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A FABRIC TENSION METER FOR USE ON AIRCRAFT

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ABSTRACT

If the fabric coverings of airplane wings or of airships do not have the proper tautness, the operation of the craft will not be satisfactory. In the case of airplane wings the fabric cover must be tight enough to prevent flapping but can exceed this minimum by considerable amounts without affecting the operation of the airplane. The cover of a rigid airship must also be tight enough to prevent excessive flapping, but it is not safe to tighten it too much because of the strain thus placed upon the metal framework.

Because of the importance of the proper adjustment of the tautness of the fabric cover of the airships *Shenandoah* and *Los Angeles*, the Bureau of Standards has constructed for the Bureau of Aeronautics of the Navy Department a convenient instrument for measuring the tautness of fabrics. The instrument consists of an open chamber having an elliptical cross section and provided with a pressure gauge and deflection meter. Around its perimeter is a hollow rim perforated with small holes, by means of which the production of a partial vacuum in the rim causes the instrument to adhere firmly to the fabric and so isolates the portion of the fabric lying within the suction rim.

Once isolated, the fabric is deflected inward by means of a small suction until its deflection reaches a certain fixed value, when the operator holds the suction constant and reads the suction gauge. Several such readings are taken with the instrument placed at the same point on the fabric, but with its major axis rotated through successive 45 or 90° angles, as the case may require. A nomogram is used to convert the suction readings into tensions.

The theory of the instrument is developed for the various cases which may arise in use, including the measurement of tensions in single-ply fabric, in which the directions of the principal stresses are known, and in multiple-ply fabric, the directions of the principal stresses being unknown initially. A method is developed for measuring the tensions when the "modulus" of the fabric is not known.

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I. INTRODUCTION

The fabric tension meter described in this paper was developed at the Bureau of Standards in 1923 for the Bureau of Aeronautics of the United States Navy Department. The instrument (see figs. 1 and 2) was designed to measure the stresses existing in the cover fabrics of rigid airships and in the envelopes of nonrigid and semi-rigid airships. The fundamental principle on which this instrument is based is this: If a portion of the fabric is deflected by a hydrostatic pressure, a relation exists between this pressure, the tensions in the fabric, and the principal radii of curvature at the center of the deflected portion of the fabric. A single relation of this kind is not sufficient to determine the unknown stresses in two given perpendicular directions. However, if the boundary conditions are changed, a sufficient number of these relations can be obtained and the stresses can be determined. The theory developed here shows how this can be done by applying hydrostatic pressure to a portion of the fabric having an elliptical contour, provided the changes in strain when the fabric is deflected are so small that they may be assumed to be either negligible or proportional to the change in stress. It will be shown that where, as in a single-ply fabric, the directions of the principal stresses are known, four pressure determinations are, in general, necessary to determine the stresses at any point. If, in addition, the change of stress produced by the pressure applied is negligible or if the stress-strain relations of the fabric are known, one-half this number—two observations—are sufficient.

Where, as in a three-ply fabric, the directions of principal stress are not known, six readings are, in general, necessary, but eight readings are more convenient. Here, also, if the change of stress produced by the pressure applied is negligible or if the stress-strain relations of the fabric are known, one-half this number—three (or more conveniently four) observations—suffice to determine the principal stresses, both in magnitude and direction.

II. PRINCIPLE OF THE INSTRUMENT

Fabric in tension may be regarded as a particular case of a deformed thin shell in which all the flexural and torsional moments are negligible. Hence, the derivation of the equilibrium condition of an elementary rectangular portion of the fabric will involve only the stresses exerted by the rest of the fabric on the edges of the rectangular portion and the applied hydrostatic pressure, if any exists (see fig. 3).

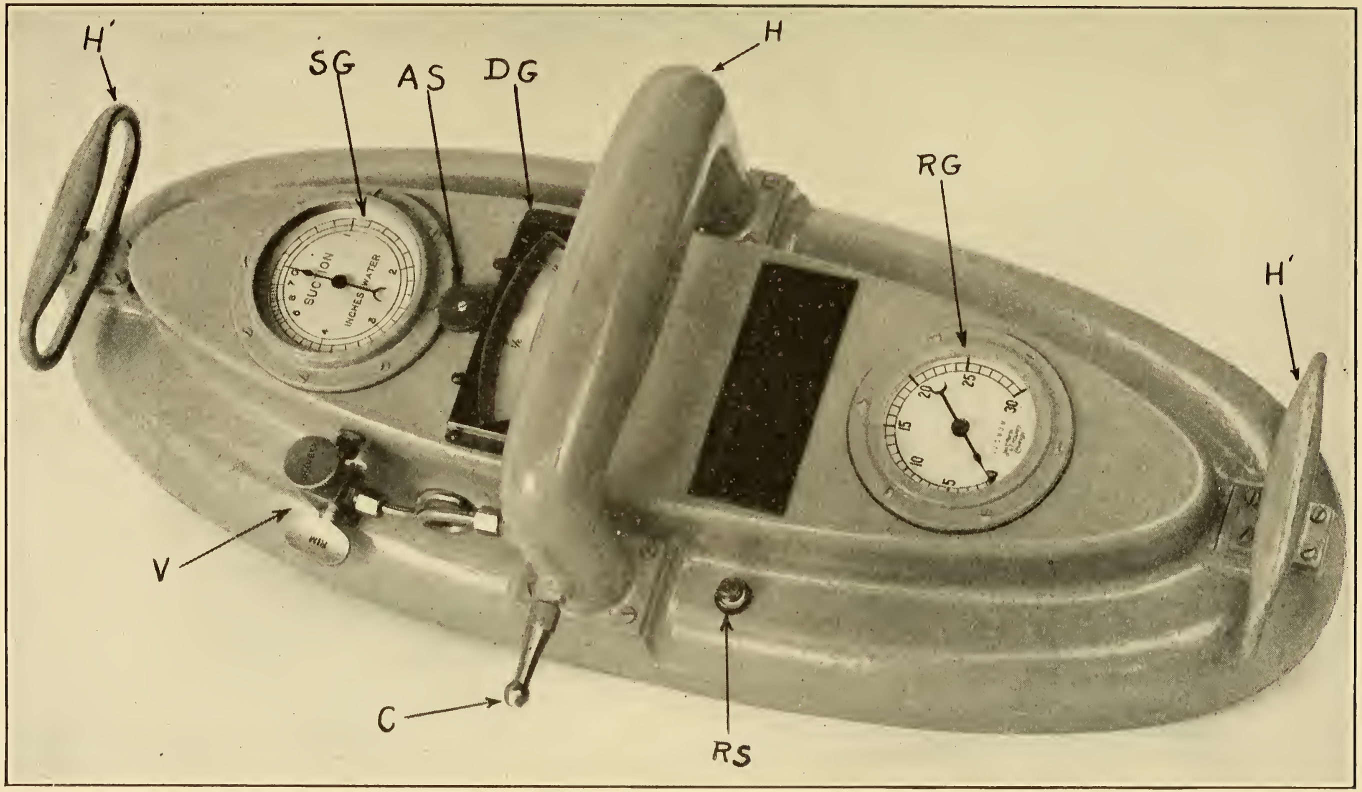

FIG. 1.—*Fabric tension meter, top view*

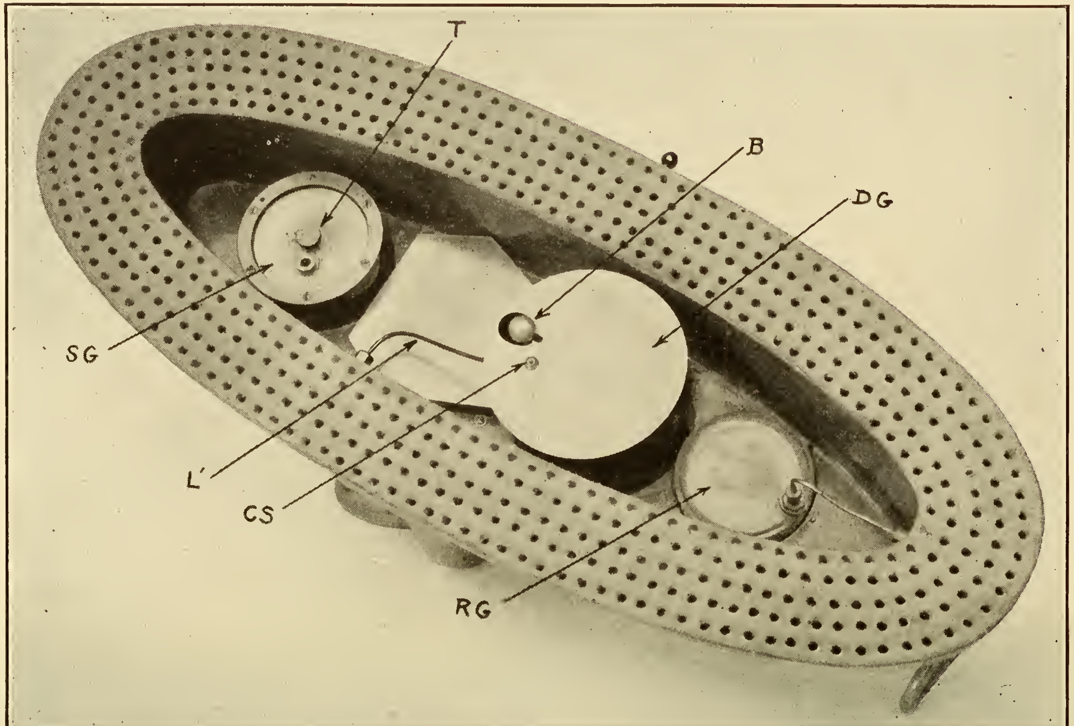

FIG. 2.—*Fabric tension meter, showing suction rim*

Let

dl_1 and dl_2 = lengths of the edges of an elementary and very nearly rectangular portion of the fabric,

R_1 and R_2 = radii of curvature of the fabric in the directions of dl_1 and dl_2, respectively,

$d\theta_1$ and $d\theta_2$ = angles subtended by dl_1 and dl_2, respectively,

S_1 and S_2 = stresses parallel to dl_1 and dl_2, respectively, and P = hydrostatic pressure.

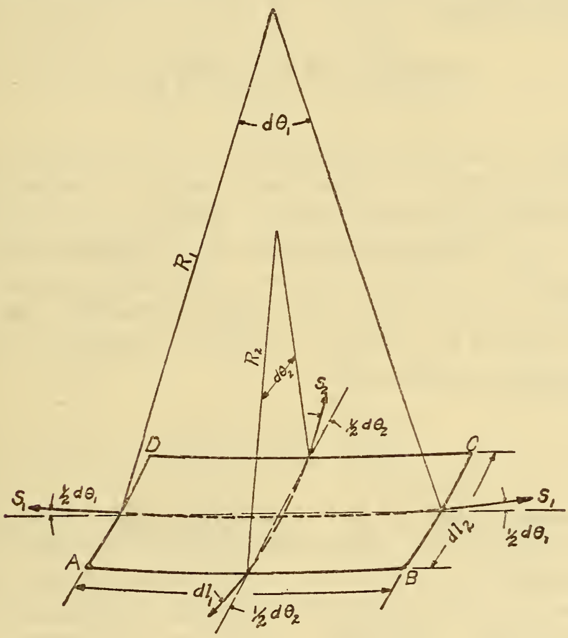

FIG. 3.—*Stresses acting on an elementary rectangle of a fabric*

It is obvious that, for equilibrium,

$$Pdl_1dl_2 = 2S_1\, dl_2 \sin\frac{d\theta_1}{2} + 2S_2\, dl_1 \sin\frac{d\theta_2}{2}$$

Now, assuming that $\sin\frac{d\theta_i}{2} = \frac{d\theta_i}{2}$, and remembering that $R_i d\theta_i = dl_i$ for $i = 1$ and 2, there results

$$P = \frac{S_1}{R_1} + \frac{S_2}{R_2} \qquad (1)^1$$

Equation (1) suggests that in order to evaluate the stresses S_1 and S_2 in any two directions perpendicular to each other it will be sufficient

[1] This relation is well known. An equivalent derivation may be found in N. A. C. A. Tech. Rep. No. 16, p. 207; 1917 Rudolf Haas and Alexander Dietzins: The Stretching of the Fabric and the Deformation of the Envelope in Nonrigid Balloons.

(*a*) to isolate a portion of the fabric of a given shape (that is, elliptical, rectangular, etc.) from the rest of the fabric, apply to it a pressure (in practice a suction), P_1', and to determine the radii of curvature R_1' and R_2' of the fabric at the center of the isolated portion in the direction of S_1 and S_2, and (*b*) to repeat this process, using a differently shaped (or differently oriented) portion of the fabric, but with the same central point, for a pressure P'' so as to obtain a second pair of radii of curvature R_1'' and R_2'' in the same directions as the first two. The equations then are

$$\begin{aligned} R_2'R_1'P' &= S_1R_2' + S_2R_1' \\ R_2''R_1''P'' &= S_1R_2'' + S_2R_1'' \end{aligned} \qquad (2)$$

In order that (2) shall have a solution, the determinant of the coefficients of S_1 and S_2 in (2) must not vanish. An especially simple case where this determinant does not vanish is: $R_1' = R_2''$ $(= R_1)$ and $R_2' = R_1''$ $(= R_2)$. The most suitable way of fulfilling this condition consists in using an elliptical portion of the fabric and in observing the pressures P' and P'' for a given fixed deflection with the minor axis of the contour first along S_1 and then along S_2, or vice versa. Equation (2) then simplifies into

$$\begin{aligned} R_1R_2P' &= S_1R_2 + S_2R_1 \\ R_1R_2P'' &= S_1R_1 + S_2R_2 \end{aligned} \qquad (3)$$

In equations (2) and (3) it was tacitly assumed that the stresses S_1 and S_2 existing in the fabric before the tension meter was applied are not changed by the deflection of the fabric into the chamber of the instrument. The symmetrical form of equation (3) is a direct consequence of this assumption. However, the stresses S_1 and S_2 are actually increased in the portion of the fabric lying within the rim of the instrument, for when the suction rim grips the fabric it isolates that portion of the fabric lying within the rim. Hence, when suction is applied to the chamber of the meter, thus deflecting the fabric inward, the stretching caused thereby occurs only in the isolated portion of the fabric, increasing the stresses there. However, it will be shown that the increase in these stresses can be taken into account by the method to be explained, and that a system of equations essentially equivalent to (3) will result.

III. EQUATION OF SURFACE WITH ELLIPTICAL CONTOUR

Consider a portion of the fabric (see fig. 4) which is isolated by the elliptical contour of the tension meter and is deflected under a differential pressure, P. Assume that the fabric is uniformly stressed initially; that is, along a straight line the stress normal to the line is constant and varies only when the direction of the line is changed.

Choose the coordinate axes OX and OY at the center of the ellipse so as to have the equation of the inner rim of the tension meter in the form

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \tag{4}$$

where 2a and 2b are, respectively, the major and minor axes of the ellipse. Let z be the deflection of any point of the isolated fabric under the hydrostatic pressure P. Let S_1 and S_2 be the stresses in the fabric along the directions OY and OX.

Now, according to the equation of equilibrium

$$P=\frac{S_1}{R_1}+\frac{S_2}{R_2}$$

where R_1 and R_2 are the radii of curvature of the deflected fabric in the normal planes cutting the fabric in OY and OX.

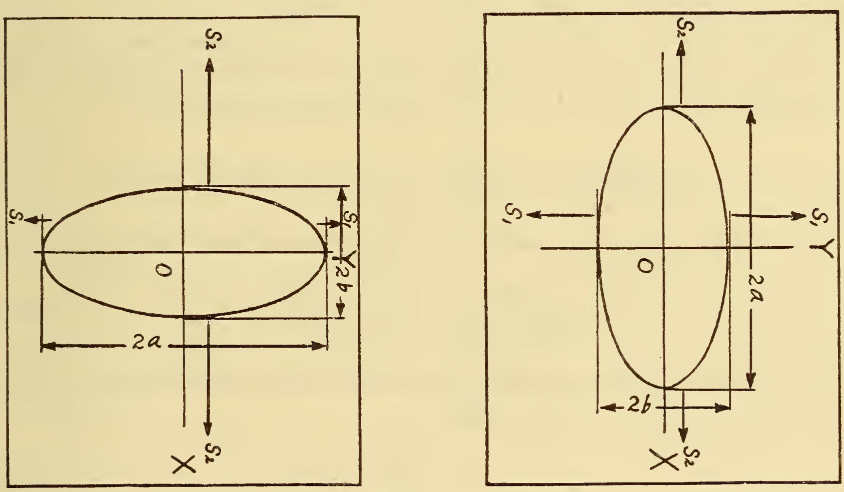

FIG 4.—*Axes of elliptical contours in the direction of the principal stresses*

If the deflection z is small in comparison to the dimensions of the ellipse, then $\frac{\delta z}{\delta y}$ is small in comparison with unity, so that it suffices to write for the curvature

$$\frac{1}{R_1}=\frac{\delta^2 z}{\delta y^2} \tag{5a}$$

instead of the exact expression

$$\frac{1}{R_1}=\frac{\frac{\delta^2 z}{\delta y^2}}{\left[1+\left(\frac{\delta z}{\delta y}\right)^2\right]^{3/2}}$$

Similarly,

$$\frac{1}{R_2}=\frac{\delta^2 z}{\delta x^2} \qquad (5b)$$

Assuming

$$S_1=k^2 S_2 \qquad (6)$$

the equation of equilibrium becomes in this case

$$\frac{\delta^2 z}{\delta x^2}+k^2\frac{\delta^2 z}{\delta y^2}=\frac{P}{S_2} \qquad (7)$$

This is a linear differential equation of the second order, with constant coefficients whose general solution in standard form is

$$z=f(y+ikx)+g(y-ikx)+\frac{P}{4k^2 S_2}(y^2+k^2 x^2) \qquad (8)$$

Since the stress was assumed to be initially uniform, z must be an even function of both x and y. Furthermore, z has no singularities within the elliptical boundary of the tension meter, and when $z=0$ (8) will reduce to the boundary curve (4). Introducing these conditions the solution of (7) becomes determinate and is

$$z=\frac{a^2b^2}{2(a^2k^2+b^2)}\cdot\frac{P}{S_2}\left(\frac{x^2}{a^2}+\frac{y^2}{b_2}-1\right)=Z_o\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right) \qquad (9)$$

where Z_o, the maximum deflection of the isolated fabric at the center is

$$Z_o=\frac{a^2b^2}{2(a^2k^2+b^2)}\cdot\frac{P}{S_2}=\frac{k^2a^2b^2}{2(a^2k^2+b^2)}\cdot\frac{P}{S_1} \qquad (10)$$

The radii of curvature R_1 and R_2 in planes perpendicular to OX and OY have the constant values

$$R_1=\frac{b^2}{2Z_o} \qquad (11)^2$$

and

$$R_2=\frac{a^2}{2Z_o} \qquad (12)^2$$

Substituting in (1) these values for R_1 and R_2, there results

$$P=\frac{2Z_o}{b^2}S_1+\frac{2Z_o}{a^2}S_2 \qquad (13)$$

Denoting $\frac{Z_o}{b^2}$ by α and $\frac{Z_o}{a^2}$ by β

$$P=2\alpha S_1+2\beta S_2 \qquad (14)$$

[2] Accordingly, the sections of the deflected fabric made by planes normal to OX and OY are arcs of circles.

Equation (13) or (14) is an expression connecting the tensions or stresses of the deflected portion of the fabric in the directions of the major and minor axes of the tension meter with the maximum deflection, the constants of the elliptical contour, and the pressure. In what follows the deflection at the center of the isolated elliptical portion will be referred to as the deflection of the fabric.

IV. DEVELOPMENT OF THEORY

1. NOTATION

In the further development of the theory the equations will be very much simplified by the adoption of the following notation for the various quantities involved. First, assume as a reference line the direction of initial maximum stress in the fabric.

The stresses in the fabric before the pressure is applied are represented by

S_η = the stress at an angle η to the direction of the maximum stress.

$S_M = S_{0°}$ = maximum stress,

$S_m = S_{90°}$ = minimum stress.

Also

P_θ is the pressure required to produce a deflection Z_0 when the minor axis of the instrument makes an angle θ with the direction of the maximum stress.

When the differential pressure P is applied, the initial stresses are changed by amounts depending upon the direction of the minor axis of the instrument, so that it is necessary to make the following distinction: When the differential pressure applied produces the deflection Z_0,

$S_{\eta,\theta}$ is the stress in the direction η when the minor axis of the instrument is in the direction θ.

$\epsilon_{\eta,\theta}$ is the corresponding extension of the fabric at the midsection in the direction η.

In the use of the instrument, only two directions η need be considered:

$$\eta = \theta, \text{ and } \eta = \theta + 90° \tag{15}$$

where θ may take on successively the values $\theta = \theta_0$, $\theta_0 + 45°$, $\theta_0 + 90°$ and $\theta_0 + 135°$.

P'_θ, $S'_{\eta,\theta}$ and $\epsilon'_{\eta,\theta}$ are the corresponding values when the pressure produces the deflection Z'_0 (in practice $Z'_0 = \sqrt{2}\, Z_0$).

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2. CASE I. DIRECTIONS OF THE MAXIMUM AND MINIMUM STRESSES ARE KNOWN

Provided the change in strain is so small that it may be assumed to be for practical purposes proportional to the change in stress

$$S_{\eta,\theta}=S_\eta+E_{\eta,\theta}\epsilon_{\eta,\theta} \tag{16a}$$

where $E_{\eta,\theta}$ is the constant of proportionality (the "extensibility") of the fabric under the given conditions. In view of equations (11) and (12), the quantities $\epsilon_{\eta,\theta}$ and $\epsilon_{\theta+90°,\theta}$ can be easily calculated:

$$\left.\begin{aligned}\epsilon_{\theta,\theta}&=\frac{2Z_o^2}{3b^2},\\ \epsilon_{\theta-90°,\theta}=\epsilon_{\theta+90°,\theta}&=\frac{2Z_o^2}{3a^2}\end{aligned}\right\} \tag{17}$$

When $Z'_o=\sqrt{2}Z_o$, by virtue of (17)

$$\epsilon_{\eta,\theta}=2\epsilon_{\eta,\theta} \tag{18}$$

Now,

$$S'_{\eta,\theta}=S_\eta+E_{\eta,\theta}\epsilon'_{\eta,\theta} \tag{16b}$$

provided this increased strain is still within the limits of practical proportionality of change of stress to change of strain.

From (18) this last equation then becomes

$$S'_{\eta,\theta}=S_\eta+2E_{\eta,\theta}\epsilon_{\eta,\theta} \tag{16c}$$

In terms of the above notation, (14) can now be written

$$P_\theta=2\alpha S_{\theta,\theta}+2\beta S_{\theta+90°,\theta} \tag{19}$$

Substituting for $S_{\theta,\theta}$ and $S_{\theta+90°,\theta}$ their values from (16a), equation (19) becomes, for the minor axis of the instrument in the direction θ_o

$$P_{\theta_o}=2\alpha(S_{\theta_o}+E_{\theta_o,\theta_o}\epsilon_{\theta_o,\theta_o})+2\beta\,(S_{\theta_o+90°}+E_{\theta_o+90°,\theta_o}\epsilon_{\theta_o+90°,\theta_o}) \tag{20a}$$

and similarly

$$\frac{P'_{\theta_o}}{\sqrt{2}}=2\alpha\,(S_{\theta_o}+2E_{\theta_o,\theta_o}\epsilon_{\theta_o,\theta_o})+2\beta\,(S_{\theta_o+90°}+2E_{\theta_o+90°,\theta_o}\epsilon_{\theta_o+90°,\theta_o}) \tag{21a}$$

Combining equations (20a) and (21a)

$$2P_{\theta_o}-\frac{P'_{\theta_o}}{\sqrt{2}}\equiv\pi_{\theta_o}=2\alpha S_{\theta_o}+2\beta S_{\theta_o+90°} \tag{22a}$$

Similarly when the minor axis of the instrument is placed in the direction $\theta_o+90°$, (20a), (21a), and (22a) become, respectively,

$$P_{\theta_o+90°}=2\alpha\,(S_{\theta_o+90°}+E_{\theta_o+90°,\theta_o+90°}\epsilon_{\theta_o+90°,\theta_o+90°})+2\beta\,(S_{\theta_o}+E_{\theta_o,\theta_o+90°}\epsilon_{\theta_o,\theta_o+90°}) \tag{20b}$$

and

$$\frac{P'_{\theta_o+90°}}{\sqrt{2}}=2\alpha\,(S_{\theta_o+90°}+2E_{\theta_o+90°,\theta_o+90°}\epsilon_{\theta_o+90°,\theta_o+90°})+2\beta\,(S_{\theta_o}+2E_{\theta_o,\theta_o+90°}\epsilon_{\theta_o,\theta_o+90°}) \quad (21b)$$

and therefore

$$\pi_{\theta_o+90°}=2\alpha S_{\theta_o+90°}+2\beta S_{\theta_o} \quad (22b)$$

since $S_{\theta_o}=S_{\theta_o+180°}$ and α and β are independent of θ.

Thus, by taking four pressure readings, P_{θ_o}, P'_{θ_o}, $P_{\theta_o+90°}$, and $P'_{\theta_o+90°}$, the effect of the stretching is eliminated, and hence a system of simultaneous equations (22a) and (22b) results which is equivalent to that of (3). Once the quantities π_{θ_o} and $\pi_{\theta_o+90°}$ are obtained, S_{θ_o} and $S_{\theta_o+90°}$ can be readily evaluated by means of a properly constructed nomogram.

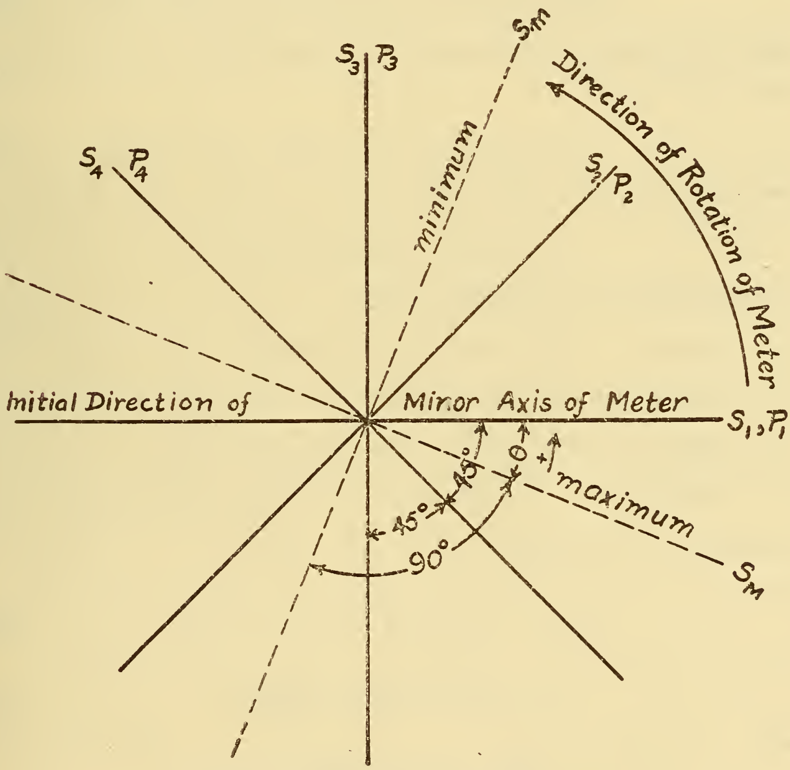

FIG. 5.—*Application to multiple-ply fabric*

3. CASE II. DIRECTIONS OF THE MAXIMUM AND MINIMUM STRESSES ARE NOT KNOWN

Let the maximum and minimum stresses be represented by S_M and S_m, respectively (see fig. 5). Then since the angle between the directions of S_M and S_{θ_o} is θ_o

$$S_m \sin^2\theta_o+S_M \cos^2\theta_o=S_{\theta_o} \quad (23)$$

This last expression can be verified by cutting an infinitesimal right prism from the fabric so that the face with the larger area is normal to S_{θ_o}, the remaining two faces are normal to S_M and S_m, while the base is subjected to the hydrostatic pressure. Consider the equilibrium of the prism, remembering at the same time that the force due to the hydrostatic pressure is of an order smaller than the forces arising from S_{θ_o}, S_M, and S_m.

Equation (23) can be also written as

$$(S_M + S_m) + (S_M - S_m) \cos 2\theta_o = 2S_{\theta_o} \tag{24}$$

Increasing θ_o by 90°

$$(S_M + S_m) - (S_M - S_m) \cos 2\theta_o = 2S_{\theta_o + 90°} \tag{25}$$

Hence (22a) becomes, on substituting in the values of S_{θ_o} and $S_{\theta_o+90°}$ from (24) and (25)

$$\pi_{\theta_o} = (\alpha + \beta)(S_M + S_m) + (\alpha - \beta)(S_M - S_m) \cos 2\theta_o \tag{26}$$

Now, replacing θ_o in (26) successively by the values (see fig. 5)

$$\theta_o + 45°,\ \theta_o + 90° \text{ and } \theta_o + 135°$$

the following set of equations results:

$$\left.\begin{aligned} \pi_1 &\equiv \pi_{\theta_o} = (\alpha + \beta)(S_M + S_m) + (\alpha - \beta)(S_M - S_m) \cos 2\theta_o \\ \pi_2 &\equiv \pi_{\theta_o + 45°} = (\alpha + \beta)(S_M + S_m) - (\alpha - \beta)(S_M - S_m) \sin 2\theta_o \\ \pi_3 &\equiv \pi_{\theta_o + 90°} = (\alpha + \beta)(S_M + S_m) - (\alpha - \beta)(S_M - S_m) \cos 2\theta_o \\ \pi_4 &\equiv \pi_{\theta_o + 135°} = (\alpha + \beta)(S_M + S_m) + (\alpha - \beta)(S_M - S_m) \sin 2\theta_o \end{aligned}\right\} \tag{27}$$

Solving for θ_o, S_M and S_m:

$$\cot \theta_o = -\frac{\pi_1 - \pi_3}{\pi_2 - \pi_4} \text{ or } \tan \theta_o = -\frac{\pi_2 - \pi_4}{\pi_1 - \pi_3} \tag{28}$$

$$S_M + S_m = \frac{\pi_1 + \pi_3}{2(\alpha + \beta)} = \frac{\pi_2 + \pi_4}{2(\alpha + \beta)} \tag{29}$$

and

$$S_M - S_m = \frac{\sqrt{(\pi_1 - \pi_3)^2 + (\pi_2 - \pi_4)^2}}{2(\alpha - \beta)} \tag{30}$$

Equation (28) gives the direction of the maximum stress with reference to the direction of the minor axis in measuring π_{θ_o}, while (29) and (30) together determine the magnitudes of maximum and minimum stresses.

It should be observed that in the system of equations (28) there are four equations but only three unknown quantities. Here the use of the additional equation is to facilitate the calculation of the unknown quantities.

V. APPLICATION OF THE THEORY

In the use of the fabric tension meter four cases may arise:

1. Single-ply fabric, directions of principal stresses known, stress-strain relation of fabric known (two readings).
2. Single-ply fabric, directions of principal stresses known, stress-strain relation of fabric not known (four readings).
3. Multiple-ply fabric, directions of principal stresses not known, stress-strain relation of fabric known (four readings).
4. Multiple-ply fabric, directions of principal stresses not known, stress-strain relations of fabric not known (eight readings).

1. CASE I

This is the case which is most common in the practical use of the instrument. For example, the tension meter has been used to investigate the tensions in the outer cover fabric of the United States Navy rigid airships *Shenandoah* and *Los Angeles*. The single-ply fabric[3] on the *Shenandoah* has been studied in the laboratory, and a characteristic value of its "extensibility" is known. Therefore, it is merely necessary to make two applications of the instrument to the fabric at the point where a knowledge of the stresses is desired, placing the minor axis of the instrument successively in the direction of each set of threads, and measuring the pressures P_{θ_o} and $P_{\theta_o+90°}$ necessary to deflect the fabric by the amount Z_o. In this case θ_o is equal either to 0 or 90°.

Equations (20a) and (20b) apply to this case:

$$P_{\theta_o} = (2\alpha S_{\theta_o} + 2\beta S_{\theta_o+90°}) + (2\alpha E_{\theta_o,\theta_o}\epsilon_{\theta_o,\theta_o} + 2\beta E_{\theta_o+90°,\theta_o}\epsilon_{\theta_o+90°,\theta_o}) \quad (20a)$$

and

$$P_{\theta_o+90°} = 2\alpha S_{\theta_o+\epsilon_o°} + 2\beta S_{\theta_o}) + (2\alpha E_{\theta_o+90°,\theta_o+90°}\epsilon_{\theta_o+99°,\theta_o+90°} + 2\beta E_{\theta_o,\theta_o+90°}\epsilon_{\theta_o,\theta_o+90°}) \quad (20b)$$

Now from (17)

$$\epsilon_{\theta_o,\theta_o} = \epsilon_{\theta_o+90°,\theta_o+90°} = \frac{2Z_o^2}{3b^2}$$

and

$$\epsilon_{\theta_o+90°,\theta_o} = \epsilon_{\theta_o,\theta_o+90°} = \frac{2Z_o^2}{3a^2}$$

and if, as is usually the case, an average value is taken for $E_{\eta,\theta}$[4].

---

[3] This was grade "BB" cotton fabric treated first with acetate clear dope and finally with aluminum acetate dope.

[4] For the outer cover fabric of the *Shenandoah* the value of E was taken as 680 pounds per inch.

Since $\frac{2Z_o^2}{3b^2}$ and $\frac{2Z_o^2}{3a^2}$ are independent of θ, it follows that

$$\left.\begin{aligned} P_{\theta_o} &= (2\alpha S_{\theta_o} + 2\beta S_{\theta_o+90^\circ}) + \left\{2\alpha E\left(\frac{2Z_o^2}{3b^2}\right) + 2\beta E\left(\frac{2Z_o^2}{3a^2}\right)\right\} \\ P_{\theta_o+90^\circ} &= (2\alpha S_{\theta_o+90^\circ} + 2\beta S_{\theta_o}) + \left\{2\alpha E\left(\frac{2Z_o^2}{3b^2}\right) + 2\beta E\left(\frac{2Z_o^2}{3b^2}\right)\right\} \end{aligned}\right\} \quad (31)$$

or

$$\left.\begin{aligned} P_{\theta_o} &= (2\alpha S_{\theta_o} + 2\beta S_{\theta_o+90^\circ}) + C \\ P_{\theta_o+90^\circ} &= (2\alpha S_{\theta_o+90^\circ} + 2\beta S_{\theta_o}) + C \end{aligned}\right\} \quad (32)$$

The value of C can be computed from (31) and proves to be equal to 3 inches head of water for the cover fabric of the *Shenandoah.* Figure 6 represents the nomogram prepared on the basis of equation (32) to use in measuring the tension of the outer cover fabric of the *Shenandoah.*

2. CASE II

If no information is available as to the "extensibility" of the fabric, equations (22a) and (22b) are applicable:

$$2P_{\theta_o} - \frac{P'_{\theta_o}}{\sqrt{2}} \equiv \pi_{\theta_o} = 2\alpha S_{\theta_o} + 2\beta S_{\theta_o+90^\circ} \quad (22a)$$

$$2P_{\theta_o+90^\circ} - \frac{P'_{\theta_o+90^\circ}}{\sqrt{2}} \equiv \pi_{\theta_o+90^\circ} = 2\alpha S_{\theta_o+90^\circ} + 2\beta S_{\theta_o} \quad (22b)$$

Now, placing the instrument in the direction of one set of threads, two readings are taken in succession of the differential pressure required to deflect the fabric by amounts Z_o and $\sqrt{2}Z_0$. These readings are called P_{θ_0} and P'_{θ_0}. The instrument is then rotated through an angle of 90° and two more readings, $P_{\theta_0+90^\circ}$ and $P'_{\theta_0+90^\circ}$ are obtained for the deflections Z_0 and $\sqrt{2}Z_0$. The computations indicated by the left members of (22a) and (22b) are made and the nomogram shown in Figure 6 is used to obtain S_{θ_0} and $S_{\theta_0+90^\circ}$.

This computation can be simplified by the expedient of having two concentric scales on the gauge which measures the reduction of pressure in the chamber of the instrument. The first scale would be graduated to give the value of P directly. The second scale would be graduated, not to give directly the value of P', but rather the value of $\frac{P'}{\sqrt{2}}$. In this way only a subtraction would be required to obtain the value of each π.

3. CASE III

This case is not important in practice and will not be discussed here.

4. CASE IV

When multiple-ply fabric is in use and when no information is available as the value of the "extensibility" of the fabric, equations (28), (29), and (30) are used to obtain the values of S_M, S_m, and the angle which S_M makes with the initial position of the minor axis of the tension meter.

The instrument is applied to the fabric with its minor axis in a convenient direction which should be marked for the purpose of

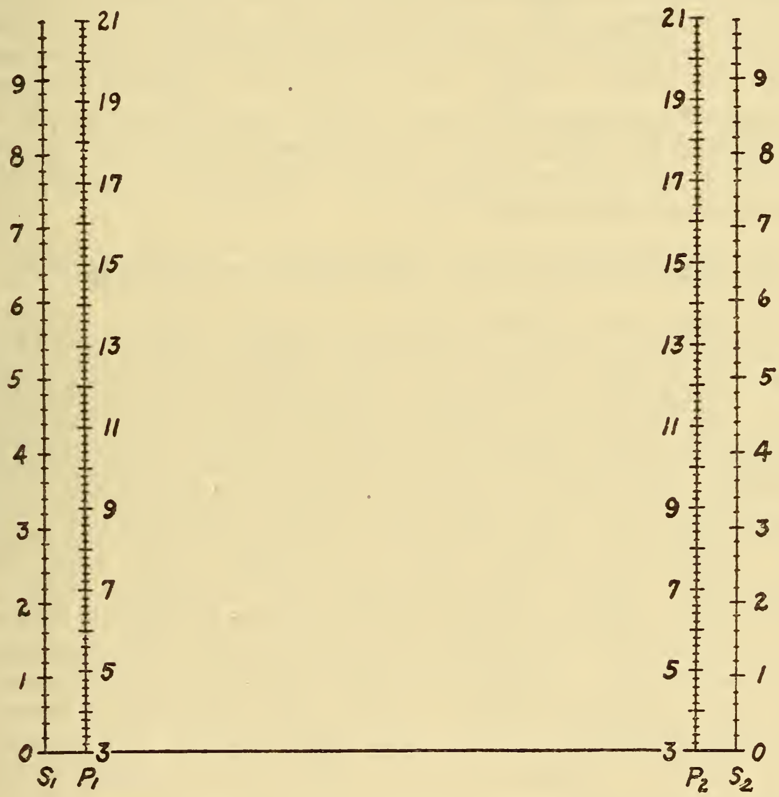

FIG. 6.—*Nomogram for computing stresses in the outer fabric of the Shenandoah*

locating the direction of the maximum principal stress. Two readings are taken as in case 2, and the value of $\pi_1 = \pi_{\theta_0}$ is computed. The instrument is then rotated through 45° and two more readings are taken, giving $\pi_2 = \pi_{\theta_0+45°}$. By rotating the instrument through 45° again $\pi_3 = \pi_{\theta_0+90°}$ is obtained, and still another 45° rotation serves to give $\pi_4 = \pi_{\theta_0+135°}$. From equations (29) and (30) the values of S_M and S_m are obtained. The value of θ_0 is determined when $\cot \theta_0$ is found from (28).

Adopting the convention that $-90° < \theta_0 < +90°$, if $\cot \theta_0$ proves to be positive, this means that the instrument was rotated in the direction of increasing positive values of θ_0. Since θ_0 was defined as the angle which the minor axis of the instrument made initially with the direction of the maximum stress, θ_0 is measured from the direction of the maximum stress to the initial direction of the minor axis. However, in laying off on the fabric the direction of the maximum stress we are measuring from the original direction of the minor axis of the instrument instead of from the direction of the maximum stress. Hence, if θ_0 proves to be positive it should be laid off in the opposite direction to that in which the fabric tension meter was rotated. Figure 4 and equation (27) illustrate this case. If θ_0 proves to be negative, then it should be laid off from the initial direction of the minor axis of the instrument in the same direction in which the instrument was rotated. It should be remembered that θ_0 is never greater than 90°.

VI. DESCRIPTION AND OPERATION OF INSTRUMENT

The fabric tension meter is shown in Figures 1 and 2. The body of the instrument consists of an elliptical chamber open at the bottom and having a hollow suction rim extending around it. A large hollow handle *H* is placed at the center of the instrument for the use of the operator and small handles *H′* are placed at the ends for the use of an assistant. The suction required to operate the instrument is obtained from a line connecting with a supply tank and pump. The line is attached to the connection *C*, mounted on the handle *H*. The suction passes to the combined rim and chamber valve *V*. When the rim valve is opened, a partial vacuum is created in the suction rim and, if the instrument is placed against the fabric properly, it will adhere there. The rim gauge *RG* indicates in inches of mercury the suction existing in the rim. A safety valve *RS* prevents the suction from increasing beyond a predetermined point, and thus protects the fabric from injury due to its deflection into the small suction holes of the rim.

When the instrument is placed against the fabric, the ball *B* on the deflection gauge *DG* is just in contact with the fabric. If the pointer of the deflection gauge then does not read zero, the scale is adjusted by means of the thumbscrew *AS*. The rim valve is opened, causing the fabic to adhere to the face of the instrument; then the chamber needle valve is opened, gradually producing a decrease in pressure in the chamber. This causes the fabric to deflect inward, carrying the ball *B* with it, thus operating the pointer of the deflection gauge *DG*. When the indicated deflection is one-eighth inch, the chamber valve is adjusted to maintain this reading constant while the operator reads the suction gauge *SG*, which measures the pressure reduction in the chamber. After the reading has been taken the operator releases the suction in the rim by turning the rim valve, and the instrument can be detached from the fabric. In order to prevent the fabric from being deflected too far into the chamber, a second safety valve *CS* is provided which is tripped by the fabric after the latter has deflected a little more than one-eighth inch.

The deflection gauge is operated by the aluminum ball *B*, which is mounted on the end of a lever. This lever operates the pointer by means of two pulleys and a nichrome strip. It is planned to redesign this gauge in order to eliminate the use of the strip, which is likely to break occasionally.

The suction gauge *SG*, used to measure the suction in the chamber, is a specially designed gauge having a slack leather diaphragm pressure element. The gauge was so designed as to have an open scale for low suctions and a more compressed scale for the higher suctions. This was done to meet the requirement of the Bureau of Aeronautics that the instrument should be capable of measuring with comparative accuracy tensions from zero to approximately 8 pounds per inch (this latter tension being a little less than that at which the dope film breaks) and indicating with fair accuracy tensions from 8 to 20 pounds per inch, these tensions being, of course, excessive. This gradual compressing of the graduations with increase in suction was accomplished by designing properly the slack diaphragm pressure element by a method recently developed at the Bureau of Standards.[5] A helical spring was used to restrain the motion of the diaphragm in the gauge. This spring proved very satisfactory, although less capable of adjustment than flat or U-shaped springs. The position of this spring could be varied by means of the thumbscrew *T*. A hole was drilled through the bezel of this gauge, so that atmospheric pressure exists in the instrument above the diaphragm. The scale shown on the suction gauge is not the one at present in use on the tension meter, since the range of the suction gauge was increased to 20 inches of water after the photograph was taken.

The two safety valves are of different types. The rim safety valve *RS* contains a ball forced against a seat by a helical spring. When the suction in the rim decreases below a certain amount, the excess of the atmospheric pressure over the pressure in the rim exerts a sufficient force on the ball to overcome the force of the spring and the ball is unseated, allowing air to flow into the chamber. The force exerted by the spring is adjustable, so that the safety valve can be set for any desired suction over a wide range. The adjustment at the time when the instrument was delivered to the Bureau of Aeronautics

[5] H. N. Eaton and C. T. Buckingham, "Nonmetallic diaphragms for instruments." National Advisory Committee for Aeronautics Technical Report No. 206; 1925.

was such that the safety valve operated at suction of approximately 15 inches of mercury. When the suction was applied to the rim, the pointer of the rim gauge *RG* would creep up to a point approximately at the graduation "15" and would remain at that point, the ball in the suction valve vibrating rapidly back and forth, thus allowing air to flow into the rim at a sufficient rate to maintain the suction constant. The valve was so designed as to be affected but slightly by changes in the position of the instrument.

The chamber safety valve *CS* was constructed differently, owing to the fact that the suction in the chamber was not sufficient to operate a ball valve in a satisfactory manner. A small plunger is set a little more than one-eighth inch above the plane of the rim so that the fabric, if deflected more than one-eighth inch, trips the plunger and opens a small valve connecting to the outer air through the line *L'*. This valve operates very well, but would be more of a safeguard if it had greater capacity.

VII. ACCURACY OF INSTRUMENT

Laboratory tests have shown the fabric-tension meter to give very satisfactory results when used on single-ply fabric for which an average value of the "modulus" was known; that is, when the instrument was used under the conditions described for Case I under "Application of the theory." The results given below may be taken as characteristic of the performance of the instrument when used under these conditions:

Tension (pounds per inch of width):	Error (per cent)
0.5–1	60–15
1–3	15–10
3–5	10– 5
5–8	5– 0

As would be expected, local variations in the "modulus" of the fabric cause relatively large errors at the lowest tensions. The tensions which have actually been measured in practice with this instrument have usually ranged from 2½ to 7 pounds per inch; so it will be seen that the accuracy obtained is quite satisfactory.

Washington, January 19, 1926.