TESTS OF HEAVILY REINFORCED CONCRETE SLAB BEAMS; EFFECT OF DIRECTION OF REINFORCEMENT ON STRENGTH AND DEFORMATION.

By Willis A. Slater and Fred B. Seely.

ABSTRACT.

The tests here reported were made as a part of the investigational work of the concrete ship section of the Emergency Fleet Corporation for the purpose of determining the relative economy of using as reinforcement for the shell of the ship bars placed at right angles to the frames which support the shell, bars placed at some other angle with the frames, and expanded metal. These three types of reinforcement were used in slab beams which were about 4 inches thick, 27½ inches wide, 5 feet between supports, and which were supported on two edges only. The loads were applied at the one-third points of the span. Analysis of this type of structure indicates that with the same percentages of reinforcement in all cases the stresses and deflections will be greater the greater the angle between the direction of the span and the direction of the reinforcement. The results agree in a general way with the indications from the analysis. To this extent they indicate that the most economical reinforcement for a slab supported on only two edges is that whose direction is the same as the direction of the span. The tests give no information on the value of diagonal reinforcement in resisting stresses parallel to the supports due to shrinkage, temperature changes, etc., or in resisting the stresses set up when the slabs are supported on four sides.

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I. INTRODUCTION.

1. PRELIMINARY STATEMENT.

This paper presents the results of tests of 26 slabs, tested as simple beams, having four types of reinforcement. The slab beams were approximately 4 inches thick and 27½ inches wide; and the four types of reinforcement consisted of (*a*) plain round bars laid direct between supports, referred to in this paper as *direct reinforcement*; (*b*) diamond mesh expanded metal with the long dimensions of the diamonds parallel to the direction of the span; (*c*) plain round bars laid in two bands making 45° with the direction of the span of the beam and 90° with each other, referred to in this paper as *two-way diagonal reinforcement*; and (*d*) plain round bars laid in one band making 45° with the direction of the span, referred to in this paper as *one-way diagonal reinforcement.*

The tests were made at the John Fritz civil engineering laboratory of Lehigh University as part of the investigative work of the concrete ship section of the Emergency Fleet Corporation. The structural laboratory investigative work of the concrete ship section was in charge of Mr. Slater, of the Bureau of Standards, who was detailed to the Emergency Fleet Corporation. The laboratory work at Lehigh University was started under the direction of Maj. W. M. Wilson and completed under the direction of Maj. A. R. Lord. The making of the tests reported in this paper was in charge of G. G. Scofield, and the report of the tests to the Emergency Fleet Corporation was prepared by H. R. Thomas. Messrs. Thomas and Scofield were structural engineers of the Emergency Fleet Corporation. Mr. Thomas's report has been of great value in the preparation of this paper.

An abstract of an article by E. Suenson, of Copenhagen, is included as an appendix. The field covered by Professor Suenson's analysis and tests is practically the same as that of this paper. No other data of tests of this kind are known to the writers, and therefore Suenson's article is of considerable interest.

2. PURPOSE OF THE TESTS.

In the design of a concrete ship, there are conditions that make it desirable to use diagonal reinforcement in the sides of the ship. But in a slab beam the stress in the bars of diagonal reinforcement for a given load is considered to be greater than the stress that would occur in the same bars if laid parallel to the direction of the span; the stress is generally assumed to be greater in proportion to the secant of the angle that the diagonal bars make with the direction of the span. This method is, of course, applicable in determining the stress in diagonal members of pin-connected structures, such as a bridge truss, and is frequently referred to as the *secant method*.

If the assumption is correct that the secant method is applicable to diagonal reinforcement in concrete beams, then, in order that a slab beam having 45° diagonal reinforcement shall develop a resisting moment equal to that in a similar beam having direct reinforcement, approximately twice [1] the weight or volume of steel will be required. This increase in weight is objectionable in a concrete ship, particularly since the percentage of reinforcement is high.

Since no test results were available to indicate whether or not diagonal reinforcement takes stress in accordance with the secant method of computation, the tests herein reported were made to obtain information on the behavior of diagonal reinforcement consisting of unconnected bars and of expanded metal in slab beams, and to obtain data by which a comparison of the value of the various types of reinforcement mentioned above could be made.

The tests were planned with the view of making the test results as directly applicable to concrete ship design as possible. There are certain features of the tests, therefore, which vary somewhat from the conditions that usually obtain in concrete structural design. For example, a very rich mix and a large percentage of reinforcement were used, and the slabs were relatively thin. These conditions have rendered it difficult to make a refined analysis of the test data, since the position of the reinforcement in the slab beam and the variation of the stress on a cross section of the beam become important factors in the analysis, but factors that can not be known with certainty. However, although the

[1] Somewhat less than twice the weight would be required since with diagonal reinforcement the value k (the proportionate depth to the neutral axis) for a given ratio of reinforcement is smaller than for direct reinforcement, and hence jd is larger and the stress correspondingly less.

conclusions suggested in this paper may not be as well established as is desirable, several methods of analysis have led to substantially the same results, and the test results give valuable information on a number of important features concerning reinforced concrete design.

3. ANALYSIS OF BEAMS WHOSE REINFORCEMENT MAKES AN ANGLE θ WITH THE SPAN.

In the analysis of beams in which diagonal reinforcement is used many of the conditions are so different from the case usually considered, in which the reinforcing bars have the same direction as the span of the beam, that it seems desirable to show here the derivation of formulas for the properties of a beam in which the reinforcement is so placed that the axis of the bar (or the axis of a strand, as in the case of expanded metal) makes an angle, θ with the direction of the span.

Usually it is assumed that all the stress in the tension reinforcement is concentrated at the center of gravity of the cross section of the reinforcement. For beams of usual proportions this is a sufficiently close approximation, but in the shallow beams here discussed in which the distance between layers of reinforcement was a considerable proportion of the depth of the beam, the error involved in such an assumption may be of importance. It was assumed in this analysis that for each layer of reinforcement the stress was constant over its section and proportional in intensity to the distance of the center of gravity of the section from the neutral axis. There were some indications in the tests that this was not strictly true, but the information available was not sufficiently extensive to afford a basis for any other assumption which would give promise of securing more exact results than the assumption used.

This analysis uses as the depth, d, the distance from the compression surface to the centroid of the area of the tension reinforcement. For exactness the depth, d, should be the distance from the compression surface to the centroid of the tensile stresses, but since in order to determine that distance it is necessary first to know the depth to the neutral axis which is dependent on the percentage of reinforcement, and since this in turn is dependent on the depth, d, it is apparent that the problem is indeterminate and for ordinary cases the use of the depth to the centroid of the tension area will be satisfactory.

A more exact value may be obtained, if desired, by successive approximations as follows: An approximate value of the proportionate depth, k, to the neutral axis may be derived from equation (9), in which p is based upon the depth to the centroid of the tension area. Making use of the value of k so obtained, the distance δ from the centroid of the areas to the centroid of the stresses may be determined very closely from equation (11). This distance added to the assumed depth (the distance from the compression surface to the centroid of the tension areas) gives very closely the desired depth to the centroid of tensile stresses. From this depth a new percentage of reinforcement may be determined from equation (1) and a more exact value of k. The values so obtained will be nearly correct. Any higher degree of accuracy may be obtained by a continued process of approximation.

Since the ratio, by volume, of reinforcement to concrete affords a direct measure of the relative quantity of the two materials required in any beam, this ratio has been used in making the comparison between beams with direct reinforcement and beams with diagonal reinforcement. Although in the analysis of reinforced concrete beams the ratio (or percentage) of reinforcement is usually considered as a ratio of cross-sectional areas, it is also, in fact, a ratio by volume, hence the procedure used in this analysis is not a departure from the usual method but an extension to cover the more general case.

In the derivations which follow it was assumed that in two beams, similar except for the direction of reinforcement, the stress in the reinforcement is proportional to the secant of the angle θ when the beams are loaded similarly and in equal amounts. Except as stated above the analysis was based upon the usual assumptions for reinforced concrete beams. These are as follows:

1. A plain section before bending remains plain after bending.
2. The modulus of elasticity in compression as well as that in tension is constant. The stress distribution is, therefore, rectilinear.
3. In calculating the moment of resistance the tensile resistance of the concrete is neglected.
4. Adhesion between the concrete and reinforcement is perfect.

The notation employed where not apparent from the figures is as follows:

1. The subscripts $_1$ and $_2$ assign to any two beams 1 and 2 the dimensions or properties to which they are attached.

A_s = right sectional area of steel in width, b, of beam, where b is measured perpendicular to the direction of the span.

$n = \frac{E_s}{E_c}$ = ratio of modulus of elasticity of steel to that of concrete.

F = total flexural stress (either tension or compression) in direction of span.

f_s = tensile unit stress in steel.

f_c = compressive unit stress in concrete.

ϵ_l = unit deformation in the direction of the span on the tension side of the beam.

ϵ_c = unit deformation in the direction of the span in the extreme fiber in compression.

M = bending moment in the direction of the span.

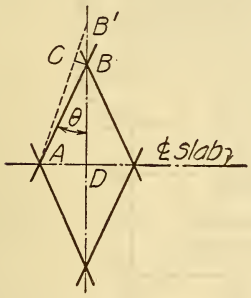


FIG. 1.—*Sketch illustrating deformation in a diagonal reinforcing bar under stress.*

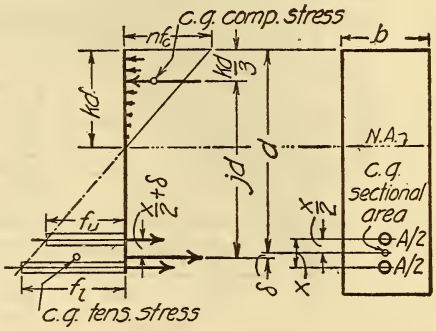


FIG. 2.—*Sketch illustrating deformation in a beam having two layers of tensile reinforcement.*

W = total load on beam.

l = length of span.

K = constant for bending moment, dependent on manner of loading.

q = constant for deflection, dependent on manner of loading.

(*a*) EXPRESSION FOR VOLUME RATIO OF REINFORCEMENT, p, AND RIGHT SECTIONAL AREA OF REINFORCEMENT, A_s (see Fig. 3).

$$p = \frac{A_s l/\cos\theta}{bdl} = \frac{A_s}{bd\cos\theta} \tag{1}$$

$$A_s = pbd\cos\theta. \tag{2}$$

(*b*) EXPRESSIONS FOR DEFORMATION ϵ_l IN THE DIRECTION OF THE SPAN IN TERMS OF THE TENSILE STRESS IN THE REINFORCEMENT, WHEN THE REINFORCEMENT MAKES AN ANGLE θ WITH THE SPAN (see Fig. 1).—Let A be a point on reinforcing bar at

the center of the span. It is evident, from symmetry, that it will not move (except vertically) when the bar is stressed due to bending. Let B be original position of any other point on the same bar. Neglecting Poisson's ratio, all points in the reinforcement will move in the direction of the span as the load comes on,

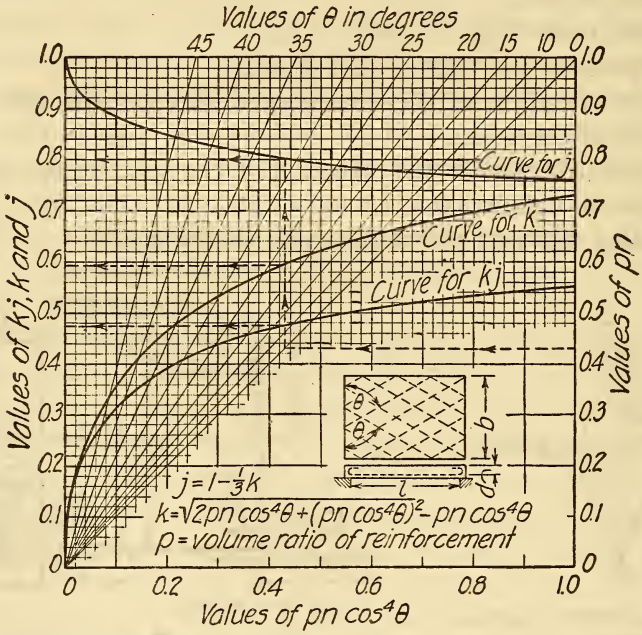


FIG. 3.—*Diagram giving values of k, j, and kj for varying values of p, n, and θ.*

The values of k and j given in Fig. 3 are from the equations given on the diagram. The values of kj will be of convenience in computing compressive stresses in the concrete by means of the usual formulas. Entering the diagram at the right with the correct value of pn proceed horizontally as indicated by the arrow to the intersection with the inclined line representing the correct value of θ. This intersection gives the value of $pn \cos^4\theta$ (shown as abscissas) Thence proceed vertically to the intersection with the curve representing the function desired The value of the function may then be read directly from the scale at the left The values of $pn \cos^4\theta$ having been included in the above operation need not be further considered in using the diagram.

and B will take the new position B'. Approximately ABC, ACB, and BCB' are right angles and $BB'C = \theta$.

$$E_l = \frac{BB'}{BD} \qquad BB' = \frac{CB'}{\cos\theta} \qquad BD = AB\cos\theta = AC\cos\theta \tag{3}$$

$$\therefore \epsilon_l = \frac{CB'}{AC}\,\frac{1}{\cos^2\theta} = \frac{f_s}{E_s\cos^2\theta}. \tag{4}$$

(*c*) EXPRESSION FOR THE PROPORTIONATE DEPTH, k, TO THE NEUTRAL AXIS (see Figs. 2 and 3).

$$F = A_s f_s \cdot \cos\theta = pbd\cos\theta f_s \cdot \cos\theta = \frac{f_c}{2}\,kbd \tag{5}$$

since total tension is equal to total compression.

$$\frac{f_c}{2}k = pf_s \cos^2\theta, \tag{6}$$

$$\frac{\frac{f_c}{E_c}}{kd} = \frac{\frac{f_s}{E_s \cos^2\theta}}{(1-k)\,d} \tag{7}$$

$$\frac{1}{2}k^2E_c = pE_s\,(1-k)\,\cos^4\theta, \text{ and} \tag{8}$$

$$k = \sqrt{2pn\cos^4\theta + (pn\cos^4\theta)^2} - pn\cos^4\theta. \text{ (See Fig. 3).} \tag{9}$$

(*d*) Expression for the Moment Arm, *jd*, for a Beam in Which Half the Reinforcement is Placed in Each of Two Horizontal Layers, Having a Vertical Distance, *x*, Between Them.—Taking moments about center of upper layer of bars (see Fig. 2).

$$xf_l\frac{A}{2} = \left(\frac{x}{2} + \delta\right)\left(\frac{f_l + f_u}{2}\right)A: \tag{10}$$

$$\delta = \frac{\frac{x}{2}\left(f_l - f_u\right)}{f_l + f_u} = \frac{x}{2}\cdot\frac{x}{2\,(d-kd)}, \text{ since} \tag{11}$$

$$\frac{f_l}{f_u} = \frac{(d-kd) + \frac{x}{2}}{(d-kd) - \frac{x}{2}} \text{ and } \frac{f_l - f_u}{f_l + f_u} = \frac{x}{2\,(d-kd)} \tag{12}$$

$$jd = d - \frac{d}{3}k + \delta = \left(1 - \frac{1}{3}k + \frac{x^2}{d^2}\cdot\frac{1}{4\,(1-k)}\right)d. \tag{13}$$

(*e*) Relation Between Direction of Reinforcement, and Tensile and Compressive Stresses in Two Beams when the Applied Moments are Equal.

Since $M = Fjd = f_s pjbd^2 \cos^2\theta$

$$f_s = \frac{M}{pjbd^2\cos^2\theta} \tag{14}$$

$$\therefore \frac{f_{s1}}{f_{s2}} = \frac{p_2j_2b_2d_2^{\,2}\cos^2\theta_2}{p_1j_1b_1d_1^{\,2}\cos^2\theta_1}. \tag{15}$$

Likewise,

$$f_c = \frac{2M}{kjbd^2} \tag{16}$$

$$\frac{f_{c1}}{f_{c2}} = \frac{k_2j_2b_2d_2^{\,2}}{k_1j_1b_1d_1^{\,2}} \tag{17}$$

$$\frac{f_c}{f_s} = \frac{2p\cos^2\theta}{k}. \tag{18}$$

Figure 4 has been prepared to show the effect of the direction of the reinforcement on the compressive stresses in beams all of which have the same computed tensile stress in the reinforcement when the applied moments are the same. A beam having 1 per cent of direct reinforcement is used as the basis of comparison. The lower curve indicates the amount of diagonal reinforcement needed to keep the tensile stress the same for all angles, θ, up to 45°. The upper curve shows the ratio of the compressive stress to the tensile stress. Since the tensile stress is constant as long as the applied bending moment is constant, the ordinates to the

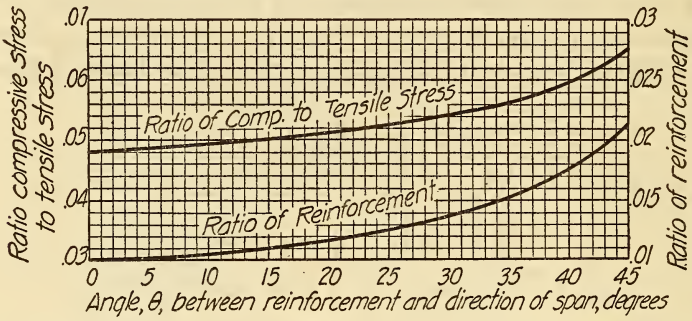


FIG. 4.—*Diagram showing ratio of compressive to tensile stresses in beams with diagonal reinforcement equivalent to 1 per cent direct reinforcement.*

upper curve give also a measure of the increase in the compressive stress with increase in the angle θ.

(*f*) RELATION BETWEEN LOADS CARRIED BY ANY TWO BEAMS,

$$M = KWl = pf_s jbd^2 \cos^2 \theta \tag{19}$$

$$\frac{W_1}{W_2} = \frac{l_2 p_1 f_{s1} j_1 b_1 d_1^2 \cos^2 \theta_1}{l_1 p_2 f_{s2} j_2 b_2 d_2^2 \cos^2 \theta_2} \tag{20}$$

(*g*) RATIO OF DEFLECTIONS, $\frac{\Delta_1}{\Delta_2}$, WHERE Δ_1 AND Δ_2 ARE THE DEFLECTIONS OF BEAMS OF DIFFERENT DIMENSIONS HAVING REINFORCEMENT AT DIFFERENT ANGLES, θ_1 AND θ_2, WITH THE DIRECTION OF THE SPAN.

$$\Delta = q\frac{l^2}{d}(\epsilon_l + \epsilon_c). \quad \text{(See footnote 2.)} \tag{21}$$

From equation (4)

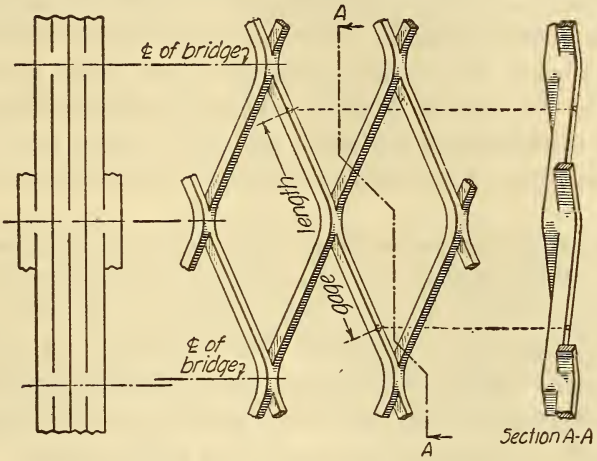
$$\epsilon_l = \frac{f_s}{E_s \cos^2 \theta} \tag{22}$$

$$\epsilon_c = \epsilon_l \frac{k}{1-k} = \frac{f_s}{E_s \cos^2 \theta} \cdot \frac{k}{1-k} \tag{23}$$

Substitute values of ϵ_l ϵ_c and $\frac{f_{s1}}{f_{s2}}$ in equation (21) and solve for $\frac{\Delta_1}{\Delta_2}$,

$$\frac{\Delta_1}{\Delta_2}=\frac{l_1^2\, b_2 d_2^3 j_2 p_2 \cos^4\theta_2\ (1-k_2)}{l_2^2 \cdot b_1 d_1^3 j_1 p_1 \cos^4\theta_1\ (1-k_1)} \tag{24}$$

(*h*) Expressions for Volume Ratio and Sectional Areas for Expanded Metal.—Let w be the weight in pounds per square foot of expanded metal of the form shown in Figure 5. Assume the weight per cubic inch of steel as 0.284 pound. The ratio of the volume of the steel in a beam reinforced with ex-



Note: Proportions shown are approximately those for expanded metal having sectional area of 1 sq. in. per foot of width.

Fig. 5.—*Portion of metal sheet illustrating deformations due to the expanding process.*

panded metal to the volume of the concrete above the reinforcement is

$$p=\frac{w}{0.284\times 144d}=\frac{w}{40.9d} \tag{25}$$

The right sectional area A_{s1} of the strands of expanded metal of the type shown in Figure 5 in the width, b, may be obtained by substituting this value of p in equation (2):

$$A_s=\frac{wb}{40.9}\cos\theta; \tag{26}$$

$$f_s=\frac{40.9\,M}{w\,j\,bd\ \cos^2\theta}. \tag{27}$$

[2] G. A. Maney, Relation between deformation and deflection in reinforced concrete beams, Proc. A. S. T. M., 14, Part II, p. 311; 1914.

In determining values of w for use in equations (25), (26), and (27) no metal should be included which is added for the purpose of securing a lap splice or which for any other reason is not effective in resisting the tensile stress.

II. TEST SPECIMENS, MATERIALS, AND METHODS OF TESTING.

1. FORMS OF TEST SPECIMENS.

The forms and dimensions of the slab beams tested are shown in Figure 6. All the beams were approximately 4 inches thick and 5½ feet long, with a span length of 5 feet. They were tested as simple beams loaded as shown in Figure 9 with two equal loads, each load being distributed across the beam along a line that is parallel to the lines of support, and each load line being 6 inches from the center line of the beam and 2 feet from the center line of the support. The distance between the loads was, therefore, 1 foot.

The beams reinforced with unconnected bars laid direct between supports (direct reinforcement) and those reinforced with expanded metal had a width of about 27½ inches for the whole length of the span as shown in Figure 6. For a distance of only about 6 inches at the center of the span, the beams reinforced with bars laid at an angle of 45° with the direction of the span had the same width as the beams having direct reinforcement. The two ends of these slabs flared out from the straight portion at angles of 45° to a width of 4 feet 3½ inches, as shown in Figure 6. The object of widening the slabs in this manner was to allow the diagonal bars to remain straight for a sufficient length to develop the necessary bond resistance without making use of the curved portion of the bars.

Each slab beam had a 4 by 4 inch rib along each end, reinforced in the direction of the support, to prevent the corners of the slab from rising from the support when the slab was being tested.

The bars of the two bands of the two-way diagonal reinforcement, as shown in Figure 8, were necessarily on different levels, and in order to permit as direct a comparison as possible of the results for the various types of reinforcement, the bars of the direct reinforcement and of the one-way diagonal reinforcement were also placed on different levels, one-half of the bars on one level and the other half on another level corresponding to the levels of the bands of bars in the two-way diagonal reinforcement. Likewise, two sheets of expanded metal were used; this was necessary

also in order to introduce a sufficient amount of reinforcement. In two of the beams the expanded metal was laid so as to form a lap or splice at the center of the beam, the two sheets of metal

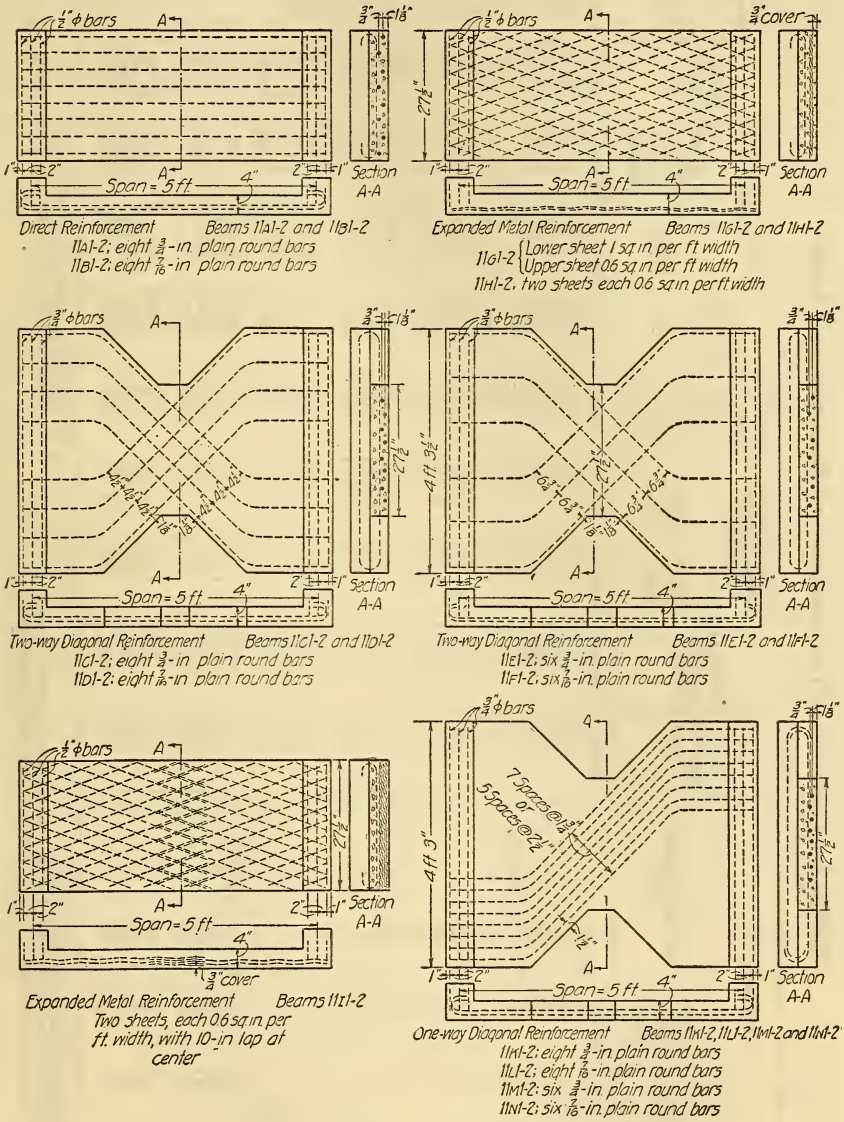


FIG. 6.—*Details of test specimens.*

in each half of the slab overlapping 10 inches as shown in Figures 6 and 7.

Two slabs of each kind were tested. A brief description of the 13 varieties of beams tested is given in Table 1 and in Figure 6.

TABLE 1.—Description of Test Specimens and Maximum Loads Carried.

Beam No.	Reinforcement.			Total thickness of beam.	Width of beam at center.	Distance from bottom surface to center of—		Distance from top of beam to center of gravity of reinforcement. (d)	Ratio of reinforcement by volume. (p)	Strength of concrete cylinders.		Maximum load.	Computed stresses for maximum load.	
	Number of bars.	Diameter of plain round bars.	Direction of reinforcement.			Lower layer of bars.	Upper layer of bars.			At age of 7 days.	At time beams were tested (42 days).		Tensile stresses.	Compressive stresses.
		Inch.		Inches.	Inches.	Inches.	Inches.	Inches.		Lbs./in.²	Lbs./in.²	Pounds.	Lbs./in.²	Lbs./in.²
11A1	8	¾	Direct	4.00	27½	1.24	1.90	2.43	0 0528	4,320	4,290	25,240	45,900	6,960
11A2	8	¾	do	4.12	27½	1.16	1.87	2 60	.0493	2,970	4,120	32,600	57,600	7,970
11B1	8	7/16	do	3.87	27½	.75	1.19	2 90	.0150	3,620	4,530	13,780	56,700	3,510
11B2	8	7/16	do	4.12	27½	.84	1.38	3 01	.0142	2,890	4,200	15,750	63,300	3,790
11C1	8	¾	Two-way diagonal	4.12	27½	1.13	1.87	2.62	.0690	3,610	.....	24,930	55,500	7,460
11C2	8	¾	do	4.25	27⅞	1.17	1.91	2.46	.0659	4,300	4,700	26,600	72,100	9,130
11D1	8	7/16	do	4.19	27⅝	.77	1.21	3.20	.0192	3,600	5,030	11,800	58,400	3,590
11D2	8	7/16	do	4.19	28	.81	1.27	3.12	.0194	2,590	4,310	13,850	71,000	4,400
11E1	6	¾	do	4.12	27⅝	1.16	1.96	2.56	.0529	3,920	4,930	19,600	58,400	6,650
11E2	6	¾	do	4.19	27¾	1.14	1.93	2.65	.0507	2,370	4,490	19,370	55,900	6,200
11F1	6	7/16	do	4.25	27 13/16	.81	1.22	3.24	.0141	3,300	2,270	11,250	72,700	2,590
11F2	6	7/16	do	4.25	27¾	.87	1.32	3.15	,0145	2,910	4,160	8,880	59,100	2,150
11G1	(1)	.....	.....	4.06	27½	[2] .62	[3] 1.68	2.90	.0443	3,000	4,480	25,300	43,600	5,400
11G2	(1)	.....	.....	4.06	27½	[2] .89	[3] 2.27	2.60	.0494	1,870	3,940	23,350	45,300	6,050
11H1	(4)	.....	.....	4.11	27⅜	[2] .83	[3] 1.89	2.76	.0331	2,710	4,030	22,750	56,600	5,690
11H2	(4)	.....	.....	4.06	27 7/16	[2] .88	[3] 2.00	2.62	.0348	2,240	3,790	19,250	50,800	5,280
11I1	(5)	.....	.....	4.06	27½	[2] .83	[3] 2.33	2.48	.0366	3,640	4,890	19,250	.....	.....
11I2	(5)	.....	.....	4.06	27½	[2] .83	[3] 2.33	2.48	.0366	2,440	4,200	20,850	.....	.....
11K1	8	¾	One-way diagonal	4.06	27⅞	1.12	1.95	2.53	.0707	3,360	4,430	8,050	.....	.....
11K2	8	¾	do	4.06	28 1/16	1.10	1.89	2.58	.0705	3,050	.....	12,550	.....	.....
11L1	8	7/16	do	4.19	27½	.75	1.23	3.20	.0194	3,350	5,230	7,220	.....	.....
11L2	8	7/16	do	4.19	28	.75	1.22	3.20	.0189	.....	.....	10,800	.....	.....
11M1	6	¾	do	4.25	27¾	1.12	1.93	2.72	.0496	3,250	5,170	10,480	.....	.....
11M2	6	¾	do	4.25	28	1.31	1.93	2.63	.0471	3,400	6,830	10,430	.....	.....
11N1	6	7/16	do	4.25	27⅞	.72	1.20	3.29	.0139	3,170	5,420	8,800	.....	.....
11N2	6	7/16	do	4.25	28⅛	.66	1.14	3.35	.0136	3,560	6,860	8,680	.....	.....

[1] Expanded metal 1.63 in.²/ft. width.
[2] Distance from bottom surface of beam to lower side of lower sheet.
[3] Distance from bottom surface of beam to upper side of upper sheet.
[4] Expanded metal 1.15 in.²/ft. width.
[5] Same as 11H1–2 but with 10-inch lap at center.

2. MATERIALS AND FABRICATION OF BEAMS.

The concrete of which the beams were made was the 1 : ⅔ : 1⅓ mix that was used in most of the investigations carried out by the concrete ship section of the Emergency Fleet Corporation. River sand varying in size from ⅛ to ¼ inch, and gravel varying in size from ¼ to ½ inch were used as aggregate. The ratio of the aggregate to the cement was about 2 to 1 and that of the gravel to the sand was also about 2 to 1. The compressive strength of the control cylinders at the ages of 7 days and 42 days are given in Table 1, each value being the average of the results from two cylinders.

The bars used as reinforcement were of two sizes, namely, $\frac{7}{16}$ inch and ¾ inch in diameter. The results of the physical tests of samples of these bars are given in Table 2. The $\frac{7}{16}$ inch bars had a yield point of 61,400 lbs./in.^{2} and an ultimate strength of 88,100 lbs./in.^{2} and the ¾ inch bars had a yield point of 54,600 lbs./in.^{2} and an ultimate strength of 93,500 lbs./in.^{2}

The sheets of expanded metal used were of two sizes, 1.0 and 0.6 square inch per foot of width. The dimensions of the strands of the larger size were approximately ¼ by ½ inch, and those of the smaller size were approximately $\frac{1}{5}$ by ⅜ inch. The dimensions of the diamonds of the mesh were approximately 3 by 8 inch for each size. The expanded metal reinforcement was furnished by the Consolidated Expanded Metal Co., of Braddock, Pa., for use in these tests. Unfortunately between the time of the completion of the tests and the working up of the results for publication the specimens of expanded metal reserved for investigation of its properties were damaged so that they could not be used for this purpose. In order to obtain values of the physical properties the company furnished new samples of material stated to be of the same grade as that used in the slab-beam test specimens. The physical properties determined from tests of strands from these samples are given in Table 2.

TABLE 2.—Physical Properties of Reinforcement.

Material.	Yield point.	Ultimate tensile strength.	Elongation in 8 inches.	Reduction of area.
	Lbs./in.^{2}	Lbs./in.^{2}	Per cent.	Per cent.
Plain round bars $\frac{7}{16}$ inch in diameter	61,400	88,100	20.9	51.2
Plain round bars ¾ inch in diameter	54,600	93,500	21.2	36.6
¼ by ½ inch strands of expanded metal[1]	57,500	58,800	15.0	67.2
⅕ by ⅜ inch strands of expanded metal	62,400	62,700	12.1	50.9

[1] The test specimens of expanded metal were not taken from the slab beams but were furnished by the manufacturers at a later date. See section II. 2.

In the fabrication of the beams the reinforcing bars were bent into the proper shape and placed in the forms. The bars were securely fastened in place, as shown in Figures 7 and 8, and blocks were attached to the forms so that holes in the concrete would occur at the points on the reinforcement where gauge holes were to be made.

The concrete was mixed for five minutes in a "Wonder" mixer having a capacity of 3 cubic feet, and after mixing was discharged from the mixer into a tight box.

The box was carried to the forms by a traveling crane and the concrete was shoveled directly from the box into the forms. The specimens were left in the forms for a period of 24 hours after which the forms were removed and the specimens stored under burlap which was kept well wetted until the time for testing. The age of the specimens at the time of testing was about 42 days.

Four 6 by 12 inch cylinders were taken from the concrete batch made for each beam and were stored with the slab beams under wet burlap until shortly before the time of test. Two of these cylinders were tested at the age of seven days and two at the time of testing the slab.

3. METHOD OF TESTING.

The slabs were tested in a vertical-screw beam-testing machine having a capacity of 300,000 pounds. Since a test specimen was too wide to be placed between the screws of the machine it was laid on two I beams that were placed across, and bolted to, one of the wings or extensions of the weighing table.

The arrangement of a beam in the machine and the method of loading are shown in Figure 9. The moving head of the testing machine applied its pressure to the center of a large H beam, one-half of the pressure causing the load on the test beam and the other half causing pressure on blocking on the opposite end of the weighing table. The load exerted on the test slab by the end of the H beam was distributed by means of a system of short beams, as shown in Figure 9, to two plates laid across the top of the slab. The plates were set in plaster of Paris to equalize the load on the two plates and to distribute the load uniformly along each line. These loading lines were 12 inches apart and each line was 24 inches from the nearest support.

In the tests of beams having one-way diagonal reinforcement (beams 11K to 11N) difficulty was experienced with the method of loading. Since the reinforcing bars in these beams were in

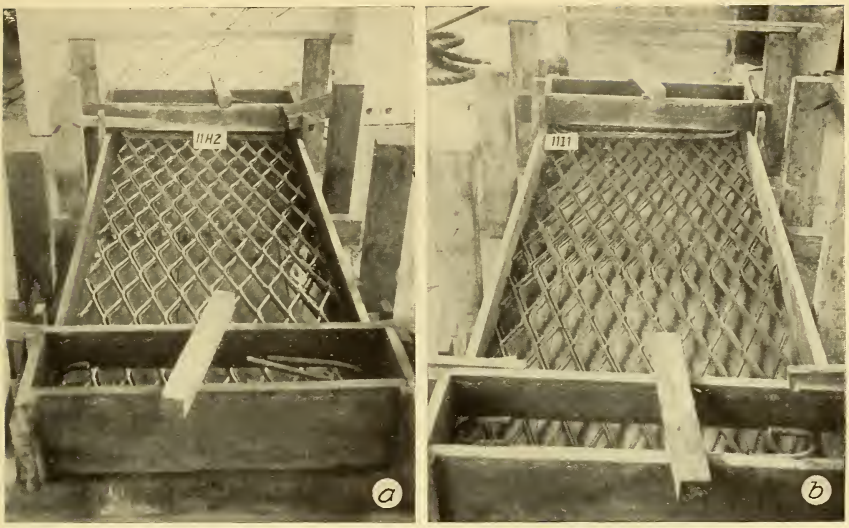


FIG. 7.—*Views of expanded metal reinforcement in forms.*

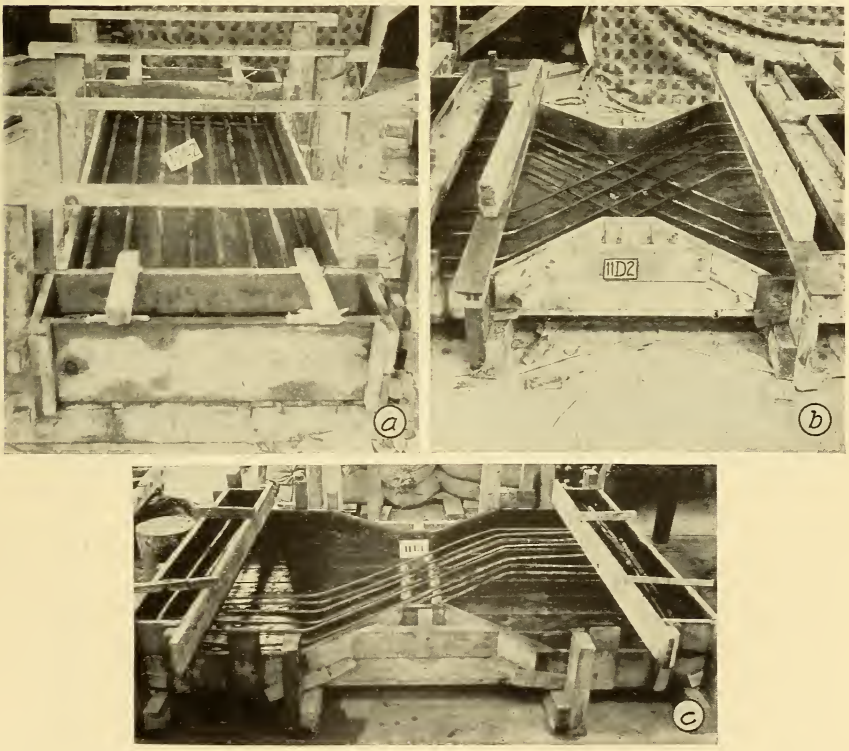


FIG. 8.—*Views of diagonal and of direct reinforcement in forms.*

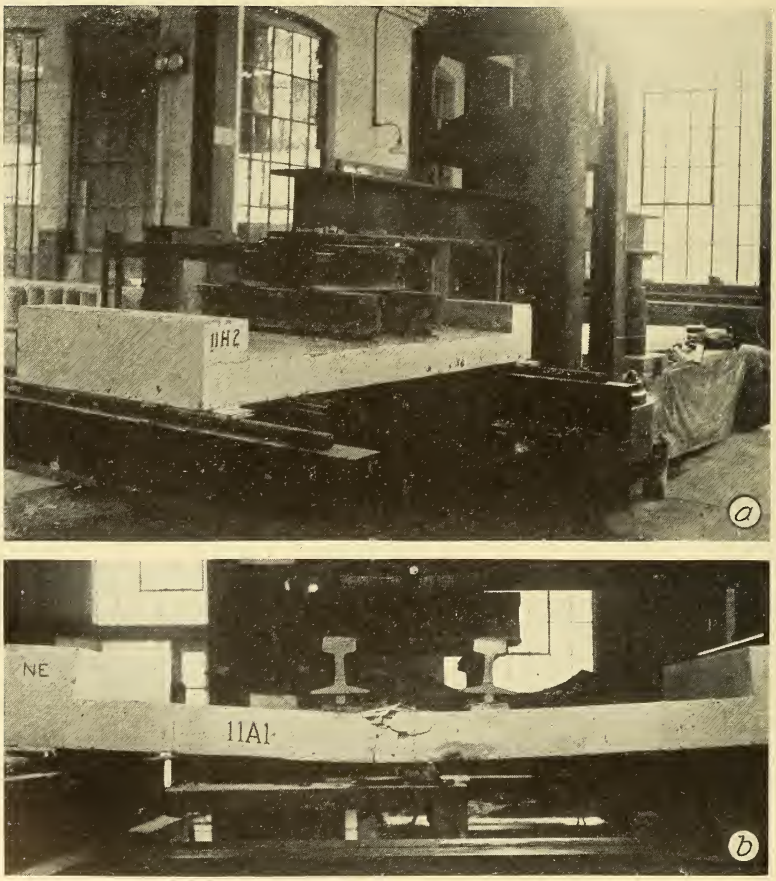


FIG. 9.—*Views showing method of testing beams.*

a diagonal band in only one direction, the line connecting the centers of the reactions was not coincident with the longitudinal center line of the beam. In the tests of the first two of these beams, 11K1 and 11L1, it was found that the I beams serving as supports for the test specimens were not attached firmly enough to the testing machine to prevent their tilting under the eccentric loading. This eccentricity of loading and lack of rigidity of the supports resulted in torsional stresses in the test specimens while under test. In the tests of the remaining beams of this type this trouble was partly avoided by changing the location of the slab and the supporting I beams so that the centers of reactions were closer to the center line of the weighing table. The positions of the slab relative to the weighing table are indicated in Figures 30 to 32. However, little use is made in this paper of the data obtained from the slabs having one-way diagonal reinforcement.

Deformation in the reinforcement was measured in all the beams by means of a Berry strain gauge having a gauge length of 6 inches. The locations of the gauge lines may be seen in Figures 17 to 19. In the beams reinforced with bars the deformation was measured in each bar, and in the beams reinforced with expanded metal, deformations were measured in eight strands in the lower sheet of metal. The two sheets of expanded metal were nested in such a way that it was practically impossible to measure the deformations in the upper sheet. The gauge length of 6 inches was greater than the length of a strand of expanded metal between joints, and hence the deformation was measured across a bridge. (See Fig. 5.) The metal, therefore, between the two ends of the gauge line was not of uniform cross section and was not quite straight. In computing stresses, however, from the deformations measured in the expanded metal, it was assumed that no portion of the measured deformation was due to a straightening out of the strands, that the unit stress in a strand was constant within the gauge length regardless of the enlargement of the cross section at the intersection with another strand, and that there was no bending in the sheet metal. For the purpose of comparing the action of the various types of reinforcement it is believed that only slight errors are introduced in the results by these assumptions.

In two of the slabs reinforced with expanded metal, laps of the sheets were made at the center of the span. In these slabs strain-gauge measurements of slip were made in order to determine whether the lap was sufficient to prevent bond failure before the

yield point stress in the steel was reached. Measurements of slip were made between the two layers of expanded metal and between the lower sheet and the concrete. A measurement of the deformation of the strand, on a gauge line alongside the gauge line at which the slip measurement was taken, permitted the correcting of the slip reading for the deformation in the steel, so that the actual slip could be determined.

The deflections of points on the bottom of the slab with respect to two other points on the slab were measured by means of the instrument shown in Figure 10. The deflections due to curvature in both the transverse and the longitudinal planes were found. The instrument shown in Figure 10 consists of a bar or frame having a leg with a conical point at each end, and to this frame at its center is attached a dial micrometer gauge head having its plunger on a line with the conical points.

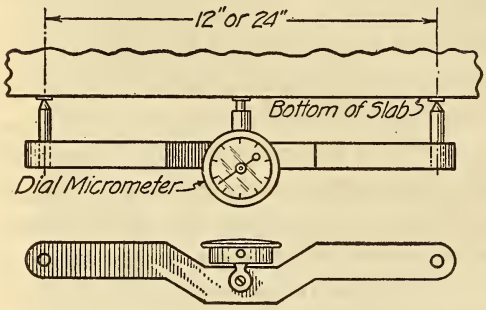


FIG. 10.—*Instrument for measuring deflections.*

In measuring the deflection on any gauge line one of the conical points is placed in a punch mark in one of the plugs of the gauge line and the other point rests in a groove in the other plug, the groove being in the direction of the gauge line. The plunger at the center rests against a plug in the same gauge line and on about the same level as the other two plugs. In taking a reading the instrument was tipped back and forth to get the maximum reading.

Two instruments of this type were used; one was used on a gauge line 12 inches long and the other on a gauge line 24 inches long. The shorter instrument was used to measure the deflection due to longitudinal curvature and the longer one to measure the deflection due to transverse curvature. Deflections were measured on three lines in the longitudinal and three in the transverse directions. The positions of the gauge lines are indicated in Figures 20 to 32.

It should be remembered that the deflections measured with this instrument are not the total deflections of the slab, but it can be shown analytically that the total deflections of the different slabs should bear the same ratio to each other as the corresponding ratios determined from the deflections which were measured.

All of the deformation and deflection measurements were made on the bottom side of the slab. In preparing a slab for testing, holes were drilled in the steel for the strain-gauge readings and plugs were set in the concrete for the deflection measurements. The specimens were then placed in the testing machine on the I beams which served as supports. Plaster of Paris was placed between the slab and its supports to assist in obtaining uniform bearing pressure. Zero-load readings were taken with both the strain gauge and the deflectometer, after which the first load increment was applied. Deformations and deflections caused by this load were then measured, and the extent of cracking was observed and marked on the specimen. A similar procedure was followed for each additional load applied. After failure, the specimen was removed from the testing machine, the cracks were painted, a photograph of the bottom side of the slab was taken showing the distribution of cracks, and the measurements of the depths of the reinforcement and of other dimensions were made. Views of the tested beams are shown in Figures 17 to 19.

III. EXPERIMENTAL DATA AND DISCUSSION OF RESULTS.

1. STRESSES IN THE VARIOUS TYPES OF REINFORCEMENT.

The stresses in the reinforcement of each slab as determined from the strain-gauge readings are shown by the load-stress curves of Figures 20 to 32. In determining the stresses from the observed deformations, the modulus of elasticity was assumed at 30,000,000 lbs./in.^{2} The deflections for at least one of each pair of slabs, as determined in the manner described in the preceding section, are also shown in the same figures. Views of the bottom sides of all but two of the slabs after failure are shown in Figures 17 to 19.

In the study of the test data, the expanded metal has been assumed to be typical of any reinforcing metal in which the strands make an angle with the direction of the span and in which the application of stress does not produce a tendency of the strand toward straightening out. The angle between the direction of the strands of the expanded metal and the direction of the span was only about one-half the corresponding angle in the beams reinforced with bars placed at 45° with the span. For purposes of analysis the angle was taken at 22½°. From the analytical considerations pointed out and from the diagrams shown in Figures 3 and 4 it will be apparent that it is not proper to make a direct comparison between the beams reinforced with

expanded metal and those reinforced with 45° diagonal bars, except for the purpose of studying the effect of direction of bars on the stresses developed. It is for this purpose that such comparisons have been made in the following pages. Less difference should be expected in the stresses between the expanded metal and those in the direct bars than between the stresses in the expanded metal and those in the 45° bars.

In order to compare the stresses in the various types of reinforcement the load-stress curves shown in Figures 11, 12, and 13 have been drawn, for which the stresses have been reduced to those which it is assumed would have occurred if all the slabs had had a value of d (depth from top of slab to center of gravity of reinforcement) and a value of p (volume of steel per unit volume

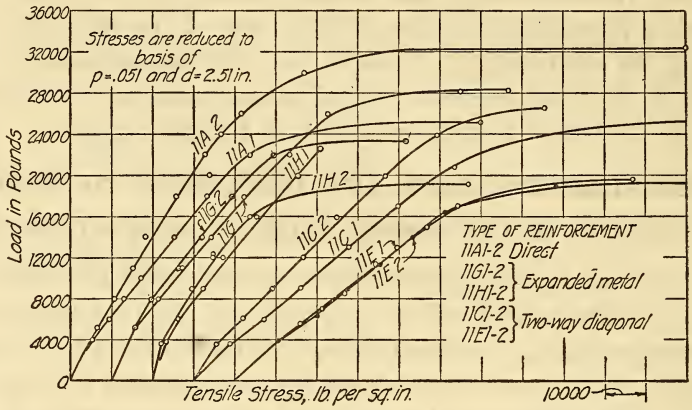


FIG. 11.—*Load-stress curves for individual beams having the larger percentages of reinforcement (Group I); average stress for all gauge lines.*

of concrete), the same as that of the d and the p of the beams having direct reinforcement.

For the purpose of comparison the slabs may be divided into two groups according to the amount of reinforcement used; one group having a ratio of reinforcement by volume of approximately 0.05 and the other group a ratio of about 0.015. Thus, in Figure 11 are shown the average load-stress curves for individual beams having the larger ratio of reinforcement and having three different types of reinforcement, namely, direct, expanded metal, and two-way diagonal. The stresses shown in the reinforcement of beams C, E, G, and H are those which, according to equation (15), should have occurred if those beams had had the same ratio of reinforcement, p, and depth, d, as the average of beams 11A1–2; that is, 0.051 and 2.51, respectively. Correspondingly, the stresses for

beams D and F in Figure 12 are those which, according to equation (15), should have occurred if those beams had had the same ratio of reinforcement and depth, respectively, as the average for beams 11B1–2. For both figures the plotted stresses were obtained by multiplying the observed stresses by the ratio $\frac{f_{s1}}{f_{s2}}$ as given by equation (15).

In Figure 13 are shown the averages of each pair of load-stress curves of Figures 11 and 12 drawn to the same set of axes, from which direct comparison of the stresses in the three types of reinforcement can be made. The maximum variation of a single value from the average was 15 per cent, in Group D. It was nearly the same in Groups A and C, but smaller in all others. This

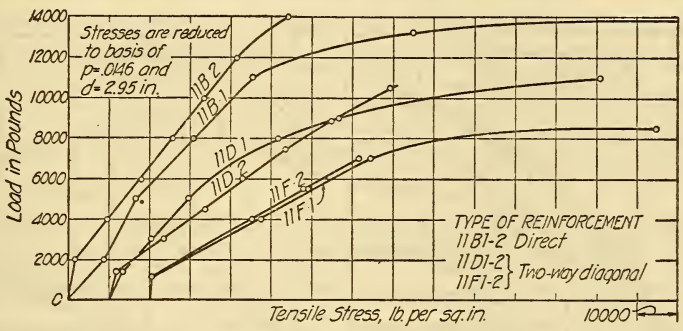


FIG. 12.—*Load-stress curves for individual beams having the smaller percentages of reinforcement (Group II); average stresses for all gauge lines.*

variation should be taken into account in giving weight to the comparison of the average values.

TABLE 3.—Ratio of Stresses in Beams Having Various Types of Reinforcement to Stresses in Beams Having Direct Reinforcement.

GROUP I.—CORRECTED TO $d=2.51$, $p=0.051$.

Type of reinforcement.	Beam number	Theoretical ratio.	Ratio from tests when stress in direct reinforcement was—		
			20,000 Lbs./in.²	25,000 Lbs./in.²	30,000 Lbs./in.²
Direct, $\theta=0°$	11A1–2	1.00	1.00	1.00	1.00
Expanded metal, $\theta=22\frac{1}{2}°$	11G1–2	1.15	.97	.....	1.01
	11H1–2		.88	.....	1.01
Two-way diagonal, $\theta=45°$	11C1–2	1.81	1.57	.....	1.56
	11E1–2		1.90	.....	1.88

GROUP II.—CORRECTED TO $d=2.95$, $p=0.0146$.

Direct, $\theta=0°$	11B1–2	1.00	1.00	1.00	1.00
Two-way diagonal, $\theta=45°$	11D1–2	1.86	1.50	1.58	.....
	11F1–2		2.25	2.28	.....

As found from Figure 13*a* and as given in Table 3, a load that caused a stress of 20,000 lbs./in.^{2} in the direct reinforcement caused a stress 3 to 12 per cent less in the expanded metal and from 57 to 90 per cent greater in the two-way diagonal reinforcement. Similarly, a load that caused a stress of 30,000 lbs./in.^{2} in the bars of the direct reinforcement caused a stress about 1 per cent greater in the expanded metal and a stress from 56 to 88 per cent greater in the bars of the two-way diagonal reinforcement. The value 88 per cent is obtained by producing the straight portion of the curve for 11E1–2 in Figure 13*a* to the load which caused a stress of 30,000 lbs./in.^{2} in beams 11A1–2.

The curves of Figure 13*b* indicate that a load that caused a stress of 20,000 lbs./in.^{2} in the bars of the direct reinforcement

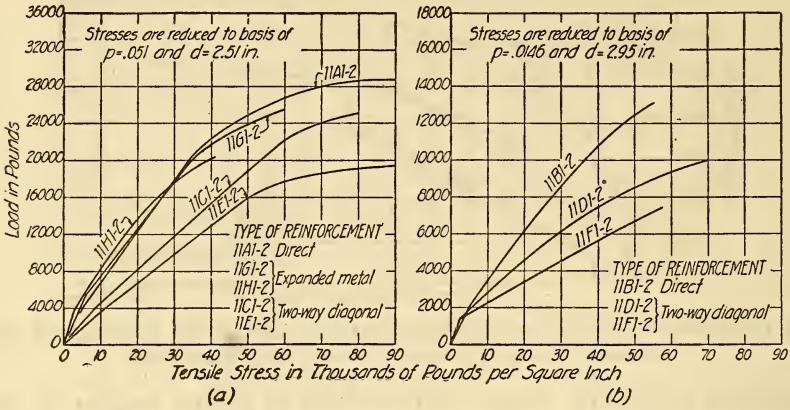


FIG. 13.—*Average load-stress curves for similar beams.*

(*a*) Beams of Group I; (*b*) Beams of Group II.

caused from 1.5 to 2.25 times as large a stress in the bars of the diagonal reinforcement.

The averages of the values in Table 3 indicate that, within the yield point of the reinforcement, the stress developed in the expanded metal reinforcement in the slab beams under a given load was practically the same as that developed in the reinforcement of a similar beam reinforced with bars laid direct between supports. And the stress developed in the reinforcement consisting of bars laid diagonally in two bands making 45° with the direction of the span was about 1.80 times as great as in a similar beam reinforced with bars laid direct between supports. By the secant method of computation developed in Section I, 3 the calculated stress in the strand of the expanded metal is approxi-

mately 1.17 times the stress in the bars of the direct reinforcement and the calculated stress in the bars of the two-way diagonal reinforcement is about 1.87 times as great as the stress in the direct reinforcement. The average stress in the expanded metal was about 15 per cent less than that called for by the analysis, and the stress in the two-way diagonal bars was about 4 per cent less than that called for by the analysis.

Most of the difference between the tensile stresses in the expanded metal and those in the 45° diagonal reinforcement may be accounted for by the difference in the angle θ, and at most only the difference between the 15 per cent and the 4 per cent variations from the theoretical ratios could be construed in favor of expanded metal. Even this advantage is somewhat offset by the fact that, as shown in Section III, 2, the load-carrying capacity of the beams reinforced with expanded metal was somewhat less than that which was computed. The smallness of the observed stresses in expanded metal may be due to a greater amount of assistance from the concrete in resisting tension during the earlier part of the test.

It will be noted that there is considerable variation in the ratios of stresses given in Table 3, and that although the average of the values is less than that called for by the secant method of computation, the ratio for beams 11F1–2 is larger than that given by the secant method. It should also be remembered that in reducing the stresses to the basis of a common p and a common d the assumption was made that the stresses in the beam varied directly as the distances from the neutral axis. For beams having the small depth and the large ratios of reinforcement possessed by the beams under discussion, there is reasonable doubt as to the correctness of this assumption. The limitations are brought out more fully in Section I, 3, and Section III, 6.

The quantitative results in Figures 11 to 14 and in Table 3 should, therefore, be considered as approximate, for, although the tests were carried out carefully and the test data are believed to be reliable, the reduction of the stresses to the basis of a common percentage of reinforcement and of a common depth to the reinforcement can not be accomplished without making assumptions which may introduce error in the results.

17153°—23——4

2. COMPARISON OF MAXIMUM LOADS.

In order to permit of a comparison of maximum loads for the beams having different types of reinforcement, it was necessary to correct the loads to those which presumably would have been carried if all the beams in each of the two groups of Table 4 had had the same depth, and had had reinforcement which was the same in quantity and in yield point. The grouping used was the same as that used in Section III, 1, except that beams 11H1–2 were used in both groups since their properties placed them midway between the two groups. The standard depths used were 2.51 and 2.95 inches, and the standard ratios of reinforcement were 0.051 and 0.0146, respectively. A standard yield point of 58,000 lbs./in.^{2} was used. Therefore, the corrected loads given in Table 4 are those which, according to the assumptions used, would have been carried if the depths of the beams and the ratios and yield point of the reinforcement had been those stated above. The basis of the computation of these loads is the assumption that the maximum load is reached when the steel becomes stressed to its yield point. Before failure could occur the reinforcement in the two layers would be stressed to almost the same intensity (the yield point) even though in the early part of the test the stresses in the layers should vary as their distances from the neutral axis. This equality of stresses in the two layers justifies the assumption used in determining the corrected maximum load; that is, that the centroid of the tensile stress lies at the centroid of the sectional area of the reinforcement. The corrected loads of Table 4 were found by solving equation (20), Section I, 3, for the load W_1. The f_{s1} of that equation represents the assumed yield point of 58,000 lbs./in.^{2} The other terms having the subscript$_1$ represent properties of the standard beams, such as $d=2.51$ and $p=0.051$. The terms having the subscripts$_2$ represent corresponding properties of the beams as they were tested.

The "computed loads" of Table 4 were obtained by using in equation (19), Section I, 3, the assumed properties of the standard beams and solving for W.

It will be seen that the corrected loads for the beams with direct reinforcement agree quite closely with the computed loads. The agreement is better for beams 11B1–2 than for 11A1–2. This may be because of the smaller amount of reinforcement in the latter than in the former beams. Although the phenomena of the tests appeared to indicate a tension failure, the amount of reinforcement in beams 11A1–2 was so great that the high com-

pressive stresses in the concrete (see Table 1) may have affected the amount of the maximum load. The fair agreement between the corrected loads and the computed loads for those beams gives confidence in the method of correcting the loads and in the assumption that the maximum load is approximately that at which the yield point stress in the reinforcement is reached. Numerous other tests are on record which show that in deeper beams the maximum load is only slightly greater than that at which the yield point stress in the reinforcement is reached.

TABLE 4.—Ratios of Corrected Maximum Loads for Beams Having Various Types of Reinforcement to Corrected Maximum Loads for Beams Having Direct Reinforcement.

GROUP I.—CORRECTED TO d=2.51, p=0.051, YIELD POINT=58,000 LBS./IN.^{2}

Beam number.	Reinforcement.			Yield point from tension tests.	Maximum load.			Ratio of corrected to computed load.	Ratio of average corrected load to average load for θ=0.	
	Type.	Ratio, p_2.	Direction, θ.		Actual.	Corrected.	Computed.		Experimental.	Theoretical.
			Degrees.	Lbs./in.^{2}	Lbs.	Lbs.	Lbs.			
11A1	Direct	0.0528	0	54,600	25,240	27,700	32,900	0.842		
11A2	do	.0493	0	54,600	32,600	34,700	32,900	1.054		
Average						31,200		.948	1.00	1.00
11G1	Expanded metal	.0443	22½	57,500	25,300	21,650	28,600	.758		
11G2	do	.0494	22½	57,500	23,350	22,500	28,600	.788		
Average						22,075		.773	.708	.870
11H1	Expanded metal	.0331	22½	62,400	22,750	25,950	28,600	.908		
11H2	do	.0348	22½	62,400	19,250	23,300	28,600	.815		
Average						24,625		.862	.790	.870
11C1	2-way diagonal	.0690	45	54,600	24,930	18,410	18,150	1.015		
11C2	do	.0659	45	54,600	26,600	23,950	18,150	1.320		
Average						21,180		1.168	.679	.552
11E1	2-way diagonal	.0529	45	54,600	19,600	19,390	18,150	1.069		
11E2	do	.0507	45	54,600	19,370	18,590	18,150	1.023		
Average						18,990		1.046	.609	.552

GROUP II.—CORRECTED TO d=2.95, p=0.0146, YIELD POINT=58,000 LBS./IN.^{2}

11B1	Direct	0.0150	0	61,400	13,780	13,180	14,250	0.925		
11B2	do	.0142	0	61,400	15,750	14,700	14,250	1.031		
Average						13,940		.978	1.00	1.00
11H1	Expanded metal	.0331	22½	62,400	22,750	10,750	11,820	.910		
11H2	do	.0348	22½	62,400	19,250	9,650	11,820	.816		
Average						10,200		.863	.732	.830
11D1	2-way diagonal	.0192	45	61,400	11,800	7,320	7,670	.955		
11D2	do	.0194	45	61,400	13,850	8,880	7,670	1.168		
Average						8,100		1.051	.581	.538
11F1	2-way diagonal	.0141	45	61,400	11,250	9,100	7,670	1.187		
11F2	do	.0145	45	61,400	8,880	7,400	7,670	.965		
Average						8,250		1.076	.592	.538

For the beams with 45° diagonal reinforcement the corrected loads were greater than the computed loads in six cases, and in two cases they were smaller. For Group I the average load carried (corrected load) by the beams with 45° diagonal reinforcement was about 10 per cent greater than the computed load. For Group II it was about 7 per cent greater. This indicates a fair agreement between the tests and the analysis based upon the secant method. The average load for the four beams reinforced with expanded metal was 82 per cent of the computed loads for beams whose reinforcement makes an angle of 22½° with the direction of the span. The difference between the observed and the computed loads for the beams with expanded metal reinforcement may be due partly to error in the assumptions as to the yield point and in the values assigned to *d*, the depth, but it is the best estimate that is available with the data

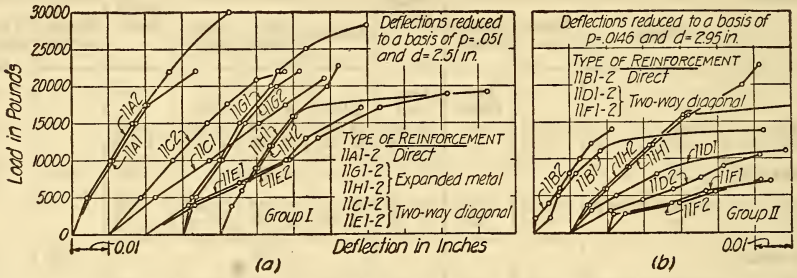


FIG. 14.—*Load-deflection curves for individual beams; average deflection for all gauge lines.*

(*a*) Beams of Group I; (*b*) Beams of Group II.

at hand. Unfortunately the yield points for the expanded metal were determined, not from the reinforcement actually used in the specimens, but from similar material furnished from stock at a later date.

It does not seem reasonable that the entire difference between the corrected observed loads and the computed loads can have been due to the use of an erroneous value for the yield point, for to explain the discrepancy on this basis would require that the yield points should have been 44,300 and 53,600 lbs./in.^{2} instead of the values used, 57,500 and 62,400 lbs./in.^{2}, respectively. From other data of yield points of expanded metal the latter values seem more likely to be correct than the former.

Another possible source of error lies in the determination of the depth, *d*. It is difficult to say to just what part of a sheet of expanded metal the measurements should be taken in arriving at the depth. This is especially true when more than one layer of

reinforcement is used, as was the case with the beams tested. Unfortunately the exact method of obtaining these measurements is not stated in the test data with sufficient detail to warrant the assurance that there may not be error in the values of d used. If the actual depth, d, were, on the average, about 10 per cent less than the depths used throughout this paper, the difference between the corrected observed loads and the computed loads would be accounted for.

The purpose in pointing out possible sources of error has been to avoid the taking of the results of these tests with too high a degree of finality. It is not intended to indicate that errors of the extent suggested are likely to have been present.

The averages of the ratios of the corrected loads to the computed loads given in column 9, Table 4, show for the three different angles θ the following values:

	Ratios for various values of the angle θ—		
	0°	22½°	45°
Ratio of corrected to computed load	0.963	0.818	1.087

In obtaining these average values the two values for 11H1–2 are averaged, and this result is averaged with the ratio for 11G1–2.

In the last two columns of Table 4 are given the proportionate loads for beams, all of which have the same volume percentage of reinforcement but which have angles θ, between the direction of the reinforcement and the direction of the span, of 0°, 22½°, and 45°. In other respects the beams are taken as identical. The values determined from the tests are the ratios of the corrected loads for beams with θ equal to 22½° and 45° to the average corrected load for beams with direct reinforcement. Under the caption "Theoretical" are given the corresponding ratios determined from the computed maximum loads. Averaging the values obtained from the tests and repeating the theoretical value for comparison the following summary is obtained:

	Ratio of maximum loads for diagonal reinforcement to loads for direct reinforcement, for various values of θ.		
	0°	22½°	45°
Computed	1.0	0.85	0.545
Experimental	1.0	.735	.615

Assuming as the basis for comparison the average corrected loads for the beams having direct reinforcement it is found (*a*) that the loads for the 22½° reinforcement were 0.865, $\left(\frac{0.735}{0.85}\right)$, as great as are given by the analysis, and (*b*) that the loads for the 45° reinforcement were 1.128, $\left(\frac{0.615}{0.545}\right)$, as great as are given by the analysis.

3. DEFLECTIONS.

The deflections measured on three gauge lines in each of two directions are shown in Figures 20 to 32 for slabs having various types of reinforcement. The gauge lines in the direction parallel to the span were 12 inches long and those transverse to the direction of the span were 24 inches long. As noted under Section II, 3, the deflections given in Figures 20 to 32 are not the total deflections of the slab, but are the deflections due to the longitudinal curvature within only a portion (12 inches) of the span and to transverse curvature of nearly the entire width of the beam. The deflection due to the longitudinal curvature, however, may be used in comparing the total deflections of the different beams, since it is found analytically that they should bear the same ratio to the total deflection for all the beams.[3]

It will be noted from Figures 20 to 32 that only the beams having one-way diagonal reinforcement show any appreciable deflection due to curvature in the transverse plane. All the other slabs within the yield point of the reinforcement acquired a curvature only in longitudinal planes, and the stress was distributed nearly uniformly to all the bars. After one or more of the bars developed a stress beyond the yield point, the load-deflection curves indicate that some of the beams acquired curvature in the transverse direction resulting from the inequality of the stresses in the different bars.

The load deflection curves for the slabs having one-way diagonal reinforcement show clearly that the slabs had considerable curva-

---

[3] In equation 21 $q=\frac{1}{12}$ and $l=60$, from which $\Delta=\frac{300}{d}(\epsilon_l+\epsilon_c)$. For a gauge length s falling between the two load-points, $\Delta_s=\frac{s^2}{8d}(\epsilon_l+\epsilon_c)=18d(\epsilon_l+\epsilon_c)$, since $s=12$ for these tests. $\therefore \frac{\Delta_s}{\Delta}=\frac{18}{300}=0.06$. The derivation of this relation is dependent upon the assumption that the beams have constant moments of inertia throughout the gauge length over which the deflection is measured. For the beams reinforced diagonally the flaring of the beams toward the supports introduced a variable moment of inertia. To a certain extent this variation prejudices the comparison, but since the width of these beams was the same at the center of the span as that of the other beams, and since the moment of inertia within the gauge length would be affected only slightly by the variation in width of the beam, it seems that this variation should not invalidate the comparison.

ture in the transverse direction. The two slabs (11K1 and 11L1,) that were tested with their longitudinal axes parallel to the center line of the weighing table had large downward deflections at the centers of the three transverse gauge lines (Figs. 29 and 30); whereas the other six slabs having one-way diagonal reinforcement, which were tested with the axes of the beams at an angle with the center line of the weighing table, show a negative (upward) deflection in one transverse gauge line and downward deflections in the other two transverse gauge lines. It is clear from these facts that the action within these beams was quite different from that usually assumed to occur in a simple beam and suggests the marked effect of placing reinforcement unsymmetrically with the structural element which it reinforces. Attention was called in Section II, 3, to the tilting of the supports when the beams having

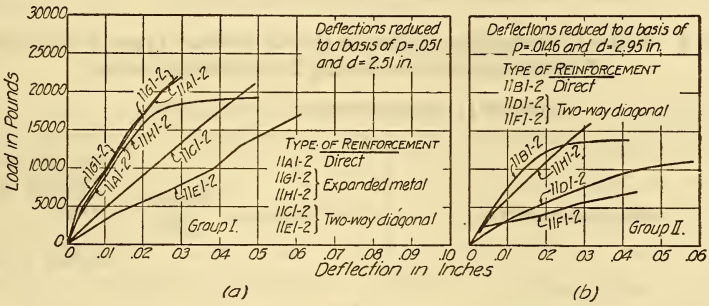


FIG. 15—*Average load-deflection curves for similar beams.*

(*a*) Beams of Group I; (*b*) Beams of Group II.

one-way diagonal reinforcement were tested. In the following analysis of the deflections the beams having one-way diagonal reinforcement are not considered.

The average load-deflection curves for individual beams for three types of reinforcement are given in Figures 14*a* and 14*b*. The deflections plotted in Figure 14*a* are those which according to equation (24) should have occurred with the beams of Group I (C, E, G, and H) if they had had the same ratio of reinforcement ($p = 0.051$) and the same depth ($d = 2.51$) as the corresponding average values for beams 11A1–2. The deflections plotted in Figure 14*b* are those which according to equation (24) should have occurred in the beams of Group II (D, F, and H) if in each case the ratio of reinforcement and the depth had been the same as the corresponding values for beams 11B1–2; that is, 0.0146 and 2.95, respectively.

In Figure 15 are given the average load-deflection curves for each pair of beams from which a comparison of the deflections of beams having different types of reinforcement can be made for a given load. Thus from Table 5 it will be noted that for the beams of Group I (having the larger percentage of reinforcement) a load that caused a deflection of 0.01 inch for the beams having direct reinforcement caused a deflection about 0.90 to 1.10 times as large for a similar beam reinforced with expanded metal and about 2 to 3.5 times as large for a beam having two-way diagonal reinforcement. Similar values are given for the beams of Group II.

The ratios of the deflections obtained analytically, assuming that the stresses were proportional to the secants of the angles which the bars made with the direction of the span are given in Table 5, under the caption "Theoretical ratio."

TABLE 5.—Ratios of Deflections of Beams Having Various Types of Reinforcement to Deflections of Beams Having Direct Reinforcement.

GROUP I.—CORRECTED TO $d=2.51$, $p=0.051$.

Type of reinforcement.	Beam number.	Theoretical ratio.	Ratio from tests when deflection of direct-reinforced beams was—	
			0.01 inch.	0.02 inch.
Direct, $\theta=0°$	11A1-2	1.0	1.0	1.0
Expanded metal, $\theta=22\frac{1}{2}°$	11G1-2 11H1-2	1.14	.90 1.10	.98 1.10
Two-way diagonal, $\theta=45°$	11C1-2 11E1-2	2.10	2.0 3.5	1.90 3.00

GROUP II.—CORRECTED TO $d=2.95$, $p=0.0146$.

Direct, $\theta=0$	11B1-2	1.0	1.00	1.00
Expanded metal, $\theta=22\frac{1}{2}°$	11H1-2	1.22	1.25	1.13
Two-way diagonal, $\theta=45°$	11D1-2 11F1-2	2.70	3.00 4.70	....

It appears from Figure 15 and from the average of the values of Table 5 that for a given load, within the yield point of the reinforcement, the deflection of the slab beams, reinforced with expanded metal was approximately the same as that of a similar beam reinforced with bars laid direct between the supports, and that the deflection of beams reinforced with two bands of bars at 45° with the span was from 2 to 4.7 times as great as that of a similar beam reinforced with bars laid direct between supports.

For some unexplained reason the beams E and F showed much greater deflections than the other beams having two-way diagonal reinforcement. Except for these beams it will be seen from the ratios given in Table 5 that the relations between the deflections for the beams having reinforcement of different types were in general agreement with the relations indicated by the analysis based upon the secant method of computing stresses. There seemed to be some advantage for the expanded metal in this comparison, as the average ratio obtained from the tests is slightly smaller than the theoretical ratios, while for the 45° reinforcement the average ratio is considerably greater than the theoretical ratios. The advantage for the expanded metal may have been due to the better distribution of bond stress and cracks referred to in Section III, 4 and 5. There is no apparent reason why the ratios found for the 45° reinforcement should be greater than the theoretical values.

4. BOND RESISTANCE OF EXPANDED METAL.

It is important to know the minimum amount of lapping of expanded metal necessary to prevent failure of a beam by slipping of the reinforcement. In order to obtain information on this question two of the beams (11I1–2) were reinforced with two layers of expanded metal with a lap or splice at the center of the beam of 10 inches or a length of about 1⅓ diamonds (see Fig. 7*b*); this length is called, commercially, a one-diamond lap, since it is not desirable to cut the metal exactly at the intersection of the strands.

In order to determine the amount of slipping of the sheets of metal at the lap, strain-gauge readings were taken within the lapped length of the metal, one gauge line having both gauge holes on the same sheet of metal and another gauge line having one gauge hole on each of the two sheets of metal. A reading on the first gauge line indicates the deformation due to the stress in the metal, while the reading on the second gauge line indicates the deformation due to the stress plus the slip between the lapped sheets. The difference, therefore, of these two readings for any load is the amount of slip between the sheets of metal for that load.

The load-slip curves for these beams are shown in Figure 16. It will be observed that slipping of the expanded metal sheets started at rather low loads increasing slowly at first and rapidly

in the later part of the test. In beam 11I1 when the total slip as shown in Figure 16 was 0.008 inch, the tensile stress was carried mainly by two strands of the metal (see Fig. 28.) and failure of the beam occurred at a slightly increased load.

In slab 11I2 the amount of stress corresponding to a given slip was less than that in beam 11I1. Thus, for beam 11I1 the stress in the reinforcement was about 20,000 lbs./in.^{2} when the slip amounted to 0.005 inch, whereas the slip in beam 11I2 did not reach this amount until the stress in the reinforcement was about 30,000 lbs./in.^{2} Beam 11I2 seemed tougher than 11I1

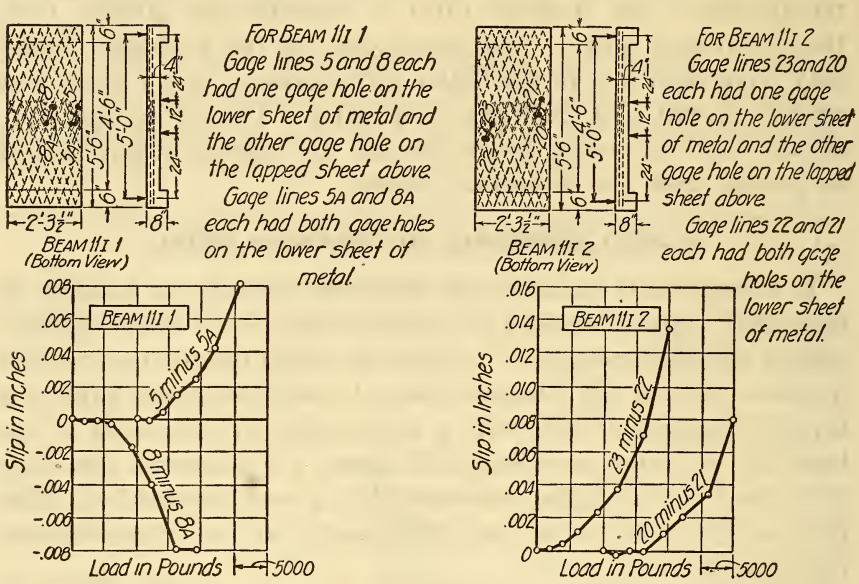


FIG. 16.—*Load-slip curves for beams 11I1 and 11I2.*

and took a greater load, resulting in a maximum slip of over 0.012 inch.

The difference in the behavior of the two beams may have been due to the fact that the concrete was not filled in well around the reinforcement of beam 11I1. An examination of beam 11I1 after test showed that the concrete around the reinforcement was somewhat porous.

It seems clear that the failure of beam 11I1 was due to slipping of the sheets of expanded metal at the lap. This fact was also indicated by the falling off of the stress along all but two of the gauge lines at an average measured stress of about 28,000 lbs./in.^{2} as shown in Figure 28. For beam 11I2, Figure 28 shows that slipping did not cause great inequality of stresses in the strands

until a stress of 40,000 lbs./in.^{2} was reached along all the gauge lines. It is clear, however, that slipping caused some of the strands to take more than their portion of the stress before the yield point of the reinforcement was reached, and hence the failure of the beam was due to slipping at the lap, although it seemed that the length of the lap was nearly large enough to develop the yield point of the expanded metal.

It is evident, therefore, that the minimum length of lap for expanded metal of this nominal size (1.2 square inches of metal per foot of width of sheet) required in concrete of rich mix to develop the yield-point stress in the strands of the metal without causing failure by slipping at the lap is somewhat more than 10 inches, or 1⅓ diamonds.

It is of value to determine the bond stress corresponding to the length of lap of 1⅓ diamonds and to determine the equivalent length of round plain bars required to develop the same bond stress per unit of cross-sectional area of reinforcing metal. The perimeter of one strand was approximately 1.15 inches and the length of a strand in the lap of 10 inches of the sheets was 10.7 inches. Hence the bond area for each strand of the lap is 12.3 square inches or 9.25 square inches per strand per diamond. Since each strand had a right cross-sectional area of approximately 0.07 square inch, the bond area within the length of the diamond is 132 square inches per square inch of cross-sectional area. With a round bar, in order to obtain the same amount of bond area per square inch of cross-sectional area of the bar, it must have a length of 33 diameters of the bar. Tests [4] have shown that with concrete having approximately the same compressive strength as that used in these beams (4,500 lbs./in.^{2} at 42 days) about 50 diameters was the length of a plain round bar in which the stress in the lapped bars became equal to that in the bars extending the whole length of the beam and lying beside the lapped bars. To meet this same requirement with expanded metal the lap of the sheets must be 1.51 diamonds.

The above analysis makes no allowance for the mechanical anchorage of the expanded metal due to the form of the sheets of metal, but it seems that the mechanical anchorage should be very effective. On the other hand, no allowance is made for a poor contact between the concrete and the expanded metal at the lap, which is apt to occur where two or more layers of expanded metal

[4] B. S. Tech. Paper No. 173; 1920.

reinforcement are used. Further, since the bond resistance varies approximately in the same ratio as the compressive strength of the concrete, the above results should not be applied directly to slab beams made of concrete having a much smaller compressive strength.

The results of these tests seem to indicate, however, that a lap of 1½ diamonds for expanded metal of the size used in these tests would be sufficient to develop the yield-point stress of the metal before failure would occur by slipping at the lap. It should be noted that increasing the length of lap from 1 to 1½ diamonds increases the mechanical anchorage by about 50 per cent, since it makes three, instead of two, "bridges" or intersections of the diamonds effective as a part of the splice. It seems reasonable to expect that less lap would be required for expanded metal of a lighter weight than that used in these tests. However, no experimental results with the lighter weights of metal were obtained in this investigation.

5. DISTRIBUTION OF CRACKS.

A study of the distribution of the cracks on the bottom or tension side of the slab (see Figs. 17 to 19) indicates that the expanded metal reinforcement was more effective in distributing the cracks in the concrete along the length of the beam than were bars laid direct between supports. This statement is true whether the beam was subjected to a "design" load only or to loads greater than the design load. In this connection it should be observed that for the same amount of reinforcement the expanded metal has a greater bond area per square inch of cross section per inch of length (per cubic inch of reinforcement) than have bars laid direct between supports. This condition may also be responsible for the apparently smaller stresses in the strands of the expanded metal than would be expected considering the diagonal direction of the strands.

6. COMPUTED TENSILE AND COMPRESSIVE STRESSES AT MAXIMUM LOAD.

All the beams discussed in this paper failed by tension in the reinforcement except some of those having one-way diagonal reinforcement in which the beam action was complicated by torsional stresses as explained in Section II, 3. In Table 1 the computed tensile stresses for maximum load are given. These were computed by equation (14) in which j was taken as $1 - \frac{1}{3}k$. That is, for the reasons given in Section III, 2, the stresses were as-

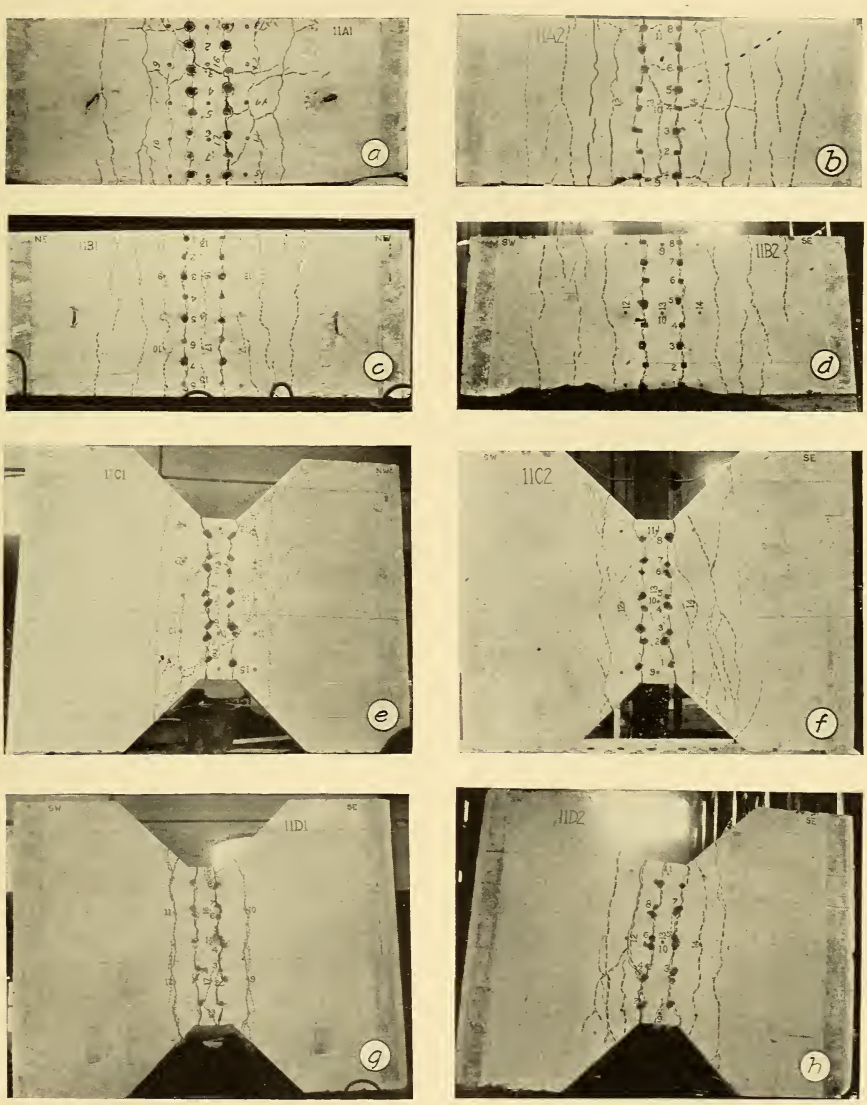


FIG. 17.—*Views showing beams 11A1–2, 11B1–2, 11C1–2, and 11D1–2 after testing.*

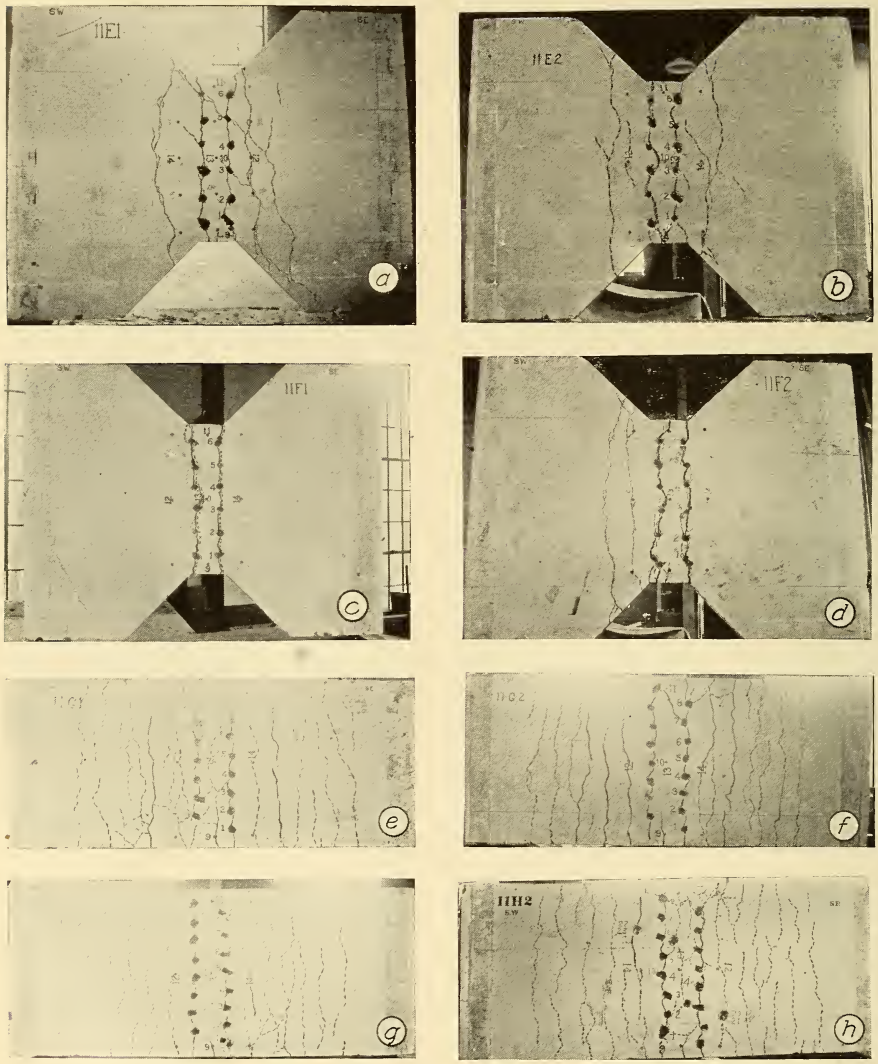


FIG. 18.—*Views showing beams 11E1–2, 11F1–2, 11G1–2, and 11H1–2 after testing.*

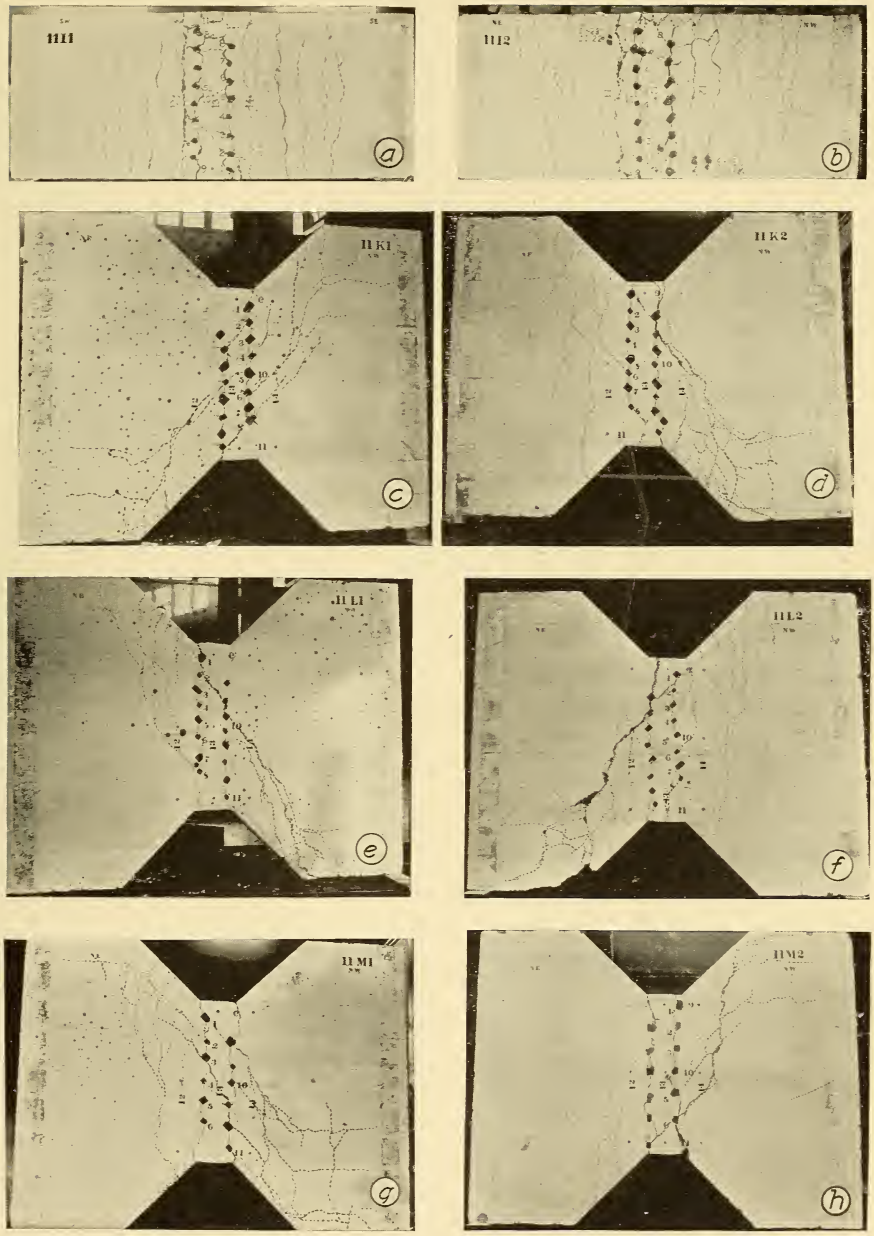


FIG. 19.—*Views showing beams 11I1–2, 11K1–2, 11L1–2, and 11M1–2 after testing.*

sumed to be equal in the two layers of reinforcement at the maximum load on the beam and the centroid of stress was taken to be at the centroid of the sectional area of the reinforcement. It will be seen that for some of the beams with direct reinforcement and for some of those with expanded metal the computed tensile stresses at failure were less than the observed stresses shown in Figures 20 to 32 for loads below the maximum load. It is possible that this behavior is brought about in part by the difference in depth to the two layers of reinforcement. The assistance from the upper layer of reinforcement may have prevented a sharp break in the load stress curve at the load causing the yield point stress in the lower layer of reinforcement.

It has already been pointed out that for the beams with two-way diagonal reinforcement the stresses in the upper bars were about the same as those in the lower bars, and likewise the average computed tensile stress at the maximum load agrees fairly well with the observed stresses at the load which caused a marked yielding of the beam. As to why the stresses were nearly equal in the two layers of bars for beams having two-way 45° diagonal reinforcement, no explanation is offered. The fact that the discrepancy between the computed tensile stress at maximum load and the yield point was greater for the expanded metal than for the other reinforcement may, perhaps, be explained by the fact that in proportion to the depth of the beam the distance between layers of reinforcement was greatest for the beams reinforced with expanded metal.

Uncertainties resulting from the unexpected behavior found in examining the tensile stresses appear to make it impossible to explain some of the phenomena of the tests. For this reason the results are not as valuable as they would have been if the compressive stresses and the tensile stresses in both layers for all beams had been measured. However, the tests indicate that reinforcement placed in more than one layer can not be expected to resist the tensile stresses as efficiently as when placed in one layer. The average distance between the centers of the two layers was 22 per cent of the depth for the direct reinforced beams and 28 per cent of the depth for those reinforced with expanded metal. The strength of some of the beams may have been limited by reaching the yield point in the lower layer of reinforcement without having developed the yield-point stress in the upper layer of reinforcement even at the maximum load.

The compressive stresses given in Table 1 were computed by the straight line formula, equation (16), using values of k and j based on the assumption that the center of tensile stresses was at the centroid of the section of the reinforcement. It will be noted that for the beams with high percentages of reinforcement the computed stresses at the maximum load were much in excess of the strength of the cylinders reported in the same table. Even the stresses computed by the parabolic formula were greater than the cylinder strengths, although the beams did not fail in compression. The uncertainties as to the correct method of analysis will, of course, have a bearing on the computed compressive stresses, but there will be less uncertainty than with respect to the tensile stresses. On almost any basis that the analysis can be made it will be found that the computed compressive stresses were greater than the cylinder strength. To this extent it will be found that the test results are in accord with those found in previous investigations.[5]

If the assumption be made that the parabolic formula gives the true compressive stress in a beam, and that the true strength of the concrete is given by the cylinder tests, Table 6 will show that for the larger percentages of reinforcement the stresses computed by the straight-line formula may be expected to be approximately 35 per cent greater than the cylinder strength of the concrete. Taking into account the additional fact that due to unevenness of bearing in cylinder tests the maximum stress in the cylinder may be considerably greater than the average stress reported as the strength, the high computed compressive stresses reported in Table 1 need not be considered as extremely unusual.

TABLE 6.—Relation between Compressive Stresses by Straight-Line Formula and by Parabolic Formula.[a]

Ratio of reinforcement. (p).	Compressive stress for tensile stress of 58,000 lbs./in.^{2}		Ratio f_c/f'_c.
	By parabolic formula, f'_c.	By straight line formula, f_c.	
	Lbs./in.^{2}	Lbs./in.^{2}	
0.01	1,810	2,660	1.47
.02	2,910	4,110	1.42
.03	3,910	5,440	1.39
.04	4,860	6,560	1.37
.05	5,800	7,840	1.35

[a] In computations by parabolic formula it is assumed that the parabola is "complete"; that is, that the vertex is at the extreme fiber in compression and, therefore, that the stresses given by the parabolic formula are the ultimate strengths of the concrete and the same as would be determined by cylinder tests. Here n was taken as 15. Smaller values of n would give slightly smaller values of f_c/f'_c.

[5] Compressive strength of concrete in flexure, Proc. Am. Conc. Inst., 16, p. 120; 1920.

IV. SUMMARY.

The tests reported in this paper were planned with the view of obtaining information, needed by the concrete ship section of the Emergency Fleet Corporation in the design of concrete ships, on the behavior of diagonal reinforcement in thin slab beams. Some of the conditions imposed in the tests vary from those which usually obtain in the construction of reinforced concrete, particularly in that the concrete mix was very rich (1 : ⅔ : 1⅓), the percentage of reinforcement was large (from 1.5 to 6.5 per cent), and the slab beams were shallow (about 4 inches thick). These conditions in turn make the position of the reinforcement in the beam and the manner of variation of the stress on a cross section in the beam important factors in the analysis of the test data, but factors that can not be known exactly. Therefore, as pointed out in the body of the paper, some of the quantitative results given should be regarded as approximate. However, although there are some uncertainties involved in the analysis of the test data, and the number of tests is not sufficient to make average results entirely reliable, several methods of interpreting the tests have led to substantially the same conclusions.

The results of these tests furnish no information on the relative merits of direct and diagonal reinforcement or of loose bars and expanded metal in slabs supported on four sides or for resisting of stresses set up by temperature changes, shrinkage, unequal settlement, etc.

The following summary of results is given:

1. The average unit stress in the expanded metal reinforcement of the slab beams was approximately the same, for a given load, as that in the reinforcement of the beam having bars laid direct between supports. The average stress in bars laid in two bands making 45° with the direction of the span was about 1.8 times the stress in the reinforcement of the beams having bars laid direct between supports, when the data had been reduced to a comparable basis.

2. The ratio of the average observed stress in the expanded metal reinforcement to the stress in the direct reinforcement, therefore, was about 15 per cent less than that given by the secant method of computation, and the average ratio for the bars of the two-way 45° diagonal reinforcement was about 4 per cent less than that given by the secant method of computation. The comparison in this and the preceding paragraph were made for loads causing stresses of 20,000 and 30,000 lbs./in.^{2} in the direct reinforcement. Assistance from the concrete in carrying the ten-

sile stresses will probably help to account for the lower stresses found in the expanded metal.

3. The ratio of the maximum loads carried by the beams reinforced with expanded metal ($\theta = 22\frac{1}{2}°$) and by those reinforced with diagonal bars ($\theta = 45°$) to the loads carried by the beams with direct reinforcement ($\theta = 0°$) were, respectively, 13 per cent less and 13 per cent greater than is indicated by the secant method of computation.

4. For the three directions of the reinforcement the ratios of the loads carried to the computed loads were 0.963, 0.818, and 1.087 where the angles θ were 0°, 22½°, and 45°, respectively.

5. The tests indicate that on the whole the stresses, deflections, and maximum loads for beams with the three types of reinforcement were approximately in accordance with the results obtained from the analysis based upon the secant relation. The shallowness of the beams and the fact that the reinforcement was in two layers give an importance to small errors in placing reinforcement and in the taking of measurements, which makes it desirable that the results be confirmed by tests not so limited in these respects.

6. The tension cracks were distributed more uniformly throughout the span for the beams reinforced with expanded metal than for those reinforced with bars.

7. The slab beams reinforced with expanded metal with a lap of 10 inches (slightly more than a length of 1 diamond) at the center of the beam failed by slipping of the sheets of expanded metal at the lap. A stress of 28,000 lbs./in.^{2} was developed in the strands of the expanded metal in one of the beams and 40,000 lbs./in.^{2} in the strands in the other beam before slipping caused great unevenness in the distribution of the stress in the various strands. A lap of 1½ diamonds for the larger sizes of expanded metal used in these tests would seem to be sufficient to develop the yield-point strength of the reinforcement before slipping at the lap occurs.

8. None of the beams tested failed in compression of the concrete, although the calculated compressive stress at the extreme fiber computed by the straight-line formula for the two beams having approximately 5 per cent of direct reinforcement was 1.73 times the compressive strength obtained from the tests of the control cylinders. This result is in conformity with the trend of other test results.

WASHINGTON, August 7, 1922.

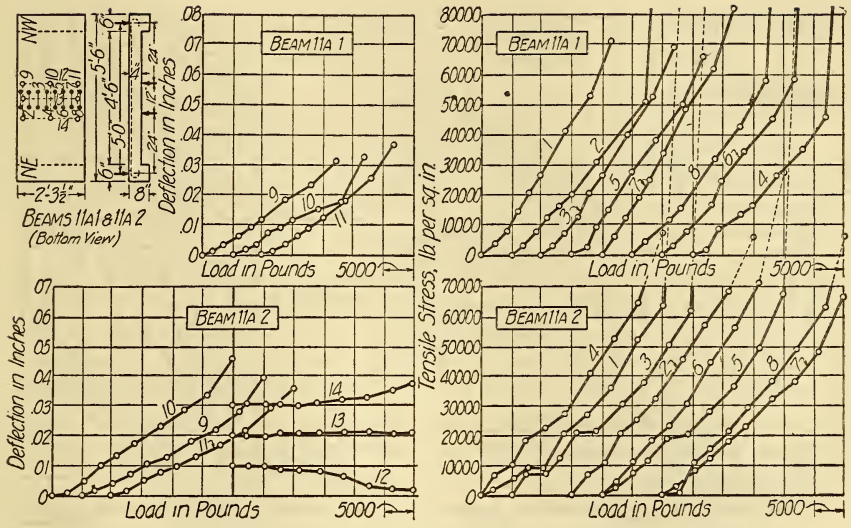


FIG. 20.—*Load-stress and load-deflection curves for beams 11A1 and 11A2.*

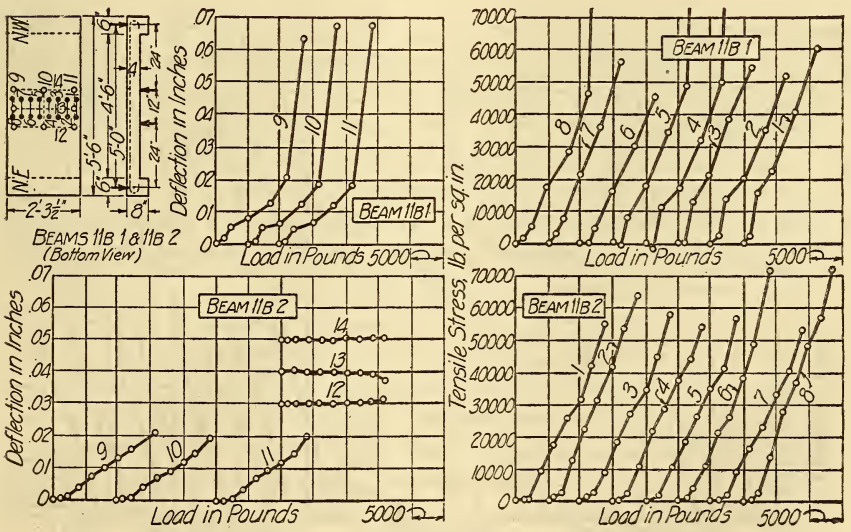


FIG. 21.—*Load-stress and load-deflection curves for beams 11B1 and 11B2.*

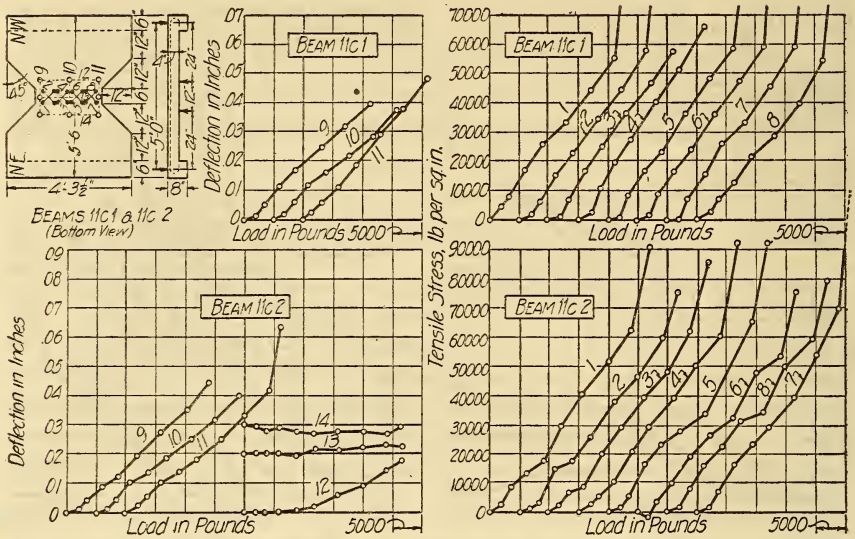


FIG. 22.—*Load-stress and load-deflection curves for beams 11C1 and 11C2.*

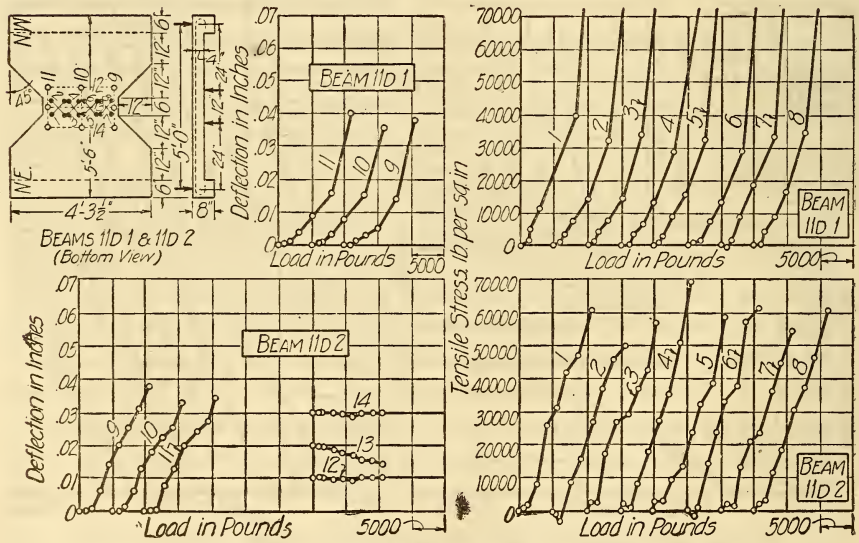


FIG. 23.—*Load-stress and load-deflection curves for beams 11D1 and 11D2.*

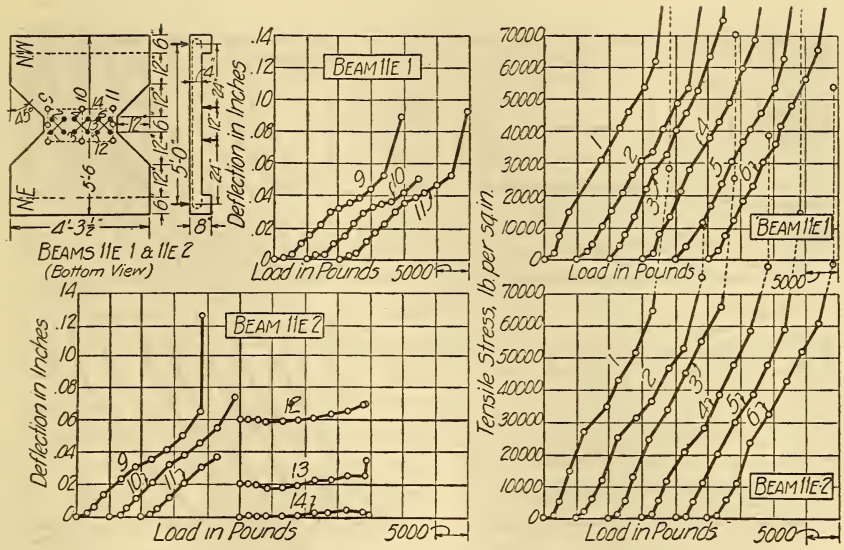


FIG. 24.—*Load-stress and load-deflection curves for beams 11E1 and 11E2.*

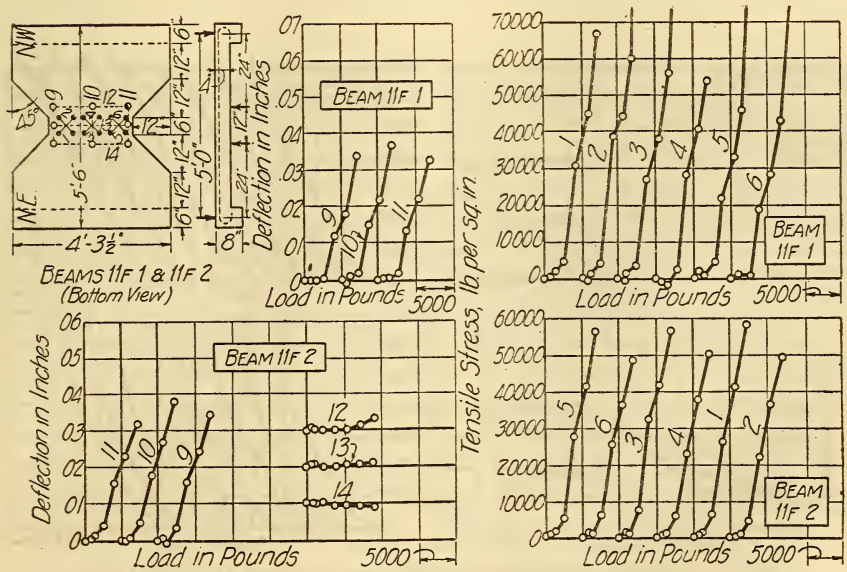


FIG. 25.—*Load-stress and load-deflection curves for beams 11F1 and 11F2.*

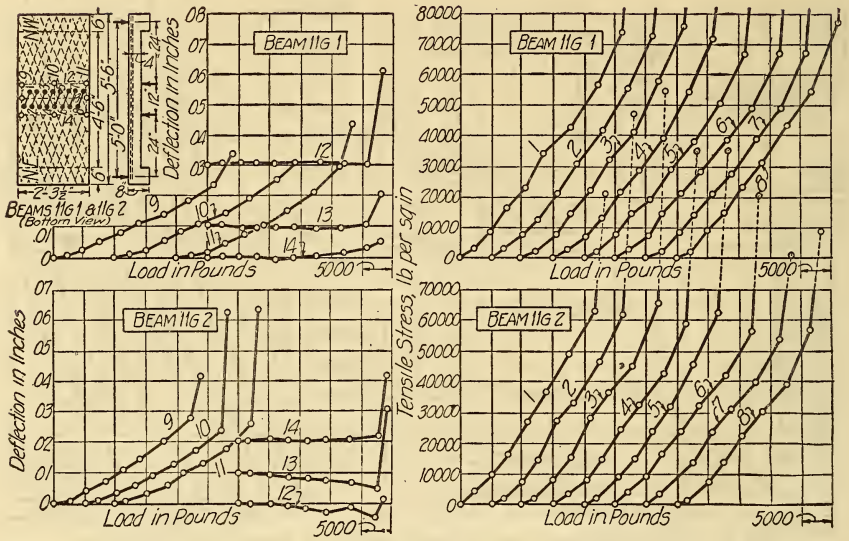


FIG. 26.—*Load-stress and load-deflection curves for beams 11G1 and 11G2.*

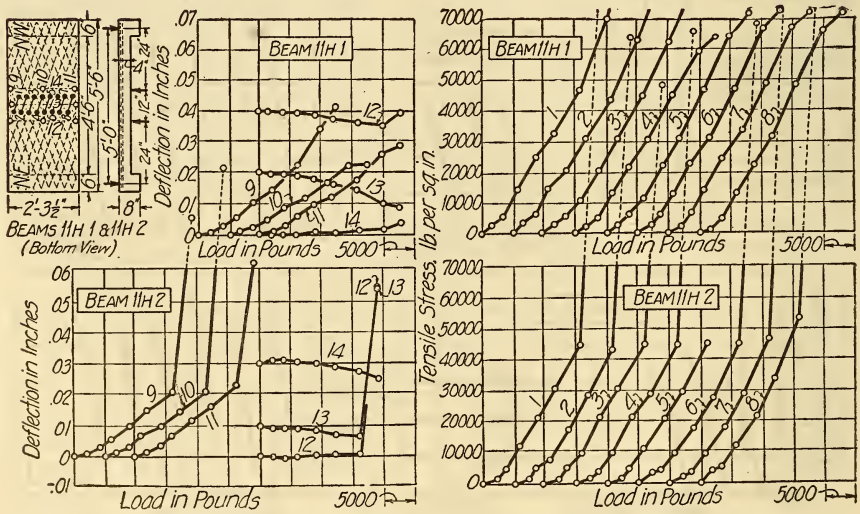


FIG. 27.—*Load-stress and load-deflection curves for beams 11H1 and 11H2.*

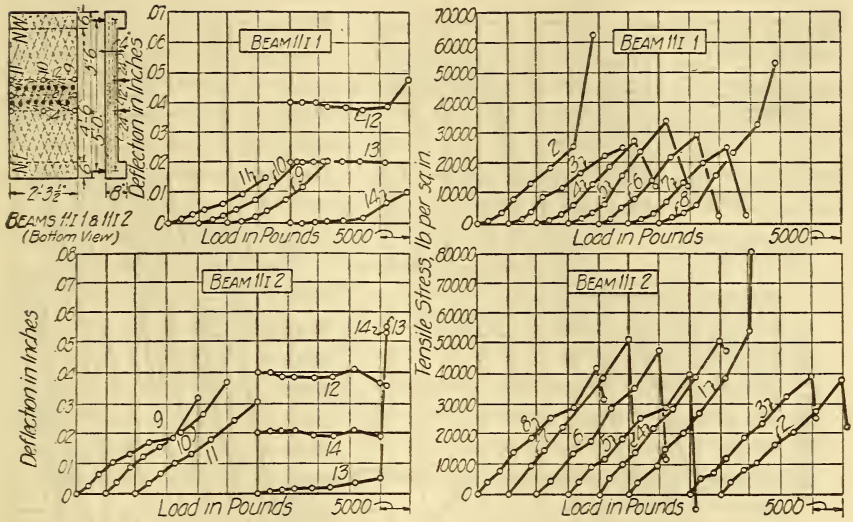


FIG. 28.—*Load-stress and load-deflection curves for beams 11I1 and 11I2.*

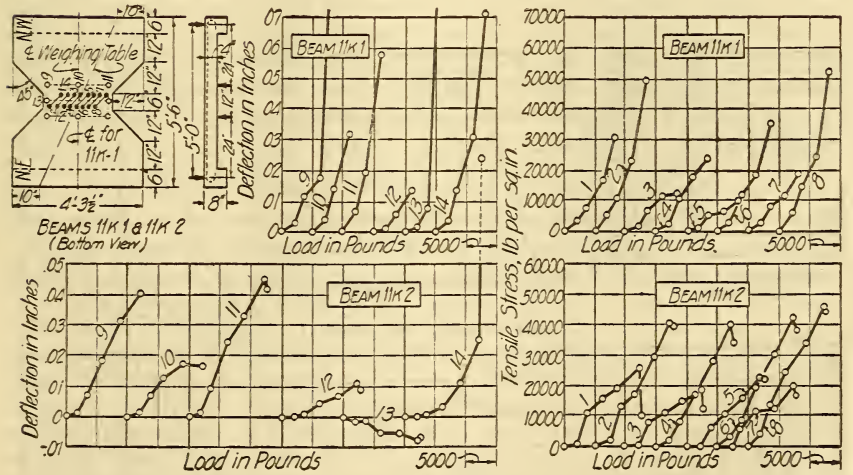


FIG. 29.—*Load-stress and load-deflection curves for beams 11K1 and 11K2.*

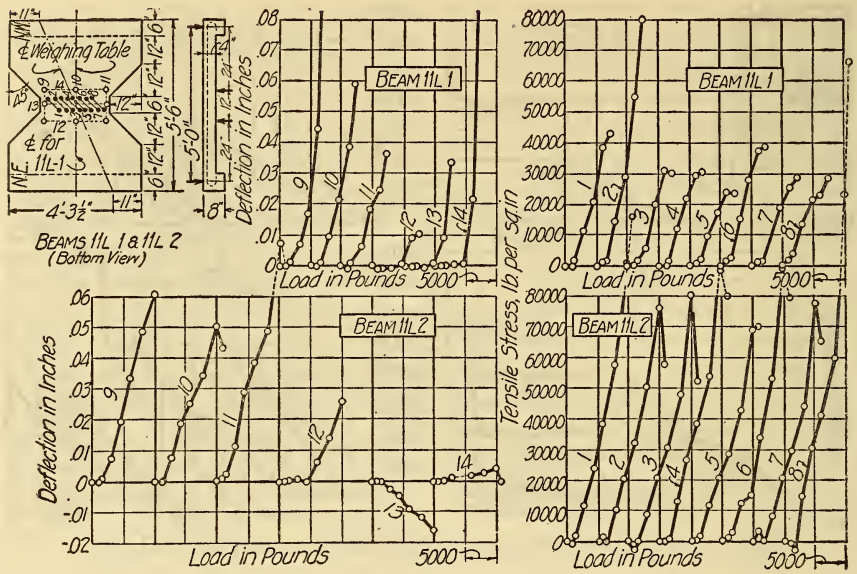


FIG. 30.—*Load-stress and load-deflection curves for beams 11L1 and 11L2.*

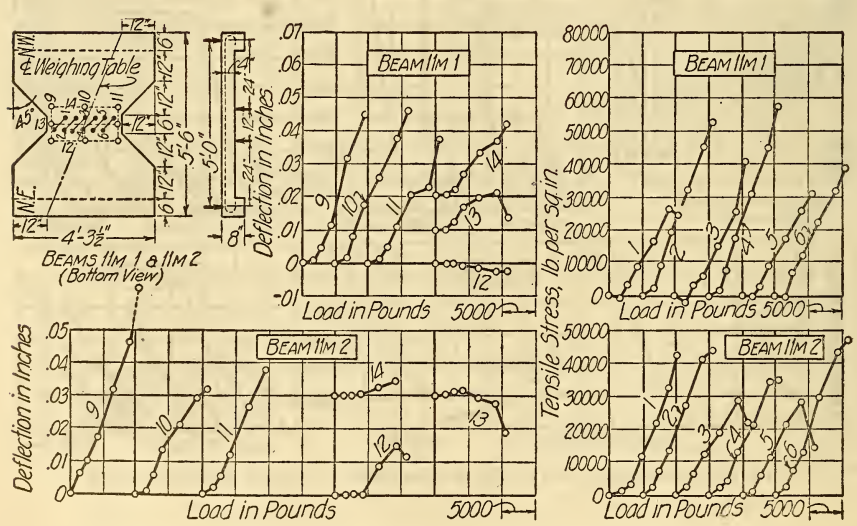


FIG. 31.—*Load-stress and load-deflection curves for beams 11M1 and 11M2.*

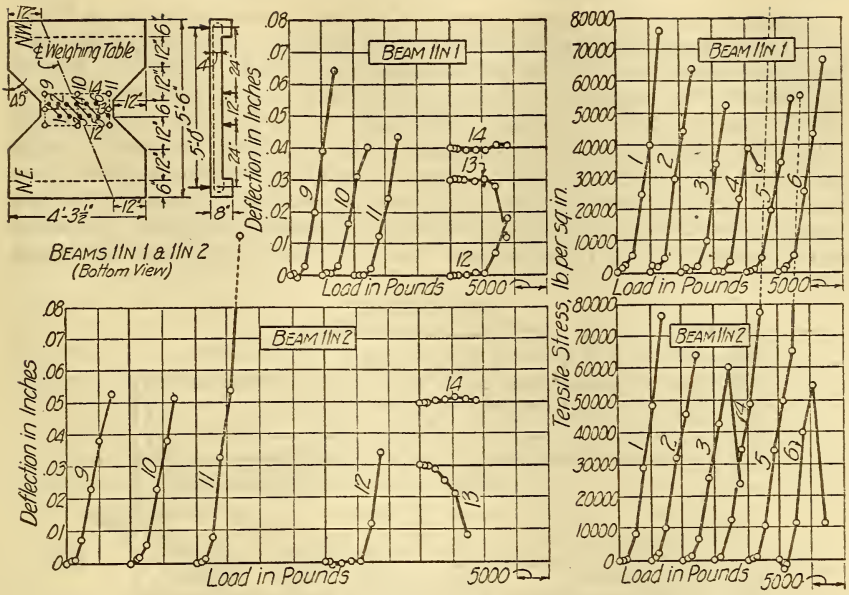


FIG. 32.—*Load-stress and load-deflection curves for beams 11N1 and 11N2.*

APPENDIX: ABSTRACT OF "CONCRETE REINFORCEMENT AT AN ANGLE WITH THE DIRECTION OF THE NORMAL FORCE," BY PROF. E. SUENSON.

Since the completion of this paper an article [6] has been published by Prof. E. Suenson, of Copenhagen, whose field and method of attack are, in general, the same as those of this paper. Professor Suenson makes an analysis of the problem and publishes the results of tests. So far as the analysis covers the same field as that in this paper the results are identical with the results given here. He considers the relative effectiveness of diagonal and of direct reinforcement for cases where both are present in the same member, and for cases where all of the reinforcement is diagonal: (*a*) In preventing the initial cracking of the concrete, (*b*) in resisting the stresses set up after the concrete has been cracked and no longer is assumed to resist any of the tensile stress, (*c*) in resisting the stresses after the yield point of the reinforcement has been passed.

The analysis leads to the conclusion that in comparison with the direct reinforcement the diagonal reinforcement is still less effective in preventing the initial cracking of the concrete than it is in resisting the stresses which are set up after the concrete has cracked. A knowledge of the relative values in resisting the stresses after the yield point of the reinforcement has been reached is of especial significance in the cases where both direct and diagonal reinforcement are present in the same beam. The analysis of the case in which both diagonal and direct reinforcement are present points out that the stress will be much greater in the direct than in the diagonal reinforcement, and that, therefore, diagonal reinforcement can not be used economically without overstressing the direct reinforcement. After the yield point of the reinforcement has been passed the stresses in the two sets of reinforcement become very nearly the same, and, therefore, the diagonal reinforcement performs a larger part proportionally in resisting the total stress than at loads below the yield point.

[6] Eisenbetonbewehrung unter einem Winkel mit der Richtung der Normalkraft. Beton und Eisen, June 15, p. 145; 1922.

The test beams had the nominal dimensions and the reinforcement shown in Figure 1A. The measured depths and widths are given in Table 1A. The reinforcement used had a very sharply defined yield point of about 44,000 lbs./in.^{2}, while at a stress of 42,500 lbs./in.^{2} the proportional limit had not been exceeded. The span used in testing was 6 feet 6.74 inches. The load points were 11.81 inches apart. Measurements of deflection and rather rough measurements of the longitudinal deformation on the tension side of the beams were taken. These measurements indicate a general agreement with the results of the analysis, but the main dependence is placed upon the loads carried at the occurrence of the first crack and at the failure of the beams. It is stated that the beams with direct reinforcement "behaved as brittle specimens which broke

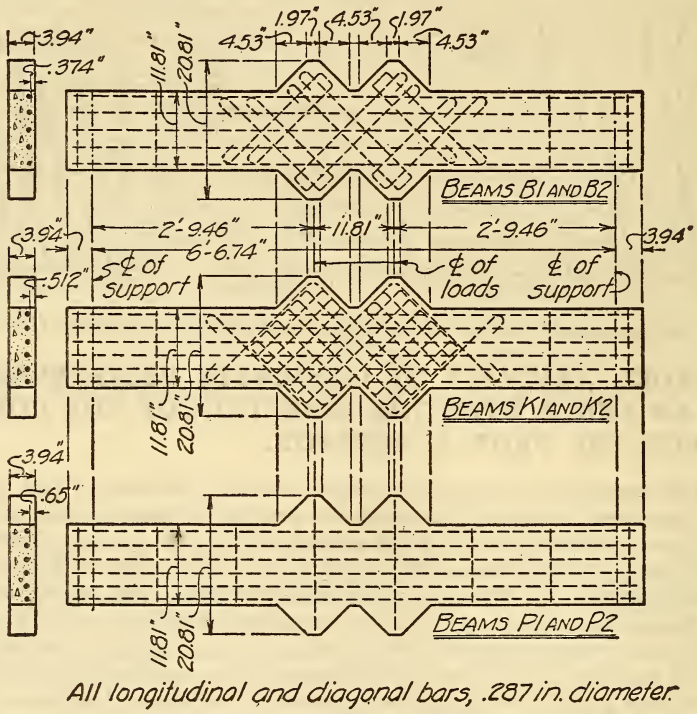


FIG. 1A.—*Design of beams tested by Suenson.*

down suddenly with a deflection of only 0.8 centimeter (0.315 inch) while the others were extraordinarily tough."

The apparent indication from the tests was that the diagonal reinforcement was less effective in preventing the occurrence of the first crack than the analysis indicated. Professor Suenson attributes this showing, partially at least, to the fact that the effectiveness of the concrete in the layer occupied by the diagonal reinforcement is impaired. The presence of the reinforcement prevents the transfer of tension by the concrete through the intersecting bars. By neglecting the concrete in this layer he shows that the computed stresses in the concrete and in the reinforcement at the time of the first crack will be more nearly equal for the three types of beams than are the values shown in Table 2A.

In Table 2A it will be noted that at the maximum load the computed stress in the reinforcement was greater than the yield point (44,000 lbs./in.^{2}) for all the beams. This was believed to be due, partially at least, to the rising of the neutral axis and

the consequent increase in the length of the moment arm, *jd*, after reaching the yield point. A correction was made by assuming that for the entire depth from the top of the beam to the neutral axis the concrete stress had reached the assumed ultimate strength of 7,125 lbs./in.^{2}, and a new moment arm, *jd*, was determined accordingly. The corrected stresses given in column 7 of Table 2A were computed in this manner. This brought the computed stress for the beams with direct reinforcement into fair agreement with the observed yield point. It will be seen, however, that the computed stresses for the beams with diagonal reinforcement were greater than those for the beams with direct reinforcement even after this correction had been made, indicating that at the maximum load the diagonal reinforcement was more effective in resisting the stresses than was found in the analysis. This difference is believed by Professor Suenson to be due to the fact that after passing the yield-point stress the elongation of the reinforcement caused the angle of the reinforcement with the span to be so much reduced as to increase appreciably the load-carrying capacity. By actual measurement the angle between the bars and the span was about 38° at the maximum load instead of 45°. Computation of the tensile stresses at the maximum load, using an angle of 38°, gave the stresses shown in column 8 of Table 2A. Obviously these values are considerably closer to the yield point of the steel than the uncorrected values given in columns 6 and 7 of Table 2A.

TABLE 1A.—Dimensions of Beams and Amounts of Bending Moment Applied.

Beam number.	Depth.		Width.	Reinforcement.[1]		Applied moment.		Ratio of the two moments.
	Total.	To c. g. of steel.		Direct.	Diagonal.	At first crack.	At maximum load.	
	Inches.	Inches.	Inches.			Lbs./in.^{2}	Lbs./in.^{2}	
P1	4.03	3.23	11.78	6 bars	.....	32,550	54,600	0.596
P2	4.04	3.26	11.81			26,980	54,100	.500
B1	4.11	3.26	11.81	3 bars	4 bars	26,980	56,300	.480
B2	4.15	3.35	11.85			26,900	56,600	.474
K1	4.12	3.23	11.85	.....	8 bars	25,200	60,000	.420
K2	4.05	3.22	11.89	.....		21,650	57,000	.378

[1] All reinforcing bars 0.287 inch in diameter.

TABLE 2A.—Computed Stresses in Beams.

Beam number.	Computed stress at first crack.			Computed stress at maximum load.			
	$\theta=45°$.[1]			$\theta=45°$.			$\theta=38°$.
	f_c.[2]	f_t.[2]	f_s.[2]	f_c.	f_t.	f_s.[2]	f_s.[3]
P1	987	901	532	2,455	50,600	45,000	44,500
P2	810	739	440	2,390	49,700	44,100	44,100
B1	792	742	425	2,740	52,100	47,100	44,800
B2	774	729	442	2,640	51,300	46,400	44,000
K1	745	714	400	3,450	56,600	52,500	47,000
K2	652	625	363	2,285	54,000	49,900	44,700

[1] θ is the angle between the axis of the bars and the direction of the span.
[2] f_c is the computed compressive stress in the concrete; f_t is the computed tensile stress in the concrete; and f_s is the computed tensile stress in the steel.
[3] These stresses computed on the assumption that the compressive stress is uniformly distributed over the part above the neutral axis and everywhere equal to 7,100 lbs./in.^{2} the ultimate strength of the concrete.

Professor Suenson recommends that where concrete in tension is considered in the computations—that is, where the loads are too small to crack the concrete—a correction should be introduced which will take account of the ineffectiveness of the concrete at the level of the diagonal reinforcement. For this purpose he proposes that the total area of concrete capable of resisting tension be reduced by the sectional area of the concrete which contains the reinforcing bars; that is, the concrete whose longitudinal continuity is broken by the reinforcement.

Where concrete is not considered to be effective, in tension Professor Suenson recommends that the indications of the analysis be made the basis of design.