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NOTES ON SMALL FLOW METERS FOR AIR, ESPECIALLY ORIFICE METERS

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NOTES ON SMALL FLOW METERS FOR AIR, ESPECIALLY ORIFICE METERS[1]

By Edgar Buckingham

ABSTRACT

The paper contains information, compiled for the use of physiologists of the Chemical Warfare Service, on the selection, design, and properties of small flow meters for air.

1. FORM OF ORIFICE

If an orifice meter is adopted, it is not advisable to use a very thin orifice plate, nor, if the plate is thick, to chamfer it on the outside so as to give the orifice a sharp edge. The thickness of the plate should be about one and one-half times the diameter of the orifice, and after a hole of the required diameter has been drilled through the plate it should be rounded off to a trumpet shape on the inlet side, so as to give an easy entrance. A suitable profile is a quarter circle of radius slightly less than the thickness of the plate, set tangent to the hole already drilled and to the entrance face of the plate, thus leaving the orifice cylindrical for a short distance in from the exit face.

An orifice of this sort, if carefully made and smoothly finished, will give a discharge which is within a few per cent of the so-called "theoretical" discharge. This is convenient because the size of orifice needed for a particular purpose may then be computed approximately from the theoretical equations, which is not the case for sharp-edged orifices.

[1] These notes were prepared in December, 1917, at the request of the Chemical Warfare Service and were intended primarily for the use of physiologists. Various calls for the same sort of information have made it seem desirable to publish them.

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2. COMPUTATION OF THE THEORETICAL DISCHARGE THROUGH A FRICTIONLESS ORIFICE

Let—

D = the diameter of the orifice in millimeters.

h = the pressure drop at the orifice.

p = the pressure of the air on the inlet side, or the "initial pressure."

$p_1 = p - h$ = the pressure on the exit side, or the "back pressure."

$r = p_1 \div p$. It is assumed that the pressure drop h is not more than 0.4 p, so that the pressure ratio r is always between 0.6 and 1.0.

T = the absolute temperature of the air on the inlet side, or the "initial temperature," measured in centigrade degrees.

V = the rate of discharge of dry air in liters per minute, measured under the initial conditions p, T.

Then the theoretical discharge is given by the equation:[2]

$$V = 2.12\, D^2 \sqrt{T} \sqrt{r^{\frac{10}{7}} - r^{\frac{12}{7}}} \tag{1}$$

It may be noted that the absolute values of the pressures do not appear explicitly in this equation. If the initial and final pressures are both doubled, the *mass* of air discharged per minute will also be doubled; but since V is now to be measured at this doubled initial pressure, its value in liters per minute will remain unchanged and may still be found from equation (1).

The volume which these V liters of air would occupy at any other temperature and pressure may be found by means of the familiar relation $pV = RT$. In particular, the volume V_0 which the air discharged per minute would occupy if measured in liters at 0° C and at a pressure of 760 mm of mercury, is given by the equation

$$V_0 = 0.761\, D^2 \frac{p}{\sqrt{T}} \sqrt{r^{\frac{10}{7}} - r^{\frac{12}{7}}} \tag{2}$$

in which p is expressed in millimeters of mercury.

[2] A deduction of this equation is given in a note at the end of the paper.

3. THE RELATION OF ACTUAL TO THEORETICAL DISCHARGE

In practice, the discharge of dry air from an orifice or nozzle of the form described in section 1 is found to be a little less than the value computed from equation (1) or (2). The ratio of the actual to the theoretical discharge is known as the discharge coefficient. For orifices of a diameter of 2 mm or more the *dsicharge coefficient* will usually be over 0.9 and may be nearly 1. Hence the theoretical equation enables us to compute beforehand about what size of orifice will be needed for a given discharge under given conditions, and so to design or to select an orifice suitable for the purpose in hand.

But it is not possible to predict the value of the discharge coefficient exactly; and, furthermore, the value is not constant for a given orifice, but varies with the pressure ratio r. Hence each orifice must be standardized by experiments at various values of r. For orifices which are large enough that accurate reproduction is mechanically practicable, a single standardization may suffice for a number of similar orifices; but for diameters of only 1 or 2 mm such accuracy is difficult of attainment, and should not be assumed without investigation.

It is not possible to give exact formulas for moist air; but it may be said that, unless the air is so moist that water is deposited in the orifice, the discharge will not differ much from that for dry air. If water is deposited on the walls of the orifice, the discharge rate is liable to be irregular. For security, the air should be dry, when practicable, or the orifice should be slightly warmed to prevent the deposition of water upon it.

4. THE EFFECT OF VARIATIONS IN THE INITIAL TEMPERATURE OF THE AIR

While the discharge coefficient of a given orifice varies somewhat with the pressure ratio, it is not sensibly affected by moderate changes in the initial temperature of the air; and for an initial temperature range of 0 to 50° C it is safe to treat the discharge coefficient as independent of the temperature. This means that when an orifice has been standardized at one temperature, the same standardization may be used for other temperatures if the changes of temperature are allowed for as indicated in the theoretical equations already given. If the air reaching the orifice is at ordinary room temperature, in the vicinity of 20° C or 293°

absolute, a change of 1° C changes the value of V or of V_0 by a trifle more than one-eighth of 1 per cent. Hence, if no greater accuracy than 1 per cent is required, an orifice that has been standardized at 1 temperature—e. g., 18° C—may be used without any temperature correction for any temperature within 5 or 6° of the temperature at which the experimental standardization was carried out.

5. THEORETICAL DISCHARGE CURVE IN TERMS OF THE SUCTION HEAD

In using an orifice meter the quantity primarily observed is the pressure drop h; the initial temperature and either the initial or the back pressure being also observed if not already known. Having the values of h and either p or p_1 we may set $r=\frac{p-h}{p}$ or $r=\frac{p_1}{p_1+h}$ and compute the theoretical discharge by means of equation (1).

But since the computation is rather cumbersome, it is convenient, if many problems relating to the same initial conditions are to be solved, to construct once for all a curve giving V in terms of h for those conditions. Furthermore, since D enters the equation very simply, it is well to construct the curve for $D=1$ mm and allow separately for changes in D.

Let us therefore set $D=1$; and let us adopt, as our fixed initial conditions, 18° C and 1 atmosphere pressure, the air being drawn through the orifice by the suction h and the back pressure being less than atmospheric. We now have $T=291$, and equation (1) reduces to

$$V=36.1\sqrt{r^{\frac{10}{7}}-r^{\frac{12}{7}}} \tag{3}$$

Let us suppose, further, that the pressure drop h is measured in centimeters of water. Then, since a head of 760 mm of mercury is equivalent to a head of 1034 cm of water, we have to substitute in equation (3) the value $r=(1034-h)/1034$. By using various values of h we may then compute the corresponding values of the theoretical discharge V and plot a curve showing the relation of V to h for an orifice of 1 mm diameter under the given initial conditions. Such a curve is appended to this paper.

6. ILLUSTRATIONS OF THE USE OF THE THEORETICAL DISCHARGE CURVE

The method of using such a curve may be illustrated by the following examples:

(*a*) Suppose we wish to know the rate at which air under the above-described initial conditions will be sucked through a 2.6 mm orifice by a head of 27 cm of water. Reading from the curve at $h=27$ we find $V=3.08$. An orifice of 2.6 mm diameter will therefore give a theoretical discharge of $3.08 \times 2.6^2 = 20.8$ liters per minute. Actually, the discharge will be a few per cent less than this.

(*b*) Suppose that in the foregoing example the initial temperature had been not 18 but 30° C; then the theoretical rate of discharge in liters per minute *measured at 30° C* would have been

$$20.8\sqrt{\frac{273+30}{273+18}}=21.2.$$

(*c*) Suppose that we want to select an orifice which shall discharge 85 liters per minute with a suction head of from 40 to 70 cm of water. From the curve we find that for $D=1$, at $h=40$ $V=3.72$, and at $h=70$ $V=4.84$. We must therefore use an orifice of such diameter D that $\frac{85}{D^2}$ lies between 3.72 and 4.84, which means that D must lie between 4.78 and 4.19 mm. An orifice of 4.5 mm diameter would give 85 liters per minute at the same head as would be required for an orifice at 1 mm diameter to discharge $\frac{85}{4.5^2}=4.2$ liters per minute; and we find from the curve that the required theoretical head is $h=52.2$ cm of water.

7. REMARKS

It will be sufficiently evident from the foregoing examples how the curve may be used for other temperatures than the one for which it was constructed, as well as for other diameters than 1 mm. But since the use at present proposed for the curve is merely for selecting or designing orifices which are then to be standardized by experiment, and since the discharge coefficient will always be in doubt by several per cent until this standardization has been effected, no great accuracy in using the curve is needed, and allowances for temperature changes may as well be disregarded so long as the initial temperature remains between 0 and 35° C.

To the degree of approximation now in question the curve may also be used for solving problems relating to the discharge of air against a constant back pressure of one atmosphere, the head h cm of water being now an excess pressure on the inlet side above one atmosphere. At $h = 80$, the value of V read from the curve is about 3.5 per cent greater than the theoretical discharge under these conditions if the volume is measured at 18° C and at the initial pressure of (1 atm. $+h$); and it is about 4.5 per cent less than if the volume is measured at 18° C and at 1 atmosphere. For smaller values of h these discrepancies are smaller; hence for rough purposes the curve may be used to solve problems relating to this second method of working.

If, finally, readings are desired in terms of V_0, the volume discharged measured under standard conditions, a curve may easily be constructed from equation (2). The time-consuming part of such work is the computation of $\sqrt{r^{\frac{10}{7}} - r^{\frac{12}{7}}} = B$. A table of values of B in terms of r from $r = 0.92$ to $r = 1.00$ is given at the end of the paper.

8. EFFECT OF THE SPEED OF APPROACH

The theoretical equations given above are deduced on the assumption that the initial velocity of the air, at the place where p and T are measured, is so small that its square is negligible in comparison with the square of the speed of the air through the orifice. Hence if the orifice plate is merely inserted as a diaphragm across a pipe which is not much larger in diameter than the orifice itself, the formulas as given will be considerably in error.

If the pressure drop at the orifice is small, so that the pressure ratio is not far from unity, the density of the air is not very different on the two sides of the orifice. Hence the speeds in the approach pipe and in the orifice will be approximately inversely proportional to the areas, and their squares, to the fourth powers of the diameters, so that it is not necessary to slow down the approaching air by introducing a large chamber at the place where the orifice is to be put. If, for example, the internal diameter of the housing is 5 times that of the orifice, the square of the speed through the orifice is some 625 times the square of the speed of approach, and a further increase of the diameter ratio could not have any appreciable effect on the rate of flow through the orifice.

9. REMARKS ON SOME OTHER SIMPLE FORMS OF FLOW METER

(*a*) Sharp-edged Orifices or holes in thin plates are often used in flow meters, instead of the trumpet-shaped orifices or nozzles recommended in section 1 above, and if properly standardized they are satisfactory. They have, however, the disadvantage that the discharge coefficient varies rapidly with the pressure ratio and may be as small as 0.6, so that the theoretical equations can not safely be used to give an approximate estimate of the rate of discharge by merely assuming a constant discharge coefficient of, say, 0.95, as may be done with an orifice which has a trumpet-shaped entrance.

(*b*) The Venturi Meter, while excellent in large sizes, could hardly be made satisfactory for rates of flow of the order of 80 liters per minute or less; and it would be difficult to construct on so small a scale as would be required unless recourse were had to the use of extremely delicate differential gages with their attendant disadvantages.

(*c*) Capillary Tube Meters in which the pressure drop to be measured is due to the resistance of a length of tube, have some defects, of which the most serious is liability to obstruction by dust or condensed water vapor. For small rates of flow the tube must be either very long or very fine. Long tubes are inconveniently fragile if made of glass, while, if the tube is opaque, the presence of an obstruction can be detected only from the behavior of the tube. On the other hand, the fineness of bore needed when the tube is short, greatly increases the liability to obstruction. And though the visibility of an obstruction in a glass tube may save the observer from relying on erroneous readings of flow, it does not remove the obstruction nor obviate the necessity of either cleaning or restandardizing.

Certain other points regarding capillary tube meters may also be worth mentioning. We shall first suppose that the tube is straight, and that its length is a large multiple of its diameter (e. g., 1000) so that the resistance is only slightly influenced by the nature of the ends and is nearly proportional to the length of the tube. Such a tube may behave in either of two quite different ways.

If the flow is slow enough, the rate of discharge is directly proportional to the pressure drop—a very simple and convenient relation; but it is also inversely proportional to the viscosity of the air in the tube; and since in the vicinity of room temperature the viscosity of air increases by about 0.3 per cent per degree C rise in temperature, no great accuracy can be achieved unless the tube is jacketed in some way, so that its temperature can be controlled and observed.

On the other hand, if the flow is very rapid, the nature of the fluid motion is entirely different; the resistance is greater and the rate of discharge increases more nearly as the 0.6 or 0.5 power of the pressure drop. And, finally, there is an ill-defined intermediate range of speeds where either of the foregoing regimes may establish itself, the nature of the motion and the relation of discharge to pressure drop sometimes changing suddenly from the first to the second. In this critical region no standardization can be relied on, and care must be taken, not only to avoid this range, but also to make sure that a standardization under one regime is not, by inadvertence, extrapolated to a point where the other regime is the actual one.

Short tubes form a transition stage between tubes which are so long that the nature of the ends is unimportant, and orifices in thin plates where only the ends remain and the middle has shrunk to nothing. We have very little information about the behavior of such tubes or even of long tubes which are not straight and cylindrical. It does not follow that such tubes may not be entirely satisfactory for use as flow meters after they have been standardized by experiment. But from the standpoint of design, they present the disadvantage that we have no simple mathematical theory which we know will represent their behavior sufficiently well to enable us to select the dimensions needed for a particular purpose with a certainty of getting approximately the desired result.

10. THE MEASUREMENT OF VERY SMALL RATES OF FLOW

For an orifice of 1 mm diameter and for the initial conditions, 18° C and 1 atmosphere pressure, we find by reading from the curve that a discharge of 1 liter per minute requires, theoretically, a suction head of about 2.8 cm of water. And since this is too small a head to be read accurately on an ordinary vertical U gage, it is evident that for rates of 1 liter per minute, or less, a more sensitive gage must be used, or, if an orifice meter is to be employed, the orifice must be less than 1 mm in diameter.

The sensitiveness of the reading of h may be increased about 10 times by means of an inclined U gage; but if much more is required, recourse must be taken to some more sensitive type of differential gage. There is no difficulty in making such gages, but they are likely to be inconvenient and sluggish; and unless both sides of the gage are equally accessible to slight accidental irregularities of pressure, such as may arise from a gusty wind or the opening and closing of doors, very sensitive gages are not easy to work with. On the other hand, orifices of much less than 1 mm diameter, even when carefully finished, are so liable to be affected by dust, dirt, or condensed water vapor that they are not to be recommended.

In view of the fact that the resistance of a tube of given diameter can be indefinitely increased by increasing the length, it seems likely that the most convenient form of meter for low rates of flow will be one in which the resistance across which the pressure drop is observed is furnished by a capillary glass tube, a long tube in coil form being preferred to a short and very fine straight tube. If the air is carefully dried and freed from dust, such a tube should be satisfactory in the sense of providing a constant resistance.

If it is not practicable or not desirable to dry the air before it reaches the meter, condensation in the tube may be prevented by keeping the tube at a temperature considerably above that of the incoming air. As already remarked in section 9(*c*), the temperature of the tube must be controlled if accurate measurements are to be made, and, if the bath in which the tube is immersed is kept well above the temperature of the incoming air, there will be no risk of condensation in the tube.

If a meter of this sort is adopted, the dimensions of the tube may be chosen so as to give a convenient pressure drop h at the desired rate of flow. Equations might be given connecting the rate of flow with the pressure drop for straight, round tubes of known dimensions, but they would not be reliable for bent or coiled tubes. In any event, such a tube must be standardized by experiment; and, if apparatus for standardization is available, it is a simple matter to select by trial a suitable tube for the purpose in hand.

It is advisable to keep the discharge rate always well below the critical range mentioned in section 9(*c*); that is, to havet he flow always so slow that the discharge rate is nearly proportional to the pressure drop *h*.

WASHINGTON, December 6, 1920.

Values of $B=\sqrt{r^{\frac{10}{7}}-r^{\frac{12}{7}}}$

h	$\frac{1034-h}{1034}=$ r	$\sqrt{r^{\frac{10}{7}}-r^{\frac{12}{7}}}=$ B	h	$\frac{1034-h}{1034}=$ r	$\sqrt{r^{\frac{10}{7}}-r^{\frac{12}{7}}}=$ B
80	0.9226	0.1424	10	0.9903	0.0523
60	.9421	.1247	7	.9932	.0438
40	.9613	.1029	4	.9961	.0332
30	.9710	.0896	2	.9981	.0235
20	.9807	.0736	1	.9990	.0166
15	.9855	.0639	0.5	.9995	.0118

NOTE

DEDUCTION OF THE "THEORETICAL EQUATION"

Let a fluid be flowing steadily along a channel from a section A, where the static pressure is p and the absolute temperature is T, to a second section A_1, where the pressure and temperature are p_1 and T_1, the sections being at the same level so that gravity may be disregarded.

Let v, K, e be, respectively, the volume, the kinetic energy, and the internal energy, of unit mass of the fluid as it passes A; and let symbols with subscripts refer to A_1. Let Q be the heat received from without by unit mass while it is between A and A_1.

By the first law of thermodynamics, the increase of energy per gram of fluid from A to A_1 is equal to the heat added plus the excess of the work done on the fluid as it enters at A over the work it does on the fluid ahead of it in issuing at A_1. Hence we have

$$(K_1+e_1)-(K+e)=Q+pv-p_1v_1,$$

Or

$$K_1-K=e-e_1+pv-p_1v_1+Q \tag{1}$$

Let A_1 be the section of a stream issuing from a nozzle or orifice, and let A be a section farther upstream and of so much larger area that the speed at A is small, and the kinetic energy K therefore negligible. Let us also suppose that Q is negligible and the flow sensibly adiabatic. Then equation (1) takes the simpler form

$$K_1=e-e_1+pv-p_1v_1 \tag{2}$$

This is general and applicable to any fluid.

Now let the fluid be ideal gas—that is, one for which the equations

$$\left.\begin{array}{l} pv=RT \\ C_v=const \\ e=TC_v+const \\ C_p=C_v+R \end{array}\right\}$$

are satisfied. Within the limits of accuracy needed here, air is such a gas. By these equations we may easily reduce equation (2) to the form

$$K_1 = C_p(T - T_1), \tag{4}$$

which says that the kinetic energy acquired, per gram, is equal to the product of the fall in temperature by the specific heat at constant pressure.

Let S_1 denote the arithmetical mean speed over the section A_1. If the speed were uniform all over A_1 and there were no cross currents or eddies, we should evidently have

$$K_1 = \tfrac{1}{2} S_1^2 \tag{5}$$

and by (4)

$$S_1 = \sqrt{2C_p(T - T_1)}. \tag{6}$$

The agreement of experiment with deductions from equation (6) is close enough to show that equation (5) is nearly fulfilled under ordinary conditions, and equation (5) is therefore adopted as an assumption, though often not mentioned as such.

Up to this point it has not been assumed that the flow was frictionless. If there is resistance due to viscosity or turbulence, the fluid will be heated by the dissipation; T_1 will be raised and S_1 diminished; but equation (6) will remain satisfied.

In short, well-formed nozzles or orifices the resistance is small, and if we neglect it altogether we may treat the expansion of the gas as not only adiabatic, but isentropic, and may then apply the familiar equations for isentropic expansion of an ideal gas, viz:

$$pv^k = \text{const}; \quad T = p^{\frac{k-1}{k}} \times \text{const}$$

where $k = C_p/C_v$. By the second of these we have (7)

$$\frac{T_1}{T} = \left(\frac{p_1}{p}\right)^{\frac{k-1}{k}} = r^{\frac{k-1}{k}} \tag{8}$$

So that (6) may be written in the form

$$S_1 = \sqrt{2TC_p\left(1 - r^{\frac{k-1}{k}}\right)} \tag{9}$$

an equation which gives the speed S_1 of the frictionless jet in terms of the initial temperature T, the specific heat at constant pressure C_p, the pressure ratio $p_1/p = r$, and the specific heat ratio k.

The volume of gas passing A_1 in unit time is $V_1 = A_1 S_1$; and the volume of this same mass measured under the initial conditions is

$$V = V_1 \frac{v}{v_1} = A_1 S_1 \frac{v}{v_1}$$

or by (9)

$$V = A_1 \frac{v}{v_1} \sqrt{2TC_p\left(1 - r^{\frac{k-1}{k}}\right)} \tag{10}$$

Let A_1 be a circle of diameter D, and by (7) set $v/v_1 = r^{\frac{1}{k}}$. Then substituting in (10) we have

$$V = \frac{\pi}{2\sqrt{2}} D^2 \sqrt{TC_p\left(r^{\frac{2}{k}} - r^{\frac{k+1}{k}}\right)} \tag{11}$$

This holds for any normal units. For example: If D is in cm, T in °C, and C_p in ergs per g. per °C, V will be in cm³ per second.

For dry air at ordinary pressures and temperatures we have $k=1.40$, $C_p=1.01\times 10^7$ erg/g/°C, so that (11) takes the form

$$V\,[\text{cm}^3/\text{sec.}]=3530\,BD^2[\text{cm}^2]\,\sqrt{T°C} \tag{12}$$

where

$$B=\sqrt{r^{\frac{10}{7}}-r^{\frac{12}{7}}} \tag{13}$$

For V in liters per minute and D in millimeters, this reduces to

$$V[\text{liters/min.}]=2.12\,BD^2\,[\text{mm}^2]\sqrt{T°C}, \tag{14}$$

which is given as equation (1) in the body of the paper.

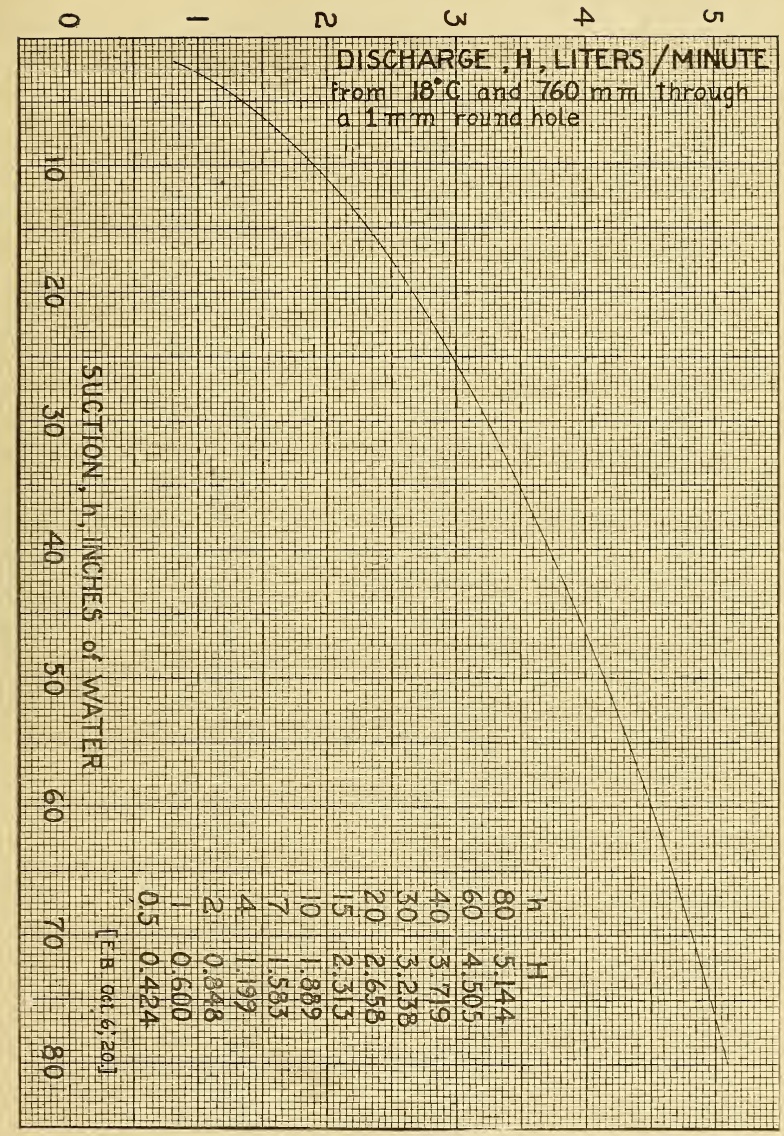

FIG. 1.—*Discharge curve*