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Digitized Low-Frequency Phasemeter Assembled from Logic Modules

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A digital phasemeter is described which is capable of operating from arbitrarily low frequencies to 10 kHz, for which the lower limit depends essentially upon the time available to the operator to make the measurement. The phasemeter involves a logic circuit which can be assembled easily from commercially available logic modules without the necessity of a proficiency in electronics. The output of the logic circuit is read into a conventional digital preset frequency-ratio meter. The determination is absolute and utilizes a time base. Accordingly, no calibration against a phase standard is necessary. The agreement between this phasemeter and a quality phase shifter was found to be within $\pm 0.01°$ at 400 Hz, which is the accuracy specified by the manufacturer of the phase shifter. Considerable ability to ignore signal imperfections is inherent in this device.

Key Words: Digital, phasemeter, logic circuit, low frequency, measurements, phase.

1. Introduction

This work is concerned with the description of a highly accurate digital phasemeter capable of operating from any arbitrarily low frequency up to 10 kHz continuously. The lower limit on frequency depends essentially upon the time available to make the measurement. The demand for a device of this type often arises in certain types of rheological measurements on substances manifesting very long relaxation times. Apparently most, if not all, commercial production phasemeters have a low-frequency cutoff in the low audio range, at best, and the more accurate ones are capable of measuring at only a single frequency. The device described here utilizes logic [1] circuitry, which can be assembled easily from commercially available logic modules [2] without the necessity of a proficiency in electronics. The output of the logic circuit is applied to a conventional digital preset frequency-ratio meter. The prototype without the frequency-ratio meter is shown in figure 1.

For two periodic signals of the same waveform, but relatively displaced in time, a phase angle may be defined as the angular separation between a pair of corresponding [3] points selected arbitrarily. Thus, using a suitable level detector, a measurement may be obtained on a time base, which has a great advantage because measurements in time may be taken to very high resolution. In the phasemeter to be described zero crossings (with both derivatives) were selected as the characteristic points. This choice is appropriate because the first derivative of a sine function is a maximum when the value of the function is zero. Thus, maximum resolution in the time of the level crossing is obtained. Consequently, only two basic ingredients are necessary to measure phase under ideal conditions, a stable level detector (for example, a high gain, or open loop, operational amplifier, or a voltage comparator) and an accurate time base generator, which is activated by the level detector. If τ is the time interval between corresponding zero crossings and ν is the frequency, the phase angle φ in radians as defined above is

$$\varphi = 2\pi\nu\tau. \tag{1}$$

The nature of errors often encountered over a one cycle measurement using a digital time interval meter (counter) is discussed in reference [2].[4]

An alternative method is to measure the output of a suitable level detector with a voltmeter over many cycles of input signal. Also, a so-called "coincident slicer" [3] has been used in this connection. Some discussion on modifying the logic circuit to use a digital integrating voltmeter in place of the frequency ratio meter is included later in this paper.

[1] "Logic" as commonly used in computer technology, applies to compatible electronic components, which when interconnected appropriately, can implement digital arithmetic and Boolean functions. For a treatise on this subject, see for example reference [1].

[2] For a list of makers of logic modules, consult the current Electronic Engineers Master Catalog, United Technical Publications, Inc., New York (annual periodic publication).

[3] Any point on a sine wave of unit amplitude is uniquely characterized over a period by the value of the function and its first derivative. Points on two different sine waves correspond if they have identical characteristic values.

[4] Figures in brackets indicate the literature references at the end of this paper.

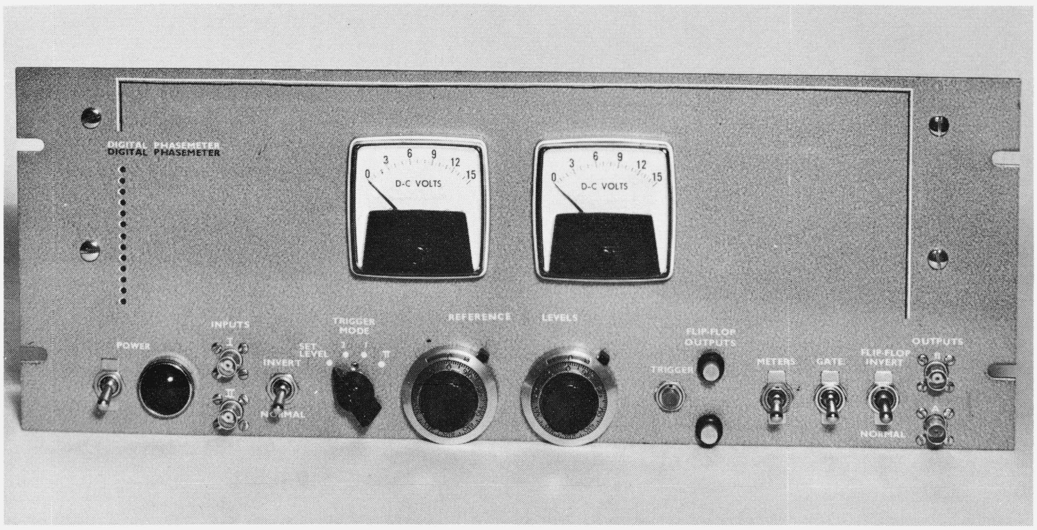

FIGURE 1. *Prototype containing the logic circuit.*

In practice ideal sine waves are never encountered, and in general the quality of the signals will not be as good as desired. Accordingly, the course which was followed in this work was to analyze the effect of signal imperfections on the shift of the zero crossings from those of ideal sine waves, and then to attempt to develop an instrument which would essentially ignore these imperfections and other potential systematic errors.

The results of this work are considered to be very satisfactory. The determination is absolute in the sense that it does not have to be calibrated against a phase standard, but may be referred to the basic frequency standard. It involves only one adjustment which can be made without the use of auxiliary equipment and which may be omitted with only a slight loss in accuracy. On comparison with a quality phase standard operating at 400 Hz with an accuracy within $\pm 0.01°$ claimed by the manufacturer, agreement was found to be slightly better than the accuracy claimed for the above instrument.

2. Design Considerations

2.1. Errors Encountered Using Zero Crossing Detectors

If the frequency of the fundamental is constant, a signal may be considered to consist of the fundamental sine wave plus imperfections, which include d-c offset, noise, and harmonic distortion. With a zero-crossing detector these imperfections will usually influence the position of the zero crossings of an otherwise ideal sine wave. Clearly, if the waveforms of the two signals are the same (including the imperfections), the zero crossings of each signal will be displaced from those of the fundamental by the same amount with no corresponding error in the apparent[5] phase angle. Unfortunately, in practice, the imperfections accompanying the fundamentals of the two signals usually will be significantly different. The imperfections which are considered are d-c offset, noise, including periodic disturbances incommensurable[6] with the signal frequency, and harmonic distortion. Also considered is frequency instability, which is a special case for which the "sine wave" itself does not exist. In order to simplify the analysis, one of the signals will be considered hypothetically to be a pure sine wave, while the other will be considered to contain only one of the following designated imperfections.

a. D-C Offset

Unless a transformer coupled output is used, the presence of a significant d-c offset voltage accompanying the signal is very likely. This is particularly true at low frequencies because of the necessity of using either direct coupled stages with "bucking" voltages, or very large electrolytic capacitors with significant dissipation.

The effect of an offset voltage on the apparent phase angle is easy to analyze. For a given offset voltage, E_{DC}, the positions of the actual zero crossings are the solutions of the equation

$$E_1 \sin \omega t + E_{DC} = 0.$$

For offset voltages small in comparison to signal amplitude, the corresponding phase error using positive-going crossings is

$$\Delta\varphi \approx -\frac{E_{DC}}{E_1}. \qquad (2)$$

Accordingly, the error introduced here may be very serious. For example, if one wishes to make a measure-

[5] In this usage "apparent" applies to the angle given by the phasemeter.

[6] Two frequencies are "commensurable" only if their ratio is a rational number. Thus, two signals with incommensurable frequencies can never have more than one common zero crossing.

ment to within 0.01°, the offset voltage may not exceed 175 μV with a 1 V signal. Since offsets of this magnitude or more are usually encountered in low-frequency measurements and their magnitudes are usually not known or even constant, it is desirable, if not necessary, for accurate measurements to incorporate a technique in the design of the phasemeter to cancel or essentially eliminate errors from this source. Two methods of accomplishing this will be mentioned in later sections. One of these is to utilize both positive- and negative-going crossings for triggering.

The effect of long time instability in level detectors may also be treated in the same fashion as offset voltages accompanying the signals.

b. Noise

Noise may be considered to be a randomly varying offset voltage for which the integral tends to go to zero over large values of the argument, time. Through reasoning similar to that in the last section, a bound on the error in the apparent phase angle measured over one cycle may be approximated by

$$|\Delta\varphi| \sim 1/r \tag{3}$$

where r is the signal to noise ratio. Since noise is usually random in character, and since the errors in time are essentially an odd function of the instantaneous noise, one would expect the phase error to be effectively cancelled if the measurement is taken over many periods of sine wave. This is accomplished by using a preset frequency-ratio meter.

Periodic fluctuations, such as 60 Hz hum, for which the frequency is incommensurable with the signal frequency have an effect similar to noise. Although these fluctuations are not random, by the specific nature of their regularity and incommensurability (with respect to the input signal frequency), the contributions of all the errors in time should cancel providing the measurement is taken over many cycles of both input signal and periodic disturbance. Commensurable periodic imperfections other than harmonics are not considered because the possibility of encountering them is remote in this application.

c. Harmonic Distortion

With phasemeters based on zero crossings, harmonic distortion appears to be the most annoying imperfection. In general, there is apparently no obvious and reliable method to remove or even significantly reduce the corresponding errors in the apparent phase angle. Oscillators with less than 0.02 percent harmonic distortion are not common; therefore, significant harmonics are usually introduced into a network which is to be investigated. Since both the phase shift and attenuation of any network are generally frequency dependent, the character of the harmonic content with respect to the fundamental will be modified so that effective cancellation of zero crossing errors is nonexistent. The only reliable solution to this problem is to reduce harmonic distortion at the source as much as possible and to maintain the network input voltage sufficiently low to linearize response. This procedure is, of course, not always possible.

In order to analyze the errors introduced by harmonic distortion, the character of the distortion must be known with respect to the amplitudes and phase relations of the significant harmonics. This kind of information is nearly impossible to obtain. The following argument reveals some semiqualitative information which may be useful, in particular, when knowledge of the harmonic content is limited.

In general, ignoring the d-c component, which was treated earlier as an offset, a periodic signal with repeat frequency $\omega/2\pi$ may be expressed in terms of its Fourier components as follows:

$$E(\omega t)=E_1 \sin \omega t+\sum_{m=2}^{M} E_m \cos m\omega t + \sum_{n=2}^{N} E_n \sin n\omega t,\ (E_1>0)$$

where the first term on the right is the undistorted signal, again arbitrarily taken as a sine function. The positions of the zero crossings are given by the solutions of the following equation in which the cosine terms have been resummed to distinguish between odd and even harmonics.

$$E_1 \sin \omega t+\sum_{j=1}^{J} E_j \cos (2j+1)\omega t+\sum_{k=1}^{K} E_k \cos 2k\omega t + \sum_{n=2}^{N} E_n \sin n\omega t=0.$$

For small harmonic distortion the behavior at the positive-going zero crossing may be approximated by

$$E_1\omega t+\sum_{j=1}^{J} E_j+\sum_{k=1}^{K} E_k=0.$$

The corresponding phase error is accordingly

$$\Delta\varphi=-\left[\sum_{j=1}^{J} E_j+\sum_{k=1}^{K} E_k\right]/E_1. \tag{4}$$

The response in the vicinity of the negative-going zero crossings may be evaluated by replacing the argument ωt by $\omega t+\pi$ and again evaluating the behavior at small ωt. The negative-going crossings are, accordingly, the solutions of the equation

$$E_1 \sin (\omega t+\pi)+\sum_{j=1}^{J} E_j \cos (2j+1)(\omega t+\pi) + \sum_{k=1}^{K} E_k \cos 2k(\omega t+\pi)=0.$$

Using multiple angle formulas and the approximations used previously, the following phase error is obtained:

$$\Delta\varphi = -\left[\sum_{j=1}^{J} E_j - 2\sum_{k=1}^{K} E_k\right] / E_1. \tag{5}$$

Equations (4) and (5) pertain to positive- and negative-going zero crossings with indices j and k pertinent to odd and even cosine harmonics. Obviously, if both zero crossings are used for triggering, the errors in phase are additive. In this case the phase error is

$$\Delta\varphi = -2\sum_{j=1}^{J} E_j, \tag{6}$$

which indicates that under the conditions imposed on this evaluation, only the cosine terms of odd order contribute to phase errors when contributions at all zero crossings are considered.

When the amplitudes of the significant harmonics are known without any knowledge with respect to their sign and zero crossings (as obtained from a wave analyzer), a useful bound on the phase error is

$$|\Delta\varphi| \leq \sum_{q=2}^{Q} |E_q| / E_1 \tag{7}$$

where q is the harmonic number. In practice the phase error is usually considerably less than that given by eq (7) because the sine terms essentially do not contribute to phase errors.

Some additional discussion on phase angle error introduced by harmonics is presented by Epstein [4] in his description of a similar device. Also, a technique which is supposed to compensate errors from even harmonics is suggested, although errors resulting from odd, out of phase, harmonics are, again, not compensated by his technique.

d. Frequency Stability

Since the apparent phase angle is linearly dependent upon the frequency (eq (1) with constant τ), the frequency must be known and stable to within the desired accuracy of the phase angle during the course of the measurement. Accordingly, for an error in frequency by the amount $\Delta\nu$, the relative error in phase is

$$\frac{\Delta\varphi}{\varphi} = \frac{\Delta\nu}{\nu}.$$

If the frequency instability is random with time, an average and more reproducible value of the phase angle may be obtained by taking the measurement over many cycles.

2.2. Principle of Operation

Basically this phasemeter is a timing circuit which obtains the information necessary to evaluate the phase angle by measuring the average time intervals between appropriate pairs of zero crossings of the two signals over an integral number of cycles. Triggering modes involving all zero crossings, or one pair per cycle, may be used. If the latter mode is used, the zero crossings may be corresponding [7] or noncorresponding. The choice of the mode is often arbitrary, but for maximum performance, depends upon the character of the signals. The following description is based upon the utilization of all zero crossings, which is the most common mode of operation for this instrument, and, for brevity, the phase angles are restricted to the first and second quadrants.

A gate which interrupts a clock signal is controlled by two devices (one for each signal) which respond to all zero crossings of the signals. The output of the gate is connected to the numerator input of a preset digital frequency-ratio meter. Another device which generates a pulse for each zero crossing of one of the signals is connected to the denominator input of the frequency-ratio meter. Accordingly, the display on the frequency-ratio meter gives the total number of clock pulses accepted by the gate for a duration of $N/2$ periods of input (the active time of the frequency-ratio meter), where N is the value of the denominator, which is preset on the frequency-ratio meter. The value of N is arbitrary, but for maximum performance, depends upon the signal frequency and the signal to noise ratio. The determination of the phase angle requires two measurements. The first, essentially determines the time between pairs of corresponding zero crossings of the two signals, whereas the second, measures the period, or frequency. (If the frequency is already known to sufficient accuracy, the second may be omitted.) In the first measurement zero crossings of one signal activate the gate while zero crossings of the other signal inactivate it. The gate is, therefore, active, two times per cycle, each for a duration of φ/ω. This procedure continues for exactly $N/2$ cycles after which the frequency-ratio meter is inactivated. Thus, the gate is active for a total duration of $N\varphi/\omega$ while the frequency-ratio meter is active. When the second measurement is taken, the gate is always held active, and its total effective active time is $\pi N/\omega$. Since both measurements are referred to the same clock, the above durations are related to the digital displays by the same factor. Accordingly, the phase angle in radians is

$$\varphi = \pi R, \qquad 0 < \varphi < \pi$$

where R is the display ratio for the first and second measurements.

Since all zero crossings are used to obtain the information, the measured phase angle is ambiguous by π. When the phase angle is in the third or fourth quadrants, φ appearing in all the durations above should be replaced by $\varphi - \pi$. Accordingly, the phase angle is

$$\varphi = \pi(R+1), \qquad \pi < \varphi < 2\pi.$$

[7] For corresponding zero crossings, the time derivatives of the fundamental have the same sign, and conversely, for noncorresponding zero crossings, have opposite signs.

As will be explained in a later section, the flip-flop switching becomes spurious using this mode when the phase angles approach 0 or π. For this reason these values are excluded from the above inequalities.

The explanation based on the utilization of one pair of zero crossings per cycle is similar. In this case the gate is only activated once per cycle, and only one pulse per cycle is applied to the denominator input of the frequency-ratio meter. Using pairs of corresponding zero crossings, the total effective active times of the gate for the first and second measurements are $N\varphi/\omega$ and $2\pi N/\omega$. The phase angle is

$$\varphi = 2\pi R.$$

Using noncorresponding zero crossings, φ appearing in the above active times should be replaced by $\varphi - \pi$. The phase angle is then

$$\varphi = \pi(2R+1).$$

It is apparent that changing between these last two modes is equivalent to introducing a phase shift of π to one of the input signals. In these modes there is no ambiguity in the phase angle, as with the utilization of all zero crossings. The merits and limitations of the various modes are discussed further in a later section.

When the frequency is known to sufficient accuracy, the frequency-ratio meter may be preset appropriately to read the phase angle directly in degrees in the first reading. The obvious requirement is that the ratio of the clock frequency to the signal frequency should be such that the value calculated for N should be an integer. Since, in general, the frequencies of these two signals are probably not commensurable, the calculated value for N should be sufficiently close to an integer to obtain the desired accuracy in the measurement.

2.3. Interconnection of the Logic Modules

In this section the logic circuit is described, which is capable of performing in the manner described in the last section. Figure 2 is the schematic diagram of the logic input to the frequency-ratio meter giving the arrangement of the designated logic modules.[8] Figure 3 gives the time or angular dependent waveforms corresponding to the points designated in figure 2 using all zero crossings. With the particular line of modules used, both logic levels are negative and are activated by positive-going pulses (positive logic) with appropriate heights and rise times only.

The two signals to be compared in phase are applied to the two essentially identical channels for appropriate processing. The initial stage of each channel is a voltage comparator. Whenever the input signal is slightly positive with respect to an arbitrary d-c reference, the comparator goes to the more positive logic level, and whenever the input signal goes slightly negative with respect to the reference, the output goes to the more negative logic level. Typically, a signal input swing of 20 mV is sufficient to swing the output to saturation at both logic levels. In order to obtain good resolution in time, however, the impput amplitude should exceed by far that need to saturate the logic levels over the course of a cycle. The excess amplitude depends, of course, upon the character of the signals, for example, the signal to noise ratio. For the performance evaluation, described later, 1 V amplitude was found to be more than sufficient.

In order to obtain a d-c reference voltage for the comparator, the simplest course is to connect the reference terminal to "common"; however, this method may introduce a slight bias, which will be, in effect, the same as a small d-c offset accompanying the signal. In general, a more satisfactory method is to apply a well regulated d-c voltage[9] necessary (usually several millivolts) to establish the comparator output at a voltage midway between the logic levels with the input shorted. This adjustment, which comprises the only one necessary in this device, is accomplished by shorting the inputs to ground and connecting the voltmeters shown in figure 2. In place of compara-

[8] The logic symbols used here are taken from reference [5].

[9] A suitable reference voltage may usually be obtained from the center tap of a multiturn voltage divider connected to the well regulated power supply such as one obtained from the manufacturer of the modules.

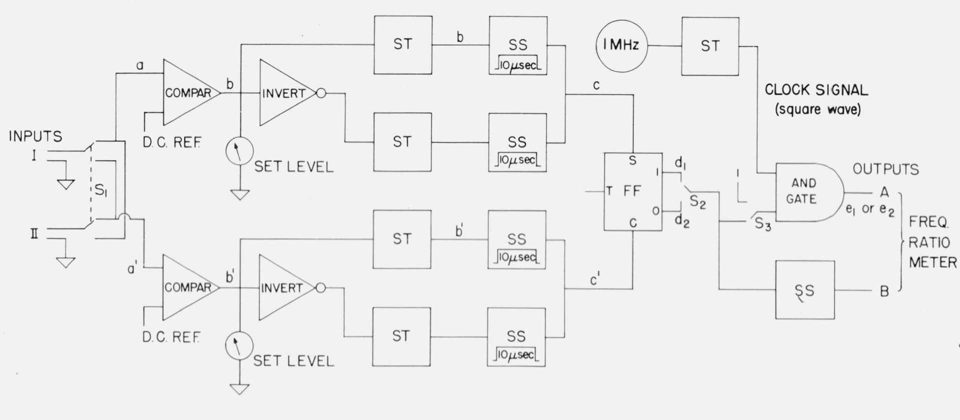

FIGURE 2. *Schematic diagram of the logic circuit.*

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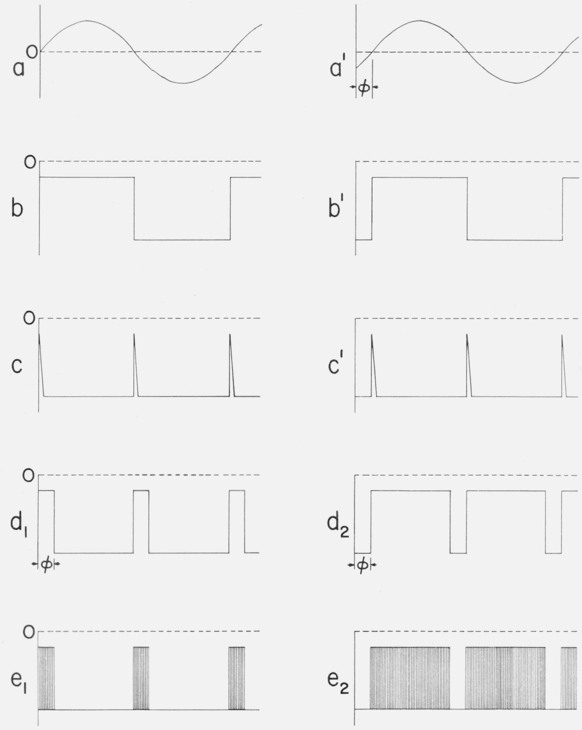

FIGURE 3. *Angular dependent wave forms at the points designated in figure 2 using all zero crossings.*

tors other types of level detectors, including certain operational amplifiers, could probably be used as well, provided they are sufficiently stable and compatible with subsequent stages. The choice of type of module or component here and certain other places in this circuit is often a matter of convenience depending upon the line of modules used, and the imput requirements.

In order to obtain information from both positive- and negative-going zero crossings, the circuit branches at the comparator outputs, as shown, with one of the branches in each channel being applied to an inverting amplifier. The next stages in each of the branches are bistable squaring circuits or Schmitt Triggers (designated ST), which decrease the rise and fall times from the comparator and inverter outputs, in particular at the lower frequencies. At the outputs of the Schmitt Trigger these times are essentially independent of the input signal frequency and now compatible with the requirements of subsequent stages or modules. The direct inputs of the Schmitt Triggers should be used here and set to trigger at a voltage approximating the average of the logic levels. This is easily accomplished by fixed biasing in accordance with the instructions supplied by the manufacturer.

The following stages are "single shots" (monostable multivibrators, designated SS). The reason for the 10 μsec pulse width shown is to reduce the occurrence of spuriousness from noise as explained in the next section.

The outputs from the "single shots" are combined as shown to form the channel outputs, which are applied to the "set" and "clear" (designated S and C) inputs of a flip-flop (bistable multivibrator, designated FF). Pulses at both flip-flop inputs correspond to all zero crossings of input signals (both positive- and negative-going). The "single shot" outputs may be disconnected appropriately in order that flip-flop is active only once per cycle and controlled by either corresponding or noncorresponding pairs of zero crossings, which are used for phase angles in the vicinity of π and 0, respectively. The merits and limitations of the above triggering modes are discussed in the following subsection. It is desirable to connect high impedance indicator lamps [10] at the flip-flop outputs to detect spuriousness or malfunctioning if either should occur. Connection to the trigger input (designated T) is not necessary for this application.

The pulses received by the flip-flop will be delayed from their corresponding zero crossings of input nearly equal to the sum of the rise times of the preceding stages. Clearly, if these delays are the same in both channels, no corresponding error will appear in the apparent phase angle. Since significant differences may appear in these delays, the corresponding error may be effectively cancelled by repeating the measurement with the channels interchanged by switching S_1 and S_2 and taking the average reading. In the prototype, the difference in these delays was found to approximate 1 μsec.

The output of the flip-flop is split into two branches. One of these is applied to the input of a "single shot" which generates pulses corresponding to zero crossings of input. These pulses are used to activate and inactivate the frequency-ratio meter. (In order to prevent spuriousness from noise, it is necessary to use this "single shot" to gate the frequency-ratio meter rather than to use any of the four "single shot" outputs which control the flip-flop.) The other branch is applied to one of the inputs of an "And" gate [11] which gates a clock signal for a duration proportional to the phase angle, with S_3 in the lower position as shown. When S_3 is in the upper position, the corresponding gate input is held at the active level (designated by 1), which means that the gate output is controlled by the clock signal only. Either passive or active "And" gates may be used here; however, the passive gates used were found to have faster rise and fall times. In order to generate the clock signal, a sufficiently stable oscillator at an appropriate frequency (depending somewhat on the anticipated signal frequency range) may be used. Usually it is desirable and convenient to use the oscillator of the frequency-ratio meter (usually 1 MHz) for this purpose. The Schmitt Trigger following the clock oscillator may be necessary depending upon input rise time requirements of the "And" gate.

Outputs A and B are applied to the A/B inputs of a

[10] Compatible transistorized or electronic high impedance indicator lamps are usually available from the manufacturer of the logic modules.

[11] The output of an "And" gate is active only when all inputs are active.

suitable preset digital frequency-ratio meter. The "preset" feature is important because it makes possible a measurement over N arbitrarily selected periods of input. Taking the measurement over a large number of periods effectively averages the influence of noise on the positions of the zero crossings and improves the phase resolution which would be limited by the clock frequency. If the "preset" feature is not available, it may be incorporated by constructing a frequency divider by appropriately interconnecting a suitable number of flip-flops.

At very low frequencies one may not wish to spend the time necessary to average the effect of noise by taking the measurement over many cycles of input. Birnboim [6] has suggested and tried a method utilizing tunnel diode level detectors to control a gate. In complete absence of noise a given tunnel diode will switch rapidly only once per sine wave zero crossing. In the presence of noise the tunnel diode may switch several times in the vicinity of the fundamental zero crossing turning "on" and "off" according to the actual sign of the input in a manner similar to a voltage comparator, but with much faster transient times. According to Birnboim, the active time of the output is nearly the same with or without noise with this method, even though the measurement is taken over only one cycle of input. Extreme caution must be taken, however, with measurements over only one cycle of input, in that long transient times in the signal are damped so that a steady state is acquired. Even at very low frequencies a network which manifests an appreciable phase angle will often manifest a long transient time of magnitude comparable to the period.

In some cases it appears possible to prevent the coincidence of pulses by introducing a small phase shift in one of the signals with an appropriate phase shifting network before one of the comparators. It is not necessary to know the exact value of the phase shift because its effect can be canceled by taking a second measurement with the channels interchanged utilizing switches S_1 and S_2 (fig. 2), which is the normal procedure even without this phase shifter. This is probably feasible at the higher frequencies; however, at very low frequencies the phase shifter would probably produce a large transient which would not damp out within the time desired to take the measurement. Accordingly, it may be desirable or necessary to use only one zero crossing per cycle which imposes certain other limitations, as discussed in the next section.

2.4. Single Versus Double Triggering

As mentioned earlier, several modes of operation are possible with this phasemeter. The phase angle can be measured by utilizing information from all zero crossings, or from one pair per cycle involving either corresponding or noncorresponding zero crossings. In this section the merits and limitations of each mode are discussed. The terms "single" and "double" triggering refer to whether the gate is active once per cycle and controlled by either corresponding or noncorresponding pairs of zero crossings, or active twice per cycle controlled by all zero crossings, respectively.

The most accurate measurements are taken with double triggering, as will become apparent; however, a limitation in this mode of operation is that phase angles must be sufficiently remote from either 0 or π. From figures 2 and 3 it is apparent that the flip-flop "set" and "clear" input pulses approach coincidence as the above angles are approached. The degree to which these angles can be approached depends upon the amount of noise present at the input, the input level detector stability, and the flip-flop response times.

A minor disadvantage of double triggering is that there is an ambiguity between the angle φ and $\varphi+\pi$. Usually a phase angle is known to within $\pm\pi/2$ before the measurement is commenced; however, if this approximate information is not known, an initial measurement with single triggering, which is not ambiguous in this respect, is useful to determine the proximity of the angle before commencing with the more accurate measurement using double triggering.

Whenever the phase angle is sufficiently remote from 0 or π, double triggering is the preferable mode and is very reliable, even when moderate amounts of noise are present in the input signals. Under these conditions, there may be more than one actual zero crossing per zero crossing of fundamental. Accordingly, the input stages, including comparators, inverters, and Schmitt Triggers, will chatter with a corresponding burst of pulses appearing at the flip-flop inputs. If the flip-flop is set by a pulse corresponding to the first zero crossing of one signal, all subsequent crossings will be ignored until the flip-flop is cleared by a pulse corresponding to the first zero crossing of the other signal. Since all zero crossings are used, no spuriousness can result from an actual zero crossing being opposite in sign from that of the fundamental in this vicinity. The noise will still perturb the positions of the zero crossings; however, when the measurement is taken over many cycles, the effect of noise on the measurement should be effectively canceled because of the random nature of noise. As will be explained in the next paragraph, complete freedom from spuriousness from noise is not inherent in the single triggering mode.

When the phase angle approximates 0 or π, and when the double triggering mode is used, the pulses appearing at the "set" and "clear" inputs of the flip-flop approach coincidence, which makes the output spurious, as mentioned earlier. Under these conditions one must proceed with single triggering using pairs of corresponding or noncorresponding zero crossings when the phase angles approach π and 0, respectively. Unfortunately, with single triggering, if the noise is sufficiently large, the output will, again, be spurious. In this mode there are unused, or ineffective, zero crossings. If a burst of noise appears in the vicinity of a zero crossing intended to be ineffective, the actual signal may reverse slope opposite to that of the fundamental one or more times within the critical region over which the circuit responds to fluctuations. Thus, zero crossings which are supposed to be ineffective become effective in an irregular manner with a corresponding spuriousness in the measurement. For a given signal to noise ratio, it has been found that spuriousness from noise was encountered more at the lower frequencies. (For example, with a one volt signal with a signal to noise ratio of 100, spuriousness was detected using the single triggering mode at frequencies less than 10 Hz.) Presumably, this is because the time of the critical region is longer at the lower frequencies. The increased pulse widths, or increased active times, of the "single shots" shown in figure 2, will in some cases eliminate the effect of this disturbance. The longer pulse simply prevents the flip-flop from chattering from more than one crossing of input in the vicinity of each zero crossing of fundamental for a duration equal to the active time of the "single shots." Ideally, this time should depend upon the signal frequency with the limitations being that it should be small in comparison with the input period and slightly larger than the duration of the critical region over which the circuit responds to fluctuations. A given pulse width on the "single shot" outputs is obtained by connecting the appropriate capacitor as instructed by the manufacturer of the logic modules. Variation of these capacities with frequency may be accomplished by suitable manual switching. This feature was not incorporated in the prototype. Since this type of spuriousness does not appear with double triggering, there is no need to increase these pulse widths, except to assist in visual display on an oscilloscope, if this need should occur.

In the author's anticipated applications for this phasemeter the occurrence of phase angles in the vicinity of 0 or π is not anticipated. Accordingly, all of the measurements are to be taken using double triggering, and not much has been done to improve the performance in the neighborhood of these angles where single triggering is necessary. Extension of performance to improve reliability in the neighborhood of these angles might be accomplished by the capacitor switching scheme mentioned above, or by incorporating in the circuit some sort of a stable trigger hysteresis limiter. Such possibilities, which could be modified to be compatible with the logic line used, are given in reference [7].

One of the principal advantages of double triggering is that errors from d-c offset voltages are canceled, providing the waveforms are symmetric about their peaks. The obvious limitation is, of course, that the offset voltage must be less than the amplitude. This cancellation is verified by the following argument. Consider an even function which obeys the above symmetry requirement,

$$F(\omega t) = F(-\omega t),$$

which has an extremum at $\Delta t = 0$. It is desired to find the change in the apparent phase angle $\Delta\varphi$ resulting from shifts of the zero crossings Δt from an offset voltage E_{DC}. Using the closest zero crossings on each side of $\omega t = 0$, we take

$$F[\omega(t+\Delta t_1)] + E_{DC} = F[-\omega(t+\Delta t_2)] + E_{DC},$$

which can be true only for

$$\Delta\varphi = \omega(\Delta t_1 - \Delta t_2) = 0.$$

It is not necessary to equate both sides of the penultimate equation to zero because the relation is valid for any pairs of values of an even function chosen by setting the argument $\omega t = -\omega t$.

The above result is illustrated in figure 4, which reveals the phase angle errors introduced from d-c offset using single triggering: (a) gives the time dependence of the symmetric input waveforms, which, for the purpose of argument, are sine waves, and the time between any pair of corresponding points is φ/ω. One of the signals with amplitude E_1 is offset by the amount E_{DC} for which the time between actual zero crossings is shifted by the amount

$$\Delta t \approx E_{DC}/\omega E_1.$$

(b) gives the corresponding wave form for the control input of the gate, which, in this case, is controlled by only positive-going zero crossings of (a). The dashed line shows what the gate input would be in the absence of offset. Clearly, the apparent phase angle is in error by the amount $\omega\Delta t$. (c) gives the gate input waveform

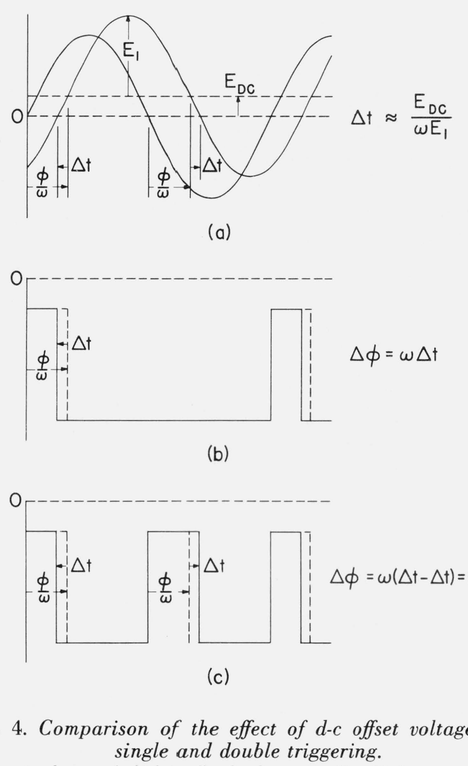

FIGURE 4. *Comparison of the effect of d-c offset voltages between single and double triggering.*

(a) Input signals, one of which is offset by the amount E_{DC}. (b) Gate control input using single triggering (positive-going zero crossings). (c) Gate control input using double triggering (all zero crossings).

using all zero crossings of (a). For symmetrical input waveforms, the errors introduced by each pair of zero crossings cancel as shown.

In addition, as illustrated in section 2.1c, the phase errors introduced by even, out of phase, harmonics are effectively eliminated only when double triggering is used.

2.5. Modification for Integrating Voltmeter

Depending on the nature of the application of the phasemeter and the availability of certain types of equipment, one may wish to use a digital integrating voltmeter in place of the digital frequency-ratio meter. A phasemeter using this technique has been described by Heydemann [8]. In order to modify the logic circuit to be used in conjunction with an integrating voltmeter, the flip-flop shown in figure 2 should be replaced by two as shown in figure 5, similar to the circuit developed by Heydemann. This arrangement converts the two logic levels, A and B, into three, A−B, 0, and −(A−B). The principal reason for doing this is for the zero output to correspond to a zero (or π) shift in phase. The flip-flop output waveforms corresponding to the designated points in figure 5 are shown in figure 6 as a function of ωt. a and b are the outputs at points a and b in figure 5 and $a-b$ is the differential input of the integrating voltmeter. The apparent phase angle φ in terms of the voltages shown is

$$\varphi = \frac{\int_0^{2\pi N} ed(\omega t)}{4\pi NE}$$

where N is the integral number of periods, selected arbitrarily, over which the measurement is taken, e is the instantaneous voltage, and E is the magnitude of the two active levels. For angles $0 < \varphi < \pi$, the output is positive as shown in (a), and for angles $0 > \varphi > -\pi$, the output is negative as shown in (b). A reversible [12] integrating digital voltmeter is preferable to the more common absolute-value type because with the former the sign of the phase angle is given as well as its magnitude. In figure 6 this distinction is illustrated by taking φ and $-\varphi$ by simply reversing the inputs as shown at the top of the figure. The author has not assembled a prototype employing an integrating voltmeter.

It is apparent that the integrating voltmeter technique might have advantages compared to the frequency-ratio meter technique at high frequency. It is not limited by the finite resolution imposed by the clock frequency and may tend to more effectively cancel errors resulting from finite flip-flop output rise and fall times. However, at low frequencies the frequency-ratio meter technique should be advantageous. For example, at a frequency of 1 Hz it is easy to establish the period, or any interval within the period, to within $\pm 10^{-6}$ sec. Under ideal conditions this would introduce a corresponding error of $\pm 0.00036°$. Comparable accuracies using voltage measurements at this frequency are difficult to obtain. With the scheme shown on figure 5 the output is not affected by offset voltages, providing the waveforms are again symmetric about their peaks, as illustrated in reference [8]. However, the possibility of spuriousness from noise is not completely removed as it is with the frequency-

[12] A reversible (sometimes called "count-up, count-down") meter integrates the signed value over a preselected time interval and indicates the sign of the definite integral in the readout. An absolute-value meter integrates the magnitude of the function over a preselected interval without respect to sign. Hence, in this application, an absolute-value meter will give the correct magnitude of the phase angle, but with no information with respect to lag or lead, whereas, the reversible meter completely specifies the phase angle.

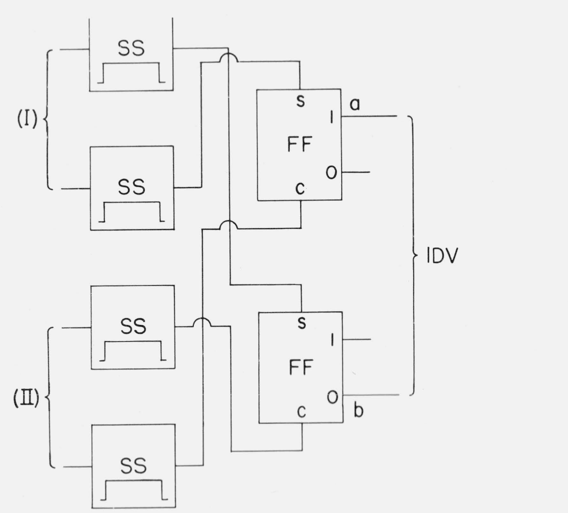

FIGURE 5. *Proposed modification of the logic circuit to use a reversible digital integrating voltmeter in place of a frequency-ratio meter.*

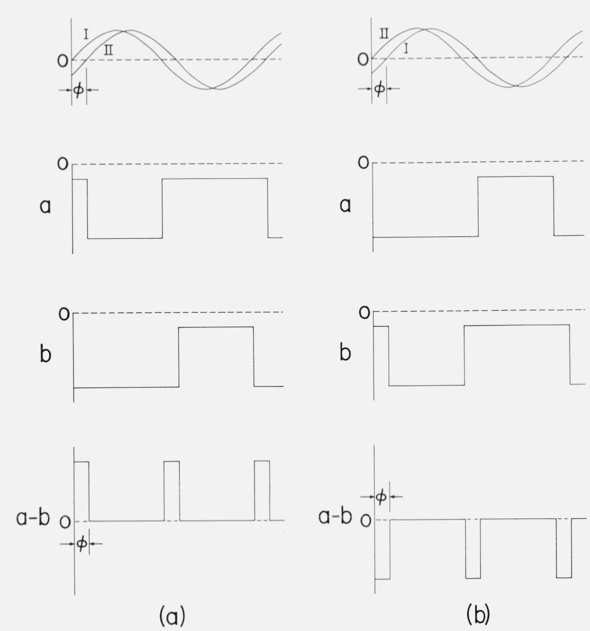

FIGURE 6. *Angular dependent wave forms at points designated in figure 5.*
(a) II leading I (b) II lagging by same amount.

ratio technique using double triggering at phase angles sufficiently remote from 0 or π.

3. Performance Evaluation

This section includes the description of tests by which the apparent phase angles obtained by this phasemeter are compared to those generated by reliable sources.

Using common laboratory techniques, it is difficult to obtain a phase "standard" operating continuously over the audio range to better than 10 min. The limitation seems to be mostly in the quality of sinewave generators because of distortion and frequency stability. Ironically, variable frequency oscillators, such as frequency synthesizers, which have excellent frequency stability, usually have considerable harmonic distortion, and those which have good wave form usually lack the stability desired for this test. If one is willing to make a comparison at a single frequency, there is at least one commercially available phase shifter capable of generating two very pure coherent sine waves from which a continuously adjustable phase angle (over 2π) can be defined to within $\pm 0.01°$.

Accordingly, two tests were used to evaluate the performance of this phasemeter. The first used a phase shifter constructed from ordinary laboratory equipment from which a phase angle can be established to within several tenths of a degree over a wide range of frequencies, and the second used a quality phase shifter of very high accuracy at a single frequency.

In order to obtain an approximate check over a frequency band, an "operational" phase shifter was assembled from available components as shown in figure 7. The decade resistors and fixed capacitor (1 μF) had accuracies specified to within 0.025 percent by the manufacturer. Two different oscillators were used: a frequency synthesizer, with a harmonic distortion of less than 1 percent, which was found to be stable in frequency to better than 1 part in 10^{-7} over the course of any of our measurements; and an RC

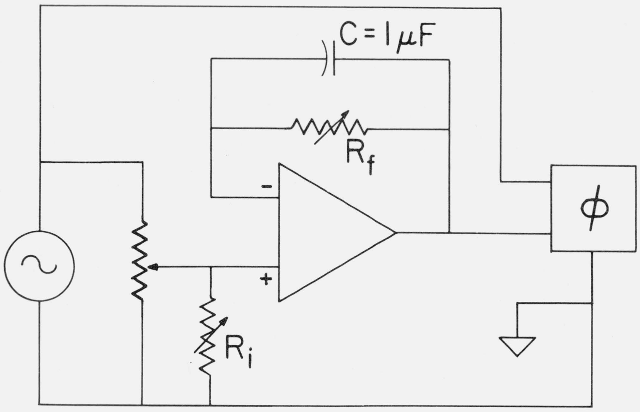

FIGURE 7. *Schematic diagram of the phase shifter circuit used to evaluate the digital phasemeter over a wide frequency band.*

oscillator, for which the corresponding numbers are 0.5 percent and 1 part in 10^4. Using hypothetical, pure sine waves, neglecting leakage resistances and stray capacitances, the magnitude of the phase angle δ evaluated from the circuit in figure 7 is

$$|\delta| = \frac{\omega R_f C}{1 + \frac{R_i}{R_f}(1 + \omega^2 R_f^2 C^2)}.$$

Measurements were made at arbitrarily selected phase angles within the first quadrant at decade intervals in frequency from 0.1 to 10,000 Hz. Agreement was always obtained to within 0.2° up to 1,000 Hz. The apparent phase angle differed by as much as 0.17° under the same settings between the two oscillators. This disagreement is attributed to their differences in harmonic content. At 10,000 Hz the agreement was within 0.7°; however, a 0.01 μF capacitor had to be substituted here in place of the 1 μF to generate an appreciable phase shift at this frequency. In this case anticipated stray capacities of approximately 100 pF could produce 0.7° difference between the observed value and that calculated from eq (8). Since all of the measured values fell within the bounds predicted from the uncertainty of the phase shifter, this is not a very meaningful test with respect to accuracy. However, this evidence does indicate reasonable performance and the absence of spuriousness except in the vicinity of 0 and π as discussed in the last section.

The second comparison was made using a high quality commercial phase shifter operating only at a single frequency. According to the manufacturer's specifications, this instrument is capable of producing two sine waves of sufficient quality to define any phase angle (0 to 2π) to within $\pm 0.01°$ at 400 Hz. With careful adjustment it is the opinion of this author that a precision to within $\pm 0.005°$ can be obtained. The manufacturer's specifications list the harmonic distortion as within 0.05 percent, and the frequency to within 4×10^{-7} over 24 hr.

The results of typical comparison are included in the following table. The first column gives the phase settings on the phase shifter. The second column gives the apparent phase angle evaluated from two sets of data. In the first set an angle δ is set on the phase shifter and the apparent phase angle φ is observed. In the second, the angle $2\pi - \delta \sim -\delta$ is set on the phase shifter, the input leads to the phasemeter are reversed, and the apparent phase angle is observed. In each determination these two apparent phase angles are averaged to give those appearing in the second column. With pure sine waves the phasemeter should see no distinction between the two methods by which these values are obtained. However, between these two sets, the apparent phase angles were found to disagree consistently with replicate measurements at a particular phase setting by as much as 0.01°, which, incidentally, is the accuracy claimed by the manufacturer

TABLE 1. *Comparison between the quality commercial phase shifter and the digital phasemeter at 400 Hz*

Setting $\delta \pm 0.01$	Apparent phase angle φ	Disagreement $\varphi - \delta$
deg.	*deg.*	*deg.*
1.5	1.503	0.003
30	30.004	.004
60	60.003	.003
90	90.006	.006
120	120.001	.001
150	150.000	.000
178.5	178.500	.000
181.5	181.501	.001
210	210.002	.002
240	239.999	−.001
270	269.997	−.003
300	299.996	−.004
330	329.995	−.005
358.5	358.498	−.002
Average error............,....		0.0004

of the phase shifter. These biased differences may be attributed to the difference in harmonic content between the primary and shifted signal of the phase shifter. The last column gives the disagreement between the corresponding values of the first two columns with the average error. The trend, in which the errors reverse sign at angles larger than 210°, also reveals the presence of a small, biased systematic error; however, the disagreement did not exceed 0.006°.

From the above evidence it is apparent that the accuracy of the digital phasemeter at 400 Hz is at least as good as that of the commercial phase shifter, which, according to the manufacturer, is ±0.01°. This uncertainty corresponds to a time interval of 0.069 μsec. Considering that the clock pulses are spaced by 1 μsec and the rise times of the components are 1 μsec or slightly less, an error in time of 0.069 μsec is probably not excessive in spite of the averages that are taken by appropriate switching and measuring over many cycles of sine wave. If the principal errors (using ideal sine waves) are from the finite resolution of the clock pulses and finite rise times, as expected, the accuracy should improve as the frequency is decreased.

4. Concluding Remarks

In evaluating the performance of a prototype of an absolute measuring technique for which all known anticipated systematic errors have been evaluated or eliminated, it is usually desired to compare the apparent values of the quantities the instrument is capable of measuring against the "true" values obtained from a suitable "standard." In this usage the meaning of a "true" value is that obtained from a well established standard for which the resolution is far superior to that of the prototype in that errors generated by the standard are essentially insignificant compared to those from the prototype. The exact "true" values of a quantity are never known except for some of the more basic standards for which the values are selected arbitrarily. In order to prevent the possibility of a mutual occurrence of an unanticipated systematic error, it is desirable for the two techniques to be disparate but still capable of measuring the same quantity.

In this particular comparison in which the digital phasemeter was compared to the commercial phase shifter used as a standard, the agreement was found always to be within ±0.006°, which is even better than the ±0.01° accuracy specified by the manufacturer of the commercial phase shifter. One should not ignore the possibility of a coincidence that the same biased systematic errors are common to both instruments. However, this possibility is considered to be extremely unlikely since the methods of measurement are very distinct. The commercial phase shifter is based on frequency division and coincidence of Lissajous patterns, whereas the digital phasemeter refers the information to a time base. Also, the commercial phase shifter utilizes all pairs of corresponding points, whereas the digital phasemeter utilizes only zero crossings ignoring all intermediate information.

In view of the agreement between these two absolute measurements, utilizing quite different principles of operation and presumably subject to different types of systematic error, it seems reasonable to claim that both this new instrument and the commercial phase shifter are accurate to something better than 0.01° at 400 Hz. As noted above, the new instrument described here should have about this same accuracy up to a frequency of about 1,000 Hz and even better accuracy at frequencies lower than 400 Hz providing the systematic errors are principally caused by uncertainties in time intervals between zero crossings. If the signals were perfect sine waves and no other systematic errors entered, an error of 0.01° at 400 Hz would correspond to an error in the measurement of time of 0.7 μsec, certainly an easily attainable accuracy. The tests described above seem to indicate that the systematic errors of the instrument described here are within this limit, and that any greater "error" is associated not with the measurement itself, but with imperfections in the signals used. One could reasonably say that this instrument measures phase angles defined as the angular separation between an arbitrary pair of corresponding points to better than 0.01° on two periodic signals of identical waveform but displaced in time. For periodic signals with the same frequency but different waveform, there is, of course, no unique definition of a phase angle. The definition based on the angular separation of zero crossings seems to be appropriate for many uses; for others a definition involving comparison over a complete cycle might be preferred. The consideration of the most commonly encountered imperfections indicates, however, that if one defines the phase angle between two voltages consisting of sine waves plus imperfections as the phase between the two pure sine waves, this phase measured here will be in error by no more than the ratio of the amplitude of the distortion to the amplitude of the basic sine wave, plus the product of the angular frequency and the timing error. This latter factor should not exceed 0.01° at 400 Hz and will be proportional to frequency.

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