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Normal Emissivity of an Isothermal, Diffusely Reflecting Cylindrical Cavity (With Top) as a Function of Inside Radius

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The normal emissivity of an isothermal cylindrical cavity (with top), with a diffusely reflecting interior of reflectivity much less than 1, is calculated approximately as a function of the inside radius of the cylinder by the DeVos method. The calculation is analytical, and considers the singly and doubly reflected radiation escaping from the cavity aperture. The results of the analysis indicate that, for cylinders whose length-to-lid aperture ratio is much larger than 1: (a), for a given cylinder length and lid aperture, the configuration with the inside diameter approximately 0.64 the length has the *smallest* normal emissivity; (b), as the inside diameter increases or decreases from the configuration of smallest normal emissivity, the normal emissivity *increases* monotonically.

Key words: Cavity; cylindrical cavity; diffusely reflecting; emissivity; normal emissivity; radiation; reflectance.

1. Introduction

It is often important to establish design criteria for cavity radiators. A frequently used configuration is the cylindrical cavity with top (fig. 1). If the interior of the cavity is *isothermal*, the normal emissivity of the cylinder may be defined as the ratio of the normal radiance of the cylinder aperture to the normal radiance of a blackbody at the temperature of the interior. If the bidirectional reflectance characteristics of the cylinder interior are known, the DeVos method [1] [1] theoretically enables the normal emissivity of an isothermal cylinder to be computed from the sum of: (a), the radiation emitted parallel to the cylinder axis by the area of the base defined by the projection of the aperture (parallel to the axis) onto the base; (b), the radiation emitted by the interior of the cylinder which is incident upon the projection of the aperture onto the base, and then reflected out the cavity aperture parallel to the axis; (c), multiply reflected radiation which escapes from the cavity aperture in a direction parallel to the axis. That is, a DeVos analysis is a series expansion of the normal emissivity of a cavity in terms involving successively greater numbers of interreflections within the cavity. This series converges rapidly if the reflectance, *r*, of the cavity interior is much less than 1; therefore, the reflectance of the cylinder interior is assumed to satisfy this condition. To simplify the analysis further, it is also assumed that the interior of the cylinder reflects and emits diffusely. [2] (Many cavity radiator materials are approximately diffuse in their reflecting and emitting characteristics.)

Let e_{CN} be the normal emissivity of the cylinder and e the diffuse emissivity of the cylinder interior. Then e_{CN} may be written,

$$e_{CN}=e[1+rA_1+r^2A_2+\ .\ .\ .], \tag{1}$$

where A_1, A_2, etc., are coefficients which depend only on the cylinder configuration. From Kirchhoff's law [2] and the assumption that the cylinder walls are opaque, it is clear that

$$r_{CN}=1-e_{CN}, \tag{2}$$

and that

$$r_{CN}=rB_1+r^2B_2+\ .\ .\ ., \tag{3}$$

where

$$B_1=1-A_1,$$

$$B_2=A_1-A_2,$$

etc.

Quinn has shown [4] that, for the cylindrical cavity of figure 1 with length L, aperture radius R_1, and inside radius R_2,

$$B_1=2[\overline{L}^2+2+\overline{L}(\overline{L}^2+4)^{0.5}]^{-1}, \tag{4}$$

where

$$\overline{L}=L/R_1. \tag{5}$$

Quinn also shows that

$$B_2=16\overline{R}_2^3\int_0^{\overline{M}}d\overline{Y}(\overline{M}^2-\overline{Y}^2)$$

[1] Figures in square brackets indicate the literature references at the end of this paper.
[2] In this paper, the terms "diffuse" and "diffusely" refer to reflection or emission that follows Lambert's cosine law [3].

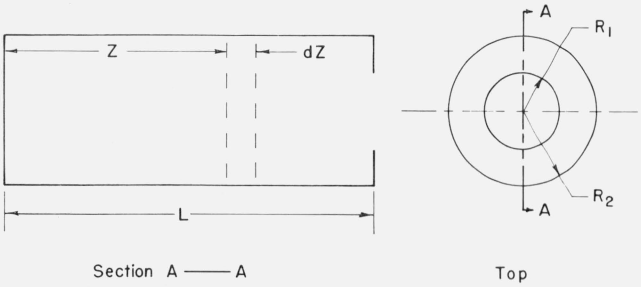

FIGURE 1. *Cross section and top view of cylindrical cavity:* L, *length of cylinder;* R₁, *radius of aperture;* R₂, *inside radius of cylinder;* Z, *height of differential element above base of cylinder.*

$$X[(\bar{M}+\bar{Y})^2+\bar{R}_2^2+1+([(\bar{M}+\bar{Y})^2+\bar{R}_2^2+1]^2 -4\bar{R}_2^2)^{0.5}]^{-1}$$

$$X[(\bar{M}-\bar{Y})^2+\bar{R}_2^2+1+([(\bar{M}-\bar{Y})^2+\bar{R}_2^2+1]^2 -4\bar{R}_2^2)^{0.5}]^{-1}$$

$$X[([(\bar{M}+\bar{Y})^2+\bar{R}_2^2+1]^2-4\bar{R}_2^2)^{0.5}]^{-1}$$

$$X[([(\bar{M}-\bar{Y})^2+\bar{R}_2^2+1]^2-4\bar{R}_2^2)^{0.5}]^{-1}, \quad (6)$$

where

$$\bar{R}_2 = R_2/R_1, \quad (7)$$

$$\bar{M} = \bar{L}/2, \quad (8)$$

$$\bar{Y} = \bar{Z} - \bar{M} \quad \text{(see fig. 1).} \quad (9)$$

2. Analysis

It is now assumed that $\bar{M}$ is much greater than 1; that is, the cylinder length $\bar{L}$ is much greater than the lid aperture, $2\bar{R}_1$. It is desired to compute B_2 for the two cases: (a), R_2 equals R_1; (b), R_2 much greater than R_1.

2.1. Calculation of B_2 for R_2 Equals R_1

Since $\bar{M}$ is much greater than 1, it is convenient to expand B_2 as a series in powers of $(\bar{M})^{-1}$. For R_2 equal to R_1, it is found that

$$B_2 = 4^{-1}(\bar{M})^{-3} + 2^{-1}(\bar{M})^{-4} + \ldots, \quad (10)$$

to order $(\bar{M})^{-4}$; that is, terms of order $(\bar{M})^{-5}$ or higher are discarded.

2.2. Calculation of B_2 for R_2 Much Greater Than R_1

If R_2 is much greater than R_1, Quinn shows [4] that B_2 may be approximated with fractional error roughly $2(\bar{M}\bar{R}_2^3)^{-1}$ by

$$B_2' = 4\bar{S}_2^3(\bar{M})^{-2}\int_0^1 d\bar{V}(1-\bar{V}^2)$$

$$X[(1+\bar{V})^2+\bar{S}_2^2]^{-2}[(1-\bar{V})^2+\bar{S}_2^2]^{-2}, \quad (11)$$

where

$$\bar{S}_2 = \bar{R}_2/\bar{M}, \quad (12)$$

$$\bar{V} = \bar{Z}/\bar{M} - 1 \quad \text{(see fig. 1).} \quad (13)$$

B_2' has the closed form solution,

$$B_2' = \bar{S}_2[16\bar{M}^2(\bar{S}_2^2+1)^3]^{-1}$$

$$X[4(1-\bar{S}_2^4) + \bar{S}_2^2(\bar{S}_2^4+3\bar{S}_2^2+6)\ln(1+4/\bar{S}_2^2)$$

$$+ 2\bar{S}_2^3(3-\bar{S}_2^2)\cot^{-1}(\bar{S}_2/2)]. \quad (14)$$

It is useful to define the function

$$F = B_2'\bar{M}^2, \quad (15)$$

which is *independent* of $\bar{M}$.

It is seen from eq (15) that

$$\lim_{\bar{S}_2\to 0} B_2' = 4^{-1}\bar{S}_2(\bar{M})^{-2} = 4^{-1}\bar{R}_2(\bar{M})^{-3}, \quad (16)$$

and that

$$\lim_{\bar{S}_2\to\infty} B_2' = (2/3)(\bar{S}_2^5\bar{M}^2)^{-1}. \quad (17)$$

Thus B_2' approaches 0 as $\bar{S}_2$ approaches 0 and as $\bar{S}_2$ becomes infinite; hence B_2' must have *at least one maximum* as $\bar{S}_2$ varies from 0 to infinity. Direct computation of B_2' (see table 1) establishes a maximum at

$$\bar{S}_{2\mathrm{MAX}} = 0.644. \quad (18)$$

The physical symmetry of the problem suggests that there is only 1 maximum of B_2' as $\bar{S}_2$ varies from 0 to infinity; however, this has *not* been proved mathematically.

TABLE 1. *The normalized, approximate second order DeVos reflection coefficient,* $F(\bar{S}_2)$, *for a diffusely reflecting cylindrical cavity with length much greater than the aperture (see fig. 1): for values of the ratio of the aperture to the length,* $\bar{S}_2$, *between 0.1 and 1.0*

$\bar{S}_2$	$F(\bar{S}_2)$
0.1	0.034350
.2	.070853
.4	.166720
.6	.208828
.644	.210088 (Maximum)
.8	.197426
1.0	.160336

3. Conclusions

From eq (3) for r_{CN}, eq (4) for B_1, and the preceding analysis of B_2, it is seen that:

a. B_1 is independent of R_2.
b. Since $\bar{M}$ is much greater than 1, to order $(\bar{M})^{-3}$,

$$B_2 \approx B_2' \approx 4^{-1}\bar{R}_2(\bar{M})^{-3}.$$

c. As R_2 varies from R_1 to $0.64R_1\overline{M}$ (for $\overline{M} \gg 1$), B_2 increases monotonically to its maximum value,

$$B_{2\mathrm{MAX}} \approx B'_{2\mathrm{MAX}} = 0.210(\bar{M})^{-2}.$$

d. As R_2 varies from $0.64R_1\overline{M}$ to infinity (for $\dot{M} \gg 1$), B_2 decreases monotonically to 0.

e. Since r is much less than 1 and $\overline{M}$ is much greater than 1, r_{CN} is approximately,

$$r_{\mathrm{CN}} \approx 4^{-1}r(\bar{M})^{-2}(1+4rB'_2\bar{M}^2).$$

f. As R_2 varies from R_1 to $0.64\mathrm{R}_1\overline{M}$ (for $M \gg 1$, $r \ll 1$), r_{CN} increases monotonically to its maximum value,

$$r_{\mathrm{CN(MAX)}} \approx 4^{-1}r(\bar{M})^{-2}(1+0.840r).$$

g. As R_2 varies from $0.64R_1\overline{M}$ to infinity (for $\overline{M} \gg 1$, $r \ll 1$), r_{CN} decreases monotonically to its minimum value.

$$r_{\mathrm{CN(MIN)}} \approx 4^{-1}r(\bar{M})^{-2}.$$

4. References

[1] DeVos, J. C., Evaluation of the quality of a blackbody, Physica **20,** 669 (1954).

[2] Jenkins, F. A., and White, H. E., Fundamentals of Optics, 2nd ed., p. 430 (McGraw-Hill Book Co., Inc., New York, N.Y., 1950).

[3] Walsh, J. W. T., Photometry, 3rd. ed., p. 137 (Dover Publications, Inc., New York, N.Y., 1958).

[4] Quinn, T. J., The calculation of the emissivity of cylindrical cavities giving near blackbody radiation, Brit. J. Appl. Phys. **18,** 1105 (1967).

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