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An Extension to the Sliding Short Method of Connector and Adaptor Evaluation*

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Sliding short methods represent a measurement tool of substantial potential for the measurement of small losses such as are associated with waveguide connectors or adaptors. Until recently, however, the use of these methods has been inhibited by the uncertainty of the error contribution due to non-ideal short behavior.

A recent analysis by Almassy has shown that by the use of proper techniques, the error contribution from this source is usually negligible, provided that the adaptor (or connector) is "well matched."

It is the purpose of this paper to eliminate this latter restriction, develop additional measurement methods, and describe further applications.

Key words: Adaptor; connector; efficiency; sliding short.

1. Introduction

A measurement problem, of continuing interest in the microwave art, is that of adaptor evaluation. With the advent of the *power equation* [1][1] methods and use of *terminal invariant parameters* [2], the dependence upon impedance properties is suppressed, and the dissipative characteristic emerges as the parameter of major interest. A similar observation may be made with regard to connectors. Here the dissipation at the connector interface, or more specifically, the lack of loss repeatability, represents a basic limitation to the attainable accuracy in many microwave measurements.

The application of sliding short methods, to this measurement problem, has been known for some time [3, 4]. These techniques were generalized in conjunction with the development of the power equation methods [1, 2]. Despite this revival of interest, however, the technique has been subject to a major limitation: an ideal sliding short is assumed.

In practice, of course, no such device exists. Moreover, the losses in the short may be of the same order (or larger) than those in the joint or adaptor to be evaluated. In the absence of further information, this loss represents a potential source of substantial error.

A recent analysis by Almassy [5] has shown that, with proper techniques, the sliding short losses can be substantially reduced, provided that the adaptor (or joint) reflection is small (i.e., $|S_{11}| \ll 1$, $|S_{22}| \ll 1$).

It is the purpose of this paper to eliminate this restriction, develop additional measurement methods, and describe further applications.

2. Background

The major thrust of this paper will be directed towards the measurement of the *efficiency* of an arbitrary two-port. In particular this two-port may be a connector, waveguide-coax adaptor, etc. By definition, the efficiency is the ratio of the (net) power output, to the (net) power input, and is a function of the load (but not of the source) impedance. Throughout this paper, the efficiency is assumed to be large (small losses). (In general the accuracy of this method decreases rapidly with losses greater than 10 dB.)

It will prove convenient to briefly review the existing theory. The recommended instrumentation takes the form shown in figure 1. The basic configuration will

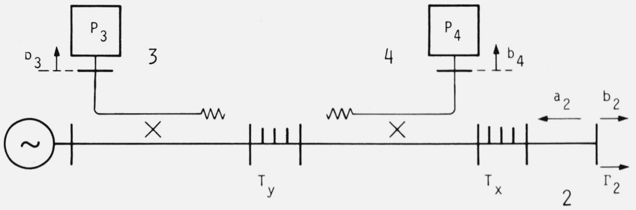

FIGURE 1. *Generalized reflectometer for use in efficiency measurement.*

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[1] Figures in brackets indicate the literature references at the end of this paper.

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be recognized as a generalized reflectometer (g-reflectometer), where the junction parameters are arbitrary except as subsequently noted. In particular, the tuning transformers T_x, T_y permit one to impose certain conditions on the measurement system. The device to be evaluated is connected to port 2, while arms 3 and 4 are terminated by power meters.

Although power meters have been specified for arms 3 and 4, it will prove convenient, momentarily, to assume an alternative detection scheme such that the complex ratio b_3/b_4 is measured, where b_3, b_4 are the emergent wave amplitudes. Let arm 2 be terminated by a moving short ($\Gamma_2 = e^{j\theta}$), and let the values $\left(\frac{b_3}{b_4}\right)$ be plotted in the complex plane as θ is permitted to vary. The resultant locus is a circle, an example of which is shown in figure 2. The parameters of this locus, in particular the radius of the circle, R, and the (absolute) distance, R_c, between its center and the origin, play a major role in the efficiency measurement.

Inspection of figure 2 indicates that [2]

$$\left|\frac{b_3}{b_4}\right|_{\max} = R + R_c, \tag{1}$$

$$\left|\frac{b_3}{b_4}\right|_{\min} = R - R_c. \tag{2}$$

and solving for R, R_c yields:

$$R = \frac{1}{2}\left(\left|\frac{b_3}{b_4}\right|_{\max} + \left|\frac{b_3}{b_4}\right|_{\min}\right), \tag{3}$$

$$R_c = \frac{1}{2}\left(\left|\frac{b_3}{b_4}\right|_{\max} - \left|\frac{b_3}{b_4}\right|_{\min}\right). \tag{4}$$

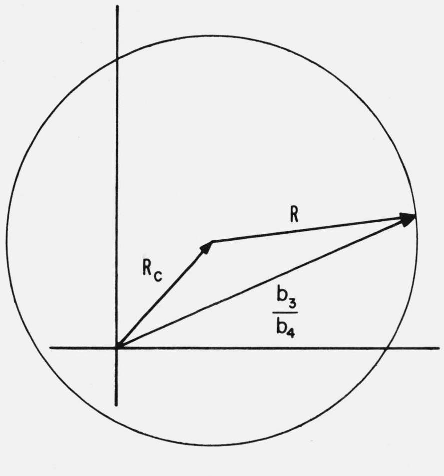

FIGURE 2. *Locus of* b_3/b_4 *as a function of* θ.

[2] It is assumed that the origin is within the circle. As long as the efficiency is high, this condition is assured. As the losses increase, however, it is possible that $R_c > R$. For a more complete discussion see [2].

The determination of R, R_c thus involves amplitudes only, the phase detection capability is not required. Finally, the power meters respond to the square of the amplitude such that $P_3 = |b_3|^2$ etc.

In order to make an efficiency measurement, the two-port is connected as shown in figure 3. Note that the designations have been chosen such that port 2 of the four arm junction mates with port 2 of the two-port. The "load" impedance, for which the efficiency is measured, is actually provided by the four arm junction.

To be more specific, it is convenient to postulate that P_4 is constant. (In practice this is often done by a "leveling" or feedback arrangement.) The source impedance, Γ_g, for the "equivalent" generator, which now obtains at port 2, depends upon T_x and the adjacent coupler [6], but is independent of T_y and the remaining coupler. The efficiency, which is obtained in the measurement described in the following paragraph, is that for an assumed power flow from terminal 1 to terminal 2, with the termination Γ_g. It will be denoted by $\eta_{21}(\Gamma_g)$.

The efficiency measurement calls for connecting a moving short to terminal 1 and determining the radius, R_1, of the resulting circular locus using (3). The two-port is then removed, and the operation repeated at terminal 2 to obtain R_2. It has been shown [2] that

$$\eta_{21}(\Gamma_g) = R_1/R_2. \tag{5}$$

In an alternative method, it is convenient (but not essential) to use the configuration shown in figure 4. Here the efficiency, $\eta_{12}(\Gamma_l)$, is measured for a power flow from terminal 2 to terminal 1, and for the terminating load Γ_l.

The measurement procedure now calls for the adjustment of T_x such that P_3 vanishes. The parameters, R, R_c, of the circular locus are then measured with the moving short connected to terminals 1 and 2. It will be shown (see appendix) that $\eta_{12}(\Gamma_l)$ is given by:

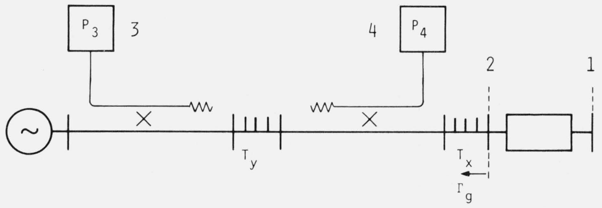

FIGURE 3. *G-reflectometer with two-port connected.*

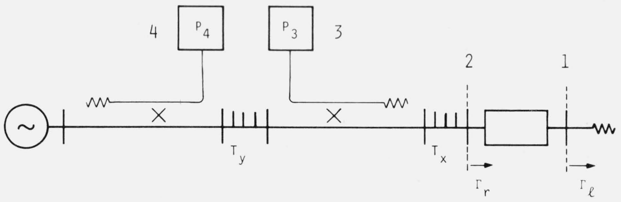

FIGURE 4. *Alternative configuration for* g-*reflectometer.*

$$\eta_{12}(\Gamma_l)=\frac{R_1}{R_2}\frac{\left(1-\left(\frac{R_{c1}}{R_1}\right)^2\right)}{\left(1-\left(\frac{R_c}{R_2}\right)^2\right)}. \tag{6}$$

It should be noted that the adjustment of T_x, in both cases, is dictated by the value of load impedance for which the efficiency is measured. Provided that the indicated configurations of couplers and tuners are used, the operation, in theory, is independent of the adjustment of T_y. Tuner T_y is useful, however, because of certain practical considerations which will be explained later. If the suggested coupler positions are interchanged, (i.e., the configuration of figure 4 is used to measure $\eta_{21}(\Gamma_g)$, or the configuration of figure 3 is used to measure $\eta_{12}(\Gamma_l)$) the indicated role of T_x is shared by T_y and the desired adjustments become interdependent and more difficult to realize.

3. Implementation

The application and implementation of these methods is perhaps best described in terms of a specific problem—an efficiency measurement of a waveguide joint or connector.

Referring to figure 5, it is convenient to first postulate a length of lossless waveguide lead, an ideal moving short, and an "ideal" reflectometer. (In terms of the description contained in (7), in the next section, an "ideal" reflectometer is one for which $b=c=0$.) The object is to measure the loss of the indicated waveguide joint. For reasons which will emerge, it is useful to record the reflectometer response as a function of short position. For the "ideal" system of figure 5 the expected response is that shown in the inset. This result is explained as follows.

As long as the short is to the left of the waveguide joint, the reflection coefficient presented to the reflectometer is of unit magnitude and variable phase. The system responds only to the magnitude; this accounts for the "straight" line section. With the short to the right, however, the joint losses will lead to reduced values for the reflection coefficient magnitude. In particular, this loss is determined by the longitudinal current component and thus will be a maximum when the short position is a multiple of a half-wavelength, and a minimum (zero [3]) when the position is an odd multiple of a quarter wavelength. In this case, the maximum and minimum values to the right of the joint are averaged, and the ratio of this average to the value left of the joint is the efficiency.

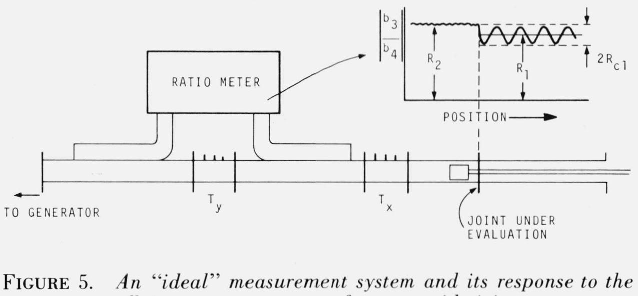

FIGURE 5. *An "ideal" measurement system and its response to the efficiency measurement of a waveguide joint.*

If this experiment is carried out in reality, a typical record is that shown in figure 6. The most obvious change is in the downward slope, this is caused by the waveguide loss. The measurement procedure now calls for a projection of the maxima and minima to the plane of the joint and the parameters R, R_c evaluated from these projections. The mathematical basis for this will be given in the following section.

Although an "ideal" reflectometer was assumed in figure 5, in principle it is possible to start with arbitrary couplers and provide only for the tuning adjustments specified in the preceding section. In general, depending upon the adjustment of T_y, the curve will show an oscillatory behavior on both sides of the joint. As a practical matter, it is generally desirable to adjust T_y such as to keep the amplitude of this oscillatory component within nominal limits. This point will be considered in greater detail in a following paragraph.

For an arbitrary two-port, it is not possible (in general) to make a continuous recording as shown here; in this case it is necessary to choose separate reference planes (usually coinciding with the waveguide flange) in the input and output arms; the determination of R, R_c follows from the projections of the maxima and minima to the reference planes as already described. At this point it is necessary to decide whether the associated joint losses are included as part of the two-port. In many cases the joint loss, or at least its reproducibility, limits the accuracy to which the two-port efficiency may be specified.

4. Analysis

The validity of the foregoing procedure was established by Almassy [5] under the condition that the two-port is "well-matched," (i.e., both $|S_{11}|$ and $|S_{22}|$ are much smaller than unity). The purpose of the present analysis is to demonstrate that the method loses very little of its accuracy, even when this condition is not satisfied.

In general the functional relationship between b_3/b_4, and a termination of reflection coefficient Γ, at either terminal 1 or terminal 2, is in the form:

$$\frac{b_3}{b_4}=\frac{a\Gamma+b}{c\Gamma+1}. \tag{7}$$

Here the complex parameters a, b, c depend upon both the reflectometer and the two-port when the termination is at terminal 1, but only on the former when the termination is at terminal 2. For an ideal short, $\Gamma=e^{j\theta}$ where θ is a function of short position.

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[3] This assumes the equivalent circuit of the joint is that of a small series resistance.

In order to account for non-ideal behavior, the model here assumed is:

$$\Gamma = re^{j\psi}e^{-2(\alpha+j\beta)l} \tag{8}$$

where r, ψ are the magnitude and phase of the plunger reflection, α, β are the loss and phase constants of the line in which the "short" moves, and l is its position with respect to the reference flange.

If (8) is substituted in (7), the resulting relation can be written as follows:

$$\frac{b_3}{b_4} = \frac{b - ac^* r^2 e^{-4\alpha l}}{1 - |c|^2 r^2 e^{-4\alpha l}} + \frac{(a - bc) re^{j\psi} e^{-2(\alpha + j\beta) l} (1 + c^* r e^{-j\psi} e^{-2(\alpha - j\beta) l})}{[1 - |c|^2 r^2 e^{-4\alpha l}] (1 + c r e^{j\psi} e^{-2(\alpha + j\beta) l})} \tag{9}$$

where (*) denotes the complex conjugate.

Although this expression is a complicated one, a great deal can be learned about the problem by inspection of this result. First, if the short is ideal, $r=1$, $\psi=\pi$, $\alpha=0$, and the expression becomes:

$$\frac{b_3}{b_4} = \frac{b - ac^*}{1 - |c|^2} - \frac{(a - bc) e^{-2j\beta l} (1 - c^* e^{2j\beta l})}{(1 - |c|^2)(1 - ce^{-2j\beta l})}. \tag{10}$$

Here it is noted that b_3/b_4 is the sum of two terms. The first is a constant (as l varies), while the second is of constant magnitude but variable phase. (Note that the last factors in the numerator and denominator of the second term are conjugates of each other.) This explicitly demonstrates the previously stated result: the locus of b_3/b_4 is a circle of radius

$$R = \left| \frac{a - bc}{1 - |c|^2} \right| \tag{11}$$

and distance to the center

$$R_c = \left| \frac{b - ac^*}{1 - |c|^2} \right| . \tag{12}$$

Next, it is desirable to return to the more general relationship contained in (9) and to consider the functional dependence of $|b_3/b_4|$ upon l. It is this relationship which is plotted in figure 6.

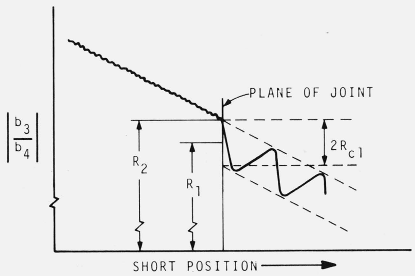

FIGURE 6. *"Typical" response in an actual measurement.*

In the discussion which follows, it will prove useful to consider not only the functional relationship of (9), but also the family of functions which is obtained from (9) for different values of ψ. Conceptually, one assumes the measurement is repeated a number of times, where the moving "shorts" have the same value for r, but differing values of ψ.

For a given l, the value of $|b_3/b_4|$ will depend upon ψ, and for some particular choice of ψ will have the maximum value:

$$\left| \frac{b_3}{b_4} \right|_{\max} = \frac{|a - bc| re^{-2\alpha l} + |b - ac^* r^2 e^{-4\alpha l}|}{1 - |c|^2 r^2 e^{-4\alpha l}}. \tag{13}$$

For some other choice of ψ, $|b_3/b_4|$ will have the minimum [4]:

$$\left| \frac{b_3}{b_4} \right|_{\min} = \frac{|a - bc| re^{-2\alpha l} - |b - ac^* r^2 e^{-4\alpha l}|}{1 - |c|^2 r^2 e^{-4\alpha l}} . \tag{14}$$

For each value of l, there will be some member of the family (value of ψ) for which these maximum and minimum values are realized, but these limits are never exceeded. Equations (13), (14) are evidently the envelope of the family. If this envelope is denoted by E, then:

$$E = \frac{|a - bc| re^{-2\alpha l} \pm |b - ac^* r^2 e^{-4\alpha l}|}{1 - |c|^2 r^2 e^{-4\alpha l}} . \tag{15}$$

Because of the oscillatory behavior of $|b_3/b_4|$, the envelope may be easily inferred from a single member of the family provided that several cycles, or more, have been recorded. From (15), the extrapolation of this envelope to the plane where $l=0$ yields:

$$E|_{l=0} = \frac{|a - bc| r \pm |b - ac^* r^2|}{1 - |c|^2 r^2} . \tag{16}$$

The use of this result to obtain approximate values for the efficiency will be evaluated in the following section. It is next instructive to consider the general behavior of E as l varies.

Returning to (15), the first term in the numerator tends to dominate the entire expression, and provides a simple exponential decay. The second term in the numerator determines the amplitude associated with the oscillatory behavior. If c vanishes, this amplitude is constant, while if b vanishes, the amplitude decays exponentially. If both these conditions obtain the oscillatory behavior is absent. The most distinctive situation occurs when b and ac^* are nominally equal. In this case there will be some value of l (which may or may not be physically realizable) for which this second term will be a minimum (possibly zero).

Finally, the second term in the denominator is usu-

[4] This assumes that the first term exceeds the second which, for the applications envisioned, is usually the case.

ally small[5] with respect to unity. As a whole the denominator increases as l increases, and the value of the complete expression decreases somewhat more rapidly than would otherwise be the case. As a rule, this effect is rather small. The most general form of the envelope is a distorted hyperbola as shown in figure 7.

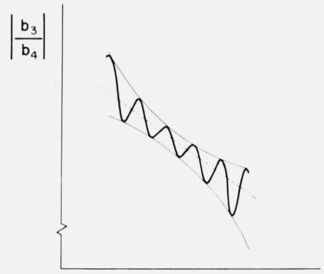

FIGURE 7. *General form of system response.*

5. Practical Considerations

As already noted, the measurement technique calls for recording $|b_3/b_4|$ as a function of l (short displacement). The envelope of this curve is then projected to the plane where $l=0$, and E_{max}, E_{min} substituted for $|b_3/b_4|$ in (3), (4). The resulting approximations to R, R_c are denoted by primes and given by:

$$R'=\frac{|a-bc|(1-\epsilon)}{1-|c|^2(1-\epsilon)^2} \tag{17}$$

$$R_c'=\frac{|b-ac^*(1-\epsilon)^2|}{1-|c|^2(1-\epsilon)^2}, \tag{18}$$

where $r=1-\epsilon$, and ϵ is usually a small quantity.

The evaluation of the error, e, due to nonzero ϵ, in an efficiency determination using (5), begins with the definition

$$e=\frac{\dfrac{R_1'}{R_2'}-\dfrac{R_1}{R_2}}{\dfrac{R_1}{R_2}}. \tag{19}$$

Substitution of (11), (17) into this result leads to

$$e\simeq 2\epsilon(|c_2|^2-|c_1|^2) \tag{20}$$

where the subscripts 1, 2, differentiate between the g-reflectometer parameters which obtain at ports 1 and 2 respectively, and only the first order terms have been retained.

If $|c_1|$, $|c_2|$, ϵ were known, their values could be substituted into (20) and e would become a "correction factor" rather than error. As a rule, however, the potential accuracy improvement does not warrant the extra effort this requires. As a general guideline, in the configuration of figure 3, $|c|\simeq|\Gamma_u|$ where Γ_u is a reflection coefficient which is "representative" of the two-port (e.g., adaptor) and the load for which the efficiency is measured. In the configuration of figure 4, $|c|$ is determined by the adjustment of T_y; if this is such as to minimize the amplitude of the oscillatory behavior of $|b_3/b_4|$ in response to the short motion, the foregoing guideline ($|c|\simeq|\Gamma_u|$) is again satisfied.[6]

For an adaptor of VSWR of 1.5, a typical value of $|c|$ is 0.2, while ϵ will ordinarily be less than 0.01. The maximum expected error, due to the nonideal short is thus 0.1 percent over a wide range of practical operating conditions, and the typical error is probably 0.01 percent or less.

To complete this discussion, there will also be an error in the determination of R_{c1}, R_{c2} due to the nonideal short. Ordinarily the error contribution from this source will be one or more magnitudes smaller than that already described and thus negligible.

Thus far, the discussion has implicitly assumed that the same short is used at both terminals 1 and 2. In some applications, the evaluation of a waveguide-coax adaptor for example, this is obviously impossible. In this case, the generalization of (20) is

$$e\simeq(\epsilon_2-\epsilon_1)+2(\epsilon_2|c_2|^2-\epsilon_1|c_1|^2). \tag{21}$$

Here ϵ_1, ϵ_2 are associated with the shorts used at ports 1 and 2 respectively.

It will be immediately recognized that the difference between ϵ_1 and ϵ_2 is now a potential source of substantial error. Fortunately, however, this error can be eliminated. The procedure is to make the measurement by each of the two described procedures (using (5) and (6)).[7] Assuming that the same pair of shorts are used for each of these measurements, it is easily shown that the geometric mean of the two efficiency determinations gives the desired result. Conversely, the square root of the ratio of these measurements gives the ratio between the reflection coefficient magnitudes of the respective plungers.

Returning to the expression for efficiency, it is convenient to write (5) in the form:

$$\eta_{21}=1-\frac{R_2-R_1}{R_2}. \tag{22}$$

This shows that the vertical displacement between R_2 and R_1 (fig. 6) is a direct measure of the *difference* between η_{21} and unity. Typically, this difference is no more than a few percent, and the performance require-

[5] It is not within the scope of this paper to consider the many "pathological" situations which could be invented, e.g., couplers without directivity or reversed in direction, extreme departures from impedance match conditions, etc. Because the mathematical formulation is general enough to include these cases, these additional qualifying statements are required.

[6] Although this guideline should prove adequate for most practical purposes, an "exact" evaluation of c_1, c_2 is not difficult to obtain. In particular, the generator in figures 3 or 4 is replaced by a variable termination which is adjusted such that b_4 vanishes when the system is excited via port 1 (or 2). The reflection coefficient observed at port 1 (or 2) is now $-c_1$ (or $-c_2$).

[7] This assumes that in the first of the described methods T_x has been adjusted such that "Γ_g" equals the load reflection coefficient for which η_{21} is desired. Moreover, it will be recognized that, during the second measurement, the two ports of the adaptor are reversed, in relation to the g-reflectometer, as compared with their positions during the first measurement.

ments on the associated measurement system are primarily for *resolution* and *stability* rather than "absolute" *accuracy*. For example, a 10% error in either R_2 or in (R_2-R_1) will cause an error of only a few tenths of a percent in η_{21}. Conversely, a system resolution of a part in 10^4 is required if (R_2-R_1) is to be observed with a resolution of one percent, etc.

The role of tuner T_y has been referred to a number of times. Although the operation is, in theory, independent of this adjustment, certain second order error considerations have been noted. In addition, however, is the practical requirement of matching the excursions of $|b_3/b_4|$ to the dynamic range limitations of the $x-y$ recorder. If these excursions are permitted to be large relative to R_2-R_1, the resolution, with which R_2-R_1 is measured, suffers. With a little experience this adjustment is not a difficult task.

Finally, it is of interest to note that the recording provides not only an indication of the joint or other losses, but also an indication of the loss of the transmission line in which the short moves. In many cases, this provides a useful confirmation of the system sensitivity and calibration. Indeed, it is possible to measure the loss in terms of the waveguide attenuation/wavelength without otherwise calibrating the system.

6. Applications

This technique represents a powerful tool for certain measurement problems. An immediate example, already alluded to, is the evaluation of connector loss. In order to gain some insight into the behavior of the Type N connector, a coaxial line was devised with a continuous outer conductor and a slip joint on the inner conductor such as is found in the Type N connector. This joint showed a loss of 0.006 dB at 9 GHz. In another test piece, the center conductor was continuous, while the outer conductor included a replica of the joint found in the Type N. This exhibited a loss of 0.05 dB.[8] The measured loss of a complete "Type N" joint, in which the slots had been deleted from the outer sleeve, was 0.01 dB.

This technique has also found application in the evaluation of certain parameters associated with a cryogenic noise source developed for the Comsat Corporation. In particular the corrected value of the temperature at the output port required both the loss per unit length in the output waveguide, and the loss in a waveguide window. Both of these quantities were easily measured via these methods.

Another use of these methods is in conjunction with the large antenna gain calibration project at the Jet Propulsion Laboratory [6]. Here a loss correction factor is obtained for the 30-40 foot waveguide run between the gain standard horn and the reference plane. The mismatch corrections are also explicitly accounted for with this technique.

[8] Although the purpose of these measurements was ostensibly that of evaluating the connector design, the real motivation was that of demonstrating the flexibility of the measurement technique. The above result is rather surprising, and may not be actually representative of the general design. In any case, it appears desirable to further pursue this subject.

7. Summary

The usefulness of the sliding short methods of "attenuation" measurement has been substantially enhanced by application of the first order correction theory, for nonideal short behavior, developed by Almassy [5], and the subsequent extension outlined in this paper.

The microwave art is rapidly approaching the place, if indeed it is not already there, where connector imperfections represent a major barrier to further advances in the accuracy of microwave measurements. Although a great deal of recent effort has been expended in the direction of improved VSWR specifications, the more important parameter, for many applications at least, is the dissipation characteristic. This method should prove a particularly valuable tool in assessing the magnitude of this problem.

The procedure may be implemented in a variety of ways, the major requirement is for a high degree of stability in the measuring system. One such system has been described in some detail by Almassy[9] [5]. At the Jet Propulsion Laboratory, a commercial version of the "precision insertion loss test set" developed at JPL [7] has been adapted for this use.

The existence of this first order correction theory also makes it possible to envision the extension of sliding short methods to much lower frequencies than has previously been feasible. In particular, because the theory explicitly takes account of line loss, it may prove useful to construct the inner and outer conductors in the form of a helix. Another possibility is that of immersing the line in a liquid dielectric.

Finally, the tuning adjustments, called out in this procedure, can be eliminated if a phase detection capability, such as is found in network analyzers, is assumed. Much of the detail for doing this has been worked out, but remains to be implemented.

8. Appendix

The purpose of this appendix is to derive (6).

Referring to figure 4, the relationship between b_3/b_4 and the reflection coefficient, Γ_r, presented at terminal 2 by the two-port and its load, Γ_l, may be written,[10]

$$\frac{b_3}{b_4}=\frac{A\Gamma_r+B}{C\Gamma_r+D} \tag{23}$$

where A, B, C, D are parameters of the g-reflectometer.

In a similar way, the relationship between Γ_r and Γ_l is given by,

$$\Gamma_r=\frac{a\Gamma_l+b}{c\Gamma_l+1} \tag{24}$$

[9] The "compensation" included in Almassy's system has not been found necessary in these measurements.

[10] The numerator and denominator could obviously be divided by D to obtain better agreement with the functional form of (7). The existing form, however, provides better continuity with earlier work in this area.

where for the moment, Γ_l is assumed to be arbitrary, and a, b, c are parameters of the two-port whose efficiency is required. (Note that this represents a change in terminology from (7).)

The required efficiency is given by [2],

$$\eta_{12}(\Gamma_l)=\frac{|a-bc|(1-|\Gamma_l|^2)}{|1+c\Gamma_l|^2-|a\Gamma_l+b|^2}. \quad (25)$$

By use of (11), (12), and a little algebra, it can be shown that

$$R_2\left(1-\left(\frac{R_{c2}}{R_2}\right)^2\right)=\frac{|A|^2-|B|^2}{|AD-BC|}. \quad (26)$$

To obtain the counterpart expression involving R_1, R_{c1} it is convenient to substitute (24) into (23) which (for an arbitrary Γ_l) yields,

$$\frac{b_3}{b_4}=\frac{(Aa+Bc)\Gamma_l+(Ab+B)}{(Ca+Dc)\Gamma_l+(Cb+D)}. \quad (27)$$

This is in the same form as (23) so,

$$R_1\left(1-\left(\frac{R_{c1}}{R_1}\right)^2\right)$$

$$=\frac{|Aa+Bc|^2-|Ab+B|^2}{|(Aa+Bc)(Cb+D)-(Ab+B)(Ca+Dc)|}. \quad (28)$$

By hypothesis, however, b_3 vanishes when Γ_l represents the load reflection for which the efficiency is desired. Therefore,

$$(Ab+B)=-(Aa+Bc)\Gamma_l, \quad (29)$$

from which,

$$\frac{B}{A}=-\frac{a\Gamma_l+b}{c\Gamma_l+1}. \quad (30)$$

Substitution of this result into (28), (26) leads to,

$$R_1\left(1-\left(\frac{R_{c1}}{R_1}\right)^2\right)$$

$$=\frac{|A|\cdot|a-bc|\cdot(1-|\Gamma_l|^2)}{|1+c\Gamma_l|\cdot|C(a\Gamma_l+b)+D(1+c\Gamma_l)|}, \quad (31)$$

$$R_2\left(1-\left(\frac{R_{c2}}{R_2}\right)^2\right)$$

$$=\frac{|A|\cdot(|1+c\Gamma_l|^2-|a\Gamma_l+b|^2)}{|1+c\Gamma_l|\cdot|C(a\Gamma_l+b)+D(1+c\Gamma_l)|}. \quad (32)$$

Finally, comparison of the ratio of (31) to (32) with (25) yields (6).

9. References

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