# AN UNRECOGNIZED PROPERTY OF THE REVERSIBLE PENDULUM

By Paul R. Heyl

ABSTRACT

In a reversible pendulum which carries two planes and is supported from either plane in turn upon the same knife-edge, it is found that the correction for a circularly rounded knife-edge eliminates itself.

The time of swing $t$ of a simple pendulum whose length is equal to the distance between the two planes of support of a reversible pendulum of times of swing $T_1$ and $T_2$ is given by the well-known formula

$$t^2 = \frac{T_1^2 h_1 - T_2^2 h_2}{h_1 - h_2} \tag{1}$$

where $h_1$ and $h_2$ are the distances from the center of gravity to the point of support in the two positions of the reversible pendulum.

The nature of this formula is such that it brings about a self-eliminating effect in the case of certain corrections to which $T_1$ and $T_2$ may be subject.

Let there be corrections $\Delta T_1$ and $\Delta T_2$ applied to $T_1$ and $T_2$ according to the following formulas:

$$\left.\begin{aligned} T_1' &= T_1 (1 + \Delta T_1) \\ T_2' &= T_2 (1 + \Delta T_2) \end{aligned}\right\} \tag{2}$$

What must be the form of $\Delta T_1$ and $\Delta T_2$ in order that the value of $t$ shall be unaltered by applying these corrections?

The condition to be fulfilled is evidently

$$T_1^2 (1 + \Delta T_1)^2 h_1 - T_2^2 (1 + \Delta T_2)^2 h_2 = T_1^2 h_1 - T_2^2 h_2 \tag{3}$$

Expanding and neglecting the second powers of $\Delta T_1$ and $\Delta T_2$, and assuming that the reversible pendulum is so closely adjusted that $T_1$ is nearly equal to $T_2$, we obtain the following condition:

$$\frac{\Delta T_1}{\Delta T_2} = \frac{h_2}{h_1} \tag{4}$$

In other words, the corrections must be inversely proportional to the $h$ distances.

Now it happens that the correction for a circularly rounded knife-edge is exactly of this form. This correction is given by the formula

$$T' = T\left(1 + \frac{\rho}{2h}\right) \tag{5}$$

where $\rho$ is the radius of curvature of the knife-edge.

The correction for the rounded knife-edge to be applied to either $T$ may be quite large under conditions likely to occur in practice. For example, with a seconds pendulum the values of $h_1$ and $h_2$ may be of the order of 750 and 250 mm, while a value of 0.01 mm for $\rho$ may often be found. The corrections for $T_1$ and $T_2$ will be, respectively, about 7 and 20 parts per 1,000,000, yet the value of $t$ remains completely unaffected.

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