NOTES ON THE ORIFICE METER; THE EXPANSION FACTOR FOR GASES

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ABSTRACT

The discharge coefficient of an orifice meter, determined with water, is applicable when the meter is used for measuring the flow of a gas, provided that the differential pressure is so small that the accompanying change of density is insignificant. But if the differential is a considerable fraction of the absolute static pressure, the water coefficient must be multiplied by an "expansion factor" which allows for the effects of change of density.

The paper contains a discussion of recent experimental data which show how the expansion factor depends on the form of the meter, the ratio of downstream to upstream pressure, and the specific heat ratio of the gas. The conclusions are summarized in an empirical equation which may be used for computing the value of the expansion factor in certain practically important cases.

A theoretical method of computing the expansion factor is developed and is shown to agree reasonably well with the facts observed under conditions that are approximately in accordance with those postulated by the theory.

CONTENTS

I. INTRODUCTION

The type of meter to which the following notes refer is illustrated diagrammatically by Figure 1, which shows some of the notation to be used, as well as certain limitations on the relative dimensions of the parts. It may be assumed that the readers to whom these notes are addressed would find a detailed description of the orifice meter superfluous.

Since the pressures p_1 and p_2, observed at the upstream and downstream side holes or pressure taps, depend on the locations of the holes, it is necessary to specify the distances, l_1 and l_2, from the center of each hole to the nearer face of the orifice plate, and some one of the following four schemes is usually adopted:

(*a*) Pipe taps, $l_1 = 2.5D$, $l_2 = 8D$.

(*b*) Throat taps, $l_1 = D$, $l_2 = 0.5D$.

(*c*) Flange taps, $l_1 = l_2 = 1$ inch for all sizes of pipe.

(*d*) Corner taps, the side holes are at the faces of the plate, or the pressures are taken off through narrow circumferential slits between the plate and the flanges, as illustrated in the lower half of Figure 1.

Combinations (*a*), (*b*), and (*c*) are in common use in the United States, and (*d*) has been adopted as standard by the Society of German Engineers (1).[1]

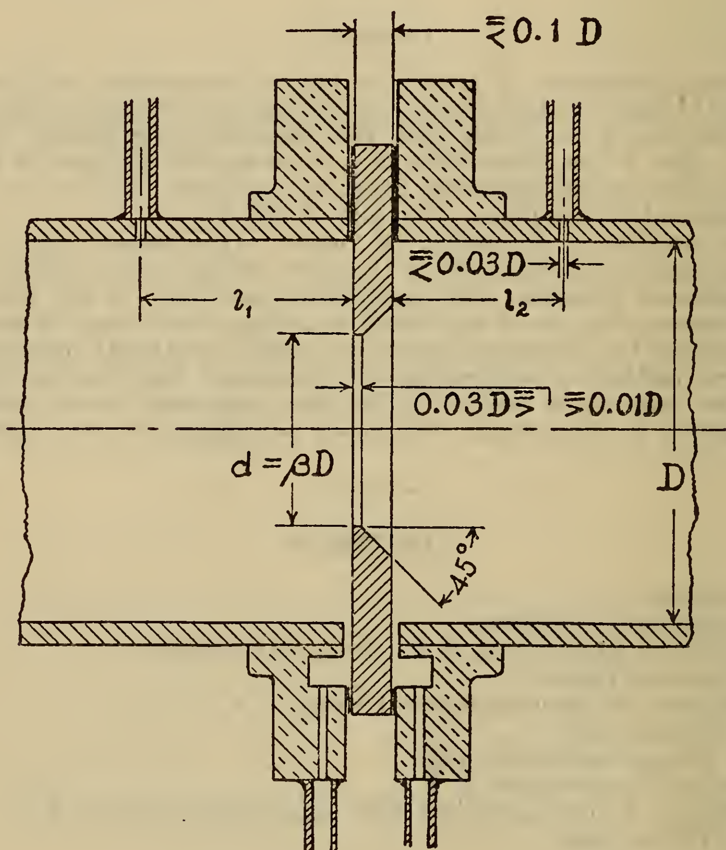

Figure 1.—*The orifice meter*

Side holes are shown above and ring slits below

There are advantages in adopting an arrangment such that meters of different sizes shall be geometrically similar as regards location of the side holes, a condition satisfied by (*a*), (*b*), and (*d*), but not by (*c*).

When the pipe diameter, D, is 8 inches or more and the orifice diameter, d, is not more than about $0.75D$, arrangements (*c*) and (*d*) give nearly the same pressure readings and may be regarded as equivalent, except in work of high precision. But with pipes as small as 4 inches in diameter, this assumption is no longer safe; and even with larger pipes, the two arrangements may give appreciably different results if the diameter ratio is as large as 0.8.

[1] Figures in parenthesis here and throughout the text indicate references given at the end of this paper.

II. THE ORIFICE METER EQUATION

The indications of an orifice meter are usually, and most conveniently, interpreted by means of some equation which is substantially equivalent to

$$M = N\, C\, A\sqrt{2\rho(p_1 - p_2)} \tag{1}$$

in which

M = the required rate of mass flow, or the mass discharged per unit time;
A = the area of the orifice;
ρ = the density of the fluid being metered;
p_1, p_2 = the pressures at the upstream and downstream taps;
N = a numerical constant dependent on the units; and
C = the discharge coefficient of the orifice, a number which does not depend on the units.

If the fluid is a gas, its density must be referred to some specified pressure and temperature, and these are most commonly taken to be the upstream pressure p_1 and the temperature t_1 of the gas approaching the orifice. We shall adopt this convention and denote the density under these conditions by ρ_1. For the sake of simplicity, it will also be supposed that all quantities are measured by a system of normal units, such as "British absolute" or cgs, because we then have $N = 1$; and with these two conventions, equation (1) takes the form

$$M = CA\sqrt{2\rho_1(p_1 - p_2)} \tag{2}$$

Before an orifice can be used as a flow meter, the value of C must be known, and this value depends on the rate of flow, the properties of the gas, the dimensions of the apparatus, the location of the pressure taps, etc. In the experimental investigations needed for the elucidation of this subject, observations of pressure and temperature at the orifice are combined with measurements of the rate of discharge by some independent method, and the equation is used in the form

$$C = \frac{M}{A\sqrt{2\rho_1(p_1 - p_2)}} \tag{3}$$

which may be regarded as a definition of C.

It may be remarked that ρ_1 denotes the true density at p_1, t_1; and if the gas in question is one that shows large departures from Boyle's law under the anticipated working conditions, the use of the familiar equation $pv = RT$, in computing the value of ρ_1 from the results of a density determination under laboratory conditions that are very different from the working conditions, may lead to large errors (2).

III. RESTRICTION TO HIGH VALUES OF THE REYNOLDS NUMBER

Let R_d be the Reynolds number defined by the equation

$$R_d = \frac{4M}{\pi d \eta} \tag{4}$$

in which η is the viscosity of the fluid.

If R_d is large, say $R_d > 200{,}000$, the value of C found by testing an orifice with water, or other liquid, is sensibly independent of the rate

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of flow (3), and this shows that the effects of viscosity have become negligible. But if the same orifice is tested with a gas, such as air, the value obtained for C varies with the rate of flow, even though R_d be high enough to make the effects of viscous forces insignificant; for the decrease of density as the pressure falls from p_1 to p_2, in contradistinction to the constancy of density of a liquid, introduces a new element into the phenomena of flow (5, 6, 7).

The condition that R_d shall have the required high value is nearly always satisfied in the commercial metering of gases, and since the object of this paper is to discuss the changes of C which are due to compressibility alone, it will be assumed, from this point onward, that the requirement is fulfilled.

IV. THE EXPANSION FACTOR

In discussing the effects of compressibility, it will be convenient to employ the following notation:

$\beta = d/D =$ the diameter ratio of the orifice, or
$m = \beta^2$ = the area ratio;
$K =$ the value found for C when the orifice is tested with a liquid at high values of R_d: so long as the installation remains unchanged, K is a constant of the orifice;
$y = p_2/p_1 =$ the pressure ratio at which the discharge coefficient determined by experiments on the gas has the value C;
$\gamma = C_p/C_v =$ the specific heat ratio of the gas; and
$Y = C/K =$ the expansion factor.

If the fall of pressure at the orifice is made so small that the accompanying decrease of density is insignificant, the gas must behave very nearly like a liquid; and experiment confirms the conclusion that $C \doteq K$ when $y \doteq 1$.
We therefore write

$$C = KY \tag{5}$$

in which the expansion factor, Y, describes, or allows for, the varying effect of compressibility on the discharge coefficient; and $Y \doteq 1$ when $y \doteq 1$.

In many important cases, the relation $Y = f\,(y)$ is very nearly linear, as is illustrated by the simultaneous values of y and C given in Table 1 and plotted in Figure 2.

Table 1.—*Relation of C to y*

Diameter ratio 0.6209. Specific heat ratio 1,283. Throat taps

$\frac{p_2}{p_1} = y$	C observed	$0.6669-0.23(1-y)$	Difference	$\frac{p_2}{p_1} = y$	C observed	$0.6669-0.23(1-y)$	Difference
0.549	0.5618	0.5632	−0.0014	0.797	0.6211	0.6202	+0.0009
.558	.5660	.5652	+.0008	.904	.6444	.6448	−.0004
.560	.5654	.5657	−.0003	.911	.6469	.6464	+.0005
.647	.5866	.5857	+.0009	.913	.6486	.6469	+.0017
.651	.5872	.5866	+.0006	.947	.6543	.6547	−.0004
.650	.5879	.5864	+.0015	.948	.6527	.6549	−.0022
.700	.5973	.5979	−.0006	.953	.6557	.6561	−.0004
.703	.5979	.5986	−.0007	.975	.6589	.6612	−.0023
.711	.5999	.6004	−.0005	.977	.6620	.6616	+.0004
.808	.6250	.6227	+.0023	.977	.6605	.6616	−.0011
.799	.6213	.6207	+.0006				

This is one of a large number of results of experiments carried out under the direction of H. S. Bean, of the Bureau of Standards, for the Committee on Gas Measurement of the Natural Gas Department of the American Gas Association. In this instance, the experimental work was done at Los Angeles in 1929, with a natural gas of specific heat ratio $\gamma = 1.283$, and the pressures were taken at throat taps.

The Los Angeles experiments included tests of 23 orifices in pipes of 4, 8, and 16 inches nominal diameter, and Table 2 gives a list of the orifices, together with the number of tests on each and the lowest value of y, for throat taps, to which the tests extended.

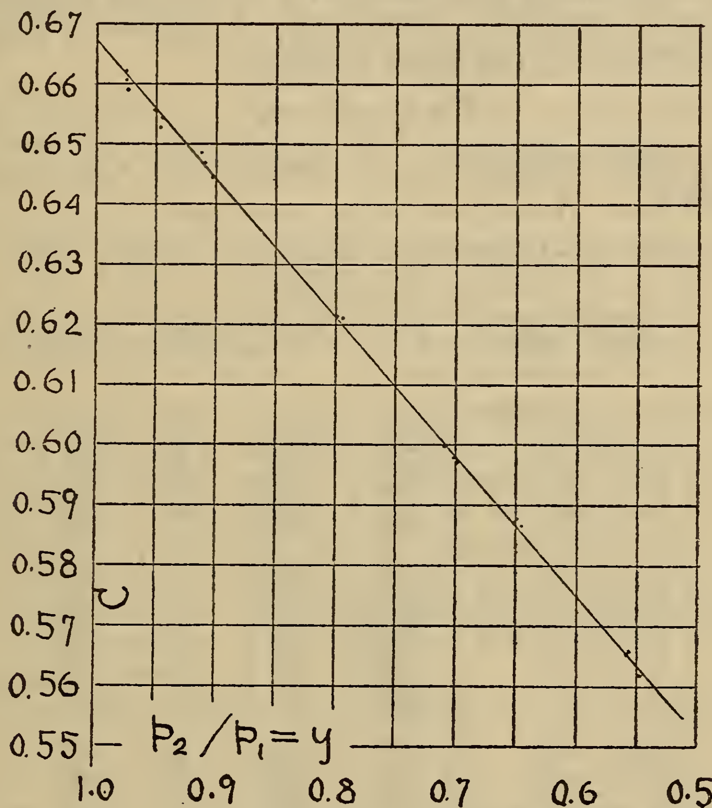

FIGURE 2.—*Relation of discharge coefficient to pressure ratio*

From Table 1

The orifice plates were one-eighth inch thick, with the edges of the orifice square and sharp at both faces. They were installed in commercial steel pipes which had been selected for smoothness, and a short nest of smaller pipes was placed in each of the three pipes at a distance of $10D$ to $15D$ ahead of the orifice, to insure straightness of flow. Pipe, throat, and flange taps were provided, and pressures were read at all three pairs. The absolute pressure was never more than 2.6 atmospheres so that departures from Boyle's law could be ignored. Since it was not practicable to measure the rates of flow by means of a gasometer, they were determined by passing the discharge from the orifice under test through standard reference orifices, of which any number up to 6 could be used in parallel. Details of the experiments and their results will be described in a later publication.

The linear relationship shown by Table 1 and Figure 2 is characteristic of the orifices for which $0.2 \lesseqgtr \beta \lesseqgtr 0.75$. In general, the departures of the plotted points from the best straight line that could be drawn among them by simple inspection were somewhat greater than in this series, but they were not systematic.

For diameter ratios of 0.8 or more, when the range of y was long enough to give a well defined band of points, the axis of the band was concave upward, the curvature increasing with β; and the tests of the orifice for which $\beta=0.1241$ indicated that a curve, slightly convex upward, would be a little better than a straight line, a result in accordance with other observations on orifices of diameter ratios below about 0.2. But the general conclusion may be drawn from the Los Angeles observations with throat taps, that, over the ranges $0.2 \lesseqgtr \beta \lesseqgtr 0.75$ and $1.0>y>0.5$, the expansion factor can be represented, within the experimental errors, by the linear equation

$$Y=1-\epsilon(1-y) \quad (6)$$

in which the slope coefficient, ϵ, is constant for any one orifice but increases with β.

TABLE 2.—*List of orifices tested at Los Angeles, 1929*

Number of orifice	Pipe diameter D	$\frac{d}{D}=\beta$	$\beta^4=m^2$	Number of tests	Lowest value of y
	Inches				
1	4.03	0.3724	0.0192	24	0.533
2		.4967	.0609	21	.541
3		.6207	.1484	20	.516
4		.7449	.3079	31	.544
5		.8069	.4239	25	.503
6		.8691	.5705	24	.607
7	8.05	.1241	.0002	21	.543
8		.3105	.0093	21	.529
9		.4967	.0609	21	.515
10		.6209	.1486	21	.549
11		.6829	.2175	28	.495
12		.7450	.3080	31	.479
13		.8070	.4241	31	.486
14		.8693	.5711	29	.550
15	15.38	.1951	.0014	26	.472
16		.3901	.0231	30	.523
17		.5527	.0933	25	.597
18		.6501	.1786	22	.741
19		.6989	.2386	21	.798
20		.7477	.3125	14	.875
21		.7963	.4021	12	.887
22		.8289	.4721	18	.933
23		.8615	.5508	12	.952

When the measurements of pressure were made with flange taps ($l_1=l_2=1$ inch instead of $l_1=D$, $l_2=0.5D$), the results were similar to those for throat taps, the only difference being that the values of ϵ were a trifle larger and that the linear relationship persisted up to higher values of β.

Experiments by R. Witte (3, 4), with corner taps, have also given a linear relation of Y to y for air and nitrogen; and the same was true of his more extensive experiments on superheated steam, except for an as yet unexplained anomaly at the highest values of y, which may be connected with the phenomena of delayed condensation. The remaining points lay along straight lines. (See reference 4, figs. 14 and 15, p. 295.)

The experimental data cited above seem to be the most extensive and trustworthy available, though not the only ones. For example,

J. L. Hodgson (5) has published curves representing the results of experiments with corner taps on air for diameter ratios $\beta = 0.421$, 0.632, and 0.843, and they are all convex upward; but for the two lower diameter ratios one can not be certain, from the small figure published, that straight lines would not do as well as the curves. For $\beta = 0.843$ there is no doubt about the curvature, but, unfortunately, there are no other published data with which this curve can be compared, so that it stands alone.

In the Los Angeles experiments with flange taps, the orifices numbered 5, 13, and 14, in Table 2 ($\beta = 0.8069$, 0.8070, and 0.8693) gave well-defined straight lines, while the observed points for number 6 ($\beta = 0.8691$) lay along a curve that was strongly concave upward. There is, however, no necessary inconsistency between these observations and Hodgson's. For when the edge of the orifice is as near the wall of the pipe as it is with these large values of β, the pressure at the wall in the vicinity of the plate varies rapidly with distance from the plate; and flange taps, which are merely near the plate, may well give quite different results from taps right in the corners. The fact that plate 14 gave a straight line, while plate 6, with the same value of β, gave a curve, was doubtless due partly, if not wholly, to the fact that the tap distances were $D/8$ in the one case and $D/4$ in the other.

The results of the early work of the Bureau of Standards on compressed air (6) were also presented as curves that were convex upward; but the observed points were so much scattered that straight lines might equally well have been used, for all but the lowest values of β. The experimental conditions at Los Angeles were more favorable, both in the steadiness of the gas supply and in the longer range of values of y that could be covered, and they permitted of so much higher precision that the Los Angeles results may be regarded as superseding the earlier ones.

Many experiments with natural gas from other fields have been carried out under the direction of H. S. Bean for the committee named above, and have given results like those already described; but the experimental conditions were generally less satisfactory and a detailed discussion of the results would not change or invalidate the conclusions drawn from the more precise data gathered at Los Angeles. These additional experiments will not be further considered here, nor will an exhaustive review of the literature be attempted; but one excellent set of observations remains to be mentioned, although the arrangement of the apparatus did not correspond exactly to any of the four usual pressure tap combinations.

H. Bachmann (7), working with air, determined the discharge coefficient of an orifice on the end of a pipe, with the jet issuing into the atmosphere. The dimensions were $D = 82.5$ mm (3.25 inches), $d = 20.032$ mm, $\beta = 0.2428$, and $l_1 = 1.82D$; and p_2 was taken to be the barometric pressure.

In view of the low value of β, this arrangement must have given very nearly the same results as if the pipe had been continued downstream and the pressures had been measured at throat taps.

The values of C from 17 experiments covering the range $0.996 \gtreqless y \gtreqless 0.535$ are reproduced without systematic error by the equation

$$C = 0.600\,[1 - 0.302\,(1 - y)] \tag{7}$$

the greatest departure being −0.6 per cent, and the mean ±0.2 per cent.

One experiment, at $y=0.997$, gave $C=0.5907$ as compared with 0.5995 from equation (7); but having regard to the admirable consistency of the other values, it seems fair to assume that this one experiment was affected by an unusually large error of some sort.

From the foregoing brief review of experimental data, it appears that, over the range $1.0 > y > 0.5$, the expansion factor may be represented, within the present accuracy of orifice meter testing, by linear equations of the form of equation (6), provided that: (*a*) The tap distances are not greater than $l_1=D$ and $l_2=0.5D$; and (*b*) the diameter ratio is within the limits $0.2 \leqq \beta \leqq 0.75$.

V. COORDINATION OF THE EXPERIMENTAL VALUES OF ϵ

Admitting the substantial correctness of the general form of equation (6), we have next to intercompare the values of the slope coefficient ϵ that fit the various sets of observations.

1. THROAT TAPS

For each of the orifices tested at Los Angeles the values of $C\sqrt{1-m^2}$ were plotted as ordinates[2] against y as abscissa; and for each orifice for which $\beta < 0.75$, the result was a more or less definite and straight band of points. By stretching a fine thread along the band and making readings at $y=0.5$ and $y=1.0$, two straight lines, of greatest and least slope, were determined, between which any line that could reasonably be drawn to represent the band must lie. The values of ϵ for these lines were computed, and their mean is recorded as ϵ_{obs} in column 5 of Table 3.

TABLE 3.—*Slope coefficient of Y for throat taps or free discharge*

Observer	D	$\frac{d}{D}=\beta$	$\beta^4=m^2$	$\epsilon_{obs.}$	$\delta\epsilon_{obs.}$	$\frac{0.41+0.33m^2}{\gamma}=\epsilon_{calc.}$	$\epsilon_{obs.}-\epsilon_{calc.}$	Column 8 −column 6	K_{obs}
1	2	3	4	5	6	7	8	9	10
H. S. Bean, throat taps, $\gamma=1.283$	*Inches* 4.03	0.3724	0.0192	0.336	±0.006	0.324	+0.012	+0.006	0.614
		.4967	.0609	*.342	.002	.335	+.007	+.005	.630
		.6207	.1484	*.363	.003	.358	+.005	+.002	.668
		.7449	.3079	.386	.006	.399	−.013	−.007	.748
	8.05	.1241	.0002	*.313	.003	.320	−.007	−.004	.595
		.3105	.0093	*.329	.003	.322	+.007	+.004	.606
		.4967	.0609	.331	.004	.335	−.004	--------	.634
		.6209	.1486	.344	.003	.358	−.014	−.011	.667
		.6829	.2175	*.383	.006	.376	+.007	+.001	.696
		.7450	.3080	*.403	.005	.399	+.004	--------	.737
	15.38	.1951	.0014	.304	.007	.320	−.016	−.009	.599
		.3901	.0231	.300	.022	.326	−.026	−.004	.613
		.5527	.0933	.307	.032	.344	−.037	−.005	.643
		.6501	.1786	.337	.017	.365	−.028	−.011	.679
		.6989	.2386	.379	.023	.381	−.002	--------	.705
		.7477	.3125	.386	.022	.400	−.014	--------	.733
H. Bachmann, free discharge, $\gamma=1.40$	3.25	.2428	.0035	.302	.010	.294	+.008	--------	.600

[2] Values of C might equally well have been used.

The number in column 6 is, in each case, one-half the difference between the two extreme values and gives a rough estimate of the uncertainty of the value of ϵ_{obs}.

The foregoing procedure evidently involves a considerable exercise of personal judgment which might have been avoided by utilizing the method of least squares. But there was no satisfactory method for weighting the separate points—which were certainly not all of equal weight—and the result of any arbitrary assignment of weights would have been no more authoritative or probable than that obtained, as described, by simple inspection.

If tests were carried out on a series of orifices which differed only in diameter ratio, the values found for ϵ should evidently lie along some smooth curve, $\epsilon=f(\beta)$, within the errors of experiment; and it appears that the relation would be approximately linear in β^4 or m^2. Column 7 of Table 3 contains values computed from the empirical equation

$$\epsilon_{calc.}=\frac{0.41+0.33m^2}{\gamma} \tag{8}$$

and column 8 contains the values of $(\epsilon_{obs.}-\epsilon_{calc.})$. Column 9 shows the positive or negative excess of $(\epsilon_{obs.}-\epsilon_{calc.})$ over the estimated uncertainty of $\epsilon_{obs.}$, given in column 6. For 5 of the 17 orifices, including Bachmann's, $(\epsilon_{obs.}-\epsilon_{calc.})$ is within the estimated uncertainty, while for the other 12 it is outside, by amounts up to 0.011.

It is quite possible that the errors in determining ϵ were larger than the admittedly rough estimates shown in column 6; and there may also have been differences of finish between the different plates, so that even if there had been no experimental errors, the points would not have lain on a smooth curve or a straight line. But in any event, the departures are not so important as might appear at first sight. An error of 0.010 in ϵ changes C by 0.4 per cent at $y=0.6$, or by 0.2 per cent at $y=0.8$, which is a lower value of y than is often encountered in practice; and it seems probable that when an orifice meter for gas is used with throat taps, the equation

$$Y_{calc.}=1-\frac{0.41+0.33m^2}{\gamma}(1-y) \tag{9}$$

will always give values of Y that are accurate enough for ordinary commercial purposes.

2. FLANGE AND CORNER TAPS

In Table 4, with the same notation as Table 3, the data in the upper part refer to Bean's observations with flange taps on the Los Angeles natural gas, the values of $\epsilon_{obs.}$ and $\delta\epsilon_{obs.}$ having been found from the observations in the manner described above for throat taps. The lower part of the table refers to Witte's (4) observations with corner taps on superheated steam, air, and nitrogen: and the values of $\epsilon_{obs.}$ were obtained by readings from the published plots of Y against y. (See reference 4, figs. 14 and 15, p. 295.)

TABLE 4.—*Slope coefficient of Y for flange and corner taps*

Observer	D	$\frac{d}{D}=\beta$	$\beta^4=m^2$	$\epsilon_{obs.}$	$\delta\epsilon_{obs.}$	$\frac{0.41+0.37m^2}{\gamma}=\epsilon_{calc.}$	$\epsilon_{obs.}-\epsilon_{calc.}$	Column 8—column 6
1	2	3	4	5	6	7	8	9
	Inches							
		0.3724	0.0192	0.333	±0.009	0.325	+0.008	--------
		.4967	.0609	.341	.003	.337	+.004	+0.001
	4.03	.6207	.1484	.367	.003	.362	+.005	+.002
		.7449	.3079	.387	.015	.408	−.021	−.006
		.8069	.4239	.441	.005	.442	−.001	--------
		.1241	.0002	.310	.006	.320	−.010	−.004
		.3105	.0093	.335	.004	.322	+.013	+.009
		.4967	.0609	.329	.005	.337	−.008	−.003
Bean, flange taps, $\gamma=1.283$	8.05	.6209	.1486	.350	.004	.362	−.012	−.008
		.6829	.2175	.393	.003	.382	+.011	+.008
		.7450	.3080	.412	.005	.408	+.004	--------
		.8070	.4241	.448	.002	.442	+.006	+.004
		.8693	.5711	.491	.003	.483	+.008	+.005
		.1951	.0014	.309	.006	.320	−.011	−.005
		.3901	.0231	.304	.012	.326	−.022	−.010
	15.38	.6501	.1786	.349	.015	.371	−.022	−.007
		.6989	.2386	.409	.018	.388	+.021	+.003
		.7477	.3125	.433	.030	.410	+.023	--------
		.20	.0016	.328	--------	.313	+.015	--------
		.4935	.0595	.356	--------	.330	+.026	--------
Witte, corner taps, $\gamma=1.31$	3.94	.58	.1136	.352	--------	.345	+.007	--------
		.70	.2401	.382	--------	.381	+.001	--------
		.76	.3329	.414	--------	.407	+.007	--------
		.152	.0005	.275	--------	.293	−.018	--------
Witte, corner taps, $\gamma=1.40$	.79	.326	.0112	.332	--------	.296	+.036	--------
	.197	.606	.1347	.312	--------	.328	−.016	--------

In Witte's experiments on steam, the rate of flow was determined by condensation and weighing, and the experimental accuracy was probably higher than could be attained with natural gas. On the other hand, in his experiments on air and nitrogen, the flow was measured by a small wet-drum meter; and while these measurements may have been accurate, the orifices were too small for exact reproduction, and comparison with larger orifices of ostensibly the same geometrical shape is of little significance. The most important result of these small-scale experiments is their satisfactory confirmation of the linear relationship between Y and y.

Since Witte's values of Y are published in the form of small plots, from which it is difficult to make accurate readings, the values of $\epsilon_{obs.}$ given in Table 4 may not do justice to the accuracy of the original data. No attempt has been made to estimate the uncertainty denoted by $\delta\epsilon_{obs}$. It is impossible to assign definite weights to the 26 values of $\epsilon_{obs.}$; but the 5 values for the largest pipe at Los Angeles and Witte's 3 values for air and nitrogen seem to be considerably less trustworthy than the others.

The values in column 7 of Table 4 were computed from the equation

$$\epsilon_{calc.}=\frac{0.41+0.37m^2}{\gamma} \tag{10}$$

and columns 8 and 9 have the same meanings as in Table 3.

As with equation (9) for throat taps, so here it appears that the slightly modified equation

$$Y_{calc.}=1-\frac{0.41+0.37m^2}{\gamma}(1-y) \tag{11}$$

represents the facts to an approximation sufficient for ordinary commercial metering, under the following conditions: (*a*) for flange taps, up to $\beta=0.8$ when $l_1=l_2 \lesseqgtr D/4$, or up to $\beta=0.87$ when $l_1=l_2 \lesseqgtr D/8$; and (*b*) for corner taps up to $\beta=0.76$, the highest value for which Witte gives data.

When an orifice for which $\beta=0.869$ was tested at Los Angeles in the 4-inch pipe, with flange taps, the resulting band of points was strongly concave upward, whereas an orifice of the same diameter ratio in the 8-inch pipe gave a well-defined straight line. The simple linear relation persisted to a higher value of β when the pressure taps were relatively closer to the orifice plate; and in Witte's measurements with corner taps, values of β above 0.76 would probably still have given the linear relation described by the general equation (6) or, in particular cases, by equations (7), (9), and (11).

VI. THEORETICAL COMPUTATION OF *Y*

Some orifice-metering devices work with more than the critical pressure drop, but in the normal meter the range of pressure is less than 2 to 1, and usually very much less. Even with gases that show considerable departures from Boyle's law over the range from 1 atmosphere up to the high pressures at which they may be metered, the departures are nearly always neglible over the range of pressure in an orifice meter; and although the use of the ideal gas equation, $pv=RT$, for computing the density at p_1 from the density at atmospheric pressure, might lead to serious errors, it is permissible to treat the expansion through the orifice as subject to this equation. It may also be stated, without discussing the details of the experimental evidence, that when R_d is large, the flow is very nearly isentropic, at least as far as the vena contracta. The changes of density in the jet may therefore, without appreciable error, be treated as conforming to the thermodynamic equations for isentropic expansion of an ideal gas.

Let us now suppose that the pressure taps are so situated that p_1 is the static pressure in the approaching stream of gas just before it has begun to converge toward the orifice, and p_2 is the static pressure in the jet at the vena contracta, where the flow has become straight and the pressure in the jet sensibly uniform and equal to the static pressure of the gas in the surrounding space.

The area of the orifice being A, let μ_a be the contraction coefficient, so that the cross section of the jet at the vena contracta is $A\mu_a$. Then by the usual, familiar train of reasoning we arrive at the equation

$$M=A\mu_a\sqrt{\frac{2\gamma}{\gamma-1}p_1\rho_1\frac{y^{\frac{2}{\gamma}}-y^{\frac{\gamma+1}{\gamma}}}{1-\mu_a{}^2m^2y^{\frac{2}{\gamma}}}} \tag{12}$$

And upon comparing this with equation (2) in the form

$$M = KYA\sqrt{2\rho_1(p_1 - p_2)} \tag{13}$$

and introducing the abbreviation

$$\frac{\gamma}{\gamma-1}\,\frac{y^{\frac{2}{\gamma}} - y^{\frac{\gamma+1}{\gamma}}}{1-y} \equiv Z \tag{14}$$

we get the equation

$$Y = \frac{1}{K}\sqrt{\frac{Z}{\frac{1}{\mu^2_a} - m^2 y^{\frac{2}{\gamma}}}} \tag{15}$$

from which Y may be computed, for any given values of m, K, γ, and y, if the value of μ_a can be determined.

In default of a solution of the equations of motion, μ_a can be found only by recourse to some plausible, simplifying assumption. In an earlier paper (8) it was assumed that, at any given mass flow, the force exerted by the gas on the upstream face of the orifice plate was the same, whether the subsequent flow through the orifice was isentropic or went on without change of density, as for liquids. If the jet issues into a space in which the static pressure is uniform, and is therefore the same all over the boundary of the jet and the downstream face of the plate as in the vena contracta, the assumption makes it possible to apply the momentum principle and obtain a relation between the contraction coefficient μ_a, and the contraction coefficient, μ, for a jet of liquid from the same orifice. The latter may readily be shown to satisfy the equation

$$\mu = \frac{K}{\sqrt{1 + m^2K^2}} \tag{16}$$

so that μ may be computed from the diameter ratio of the orifice and the value of K, which is accessible to measurement, either by experiments with a liquid or as the limiting value of C in experiments with a gas.

The relation in question (equation (20) of reference 8) may be put into the form

$$\mu_a = \frac{Z}{y^{\frac{1}{\gamma}}B}\left(1 - \sqrt{1 - \frac{y^{\frac{2}{\gamma}}B}{Z^2}}\right) \tag{17}$$

where

$$B = \left(m^2 + \frac{2}{\mu} - \frac{1}{\mu^2}\right)Z - m^2 y^{\frac{2}{\gamma}} \tag{18}$$

and Z is defined by equation (14).

The value, or lack of value, of the assumption on which equation (17) is based is to be determined by comparing the resulting "theoretical" values of Y with values found by experiments with a gas on an orifice which is so installed that the conditions regarding p_1 and p_2 are satisfied.

The value of K, found either by testing with a liquid or by extrapolation to $y=1$ from the experiments with the gas, is substituted in equation (16), together with the measured value of $\beta^2=m$, to give the value of μ; a value of y is selected, and with the given value of γ the value of Z is computed from equation (14); and after these preliminaries, B, μ_a, and Y are computed, successively, from equations (18), (17), and (15). The value of Y may then be compared with the value found by experiment at the selected value of y.

VII. COMPARISON OF THEORETICAL AND EXPERIMENTAL VALUES OF Y

In the deduction of equation (12), p_1 and p_2 represent the static pressure in the stream just before it begins to converge toward the orifice, and the static pressure in the jet at the vena contracta; and if the theory is to be tested by comparing values of Y from equation (15) with values obtained by experiment, the pressure taps in the experimental apparatus must be so placed as to conform to the requirements of the theory. For diameter ratios up to 0.75, the former condition may be satisfied, within the precision of all but the most refined measurements, by placing the upstream side hole anywhere within the limits $0.5D \lesseqgtr l_1 \lesseqgtr 2D$; but the location of the downstream side hole requires more care.

Visual observations with orifices installed in glass pipes have shown that the vena contracta occurs at about the same cross section of the pipe as the minimum static pressure at the wall, and it is commonly assumed that this minimum pressure is identical with the pressure in the jet at the vena contracta. No direct experimental proof of this is known to the present writer, but there is no obvious reason for doubting that the assumption is substantially correct, and it will be accepted here.

For low values of β, the downstream minimum of pressure is about one pipe diameter from the orifice, but is too flat to be located accurately. As β is increased, the minimum becomes more pronounced and moves closer to the orifice, but its position also depends to some extent on the rate of flow, being blown farther downstream if the speed of the jet is raised (9). Nevertheless, the pressure in a fixed side hole at the distance $l_2=0.5D$ from the orifice plate is only very slightly higher than the minimum pressure, unless β is large; and up to $\beta=0.75$ the difference is not more than 0.005 (p_1-p_2), which corresponds to a change in C of only 0.25 per cent. It is therefore evident that measurements of p_1 and p_2 with throat taps will come very close to satisfying the conditions presupposed in the deduction of equation (15).

The deduction of equation (17) is subject to the further condition that the static pressure on the downstream face of the orifice plate, and over the bounding surface of the jet as far as the vena contracta, shall be uniform and equal to the pressure inside the jet at the vena contracta. This requirement is satisfied when the jet discharges into the open air, as in Bachmann's experiments (7), or in the ordinary installation when β is small. As β is increased, the static pressure in the region about the jet becomes less uniform, if we may judge by observations at a series of small side holes distributed along the wall of the pipe, and the conditions for the validity of equation (17) are less nearly satisfied.

1. THROAT TAPS

It is impossible to form a quantitative estimate of the effects of the departures from the theoretical conditions just considered, and the comparison of theory with experiment will therefore be carried up to $\beta = 0.75$, which is as far as the Los Angeles experiments with throat taps gave well-determined values of Y.

To cover this range of diameter ratios six of the Los Angeles orifices were selected as having particularly well determined values of ϵ_{obs}; they are marked with asterisks (*) in column 5 of Table 3. Values of Y were computed from equation (15) at $y = 0.5 - - - 0.9$, with $\gamma = 1.283$ and the values of K shown in column 10 of Table 3, which were found graphically at the same time as the values of ϵ_{obs}.

The first result to be noted is that the theoretical curves, $Y = f(y)$, are slightly convex upward, the curvature being greatest for low values of β. For $y \gtreqless 0.6$, the computed points are not far from the straight line drawn from the point ($y = 1$, $Y = 1$) through the point computed for $y = 0.7$. The slope of this line will be denoted by $\epsilon_{0.7}$, and the ordinates by

$$Y_{0.7} = 1 - \epsilon_{0.7}\ (1 - y) \tag{19}$$

The value of $(Y - Y_{0.7})$ at any value of y, is the amount by which the computed theoretical curve is above the straight line at that value of y, and these amounts are shown in Table 5.

TABLE 5.—*Curvature of the computed curve $Y = f(y)$*

γ	D	β	$K_{obs.}$	Values of $(Y - Y_{0.7})$				
				$y = 0.5$	0.6	0.7	0.8	0.9
	Inches							
1.283	8.05	0.1241	0.595	−0.0083	−0.0030	±0.0000	+0.0012	+0.0011
	8.05	.3105	.606	−.0080	−.0028	±.0000	+.0011	+.0011
	4.03	.4967	.630	−.0071	−.0026	±.0000	+.0010	+.0008
	4.03	.6207	.668	−.0059	−.0020	±.0000	+.0008	+.0006
	8.05	.6829	.696	−.0049	−.0016	±.0000	+.0006	+.0004
	8.05	.7450	.737	−.0036	−.0011	±.0000	+.0003	+.0001
1.40	3.25	.2428	.600	−.0076	−.0028	±.0000	+.0012	+.0011

The fourth figure in Y is not certain, but the table suffices to give an idea of the degree of curvature and its regular increase as β decreases. As already noted, experiments indicate that when β is small the true curve is slightly convex upward, although for larger values of β it is sensibly a straight line.

The computations were also carried out for Bachmann's (7) orifice with $\beta = 0.2428$, $K = 0.600$, and $\gamma = 1.40$; and Table 6 contains values of the following quantities for each of the seven orifices:

Y computed from equation (15);

$Y_{0.7}$ computed from equation (19); and

$Y_{obs.}$ computed from equation (6), with $\epsilon_{obs.}$ taken from column 5 of Table 3.

TABLE 6.—*Computed and observed values of Y for throat taps (Bean, $\gamma = 1.283$) and free discharge (Bachmann, $\gamma = 1.40$)*

$\epsilon_{0.7}$ ϵ_{obs} $\epsilon_{calc.}$	β	K		$y=0.5$	0.6	0.7	0.8	0.9
0.303			Y	0.841	0.876	0.9094	0.941	0.971
.313	0.1241	0.595	$Y_{0.7}$	.849	.879	.909	.940	.970
.320			Y_{obs}	.844	.875	.906	.937	.969
.313			Y	.836	.872	.9062	.939	.970
.329	.3105	.606	$Y_{0.7}$	.844	.875	.906	.937	.969
.322			Y_{obs}	.836	.868	.901	.934	.967
.333			Y	.826	.864	.9000	.934	.968
.342	.4967	.630	$Y_{0.7}$	.834	.867	.900	.933	.967
.335			Y_{obs}	.829	.863	.897	.932	.966
.365			Y	.812	.852	.8904	.928	.964
.363	.6207	.668	$Y_{0.7}$	.818	.854	.890	.927	.963
.358			Y_{obs}	.819	.855	.891	.927	.964
.387			Y	.801	.843	.8838	.923	.962
.383	.6829	.696	$Y_{0.7}$	.807	.845	.884	.923	.961
.376			Y_{obs}	.809	.847	.885	.923	.962
.419			Y	.787	.831	.8742	.916	.958
.403	.7450	.737	$Y_{0.7}$	.791	.832	.874	.916	.958
.399			Y_{obs}	.799	.839	.879	.919	.960
.283			Y	.851	.884	.9151	.945	.973
.302	.2428	.600	$Y_{0.7}$	.859	.887	.915	.943	.972
.294			Y_{obs}	.849	.879	.909	.940	.970

In the first column, values of $\epsilon_{0.7}$ are given for comparison with those of $\epsilon_{obs.}$ and $\epsilon_{calc.}$, repeated from columns 5 and 7 of Table 3.

Small discrepancies between Tables 6 and 5 are due to the dropping of subsequent figures.

2. CORNER TAPS

Observations of pressure at corner taps do not quite satisfy the conditions for which equation (15) was deduced, and if the results of such observations are to be used for testing the value of that equation, the experimental values of K must first be reduced to what they would have been if the pressures had been observed at throat taps, which conform more nearly to the requirements of the theory. This can not, at present, be done with any great accuracy, but in order not to neglect the opportunity offered by the publication of Witte's (4) observations on steam, the reduction will be attempted. It might be effected by means of Witte's observations on the longtitudinal distribution of pressure at the wall of the pipe near the orifice plate; but the uncertainty of readings from the rather small-scale curves by which the results are represented (see reference 4, Pt. II) has led me to prefer using the somewhat similar data obtained at Chicago in 1924 (9).

Letting K_c denote the value of K for corner taps and writing

$$K = b\,K_c \tag{20}$$

values of the reduction factor b were found, by interpolation in Table 19 of reference 9, from the equation

$$b = \frac{C(24,12)}{C(1,1)} \tag{21}$$

in which $C(24,12)$ and $C(1,1)$ represent the values of C for an orifice installed in a smooth pipe of 23.3 inches inside diameter, when the distances from the orifice plate to the side holes were, respectively, 24 and 12 inches, and 1 and 1 inch, the ratio being deduced from the observed longitudinal distribution of pressure.

The identification of $C(1,1)$ with the value $C(0,0)$ that would be obtained with the side holes right at the faces of the plate instead of $D/23.3$ away, is of course not exact; the difference is small but not yet accurately known, and it varies with the diameter of the side holes (4). Furthermore, the pressure distribution is not entirely independent of the pressure ratio. The values of b found as described above are therefore slightly uncertain, but in default of a detailed tabulation of Witte's measurements it appears that we can do no better at present.

Table 7 refers to the five orifices for which Witte gives values of $Y=f(y)$ determined with superheated steam (see reference 4, fig. 15): the notation and arrangement are the same as in Table 6, with the addition of two columns containing K_c, as given by Witte, and b, obtained as already described. The values of Y were computed from equation (15) with $\gamma=1.31$ and $K=bK_c$; and those of $\epsilon_{obs.}$ and $\epsilon_{calc.}$ in the first column are repeated from Table 4.

Table 7.—*Computed and observed values of Y for corner taps (Witte, $\gamma=1.31$)*

$\epsilon_{0.7}$ ϵ_{obs} $\epsilon_{calc.}$	β	K_c	b	$bK_c=K$		$y=0.5$	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9	10	11
0.304					Y	0.840	0.875	0.9087	0.940	0.971
.328	0.20	0.604	0.9984	0.6030	$Y_{0.7}$	.848	.878	.909	.939	.970
.313					Y_{obs}	.836	.869	.902	.934	.967
.318					Y	.833	.870	.9046	.937	.969
.356	.4935	.622	.9991	.6215	$Y_{0.7}$	.841	.873	.905	.936	.968
.330					Y_{obs}	.822	.858	.893	.929	.964
.336					Y	.825	.863	.8992	.934	.967
.352	.58	.644	.9998	.6439	$Y_{0.7}$	.832	.866	.899	.933	.966
.345					Y_{obs}	.824	.859	.894	.929	.965
.378					Y	.805	.847	.8866	.925	.963
.382	.70	.692	1.0081	.6976	$Y_{0.7}$	.811	.849	.887	.924	.962
.381					Y_{obs}	.809	.847	.885	.924	.962
.416					Y	.788	.832	.8753	.917	.959
.414	.76	.730	1.0202	.7447	$Y_{0.7}$	.792	.834	.875	.917	.958
.407					Y_{obs}	.793	.834	.876	.917	.959

A question might arise here concerning the values of ϵ_{obs}. Let p_1' and p_2' be the pressures measured at corner taps, while p_1 and p_2 are the upstream and downstream minimum pressures dealt with by the theory. To be comparable with values of Y computed from the reduced values of K, the observed values of Y should be plotted against the simultaneous values of p_2/p_1; and if, by an oversight, the abscissa in the plot were p_2'/p_1', the values of ϵ_{obs}. read from the plot would need slight corrections, which would, however, be negligible except for the two largest orifices. In reality, the abscissa in the figure is stated to be p_2/p_1, and since it must be assumed that the statement is correct, no further reduction has been undertaken.

In view of the uncertainties involved in the foregoing reduction of Witte's experimental data, to say nothing of the difficulty of making accurate readings from his published figure, the surprisingly close agreement of Y and Y_{obs} for the two largest orifices is not to be taken too seriously. Nevertheless, it appears that, over the range $0.6 < y < 1.0$ and $0.2 \lesseqgtr \beta \lesseqgtr 0.76$, equation (15), developed by theoretical reasoning from an initial approximating assumption, does give a fairly good representation of the best established experimental facts for steam ($\gamma = 1.31$), as well as for natural gas ($\gamma = 1.283$) and for air ($\gamma = 1.40$).

VIII. SUMMARY

1. NOTATION

With all quantities expressed in terms of normal units, such as "British absolute" or cgs, let C be the discharge coefficient of an orifice meter of the type illustrated by Figure 1, as defined by the equation

$$M = CA\sqrt{2\rho_1(p_1 - p_2)} \tag{A}$$

in which:

M = the mass discharged per unit time;
A = the area of the orifice;
p_1, p_2 = the static pressures observed at the upstream and down-stream side holes or pressure taps; and
ρ_1 = the density of the gas at p_1 and the upstream temperature.

In addition to the foregoing notation, let

d = the diameter of the orifice;
D = the diameter of the pipe in which it is installed;
$\beta = d/D$ = the diameter ratio, or
$m = \beta^2$ = the area ratio;
l_1, l_2 = the distances from the orifice plate to the centers of the upstream and downstream side holes;
$y = p_2/p_1$ = the pressure ratio;
$\gamma = C_p/C_v$ = the specific heat ratio of the gas;
η = the viscosity of the gas;
$R_d = 4M/\pi d\eta$ = the Reynolds number;
K = the value obtained for C when the meter is tested with water or other liquid under conditions that make $R_d > 200{,}000$—M, ρ_1, and η now referring to the liquid;
$Y = C/K$ = the expansion factor for the gas; and
$\epsilon = (1 - Y)/(1 - y)$, so that

$$C = KY \tag{B}$$

and

$$Y = 1 - \epsilon\,(1 - y) \tag{C}$$

2. CONCLUSIONS

The following statements and conclusions are subject to the restriction that $R_d > 200{,}000$, a condition which is nearly always satisfied in the commercial measurement of gas by orifice meters except when the orifices are very small.

1. The water coefficient, K, is sensibly constant for any one orifice when installed and operated in a prescribed manner.
2. When the meter is used for gas, $C \doteq K$ or $Y \doteq 1$, when $y \doteq 1$.

These two facts are already familiar from the published work of Witte and others. The condition $R_d > 200{,}000$ results from Witte's experiments (3). The following statements are conclusions from the discussion in the present paper of experimental data obtained with pressure taps located within the limits $l_1 \lesseqgtr 2D$ and $l_2 \lesseqgtr 0.5D$: they may or may not be true outside those limits.

3. When $\beta < 0.2$, the curve $Y = f(y)$ is slightly concave toward the y axis, but the data for low values of β are scanty and no more specific statement is possible.

4. When $0.2 \lesseqgtr \beta \lesseqgtr 0.75$, $Y = f(y)$ is linear within the present accuracy of orifice meter measurements, at least as far down as the critical value of y; in other words, ϵ is a constant for any one orifice meter.

5. When $\beta > 0.75$, the linearity of $Y = f(y)$ may or may not persist, according to the location of the pressure taps.

(*a*) For $l_1 = D$ and $l_2 = 0.5D$ (throat taps), the curve is convex toward the y axis at $\beta = 0.8$ and still more so when $\beta = 0.87$. Presumably, the linear relationship ceases to hold soon after β exceeds 0.75.

(*b*) For $l_1 = l_2 = D/4$ (flange taps in a 4-inch pipe), $Y = f(y)$ is still represented by a straight line at $\beta = 0.8$; but at $\beta = 0.87$ the curve is strongly convex toward the y axis.

(*c*) For $l_1 = l_2 = D/8$ (flange taps in an 8-inch pipe) the linear relationship still persists at $\beta = 0.87$.

(*d*) For corner taps there are no satisfactory data above $\beta = 0.76$; but (*b*) and (*c*), above, indicate that $Y = f(y)$ would still be linear at considerably higher values of β.

6. Within the limits $0.2 \lesseqgtr \beta \lesseqgtr 0.75$, where each of the foregoing arrangements of the pressure taps gives a linear change of Y with y, or a constant ϵ for each orifice, the values of ϵ vary systematically with β and γ, and the values of Y are given approximately by the equation

$$Y = 1 - \frac{0.41 + 0.35m^2}{\gamma}(1 - y) \qquad \text{(D)}$$

The available observations may be slightly better represented by using separate equations for throat, and for corner and flange taps; but the difference is little, if at all, greater than the experimental uncertainties. And it seems probable that when the pressure ratio is greater than 0.8, as it is in the vast majority of practical metering operations, the mean equation (D) will always give the value of Y correctly within 0.5 per cent, and usually much closer than that, provided that the pressure taps are located within the limits $l_1 \lesseqgtr 2D$ and $l_2 \lesseqgtr 0.5D$.

Further accumulation of experimental data may require some modification of the numerical coefficients of equation (D), but it seems improbable that the changes will be of serious importance to gas engineers.

Although the variations of the limiting or liquid coefficient, K, have not been discussed in this paper, it may be stated here that the values of Y, or of the slope coefficient ϵ, are much less sensitive to changes of tap location or roughness of the pipe than the values of K.

7. In continuation of an earlier paper (8), a theoretical method for computing Y has been developed, and has been shown to be in fair agreement with the experimental facts in a number of typical cases.

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