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Viscoelastic Behavior of Dental Amalgam*

P. L. Oglesby, G. Dickson, M. L. Rodriguez,**
R. M. Davenport, and W. T. Sweeney

Institute for Materials Research, National Bureau of Standards, Washington, D.C. 20234

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Measurements made on dental amalgam in tension indicate that amalgam exhibits three types of viscoelastic phenomena: (1) instantaneous elastic strain, (2) retarded elastic strain (transient creep), and (3) viscous strain (steady state creep). The combination of elastic plus retarded strain can be represented by an equation of the form $\epsilon = A\sigma + B^2\sigma^2$ where A and B are functions of time but not of the stress, σ. The viscous strain rate can be represented by an equation of the form $\dot{\epsilon}_v = K\sigma^m$ where K and m are constants of the material. By applying a nonlinear generalization of the Boltzmann superposition principle to a general equation describing the creep behavior of amalgam, the results of creep tests can be directly related to the results of stress-strain tests.

Key Words: Amalgam; creep; dental; steady-state creep; stress-strain; transient creep; viscoelastic.

1. Introduction

There is no information in the literature on the viscoelastic behavior of dental amalgam except the limited amount that can be obtained from stress-strain relationships [1, 2].[1] The stress-strain data indicate that amalgam exhibits a nonlinear relation between stress and strain over the entire range of stress investigated; this has been found to be true in both tension and compression. It has been further noted that the shapes of the stress-strain curves vary with the strain rate indicating that the strain developed is not only dependent on the applied stress but also upon time of application of the stress. The results of tensile stress-strain investigation by Rodriguez and Dickson [1] indicated that dental amalgam might be a viscoelastic material. It was, therefore, the objective of this study (1) to make an exploratory investigation of the types of viscoelastic phenomena exhibited by dental amalgam in tension, (2) to describe the viscoelastic phenomena exhibited by amalgam in terms of available viscoelastic theory and (3) to make available a practical example and method of application of viscoelastic theory to a dental material.

2. Theory

In general a strain-hardened material will exhibit one or more of three aspects of viscoelastic phenomena: (1) elastic behavior, (2) viscous behavior (steady-state creep), and (3) retarded elastic behavior (transient creep).

In a material which exhibits all three phenomena linearly related to the applied stress and in which the responses are additive (obey a superposition relation), the strain at any time may be represented by the equation

$$\epsilon = J_0\sigma + \sigma \int_0^\infty J'(\tau)(1-e^{-t/\tau})d\tau + \frac{\sigma t}{\eta}, \tag{1}$$

where

ϵ is the creep strain,
$J_0\sigma$ is the instantaneous elastic response,
$\sigma \int_0^\infty J'(\tau)(1-e^{-t/\tau})d\tau$ is the retarded elastic response,
$\frac{\sigma t}{\eta}$ is the viscous response,
σ is the applied stress,
J_0 is the elastic compliance, the reciprocal of the elastic modulus,
$J'(\tau)$ is a continuous distribution of retarded elastic compliances as a function of the variable retardation time τ of a continuous distribution of responses
t is the time after application of the stress, and
η is the coefficient of viscosity.

The relationship between creep strain and stress may not be linear as assumed above. For example, a

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**Present address: Dental School, Loyola University, 6363 St. Charles Ave., New Orleans, Louisiana. 70118.
[1] Figures in brackets indicate the literature references at the end of this paper.

material may obey the following equation:

$$\epsilon = J_0\sigma + \sigma \int_0^\infty J'(\tau)(1-e^{-t/\tau})d\tau$$

$$+\sigma^2 \left[\int_0^\infty \beta(\gamma)(1-e^{-t/\gamma})d\gamma\right]^2 + K(\sigma-\sigma_0)^m t, \quad (2)$$

where $K(\sigma-\sigma_0)^m t = 0$ for all values of $\sigma \leqslant \sigma_0$ and

$\beta(\gamma)$ is a continuous distribution of retarded elastic compliances as a function of the variable retardation time γ of a continuous distribution of responses in the nonlinear second degree stress response of the material,

σ_0 is the applied stress level at which the material begins to exhibit a viscous flow, and

K and m are constants at any given temperature.

Where the creep strain is linearly related to the applied stress, the strain can be written as a product of a general creep compliance, which is a function of time only, and the applied stress as given by the following equation:

$$\epsilon = J(t)\sigma, \quad (3)$$

where

ϵ is the creep strain developed,
σ is the applied stress, and
$J(t)$ is the general creep compliance.

For example, in the linear creep behavior as seen in equation (1), the general creep compliance $J(t)$ is of the form:

$$J(t) = J_0 + \int_0^\infty J'(\tau)(1-e^{-t/\tau})d\tau + \frac{t}{\eta}. \quad (4)$$

In the case that creep strain is linearly related to the applied stress, the stress-strain behavior for a material may be related to the creep behavior by means of the linear superposition principle developed by Boltzmann [3] as shown below:

$$E(\sigma_t) = \int_0^{\sigma_t} J(T-\theta)d\sigma(\theta), \quad (5)$$

or

$$E(T) = \int_0^T J(T-\theta)\frac{d\sigma(\theta)}{d\theta}d\theta, \quad (6)$$

where

σ is the applied stress related to the experimental time θ in a prescribed experimental functional relation,

$E(T)$ is the strain of a stress-strain relation measured at a time T subsequent to the variable time θ, and

$\frac{d\sigma(\theta)}{d\theta}d\theta$ is the increment of stress from time θ to time $(\theta+d\theta)$.

For example, applying the Boltzmann superposition principle to the example of creep given in eq (1) gives the following relation between creep behavior and the strain measured in a stress-strain test [4]:

$$E(T) = J_0\sigma + \int_0^T \int_0^\infty J'(\tau)(1-e^{-\frac{(T-\theta)}{\tau}})d\tau \frac{d\sigma(\theta)}{d\theta}d\theta$$

$$+\int_0^T \frac{\sigma(\theta)}{\eta}d\theta. \quad (7)$$

A discussion of the application of the Boltzmann superposition principle as applied to linear viscoelastic phenomena is given by Leaderman [5]. However, if a material exhibits a nonlinear relation in its retarded strain (transient creep) and the applied stress, as discussed earlier, the linear Boltzmann superposition principle cannot be applied to relate strain of a stress-strain test on the material to the creep strain. A generalization of Boltzmann's superposition principle developed by Nakada [6] may be used to relate nonlinear retarded elastic creep behavior to the stress-strain behavior of the material as given:

$$E_R(\sigma) = \int_0^{\sigma(T)} \Psi(T-\theta)d\sigma(\theta) +$$

$$\int_0^{\sigma(T)}\int_0^{\sigma(T)} \Phi(T-\theta, T-\phi)d\sigma(\theta)d\sigma(\phi)$$

$$+\int_0^{\sigma(T)}\int_0^{\sigma(T)}\int_0^{\sigma(T)} F(T-\theta,$$

$$T-\phi, T-\alpha)d\sigma(\theta)d\sigma(\phi)d\sigma(\alpha) + \ldots \quad (8)$$

Now for example, consider the application of the nonlinear superposition principle of Nakada as applied to the case of retarded strain (transient creep) related to the applied stress by a second degree equation as shown earlier:

$$\epsilon_R = \sigma\Psi(t) + \sigma^2\Phi^2(t) \quad (9)$$

where $\Psi(t)$ and $\Phi(t)$ could be represented by the equation shown below in accordance with the Voigt model

$$\Psi(t) = \int_0^\infty J'(\tau)(1-e^{-t/\tau})d\tau,$$

$$\Phi(t) = \int_0^\infty \beta(\gamma)(1-e^{-t/\gamma})d\gamma.$$

If one applies an approximation of Nakada's general formulation described by Leaderman, McCrackin, and Nakada [7], the equation representing the retarded elastic response takes the following form:

$$E_R(\sigma(T)) = \int_0^{\sigma(T)} \Psi(T-\theta)d\sigma(\theta)$$

$$+\int_0^{\sigma(T)}\int_0^{\sigma(T)}\Phi(T-\theta)\Phi(T-\phi)d\sigma(\theta)d\sigma(\phi). \tag{10}$$

Considering an example of a material which exhibits all three phenomena but in which the creep strain is a nonlinear function of the applied stress according to eq (2), then applying the above approximation of the nonlinear superposition principle, the stress-strain behavior of the material may be related to the creep behavior by the following equation:

$$E=\sigma J_0+\int_0^T \Psi(T-\theta)d\sigma(\theta)$$

$$+\int_0^T\int_0^T \Phi(T-\theta)\Phi(T-\phi)d\sigma(\theta)d\sigma(\phi)$$

$$+\int_0^T K(\sigma(\theta)-\sigma_0)^m d\theta. \tag{11}$$

However, as will be discussed later, it was found in the investigation that a different approximation is required to describe the behavior of dental amalgam.

3. Materials

The specimens were prepared using a commercial alloy for dental amalgam certified to comply with American Dental Association Specification No. 1. This alloy (composition approximately Ag 70%, Sn 26%, Cu 3.5%, Zn 0.5%) was mixed with mercury and condensed into a mold as described by Rodriguez and Dickson [1] to produce a specimen with dimensions as shown in figure 1. Specimens were aged for at least one week to obtain essentially full mechanical strength [8].

4. Procedure

The dumbbell-shaped specimen was placed in the grips and Tuckerman optical strain gages were mounted on opposite sides of the specimen as shown in figure 2. To obtain the creep curve, readings were taken on the strain gages, a weight was suspended from the lower grip and a second strain gage reading was taken immediately. Strain readings were then made at 15 s intervals for 4 min and at increasingly longer intervals until the strain rate became constant (usually after approximately 1.5 hr). At the end of this period, the load was removed, a strain reading was taken immediately and the recovery curve was followed by reading first at 15 s intervals and then at longer intervals until the strain became constant.

Strain readings obtained on the two sides of the specimen were averaged and strain was plotted against time to obtain the loaded creep and unloaded recovery curves. Readings on the strain gages were normally made to the nearest 2×10^{-5} in. Since a gage length of 0.25 in was used this is equivalent to a strain of 8×10^{-5}. Thus, in the results given below, differences in strain of 1×10^{-4} are approximately equal to the minimum reading difference.

Loads placed on the specimen (with a nominal 0.01 in^2 cross-sectional area) varied from 5 to 40 lb giving stresses from approximately 500 to 4,000 psi. Most specimens were used for several runs, first at high and then at lower stresses. The first loaded creep run was considered a strain hardening treatment and data obtained on these runs were not used in the calculation of results other than for viscous strain rate. All runs were made at 23 ± 1 °C.

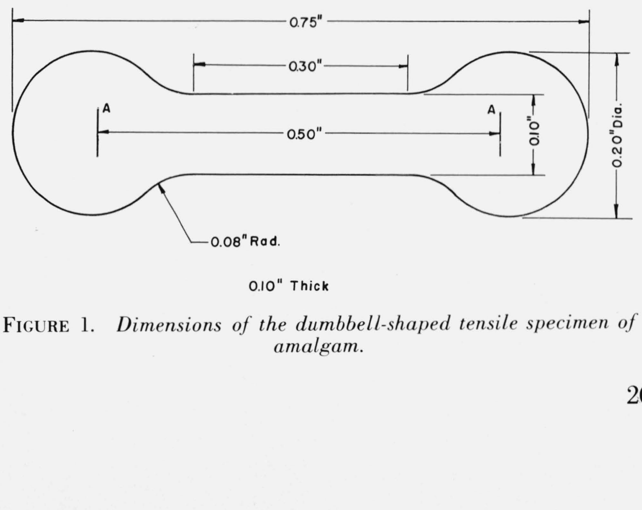

FIGURE 1. *Dimensions of the dumbbell-shaped tensile specimen of amalgam.*

5. Results and Discussion

The creep curves (both loaded and recovery curves) of strain hardened dental amalgam as shown for a

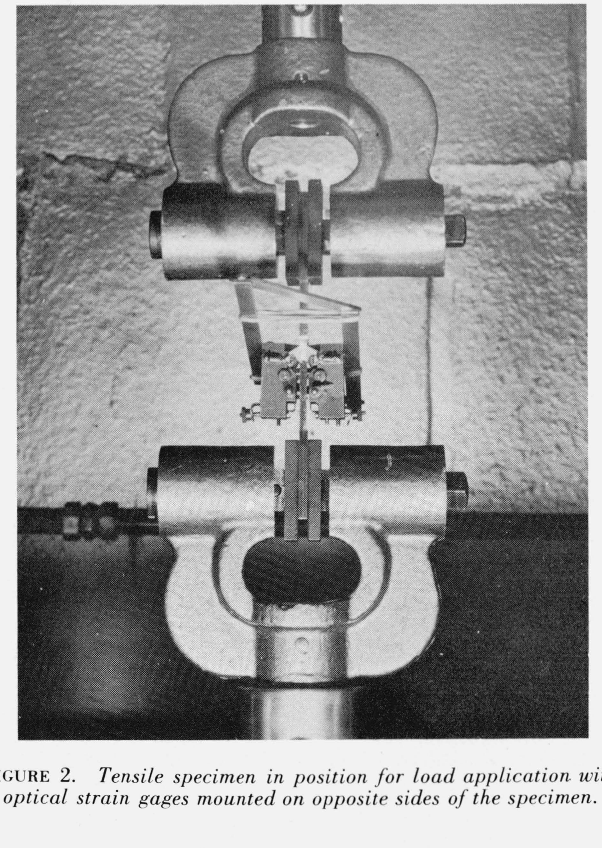

FIGURE 2. *Tensile specimen in position for load application with optical strain gages mounted on opposite sides of the specimen.*

number of different stress levels in figure 3 indicate that at room temperature amalgam exhibits three different types of viscoelastic phenomena: (1) instantaneous elastic strain, (2) retarded elastic strain (transient creep), and (3) viscous strain (steady-state creep).

The viscous strain rate was determined from the loaded portion of the creep curve by taking the slope of the straight line portion of the curve, and was also determined from the recovery portion of the creep curve by dividing the value of the recovery strain (the permanent strain in the specimen) by the total time the load was on the specimen. The viscous strain rates for any given creep curve as calculated from the loaded and recovery portions of the curve were found to agree fairly well as shown in table 1. The log of the viscous strain rate was found to be a linear function of the log of the applied stress, as shown in figure 4; thus the viscous strain rate could be related to the applied stress by the following equation:

$$\dot{\epsilon}_v = K\sigma^m, \qquad 12)$$

where

$\dot{\epsilon}_v$ is the viscous strain rate,
σ is the applied stress, and
K and m are constants of the material.

The value of m for amalgam is the value of the slope of the curve in figure 4, while the value of K is the antilog of the viscous strain rate value at a value of applied stress σ of 1 psi. For the dental amalgam used in this investigation values for K of 2.85×10^{-19} and 4.98×10^{-19} were obtained from loaded and unloaded data respectively, and values of 3.99 and 3.92 were obtained for m.

The strain developed in amalgam due to the other two phenomena (1) instantaneous elastic strain and (2) retarded strain can be determined from the strain recovery since these two types of strain are recoverable while the viscous strain is not. Thus, at any given load the strain values taken from the creep curve after the sample has been unloaded (that is in the recovery portion of the creep curve) are subtracted from the strain value on the creep curve at the instant just before unloading of the specimen. This difference is plotted against recovery time $t_i = T_i - T_u$ where T_u is the time at which the specimen was unloaded and

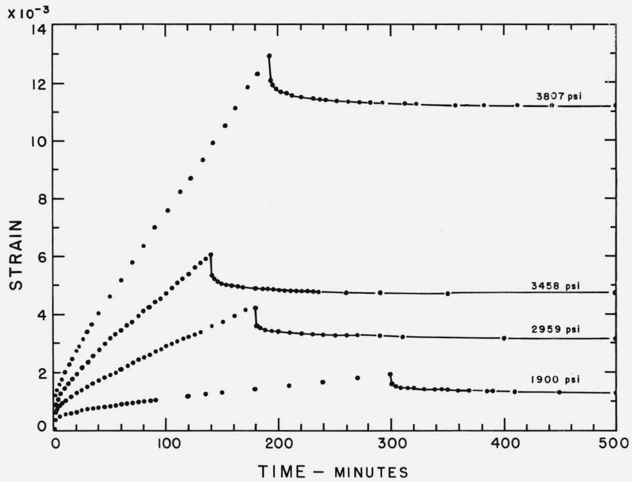

FIGURE 3. *Creep and recovery of amalgam loaded in tension.*

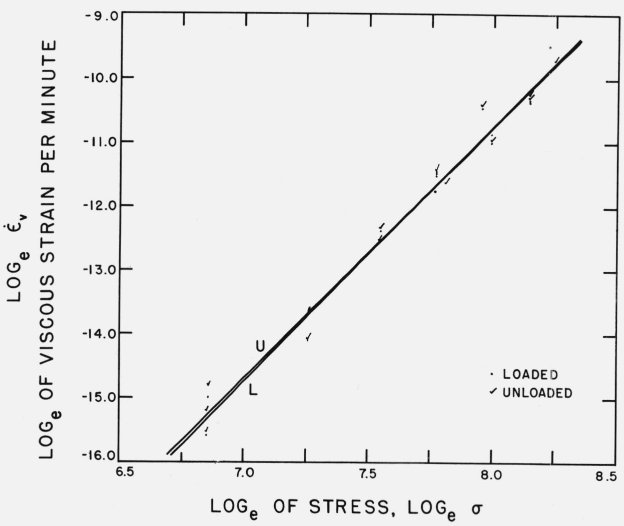

FIGURE 4. *Relationships between viscous strain rate and stress.*
Straight lines are least-squares fits of equation $\log \dot{\epsilon}_v = \log K + m \log \sigma$ to the data, with errors assumed to be in $\log \dot{\epsilon}_v$ only. Loaded values are from straight portion of creep curve; unloaded values are from strain remaining in specimen after unloading and recovery.

TABLE 1. *Viscous strain rates*

Specimen — run	Stress lb/in²	Strain per minute: Loaded $\times 10^{-5}$ [a]	Strain per minute: Unloaded $\times 10^{-5}$ [b]
41–40–1	3807	5.65	[c] 6.35
41–40–2	3807	5.98	5.80
39–40–1	3731	4.92	[c] 5.74
39–40–2	3731	5.23	5.08
40–40–1	3731	7.45	[c] 8.04
43–35–1	3458	3.12	[c] 3.71
43–35–2	3458	3.23	3.28
43–35–3	3458	3.42	3.44
44–30–1	2959	1.92	[c] 1.98
44–30–2	2959	1.68	1.68
44–30–3	2959	1.74	1.72
41–30–3	2855	2.84	2.96
44–25–4	2465	0.91	0.89
35–25–1	2375	1.05	[c] 1.33
35–25–2	2375	0.98	1.06
35–25–3	2375	1.03	1.08
38–25–1	2366	1.09	[c] 1.40
38–25–2	2366	0.783	0.821
38–25–3	2366	.785	.806
35–20–4	1900	.384	....
35–20–5	1900	.419	0.435
38–20–4	1892	.349	.358
35–15–6	1425	.111	.113
38–15–5	1419	.076	.077
35–10–7	950	.031	.034
38–10–6	946	.023	.025
38–10–7	946	.017	.018
35–05–8	475	.008	.007

[a] From slope of straight portion of loaded creep curve.
[b] From strain remaining in specimen after unloading and recovery.
[c] These values include effects of strain hardening and were not used in calculating the relation between stress and viscous strain rate.

T_i is the time of the strain value on the recovery portion of the curve. These difference values, ϵ', are seen plotted against the recovery time for each load or stress in figure 5. These plots are a measure of the combination of the elastic and retarded elastic strain behavior of dental amalgam as a function of time for various stress levels. A measure of the combination of elastic and retarded strain may also be obtained from the loaded portion of the creep curve by taking values off the loaded creep curve and subtracting the viscous strain accumulated in the specimen at that time. The accumulated viscous strain at any time may be calculated by multiplying the viscous strain rate by the time corresponding to that value on the creep curve. Thus the difference between the creep curve value on the loaded portion and the viscous strain value at a corresponding time is a measure of the combination of the instantaneous and retarded elastic strain. However, a small error in the viscous strain rate causes a large error in the difference value. Therefore, the plot of the combination of elastic and retarded elastic strain versus time as obtained from the loaded creep curve is subject to large possible error.

Using the approximation of Nakada's general formulation described by Leaderman, McCrackin, and Nakada [7], the ordinates of the inverted recovery curves as described above should be greater in the early portion of the time scale than the ordinates of the loaded portion of the creep curves minus the viscous strain. The difference between the curves should rise to a maximum within the first few minutes and gradually decrease to zero as time increases. However, as shown in figure 6, the loaded curves appear to lie above the recovery curves. Fitting such data to Nakada's formulation would be very difficult. Moreover, the differences are not large and the method of obtaining values for the loaded curves is subject to considerable error because the loaded curves had to be corrected by subtracting the viscous component. Thus it is believed that the differences between the curves are not significant and so the curves were treated as though they are superimposable. Therefore, the data reported for the combination of elastic and retarded elastic strain were obtained from the recovery portion of the creep curves.

The combination elastic strain (instantaneous and retarded elastic strain) becomes asymptotic with time in accordance with theory as seen in figure 5. The combination elastic strain values were plotted as a function of the various stress levels for corresponding time, as shown in figure 7. The combination elastic strain is seen to be a nonlinear function of the applied stress. When the combination strain values were divided by their corresponding stresses and then plotted against the corresponding stress for a fixed time, a linear plot was obtained for each fixed time as illustrated in figure 8; this result indicated that the combination elastic behavior of amalgam as a function of applied stress under the test conditions could be represented by an equation of the form:

$$\epsilon' = A(t)\sigma + B^2(t)\sigma^2, \tag{13}$$

where

ϵ' is the combination of elastic and retarded elastic strain, and
σ is the applied stress.

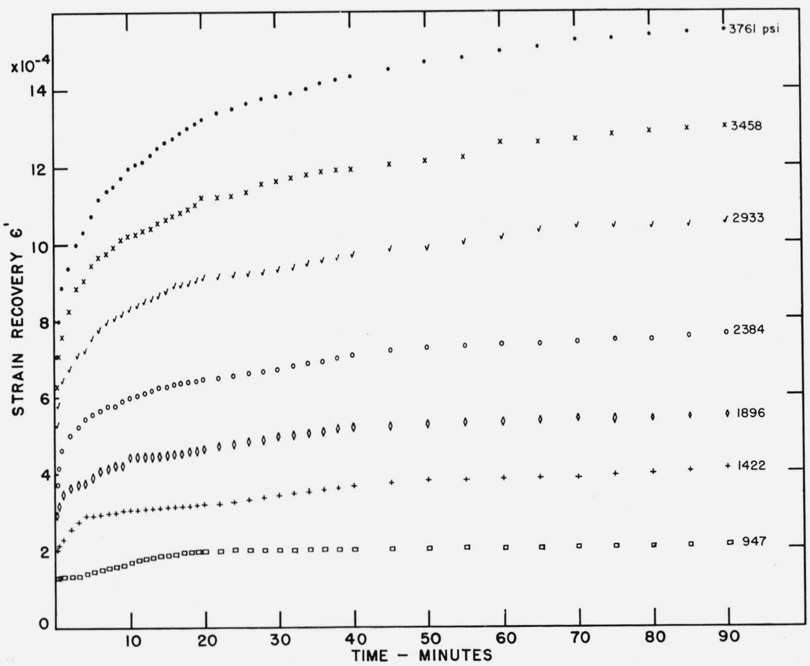

FIGURE 5. *Strain recovery after release of tensile stress.*
Each curve is an average of 2 to 7 runs.

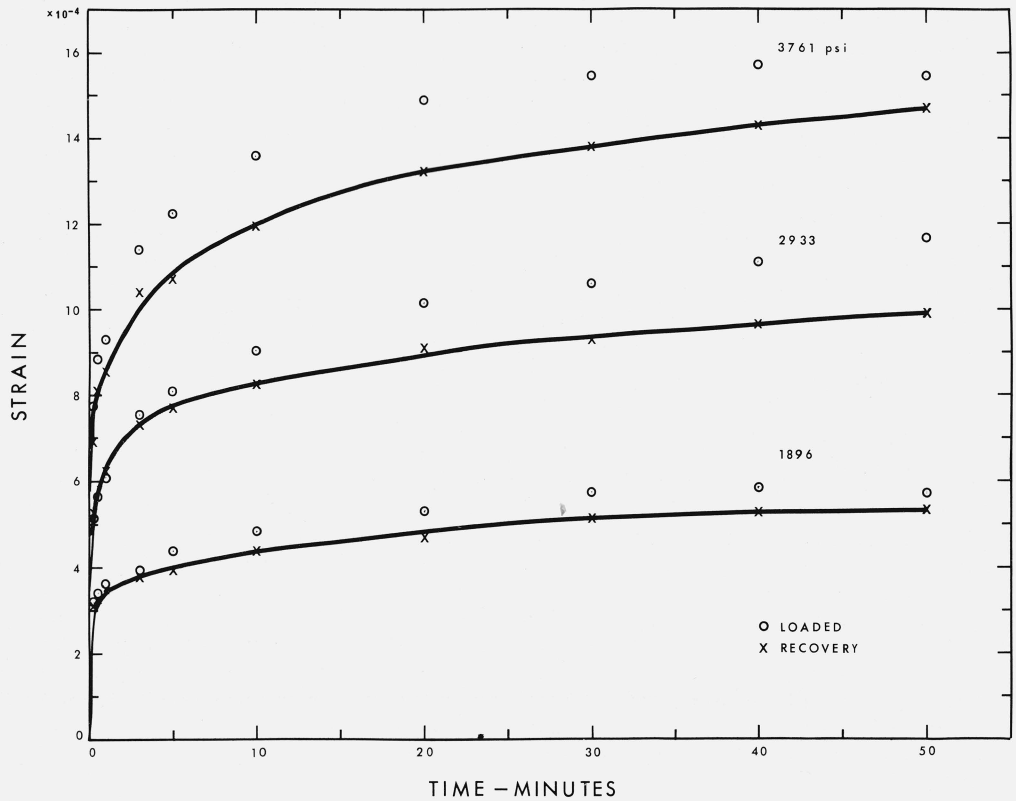

FIGURE 6. *Comparison of inverted recovery curves with loaded creep curves minus the viscous strain.*

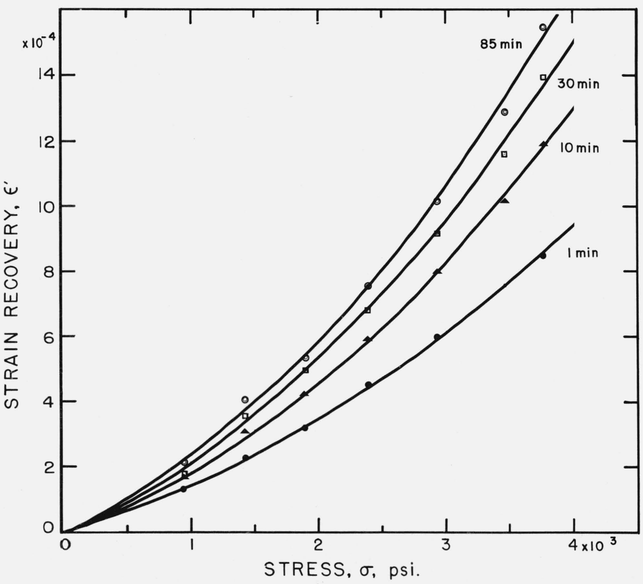

FIGURE 7. *Relationships between recovery strain, stress and time.*

Plotted points are averages of 2 to 7 determinations. Curves are calculated from least-squares fits of the equation $\epsilon'/\sigma = A(t) + B^2(t)\sigma$ to the data with error assumed to be in ϵ'/σ only.

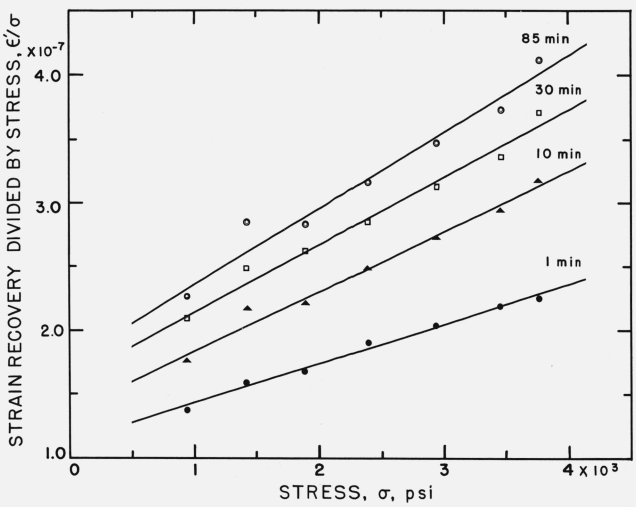

FIGURE 8. *Relationships between recovery strain, stress and time.*

Plotted points are averages of 2 to 7 determinations. Straight lines represent the constants obtained by least-squares fit of the equation $\epsilon'/\sigma = A(t) + B^2(t)\sigma$ to the data with error assumed to be in ϵ'/σ only.

$A(t)$ and $B^2(t)$ are functions [2] of time but not of stress. The value of $A(t)$ for any time value is the intercept at $\sigma=0$ of the plot for that time value as shown in figure 8 while $B^2(t)$ is the slope of the straight line for that time value. It is also noted in figure 8 that as a function of stress the combination strain divided by the stress is a straight line for all values of t. This indicated that over all ranges of t, the combination elastic strain obeys the same functional relation to the stress. Since the recovery curves were found to be superimposable upon the loading curves minus the viscous portion, it appeared that the nonlinear material did not obey the approximation of nonlinear generalization previously noted [7]. It was then questioned whether the retarded behavior of amalgam was truly nonlinear or whether the nonlinear behavior could be attributed to the geometry of the specimen being observed. However, longer specimens were tested and found to give the same result, also photoelastic specimens were made and the stress distribution over the area observed was found to be uniform. It is therefore concluded that the observed nonlinear behavior of dental amalgam is not a geometric artifact, but an intrinsic phenomenon in the material. The difference between the recovery curves and the loading curves minus the viscous portion is given by [7]:

$$2\sigma_0^2\,[\Phi(\Delta T,T)-\Phi(T,T)],$$

where ΔT is the time the specimen was under the load σ_0. For amalgam, this difference is found to be zero, so:

$$\Phi(\Delta T,T)=\Phi(T,T) \tag{14}$$

for creep recovery from equilibrium, i.e., for ΔT large. McCrackin [9] has suggested that this condition be satisfied by assuming that the value of Φ is only dependent on the value of the least of its arguments. That is,

$$\Phi(T_1,T_2)=B^2(T_1)\text{ if }T_1\leq T_2. \tag{15}$$

With this condition, the quadratic term of eq (8) is shown in the Appendix to reduce to

$$\int_0^{\sigma(T)} B^2(T-\theta)\,d[\sigma^2(\theta)] \tag{16}$$

so, eq (8) reduces to:

$$E_R(T)=\int_0^{\sigma(T)} A(T-\theta)\,d\sigma(\theta)+\int_0^{\sigma(T)} B^2(T-\theta)\,d[\sigma^2(\theta)]. \tag{17}$$

[2] The notation $B^2(t)$ rather than $B(t)$ is used for consistency with the form of the notation of Leaderman, McCrackin, and Nakada [7].

TABLE 2. *Values of* A(t) *and* B²(t) *in equation*

$\epsilon'=A(t)\sigma+B^2(t)\sigma^2$

Time – min	$A(t)\times10^{-7}$	$B^2(t)\times10^{-11}$
0.25	1.130	1.906
.50	1.120	2.468
1.00	1.132	3.065
2.00	1.183	3.548
3.00	1.206	3.876
5.00	1.248	4.285
10.00	1.356	4.737
20.00	1.518	5.100
30.00	1.598	5.364
40.00	1.626	5.636
50.00	1.648	5.839
60.00	1.668	5.970
70.00	1.693	6.031
80.00	1.727	6.050
90.00	1.761	6.064
120.00	1.812	6.223
150.00	1.877	6.224
200.00	1.952	6.210
300.00	2.074	6.082
600.00	2.103	6.290
1000.00	2.079	6.497

ϵ' is combination of instantaneous and retarded elastic strain.
σ is stress.
$A(t)$ and $B^2(t)$ are constants for any time and are functions of time but not of stress.

The values for $A(t)$ and $B^2(t)$ were determined by fitting curves to the data by the method of least squares and were tabulated as a function of time as shown in table 2.[3] The $A(t)$ values were plotted as a function of the corresponding t values as shown in figure 9. The $A(t)$ values are seen to approach an asymptote, A_{as} as $t\to\infty$. Thus, the curve as shown in figure 9 could be represented by the following equation from linear viscoelastic theory since $A(t)$ is the linear term in stress:

$$A(t)=J_0+\int_0^{\infty} J'(\tau)(1-e^{-t/\tau})\,d\tau, \tag{18}$$

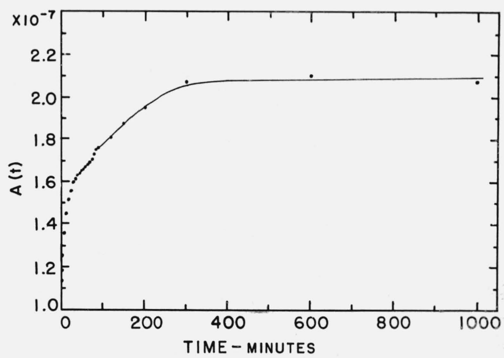

FIGURE 9. *Variation with time of the linear creep compliance term,* A(t), *in the equation* $\epsilon'=A(t)\sigma+B^2(t)\sigma^2$.

[3] Strain values for different stresses were obtained at each of the specific times listed in table 2 (and for many other times not listed in the table) by interpolation between the recorded values of strain along each of the 25 to 30 creep and recovery curves. Then the strain values for a specific time were fitted to the equation $\epsilon'/\sigma=A(t)+B^2(t)\sigma$ to obtain the values of $A(t)$ and $B^2(t)$ for that specific time.

$$A(t)=J_0+\int_0^\infty J'(\tau)d\tau-\int_0^\infty J'(\tau)e^{-t/\tau}d\tau, \tag{19}$$

$$A(t)=A_{as}-\int_0^\infty J'(\tau)e^{-t/\tau}d\tau, \tag{20}$$

$$A_{as}-A(t)=\int_0^\infty J'(\tau)e^{-t/\tau}d\tau, \tag{21}$$

where

$A(t)$ is the creep compliance term which is linear in stress,
J_0 is instantaneous elastic compliance,
$J'(\tau)$ is the retarded elastic compliance spectrum as a function of the retardation time τ, and
A_{as} is the asymptote value of $A(t)$ as t becomes very large.

Thus, eq (20) does describe the behavior of $A(t)$ as a function of time as seen in figure 9. The linear creep compliance term $A(t)$ is plotted as a function of log t to obtain a signoidal curve as shown in figure 10. The first plateau of the signoidal plot at very small values of t should correspond to J_0 (the instantaneous elastic compliance) [10]. The value of J_0 calculated from the curve is 1.13×10^{-7} in^2/lb, equivalent to a modulus of elasticity of 8.9×10^6 lb/in^2. This value is in good agreement with a value of 9.1×10^6 reported for the modulus of dental amalgam determined by an ultrasonic method [11].

The nonlinear creep compliance term $B^2(t)$ was plotted against log t to obtain a sigmoidal curve as shown in figure 11. Thus, in the nonlinear theory of Nakada [6] the experimental $B(t)$ for amalgam might be represented by the following equation:

$$B(t)=\sqrt{B^2(t)}=\int_0^\infty \beta(\gamma)(1-e^{-t/\gamma})\,d\gamma \tag{22}$$

which would indeed describe the behavior of the curves seen in figure 11. The combination elastic behavior of dental amalgam in creep under the test conditions used could then be described by means of the following equation from viscoelastic theory [6, 10]:

$$\epsilon'=A(t)\sigma+B^2(t)\sigma^2,$$

$$\epsilon'=J_0\sigma+\sigma\int_0^\infty J'(\tau)(1-e^{-t/\tau})\,d\tau$$
$$+\left[\int_0^\infty \beta(\gamma)(1-e^{-t/\gamma})\,d\gamma\right]^2\sigma^2. \tag{24}$$

and since the loaded portion of the creep curve for amalgam is also composed of viscous strain ϵ_v as well as combination elastic strain ϵ' then the strain on the loaded portion is composed of the sum of the two

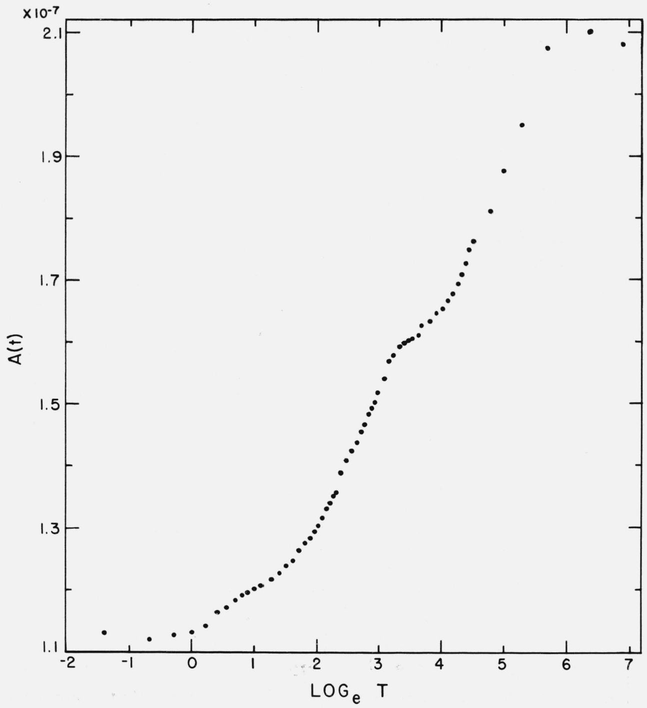

FIGURE 10. *Variation with log of time of the linear creep compliance term,* A(t), *in the equation* $\epsilon'=A(t)\sigma+B^2(t)\sigma^2$.

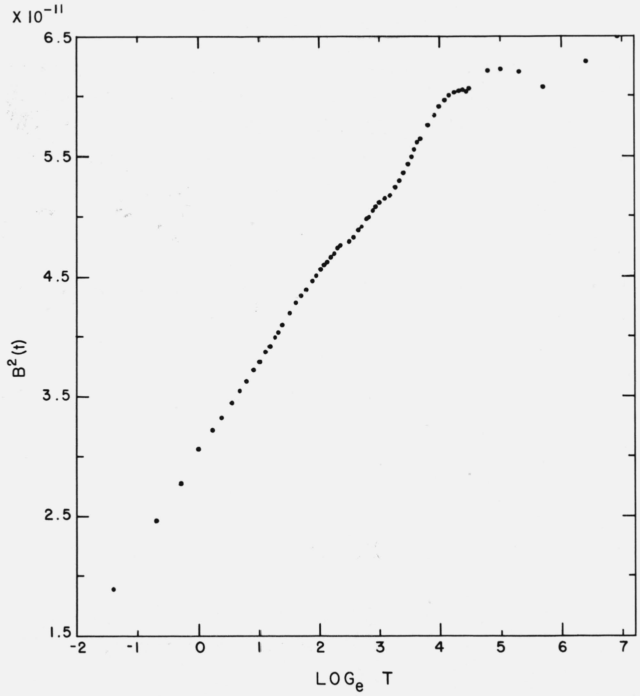

FIGURE 11. *Variation with log of time of nonlinear creep compliance term,* $B^2(t)$, *in the equation* $\epsilon'=A(t)\sigma+B^2(t)\sigma^2$.

as follows:

$$\epsilon = \epsilon' + \epsilon_v. \tag{25}$$

Thus

$$\epsilon = J_0 \sigma + \sigma \int_0^\infty J'(\tau)(1 - e^{-t/\tau})\, d\tau$$

$$+ \left[\int_0^\infty \beta(\gamma)(1 - e^{-t/\gamma})\, d\gamma\right]^2 \sigma^2 + K\sigma^m t. \tag{26}$$

Therefore, applying both linear and nonlinear viscoelastic theory to the experimental behavior of amalgam under the test conditions used, a general equation describing the creep behavior of amalgam as given by eq (26) above is obtained.

Applying the nonlinear generalization of the Boltzmann superposition principle as developed by Nakada [6], to the above creep equation for amalgam, the stress-strain curves for various stress rate conditions were calculated for amalgam from the creep data and compared to the experimental stress-strain curves obtained under those conditions. The stress-strain curve corresponding to the experimental stress rate was calculated from the following equation:

$$E_T(n\Delta t) = \sum_{i=1}^{n} (\sigma_i - \sigma_{i-1}) \left(\frac{A_{n-1} + A_{n+1-i}}{2}\right)$$

$$+ \sum_{i=1}^{n} (\sigma_i^2 - \sigma_{i-1}^2) \left(\frac{B_{n-1}^2 + B_{n+1-i}^2}{2}\right)$$

$$+ \Delta t \sum_{i=1}^{n} K \left(\frac{\sigma_{i-1} + \sigma_i}{2}\right)^m \tag{27}$$

which is a finite difference approximation to eqs (17) and (12). For the experimental data, the testing machine was run at a constant head speed which produced a varying stress rate over a total time of 3½ min. From the stress versus time curve, observed stress values were obtained at 15-s intervals. Using the values of $A(t)$ and $B(t)$ calculated for 15-s intervals from the creep data and also the viscous constants from these data, a curve was calculated for comparison with the experimental curve. Figure 12 shows the contribution of instantaneous elastic, retarded elastic and viscous components to the total calculated strain. It is seen in figure 13 that good agreement is obtained between the calculated and experimental stress-strain curves. Thus, it is concluded that in the case of dental amalgam the results of creep tests can be related to those of stress-strain tests by use of viscoelastic theory [6, 10].

6. Conclusion

Dental amalgam was found to exhibit three types of viscoelastic phenomena: (1) instantaneous elastic strain, (2) retarded elastic strain (transient creep), and (3) viscous strain (steady state creep).

The combination of elastic plus retarded strain in tension was found to be a nonlinear function of

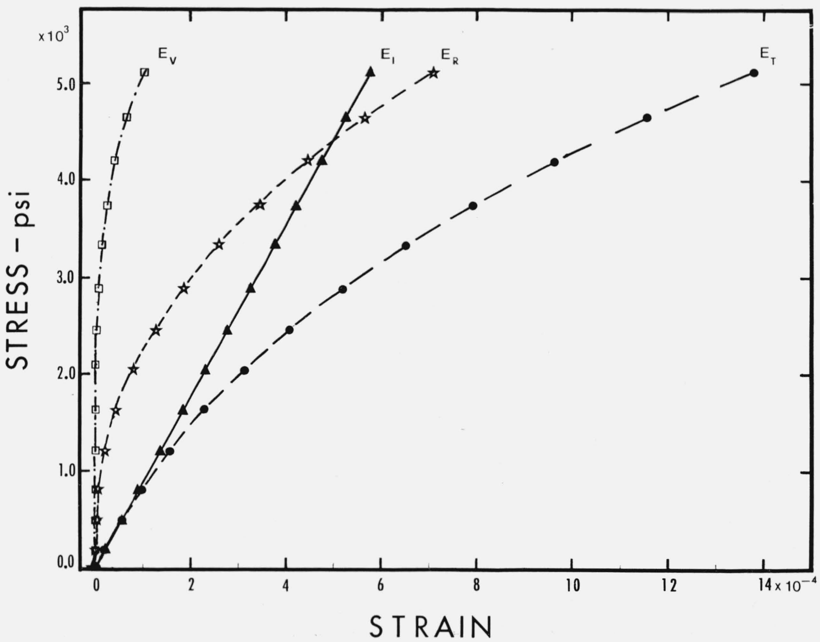

FIGURE 12. *Calculated instantaneous elastic strain E_I, retarded elastic strain E_R, viscous strain E_V, and total strain E_T occurring during a stress-strain test of dental amalgam.*

Head speed of testing machine was constant, and total time was 3½ min.

stress and could be represented by an equation of the following form:

$\epsilon' = A\sigma + B^2\sigma^2$ where A and B are functions of time, but not of stress.

The viscous strain rate is also a nonlinear function of stress and can be represented by an equation of the form:

$\epsilon_v = K\sigma^m$ where K and m are constants of the material.

An equation describing the creep behavior of amalgam was obtained by the application of nonlinear viscoelastic theory to the experimental behavior of amalgam under the test conditions used. By applying the nonlinear generalization of the Boltzmann superposition principle to this equation the results of creep tests were related to results of stress-strain tests.

7. Appendix

The derivation of eq (16) is as follows [9]:

$$\int_0^{\sigma(T)}\int_0^{\sigma(T)} \Phi(T-\theta, T-\phi)\, d\sigma(\theta)\, d\sigma(\phi)$$

$$= \int_0^{\sigma(T)}\int_0^{\sigma(\theta)} B^2(T-\theta)\, d\sigma(\phi)\, d\sigma(\theta)$$

$$+ \int_0^{\sigma(T)}\int_{\sigma(\theta)}^{\sigma(T)} B^2(T-\phi)\, d\sigma(\phi)\, d\sigma(\theta).$$

The first integral is

$$\int_0^{\sigma(T)} B^2(T-\theta)\left[\int_0^{\sigma(\theta)} d\sigma(\phi)\right] d\sigma(\theta)$$

$$= \int_0^{\sigma(T)} B^2(T-\theta)\sigma(\theta)\, d\sigma(\theta)$$

$$= \frac{1}{2}\int_0^{\sigma(T)} B^2(T-\theta)\, d[\sigma^2(\theta)].$$

Applying Dirichlet's formula [12] to the second integral,

$$\int_{\theta=0}^{T}\left[\int_{\phi=\theta}^{T} B^2(T-\phi)\frac{d\sigma(\phi)}{d\phi}\frac{d\sigma(\theta)}{d\theta}\, d\phi\right] d\theta$$

gives

$$\int_{\phi=0}^{T}\left[\int_0^{\phi} B^2(T-\phi)\frac{d\sigma(\phi)}{d\phi}\frac{d\sigma(\theta)}{d\theta}\, d\theta\right] d\phi$$

$$= \int_0^{T}\left[B^2(T-\phi)\frac{d\sigma(\phi)}{d\phi}\int_0^{\phi}\frac{d\sigma(\theta)}{d\theta}\, d\theta\right] d\phi$$

$$= \int_0^{T} B^2(T-\phi)\frac{d\sigma(\phi)}{d\phi}\sigma(\phi)\, d\phi$$

$$= \frac{1}{2}\int_0^{T} B^2(T-\phi)\frac{d[\sigma^2(\phi)]}{d\phi}\, d\phi.$$

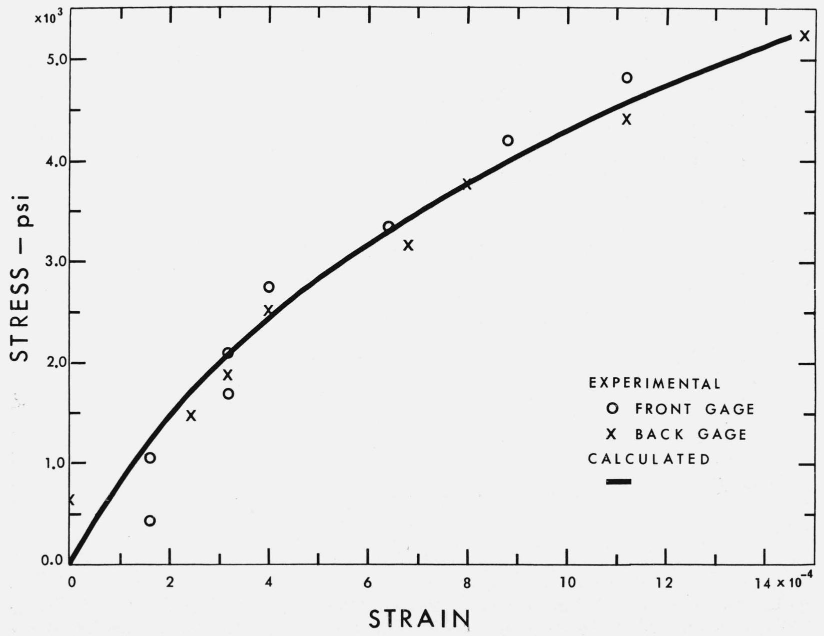

FIGURE 13. *Comparison of calculated and experimental stress-strain curves.*

However, this is the same as the first integral as θ and ϕ are dummy variables of integration so the sum of the integrals is

$$\int_0^T B^2(T-\theta)\frac{d[\sigma^2(\theta)]}{d\theta}d\theta=\int_0^{\sigma(T)} B^2(T-\theta)d[\sigma^2(\theta)].$$

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