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# Valence-Only Correlation in LiH and BeH<sup>+\*</sup>

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The pseudonatural orbital procedure has been applied to the calculation of the potential energy curve of LiH and the dissociation energy of BeH<sup>+</sup>. Only the two-electron bonding pair is correlated and estimates of  $\sigma$  and  $\pi$  type correlation are obtained. The results for LiH are in good agreement with the most accurate previously published calculations. Comparison with experimental results for LiH indicates that the calculated dissociation energies are accurate to about 0.15 to 0.2 eV.

Key Words: BeH+, correlation energy, dissociation energy, LiH, molecular orbital, potential energy curve.

# 1. Introduction

The calculation of accurate potential energy curves of small molecules cannot be achieved within the framework of the Hartree-Fock one-electron model. Consideration of the correlation is necessary to insure both the correct asymptotic dependence and the depth and shape of the curve near the equilibrium separation. In order to simplify the problem correlation considerations should be limited to only those electrons intimately involved in the binding process, i.e., the valence electrons. This permits the utilization of localization techniques which limit the number of relevant electrons that must be correlated. Prior to the introduction of localization techniques it is necessary to show that, as expected, only the valence electrons need be considered. Evidence for the validity of this concept has been accumulated by extended Hartree-Fock (H. F.) calculations  $[1]^1$  on the molecules  $H_2$  and  $Li_2$ , for which the bonding valence electrons are a simple pair. Additional work has been done on systems for which intershell pairs are significant [2]. However, such complicated systems fall beyond the range of the present investigation, which is to add to the evidence for the accuracy of potential curves calculated by correlating only the two-electron valence shell.

The simplest case beyond  $H_2$  is the isoelectronic LiH sequence. Numerous calculations [3] of this molecule have appeared over the years but the relative simplicity of the model has enticed all the investigators to do as complete a calculation as possible within their model. Those studies which consider correlation therefore apply a correlated trial function to both the Li K shell and the bonding shell. Although the probability of success for a valence-shell-only treatment is predictably high, we feel that such a calculation is required as a basis for future work in this area.

The calculation in this paper is equivalent to the extended H. F. calculation with a frozen  $1\sigma(K \text{ shell})$ H. F. molecular orbital. The actual procedure follows the pseudonatural orbital (PNO) procedure previously applied to the three-electron system [2]. In effect the virtual H. F. orbitals are transformed into approximate natural orbitals [4] which span the same region of space as the occupied H. F. valence orbitals. Such orbitals provide for rapid convergence in the superposition of configurations (SOC) calculation. A number of points of the LiH curve were calculated in this way. The results are compared in particular to the natural orbital based calculation of Bender and Davidson [3f] which is the most accurate calculation for LiH. We can anticipate our conclusion by noting that the outer-shell correlation energies for the two calculations are quite comparable.

The isoelectronic system BeH<sup>+</sup> is also considered only at the equilibrium internuclear separation. Since the electronic distributions in the H. F. solution for LiH and BeH<sup>+</sup> are quite different, the correlation

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results shed light on different bonding cases ranging from the ionic or strongly polarized through the covalent. For the LiH and BeH<sup>+</sup> molecules the techniques utilized have proved convenient and quantitative.

### 2. Pseudonatural Orbitals and Basis Functions

All calculations are based on trial functions of the form

$$\psi(1, 2, 3, 4) = A\psi_1(1)\psi_1(2)\psi(3, 4)$$

where  $\psi_1$  is an accurate approximation to the  $1\sigma$  H. F. molecular orbital,  $\varphi$  is a two-electron pair function including a singlet spin function, and A is the antisymmetrizer. The pair function is held strongly orthogonal to the  $\psi_1$  orbital. The overall symmetry is  ${}^{1}\Sigma^{+}$ and the wave function would agree with the result of an extended H. F. calculation if the  $\psi_1$  orbital were permitted to relax in the field of the pair function.

The  $\varphi(3, 4)$  is determined here in two steps. Using the  $\sigma$  virtual H. F. solutions, a superposition of all configurations formed by all single and double excitations from the valence shell molecular orbital is diagonalized. Similarly, an arbitrarily orthonormalized set of  $\pi$  orbitals is diagonalized. The first-order density matrices for both the  $\sigma$  and  $\pi$  substituted wave functions are diagonalized by determining the equivalent transformation which diagonalized the coefficient matrix for the SOC expansion. The approximate or pseudonatural orbitals are now available for a complete SOC involving both  $\sigma$  and  $\pi$  excitations. Only diagonal excitations are now considered and the final SOC requires a limited number of interaction integrals in the new basis, thereby simplifying the calculation.

The use of pseudonatural orbitals is most convenient when intershell pairs are significant. Extended H. F. procedures would actually be better in the present case, since only one pair is being correlated. However, the results are essentially equivalent and are obtained with comparable effort.

The basis set is built around the best atom Gaussiantype function (GTF) H. F. solutions of Huzinaga [5]. As is widely known [6], the natural orbitals are localized in the same region of space as the respective H. F. orbitals. Except for polarization orbitals of new symmetry type the H. F. basis should suffice. Polarization orbitals were added by scaling GTF fits to  $p\sigma$  and  $p\pi$ functions and varying the scale factor to determine the best energy. The variations were by no means exhaustive and improvements are possible. The variations were done only for LiH at 3.0 a.u. and the same basis was used for the entire curve (or appropriately scaled for the BeH<sup>+</sup> case).

Small exponent s and p GTF are centered on the Li atom. They are intended to account for the polarization of the charge distribution toward the H atom. Whether the LiH charge density can be represented by an ionic model is somewhat debatable [7] with the present evidence leaning toward the polarization of a diffuse Li 2s function. The large set on the Li actually

contributes to the wavefunction near H with the heavy weight to the small exponent functions.

The accuracy of the basis is best judged in this light by comparison with the results using significantly different bases. The BeH<sup>+</sup> charge density has even less resemblance to an ionic model and the basis should be adequate.

The basis sets are defined in table 1 and the PNO coefficients are given in table 2. The first two orbitals are exactly the undisturbed  $1s_{\text{Li}}$  H. F. molecular orbital and the bonding PNO which is very similar to the second occupied H.F. molecular orbital. This orbital has been referred to as a  $1s_{\text{H}^-}$  orbital because of the ionicity of the LiH bond [8]. However, the  $2s_{\text{Li}}$  and  $2p_{\text{Li}}$  densities are far from insignificant. From our results and those of Bender and Davidson on the partitioning of the correlation into  $\sigma$  and  $\pi$  type terms, the identification of this orbital as a  $1s_{\text{H}^-}$  is obviously oversimplified. The basis set that was chosen to represent both the H. F. and correlation of the H centered density.

TABLE 1. Parameters for basis orbitals,  $f(x, y, z) exp(-al^{-2})$ 

(a) LiH	Atomic orbital basis					
Orbital No.	Туре	Center	Exponent			
$1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \\ 22 \\ 23 \\ 24 \\ 10 \\ 10 \\ 10 \\ 11 \\ 12 \\ 12 \\ 22 \\ 23 \\ 24 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 1$	$P \stackrel{\sigma}{P} \stackrel{\sigma}{P} \stackrel{\sigma}{P} \stackrel{\sigma}{P} \stackrel{\sigma}{S} \stackrel{S}{S} $		$\begin{array}{c} 1.34829\\ .31932\\ .098736\\ .75\\ .028643\\ .076663\\ .44462\\ 1.15685\\ .3.15789\\ .9.35328\\ .319415\\ .38.73\\ .921.271\\ .0.101309\\ .321144\\ .1.1468\\ .5.05796\\ .33.6444\\ .29353\\ .0.696844\\ .222852\\ .08074\\ .557104\\ .129568\end{array}$			
(b) $BeH^+$	Atomic ork	oital basis				
$ \begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15\\16\\17\\18\\19\\20\\21\\22\\23\\24\end{array} $	$\begin{array}{c} P_{\sigma}^{\sigma} P_{\rho}^{\sigma} P_{\sigma}^{\sigma} \\ P_{\rho}^{\sigma} P_{\sigma}^{\sigma} S S S S S S S S S S S S S S S S S S S$	Be Be Be Be Be Be Be Be Be H H H H H H H	$\begin{array}{c} 1.483\\351\\109\\75\\0583\\1806\\8589\\1847\\59326\\17.6239\\603255\\262, 139\\1741.38\\1468\\ .$			

1 a.u. (length) = 0.529172 Å

	(a) PNO Coefficients, LiH $R = 2.0$						(c) PNO Coefficients, LiH $R = 3.0$							
Orbital No.	1σ H. F.	2σ H. F.	$2\sigma$	Зσ	$4\sigma$	·5σ	Orbital No:	1σ H. F.	2σ H. F.	$2\sigma$	$3\sigma$	$4\sigma$	$5\sigma$	
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	$\begin{array}{c} 0.004407\\.007878\\002017\\.004964\\000002\\.002588\\.424381\\.341843\\.4160068\\049580\\010380\\010380\\001360\\001627\\010437\\009115\\002351\\000373\end{array}$	0.033912 .091818 .117561 021768 073225 172702 .097015 .118228 .066803 .029541 .008663 .029541 .008663 .001814 .000235 317569 360808 164161 038021 005348	$\begin{array}{c} 0.034077\\.088182\\.114025\\-0.21483\\-0.99539\\-1.74916\\.005932\\.118447\\.067831\\.029685\\.008762\\.001826\\.000876\\010826\\.000827\\310838\\346714\\173403\\3346714\\173403\\038413\\005422\end{array}$	0.008329 068465 231170 072883 .252490 .317555 069496 .015033 .003243 .000617 .0000175 .000000000000000000000000000000000000	$\begin{array}{c} -0.032827\\477243\\554928\\48458\\48458\\225179\\225179\\568379\\568379\\568379\\00212\\069599\\003246\\000279\\030246\\030246\\030246\\030246\\030246\\030246\\030246\\000279\\598575\\378740\\ .012201\\010818\\ .000229\end{array}$	-0.017292 -0.12876 .367876 -234911 -323240 -693758 -0.05188 -1.02560 -0.07132 -0.017122 -0.002056 -0.00788 -0.000788 -0.000788 -1.736012 -1.266605 -0.165061 -0.10251	$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \end{array}$	$\begin{array}{c} 0.006264\\ .005319\\001506\\ .000962\\000437\\ .002998\\ .170797\\ .423435\\ .344369\\ .160459\\ .010409\\ .001366\\ .000747\\ .002781\\ .001290\\ .000281\\ .000050\\ \end{array}$	$\begin{array}{c} 0.016858\\ .069495\\ .107464\\016321\\085497\\207120\\ .042737\\ .086114\\ .03994\\ .019650\\ .005456\\ .001184\\ .000150\\364782\\318529\\140493\\034264\\004698 \end{array}$	$\begin{array}{c} 0.016266\\ .068472\\ .102147\\014973\\109990\\220749\\ .042031\\ .089866\\ .040816\\ .020300\\ .005598\\ .001220\\ .000154\\342419\\313586\\149304\\19304\\034721\\004787\end{array}$	$\begin{array}{c} 0.011714\\ .067528\\ .276211\\ .018269\\249975\\576710\\ .092888\\ .040843\\ .018641\\ .005257\\ .001118\\ .000143\\303882\\ .33882\\ .33882\\ .33882\\ .00248\\ .007121\\ \end{array}$	$\begin{array}{c} - 0.007152\\380256\\378214\\ .443654\\280662\\ .323136\\326297\\ .003043\\004792\\001997\\000174\\ - 1.304840\\ 0.245100\\ .003527\\ .001194\\ \end{array}$	$\begin{array}{c} - 0.000853\\ - 1.74121\\434746\\231176\\370475\\ .728993\\289679\\030252\\012447\\000831\\ .000431\\ .0000431\\ .0000431\\ .0000431\\654657\\ - 1.045312\\ 1.112123\\ 0.105760\\ .011175\end{array}$	
	$1\pi$	$2\pi$						$1\pi$	$2\pi$					
19 20 21 22 23 24	$\begin{array}{c} 0.006254\\010794\\049588\\049588\\ .499923\\ .591811 \end{array}$	$\begin{array}{r} 0.017242\\ .047861\\ .130557\\026074\\ -1.089518\\ .934826\end{array}$					19 20 21 22 23 24	$\begin{array}{c} 0.005663 \\005169 \\ .161447 \\ .022901 \\ .413339 \\ .622294 \end{array}$	$\begin{array}{r} -\ 0.007912\\ .005612\\ -\ .209146\\ -\ .193739\\ 1.031171\\ -\ 0.630077\end{array}$					
		(b) PNO	Coefficients,	LiH $R = 2.6$	×				(d) PNO	D Coefficients, LiH $R = 3.4$				
$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \end{array}$	$\begin{array}{c} 0.006374\\ .005588\\001458\\ .001650\\000365\\ .002908\\ .171372\\ .423021\\ .343919\\ .160316\\ .049759\\ .010401\\000849\\ .001364\\000849\\ .000437\\ .002763\\ .000597\\ .000109 \end{array}$	$\begin{array}{c} 0.022623\\ .078845\\107365\\018462\\076565\\97091\\ .048569\\ .022722\\ .006484\\ .001382\\ .000177\\342670\\334963\\ .00177\\342670\\342670\\344963\\ .00177\\342670\\35569\\004919\\ .004919\\ \end{array}$	$\begin{array}{c} 0.022271\\ .077196\\ .102291\\017715\\ .100904\\203380\\ .051759\\ .099854\\ .049199\\ .023160\\ .006589\\ .001406\\ .000180\\328782\\325985\\155785\\035967\\005002 \end{array}$	$\begin{array}{c} 0.006199\\ .059848\\ .254043\\ .052101\\ -249082\\484031\\ .101569\\ .048487\\ .027676\\ .011783\\ .003410\\ .0009710\\ .0009710\\ .000091\\458568\\ .829587\\ .383794\\ .053933\\ .007591 \end{array}$	$\begin{array}{c} -0.028556\\398499\\428187\\ .447592\\ .231610\\ 1.190100\\ .336591\\376673\\011197\\050662\\006556\\0002475\\0002475\\0002475\\0002475\\0002475\\000245\\000245\\000253\\003370\\007512\\ .000534\end{array}$	$\begin{array}{c} -0.014421\\111385\\ .458737\\ .198343\\327268\\33528\\ .224806\\134147\\090049\\015508\\000927\\000045\\ .505913\\ 1.280689\\1.74829\\1.74829\\1.74829\\1.74829\\0.127366\\010907\end{array}$	$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \end{array}$	$\begin{array}{c} 0.005777\\.005095\\001441\\.000646\\000436\\002847\\.170049\\.423967\\.344544\\.160571\\.049833\\.010415\\001366\\000574\\.001807\\.000898\\.000151\\.0000023\end{array}$	$\begin{array}{c} 0.012993\\ .058102\\ .110553\\014262\\097410\\213738\\ .040677\\ .075020\\ .034459\\ .017156\\ .004721\\ .001030\\00130\\388998\\302829\\136544\\033149\\004542 \end{array}$	$\begin{array}{c} 0.012383\\ .057180\\ .104984\\012337\\123429\\235591\\ .041490\\ .079468\\ .035914\\ .018024\\ .004935\\ .000136\\356436\\302215\\145546\\033677\\004638\\ \end{array}$	$\begin{array}{c} -0.012011\\ -0.07785\\285139\\ .004920\\ .255716\\ .615901\\088486\\100022\\046361\\022163\\006135\\006135\\000132\\ .184030\\768513\\296663\\296663\\050383\\006625\\ \end{array}$	$\begin{array}{c} 0.015253\\353559\\378649\\ .447394\\ .903941\\ .236146\\268000\\ .014584\\268000\\ .014584\\268000\\ .014584\\004262\\002916\\0001572\\0001572\\0001523\\ -1.297227\\ 0.438748\\ .073463\\ .014132\\ .001653\\ \end{array}$	$\begin{array}{c} 0.018591\\198672\\ .347227\\ .291657\\422054\\562947\\ .272905\\129519\\ .047560\\008843\\ .002546\\000172\\ .000036\\ .650819\\ .853499\\1058826\\ -0.088454\\011337\end{array}$	
	$1\pi$	$2\pi$						$1\pi$	$2\pi$					
19 20 21 22 23 24	$\begin{array}{c} 0.005296\\ .005507\\ .154002\\011169\\ .449336\\ .599113\end{array}$	- 0.009818 002790 187711 115084 1.057181 - 0.739935					19 20 21 22 23 24	$\begin{array}{c} 0.005932 \\012050 \\ .149304 \\ .063263 \\ .382200 \\ .646210 \end{array}$	$\begin{array}{r} 0.007031 \\009586 \\ .216540 \\ .263758 \\ - 1.000147 \\ 0.545015 \end{array}$					

TABLE 2.	Expansion coefficients for the Hartree-Fock and pseudo-
	natural orbitals-Continued

#### 1 a.u. (length)=0.529172 Å

1 a.u. (length) = 0.529172 Å

(e) PNO Coefficients, LiH $R = 4.0$							(g) PNO	Coefficients, I	iH R = 8.0		-		
Orbital No.	1σ H. F.	2σ H. F.	$2\sigma$	- 3σ	$4\sigma$	$5\sigma$	Orbital No.	1σ H. F.	2σ H. F.	2σ	3	-	
$ \begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ \end{array} $	$\begin{array}{c} 0.004917\\.004625\\001250\\.000359\\000404\\.002553\\.342585\\.344632\\.160685\\.049852\\.010420\\.001367\\000350\\.000985\\.0000153\\.000070\\.000008\end{array}$	0.009900 .039332 .116407 010693 119925 215459 .042781 .060143 .029858 .014223 .004007 .0000861 .000109 426049 426049 281052 134137 031798 001401	$\begin{array}{c} 0.009271\\ 0.038823\\ -1.008902\\ -0.007997\\151321\\251724\\ .046063\\ .066555\\ .032587\\ .015584\\ .0004378\\ .000942\\ .000120\\375174\\288216\\143327\\032398\\004500 \end{array}$	$\begin{array}{c} -0.010910\\ -0.048059\\048059\\276563\\ 0.15361\\ .282965\\ .606841\\096618\\097040\\050435\\022909\\006559\\00178\\082667\\693330\\248696\\046355\\0065993\end{array}$	$\begin{array}{c} 0.026950\\251935\\471289\\455548\\429119\\ .890824\\ .044140\\137987\\0002060\\0002060\\000081\\000081\\1273220\\ 0.596246\\ .082459\\ .025582\\ .001990\\ \end{array}$	$\begin{array}{c} 0.031569\\134434\\ .122146\\503933\\275745\\ .117570\\028820\\ .001788\\ .003494\\ .000312\\ .000075\\ .631880\\ .572505\\966482\\071228\\011228\\011228\\011296\end{array}$	$ \begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15\\16\\17\\18\end{array} $	0.001449 001564 .000283 000001 000307 .001755 .168259 .425437 .344690 .344690 .160840 .010429 .001366 0000065 0000000 .000003	0.004125 006912 .084680 003467 .306383 1.147139 036650 041815 0022121 010078 002912 000015 000015 000015 000015 000015 000012 000012 000012 000012 000012 000012 000012 000012 000012 000012 00012 00012 00012 00012 00012 00012 00012 00012 00012 000012 00012 00012 00012 00012 00012 00012 00012 00012 00012 00012 00012 00012 0000000 00000000	0.000824 .00074 .016580 -0.00248 .365248 .355322 -0.69391 -0.71124 -0.71124 -0.071841 -0.07341 -0.07082 -0.00162 -0.00162 -0.00162 -0.00162 -0.00162 -0.00162 -0.00162 -0.00162 -0.00162 -0.00162 -0.00248 -0.00162 -0.0016 -	$\begin{array}{c} -0.00091\\ .00087\\ .00087\\ .00041\\ .00041\\ .08076\\ .0819\\ .04521\\ .01984\\ .00583\\ .00121\\ .01984\\ .00015\\ .22727\\ .39848\\ .15021\\ .03385\\ .00443\\ .00443\end{array}$	- 6 1 2 2 6 3 0 6 8 5 9 7 8 1 5 9 7 8 1 5 9 7 8 1 2 8 1 2 2 2 2 2 2 2 2	
	$1\pi$	2π						1π				-	
19 20 21 22 23 24	$\begin{array}{c} 0.005310 \\012126 \\ .108804 \\ .123893 \\ .347961 \\ .677194 \end{array}$	$\begin{array}{c} 0.006601 \\013210 \\ .203931 \\ .363187 \\950778 \\ .458483 \end{array}$					19 20 21 22 23 24	$\begin{array}{c} -\ 0.000368\\ .009481\\ -\ .028502\\ .134216\\ .349310\\ .755300\end{array}$					
		(f) PNO	Coefficients,	LiH $R = 6.0$					(h) PNO C	oefficients, B	$eH^{+}R = 2.4$	8	
$ \begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ \end{array} $	0.002703 .002877 000616 000011 000319 .001914 .168348 .425325 .344690 .160819 .049869 .010426 .001367 000004 .000089 .000033 .000009	$\begin{array}{c} -0.006172\\ -0.00471\\111914\\ .005416\\ .218056\\039456\\039456\\039456\\023180\\010249\\0003010\\000081\\000081\\ .20038\\ .240391\\ .136112\\ .029438\\ .004280\\ \end{array}$		0.005354 .003205 .121849 0053711 418685 .0467353 .085498 .048901 .021993 .001298 .000169 .104801 .496591 .168076 .037554 .004759	0.025793 152044 .740686 553481 .450527 695251 .253192 114445 0.00057 .000059 000057 .000059 550843 130697 029655 002595	$\begin{array}{c} -0.013727\\.095237\\312469\\.060749\\.100854\\.115331\\006532\\.016090\\.0001180\\.000733\\.000054\\.897501\\220667\\646153\\052456\\009329\\\end{array}$	Orbital No. 1 2 3 4 4 5 6 6 7 8 9 9 10 11 12 13 11 12 13 14 15 16 17 18	$\begin{array}{c} 1 \sigma \ \mathrm{H.\ F.} \\ \hline 0.007213 \\000225 \\ .000239 \\ .000557 \\002563 \\161356 \\425490 \\351194 \\158475 \\047991 \\009950 \\001302 \\0001302 \\000128 \\000128 \\000128 \\000128 \\000124 \\000128 \\000124 \\0000124 \\000124 \\000124 \\000124 \\000124 \\000$	$\begin{array}{c} 2\sigma \ \text{H. F.} \\ \hline 0.047173 \\ .147334 \\ .085880 \\083569 \\ .103897 \\ .434705 \\044723 \\120599 \\056745 \\025758 \\025758 \\007002 \\001494 \\000189 \\ .103976 \\ .341322 \\ .142022 \\ .034818 \\ .004820 \end{array}$	$\begin{array}{c} 2\sigma \\ \hline 0.045504 \\ .137219 \\ .074154 \\031121 \\ .107463 \\ .447353 \\042953 \\ .124988 \\057854 \\026487 \\007162 \\007162 \\001533 \\000194 \\ .102979 \\ .335900 \\ .150177 \\ .035313 \\ .004999 \end{array}$	$3\sigma$ - 0.015329 176527 131239 093298 182799 .065945 .0030270 .008003 .001724 .000216 068896 .760629 .385022 .049967 .007649	$\begin{array}{c} 4\sigma \\ \hline \\ -0.113542 \\470720 \\ .149033 \\ .149033 \\ .149033 \\ .149033 \\ .39832 \\ .339832 $	$\begin{array}{c} 5\sigma \\ \hline \\ 0.061338 \\ .249390 \\043572 \\ .058923 \\369361 \\ .136442 \\044554 \\ .018136 \\ .000387 \\ .001269 \\ .0001269 \\ .0001269 \\ .000024 \\ .175235 \\ .1146932 \\1170663 \\0117906 \\0117906 \\0117933 \\011793$
	1π	ł					18	000023	.004820	.004899	.007649	002056	011233
19 20 21 22 23 24	0.000790 .007953 010902 .197279 .326293 .739552						19 20 21 22 23 24	$\begin{array}{c} 1\pi \\ 0.015553 \\ .115097 \\ .500244 \\067143 \\ .439962 \\ .304166 \end{array}$	$2\pi$ 0.011738 .138439 .582119 002065 958014 .204944				

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The final wavefunctions are obtained with a trial function based on the PNO. The square of the coefficients are listed in table 3. Since only one pair of electrons is excited the C<sup>2</sup> are equivalent to the occupation number for the natural spin orbital. Only configurations with C<sup>2</sup> > 0.0001 are included.

# 3. Analysis of the Correlation

The initially occupied  $1\sigma$  and  $2\sigma$  H. F. orbitals are listed in table 2. Comparison of the  $2\sigma$  H. F. orbitals with the  $2\sigma$  NO shows no appreciable difference until R exceeds 6 a.u. The first NO and the Hartree-Fock orbital are very similar at the equilibrium configuration [9].

The asymptotic behavior of the  $2\sigma$  and  $3\sigma$  PNO results in the correct atom products, the H. F. ground states of H and Li. No other  $\sigma$ PNO contributes to the correlation as R approaches infinity. Of the  $\pi$ orbitals only the  $1\pi$  PNO still contributes significantly at 8 a.u.

The  $3\sigma$  PNO in the neighborhood of the equilibrium separation acts mostly as an in-out type of correlation on the H<sup>-</sup> bonding distribution. Bender and Davidson note that their  $3\sigma$  orbital resembles a 2s hydrogen atom orbital orthogonalized to the 1s Li orbital. In our case the Li portion of the  $3\sigma$  orbital resembles that part of the  $2\sigma$  which serves as a hybridized 2s Li orbital. However, the left-right type of correlation required to assure proper asymptotic dependence is also present. As R increases this type dominates and accounts for the rapid rise in the  $\sigma$  correlation energy.

TABLE 3. Square of the SOC expansion coefficient for a. LiH as a function of the internuclear distance and b. BeH<sup>+</sup> at R = 2.48 a.u.

		(a	) LiH Wave	e Function								
Configu- ration	С²( <i>R</i> )											
	R = 2.0	2.6	3.0	3.4	4.0	6.0	8.0					
$\begin{array}{c} 2\sigma^2 \\ 3\sigma^2 \\ 4\sigma^2 \\ 5\sigma^2 \end{array}$ $\begin{array}{c} 1\pi^2 \\ 2\pi^2 \end{array}$	0.97166 .01398 .00208 .00016 .01176 .00025	0.97175 .01396 .00303 .00017 .01070 .00026	0.96989 .01609 .00329 .00018 .01014 .00026	0.96640 .02032 .00309 .00019 .00959 .00026	0.95723 .03119 .00238 .00020 .00861 .00024	0.85232 .14324 .00052 .00021 .00355 .00007	0.64815 .35131 .00004 .00003 .00045 .00000					
				R = 2.48								
$2\sigma^{2}$ $3\sigma^{2}$ $4\sigma^{2}$ $5\sigma^{2}$ $1\pi^{2}$ $2\pi^{2}$				0.97320 .01428 .00263 .00017 .00930 .00031								

The  $p\sigma$  contribution is significant in the  $4\sigma$  PNO but without plots of the orbitals it becomes increasingly difficult to ascertain the dominant correlating effects. This orbital tends to shift charge along the bond toward the Li.

The only other correlating orbital with a simple effect is the  $1\pi$  PNO, which dominates the angular correlation. As R increases and charge transfers back toward the Li, the angular correlation also becomes more diffuse and eventually goes to zero. The dependence of the  $\sigma$  and  $\pi$  type correlation as a function of R is shown in table 4 along with the total and correlation energy.

The  $\sigma$  and  $\pi$  type correlation are determined by considering the  $\sigma$  and  $\pi$  SOC independently. As a result these estimates do not add up to the total correlation which is the difference between the total energy, for the combined  $\sigma$  and  $\pi$  SOC, and the H. F. energy. Such estimates are always found to exceed the true value as do the C<sup>2</sup> coefficients. Comparison with Bender and Davidson's breakdown of the correlation is encouraging. The  $\sigma$  and  $\pi$  results are essentially identical. As Bender and Davidson note, this correlation breakdown does not correspond to that expected for H<sup>-</sup>.

The dissociation energy relative to the Hartree-Fock products is 2.36 eV. This is within 0.15 eV of the experimental value of 2.51 eV [10]. The neglect of intershell and  $\delta$ -type correlations cannot account for such an error. The error is also not in the convergence properties of the NO expansion for this basis. The results quoted in table 4 are for the entire basis. The PNO listed in table 2 subsume all but about 0.05 eV of the energy contributions from this basis. The basis set is poor and will require further experimentation if an improvement is to be had.

<b>FABLE</b>	4.	Hartree-Fock,	total,	and	correlation	energies	for	LiH
			and	BeH	+			
		1 a.1	u. (energy	= 27.2	20976 eV.			

		Li	Н		
R	-E(H. F.)	-E(tot.)	$-E\sigma$	$-E\pi$	-E(corr.)
2.0	7.911640	7.946792	0.02125	0.01668	0.0351
2.6	7.977167	8.010911	.02138	.01507	.0337
3.4	7.981539	8.015129	02333	01324	0335
4.0	7.966088	8.000858	.02598	.01220	.0347
6.0	7.904227	7.951813	.04407	.01004	.0475
8.0 ∞	7.859542 a 7.932088	7.934728	.07470	.00925	.0751
		Bel	-I +		
2.48	14.85108	14.88497	0.02357	0.01510	0.0348

<sup>a</sup> This result is the approximate Hartree-Fock energies of the atoms for the Gaussian basis used; the accurate sum of H. F. atom energies is -7.9327257 a.u. (See E. Clementi, Tables of Atomic Functions, Supplement to IBM Journal of Research and Development **9**, 2 (1965).)

BeH<sup>+</sup> at the equilibrium configuration presents results comparable to that for equilibrium LiH. The correlation breakdown in terms of  $\sigma$  and  $\pi$  contributions are essentially the same and nearly independent of the bonding characteristics of the respective Hartree-Fock molecules. The dissociation energy is 2.93 eV and on the basis of the LiH calculation should be good to about 0.2 eV.

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