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Complete Elliptic Integrals Resulting from Infinite Integrals of Bessel Functions*

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Infinite integrals of Bessel and modified Bessel functions reducible to complete elliptic integrals are compiled. These formulas are of great use in solving problems of applied mathematics, physics and engineering.

Key words: Applied mathematics; Bessel functions; complete elliptic integrals; engineering; infinite integrals; modified Bessel functions; physics; signal statistics.

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1. Introduction

This table, which is an outgrowth of the author's experience in the theoretical study of signal statistics, contains infinite integrals of Bessel and modified Bessel functions reducible to complete elliptic integrals.[1] The formulas listed below are those important in applications and all results are expressed in conviently compact forms.

The materials were first extracted from the author's own memorandum which were then thoroughly augmented and rearranged in the present form by scrutinizing various published books and papers.

The parameters used in this table are usually positive real and notations occurring several times on a section are explained at the top of the section.

2. Integrands Involving Bessel Functions of the First Kind

2.1. $k^2=\dfrac{\sqrt{p^2+a^2}-p}{2\sqrt{p^2+a^2}}$

$$\int_0^\infty e^{-px^2}J_0(ax^2)dx=\frac{\sqrt{1-2k^2}}{\sqrt{\pi p}}K(k). \qquad \text{OL 119 (13.2)} \qquad (1)$$

$$\int_0^\infty e^{-px^2}J_0(ax^2)x^2dx=\frac{\sqrt{(1-2k^2)^3}}{2\sqrt{\pi p^3}}[2E(k)-K(k)]. \qquad \text{OL 119 (13.1)} \qquad (2)$$

$$\int_0^\infty e^{-px^2}J_0(ax^2)x^4dx=\frac{\sqrt{(1-2k^2)^5}}{4\sqrt{\pi p^5}}[8(1-2k^2)E(k)-(5-8k^2)K(k)]. \qquad (3)$$

[1] Including Bierens de Haan, D., Nouvelles Tables d'Intégrales Définies (Leide, Amsterdam 1867), there exists a considerable literature on complete elliptic integrals involving algebraic, trigonometric, hyperbolic or logarithmic integrands.

$$\int_0^\infty e^{-px^2}J_0(ax^2)x^6dx=\frac{\sqrt{(1-2k^2)^7}}{8\sqrt{\pi p^7}}[2(23-128k^2+128k^4)E(k)-(31-144k^2+128k^4)K(k)]. \tag{4}$$

$$\int_0^\infty e^{-px^2}J_1(ax^2)dx=\frac{\sqrt{1-2k^2}}{k\sqrt{\pi p(1-k^2)}}[E(k)-(1-k^2)K(k)]. \tag{5}$$

$$\int_0^\infty e^{-px^2}J_1(ax^2)x^2dx=\frac{\sqrt{(1-2k^2)^3}}{2k\sqrt{\pi p^3(1-k^2)}}[(1-k^2)K(k)-(1-2k^2)E(k)]. \tag{6}$$

$$\int_0^\infty e^{-px^2}J_1(ax^2)x^4dx=\frac{\sqrt{(1-2k^2)^5}}{4k\sqrt{\pi p^5(1-k^2)}}[(1-k^2)(1-8k^2)K(k)-(1-16k^2+16k^4)E(k)]. \tag{7}$$

$$\int_0^\infty e^{-px^2}J_1(ax^2)x^{-2}dx=\frac{2\sqrt{p}}{3k\sqrt{\pi(1-k^2)(1-2k^2)}}[(1-k^2)K(k)-(1-2k^2)E(k)]. \tag{8}$$

$$\int_0^\infty e^{-px^2}J_2(ax^2)dx=\frac{\sqrt{1-2k^2}}{3k^2(1-k^2)\sqrt{\pi p}}[(1-k^2)(2-3k^2)K(k)-2(1-2k^2)E(k)]. \tag{9}$$

$$\int_0^\infty e^{-px^2}J_2(ax^2)x^2dx=\frac{\sqrt{(1-2k^2)^3}}{2k^2(1-k^2)\sqrt{\pi p^3}}[2(1-k^2+k^4)E(k)-(1-k^2)(2-k^2)K(k)]. \tag{10}$$

$$\int_0^\infty e^{-px^2}J_2(ax^2)x^4dx=\frac{\sqrt{(1-2k^2)^5}}{4k^2(1-k^2)\sqrt{\pi p^5}}[(2+5k^2-8k^4)$$

$$\times(1-k^2)K(k)-2(1-2k^2)(1+4k^2-4k^4)E(k)]. \tag{11}$$

$$\int_0^\infty e^{-px^2}J_2(ax^2)x^6dx=\frac{\sqrt{(1-2k^2)^7}}{8k^2(1-k^2)\sqrt{\pi p^7}}[(2+15k^2-144k^4+128k^6)$$

$$\times(1-k^2)K(k)-2(1+7k^2-135k^4+256k^6-128k^8)E(k)]. \tag{12}$$

$$\int_0^\infty e^{-px^2}J_2(ax^2)x^{-2}dx=\frac{2\sqrt{p}}{15k^2(1-k^2)\sqrt{\pi(1-2k^2)}}$$

$$\times[2(1-k^2+k^4)E(k)-(1-k^2)(2-k^2)K(k)]. \tag{13}$$

$$\int_0^\infty e^{-px^2}J_3(ax^2)dx=\frac{\sqrt{1-2k^2}}{15k^3\sqrt{\pi p(1-k^2)^3}}[(8-23k^2+23k^4)E(k)$$

$$-(1-k^2)(8-19k^2+15k^4)K(k)]. \tag{14}$$

$$\int_0^\infty e^{-px^2}J_3(ax^2)x^2dx=\frac{\sqrt{(1-2k^2)^3}}{6k^3\sqrt{\pi p^3(1-k^2)^3}}[(8-15k^2+3k^4)$$

$$\times(1-k^2)K(k)-(8-19k^2+9k^4-6k^6)E(k)]. \tag{15}$$

$$\int_0^\infty e^{-px^2} J_3(ax^2) x^{-2} dx = \frac{2\sqrt{p}}{105k^3\sqrt{\pi(1-k^2)^3(1-2k^2)}} [(1-k^2)$$

$$\times (8-15k^2+3k^4)K(k) - (8-19k^2+9k^4-6k^6)E(k)]. \qquad (16)$$

2.2. $k_1^2 = \frac{1}{2}(1-\sqrt{1-a^2\gamma^2}),\qquad k_2^2 = \frac{1}{2}(1-\sqrt{1-b^2\gamma^2}),$

$$\gamma^2 = \frac{2}{p^2+a^2+b^2+\sqrt{(p^2+a^2+b^2)^2-4a^2b^2}}$$

$$\int_0^\infty e^{-px^2} J_0(ax^2) J_0(bx^2) dx = \frac{2\sqrt{\gamma}}{\sqrt{\pi^3}} K(k_1)K(k_2). \qquad (1)$$

$$\int_0^\infty e^{-px^2} J_1(ax^2) J_0(bx^2) dx = \frac{2\sqrt{\gamma}}{k_1\sqrt{\pi^3(1-k_1^2)}}$$

$$\times [E(k_1) - (1-k_1^2)K(k_1)] [2E(k_2) - K(k_2)]. \qquad (2)$$

$$\int_0^\infty e^{-px^2} J_1(ax^2) J_1(bx^2) dx = \frac{2\sqrt{\gamma}}{3k_1k_2\sqrt{\pi^3(1-k_1^2)(1-k_2^2)}}$$

$$\times [(1-2k_1^2)E(k_1) - (1-k_1^2)K(k_1)]$$

$$\times [(1-2k_2^2)E(k_2) - (1-k_2^2)K(k_2)]. \qquad (3)$$

$$\int_0^\infty e^{-px^2} J_2(ax^2) J_0(bx^2) dx = \frac{2\sqrt{\gamma}}{9k_1^2(1-k_1^2)\sqrt{\pi^3}}$$

$$\times [(1-k_1^2)(2-3k_1^2)K(k_1) - 2(1-2k_1^2)E(k_1)]$$

$$\times [8(1-2k_2)E(k_2) - (5-8k_2^2)K(k_2)]. \qquad (4)$$

$$\int_0^\infty e^{-px^2} J_2(ax^2) J_1(bx^2) dx = \frac{2\sqrt{\gamma}}{15k_1^2k_2(1-k_1^2)\ \sqrt{\pi^3(1-k_2^2)}}$$

$$\times [2(1-k_1^2+k_1^4)E(k_1) - (1-k_1^2)(2-k_1^2)K(k_1)]$$

$$\times [(1-k_2^2)(1-8k_2^2)\,K(k_2) - (1-16k_2^2+16k_2^4)E(k_2)]. \qquad (5)$$

$$\int_0^\infty e^{-px^2} J_2(ax^2) J_2(bx^2) dx = \frac{2\sqrt{\gamma}}{105k_1^2k_2^2(1-k_1^2)(1-k_2^2)\ \sqrt{\pi^3}}$$

$$\times [(1-k_1^2)(2+5k_1^2-8k_1^4)K(k_1) - 2(1-2k_1^2)(1+4k_1^2-4k_1^4)E(k_1)]$$

$$\times [(1-k_2^2)(2+5k_2^2-8k_2^4)K(k_2) - 2(1-2k_2^2)(1+4k_2^2-4k_2^4)E(k_2)]. \qquad (6)$$

2.3. $k^2 = \frac{a^2}{p^2+a^2}$

$$\int_0^\infty e^{-2px} J_0^2(ax) x^2 dx = \frac{k}{4\pi p^2 a} [\ (3-2k^2)E(k) - (1-k^2)K(k)\ ]. \qquad (1)$$

$$\int_0^\infty e^{-2px} J_1^2(ax) x^2 dx = \frac{k}{4\pi p^2 a} [\ (1-k^2)K(k) - (1-2k^2)E(k)\ ]. \qquad \text{BY 251 (561.08)}^2 \qquad (2)$$

$$\int_0^\infty e^{-2px} J_1^2(ax) x^{-2} dx = \frac{4a}{3\pi k^3} [\ (1-k^2)K(k) - (1-2k^2)E(k)\ ] - p. \qquad (3)$$

$$\int_0^\infty e^{-2px} J_2^2(ax) x dx = \frac{1}{2\pi p a k^3} [\ (16-16k^2+k^4)E(k) - 8(1-k^2)(2-k^2)K(k)\ ]. \qquad (4)$$

$$\int_0^\infty e^{-2px} J_2^2(ax) x^{-2} dx = \frac{4a}{15\pi k^5} [\ (1-k^2)(4+3k^2)K(k) - (4+k^2-6k^4)E(k)\ ] - \frac{p}{2}. \qquad (5)$$

2.4. $k^2 = \frac{a^2}{p^2+a^2}$

$$\int_0^\infty e^{-2px} J_2(ax) J_1(ax) x^{-1} dx = \frac{2}{3\pi k^3} [4(1-k^2)K(k) - (4-5k^2)E(k)] - \frac{p}{a}. \qquad (1)$$

$$\int_0^\infty e^{-2px} J_3(ax) J_1(ax) dx = \frac{1}{3\pi a k^3} [(32-38k^2+3k^4)K(k) - 2(16-23k^2)E(k)] - \frac{4p}{a^2}. \qquad (2)$$

$$\int_0^\infty e^{-2px} J_3(ax) J_2(ax) x^{-1} dx = \frac{2}{15\pi k^5} [4(1-k^2)(8+k^2)K(k) - (32-12k^2-23k^4)E(k)] - \frac{p}{a}. \qquad (3)$$

2.5. $k^2 = \frac{4ab}{p^2+(a+b)^2}$

$$\int_0^\infty e^{-px} J_0(ax) J_0(bx) dx = \frac{k}{\pi\sqrt{ab}} K(k). \qquad \text{BY 248 (560.01)} \qquad (1)$$

$$\int_0^\infty e^{-px} J_0(ax) J_0(bx) x dx = \frac{pk^3}{4\pi(1-k^2)\sqrt{(ab)^3}} E(k). \qquad \text{LU 316 (19)} \qquad (2)$$

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[2]The right-hand side in BY 251 (561.08) is incorrect.

$$\int_0^\infty e^{-px}J_1(ax)J_0(bx)x\,dx=\frac{k}{8\pi(1-k^2)a\sqrt{(ab)^3}}[k^2(a^2-b^2-p^2)E(k)+4ab(1-k^2)K(k)].$$

LU 317 (21) (3)

$$\int_0^\infty e^{-px}J_1(ax)J_1(bx)dx=\frac{1}{\pi k\sqrt{ab}}[(2-k^2)K(k)-2E(k)].$$

BY 249 (560.02) (4)

$$\int_0^\infty e^{-px}J_1(ax)J_1(bx)x\,dx=\frac{pk}{4\pi(1-k^2)\sqrt{(ab)^3}}[(2-k^2)E(k)-2(1-k^2)K(k)].$$

LU 316 (20) (5)

$$\int_0^\infty e^{-px}J_2(ax)J_2(bx)dx=\frac{1}{3\pi k^3\sqrt{ab}}[(4-k^2)(4-3k^2)K(k)-8(2-k^2)E(k)].$$

BY 248 (560.03) (6)

$$\int_0^\infty e^{-px}J_3(ax)J_3(bx)\,dx=\frac{1}{15\pi k^5\sqrt{ab}}[(128-128k^2+15k^4)\times(2-k^2)K(k)-2(128-128k^2+23k^4)E(k)]. \quad (7)$$

$$\int_0^\infty e^{-px}J_4(ax)J_4(bx)\,dx=\frac{1}{105\pi k^7\sqrt{ab}}[(6144-12288k^2+8000k^4-1856k^6+105k^8)K(k)-32(2-k^2)(96-96k^2+11k^4)E(k)]. \quad (8)$$

2.6 $k^2=\dfrac{4ab}{p^2+(a+b)^2}=\sin^2\alpha,\qquad \sin\beta=\dfrac{p}{\sqrt{p^2+(a-b)^2}}$

$\Lambda_0(\alpha,\beta)$ is Heuman's Lambda function.[3]

$$\int_0^\infty e^{-px}J_1(ax)J_0(bx)\,dx=-\frac{pk}{2\pi a\sqrt{ab}}K(k)-\frac{1}{2a}\Lambda_0(\alpha,\beta)+\frac{1}{a},\qquad a>b;$$

$$=-\frac{pk}{2\pi a^2}K(k)+\frac{1}{2a},\qquad a=b;$$

$$=-\frac{pk}{2\pi a\sqrt{ab}}K(k)+\frac{1}{2a}\Lambda_0(\alpha,\beta),\qquad a<b.$$

LU 317 (22) (1)

$$\int_0^\infty e^{-px}J_1(ax)J_0(bx)\,x^{-1}dx=\frac{2\sqrt{ab}}{\pi ak}E(k)+\frac{k(a^2-b^2)}{2\pi a\sqrt{ab}}K(k)+\frac{p}{2a}\Lambda_0(\alpha,\beta)-\frac{p}{a},\qquad a>b;$$

$$=\frac{2}{\pi k}E(k)-\frac{p}{2a},\qquad a=b;$$

[3] In connection with the Heuman's Lambda function the reader should consult BY pp. 35–37.

$$=\frac{2\sqrt{ab}}{\pi ak}E(k)+\frac{k(a^2-b^2)}{2\pi a\sqrt{ab}}K(k)-\frac{p}{2a}\Lambda_0(\alpha,\beta), \qquad a<b.$$

LU 318 (24) (2)

$$\int_0^\infty e^{-px}J_1(ax)J_1(bx)x^{-1}dx=\frac{p}{\pi k\sqrt{ab}}E(k)-\frac{pk(p^2+2a^2+2b^2)}{4\pi\sqrt{(ab)^3}}K(k)+\frac{(a^2-b^2)}{4ab}\Lambda_0(\alpha,\beta)$$

$$+\frac{b}{2a}, \qquad a>b;$$

$$=\frac{p}{\pi ak}E(k)-\frac{pk(p^2+4a^2)}{4\pi a^3}K(k)+\frac{1}{2}, \qquad a=b;$$

$$=\frac{p}{\pi k\sqrt{ab}}E(k)-\frac{pk(p^2+2a^2+2b^2)}{4\pi\sqrt{(ab)^3}}K(k)-\frac{(a^2-b^2)}{4ab}\Lambda_0(\alpha,\beta)$$

$$+\frac{a}{2b}, \qquad a<b.$$

LU 318 (25) (3)

$$\int_0^\infty e^{-px}J_2(ax)J_0(bx)dx=\frac{4\sqrt{ab}}{\pi a^2k}E(k)-\frac{kb^2}{\pi a^2\sqrt{ab}}K(k)+\frac{p}{a^2}\Lambda_0(\alpha,\beta)-\frac{2p}{a^2}, \qquad a>b;$$

$$=\frac{4}{\pi ak}E(k)-\frac{k}{\pi a}K(k)-\frac{p}{a^2}, \qquad a=b;$$

$$=\frac{4\sqrt{ab}}{\pi a^2k}E(k)-\frac{kb^2}{\pi a^2\sqrt{ab}}K(k)-\frac{p}{a^2}\Lambda_0(\alpha,\beta), \qquad a<b. \qquad (4)$$

$$\int_0^\infty e^{-px}J_2(ax)J_0(bx)xdx=-\frac{pk}{\pi a^2\sqrt{ab}}K(k)-\frac{pk^3}{4\pi(1-k^2)\sqrt{(ab)^3}}E(k)-\frac{1}{a^2}\Lambda_0(\alpha,\beta)$$

$$+\frac{2}{a^2}, \qquad a>b;$$

$$=-\frac{pk}{\pi a^3}K(k)-\frac{pk^3}{4\pi(1-k^2)a^3}E(k)+\frac{1}{a^2}, \qquad a=b;$$

$$=-\frac{pk}{\pi a^2\sqrt{ab}}K(k)-\frac{pk^3}{4\pi(1-k^2)\sqrt{(ab)^3}}E(k)$$

$$+\frac{1}{a^2}\Lambda_0(\alpha,\beta), \qquad a<b. \qquad (5)$$

$$\int_0^\infty e^{-px}J_2(ax)J_1(bx)\,dx=\frac{2p}{\pi ka\sqrt{ab}}\,E(k)-\frac{pk(p^2+a^2+2b^2)}{2\pi a\sqrt{(ab)^3}}\,K(k)-\frac{b}{2a^2}\,\Lambda_0(\alpha,\beta)$$

$$+\frac{b}{a^2},\qquad a>b;$$

$$=\frac{2p}{\pi a^2k}\,E(k)-\frac{pk(p^2+3a^2)}{2\pi a^4}\,K(k)+\frac{1}{2a},\qquad a=b;$$

$$=\frac{2p}{\pi ka\sqrt{ab}}\,E(k)-\frac{pk(p^2+a^2+2b^2)}{2\pi a\sqrt{(ab)^3}}\,K(k)+\frac{b}{2a^2}\,\Lambda_0(\alpha,\beta)$$

$$+\frac{1}{b},\qquad a<b. \tag{6}$$

2.7. $k^2=\dfrac{4ab}{(a+b)^2}$

$$\int_0^\infty J_1(ax)J_1(bx)\,x^{-2}dx=\frac{a+b}{3\pi ab}\,[(a^2+b^2)E(k)-(a-b)^2K(k)]. \tag{1}$$

$$\int_0^\infty J_2(ax)J_1(bx)\,x^{-1}dx=\frac{1}{3\pi a^2b}\,[(a-b)(a^2+2b^2)K(k)-(a+b)(a^2-2b^2)E(k)]. \tag{2}$$

$$\int_0^\infty J_3(ax)J_1(bx)\,dx=\frac{1}{3\pi a^3b(a+b)}\,[(a^4+a^2b^2-8b^4)K(k)-(a+b)^2(a^2-8b^2)E(k)]. \tag{3}$$

$$\int_0^\infty J_3(ax)J_2(bx)\,x^{-1}dx=\frac{1}{15\pi a^3b^2}\,[(a-b)(2a^4+5a^2b^2+8b^4)K(k)$$

$$-(a+b)(2a^4+3a^2b^2-8b^4)E(k)]. \tag{4}$$

$$\int_0^\infty J_4(ax)J_2(bx)\,dx=\frac{2}{15\pi a^4b^2(a+b)}\,[(a^6+4a^4b^2+4a^2b^4-24b^6)$$

$$\times K(k)-(a+b)^2(a^4+4a^2b^2-24b^4)E(k)]. \tag{5}$$

2.8. $k^2=\dfrac{b-\sqrt{b^2-a^2}}{2b},\qquad b>a$

$$\int_0^\infty J_0^2(ax)J_0(2bx)\,dx=\frac{2}{\pi^2b}\,[K(k)]^2. \tag{1}$$

$$\int_0^\infty J_0^2(ax)J_2(2bx)dx=\frac{2}{\pi^2b}\,[2E(k)-K(k)]^2. \tag{2}$$

$$\int_0^\infty J_1^2(ax)J_0(2bx)dx=-\frac{8b}{\pi^2a^2}\,[E(k)-(1-k^2)K(k)]^2. \tag{3}$$

$$\int_0^\infty J_1^2(ax)J_2(2bx)dx=\frac{8b}{3\pi^2a^2}\,[(1-k^2)K(k)-(1-2k^2)E(k)]^2. \tag{4}$$

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$$\int_0^\infty J_{\frac{2}{2}}^2(ax)J_0(2bx)dx=\frac{32b^3}{9\pi^2a^4}[(2-3k^2)(1-k^2)K(k)-2(1-2k^2)E(k)]^2. \tag{5}$$

$$\int_0^\infty J_{\frac{2}{2}}^2(ax)J_2(2bx)dx=-\frac{32b^3}{15\pi^2a^4}[2(1-k^2+k^4)E(k)-(1-k^2)(2-k^2)K(k)]^2. \tag{6}$$

$$\int_0^\infty J_{\frac{2}{2}}^2(ax)J_4(2bx)dx=\frac{32b^3}{105\pi^2a^4}[(2+5k^2-8k^4)(1-k^2)K(k)$$
$$-2(1-2k^2)(1+4k^2-4k^4)E(k)]^2. \tag{7}$$

$$\int_0^\infty J_{\frac{3}{3}}^2(ax)J_0(2bx)dx=-\frac{128b^5}{225\pi^2a^6}[(8-23k^2+23k^4)E(k)-(1-k^2)(8-19k^2+15k^4)K(k)]^2. \tag{8}$$

$$\int_0^\infty J_{\frac{3}{3}}^2(ax)J_2(2bx)dx=\frac{128b^5}{315\pi^2a^6}[(8-15k^2+3k^4)(1-k^2)K(k)-(8-19k^2+9k^4-6k^6)E(k)]^2. \tag{9}$$

2.9. $k_1^2=\dfrac{abcd}{\Delta^2}, \qquad k_2^2=\dfrac{\Delta^2}{abcd}$

$$16\Delta^2=(a+b+c-d)(a+b+d-c)(a+c+d-b)(b+c+d-a)$$

$$\int_0^\infty J_0(ax)J_0(bx)J_0(cx)J_0(dx)xdx=\frac{1}{\pi^2\Delta}K(k_1), \qquad \Delta^2>abcd;$$

$$=\frac{1}{\pi^2\sqrt{abcd}}K(k_2), \qquad \Delta^2<abcd. \qquad \text{WA 414 (9)} \tag{1}$$

3. Integrands Involving Bessel Functions of the Second Kind

3.1. $k^2=\dfrac{\sqrt{p^2+a^2}+p}{2\sqrt{p^2+a^2}}$

$$\int_0^\infty e^{-px^2}Y_0(ax^2)dx=-\frac{\sqrt{2k^2-1}}{\sqrt{\pi p}}K(k). \tag{1}$$

$$\int_0^\infty e^{-px^2}Y_0(ax^2)x^2dx=\frac{\sqrt{(2k^2-1)^3}}{2\sqrt{\pi p^3}}[2E(k)-K(k)]. \tag{2}$$

$$\int_0^\infty e^{-px^2}Y_0(ax^2)x^4dx=-\frac{\sqrt{(2k^2-1)^5}}{4\sqrt{\pi p^5}}[8(1-2k^2)E(k)-(5-8k^2)K(k)]. \tag{3}$$

$$\int_0^\infty e^{-px^2}Y_0(ax^2)x^6dx=\frac{\sqrt{(2k^2-1)^7}}{8\sqrt{\pi p^7}}[2(23-128k^2+128k^4)E(k)-(31-144k^2+128k^4)K(k)]. \tag{4}$$

$$\int_0^\infty e^{-px^2} Y_1(ax^2)x^2 dx = -\frac{\sqrt{(2k^2-1)^3}}{2k\sqrt{\pi p^3(1-k^2)}}[(1-k^2)K(k)-(1-2k^2)E(k)]. \tag{5}$$

$$\int_0^\infty e^{-px^2} Y_1(ax^2)x^4 dx = \frac{\sqrt{(2k^2-1)^5}}{4k\sqrt{\pi p^5(1-k^2)}}$$
$$\times[(1-k^2)(1-8k^2)K(k)-(1-16k^2+16k^4)E(k)]. \tag{6}$$

$$\int_0^\infty e^{-px^2} Y_1(ax^2)x^6 dx = -\frac{\sqrt{(2k^2-1)^7}}{8k\sqrt{\pi p^7(1-k^2)}}$$
$$\times[(3-80k^2+128k^4)(1-k^2)K(k)-(3-134k^2+384k^4-256k^6)E(k)]. \tag{7}$$

$$\int_0^\infty e^{-px^2} Y_2(ax^2)x^4 dx = -\frac{\sqrt{(2k^2-1)^5}}{4k^2(1-k^2)\sqrt{\pi p^5}}$$
$$\times[(2+5k^2-8k^4)(1-k^2)K(k)-2(1-2k^2)(1+4k^2-4k^4)E(k)]. \tag{8}$$

$$\int_0^\infty e^{-px^2} Y_2(ax^2)x^6 dx = \frac{\sqrt{(2k^2-1)^7}}{8k^2(1-k^2)\sqrt{\pi p^7}}$$
$$\times[(2+15k^2-144k^4+128k^6)(1-k^2)K(k)-2(1+7k^2-135k^4+256k^6-128k^8)E(k)]. \tag{9}$$

3.2. $k^2=\frac{p^2}{p^2+a^2}$

$$\int_0^\infty e^{-2px} Y_0(ax)J_0(ax)dx = -\frac{k}{\pi p}K(k)\cdot \qquad \text{OL 122 (13.18)} \tag{1}$$

$$\int_0^\infty e^{-2px} Y_0(ax)J_0(ax)x\,dx = -\frac{k}{2\pi p^2}[K(k)-E(k)]. \tag{2}$$

$$\int_0^\infty e^{-2px} Y_0(ax)J_0(ax)x^2 dx = -\frac{k}{4\pi p^3}[(1+k^2)K(k)-(1+2k^2)E(k)]. \tag{3}$$

$$\int_0^\infty e^{-2px} Y_0(ax)J_0(ax)x^3 dx = -\frac{k}{8\pi p^4}[2(1-2k^4)K(k)+(2+k^2+8k^4)E(k)]. \tag{4}$$

3.3. $k^2=\frac{a-\sqrt{a^2-b^2}}{2a}, \qquad a>b$

$$\int_0^\infty Y_0(ax^2)J_0(bx^2)dx = -\frac{\Gamma^2\left(\frac{1}{4}\right)}{2\pi^2\sqrt{a}}K(k). \tag{1}$$

$$\int_0^\infty Y_0(ax^2)J_0(bx^2)x^2 dx = \frac{\Gamma^2\left(\frac{3}{4}\right)}{\pi^2\sqrt{a(a^2-b^2)}}K(k). \tag{2}$$

$$\int_0^\infty Y_0(ax^2)J_1(bx^2)\,dx = \frac{4\Gamma^2\left(\frac{3}{4}\right)\sqrt{a}}{\pi^2 b}\,[E(k)-(1-k^2)K(k)]. \tag{3}$$

$$\int_0^\infty Y_0(ax^2)J_2(bx^2)\,dx = -\frac{2\Gamma^2\left(\frac{1}{4}\right)\sqrt{a^3}}{9\pi^2 b^2}\,[(2-3k^2)(1-k^2)K(k)-2(1-2k^2)E(k)]. \tag{4}$$

$$\int_0^\infty Y_1(ax^2)J_1(bx^2)\,dx = -\frac{\Gamma^2\left(\frac{1}{4}\right)\sqrt{a}}{3\pi^2 b}\,[(1-k^2)K(k)-(1-2k^2)E(k)]. \tag{5}$$

$$\int_0^\infty Y_1(ax^2)J_1(bx^2)\,x^2dx = \frac{2\Gamma^2\left(\frac{3}{4}\right)\sqrt{a}}{\pi^2 b\sqrt{a^2-b^2}}\,[(1-k^2)K(k)-(1-2k^2)E(k)]. \tag{6}$$

$$\int_0^\infty Y_1(ax^2)J_2(bx^2)\,dx = \frac{8\Gamma^2\left(\frac{3}{4}\right)\sqrt{a^3}}{5\pi^2 b^2}\,[2(1-k^2+k^4)E(k)-(2-k^2)(1-k^2)K(k)]. \tag{7}$$

$$\int_0^\infty Y_2(ax^2)J_2(bx^2)\,dx = -\frac{2\Gamma^2\left(\frac{1}{4}\right)\sqrt{a^3}}{21\pi^2 b^2}\,[(2+5k^2-8k^4)\;(1-k^2)K(k)$$
$$-2(1-2k^2)(1+4k^2-4k^4)E(k)]. \tag{8}$$

3.4. $k^2=\dfrac{b^2}{a^2+b^2}$

$$\int_0^\infty Y_0(ax)K_0(bx)\,dx = -\frac{k}{b}K(k).$$ OB 152 (2.29) (1)

4. Integrands Involving Modified Bessel Functions of the First Kind [4]

4.1. $k^2=\dfrac{2a}{p+a}, \quad p>a$

$$\int_0^\infty e^{-px^2}I_0(ax^2)\,dx = \frac{k}{\sqrt{2\pi a}}K(k).$$ OL 119 (13.5) (1)

$$\int_0^\infty e^{-px^2}I_0(ax^2)\,x^2dx = \frac{k^3}{4(1-k^2)\sqrt{2\pi a^3}}E(k).$$ NM 151 (2)

$$\int_0^\infty e^{-px^2}I_0(ax^2)\,x^4dx = \frac{k^5}{16(1-k^2)^2\sqrt{2\pi a^5}}[2(2-k^2)E(k)-(1-k^2)K(k).]$$
NM 151 (3)

[4] The integrals of the same types as compiled in 2.1–2.6, 6.1–6.7 which involve $I_n(x)$ in place of $J_n(x)$ are also expressible in terms of $K(k)$ and $E(k)$, whose results may be deduced from those without difficulty by using the relation $J_n(ix)=i^nI_n(x)$. Some of those results are also found in: WA 391 (5); OL 122 (13.21)–(13.23), 123 (13.24)–(13.27); OB 152 (2.30), (2.31), 153 (2.38), 154 (2.40), (2.41); BY 251 (562.01)–(562.04).

$$\int_0^\infty e^{-px^2} I_0(ax^2) x^6 dx = \frac{k^7}{64(1-k^2)^3 \sqrt{2\pi a^7}} [(23-23k^2+8k^4)E(k) - 4(1-k^2)(2-k^2)K(k)]. \tag{4}$$

$$\int_0^\infty e^{-px^2} I_1(ax^2) dx = \frac{1}{k\sqrt{2\pi a}} [(2-k^2)K(k) - 2E(k)]. \tag{5}$$

$$\int_0^\infty e^{-px^2} I_1(ax^2)\, x^2 dx = \frac{k}{4(1-k^2)\sqrt{2\pi a^3}} [(2-k^2)E(k) - 2(1-k^2)K(k)]. \tag{6}$$

$$\int_0^\infty e^{-px^2} I_1(ax^2)\, x^4 dx = \frac{k^3}{16(1-k^2)^2\sqrt{2\pi a^5}} [2(1-k^2+k^4)E(k) - (2-k^2)(1-k^2)K(k)].$$

NM 151 (7)

$$\int_0^\infty e^{-px^2} I_1(ax^2)\, x^{-2} dx = \frac{2\sqrt{2a}}{3\sqrt{\pi k^3}} [(2-k^2)E(k) - 2(1-k^2)K(k)]. \tag{8}$$

$$\int_0^\infty e^{-px^2} I_2(ax^2)\, dx = \frac{1}{3k^3\sqrt{2\pi a}} [(16-16k^2+3k^4)K(k) - 8(2-k^2)(1-k^2)E(k)]. \tag{9}$$

$$\int_0^\infty e^{-px^2} I_2(ax^2) x^2 dx = \frac{1}{4k(1-k^2)\sqrt{2\pi a^3}} [(16-16k^2+k^4)E(k) - 8(1-k^2)(2-k^2)K(k)]. \tag{10}$$

$$\int_0^\infty e^{-px^2} I_2(ax^2)\, x^4 dx = \frac{k}{16(1-k^2)^2\sqrt{2\pi a^5}} [(16-16k^2-k^4) \times (1-k^2)K(k) - 2(2-k^2)(4-4k^2-k^4)E(k)]. \tag{11}$$

$$\int_0^\infty e^{-px^2} I_2(ax^2) x^6 dx = \frac{k^3}{64(1-k^2)^3\sqrt{2\pi a^7}} [4(2-2k^2-k^4) \times (2-k^2)(1-k^2)K(k) - (16-32k^2+9k^4+7k^6-8k^8)E(k)]. \tag{12}$$

$$\int_0^\infty e^{-px^2} I_2(ax^2) x^{-2} dx = \frac{2\sqrt{2a}}{15\sqrt{\pi k^5}} [(16-16k^2+k^4)E(k) - 8(2-k^2)(1-k^2)K(k)]. \tag{13}$$

$$\int_0^\infty e^{-px^2} I_3(ax^2) dx = \frac{1}{15k^5\sqrt{2\pi a}} [(128-128k^2+15k^4) \times (2-k^2)K(k) - 2(128-128k^2+23k^4)E(k)]. \tag{14}$$

$$\int_0^\infty e^{-px^2} I_3(ax^2) x^2 dx = \frac{1}{12k^3(1-k^2)\sqrt{2\pi a^3}} [(128-128k^2+3k^4) \times (2-k^2)E(k) - 2(1-k^2)(128-128k^2+27k^4)K(k)]. \tag{15}$$

$$\int_0^\infty e^{-px^2} I_3(ax^2) x^4 dx = \frac{1}{16k(1-k^2)^2\sqrt{2\pi a^5}} [(128-128k^2-k^4) \times (2-k^2)(1-k^2)K(k) - 2(128-256k^2+135k^4-7k^6-k^8)E(k)]. \tag{16}$$

$$\int_0^\infty e^{-px^2} I_3(ax^2) x^6 dx = \frac{k}{64(1-k^2)^3 \sqrt{2\pi a^7}} [(128-256k^2+99k^4+29k^6+8k^8)(2-k^2)E(k)$$
$$-2(1-k^2)(128-256k^2+123k^4+5k^6+2k^8)K(k)]. \qquad (17)$$

$$\int_0^\infty e^{-px^2} I_3(ax^2)\, x^{-2} dx = \frac{2\sqrt{2a}}{105\sqrt{\pi k^7}} [(128-128k^2+3k^4)$$
$$\times (2-k^2)E(k) - 2(1-k^2)(128-128k^2+27k^4)K(k)]. \qquad (18)$$

$$\int_0^\infty e^{-px^2} I_3(ax^2)\, x^{-4} dx = \frac{8\sqrt{2a^3}}{945\sqrt{\pi k^9}} [(128-128k^2-k^4)(1-k^2)$$
$$\times (2-k^2)K(k) - 2(128-256k^2+135k^4-7k^6-k^8)E(k)]. \qquad (19)$$

$$\int_0^\infty e^{-px^2} I_3(ax^2)\, x^{-6} dx = \frac{32\sqrt{2a^5}}{10395\sqrt{\pi k^{11}}} [(128-256k^2+99k^4+29k^6+8k^8)$$
$$\times (2-k^2)E(k) - 2(1-k^2)(128-256k^2+123k^4+5k^6+2k^8)K(k)]. \qquad (20)$$

4.2 $k^2=\frac{1}{b^2}(\sqrt{a+b}-\sqrt{a})^2(\sqrt{a+2b}-\sqrt{a+b})^2, \qquad k^2+k'^2=1$

$$\int_0^\infty e^{-2(a+b)x} I_0^2(ax) I_0(2bx)\, dx = \frac{4}{\pi^2 b\sqrt{a}}(\sqrt{a+2b}-\sqrt{a+b})\; K(k)K(k'). \qquad \text{MV 228} \qquad (1)$$

4.3. $k^2=\left(\frac{p-a}{p+a}\right)^2$

$$\int_0^\infty e^{-2px} I_0(ax) K_0(ax)\, dx = \frac{1}{p+a} K(k). \qquad \text{ET II 370 (48)} \qquad (1)$$

$$\int_0^\infty e^{-2px} I_0(ax) K_0(ax) x dx = \frac{1}{4p(a^2-p^2)} [(p+a)E(k) - 2pK(k)]. \qquad (2)$$

$$\int_0^\infty e^{-2px} I_0(ax) K_0(ax)\, x^2 dx = \frac{1}{8p^2(a+p)(a-p)^2} [(a^2-3p^2)E(k)+2p(2p-a)K(k)]. \qquad (3)$$

4.4. $k^2=\left(\frac{b}{\sqrt{a^2+b^2}+a}\right)^2, \qquad k^2+k'^2=1$

$$\int_0^\infty I_0(ax)K_0(ax)J_0(2bx)dx = \frac{k'^2}{\pi b} K(k)K(k'). \qquad \text{OB 23 (2.116)} \qquad (1)$$

$$\int_0^\infty I_0(ax)K_0(ax)J_2(2bx)dx = \frac{1}{\pi bk'^2}[2E(k)-(1-k^2)K(k)][(2-k'^2)K(k')-2E(k')]. \qquad (2)$$

$$\int_0^\infty I_1(ax)K_1(ax)J_0(2bx)dx = \frac{k'^2}{\pi bk^2} E(k')[K(k)-E(k)]. \qquad (3)$$

$$\int_0^\infty I_1(ax)K_1(ax)J_2(2bx)dx = \frac{1}{3\pi bk^2k'^2}[(1+k^2)E(k)$$

$$-(1-k^2)K(k)][(2-k'^2)E(k')-2(1-k'^2)K(k')]. \quad (4)$$

$$\int_0^\infty I_2(ax)K_2(ax)J_0(2bx)dx = \frac{k'^2}{9\pi bk^4}[(2+k^2)K(k)$$

$$-2(1+k^2)E(k)][2(2-k'^2)E(k')-(1-k'^2)K(k')]. \quad (5)$$

5. Integrands Involving Modified Bessel Functions of the Second Kind

5.1.[5] $k_1^2 = \dfrac{p-a}{p+a}, \qquad k_2^2 = \dfrac{a-p}{2a}$

$$\int_0^\infty e^{-px^2}K_0(ax^2)\,dx = \frac{\sqrt{\pi}}{\sqrt{p+a}}K(k_1), \qquad p > a;$$

$$= \frac{\sqrt{\pi}}{\sqrt{2a}}K(k_2), \qquad p < a. \quad (1)$$

$$\int_0^\infty e^{-px^2}K_0(ax^2)\,x^2dx = \frac{\sqrt{\pi(p+a)}}{2(p^2-a^2)}[K(k_1)-E(k_1)], \qquad p > a;$$

$$= \frac{\sqrt{\pi}}{2(a^2-p^2)\sqrt{2a}}[2aE(k_2)-(p+a)K(k_2)], \qquad p < a. \quad (2)$$

$$\int_0^\infty e^{-px^2}K_0(ax^2)\,x^4dx = \frac{\sqrt{\pi(p+a)}}{4(p^2-a^2)^2}[(3p+a)K(k_1)-4pE(k_1)], \qquad p > a;$$

$$= \frac{\sqrt{\pi}}{4(a^2-p^2)^2\sqrt{2a}}[(p+a)(3p+a)K(k_2)-8paE(k_2)], \qquad p < a. \quad (3)$$

$$\int_0^\infty e^{-px^2}K_0(ax^2)x^6dx = \frac{\sqrt{\pi(p+a)}}{8(p^2-a^2)^3}[(15p^2+8pa+9a^2)K(k_1)-(23p^2+9a^2)E(k_1)], \qquad p > a;$$

$$= \frac{\sqrt{\pi}}{8(a^2-p^2)^3\sqrt{2a}}[2a(23p^2+9a^2)E(k_2)-(p+a)(15p^2+8pa+9a^2)K(k_2)], \qquad p < a. \quad (4)$$

$$\int_0^\infty e^{-px^2}K_1(ax^2)x^2dx = \frac{\sqrt{\pi(p+a)}}{2a(p^2-a^2)}[pE(k_1)-aK(k_1)], \qquad p > a;$$

$$= \frac{\sqrt{\pi}}{2(a^2-p^2)\sqrt{2a}}[(p+a)K(k_2)-2pE(k_2)], \qquad p < a. \quad (5)$$

$$\int_0^\infty e^{-px^2}K_1(ax^2)x^4dx = \frac{\sqrt{\pi(p+a)}}{4a(p^2-a^2)^2}[(p^2+3a^2)E(k_1)-a(p+3a)K(k_1)], \qquad p > a;$$

$$= \frac{\sqrt{\pi}}{4(a^2-p^2)^2\sqrt{2a}}[2(p^2+3a^2)E(k_2)-(p+a)(p+3a)K(k_2)], \qquad p < a. \quad (6)$$

[5] For negative p, the second results are available.

$$\int_0^\infty e^{-px^2}K_1(ax^2)x^6dx = \frac{\sqrt{\pi(p+a)}}{8a(p^2-a^2)^3}[p(3p^2+29a^2)E(k_1)$$

$$-a(3p^2+24pa+5a^2)K(k_1)], \qquad p>a;$$

$$= \frac{\sqrt{\pi}}{8(a^2-p^2)^3\sqrt{2a}}[(p+a)(3p^2+24pa+5a^2)K(k_2)$$

$$-2p(3p^2+29a^2)E(k_2)], \qquad p<a. \tag{7}$$

$$\int_0^\infty e^{-px^2}K_2(ax^2)x^4dx = \frac{\sqrt{\pi(p+a)}}{4a^2(p^2-a^2)^2}[a(5a^2+3pa-4p^2)K(k_1)$$

$$-4p(2a^2-p^2)E(k_1)], \qquad p>a;$$

$$= \frac{\sqrt{\pi}}{4a(a^2-p^2)^2\sqrt{2a}}[(5a^2+3pa-4p^2)$$

$$\times(p+a)K(k_2)-8p(2a^2-p^2)E(k_2)], \qquad p<a. \tag{8}$$

5.2. $k_1^2 = \frac{a-\sqrt{a^2-b^2}}{2a}, \qquad k_2^2 = \frac{b-\sqrt{b^2-a^2}}{2b}$

$$k_1^2 + k_1'^2 = 1, \qquad k_2^2 + k_2'^2 = 1$$

$$\int_0^\infty K_0(ax^2)K_0(bx^2)\,dx = \frac{\Gamma^2\left(\frac{1}{4}\right)}{4\sqrt{2a}}[K(k_1)+K(k_1')], \qquad a>b. \qquad \text{OB 153 (2.33)} \tag{1}$$

$$\int_0^\infty K_0(ax^2)K_0(bx^2)\,x^2dx = \frac{\Gamma^2\left(\frac{3}{4}\right)}{2\sqrt{2a^3}\,(k_1'^2-k_1^2)}[K(k_1')-K(k_1)], \qquad a>b. \tag{2}$$

$$\int_0^\infty K_1(ax^2)\;K_0(bx^2)\;x^2dx$$

$$= \frac{\Gamma^2\left(\frac{1}{4}\right)}{8\sqrt{2a^3}\,(k_1'^2-k_1^2)}[2E(k_1)-K(k_1)+K(k_1')-2E(k_1')], \qquad a>b;$$

$$= \frac{a\Gamma^2\left(\frac{1}{4}\right)}{16\sqrt{2b^5}\,(k_2'^2-k_2^2)\,(k_2k_2')^2}[(1-k_2^2)K(k_2)-E(k_2)+E(k_2')-(1-k_2'^2)K(k_2')], \qquad a<b. \tag{3}$$

5.3 $k^2 = \left(\frac{a-b}{a+b}\right)^2$

$$\int_0^\infty K_0(ax)K_0(bx)dx = \frac{\pi}{a+b}K(k). \tag{1}$$

$$\int_0^\infty K_0(ax)K_0(bx)x^2dx=\frac{\pi}{(a+b)(a-b)^2}[K(k)-E(k)]. \tag{2}$$

$$\int_0^\infty K_1(ax)K_0(bx)xdx=\frac{\pi}{2a(a^2-b^2)}[2aK(k)-(a+b)E(k)]. \tag{3}$$

$$\int_0^\infty K_1(ax)K_0(bx)x^3dx=\frac{\pi}{2a(a+b)^2(a-b)^3}[2a(3a+b)K(k)-(7a^2+b^2)E(k)]. \tag{4}$$

$$\int_0^\infty K_1(ax)K_1(bx)x^2dx=\frac{\pi}{2ab(a+b)(a-b)^2}[(a^2+b^2)E(k)-2abK(k)]. \tag{5}$$

$$\int_0^\infty K_2(ax)K_0(bx)x^2dx=\frac{\pi}{a^2(a+b)(a-b)^2}[a(3a-2b)K(k)-(2a^2-b^2)E(k)]. \tag{6}$$

$$\int_0^\infty K_2(ax)K_1(bx)x^3dx=\frac{\pi}{2a^2b(a+b)^2(a-b)^3}[(3a^4+7a^2b^2$$

$$-2b^4)E(k)-2ab(3a^2+3ab-2b^2)K(k)]. \tag{7}$$

5.4 $k_1^2=\frac{a-\sqrt{a^2-b^2}}{2a},\qquad k_2^2=\frac{b-\sqrt{b^2-a^2}}{2b}$

$k_1^2+k_1'^2=1,\qquad k_2^2+k_2'^2=1$

$$\int_0^\infty K_0^2(ax)K_0(2bx)dx=\frac{\pi}{2a}K(k_1)K(k_1'),\qquad a>b;$$

$$=\frac{\pi}{4b}[K^2(k_2)+K^2(k_2')],\qquad a<b.$$

OB 154 (2.39) (1)

6. Integrands Involving Products of Bessel and Modified Bessel Functions

6.1 $k_1^2=\frac{2}{1+\sqrt{1+a^2\gamma^2}},\qquad k_2^2=\frac{1-\sqrt{1-b^2\gamma^2}}{2}$

$\gamma^2=\frac{2}{p^2-a^2+b^2+\sqrt{(p^2-a^2+b^2)^2+4a^2b^2}}$

$$\int_0^\infty e^{-px^2}K_0(ax^2)J_0(bx^2)dx=\frac{k_1\sqrt{\gamma}}{\sqrt{\pi}}K(k_1)K(k_2). \tag{1}$$

$$\int_0^\infty e^{-px^2}K_0(ax^2)J_1(bx^2)dx=\frac{\sqrt{\gamma}}{k_1k_2\sqrt{\pi(1-k_2^2)}}[(2-k_1^2)K(k_1)$$

$$-2E(k_1)][E(k_2)-(1-k_2^2)K(k_2)]. \tag{2}$$

$$\int_0^\infty e^{-px^2}K_0(ax^2)J_2(bx^2)dx=\frac{\sqrt{\gamma}}{9\sqrt{\pi}k_1^3k_2^2(1-k_2^2)}$$
$$\times[(16-16k_1^2+3k_1^4)K(k_1)-8(2-k_1^2)E(k_1)]$$
$$\times[(1-k_2^2)(2-3k_2^2)K(k_2)-2(1-2k_2^2)E(k_2)]. \qquad (3)$$

$$\int_0^\infty e^{-px^2}K_1(ax^2)J_1(bx^2)dx=\frac{\sqrt{\gamma}}{3k_1k_2\sqrt{\pi(1-k_1^2)(1-k_2^2)}}$$
$$\times[(2-k_1^2)E(k_1)-2(1-k_1^2)K(k_1)][(1-k_2^2)K(k_2)-(1-2k_2^2)E(k_2)]. \qquad (4)$$

$$\int_0^\infty e^{-px^2}K_1(ax^2)J_2(bx^2)dx=\frac{\sqrt{\gamma}}{15\sqrt{\pi}k_1^3k_2^2(1-k_2^2)\sqrt{1-k_1^2}}$$
$$\times[(16-16k_1^2+k_1^4)E(k_1)-8(1-k_1^2)(2-k_1^2)K(k_1)]$$
$$\times[2(1-k_2^2+k_2^4)E(k_2)-(1-k_2^2)(2-k_2^2)K(k_2)]. \qquad (5)$$

$$\int_0^\infty e^{-px^2}K_2(ax^2)J_2(bx^2)dx=\frac{\sqrt{\gamma}}{105\sqrt{\pi}k_1^3k_2^2(1-k_1^2)(1-k_2^2)}$$
$$\times[(1-k_1^2)(16-16k_1^2-k_1^4)K(k_1)-2(2-k_1^2)(4-4k_1^2-k_1^4)E(k_1)]$$
$$\times[(1-k_2^2)(2+5k_2^2-8k_2^4)K(k_2)-2(1-2k_2^2)(1+4k_2^2-4k_2^4)E(k_2)]. \qquad (6)$$

6.2. $k^2=\left(\frac{b}{\sqrt{a^2+b^2}+a}\right)^2$

$$\int_0^\infty K_0(ax^2)J_0(bx^2)dx=\frac{\Gamma^2\left(\frac{1}{4}\right)\sqrt{k}}{2\pi\sqrt{b}}K(k). \qquad \text{OB 22 (2.108)} \qquad (1)$$

$$\int_0^\infty K_0(ax^2)J_0(bx^2)x^2dx=\frac{\Gamma^2\left(\frac{3}{4}\right)\sqrt{k}}{\pi\sqrt{b(a^2+b^2)}}K(k). \qquad \text{OB 21 (2.107)} \qquad (2)$$

$$\int_0^\infty K_0(ax^2)J_1(bx^2)dx=\frac{2\Gamma^2\left(\frac{3}{4}\right)}{\pi\sqrt{bk}}[K(k)-E(k)]. \qquad (3)$$

$$\int_0^\infty K_0(ax^2)J_2(bx^2)dx=\frac{\Gamma^2\left(\frac{1}{4}\right)}{18\pi\sqrt{bk^3}}[(2+k^2)K(k)-2(1+k^2)E(k)]. \qquad (4)$$

$$\int_0^\infty K_1(ax^2)J_1(bx^2)dx=\frac{\Gamma^2\left(\frac{1}{4}\right)\sqrt{b}}{12\pi a\sqrt{k^3}}[(1+k^2)E(k)-(1-k^2)K(k)]. \qquad (5)$$

$$\int_0^\infty K_1(ax^2)J_1(bx^2)x^2dx=\frac{\Gamma^2\left(\frac{3}{4}\right)\sqrt{b}}{2\pi a\sqrt{(a^2+b^2)k^3}}[(1+k^2)E(k)-(1-k^2)K(k)]. \qquad (6)$$

$$\int_0^\infty K_1(ax^2)J_2(bx^2)\,dx=\frac{\Gamma^2\left(\frac{3}{4}\right)\sqrt{b}}{5\pi a\sqrt{k^5}}\,[2(1-k^2+k^4)E(k)-(1-k^2)\,(2-k^2)K(k)]. \qquad (7)$$

$$\int_0^\infty K_2(ax^2)J_2(bx^2)\,dx=\frac{\Gamma^2\left(\frac{1}{4}\right)\sqrt{b^3}}{168\pi a^2\sqrt{k^7}}\,[(2-9k^2-k^4)$$

$$\times(1-k^2)K(k)-2(1+k^2)\,(1-6k^2+k^4)E(k)]. \qquad (8)$$

6.3. $k^2=\dfrac{b^2}{a^2+b^2}$

$$\int_0^\infty K_0(ax)J_0(bx)dx=\frac{k}{b}K(k). \qquad \text{OB 22 (2.109), 152 (2.28)} \qquad (1)$$

$$\int_0^\infty K_0(ax)J_0(bx)\,x^2dx=\frac{k^3}{b^3}\,[2E(k)-K(k)]. \qquad (2)$$

$$\int_0^\infty K_0(ax)J_0(bx)\,x^4dx=\frac{3k^5}{b^5}\,[8(1-2k^2)E(k)-(5-8k^2)K(k)]. \qquad (3)$$

$$\int_0^\infty K_0(ax)J_0(bx)x^6dx=\frac{15k^7}{b^7}[2(23-128k^2+128k^4)E(k)-(31-144k^2+128k^4)K(k)]. \qquad (4)$$

$$\int_0^\infty K_0(ax)J_1(bx)\,x\,dx=\frac{k}{b^2}\,[K(k)-E(k)]. \qquad (5)$$

$$\int_0^\infty K_0(ax)J_1(bx)\,x^3dx=\frac{k^3}{b^4}\,[(1-4k^2)K(k)-(1-8k^2)E(k)]. \qquad (6)$$

$$\int_0^\infty K_0(ax)J_1(bx)\,x^5dx=\frac{3k^5}{b^6}\,[(3-4k^2)(1-16k^2)K(k)-(3-88k^2+128k^4)E(k)]. \qquad (7)$$

$$\int_0^\infty K_0(ax)J_1(bx)\,x^{-1}dx=\frac{1}{k}\,[K(k)-E(k)]. \qquad (8)$$

$$\int_0^\infty K_0(ax)J_2(bx)\,dx=\frac{1}{bk}\,[(2-k^2)K(k)-2E(k)]. \qquad (9)$$

$$\int_0^\infty K_0(ax)J_2(bx)\,x^2dx=\frac{k}{b^3}\,[(2+k^2)K(k)-2(1+k^2)E(k)]. \qquad (10)$$

$$\int_0^\infty K_0(ax)J_2(bx)\,x^4dx=\frac{k^3}{b^5}\,[(2+7k^2-24k^4)K(k)-2(1+4k^2-24k^4)E(k)]. \qquad (11)$$

$$\int_0^\infty K_0(ax)J_2(bx)\,x^{-2}dx=\frac{b}{9k^3}\,[(2+k^2)K(k)-2(1+k^2)E(k)]. \qquad (12)$$

$$\int_0^\infty K_0(ax)J_3(bx)\,x\,dx=\frac{1}{b^2k}\,[(8-5k^2)K(k)-(8-k^2)E(k)]. \qquad (13)$$

$$\int_0^\infty K_0(ax)J_3(bx)\,x^3dx=\frac{k}{b^4}\,[(8+3k^2+4k^4)K(k)-(8+7k^2+8k^4)\,E(k)]. \qquad (14)$$

$$\int_0^\infty K_0(ax)J_3(bx)\ x^5dx=\frac{k^3}{b^6}\ [(8+19k^2+60k^4-192k^6)K(k)-(8+23k^2+72k^4-384k^6)E(k)]. \quad (15)$$

$$\int_0^\infty K_0(ax)J_3(bx)\ x^{-1}dx=\frac{1}{9k^3}\ [(8-5k^2)K(k)-(8-k^2)E(k)]. \quad (16)$$

$$\int_0^\infty K_0(ax)J_3(bx)\ x^{-3}dx=\frac{b^2}{225k^5}\ [(8+3k^2+4k^4)K(k)-(8+7k^2+8k^4)E(k)]. \quad (17)$$

$$\int_0^\infty K_1(ax)J_0(bx)\ x\ dx=\frac{k}{ab}\ E(k). \quad (18)$$

$$\int_0^\infty K_1(ax)J_0(bx)\ x^3dx=\frac{k^3}{ab^3}\ [(7-8k^2)E(k)-4(1-k^2)K(k)]. \quad (19)$$

$$\int_0^\infty K_1(ax)J_0(bx)\ x^5dx=\frac{3k^5}{ab^5}\ [(43-168k^2+128k^4)E(k)-4(1-k^2)\ (7-16k^2)K(k)]. \quad (20)$$

$$\int_0^\infty K_1(ax)J_0(bx)\ x^7dx=\frac{15k^7}{ab^7}\ [(337-2784k^2+5504k^4-3072k^6)E(k)$$
$$-8(1-k^2)\ (29-176k^2+192k^4)K(k)]. \quad (21)$$

$$\int_0^\infty K_1(ax)J_1(bx)\ dx=\frac{1}{ak}\ [E(k)-(1-k^2)K(k)]. \quad (22)$$

$$\int_0^\infty K_1(ax)J_1(bx)\ x^2dx=\frac{k}{ab^2}\ [(1-k^2)K(k)-(1-2k^2)\ E(k)]. \quad (23)$$

$$\int_0^\infty K_1(ax)J_1(bx)\ x^4dx=\frac{3k^3}{ab^4}\ [(1-8k^2)(1-k^2)K(k)-(1-16k^2+16k^4)E(k)]. \quad (24)$$

$$\int_0^\infty K_1(ax)J_1(bx)\ x^6dx=\frac{15k^5}{ab^6}\ [(1-k^2)(3-80k^2+128k^4)K(k)-(3-134k^2+384k^4-256k^6)E(k)]. \quad (25)$$

$$\int_0^\infty K_1(ax)J_2(bx)\ x\ dx=\frac{1}{abk}\ [(2-k^2)E(k)-2(1-k^2)K(k)]. \quad (26)$$

$$\int_0^\infty K_1(ax)J_2(bx)\ x^3dx=\frac{k}{ab^3}\ [2(1-k^2)(1+2k^2)K(k)-(2+3k^2-8k^4)E(k)]. \quad (27)$$

$$\int_0^\infty K_1(ax)J_2(bx)\ x^5dx=\frac{3k^3}{ab^5}\ [2(1-k^2)(1+6k^2-32k^4)K(k)-(2+11k^2-136k^4+128k^6)E(k)]. \quad (28)$$

$$\int_0^\infty K_1(ax)J_2(bx)\ x^{-1}dx=\frac{b}{3ak^3}\ [(2-k^2)E(k)-2(1-k^2)\ K(k)]. \quad (29)$$

$$\int_0^\infty K_1(ax)\ J_3(bx)\ dx=\frac{1}{3ak^3}\ [(8-7k^2)E(k)-(8-3k^2)\ (1-k^2)K(k)]. \quad (30)$$

$$\int_0^\infty K_1(ax)\ J_3(bx)\ x^2dx=\frac{1}{ab^2k}\ [(8-3k^2-2k^4)E(k)-(1-k^2)\ (8+k^2)K(k)]. \quad (31)$$

$$\int_0^\infty K_1(ax)\ J_3(bx)\ x^4dx=\frac{k}{ab^4}\ [(1-k^2)\ (8+13k^2+24k^4)K(k)-(8+9k^2+16k^4-48k^6)E(k)]. \quad (32)$$

$$\int_0^\infty K_1(ax)J_3(bx)\,x^{-2}dx=\frac{b^2}{45ak^5}\,[(8-3k^2-2k^4)E(k)-(1-k^2)\,(8+k^2)K(k)]. \tag{33}$$

$$\int_0^\infty K_2(ax)J_0(bx)\,x^2dx=\frac{k}{a^2b}\,[2(2-k^2)E(k)-(1-k^2)K(k)]. \tag{34}$$

$$\int_0^\infty K_2(ax)J_0(bx)\,x^4dx=\frac{k^3}{a^2b^3}\,[2(19-44k^2+24k^4)E(k)-(1-k^2)\,(23-24k^2)K(k)]. \tag{35}$$

$$\int_0^\infty K_2(ax)J_0(bx)\,x^6dx=\frac{3k^5}{a^2b^5}\,[2(158-923k^2+1408k^4-640k^6)\times E(k)-(1-k^2)(211-848k^2+640k^4)K(k)]. \tag{36}$$

$$\int_0^\infty K_2(ax)J_1(bx)\,x\,dx=\frac{1}{a^2k}\,[(1+k^2)E(k)-(1-k^2)K(k)]. \tag{37}$$

$$\int_0^\infty K_2(ax)J_1(bx)\,x^3dx=\frac{k}{a^2b^2}\,[(1-k^2)(3-4k^2)K(k)-(3-13k^2+8k^4)E(k)]. \tag{38}$$

$$\int_0^\infty K_2(ax)J_1(bx)\,x^5dx=\frac{3k^3}{a^2b^4}\,[(1-k^2)(5-68k^2+64k^4)K(k)-(5-123k^2+248k^4-128k^6)E(k)]. \tag{39}$$

$$\int_0^\infty K_2(ax)J_2(bx)\,dx=\frac{b}{3a^2k^3}\,[(1-k^2)(2-3k^2)K(k)-2(1-2k^2)E(k)]. \tag{40}$$

$$\int_0^\infty K_2(ax)J_2(bx)\,x^2dx=\frac{1}{a^2bk}\,[2(1-k^2+k^4)E(k)-(1-k^2)(2-k^2)K(k)]. \tag{41}$$

$$\int_0^\infty K_2(ax)J_2(bx)\,x^4dx=\frac{3k}{a^2b^3}\,[(1-k^2)(2+5k^2-8k^4)K(k)-2(1-2k^2)(1+4k^2-4k^4)E(k)]. \tag{42}$$

$$\int_0^\infty K_2(ax)J_2(bx)\,x^6dx=\frac{15k^3}{a^2b^5}\,[(2+15k^2-144k^4+128k^6)\,(1-k^2)K(k)-2(1+7k^2-135k^4+256k^6-128k^8)E(k)]. \tag{43}$$

$$\int_0^\infty K_2(ax)J_3(bx)\,x\,dx=\frac{1}{3a^2k^3}\,[(1-k^2)\,(8-9k^2)K(k)-(8-13k^2+3k^4)E(k)]. \tag{44}$$

$$\int_0^\infty K_2(ax)J_3(bx)\,x^3dx=\frac{1}{a^2b^2k}\,[(1+k^2)\,(8-13k^2+8k^4)E(k)-(1-k^2)\,(8-k^2-4k^4)K(k)]. \tag{45}$$

$$\int_0^\infty K_2(ax)J_3(bx)\,x^{-1}dx=\frac{b^2}{15a^2k^5}\,[(1-k^2)\,(8-9k^2)K(k)-(8-13k^2+3k^4)E(k)]. \tag{46}$$

$$\int_0^\infty K_3(ax)J_0(bx)\,x^3dx=\frac{k}{a^3b}\,[(23-23k^2+8k^4)E(k)-4(2-k^2)\,(1-k^2)K(k)]. \tag{47}$$

$$\int_0^\infty K_3(ax)J_0(bx)\,x^5dx=\frac{k^3}{a^3b^3}\,[(281-985k^2+1080k^4-384k^6)E(k)-4(1-k^2)\,(44-93k^2+48k^4)K(k)]. \tag{48}$$

$$\int_0^\infty K_3(ax)J_1(bx)\, x^2 dx = \frac{1}{a^3 k}\,[(3+7k^2-2k^4)E(k) - (1-k^2)\,(3+k^2)K(k)]. \tag{49}$$

$$\int_0^\infty K_3(ax)J_1(bx)\, x^4 dx = \frac{k}{a^3 b^2}\,[(15-43k^2+24k^4)(1-k^2)K(k)$$
$$-(15-103k^2+128k^4-48k^6)E(k)]. \tag{50}$$

$$\int_0^\infty K_3(ax)J_2(bx)\, x\, dx = \frac{b}{3a^3k^3}\,[2(1-k^2)\,(1-3k^2)K(k) - (2-7k^2-3k^4)E(k)]. \tag{51}$$

$$\int_0^\infty K_3(ax)J_2(bx)\, x^3 dx = \frac{1}{a^3bk}\,[(6-9k^2+19k^4-8k^6)E(k) - 2(1-k^2)\,(3-3k^2+2k^4)K(k)]. \tag{52}$$

$$\int_0^\infty K_3(ax)J_3(bx)\, dx = \frac{b^2}{15a^3k^5}\,[(8-23k^2+23k^4)E(k) - (1-k^2)\,(8-19k^2+15k^4)K(k)]. \tag{53}$$

$$\int_0^\infty K_3(ax)J_3(bx)\, x^2 dx = \frac{1}{3a^3k^3}\,[(8-15k^2+3k^4)\,(1-k^2)K(k) - (8-19k^2+9k^4-6k^6)E(k)]. \tag{54}$$

$$\int_0^\infty K_4(ax)J_0(bx)\, x^4 dx = \frac{k}{a^4 b}\,[8(11-11k^2+6k^4)$$
$$\times(2-k^2)E(k) - (1-k^2)\,(71-71k^2+24k^4)K(k)]. \tag{55}$$

$$\int_0^\infty K_4(ax)J_1(bx)\, x^3 dx = \frac{1}{a^4 k}\,[(15+58k^2-33k^4+8k^6)E(k) - (1-k^2)\,(15+13k^2-4k^4)K(k)]. \tag{56}$$

$$\int_0^\infty K_4(ax)J_2(bx)\, x^2 dx = \frac{b}{a^4k^3}\,[(1-k^2)\,(2-9k^2-k^4)K(k) - 2(1+k^2)\,(1-6k^2+k^4)E(k)]. \tag{57}$$

$$\int_0^\infty K_4(ax)J_3(bx)\, x\, dx = \frac{b^2}{15a^4k^5}\,[(8-33k^2+58k^4+15k^6)$$
$$\times E(k) - (1-k^2)\,(8-29k^2+45k^4)K(k)]. \tag{58}$$

6.4. $k^2 = \left(\frac{b}{\sqrt{a^2+b^2}+a}\right)^2$

$$\int_0^\infty K_0(2ax)J_0^2(bx)\, dx = \frac{2k}{\pi b}\,[K(k)]^2. \qquad \text{OB 153 (2.37)} \tag{1}$$

$$\int_0^\infty K_0(2ax)J_1^2(bx)\, dx = \frac{2}{\pi bk}\,[K(k)-E(k)]^2. \tag{2}$$

$$\int_0^\infty K_0(2ax)J_2^2(bx)\, dx = \frac{2}{9\pi bk^3}\,[(2+k^2)K(k) - 2(1+k^2)E(k)]^2. \tag{3}$$

$$\int_0^\infty K_0(2ax)J_3^2(bx)\, dx = \frac{2}{225\pi bk^5}\,[(8+3k^2+4k^4)K(k) - (8+7k^2+8k^4)E(k)]^2. \tag{4}$$

$$\int_0^\infty K_1(2ax)J_0^2(bx)\, x\, dx = \frac{1}{2\pi a\sqrt{a^2+b^2}}\,K(k)[2E(k) - (1-k^2)K(k)]. \tag{5}$$

$$\int_0^\infty K_1(2ax)J_1^2(bx)\, x\, dx = \frac{1}{2\pi k^2 a\sqrt{a^2+b^2}}\,[K(k)-E(k)][(1+k^2)E(k) - (1-k^2)K(k)]. \tag{6}$$

$$\int_0^\infty K_1(2ax)J_2^2(bx)\,x\,dx = \frac{1}{6\pi k^4 a\sqrt{a^2+b^2}}\,[2(1-k^2+k^4)E(k)-(1-k^2)(2-k^2)K(k)]$$
$$\times[(2+k^2)K(k)-2(1+k^2)E(k)]. \qquad (7)$$

$$\int_0^\infty K_2(2ax)J_1^2(bx)\,dx = \frac{b}{6\pi a^2 k^3}\,[(1+k^2)E(k)-(1-k^2)K(k)]^2. \qquad (8)$$

$$\int_0^\infty K_2(2ax)J_2^2(bx)dx = \frac{b}{30\pi a^2 k^5}\,[2(1-k^2+k^4)E(k)-(1-k^2)(2-k^2)K(k)]^2. \qquad (9)$$

$$\int_0^\infty K_2(2ax)J_3^2(bx)dx = \frac{b}{630\pi a^2 k^7}\,[(8-13k^2+8k^4)(1+k^2)E(k)$$
$$-(1-k^2)(8-k^2-4k^4)K(k)]^2. \qquad (10)$$

$$\int_0^\infty K_3(2ax)J_1^2(bx)xdx = \frac{b^2}{24\pi k^4 a^3\sqrt{a^2+b^2}}\,[(1+14k^2+k^4)E(k)$$
$$-(1-k^2)(1+7k^2)K(k)][(1+k^2)E(k)-(1-k^2)K(k)]. \qquad (11)$$

6.5. $k^2 = \left(\frac{b}{\sqrt{a^2+b^2}+a}\right)^2$

$$\int_0^\infty K_0^2(ax)J_0(2bx)dx = \frac{2k}{b}\,[K(k)]^2. \qquad \text{OB 23 (2.117)} \qquad (1)$$

$$\int_0^\infty K_0^2(ax)J_1(2bx)xdx = \frac{1}{b\sqrt{a^2+b^2}}\,K(k)[K(k)-E(k)]. \qquad (2)$$

$$\int_0^\infty K_0^2(ax)J_2(2bx)dx = \frac{2}{bk}\,[K(k)-E(k)]^2. \qquad (3)$$

$$\int_0^\infty K_0^2(ax)J_3(2bx)xdx = \frac{1}{bk^2\sqrt{a^2+b^2}}\,[K(k)-E(k)][(2+k^2)K(k)-2(1+k^2)E(k)]. \qquad (4)$$

$$\int_0^\infty K_0^2(ax)J_4(2bx)dx = \frac{2}{9bk^3}\,[(2+k^2)K(k)-2(1+k^2)E(k)]^2. \qquad (5)$$

$$\int_0^\infty K_1^2(ax)J_1(2bx)xdx = \frac{b}{4k^2a^2\sqrt{a^2+b^2}}$$
$$\times[(1+k^2)E(k)-(1-k^2)K(k)][2E(k)-(1-k^2)K(k)]. \qquad (6)$$

$$\int_0^\infty K_1^2(ax)J_2(2bx)dx = \frac{b}{6k^3a^2}\,[(1+k^2)E(k)-(1-k^2)K(k)]^2. \qquad (7)$$

6.6 $k^2=\left(\frac{b}{\sqrt{a^2+b^2}+a}\right)^2$

$$\int_0^\infty K_0(2ax)J_1(bx)J_0(bx)xdx=\frac{\sqrt{a^2+b^2}}{\pi a^2 b}K(k)[K(k)-E(k)]. \tag{1}$$

$$\int_0^\infty K_0(2ax)J_2(bx)J_1(bx)xdx=\frac{\sqrt{a^2+b^2}}{\pi a^2 bk^2}[K(k)-E(k)][(2+k^2)K(k)-2(1+k^2)E(k)]. \tag{2}$$

$$\int_0^\infty K_0(2ax)J_3(bx)J_2(bx)xdx=\frac{\sqrt{a^2+b^2}}{9\pi a^2 bk^4}$$
$$\times[(2+k^2)K(k)-2(1+k^2)E(k)][(8+3k^2+4k^4)K(k)-(8+7k^2+8k^4)E(k)]. \tag{3}$$

$$\int_0^\infty K_2(2ax)J_1(bx)J_0(bx)xdx=\frac{b\sqrt{a^2+b^2}}{4\pi a^4k^2}[(1+k^2)E(k)-(1-k^2)K(k)][2E(k)-(1-k^2)K(k)]. \tag{4}$$

$$\int_0^\infty K_2(2ax)J_2(bx)J_1(bx)xdx=\frac{b\sqrt{a^2+b^2}}{12\pi a^4k^4}$$
$$\times[(1+k^2)E(k)-(1-k^2)K(k)][2(1-k^2+k^4)E(k)-(1-k^2)(2-k^2)K(k)]. \tag{5}$$

$$\int_0^\infty K_2(2ax)J_3(bx)J_2(bx)xdx=\frac{b\sqrt{a^2+b^2}}{60\pi a^4k^6}[2(1-k^2+k^4)E(k)-(1-k^2)(2-k^2)K(k)]$$
$$\times[(1+k^2)(8-13k^2+8k^4)E(k)-(1-k^2)(8-k^2-4k^4)K(k)]. \tag{6}$$

6.7. $k^2=\left(\frac{b}{\sqrt{a^2+b^2}+a}\right)^2$

$$\int_0^\infty K_1(ax)K_0(ax)J_0(2bx)xdx=\frac{1}{2a\sqrt{a^2+b^2}}K(k)[2E(k)-(1-k^2)K(k)]. \tag{1}$$

$$\int_0^\infty K_1(ax)K_0(ax)J_2(2bx)xdx=\frac{1}{2k^2a\sqrt{a^2+b^2}}[K(k)-E(k)][(1+k^2)E(k)-(1-k^2)K(k)]. \tag{2}$$

$$\int_0^\infty K_1(ax)K_0(ax)J_4(2bx)xdx=\frac{1}{6k^4a\sqrt{a^2+b^2}}[(2+k^2)K(k)-2(1+k^2)E(k)]$$
$$\times[2(1-k^2+k^4)E(k)-(1-k^2)(2-k^2)K(k)]. \tag{3}$$

$$\int_0^\infty K_2(ax)K_1(ax)J_2(2bx)xdx=\frac{b^2}{24k^4a^3\sqrt{a^2+b^2}}[(1+k^2)E(k)-(1-k^2)K(k)]$$
$$\times[(1+14k^2+k^4)E(k)-(1-k^2)(1+7k^2)K(k)]. \tag{4}$$

$$\int_0^\infty K_3(ax)K_2(ax)J_4(2bx)xdx=\frac{b^4}{3360k^8a^5\sqrt{a^2+b^2}}[(1-k^2)(2-9k^2-k^4)K(k)$$
$$-2(1+k^2)(1-6k^2+k^4)E(k)][(1-k^2)(2-21k^2-108k^4-k^6)K(k)$$
$$-2(1-11k^2-108k^4-11k^6+k^8)E(k)]. \tag{5}$$

6.8. $k^2 = \left(\frac{a^2 - b^2}{a^2 + b^2}\right)^2$

$$\int_0^\infty K_0(ax)K_0(bx)J_0(ax)J_0(bx)xdx = \frac{1}{2(a^2+b^2)} K(k). \quad (1)$$

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7. References

For preparation of this paper the author referred to general expressions of these types of integrals in terms of hypergeometric functions or Legendre functions. The section numbers and the general expressions referred there are:

2.1 WA 385 (2)(3),ET II 29 (6); 2.2 deduced from ET I 196 (12); 2.3,2.4 LU 319 (28); 2.5,2.6 WA 389 (1)(2), LU 319 (26)(27); 2.7 WA 401 (2),407 (1); 2.8 ET II 52 (33); 3.1 WA 385 (4),ET II 105 (2); 3.3 ET I 332 (37); 4.1 GW 199 (6a),OL 149 (15.8); 5.1 ET II 131 (23); 5.2,5.3 ET II 145 (49); 6.1 deduced from ET I 198 (30); 6.2,6.3 WA 410 (1); 6.4 ET I 138 (18)(19); 6.5 ET II 66 (27)(28); 6.6 ET II 138 (20); 6.7 ET II 67 (29); 6.8 ET II 373 (10).

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9. Supplementary References

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