3.30 a. m.	205	275
2.30 a. m.	205	275			

(*b*) *Current-Leakage Data.*—In Table 7 are shown the results of this test arranged as in the previous tables, showing time, dis-

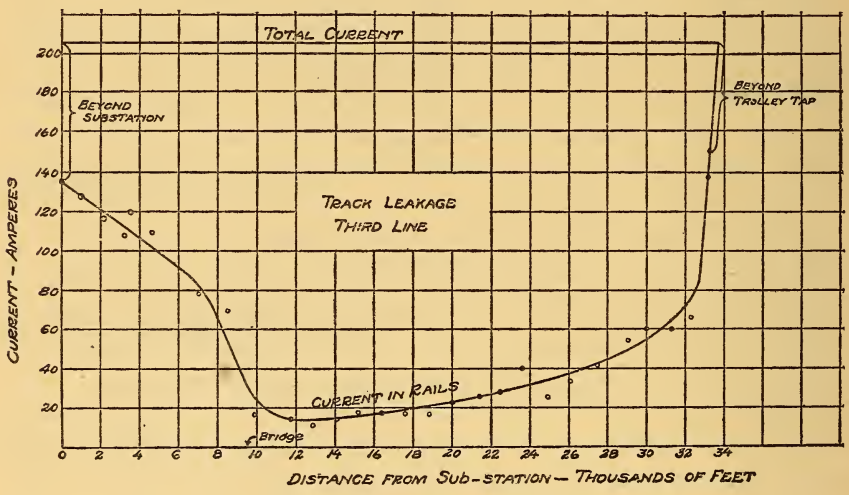

FIG. 7.—*Track leakage, third line*

tance from substation, millivolt drop on rails, and the calculated current. It will be noted that 70 amperes is flowing into the earth at the outer end and that 85 amperes is finding its way to the track beyond the substation. The distribution of current between rails is very poor at times, as shown by zero readings on one and the total on the other, indicating bad bonding and cross-bonding on these sections. The curve (Fig. 7) shows very graphically the small current left in the track at about the middle point, the lowest current recorded being 12.2 amperes, or only 6 per cent of the total and about 9 per cent of that actually flowing in this direction.

TABLE 7

Current Leakage

[Nov. 8, 1913, third line]

Time	Distance from sub-station	Millivolt drop on 4 feet of rail			Weight of rail	Current
		East rail	West rail	Total		
	Feet				Lbs./yd.	Amperes.
1.29 a. m.	33 415	4.0	0.0	4.0	60	[a] 70.0
1.37 a. m.	33 100	6.2	1.6	7.8	60	136.4
1.43 a. m.	32 300	1.8	2.0	3.8	60	66.5
1.46 a. m.	31 100	1.6	1.8	3.4	60	59.5
1.50 a. m.	29 900	1.6	1.8	3.4	60	59.5
1.57 a. m.	29 000	1.6	1.6	3.2	60	56.0
2.00 a. m.	29 000	.8	.6	.2	60	3.5
2.04 a. m.	28 700	1.4	1.3	2.7	60	47.2
2.09 a. m.	27 500	1.2	1.2	2.4	60	42.0
2.12 a. m.	26 100	.8	1.0	1.8	60	31.5
2.14 a. m.	24 900	.7	8.0	1.5	60	26.2
2.16 a. m.	23 650	1.1	1.2	2.3	60	40.2
2.18 a. m.	22 400	.0	1.6	1.6	60	28.0
2.20 a. m.	21 200	.7	.8	1.5	60	26.2
2.22 a. m.	20 000	.5	.8	1.3	60	22.7
2.24 a. m.	18 800	1.0	.0	1.0	60	17.5
2.35 a. m.	17 600	.0	1.0	1.0	60	17.5
2.42 a. m.	16 400	.5	.8	1.3	60	22.7
2.47 a. m.	15 200	1.0	.0	1.0	60	17.5
2.52 a. m.	14 100	.9	.0	.9	60	15.7
2.58 a. m.	12 900	.7	.0	.7	60	12.2
3.05 a. m.	11 800	.9	.0	.9	60	15.7
3.12 a. m.	10 800	.6	.4	1.0	60	17.5
3.16 a. m.	9 700	.0	1.0	1.0	60	17.5
3.19 a. m.	8 500	2.0	2.0	4.0	60	70.0
3.22 a. m.	7 100	2.2	2.3	4.5	60	78.8
3.24 a. m.	5 800	3.1	3.2	6.3	60	110.2
3.26 a. m.	4 600	2.3	4.6	6.9	60	120.2
3.28 a. m.	3 200	3.0	3.2	6.2	60	108.5
3.30 a. m.	2 200	4.2	2.5	6.7	60	117.2
3.34 a. m.	1 000	7.3	.0	7.3	60	127.7
3.40 a. m.	150	3.4	4.0	7.4	60	129.5
3.42 a. m.	50	3.5	2.6	6.1	48	85.0

[a] Beyond point of connection of trolley wire to track. Current flowing away from power house to earth.

(*c*) *Calculation of Track and Roadbed Resistance.*—From the total current and voltage and resistance of the positive feeders the drop on the track can be calculated. This is found to be 87.4 volts.

Using the same symbols as before, the half of the track nearest the power house has these values.

$$E_A = 43.7 \text{ volts}$$
$$I_A = 135 \text{ amperes}$$
$$I_B = 14 \text{ amperes}$$
$$L = 16\ 600 \text{ feet}$$

Substituting these values in the equations previously used

$$\text{Cosh}\ \Theta = \frac{135}{14} = 9.64$$

$$\Theta = 2.957 = L\sqrt{rg} \tag{7}$$

$$E_A = I_B\sqrt{\frac{r}{g}}\ sinh\ \Theta$$

Substituting above values,

$$\sqrt{\frac{r}{g}} = 0.3254 \tag{8}$$

Solving for r and g in (7) and (8)

$$g = 0.547 \text{ mho per 1,000 feet}$$

or resistance to ground is

$$\frac{1}{g} = 1.828 \text{ ohms per 1,000 feet}$$

$$r = 0.0580 \text{ ohm per 1,000 feet}$$

For the other half of the line the values are practically the same.

$$E_A = 43.7 \text{ volts}$$
$$I_A = 137 \text{ amperes}$$
$$I_B = 14 \text{ amperes}$$
$$L = 16\ 600 \text{ feet}$$

Substituting

$$\cosh\ \Theta = \frac{137}{14} = 9.78$$

$$\Theta = 2.971 = L\sqrt{rg}$$

$$E_A = I_B\sqrt{\frac{r}{g}}\ \sinh\ \Theta. \tag{9}$$

Substituting the above values

$$\sqrt{\frac{r}{g}} = 0.321 \tag{10}$$

Solving for g and r in (9) and (10)

$$g = 0.558 \text{ mho per 1000 feet}$$

or roadbed resistance

$$1/g = 1.79 \text{ ohms for 1000 feet}$$
$$r = 0.0573 \text{ ohm per 1000 feet}$$

The averages for the two halves of the line are:

$$g = 0.553 \text{ mho for 1000 feet}$$
$$1/g = 1.81 \text{ ohms for 1000 feet}$$
$$r = 0.0576 \text{ ohm per 1000 feet.}$$

This line has a very low elevation, and is surrounded by marshes, and its resistance to ground is unusually low; while the rail resistance is very high, being 15.7 times that of solid rail for this weight. In other words, the average resistance of each joint is equal to that of 470 feet of rail. For a considerable per cent of the distance only one rail was carrying any current, and it also undoubtedly had frequent high-resistance joints.

III. SUMMARY

Table 8 contains a summary of the above data and certain interesting conclusions may be drawn from it when the characteristics of the lines are considered. The Chevy Chase line is the shortest of the three, and only 21 volts was required to maintain 300 amperes on this length. The total rail resistance of line No. 2 is about eight times that of the Chevy Chase line, and the potential difference required per ampere is about six times as great, so that the greater length of line in No. 2 has about offset the effect of the much higher leakage resistance. City lines seldom have great feeding distances; the per cent of current leaking to earth will, therefore, generally be far less than in suburban lines except in special cases where underground pipes are metallically connected to the track.

TABLE 8

Summary of Track-Leakage Data

Line	Length	Weight of rail	Over-all potential drop	Current		Resistance of return single track			Leakage resistance	
				Maximum in rails	Per cent to earth	Ohms for 1000 feet	Total ohms	Per cent of solid rail	Ohms for 1000 feet	Ohms for a mile
	Feet	Lbs./yd.	Volts	Amp.						
Chevy Chase (double track)	28 600	4–80	21.0	250	13	0.00636	0.091	115	1.76	0.333
Line No. 2 (single track)	56 000	2–48	66.7	143	42	.0128	.717	140	14.57	2.75
Line No. 3 (single track)	33 200	2–60	87.4	136	90	.0576	1.913	1570	1.81	.343

Although most of the joints on all these lines are better than indicated by the per cent increase of resistance, there are certain bad places on each line which bring this value up to a remarkably high figure, especially on line No. 3. The need of occasional tests and of cross-bonding is emphasized by such conditions.

The leakage resistance of line No. 2 is much higher than either of the others, being about eight times that of both the Chevy Chase line and line No. 3. This indicates that rails clear of soil, and ties embedded in well-drained sand and gravel have a higher resistance than any track with rails in contact with macadam-brick, or asphalt pavement. When it is considered that a leakage resistance of 14.5 ohms for each 1,000 feet of roadbed indicates a leakage resistance of over 29 000 ohms for 1 foot, the high insulating properties of this roadbed are more evident. This line probably represents the upper extreme of roadbed resistance that will be found under ordinary conditions.

WASHINGTON, April 14, 1916.