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Precise Continuous Optical Attenuator*

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The construction of a precision photometric attenuator is described, which uses linear birefringent polarizers. If properly calibrated, the instrument has relative error not exceeding 2.4×10^{-4} within a relative transmittance range down to 8×10^{-3}. The calibration takes into account stray light, provided the ratio of stray light intensity to total intensity remains constant and low in the whole range. The construction and calibrating procedure apply equally well to the near infrared region.

Key words: Dichroic polarizers; optics; pyrometry; radiometry; relative transmittance factor.

1. Introduction

Very accurate radiometric and pyrometric measurements often require relative transmittance factors to be measured with an accuracy of the order of 10^{-4}. Fixed values of these factors having this order of accuracy may be realized with rotating sectors, where the angular openings of the sector gives the attenuation.

Suitable means for producing a continuously variable relative transmittance factor which may be measured as a function of a mechanical quantity can be constructed with a set of light polarizers. A good arrangement consists of three linear dichroic polarizers lying in parallel planes, the first and third of them having fixed parallel orientation (fig. 1).

The middle polarizer can rotate in its plane, α being the angle between the rotating and fixed axes. R. Clark Jones [1][1] calculated the transmittance of such a train of three birefringent polarizers, which, for the most general case of three nonidentical polarizers, is given by the following expression:

$$T(\alpha) = S_1S_2S_3 + D_2(S_1D_3 + S_3D_1)\ \cos 2\alpha + S_3D_1D_3 \cos^2 2\alpha + P_2D_1D_3 \cos\delta_2 \sin^2 2\alpha \qquad (1)$$

where the coefficients are functions of the two principal transmittances of the polarizers—the subscripts applying to each of them—and δ_2 is the phase retardation of the middle polarizer.

H. E. Bennett [2] described a mechanical arrangement of three polarizers in series for producing optical attenuation. He showed that, if high grade identical polarizers are selected, the relative transmittance factor for the angles α and 0 is given by the simple relation:

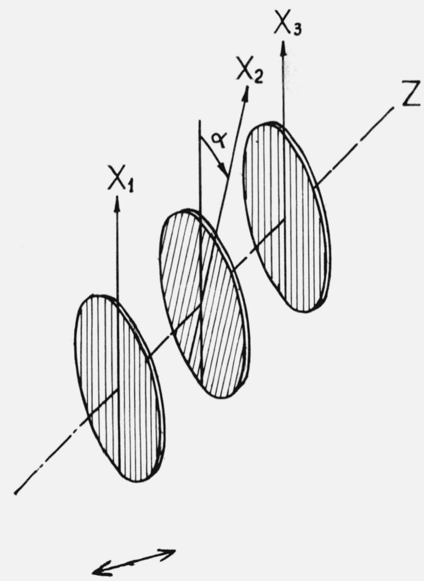

FIGURE 1. *The triple-polaroid optical attenuator.*

$$\frac{T(\alpha)}{T(0)} = \cos^4\alpha. \qquad (2)$$

For this to hold with an accuracy better than 10^{-3}, the principal transmittance ratio of the birefringent polarizers must be $\leq 4 \times 10^{-6}$.

In connection with the development of a photoelectric pyrometer a polaroid attenuator has been constructed, which is basically similar to the one described by Bennett, with provisions for meeting the following major requirements:

(1) the middle polarizer should have the highest resetability;

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[1] Figures in brackets indicate the literature references at the end of this paper.

(2) its rotation angle must be measured with high precision;

(3) the attenuators may be calibrated at a discrete set of points, yielding the coefficients of a function relating the angle with the relative transmittance factor, without any special requirement for the quality of the polarizers.

2. Theory

Equation (1) may be arranged in the following form:

$$T(\alpha)=a-b\sin^2\alpha-c\sin^2 2\alpha \tag{3}$$

with: $a=S_1S_2S_3+D_2(S_1D_3+S_3D_1)+S_3D_1D_3$
$b=2D_2(S_1D_3+S_3D_1)$
$c=D_1D_3(S_3-P_2\cos\delta_2)$.

The transmittance with parallel axes is

$$T(0)=a. \tag{4}$$

Let Φ be the radiant flux of a beam entering the system, Φ_0 the emerging flux with the polaroid set with parallel axes and Φ_α the emerging flux corresponding to the rotation α of the middle polarizer. Then the relative transmittance factor angle α will be:

$$T_r(\alpha)=\frac{\Phi_\alpha}{\Phi_0}=\frac{\Phi_\alpha/\Phi}{\Phi_0/\Phi}=\frac{T(\alpha)}{T(0)}. \tag{5}$$

Referring to (3) and putting $A=\frac{b}{a}$ and $B=\frac{c}{a}$, it is easily seen that

$$T_r(\alpha)=1-A\sin^2\alpha-B\sin^2 2\alpha \tag{6}$$

which gives the relative transmittance factor as a function of α, provided the coefficients A and B are known.

We may expect that not all the incident flux will be attenuated according to eq (3). Then, for every angle, we suppose the incident flux Φ to be divided in two portions: the first Φ^*, which is attenuated according to (3) and the second $\Delta\Phi$ attenuated by a different amount $T'(\alpha)$. We call $\Delta\Phi$ the stray flux, or "stray light."

Now we set two limitations to this stray flux:

(1) it is a small fraction, which will be defined later, of the total flux;

(2) the proportion between stray and total flux is constant for any angle.

The second hypothesis leads to the following expressions for the emerging flux:

$$\Phi_0'=T(0)\,\Phi^*+T'(0)\,\Delta\Phi \tag{7}$$

$$\Phi_\alpha'=T(\alpha)\,\Phi^*+T'(\alpha)\,\Delta\Phi.$$

The transmittance ratio is then:

$$T_r'(\alpha)=\frac{T(\alpha)\,\Phi^*+T'(\alpha)\,\Delta\Phi}{T(0)\,\Phi^*+T'(0)\,\Delta\Phi} \tag{8}$$

$$\simeq\frac{T(\alpha)}{T(0)}\left[1+\frac{T'(\alpha)}{T(\alpha)}\frac{\Delta\Phi}{\Phi^*}-\frac{T'(0)}{T(0)}\frac{\Delta\Phi}{\Phi^*}\right]$$

$$=T_r(\alpha)\left[1-\frac{T'(0)}{T(0)}\frac{\Delta\Phi}{\Phi^*}\right]+\frac{T'(\alpha)}{T(0)}\frac{\Delta\Phi}{\Phi^*}.$$

The approximation involved in the second equation is acceptable provided

$$\left|\left[\frac{T'(\alpha)}{T(\alpha)}-\frac{T'(0)}{T(0)}\right]\frac{T'(0)}{T(0)}\left(\frac{\Delta\Phi}{\Phi^*}\right)^2\right|<<1$$

which gives a quantitative expression to the first hypothesis. Putting

$$\frac{T'(0)}{T(0)}\frac{\Delta\Phi}{\Phi^*}=\kappa$$

and taking into account eq (6), eq (8) may be written:

$$T_r'(\alpha)=1-(1-\kappa)\,A\sin^2\alpha-(1-\kappa)\,B\sin^2 2\alpha+\frac{T'(\alpha)-T'(0)}{T(0)}\frac{\Delta\Phi}{\Phi^*}. \tag{9}$$

If the last term is

$$\left|\frac{T'(\alpha)-T'(0)}{T(0)}\frac{\Delta\Phi}{\Phi^*}\right|<<1$$

then it may be neglected and the transmittance ratio becomes:

$$T_r'(\alpha)=1-(1-\kappa)\,A\sin^2\alpha-(1-\kappa)\,B\sin^2 2\alpha \tag{10}$$

or:

$$T_r^*(\alpha)=1-A^*\sin^2\alpha-B^*\sin^2 2\alpha. \tag{11}$$

Therefore the stray light, at least as defined here, affects only the value of the constant coefficients of the expression of the relative transmittance factor.

The conclusion is that eq (6) yields the relative transmittance factor of a train of three birefringent polarizers in the most general case in which they are not equal, their extinction is not null, and stray flux, under specified conditions, is present in the system.

3. Mechanical Design

An attenuator using sheet polarizers [2] and embodying the above theory has been constructed. A longitudinal section is represented in figure 2. The instrument consists of five principal parts:

(1) A rotor R holds the middle polarizer P_2 and a glass graduated circle C centered on the rotation axis. The latter is divided in degrees and is a component of a precision survey instrument.[3]

[2] Type HN-22, selected to have a principal transmittance ratio $\leq 4\times10^{-6}$.
[3] Made by Salmoiraghi, Italy.

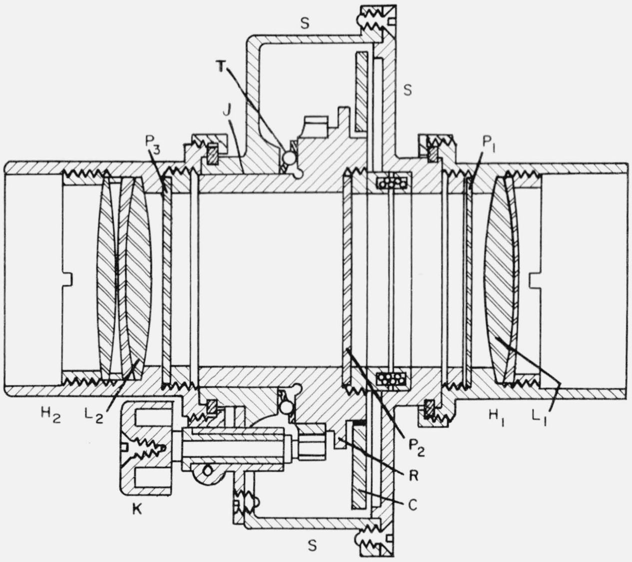

FIGURE 2. *Mechanical design of the attenuator.*

(2) A stator S acts as a support for the instrument and is coupled to the rotor through a journal J, and a thrust bearing T. The journal is made with hardened and ground cylindrical surfaces, which are coupled with a tight tolerance in order to prevent eccentricity and give the highest resetability to the rotor. The thrust bearing consists of two hardened and ground flats, the one in the stator, the other in the rotor, both perpendicular to the rotation axis, holding between them a row of steel balls. A set of springs located in the stator cover presses the rotor against the opposite face. The stator also accommodates a gear train allowing the rotation of the middle polarizer by means of an external knob K. Both stator and rotor are fabricated in hardened steel in order to insure precision coupling. Particular care is required in the choice of the material, which must not be deformed through hardening by an amount which could not be corrected by the precision machining which follows the hardening.

(3) Two tubular pieces, H_1 and H_2, are clamped at the end of the stator, each of them holding one external polaroid P_1 (or P_3) and an objective lens L_1 (or L_2). Each holder, before being clamped, can be independently adjusted for achieving the alinement of the principal optical axes of the polarizers.

(4) A micrometric microscope (not shown in the figure) allows the reading of the scale to be made with the resolution of 0.1 min.

4. Calibration Procedure

The attenuator has been mounted in a photoelectric pyrometer and therefore it was convenient to make use of all parts of the pyrometer for its calibration. Figure 3 represents the optical set up.

The objective lens L_1L_2, embodying the optical attenuator, forms the image of a pyrometric lamp ribbon, acting as a source S, on the slit D_1. A second objective L_3L_4 projects the image of the slit on a plane containing the flat surface of the straight filament of a pyrometer lamp RS.[4] A third objective L_5L_6 forms the image of the filament and of the source on the plane of an oscillating slit D_2. The slit D_1, the filament and the oscillating slit are parallel to each other, the first being 0.15 mm and the latter 0.05 mm wide.

The oscillating slit scans the image of the source, on which is superimposed the filament image. In this way any difference in the radiances of the images gives rise to an alternating component in the radiation flux emerging from the oscillating slit. The radiation flux is detected by a photomultiplier, PM (an RCA type 7265).

The oscillating slit acts as a light chopper and is driven by a cam mounted on the same shaft which actuates synchronous contacts demodulating the output signal of the photomultiplier. In this way the zeroing of this signal reveals the radiance match.

The calibration was performed with an interference filter peaked at 650 nm, with half bandwidth of 10 nm, located between the optical elements of objective L_5L_6.

The flux addition method has been used for calibration [3]. In figure 4 the light beam coming from the source SL (a ribbon lamp) is split by the semireflecting mirror M_1 in two beams which, after reflection in M_2 and M_3, merge in the semireflecting plate M_4, where they are partially superimposed. The two fluxes leaving M_4 are balanced to equality by adjusting the inclination on the optical path of the plates P_1 and P_2. Each of the beams may be intercepted by the shutters D_1 and D_2 and in this way two light fluxes may be produced having a magnitude ratio of two. The fluxes are intercepted by the attenuator and are balanced at the appropriate values of the current of the reference lamp.

The calibration is performed through the following steps:

(1) With parallel polaroid axes ($\alpha=0$) and one shutter closed, the flux Φ_1 is balanced with the reference lamp current I_1. The flux may have any value but must be kept constant within 5×10^{-5} during each step by means of a suitable power supply.

(2) Both shutters are open giving flux $\Phi_2=2\ \Phi_1$ which is matched with I_2.

In this way two values of the reference lamp current are given which correspond to light fluxes having a given ratio. Experience has shown that, after careful degassing of the lamp, and with low filament temperature (about 1300 K), the flux ratio corresponding to current I_1 and I_2 remained appreciably unchanged during several weeks of operation. The same ratio holds when the flux adder is removed, only the absolute values of Φ_1 and Φ_2 being changed.

(3) The flux $\Phi_2=2\ \Phi_1$ is balanced with current I_1 and an appropriate angle α_1 of the middle polarizer. This angle gives the relative transmittance factor, 1/2.

(4) With the attenuator set at the angle α_1 the source radiance is raised until it is balanced with current I_2. Setting now the current at I_1, the attenuator is turned to balance at the angle α_2, which corresponds to the relative transmittance factor 1/4.

The same procedure is repeated, generating in this

[4] The setup described is the one which was in use during calibration. However there are many advantages in interchanging the position of systems $L_1P_1P_2P_3L_2$ and L_3L_4.

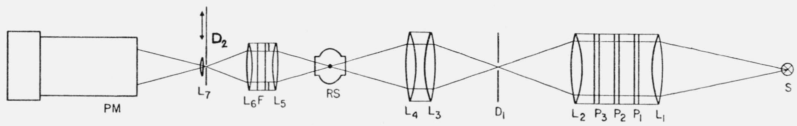

FIGURE 3. *Optical setup.*

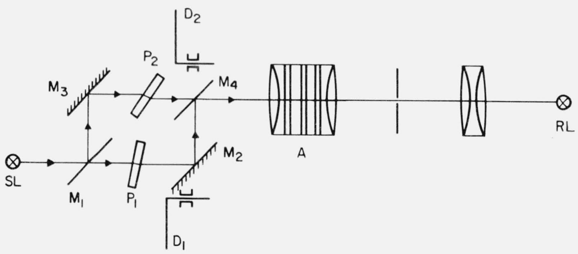

FIGURE 4. *The calibration arrangement.*

way a geometric series of values of the relative transmittance factors having seven terms.

The actual calibration was performed by sweeping the entire relative transmittance range seven times, proceeding stepwise from the lowest value, allowing due time for stabilization, and always using the same two values of the reference currents. For each point two readings were made so that for each nominal value of the relative transmittance factor a total of 14 readings were taken. Table 1 reports, in successive columns, the nominal values of the relative transmittance factor, the mean value of the angle, the estimated standard deviation of a reading and the estimated standard deviation of the mean.

TABLE 1

T_r	$\bar{\alpha}$	s	s_m
2^{-1}	32° 48.44′	0.54′	0.16′
4^{-1}	45° 5.70′	.50′	.14′
8^{-1}	53° 40.32′	.41′	.11′
16^{-1}	60° 14.11′	.32′	.09′
32^{-1}	65° 28.42′	.25′	.07′
64^{-1}	69° 46.04′	.29′	.08′
128^{-1}	73° 21.55′	.22′	.07′

5. Treatment of Calibration Data

The constant coefficients of eq (6) now may be determined through the least squares method on the basis of the calibration data. But first two remarks have to be made:

(1) The first attenuation step, T_{r1}, is affected by an error which is mainly due to inequality of the two fluxes which are added to create the ratio 1/2. Any other value of the attenuation is generated by multiplying the preceding one by T_{r1}, expressed as

$$T_{ri}=T_{r1}^{i} \tag{12}$$

in which i is the serial number of the step. Obviously the multiplication by T_{r1} propagates the error by which it itself is affected.

(2) The transmission ratio decreases rapidly with increasing angles. Therefore equal deviations of their calculated values from the experimental ones may lead to enormous relative errors for the lowest transmittance ratios. This must be avoided.

On this basis the coefficient A and B are determined according to the following procedure.

(1) We form a series of relative transmittance factors, each term being formed according to eq (12). Giving to the first term T_{r1} values which are closely and equally spaced around the expected value, for each of them we form a series of relative transmittance factors. Then, for each series, we minimize the function:

$$\Phi\ (A,B)=\sum_{i=1}^{7}\left[\frac{T_{ri}-(1-A\sin^2\alpha-\mathrm{B}\sin^2 2\alpha))}{T_{ri}}\right]^2. \tag{13}$$

Each term of this function is the square of the relative deviation, which is the difference between experimental and computed values divided by the experimental value. Each value of T_{r1} yields a value of A and of B.

(2) With each set of values of A and B the following quantities are computed:

(a) $T_{rc}=1-A\sin^2\alpha-B\sin^2 2\alpha$

(b) $\varepsilon_c=T_r-T_{rc}$

(c) $\eta_c=\dfrac{\varepsilon_c}{T_r}$

(d) $\varepsilon_e=\dfrac{dT_{rc}}{d\alpha}s_m$ (experimental absolute error)

(e) $\eta_e=\dfrac{\varepsilon_e}{T_r}$ (experimental relative error).

(3) Finally, among the ensemble of the groups of T_{rc}, A and B values, the one is chosen for which $\varepsilon_c<\varepsilon_e$. In this way the errors of the relative transmittance factor given by the fitting formula lies within the uncertainty of the experimental values. A program embodying this procedure has been put in a computer and the results of the best fit are collected in table 2.

The equation of the relative transmittance factor for the polaroid train tested is

$$T_r=1-0.999655\sin^2\alpha-0.246799\sin^2 2\alpha \tag{14}$$

TABLE 2

α	T_r	T_{rc}	ε_c	η_c	ε_e	η_e
32° 48.44′	0.50175	0.50181	-5.8×10^{-5}	-1.2×10^{-4}	6.0×10^{-5}	1.2×10^{-4}
45° 5.70′	.25175	.25172	3.5×10^{-5}	1.4×10^{-4}	4.1×10^{-5}	1.6×10^{-4}
53° 40.32′	.12632	.12630	1.4×10^{-5}	1.1×10^{-4}	2.2×10^{-5}	1.7×10^{-4}
60° 12.11′	.063380	.063377	0.2×10^{-5}	0.4×10^{-4}	1.1×10^{-5}	1.8×10^{-4}
65° 28.42′	.031801	.031805	$-.4 \times 10^{-5}$	-1.3×10^{-4}	0.5×10^{-5}	1.7×10^{-4}
69° 46.04′	.015956	.0159560	0	0	$.4 \times 10^{-5}$	2.4×10^{-4}
73° 21.55′	.0080059	.00080057	$.2 \times 10^{-6}$	0.2×10^{-4}	$.2 \times 10^{-5}$	2.4×10^{-4}

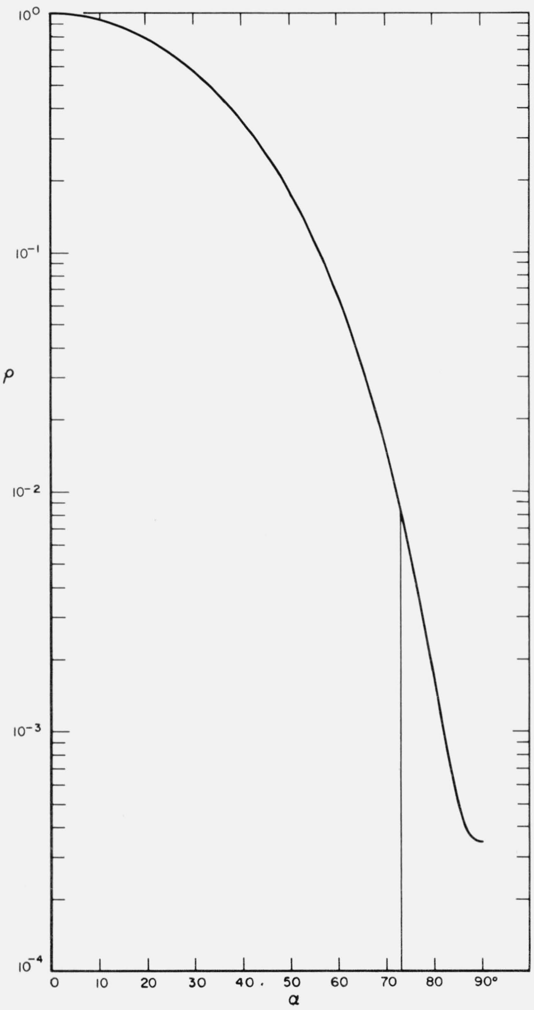

FIGURE 5. *Optical attenuation versus angular setting.*

which is plotted in figure 5 (the vertical line in the diagram represents the upper limit of calibrated range).

6. Discussion

From the last column of table 2 it is apparent that the optical attenuator here described may be calibrated in the range from 1 to 8×10^{-3} of the transmission ratio, with a maximum random error of 2.4×10^{-4}. From the calibration data are computed the constant coefficients of the theoretical interpolating equation, which gives the intermediate values well within the experimental errors.

Very important is the fact that the formula giving the attenuation as a function of the angle, under certain general conditions, takes into account the stray light.

From the operation and the calibration procedure of the instrument it is apparent that its precision is subject to a few limitations:

(1)—Unfortunately the transmittances and their ratio in birefringent polarizers are not wavelength independent. Polarizer sheets are manufactured to have roughly constant parameters over the entire visible radiation band and in the near infrared. Therefore, for the highest precision, they must be calibrated within a narrow bandwidth at each wavelength for which they are intended to be used.

(2)—It is well known that several sources, such as incandescent lamps, emit partially polarized radiation. In this case the present device attenuates the polarized component according to the angle between the polarization plane and the optical axis of the outer polarizers. This fact is not objectionable when the instrument is used for generating different flux values from a constant source, since the relative transmittance factor is independent of the polarization state of the entering radiation.

(3)—The attenuator has been calibrated with a parallel radiation beam. Since the transmittance and the retardation depend on the propagation direction of the radiation through the polarizing sheets, calibration precision will be assured only if the attenuator is operated with plane waves. Therefore the target area should have small dimensions and the source should be placed in the focal plane of the front lens of the instrument.

On the other hand, this device has an important advantage in the fact that the interpolating equation does not make any assumption about the ratio between the principal transmittances of the birefringent polarizers. This means that an accurate calibration may be performed even when that ratio is rather high, which opens the way to the use of accurate optical attenuators in the near infrared.

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7. References

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