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Student-*t* Deviate Corresponding to a Given Normal Deviate

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A table is given of the t deviate that corresponds to a given normal deviate z in the sense that the probabilities outside the t value and outside the z value are identical. This table enables those who are accustomed to expressing uncertainties in terms of 1σ, 2σ, 3σ, or 4σ limits to give statistically equivalent limits when the standard deviation σ is not known and consequently must be estimated from small samples.

Key words: Standard deviation; statistics; Student-t; table of Student-t; uncertainties.

In many fields experimenters have become accustomed to expressing limits to the random error in a reported value in terms of (for example) 3σ limits. Obviously such a procedure requires a knowledge of the true underlying standard deviation σ of the reported value and while such knowledge is always to be desired, it may not always be available. It may happen that an estimate S of the standard deviation, based on the information in a small sample, is all that is available and the problem then is to compute uncertainty limits using a t factor that have the same chance of being correct as the 3σ limits one might otherwise use.

The table gives the t deviate that corresponds to a given normal deviate in the sense that the probabilities inside the t value and inside the z value are identical. For example, the probability of a normal deviate not exceeding 2 in absolute value is the same as the probability of a t deviate based on 4 degrees of freedom not exceeding 2.869 in absolute value. The entries in the columns of the table headed $z=1$, 2, 3, and 4 are, therefore, the values of t not exceeded in absolute value with probabilities 0.68269, 0.95450, 0.99730, and 0.99994, respectively. These t values could, in principle, be determined from existing tables but rather extensive interpolation would be necessary since the probabilities are not round numbers.

Natrella [1963] [1] in section 2–1.5 describes the usual procedures for setting confidence limits in the case of a known standard deviation and in section 2–1.4.1 describes the procedure in the case of an estimated standard deviation based on ν degrees of freedom. In the special case considered there of setting a confidence limit on the mean, ν is always one less than the sample size.

As an example of the use of the table, suppose that the only information available on σ were in terms of the sample standard deviation, S, computed using the formula $S^2 = \text{sum } (x_i - \bar{x})^2/9$ for a sample of size 10 and that one was accustomed to reporting two-sigma limits. Then one could use $\bar{x} \pm 2.320\, S/\sqrt{10}$ rather than $\bar{x} \pm 2\sigma/\sqrt{10}$ as confidence limits for μ. One might state in such a case that, "based on the observed data, $\bar{x} - 2.320\, S/\sqrt{10}$ and $\bar{x} + 2.320\, S/\sqrt{10}$ correspond in probability to the usual two-sigma uncertainty limits."

The values in the table were computed using the computer programs of Bargmann and Ghosh [1963] and are believed to be accurate to the last digit given. The program was checked using exact formulas for small values of ν and the tables of Federighi [1959].

Computation of this table was suggested by Rolf B. F. Schumacher of Autonetics.

[1] Figures in brackets indicate the literature references at the end of this paper.

References

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*Student-*t *deviate corresponding to a given normal deviate* z, *for use with an estimated standard deviation based on* ν *degrees of freedom*

$t=(x-\mu)/s$	$z=(x-\mu)/\sigma$			
ν	$z=1$	$z=2$	$z=3$	$z=4$
1	1.837	13.968	235.80	10050.
2	1.321	4.527	19.207	125.64
3	1.197	3.307	9.219	32.616
4	1.142	2.869	6.620	17.448
5	1.111	2.649	5.507	12.281
6	1.091	2.517	4.904	9.844
7	1.077	2.429	4.530	8.467
8	1.067	2.366	4.277	7.595
9	1.059	2.320	4.094	6.999
10	1.053	2.284	3.957	6.567
11	1.048	2.255	3.850	6.242
12	1.043	2.231	3.764	5.988
13	1.040	2.212	3.694	5.785
14	1.037	2.195	3.636	5.619
15	1.034	2.181	3.586	5.480
16	1.032	2.169	3.544	5.364
17	1.030	2.158	3.507	5.264
18	1.029	2.149	3.475	5.177
19	1.027	2.140	3.447	5.102
20	1.026	2.133	3.422	5.036
25	1.020	2.105	3.330	4.795
30	1.017	2.087	3.270	4.645
35	1.014	2.074	3.229	4.543
40	1.013	2.064	3.199	4.468
45	1.011	2.057	3.176	4.412
50	1.010	2.051	3.157	4.367
60	1.008	2.043	3.130	4.302
70	1.007	2.036	3.111	4.256
80	1.006	2.032	3.097	4.223
90	1.006	2.028	3.086	4.197
100	1.005	2.025	3.077	4.177
200	1.003	2.013	3.038	4.087
500	1.001	2.005	3.015	4.034
∞	1.000	2.000	3.000	4.000

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