JOURNAL OF RESEARCH of the National Bureau of Standards—A. Physics and Chemistry
Vol. 78A, No. 4, July–August 1974

Theoretical and Experimental Compton Scattering Cross Sections at 1.12 MeV in the Case of Strongly Bound *K*-Shell Electrons*

P. N. Baba Prasad and P. P. Kane****

(March 4, 1974)

Measurements are reported for the differential cross sections for Compton scattering of 1.12 MeV gamma rays by the *K*-shell electrons of tin, tantalum, gold, and thorium. A few discrepancies between approximate theoretical calculations and the experimental results for different energies are pointed out. The need for an exact relativistic calculation is indicated.

Key words: Compton scattering; differential cross section; electron binding; gamma rays; *K*-shell; photons.

The Compton scattering of gamma rays by strongly bound atomic *K*-shell electrons has been studied experimentally at 0.279 MeV [1],[1] at 0.320 MeV [2], very intensively at 0.662 MeV [3 . . . 11] and at an average energy of 1.002 MeV [12]. We have given preliminary reports on similar measurements made with a zinc-65 source at an energy of 1.12 MeV [13].

Motz and Missoni [3] suggested that the results for large momentum transfers can be understood within the framework of an impulse approximation. Recently, a justification [14] of this approximation within the framework of nonrelativistic quantum mechanics has been given. If the target electron momentum $\mathbf{p}$ makes angles α and α' with the momenta $\boldsymbol{\eta}$ and $\boldsymbol{\eta}'$ of the incident and the scattered photons respectively, the energy k of the photon scattered through an angle θ is given by eq (1).

$$\frac{k}{k_0}=\frac{1-\beta\cos\alpha}{k_0E^{-1}(1-\cos\theta)+(1-\beta\cos\alpha')} \tag{1}$$

where k_0 is the incident photon energy, β is the ratio of the initial electron velocity to the velocity of light and E is the sum of the rest energy and the kinetic energy T of the target electron. In view of the virial theorem, a convenient measure of T is taken[2] to be the binding energy of the electron. Unfortunately, in the work of Jauch and Rohrlich [15], there was a misprint so that α appeared in the denominator instead of α'. The final result for the cross section $\left(\frac{d\sigma_K}{d\Omega}\right)_{\text{Imp.}}$ is obtained in terms of an average, as in eq (2).

$$\left(\frac{d\sigma_K}{d\Omega}\right)_{\text{Imp.}}=\frac{\int_{-1}^{+1}\int_{-1}^{+1}\left(\frac{d\sigma}{d\Omega}\right)d(\cos\alpha)\,d(\cos\phi)}{\int_{-1}^{+1}\int_{-1}^{+1}d(\cos\alpha)\,d(\cos\phi)} \tag{2}$$

where ϕ is the angle between the planes formed by the pairs of vectors $(\boldsymbol{\eta}, \mathbf{p})$ and $(\boldsymbol{\eta}', \mathbf{p})$ and the relativistic expression for $d\sigma/d\Omega$ in the case of a free but fast electron has been given by Jauch and Rohrlich.

In the calculation of Motz and Missoni for 0.662 MeV, the misprinted formula with α instead of α' was used and so the average over ϕ was not evaluated. On the basis of the correct eq (2), Shimizu et al. [7] proceeded to derive an extremely complicated expression in closed form. A graphical integration of (2) was performed by Ramalinga Reddy et al. [2, 12] for 0.320 MeV and 1.002 MeV gamma rays. However, on the basis of a repetition of their calculations, it appears to us that they might have used the misprinted formula. Pingot [9] evaluated the averages for 0.662 MeV by a numerical integration procedure. His theoretical values for gold were in disagreement with the experimental data and with the first two calculations.

In a few recent reports [1, 9, 10], an attempt has been made to compare the experimental results for $d\sigma_K/d\sigma_{KN}$ at all angles with the calculated values of the incoherent scattering function S_K for *K*-shell electrons. Here, $d\sigma_K/d\sigma_{KN}$ is the ratio R of the scatter-

*Work supported in part by a grant from the National Bureau of Standards, Washington, D.C. under the PL–480 programme.
**Present address: Physics Department, Indian Institute of Technology, Powai, Bombay–76, India.
[1] Figures in brackets indicate the literature references at the end of this paper.
[2] According to the virial theorem, the average kinetic energy of an electron bound by the attractive Coulomb potential in an atom turns out to be equal to the negative of half the average potential energy. Therefore, the average kinetic energy is the negative of the sum of the potential and the kinetic energies. Thus, the kinetic energy may be put equal to the binding energy of the electron.

ing cross section of an electron bound in the K-shell to the Klein-Nishina prediction for an electron initially free and at rest.[3] On the basis of a nonrelativistic treatment and the incoherent scattering function approximation, Shimizu et al. [7] have given an analytical expression for S_K, which should be really valid only for small momentum transfers. But the same expression was used in the above-mentioned reports even at large scattering angles. The resulting S_K values for gold at 0.279 MeV and 0.662 MeV were in approximate agreement with the measured values of the ratio R. However, on the other hand, the data for lead at 0.662 MeV do not agree well with the S_K values computed in this way or according to an earlier relativistic treatment [4].

We have repeated the numerical evaluation associated with eq 2 and with the well-known Klein-Nishina formula for $\frac{d\sigma_{KN}}{d\Omega}$, for five gamma energies including 1.12 MeV. The resulting estimates for the ratio $R_{\mathrm{Imp.}}$ are less than unity at all angles and are in disagreement with several experimental values of R at 0.279 MeV, 0.320 MeV, 0.662 MeV, and 1.002 MeV. Some of the new experimental results for R at 1.12 MeV, outlined briefly in the following paragraph, are also in disagreement with the theoretical estimates of $R_{\mathrm{Imp.}}$. Our estimates for a gold scatterer and 0.662 MeV gamma rays agree extremely well with those reported by Pingot and thus confirm the correctness of the estimation procedure.

The experimental results at 1.12 MeV were obtained in a conventional coincidence experiment with 14.9 mg/cm² thorium, 12.9 mg/cm² gold, 22.1 mg/cm² tantalum and 18.8 mg/cm² tin scatterers. The bias level in the gamma channel was chosen to correspond to about one third of the photon energy calculated for the Compton scattering from a free and stationary electron. However, an additional consideration determined the final choice. It was desirable to keep the bias substantially above the target atom K_α x-ray energy in order to minimize the spurious coincidences arising from K x-rays registering in the gamma detector and the scattered gamma rays simultaneously registering in the thin x-ray detector. The final bias values were 0.300 MeV, 0.165 MeV, 0.165 MeV, and 0.100 MeV in the case of measurements made at scattering angles of 25, 60, 90, and 120° respectively except that for thorium at 120°, the bias was 0.165 MeV. Measurements of random coincidences were made as usual. In addition, counts due to the natural radioactivity of the thorium foil were determined in an auxiliary experiment. The experimental results obtained by us are summarized in the third column of table 1. The values estimated according to

TABLE 1. *Experimental and theoretical values of* $\mathrm{R}=\frac{d\sigma_K}{d\sigma_{KN}}$ *for 1.12 MeV gamma rays from a zinc-65 source*

Element	θ	$R_{\mathrm{expt.}}$	$R_{\mathrm{Imp.}}$ [a]	$S_{K(\mathrm{Shimizu})}$ [b]
	25°	0.39 ± 0.13	0.896	0.420
	60°	$.96 \pm 0.13$	.918	1.149
Thorium	90°	1.35 ± 0.13	.934	1.489
14.9 mg/cm²	120°	0.92 ± 0.18	.869	1.655
	25°	$.67 \pm 0.09$	.925	0.560
	60°	$.82 \pm 0.12$	.937	1.575
Gold	90°	1.39 ± 0.12	.952	2.046
12.9 mg/cm²	120°	1.07 ± 0.16	.898	2.277
Tantalum 22.1 mg/cm²	120°	0.96 ± 0.26	.914	2.735
	25°	$.78 \pm 0.07$	.967	1.491
Tin	60°	$.97 \pm 0.08$	.975	4.406
18.8 mg/cm²	90°	1.09 ± 0.09	.985	5.765

[a] The values of $R_{\mathrm{Imp.}}$ are estimated for K-shell electrons on the basis of eqs (1) and (2) in the text, and the Klein-Nishina formula.

[b] S_K is computed from an equation given by Shimizu et al (reference [7]). In their equation, b and not b^2 should be put equal to $Z\frac{\hbar}{a_0}$.

impulse approximation mentioned above are listed in the fourth column. S_K values calculated according to the formula of Shimizu et al. are tabulated in the last column. It should be noted that several $S_{K(\mathrm{Shimizu})}$ values are larger than unity and much larger than the

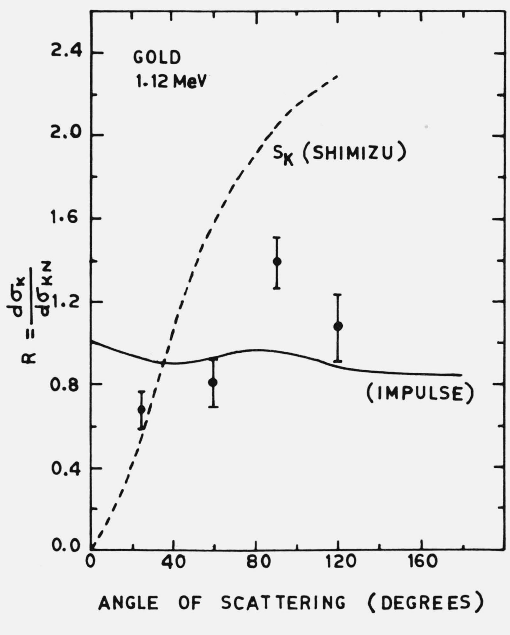

FIGURE 1. *Angular distribution of the cross section ratio* $\mathrm{R}=\frac{d\sigma_K}{d\sigma_{KN}}$.

ϕ—Present experimental results at 1.12 MeV for a gold scatterer.
.... S_K calculated according to the formula of Shimizu et al.
———= The theoretical values of $R_{\mathrm{Imp.}}$ calculated according to eqs (1) and (2), and the Klein-Nishina formula.

As pointed out in the text, the impulse approximation is expected to be valid for large transfers; whereas the $S_{K\,(\mathrm{Shimizu})}$ formula has been derived in a nonrelativistic treatment and for small momentum transfers.

[3] The two symbols R and S_K are introduced, for the purposes of this paper, to distinguish between (1) an operational definition of the ratio R of bound-electron incoherent scattering to free-electron (Klein-Nishina) scattering obtained either from (a) experiment or (b) a theoretical model not restricted to small scattering angles, and (2) $S(q,Z)$ (here S_K for K-shell electrons only), the same ratio, but defined as the probability of atomic excitation or ionization resulting from any impulsive action imparting a recoil momentum q to an atomic electron (see, e.g., Grodstein [16], Evans [17], Davisson [18], Hubbell [19], or Storm and Israel [20]), calculated up to now from models claiming validity only for applications to small angles.

corresponding experimental values of R. In figure 1, the experimental results for a gold scatterer are compared with the theoretical values computed according to the above mentioned approximations.

The detailed spectral distributions of 1.12 MeV photons Compton scattered by the K-shell electrons of different atoms are being determined at present. The results of this study will be reported in a later communication.

For 0.662 MeV gamma rays, a relativistic calculation [16] employing Dirac eigenfunctions for the initial and final electron states and the bound electron propagator for intermediate states has been done. It is clearly necessary to extend this type of calculation to other gamma ray energies. A similar suggestion has been made recently by other workers [17].

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(Paper 78A4–826)

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