JOURNAL OF RESEARCH of the National Bureau of Standards—B. Mathematical Sciences
Vol. 73B, No. 4, October–December 1969

Highly Restricted Partitions*

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(May 20, 1969)

The function $g(n, m, h, k)$, which enumerates the number of partitions of n into exactly k summands each less than or equal to m and in which the number of different summands is exactly h, is here tabulated and studied.

Key words: Generating functions; graph of a partition; partitions.

1. Introduction

The problem is to find the number $g(n, m, h, k)$ of partitions of n, into exactly k summands each $\leq m$, just h (and any h) of the natural numbers $\leq m$ being used as summands, in any partition. Evidently, we must have $h \leq k$ and also $h \leq m$.

EXAMPLE: Let $n=10$, $m=3$, $h=2$, and $k=4$. Then, there are just two acceptable partitions, viz.,

$$1, 3, 3, 3 \text{ and } 2, 2, 3, 3.$$

For $n=11$, there is only one acceptable partition: 2, 3, 3, 3. For $n \leq 4$ or $n \geq 12$, there is no partition of the desired type.

In what follows, we shall denote by $g^*(n, m, h, k)$, the number of those g-type partitions of n in which the greatest summand used is m, and by $G(n, m, h, k)$ the number of those in which the number of summands is at most k $(\geq h)$. Thus

$$g^*(n, m, h, k) = g(n, m, h, k) - g(n, m-1, h, k); \tag{1.1}$$

and

$$G(n, m, h, k) = \sum_{j=1}^{k} g(n, m, h, j). \tag{1.2}$$

2. Some Useful Relations

Let

$$\begin{array}{ccccc} * & * & & & \\ * & * & & & \\ * & * & & & \\ * & * & * & & \\ * & * & * & & \\ * & * & * & * & * \end{array}$$

be the graph of a g-type partition of n. (In the graph, we have $n=17$, $h=3$, $k=6$ and we take $m=7$.) Place crosses in each row so that the total number of stars and crosses in each row is $(m+1)$. Then the graph takes the form:

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*	*	* X	* X	* X	* X	* X	* X
*	*	* X	* X	* X	* X	* X	* X
*	*	* X	* X	* X	* X	* X	* X
*	*	*	* X	* X	* X	* X	* X
*	*	*	* X	* X	* X	* X	* X
*	*	*	*	*	* X	* X	* X

It will be readily seen that the crosses provide an acceptable partition of $k(m+1)-n$. The correspondence being one-one onto, we have:

$$g(n, m, h, k) = g(k(m+1)-n, m, h, k). \tag{2.1}$$

(The two partitions may be spoken of as complementary.)

Again, if m be the largest summand in a g-type partition of n, then in the conjugate partition (obtained by interchanging rows and columns in the graph) the largest summand is k and the number of summands exactly m. Hence

$$g^*(n, m, h, k) = g^*(n, k, h, m). \tag{2.2}$$

Similar reasoning gives

$$G(n, m, h, k) = G(n, k, h, m). \tag{2.3}$$

3. The Generating Function for g

It is not difficult to see that $g(n, m, h, k)$ is the coefficient of $x^n z^k$ in

$$\sum_{j=1}^{\binom{m}{h}} \prod_{i \epsilon C_j} x^i z (1 - x^i z)^{-1}, \tag{3.1}$$

where C_j runs through each of the $\binom{m}{h}$ sets of h distinct natural numbers $\leq m$. In other words, $g(n, m, h, k)$ is the coefficient of $x^n z^k t^h$ in

$$\prod_{i=1}^{m} \left(1 + \frac{x^i z}{1 - x^i z} t\right), \tag{3.2}$$

i.e., in

$$\prod_{i=1}^{m} \frac{1 + x^i z(t-1)}{1 - x^i z} = f(t)/f(0), \tag{3.3}$$

where

$$f(t) = 1 + \begin{bmatrix} m \\ 1 \end{bmatrix} z(t-1) + \begin{bmatrix} m \\ 2 \end{bmatrix} z^2 (t-1)^2 + \ldots + \begin{bmatrix} m \\ m \end{bmatrix} z^m (t-1)^m,$$

with

$$\begin{bmatrix} m \\ i \end{bmatrix} = \frac{(1-x^m)(1-x^{m-1})(1-x^{m-2}) \ldots (1-x^{m-i+1})}{(1-x)(1-x^2)(1-x^3) \ldots (1-x^i)} x^{i(i+1)/2}.$$

The coefficient of t^h in $f(t)$ is

$$\begin{bmatrix} m \\ h \end{bmatrix} z^h - \binom{h+1}{1} \begin{bmatrix} m \\ h+1 \end{bmatrix} z^{h+1} + \ldots + (-1)^{m-h} \binom{m}{m-h} \begin{bmatrix} m \\ m \end{bmatrix} z^m = z^h A_h(x, z), \text{ say.} \tag{3.4}$$

Breaking into partial fractions, we have

$$A_h(x, z) \prod_{i=1}^{m} (1-x^i z)^{-1} = \sum_{i=1}^{m} B_i^{(h)}(x)/(1-x^i z), \tag{3.5}$$

with

$$B_i^{(h)}(x) = A_h(x, x^{-i})/(1-x^{-1}) \ldots (1-x^{1-i})(1-x) \ldots (1-x^{m-i}), \qquad \text{for } i \geqslant 2;$$

and

$$B_1^{(h)}(x) = A_h(x, x^{-i})/(1-x)(1-x^2) \ldots (1-x^{m-1}).$$

The coefficient of z^{k-h} in (3.5) is given by

$$F(x, m, h, k) = \sum_{i=1}^{m} B_i^{(h)}(x) x^{i(k-h)}. \tag{3.6}$$

Hence, $g(n, m, h, k)$ is the coefficient of x^n in $F(x, m, h, k)$.

In particular, it can be shown that

$$F(x, 3, 2, 4) = x^5 + 2x^6 + x^7 + x^8 + x^9 + 2x^{10} + x^{11}.$$

Hence,

$$g(10, 3, 2, 4) = \text{coefficient of } x^{10} \text{ in } F(x, 3, 2, 4) = 2.$$

It will be seen that the coefficient of $z^k t$ in (3.2), is

$$x^k + x^{2k} + x^{3k} + \ldots + x^{mk} = x^k(1-x^{mk})/(1-x^k).$$

Hence,

$$F(x, m, 1, k) = x^k(1-x^{mk})/(1-x^k). \tag{3.7}$$

THEOREM: *$F(x, m, h, k)$ is a polynomial in x of degree exactly $mk - \frac{h(h-1)}{2}$, in which the coefficients of terms equidistant from the beginning and the end are equal.*

It will be readily seen that

$$g(n, m, h, k) = 0 \text{ for } n < k + \frac{h(h-1)}{2}, \text{ and also}$$

$$\text{for } n > mk - \frac{h(h-1)}{2}.$$

Moreover,

$$g\left(mk - \frac{h(h-1)}{2}, m, h, k\right) = 1,$$

because the only g-type partition of $mk-\dfrac{h(h-1)}{2}$ is

$$m-1,\ m-2,\ m-3, \ldots, m-(h-1),\ m,\ m, \ldots, m.$$

The rest of the theorem, follows immediately from (2.1).

4. Reduction Formulas

We observe that $g(n, m, h, k)$ is also the number of solutions of the equation:

$$a_1x_1+a_2x_2+\ldots+a_hx_h=n, \tag{4.1}$$

such that

$$1\leqslant a_1<a_2<\ldots<a_h\leqslant m,$$

and

$$x_1+x_2+\ldots+x_h=k,$$

a's and x's being positive integers.
Since (4.1) can be written in the form:

$$(a_2-a_1)x_2+(a_3-a_1)x_3+\ldots+(a_h-a_1)x_h=n-ka_1,$$

we have for $h>1$,

$$g(n, m, h, k)=\sum_{a_1=1}^{[(n-1)/k]}\ \sum_{x_1=1}^{k-1} g(n-ka_1, m-a_1, h-1, k-x_1),$$

$$=\sum_{j=1}^{[(n-1)/k]} G(n-kj, m-j, h-1, k-1). \tag{4.2}$$

This provides a reduction formula for $g(n, m, h, k)$.
From (4.2), we readily obtain:

$$g(n, m, h, k)=G(n-k, m-1, h-1, k-1)+g(n-k, m-1, h, k), \tag{4.2$'$}$$

which gives

$$G(n-k, m-1, h-1, k-1)=g(n, m, h, k)-g(n-k, m-1, h, k).$$

Since

$$G(n-k, m-1, h-1, k-1)=g(n-k, m-1, h-1, k-1)+G(n-k, m-1, h-1, k-2),$$

we get

$$g(n, m, h, k)=g(n-k, m-1, h, k)+g(n-k, m-1, h-1, k-1)$$
$$+g(n-1, m, h, k-1)-g(n-k, m-1, h, k-1). \tag{4.3}$$

For $h=1$, we have

$$g(n, m, 1, k)=1, \text{ when } k|n \text{ and } n\leqslant km;$$
$$=0, \text{ otherwise.} \tag{4.4}$$

As an alternative proof of (4.2′), we offer the following:

The number of g-type partitions of n in which 1 appears as a summand is given by $G(n-k, m-1, h-1, k-1)$, because reducing each summand by 1, we get a partition of $(n-k)$ into at most $(k-1)$ parts with each summand $\leq (m-1)$ and using $(h-1)$ distinct natural numbers $\leq (m-1)$ as summands. The number of partitions in which 1 does not appear as a summand is given by $g(n-k, m-1, h, k)$. For, diminishing each summand by 1, as before, we are this time left with a partition of $(n-k)$ into exactly k summands each $\leq (m-1)$ and utilizing just h natural numbers $\leq (m-1)$ as summands.

Hence the result.

As an illustration of the use of (4.2), we evaluate $g(16, 6, 3, 5)$ at some length. We have

$$g(16, 6, 3, 5) = G(11, 5, 2, 4) + G(6, 4, 2, 4) + G(1, 3, 2, 4).$$

Now,

$$G(11, 5, 2, 4) = g(11, 5, 2, 4) + G(11, 5, 2, 3);$$

$$\begin{aligned} g(11, 5, 2, 4) &= G(7, 4, 1, 3) + G(3, 3, 1, 3), \\ &= 0 + g(3, 3, 1, 1) + g(3, 3, 1, 2) + g(3, 3, 1, 3), \\ &= 0 + 1 + 0 + 1 = 2. \end{aligned}$$

$$\begin{aligned} G(11, 5, 2, 3) &= g(11, 5, 2, 1) + g(11, 5, 2, 2) + g(11, 5, 2, 3), \\ &= 0 + 0 + G(8, 4, 1, 2) + G(5, 3, 1, 2) + G(2, 2, 1, 2); \end{aligned}$$

$$G(8, 4, 1, 2) = g(8, 4, 1, 1) + g(8, 4, 1, 2) = 0 + 1 = 1;$$

$$G(5, 3, 1, 2) = g(5, 3, 1, 1) + g(5, 3, 1, 2) = 0 + 0 = 0;$$

$$G(2, 2, 1, 2) = g(2, 2, 1, 1) + g(2, 2, 1, 2) = 1 + 1 = 2.$$

Again,

$$\begin{aligned} G(6, 4, 2, 4) &= g(6, 4, 2, 1) + g(6, 4, 2, 2) + g(6, 4, 2, 3) + g(6, 4, 2, 4) \\ &= 0 + G(4, 3, 1, 1) + G(2, 2, 1, 1) + G(3, 3, 1, 2) + G(2, 3, 1, 3), \end{aligned}$$

$$G(4, 3, 1, 1) = g(4, 3, 1, 1) = 0,$$

$$G(2, 2, 1, 1) = g(2, 2, 1, 1) = 1,$$

$$G(3, 3, 1, 2) = g(3, 3, 1, 1) + g(3, 3, 1, 2) = 1 + 0 = 1,$$

$$\begin{aligned} G(2, 3, 1, 3) &= g(2, 3, 1, 1) + g(2, 3, 1, 2) + g(2, 3, 1, 3) \\ &= 1 + 1 + 0 = 2. \end{aligned}$$

Finally,

$$G(1, 3, 2, 4) = 0.$$

Hence

$$g(16, 6, 3, 5) = 9.$$

Tracing backward, each one of the nine paths of reduction, this method enables us to write down the corresponding partitions as well.

Thus, one path of reduction is:

$$g(2, 2, 1, 1) \leftarrow g(11, 5, 2, 3) \leftarrow g(16, 6, 3, 5).$$

This gives in (4.1),

$$a_1 = 1,\ x_1 = 2;\ a_2 - a_1 = 3,\ x_2 = 2;\ a_3 - a_2 = 2,\ x_3 = 1;$$

which leads to the partition: 1, 1, 4, 4, 6 of 16. The other partitions viz.,

1, 1, 2, 6, 6; 1, 1, 4, 5, 5; 1, 3, 3, 3, 6; 1, 3, 4, 4, 4;

2, 3, 3, 4, 4; 2, 3, 3, 3, 5; 2, 2, 3, 3, 6; 2, 2, 2, 4, 6;

can be similarly obtained.

As another reduction formula, we have the following:

$$g(n, m, h, k) = g(n, m-1, h, k) + \sum_{j=1}^{[(n-1)/m]} g(n-jm, m-1, h-1, k-j), \tag{4.5}$$

because $g(n, m-1, h, k)$ is the number of those partitions of n in which m does not at all occur as a part, while the sigma provides the rest.

From (4.5), we can easily deduce that

$$g(n, m, h, k) = g(n, m-1, h, k) + g(n-m, m-1, h-1, k-1) + g(n-m, m, h, k-1) - g(n-m, m-1, h, k-1). \tag{4.6}$$

When $m \neq k$, (2.2) gives the simpler result:

$$\begin{aligned} g(n, m, h, k) &= g(n, m-1, h, k) + g^*(n, m, h, k), \\ &= g(n, m-1, h, k) + g^*(n, k, h, m), \\ &= g(n, m-1, h, k) + g(n, k, h, m) - g(n, k-1, h, m). \end{aligned} \tag{4.7}$$

In computing the tables that are appended, we have made use of (4.6). The other formulas have served as checks. In view of (2.1), the tables give values of $g(n, m, h, k)$ for

$$k + \frac{h(h-1)}{2} \leq n \leq [k(m+1)/2]; \qquad m, h, k \leq 10.$$

The plan of the tables is easy to follow. For even values of h, the values of $g(n, m, h, k)$ appear in the right-hand triangles and those for odd values of h in the left-hand ones.

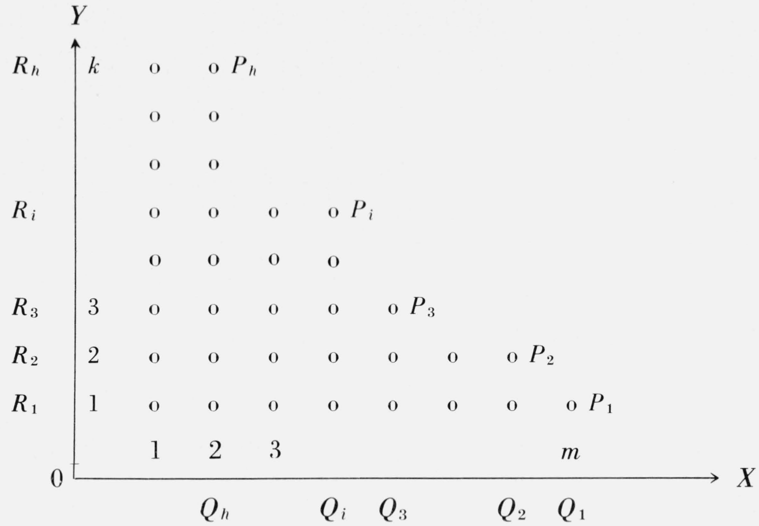
For fixed n, h, k, the values of $g(n, m, h, k)$ are nondecreasing for increasing m. This is what could be expected. But, for fixed m, h, k and increasing n, the function $g(n, m, h, k)$ behaves quite irregularly. This is rather unusual.

A close study of the tables, has led to the

THEOREM: *For any fixed m, h and k,*

$$\sum_{n=1}^{\infty} g^*(n, m, h, k) = \binom{m-1}{h-1}\binom{k-1}{h-1}. \tag{4.8}$$

We offer a combinatorial proof.
Let



be the graph of a g^*-partition for some n. The graph determines uniquely and is uniquely determined by its corner points: $P_1, P_2, P_3, \ldots, P_h$. These, in turn, are uniquely determined by and uniquely determine, two sets of h points each viz., $Q=\{Q_1, Q_2, \ldots, Q_h\}$, and $R=\{R_1, R_2, \ldots, R_h\}$ obtained from the projection of the points $P_1, P_2, \ldots, P_h$ on the axes of X and Y. The points $Q_2, Q_3, \ldots, Q_h$ lie strictly between 0 and m $(=Q_1)$ on the X axis. The points $R_1, R_2, \ldots, R_{h-1}$ lie strictly between 0 and k $(=R_h)$ on the Y axis. The total number of ways in which the $(h-1)$ points $Q_2, Q_3, \ldots, Q_h$ can be placed between 0 and m, is $\binom{m-1}{h-1}$. Similarly, the $(h-1)$ points $R_1, R_2, \ldots, R_{h-1}$ can be located between 0 and k on the Y axis in $\binom{k-1}{h-1}$ ways. Hence the total number of g^*-partitions must be $\binom{m-1}{h-1}\binom{k-1}{h-1}$ and the theorem follows.

Proceeding on the same lines, one can prove that

$$\sum_{n=1}^{\infty} g(n, m, h, k)=\binom{k-1}{h-1}\binom{m}{h}, \tag{4.9}$$

and

$$\sum_{n=1}^{\infty} G(n, m, h, k)=\binom{k}{h}\binom{m}{h}. \tag{4.10}$$

Result (4.9) can be derived inductively from (4.6). The results (4.9) and (4.10) can both be derived also from the theorem itself.

The points of sets Q and R determine uniquely another set of h points $T=\{T_1, T_2, \ldots, T_h\}$; where T_j is the point of which Q_{h-j+1} is the projection on the X axis and R_j the projection on the Y axis.

The broken line $OT_1\ T_2 \ldots T_h$ corresponds to an ordered partition of the vector (m, k) into exactly h nondegenerate parts.[1] Hence the sum in (4.8) can be interpreted as the number of such partitions of the bipartite number (m, k).

[1] Gupta, H., Graphic Representation of a Partition of a j-Partite Number. Research Bulletin (N.S.), Panjab University, **10**, 189–196 (1959).

$h=2$

***	**	**	**	**	**	**	**	**	**	**	***	
		2	3	4	5	6	7	8	9	10	m / n	k
		1	1	1	1	1	1	1	1	1	3	2
			1	1	1	1	1	1	1	1	4	
				2	2	2	2	2	2	2	5	
					2	2	2	2	2	2	6	
						3	3	3	3	3	7	
							3	3	3	3	8	
								4	4	4	9	
									4	4	10	
										5	11	
		1	1	1	1	1	1	1	1	1	8	7
		1	2	2	2	2	2	2	2	2	9	
		1	1	2	2	2	2	2	2	2	10	
			2	2	3	3	3	3	3	3	11	
38	47		1	1	1	2	2	2	2	2	12	
37	54		2	3	3	3	4	4	4	4	13	
36	51		0	0	0	0	0	1	1	1	14	
35	54	50		2	3	3	3	3	4	4	15	
34	49	38		3	3	3	3	3	3	4	16	
33	47	40		2	3	4	4	4	4	4	17	
32	45	35			2	3	3	3	3	3	18	
31	47	38	28		4	4	6	6	6	6	19	
30	44	38	30		3	3	3	4	4	4	20	
29	45	40	31		0	0	0	1	2	2	21	
28	47	39	37	31		5	5	5	5	6	22	
27	39	35	26	21		4	4	4	5	5	23	
26	34	28	24	18		3	4	4	4	4	24	
25	38	35	28	23			5	5	5	6	25	
24	34	29	25	18	16		4	6	6	6	26	
23	33	31	27	22	14		5	5	6	6	27	
22	26	23	21	16	11		0	1	2	3	28	
21	32	30	27	26	20	14		5	5	5	29	
20	24	21	19	16	14	8		4	4	5	30	
19	21	19	16	14	12	9		6	7	7	31	
18	20	18	16	13	11	8			6	6	32	
17	18	17	15	13	10	9	5		6	6	33	
16	14	14	13	11	9	6	4		5	7	34	
15	12	12	12	11	9	7	5		2	3	35	
14	11	11	11	11	10	8	6	3		5	36	
13	6	6	6	6	6	5	3	2		6	37	
12	5	5	5	5	5	5	4	2		7	38	
11	2	2	2	2	2	2	2	1				
10	1	1	1	1	1	1	1	1				
***												
n / m	10	9	8	7	6	5	4	3				

****

$h=3$

$h=2$

***	**	** 2	** 3	** 4	** 5	** 6	** 7	** 8	** 9	** 10	*** m / n	k
		1	1	1	1	1	1	1	1	1	4	3
			2	2	2	2	2	2	2	2	5	
16	10		0	1	1	1	1	1	1	1	6	
15	10	8		2	3	3	3	3	3	3	7	
14	9	8			2	3	3	3	3	3	8	
13	8	7	6		2	2	3	3	3	3	9	
12	7	7	6	5		3	3	4	4	4	10	
11	5	5	5	4			4	4	5	5	11	
10	4	4	4	4	3		2	3	3	4	12	
9	3	3	3	3	3	2		4	5	5	13	
8	2	2	2	2	2	2			4	5	14	
7	1	1	1	1	1	1	1		4	4	15	
6	1	1	1	1	1	1	1	1		5	16	
		1	1	1	1	1	1	1	1	1	7	6
		1	2	2	2	2	2	2	2	2	8	
		1	1	2	2	2	2	2	2	2	9	
33	32		2	2	3	3	3	3	3	3	10	
32	44		1	1	1	2	2	2	2	2	11	
31	42		1	2	2	2	3	3	3	3	12	
30	38	30		1	1	1	1	2	2	2	13	
29	40	33		3	4	4	4	4	5	5	14	
28	41	35		2	3	3	3	3	3	4	15	
27	32	24	18		3	5	5	5	5	5	16	
26	38	32	25		1	1	2	2	2	2	17	
25	36	33	27		4	4	5	6	6	6	18	
24	31	27	23	18		1	1	1	2	2	19	
23	34	29	24	20		4	4	5	5	6	20	
22	29	26	22	16		5	5	5	5	5	21	
21	24	20	17	12	6		6	6	7	7	22	
20	26	25	21	19	13		1	2	2	2	23	
19	26	23	22	18	15		5	6	7	8	24	
18	19	17	14	13	10	6		1	1	2	25	
17	21	19	17	14	13	9		6	7	7	26	
16	16	15	13	11	8	8		6	6	6	27	
15	13	13	12	10	8	5	4		6	7	28	
14	10	10	10	9	7	5	3		2	2	29	
13	10	10	10	10	9	7	5		8	9	30	
12	6	6	6	6	6	5	3	2		2	31	
11	5	5	5	5	5	5	4	2		7	32	
10	2	2	2	2	2	2	2	1		7	33	
9	1	1	1	1	1	1	1	1				
n / m	10	9	8	7	6	5	4	3				

****

$h=3$

h=2

***	**	**	**	**	**	**	**	**	**	**	***	
		2	3	4	5	6	7	8	9	10	m / n	k
		1	1	1	1	1	1	1	1	1	5	4
		1	2	2	2	2	2	2	2	2	6	
22	16		1	2	2	2	2	2	2	2	7	
21	22		1	1	2	2	2	2	2	2	8	
20	18	14		1	1	2	2	2	2	2	9	
19	20	17		4	4	4	5	5	5	5	10	
18	16	13	10		2	2	2	3	3	3	11	
17	19	16	13		2	3	3	3	4	4	12	
16	14	13	10	7		2	3	3	3	4	13	
15	16	14	13	10		5	5	6	6	6	14	
14	11	10	8	7	4		2	2	3	3	15	
13	11	11	10	8	7		5	5	5	6	16	
12	8	8	8	7	5	4		3	3	3	17	
11	7	7	7	7	6	4		6	7	7	18	
10	3	3	3	3	3	2	0		3	4	19	
9	4	4	4	4	4	4	3		6	6	20	
8	2	2	2	2	2	2	2	1		3	21	
7	1	1	1	1	1	1	1	1		9	22	
		1	1	1	1	1	1	1	1	1	6	5
		1	2	2	2	2	2	2	2	2	7	
			1	2	2	2	2	2	2	2	8	
27	31		2	2	3	3	3	3	3	3	9	
26	28		0	0	0	1	1	1	1	1	10	
25	31	26		3	3	3	4	4	4	4	11	
24	29	22		2	2	2	2	3	3	3	12	
23	28	25			4	4	4	4	5	5	13	
22	25	21	17		3	4	4	4	4	5	14	
21	25	22	19		0	1	2	2	2	2	15	
20	26	21	18	14		3	3	4	4	4	16	
19	23	21	16	14		4	5	5	6	6	17	
18	20	18	16	11			4	4	4	5	18	
17	18	16	14	12	8		4	5	5	5	19	
16	17	15	13	11	9		2	3	3	3	20	
15	18	17	15	13	11	10		4	6	6	21	
14	11	11	10	8	6	4		5	5	6	22	
13	9	9	9	8	6	4			5	6	23	
12	8	8	8	8	7	5	3		5	5	24	
11	5	5	5	5	5	4	2		4	4	25	
10	5	5	5	5	5	5	4	2		7	26	
9	2	2	2	2	2	2	2	1		5	27	
8	1	1	1	1	1	1	1	1				
n / m	10	9	8	7	6	5	4	3				

****

h=3

$h=2$

***	**	**	**	**	**	**	**	**	**	**	***	
		2	3	4	5	6	7	8	9	10	m \ n	k
		1	1	1	1	1	1	1	1	1	9	8
		1	2	2	2	2	2	2	2	2	10	
		1	1	2	2	2	2	2	2	2	11	
		1	2	2	3	3	3	3	3	3	12	
44	56		1	1	1	2	2	2	2	2	13	
43	54		2	3	3	3	4	4	4	4	14	
42	72		1	1	1	1	1	2	2	2	15	
41	61		1	1	2	2	2	2	3	3	16	
40	66	56		2	2	2	2	2	2	3	17	
39	55	45		3	3	4	4	4	4	4	18	
38	66	56		1	2	2	2	2	2	2	19	
37	51	41		4	5	6	7	7	7	7	20	
36	52	40	26		1	1	2	2	2	2	21	
35	56	48	38		4	4	4	6	6	6	22	
34	63	54	42		2	3	3	3	4	4	23	
33	48	41	33		2	3	3	3	4	5	24	
32	56	48	40	29		2	2	2	2	2	25	
31	43	39	32	23		4	6	6	6	7	26	
30	53	46	42	32		2	2	2	2	2	27	
29	43	35	29	28		7	8	9	9	9	28	
28	40	35	26	20	14		2	4	4	4	29	
27	41	36	31	23	18		4	4	6	6	30	
26	41	38	32	26	20		3	3	3	4	31	
25	37	33	29	23	18		5	5	6	7	32	
24	35	34	31	26	19	14		3	3	3	33	
23	29	25	24	20	15	11		6	6	6	34	
22	28	26	22	22	17	11		2	2	3	35	
21	27	24	22	18	17	12		10	11	11	36	
20	20	18	15	13	10	8	4		4	4	37	
19	22	20	18	15	13	9	7		6	8	38	
18	19	18	16	14	11	10	5		3	3	39	
17	16	16	15	13	11	8	6		6	7	40	
16	13	13	13	12	10	8	6	3		3	41	
15	11	11	11	11	10	8	6	3		6	42	
14	6	6	6	6	6	5	3	2		4	43	
13	5	5	5	5	5	5	4	2		13	44	
12	2	2	2	2	2	2	2	1				
11	1	1	1	1	1	1	1	1				
n \ m	10	9	8	7	6	5	4	3				

****

$h=3$

h=2

***	**	** 2	** 3	** 4	** 5	** 6	** 7	** 8	** 9	** 10	*** m/n	k
		1	1	1	1	1	1	1	1	1	10	9
		1	2	2	2	2	2	2	2	2	11	
		1	1	2	2	2	2	2	2	2	12	
		1	2	2	3	3	3	3	3	3	13	
			1	1	1	2	2	2	2	2	14	
49	73		2	3	3	3	4	4	4	4	15	
48	65		1	1	1	1	1	2	2	2	16	
47	74		2	2	3	3	3	3	4	4	17	
46	74		0	1	1	1	1	1	1	2	18	
45	82	72		2	2	3	3	3	3	3	19	
44	66	51		2	2	2	2	2	2	2	20	
43	70	60		3	5	5	6	6	6	6	21	
42	63	49		2	2	3	3	3	3	3	22	
41	67	60			2	2	3	4	4	4	23	
40	70	57	47		4	5	5	6	6	6	24	
39	58	46	38		3	3	3	3	5	5	25	
38	66	52	42		2	3	3	3	3	4	26	
37	64	57	39		2	2	3	3	3	4	27	
36	64	55	48	34		2	3	3	3	3	28	
35	64	58	46	36		4	4	4	4	4	29	
34	51	46	40	28		6	6	8	8	8	30	
33	48	40	34	26		2	3	3	3	3	31	
32	51	43	39	31			2	3	4	4	32	
31	56	51	41	34	24		8	8	10	10	33	
30	41	32	28	24	16		4	4	4	6	34	
29	50	46	36	30	21		3	3	3	3	35	
28	43	38	34	25	23		2	3	3	4	36	
27	44	42	36	31	22	16		5	5	5	37	
26	36	33	30	24	20	13		4	4	4	38	
25	36	34	32	28	21	15		8	10	10	39	
24	26	22	20	17	13	9		3	3	3	40	
23	31	29	25	24	20	15			5	6	41	
22	26	23	21	17	15	11	7		8	10	42	
21	22	20	17	15	12	9	6		4	4	43	
20	23	21	19	16	14	10	7		5	5	44	
19	21	20	18	16	13	12	7		4	5	45	
18	17	17	16	14	12	9	7	4		5	46	
17	13	13	13	12	10	8	6	3		4	47	
16	11	11	11	11	10	8	6	3		10	48	
15	6	6	6	6	6	5	3	2		5	49	
14	5	5	5	5	5	5	4	2				
13	2	2	2	2	2	2	2	1				
12	1	1	1	1	1	1	1	1				
n/m	10	9	8	7	6	5	4	3				

****

h=3

h = 2

***	**	** 2	** 3	** 4	** 5	** 6	** 7	** 8	** 9	** 10	*** m \ n	k
		1	1	1	1	1	1	1	1	1	11	10
		1	2	2	2	2	2	2	2	2	12	
		1	1	2	2	2	2	2	2	2	13	
		1	2	2	3	3	3	3	3	3	14	
		1	1	1	1	2	2	2	2	2	15	
55	56		2	3	3	3	4	4	4	4	16	
54	97		1	1	1	1	1	2	2	2	17	
53	78		2	2	3	3	3	3	4	4	18	
52	91		1	2	2	2	2	2	2	3	19	
51	74		1	1	1	2	2	2	2	2	20	
50	84	68		1	1	1	1	1	1	1	21	
49	84	62		4	5	5	6	6	6	6	22	
48	87	69		1	2	2	2	2	2	2	23	
47	76	64		3	3	4	4	5	5	5	24	
46	89	77		2	2	3	4	4	4	4	25	
45	51	42	32		5	5	5	6	7	7	26	
44	78	69	54		1	1	1	1	2	2	27	
43	78	60	50		4	5	6	6	6	8	28	
42	79	67	53		2	2	2	2	2	2	29	
41	64	56	49		2	3	4	4	4	4	30	
40	82	65	52	42		2	2	3	3	3	31	
39	63	55	43	33		5	5	6	6	6	32	
38	69	61	47	37		2	2	2	2	2	33	
37	59	54	47	37		5	7	7	9	9	34	
36	65	59	50	38		3	4	5	5	5	35	
35	47	40	34	26	22		5	5	6	7	36	
34	60	50	45	36	25		2	2	2	4	37	
33	51	45	37	30	23		6	8	8	8	38	
32	57	51	43	35	25		2	2	2	2	39	
31	50	42	37	33	24		5	6	6	6	40	
30	52	48	39	32	26	18		2	3	3	41	
29	43	39	35	27	22	15		6	8	8	42	
28	41	40	35	30	22	16		2	2	2	43	
27	38	34	32	27	23	15		7	7	9	44	
26	33	31	28	25	19	13		6	7	7	45	
25	29	25	23	19	16	13	8		7	9	46	
24	30	28	24	23	18	14	7		2	2	47	
23	28	25	23	19	17	12	9		8	8	48	
22	23	21	18	16	13	10	6		2	2	49	
21	25	23	21	18	16	12	9		8	8	50	
20	22	21	19	17	14	13	8	4		3	51	
19	17	17	16	14	12	9	7	4		10	52	
18	13	13	13	12	10	8	6	3		2	53	
17	11	11	11	11	10	8	6	3		7	54	
16	6	6	6	6	6	5	3	2		9	55	
15	5	5	5	5	5	5	4	2				
14	2	2	2	2	2	2	2	1				
13	1	1	1	1	1	1	1	1				
n \ m	10	9	8	7	6	5	4	3				

****

h = 3

h=4

***	***	***	***	***	***	***	***	***	***	
		4	5	6	7	8	9	10	*m* \ *n*	*k*
		1	1	1	1	1	1	1	10	4
			1	1	1	1	1	1	11	
			1	2	2	2	2	2	12	
				2	3	3	3	3	13	
				3	4	5	5	5	14	
					4	5	6	6	15	
					5	7	8	9	16	
						7	9	10	17	
						8	11	13	18	
							11	14	19	
							12	16	20	
								16	21	
								18	22	
		1	1	1	1	1	1	1	13	7
		1	2	2	2	2	2	2	14	
		2	4	5	5	5	5	5	15	
		3	7	9	10	10	10	10	16	
		3	6	11	13	14	14	14	17	
			9	14	19	21	22	22	18	
38	170		8	15	21	26	28	29	19	
37	176		10	16	25	31	36	38	20	
36	168		6	13	20	30	36	41	21	
35	164	104		23	32	41	51	57	22	
34	159	102		21	31	41	51	61	23	
33	143	92		20	38	50	62	72	24	
32	136	94			35	51	64	77	25	
31	129	85	50		43	61	79	94	26	
30	116	87	50		42	65	81	100	27	
29	103	76	51		26	53	80	98	28	
28	98	75	53	35		62	88	113	29	
27	76	59	38	20		67	92	122	30	
26	64	52	36	20		69	100	125	31	
25	56	47	35	21			103	137	32	
24	43	38	29	18	9		105	143	33	
23	33	31	26	17	8		103	141	34	
22	22	21	19	14	6		94	144	35	
21	17	17	16	14	9	3		150	36	
20	9	9	9	8	6	2		146	37	
19	5	5	5	5	4	2		158	38	
18	2	2	2	2	2	1				
17	1	1	1	1	1	1				
n \ *m*	10	9	8	7	6	5				

****

h=5

$h=4$

***	***	***	***	***	***	***	****	***	***	
		4	5	6	7	8	9	10	m \ n	k
		1	1	1	1	1	1	1	11	5
		1	2	2	2	2	2	2	12	
			3	4	4	4	4	4	13	
27	20		4	6	7	7	7	7	14	
26	19		0	4	6	7	7	7	15	
25	18	12		6	10	12	13	13	16	
24	16	11		7	10	14	16	17	17	
23	14	11			13	16	20	22	18	
22	11	9	6		11	18	21	25	19	
21	9	8	6		12	19	26	29	20	
20	7	6	5	3		19	27	34	21	
19	5	5	4	3		21	31	39	22	
18	3	3	3	2			29	40	23	
17	2	2	2	2	1		33	43	24	
16	1	1	1	1	1		30	44	25	
15	1	1	1	1	1	1		47	26	
								46	27	
		1	1	1	1	1	1	1	12	6
		1	2	2	2	2	2	2	13	
		2	4	5	5	5	5	5	14	
		2	6	8	9	9	9	9	15	
33	66		4	9	11	12	12	12	16	
32	75		5	9	14	16	17	17	17	
31	68		6	11	16	21	23	24	18	
30	69	42		10	17	22	27	29	19	
29	61	40		10	16	24	29	34	20	
28	62	42		20	27	35	43	48	21	
27	50	34	18		23	31	40	48	22	
26	50	37	23		23	37	47	56	23	
25	39	32	20		22	35	50	61	24	
24	35	28	21	11		39	53	70	25	
23	27	23	16	10		37	53	68	26	
22	22	20	16	9		48	68	85	27	
21	13	12	10	6	0		56	76	28	
20	12	12	11	9	5		64	89	29	
19	7	7	7	6	4		62	86	30	
18	4	4	4	4	3	1		93	31	
17	2	2	2	2	2	1		85	32	
16	1	1	1	1	1	1		104	33	
n \ m	10	9	8	7	6	5				

****

$h=5$

$h=4$

***	***	***	***	***	***	***	***	***	***	k
		4	5	6	7	8	9	10	m / n	
		1	1	1	1	1	1	1	14	8
		1	2	2	2	2	2	2	15	
		2	4	5	5	5	5	5	16	
		3	7	9	10	10	10	10	17	
		4	7	12	14	15	15	15	18	
		4	11	16	21	23	24	24	19	
		5	12	20	26	31	33	34	20	
44	324		12	20	30	36	41	43	21	
43	337		12	20	29	40	46	51	22	
42	339		14	28	38	49	60	66	23	
41	340		11	24	36	47	59	70	24	
40	321	200		30	46	60	73	85	25	
39	311	196		26	44	59	74	88	26	
38	297	194		33	56	75	92	109	27	
37	279	181		33	60	82	100	118	28	
36	255	164	78		53	89	113	133	29	
35	256	183	106		50	75	108	131	30	
34	237	170	107		64	96	127	162	31	
33	204	151	92		55	85	122	151	32	
32	194	142	96	47		103	140	181	33	
31	162	126	83	43		86	127	166	34	
30	152	116	85	48		97	139	187	35	
29	127	99	68	46		118	160	203	36	
28	108	86	60	34	16		161	215	37	
27	89	75	54	33	15		143	200	38	
26	77	67	53	34	18		158	223	39	
25	56	51	41	28	14		144	212	40	
24	43	41	36	26	15	5		221	41	
23	27	26	24	19	10	4		212	42	
22	19	19	18	16	11	4		239	43	
21	10	10	10	9	7	3		236	44	
20	5	5	5	5	4	2				
19	2	2	2	2	2	1				
18	1	1	1	1	1	1				
n / m	10	9	8	7	6	5				

****

$h=5$

$h=4$

***	***	***	***	***	***	***	***	***	***	
		4	5	6	7	8	9	10	m / n	k
		1	1	1	1	1	1	1	15	9
		1	2	2	2	2	2	2	16	
		2	4	5	5	5	5	5	17	
		3	7	9	10	10	10	10	18	
		4	7	12	14	15	15	15	19	
		5	12	17	22	24	25	25	20	
		6	14	22	28	33	35	36	21	
		6	16	25	35	41	46	48	22	
			14	24	34	45	51	56	23	
49	581		20	35	47	59	70	76	24	
48	580		16	30	43	56	69	80	25	
47	573		19	35	53	68	83	96	26	
46	577		16	36	52	69	85	101	27	
45	590	378		38	65	83	102	120	28	
44	520	326		41	67	91	108	128	29	
43	507	314		47	72	105	132	151	30	
42	508	334		41	72	103	131	157	31	
41	471	308			75	112	147	178	32	
40	460	316	177		85	122	165	196	33	
39	419	278	172		82	123	165	210	34	
38	407	285	165		79	121	168	213	35	
37	366	260	152		74	116	164	216	36	
36	354	270	174	88		143	187	240	37	
35	305	227	154	77		139	192	245	38	
34	272	211	141	78		140	215	270	39	
33	240	179	127	68		134	197	266	40	
32	216	170	119	73			211	285	41	
31	189	148	107	64	29		210	287	42	
30	163	130	94	63	26		222	304	43	
29	138	114	83	52	27		223	316	44	
28	112	97	74	48	26		184	280	45	
27	91	81	66	45	24	8		306	46	
26	66	61	51	37	21	7		317	47	
25	48	46	41	31	19	7		323	48	
24	29	28	26	21	12	5		321	49	
23	20	20	19	17	12	5				
22	10	10	10	9	7	3				
21	5	5	5	5	4	2				
20	2	2	2	2	2	1				
19	1	1	1	1	1	1				
n / m	10	9	8	7	6	5				

****

$h=5$

362-120 O - 69 - 5

$h=4$

*** *** *** *** *** *** *** *** *** ***

		4	5	6	7	8	9	10	m \ n	k
		1	1	1	1	1	1	1	16	10
		1	2	2	2	2	2	2	17	
		2	4	5	5	5	5	5	18	
		3	7	9	10	10	10	10	19	
		4	7	12	14	15	15	15	20	
		5	12	17	22	24	25	25	21	
		7	15	23	29	34	36	37	22	
		7	18	27	37	43	48	50	23	
		8	18	29	39	50	56	61	24	
		8	22	39	52	64	75	81	25	
55	864		22	37	52	66	79	90	26	
54	940		24	41	60	77	93	106	27	
53	955		22	42	60	78	96	113	28	
52	895		25	50	75	95	115	135	29	
51	915		22	46	76	99	118	139	30	
50	855	514		54	79	114	140	161	31	
49	897	541		51	85	114	145	170	32	
48	825	511		60	98	139	170	204	33	
47	833	540		54	92	131	176	204	34	
46	771	523		62	112	160	205	250	35	
45	714	468	256		101	144	196	242	36	
44	692	466	272		101	155	208	266	37	
43	700	474	281		106	163	216	272	38	
42	632	439	265		112	177	232	298	39	
41	603	431	268		100	168	237	288	40	
40	558	393	249	128		175	249	326	41	
39	512	380	239	124		175	252	326	42	
38	462	345	223	120		179	261	332	43	
37	424	329	223	125		177	270	361	44	
36	375	287	203	112		212	308	411	45	
35	324	253	175	102	42		260	367	46	
34	302	231	170	103	46		274	384	47	
33	257	206	145	92	41		288	393	48	
32	230	184	138	86	44		285	386	49	
31	195	160	119	83	39		292	418	50	
30	162	137	104	68	39	12		419	51	
29	126	111	87	59	32	11		414	52	
28	101	91	76	54	31	11		416	53	
27	71	66	56	42	25	9		404	54	
26	50	48	43	33	21	8		476	55	
25	30	29	27	22	13	6				
24	20	20	19	17	12	5				
23	10	10	10	9	7	3				
22	5	5	5	5	4	2				
21	2	2	2	2	2	1				
20	1	1	1	1	1	1				
n \ m	10	9	8	7	6	5				

****

$h=5$

$h=6$

***	***	***	***	***	***	***	***	
		6	7	8	9	10	m \ n	k
		1	1	1	1	1	21	6
			1	1	1	1	22	
			1	2	2	2	23	
			1	2	3	3	24	
				3	4	5	25	
				3	5	6	26	
				4	7	9	27	
					7	10	28	
					8	13	29	
					8	14	30	
						16	31	
						16	32	
						18	33	
		1	1	1	1	1	24	9
		1	2	2	2	2	25	
		2	4	5	5	5	26	
		3	7	9	10	10	27	
		4	11	16	18	19	28	
		5	16	25	30	32	29	
		6	16	32	42	47	30	
		6	22	41	59	69	31	
			22	48	72	91	32	
49	171		28	57	93	119	33	
48	170		26	62	103	144	34	
47	167		29	67	121	172	35	
46	165		24	66	122	189	36	
45	164	84		89	154	230	37	
44	131	56		88	165	253	38	
43	123	56		88	189	293	39	
42	116	57		88	190	315	40	
41	101	52			210	353	41	
40	90	49	18		209	372	42	
39	74	42	17		231	413	43	
38	63	38	15		233	441	44	
37	48	32	12		186	416	45	
36	39	30	16	4		450	46	
35	25	20	12	3		473	47	
34	16	14	9	3		482	48	
33	9	8	6	2		489	49	
32	5	5	4	2				
31	2	2	2	1				
30	1	1	1	1				
n \ m	10	9	8	7				

****

$h=7$

h = 6

***	***	***	***	***	***	***	***	
		6	7	8	9	10	*m* \ *n*	*k*
		1	1	1	1	1	22	7
		1	2	2	2	2	23	
		1	3	4	4	4	24	
			4	6	7	7	25	
38	10		5	9	11	12	26	
37	10		6	12	16	18	27	
36	9		0	9	16	20	28	
35	8	4		12	23	30	29	
34	7	4		14	24	36	30	
33	5	3		15	31	43	31	
32	4	3			31	51	32	
31	3	2	1		36	59	33	
30	2	2	1		33	61	34	
29	1	1	1		34	67	35	
28	1	1	1	1		71	36	
						72	37	
						76	38	
		1	1	1	1	1	23	8
		1	2	2	2	2	24	
		2	4	5	5	5	25	
		2	6	8	9	9	26	
		3	9	14	16	17	27	
		3	12	20	25	27	28	
44	48		9	23	32	37	29	
43	54		11	25	41	50	30	
42	50		13	30	49	66	31	
41	52		13	32	58	79	32	
40	44	20		36	65	96	33	
39	43	20		35	71	109	34	
38	34	18		35	73	120	35	
37	32	16		56	98	151	36	
36	23	12	0		95	156	37	
35	23	16	7		97	176	38	
34	16	12	6		96	179	39	
33	11	9	5		98	197	40	
32	7	6	4	1		193	41	
31	4	4	3	1		211	42	
30	2	2	2	1		210	43	
29	1	1	1	1		228	44	
n \ *m*	10	9	8	7				

****

h = 7

h = 6

***	***	***	***	***	***	***	***	
		6	7	8	9	10	m / n	k
		1	1	1	1	1	25	10
		1	2	2	2	2	26	
		2	4	5	5	5	27	
		3	7	9	10	10	28	
		5	12	17	19	20	29	
		6	18	27	32	34	30	
		8	20	37	47	52	31	
		9	29	50	69	79	32	
		11	34	65	91	111	33	
		11	39	77	118	146	34	
		12	45	92	143	189	35	
55	388		49	102	169	230	36	
54	458		47	108	184	265	37	
53	454		54	133	221	319	38	
52	420		53	136	240	356	39	
51	418		54	153	278	413	40	
50	385	154		150	293	451	41	
49	380	159		168	331	520	42	
48	342	152		162	344	549	43	
47	326	156		179	387	639	44	
46	281	150		182	412	690	45	
45	248	122	42		396	733	46	
44	221	116	41		404	740	47	
43	206	112	45		441	814	48	
42	170	98	39		429	811	49	
41	148	89	40		452	893	50	
40	115	73	32	8		880	51	
39	93	63	31	8		938	52	
38	66	48	23	7		900	53	
37	49	39	23	7		930	54	
36	30	25	16	5		1020	55	
35	18	16	11	4				
34	10	9	7	3				
33	5	5	4	2				
32	2	2	2	1				
31	1	1	1	1				
n / m	10	9	8	7				

****

h = 7

$h=8$

		8	9	10	$m \diagup n$	k
		1	1	1	36	8
			1	1	37	
			1	2	38	
			1	2	39	
			1	3	40	
				3	41	
				4	42	
				4	43	
				5	44	
		1	1	1	37	9
		1	2	2	38	
		1	3	4	39	
		1	4	6	40	
			5	9	41	
49	1		6	12	42	
48	1		7	16	43	
47	1		8	20	44	
46	1		0	16	45	
45	1	1		20	46	
				23	47	
				25	48	
				26	49	
		1	1	1	38	10
		1	2	2	39	
		2	4	5	40	
		2	6	8	41	
		3	9	14	42	
		3	12	20	43	
		4	16	30	44	
		4	20	40	45	
55	0		16	46	46	
54	9		19	51	47	
53	8		22	60	48	
52	7		23	66	49	
51	6		24	75	50	
50	5	1		79	51	
49	4	1		85	52	
48	3	1		84	53	
47	2	1		84	54	
46	1	1		120	55	
$n \diagup m$	10	9				

$h=9$

$h=10 \vert = k$

10	$m \diagup n$
1	55

I thank Professor D. H. Lehmer for his kindness in going through this paper before it was submitted for publication.

(Paper 73B4–312)

Publications of the National Bureau of Standards*

Selected Abstracts

Haber, S., Osgood, C. F., **On a theorem of Piatetsky-Shapiro and approximation of multiple integrals,** *Math. Comput.* **23,** *No. 105, 165–168 (Jan. 1969).*

Key words: Banach-Steinhaus theorem; fourier series; multiple integrals; numerical analysis; number theory; periodic functions; quadrature.

Quadrature formulas are derived for certain classes of functions of several variables having absolutely convergent Fourier Series. A limitation on formulas of the type considered is proven.

Newman, M., **Maximal normal subgroups of the modular group,** *Proc. Am. Math. Soc.* **19,** *No. 5, 1138–1144 (Oct. 1968).*

Key words: Linear fractional groups; maximal normal subgroups; simple groups.

The principal results of this paper are: (1) The modular group Γ contains infinitely many maximal normal subgroups of finite index which are not congruence groups, and (2) any element of $LF(2, q)$ other than $\pm I$ may be written as the product of an element of period 2 and an element of period 3, provided that $p > 3$, where $q = p^n$.

Olver, F. W. J., **Error bounds for the Laplace approximation for definite integrals,** *J. Approx. Theory* **1,** *293–313 (1968).*

Key words: Asymptotic expansions; Bessel functions; definite integrals; error bounds; gamma function; Laplace approximation.

Explicit error bounds are obtained for the well-known asymptotic expansion of integrals of the form

$$\int_a^b e^{-\lambda p(x)} q(x)\, dx$$

in which λ is a large positive parameter, $p(x)$ and $q(x)$ are real differentiable functions, and $p'(x)$ has a simple zero in the finite or infinite range $[a, b]$. The bounds are expressed in terms of the supremum of a certain function taken over $[a, b]$, and are asymptotic to the absolute value of the first neglected term in the expansion as $\lambda \to \infty$. Several illustrative examples are given, including modified Bessel functions and the gamma function.

Other NBS Publications

J. Res. Nat. Bur. Stand. (U.S.), 73A (Phys. and Chem.) No. 5, 451–561 (Sept.–Oct. 1969), $1.50.

Conversion of existing calorimetrically determined thermodynamic properties to the basis of the International Practical Temperature Scale of 1968. T. B. Douglas.

Measured enthalpy and derived thermodynamic properties of alpha beryllium nitride, Be_3N_2, from 273 to 1200 K. T. B. Douglas and W. H. Payne.

Measured enthalpy and derived thermodynamic properties of solid and liquid lithium tetrafluoroberyllate, Li_2BeF_4, from 273 to 900 K. T. B. Douglas and W. H. Payne.

Nonanalytic vapor pressure equation with data for nitrogen and oxygen. R. D. Goodwin.

Some precise measurements of the vapor pressure of water in the range from 25 to 100 °C. H. F. Stimson.

The configurations $(3d+4s)^n 4p$ in neutral atoms of calcium, scandium, and titanium. C. Roth.

Absolute isotopic abundance ratio and atomic weight of terrestrial rubidium. E. J. Catanzaro, T. J. Murphy, E. L. Garner, and W. R. Shields.

J. Res. Nat. Bur. Stand. (U.S.), 73C (Eng. and Instr.) Nos. 1 & 2, 1–46 (Jan.–June 1969), $1.25.

An interferometer for measuring gradients in both refractive index and thickness of large or small optics. J. B. Saunders.

A Kerr electro-optical technique for observation and analysis of high-intensity electric fields. Esther C. Cassidy and Harold N. Cones.

Student-*t* deviate corresponding to a given normal deviate. Brian L. Joiner.

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Laboratory measurements of air cavity temperature in a passenger car tire. B. G. Simson and J. Mandel.

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