A 2:1 Ratio Inductive Voltage Divider With Less Than 0.1 PPM Error to 1 MHz

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A simple ratio transformer having a 2:1 ratio of input voltage to output voltage can be made with a ratio error less than 0.1 ppm from 1 kHz to above 1 MHz. Applications and sources of error are discussed. Experimental results leading to an optimum transformer design are given. A bridge to measure the ratio error to 0.025 ppm from 1 kHz to 1 MHz is also described.

Key Words: Attenuator, bridge, high frequency, inductive voltage divider, ratio transformer.

1. Introduction

Techniques for winding inductive voltage dividers with adjustable ratios have been improved during the last few years to where dividers having errors less than 0.1 ppm at frequencies around 1 kHz are not too difficult to make. Unfortunately, as the frequency increases, the ratio error increases rapidly so that this accuracy cannot be maintained above about 10 kHz. At frequencies above 10 kHz the most accurate ratio device is an inductive voltage divider which has a fixed voltage ratio of 2:1; that is, one dividing an input voltage into two equal parts such as is shown in figure 1. Such dividers have recently been made with errors less than 0.1 ppm from 1 kHz to above 1 MHz. This paper summarizes the research that was done to develop this accurate high frequency divider.
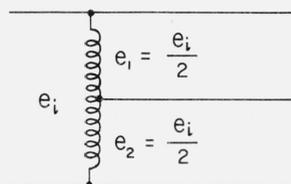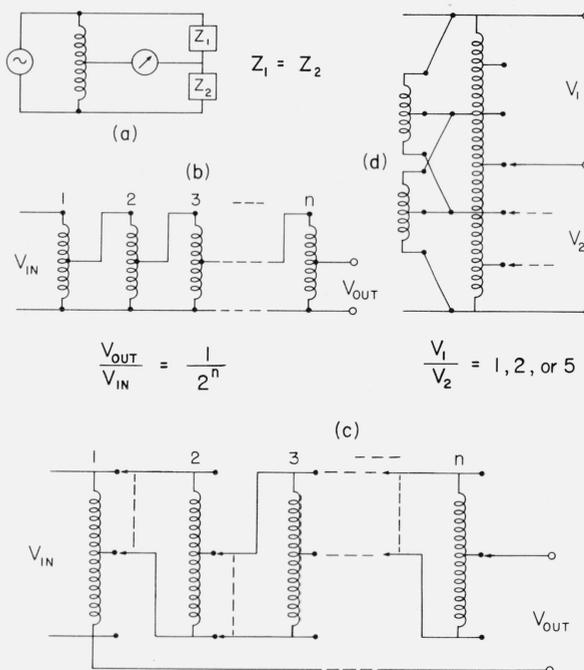

FIGURE 1.   *A 2:1 ratio inductive voltage divider.*

2. Applications

A few of the many possible applications for this type of divider are shown in figure 2. Figure 2a is a bridge for comparing two impedances, z_1 and z_2.

Figure 2b shows n basic 2:1 ratio dividers connected to give a voltage ratio

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{2^n}.$$

FIGURE 2.   *Some possible applications for 2:1 ratio dividers.*
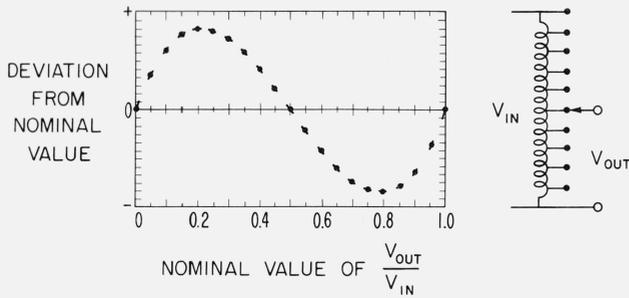
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FIGURE 3. *Typical error curve for one decade of a low-frequency decade voltage divider.*

Each section of this divider accurately divides its input voltage in half to give an attenuation of 6 dB.[1] An attenuator made with n sections will have approximately $6n$dB.

Figure 2c shows an adjustable ratio "binary" inductive voltage divider. The principle of operation is the same as for low frequency decade voltage dividers except that each section divides its input voltage into only two equal parts instead of 10 equal parts. T. L. Zapf et al., have shown that the error curve for any one isolated decade of a low frequency decade voltage divider has an "S" shape such as shown in figure 3 [1].[2] As the frequency increases, the magnitude of the error increases but the curve still tends to go through zero at a ratio of 0.5. It is this error curve that suggested that 2 : 1 ratio dividers would be accurate at high frequencies as well as low frequencies. If n of these 2 : 1 ratio dividers are connected as shown in figure 2c the voltage ratio is

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \sum_{i=1}^{n} \frac{\delta_i}{2^i}, \quad \delta_i = \begin{cases} 1 \text{ if the } i\text{th switch is up} \\ 0 \text{ if the } i\text{th switch is down.} \end{cases}$$

The input of each section is connected either across the top half or across the bottom half of the preceeding section. To obtain the same resolution with a binary divider as with a decade divider requires that the binary divider have approximately 3 times as many sections as the decade divider has.

Figure 2d shows three basic 2 : 1 ratio dividers connected in series with two more 2 : 1 ratio dividers connected so as to assure that the same voltage is applied to each of the three in series. Any number could be connected in series, however the combination shown gives the very useful ratios

$$\frac{V_1}{V_2} = 1, 2, \text{ and } 5.$$

This type of divider could be used in a bridge such as shown in figure 2a to compare standards of impedance which are often built to have nominal values in a 1-2-5 sequence.

3. The 2 : 1 Ratio Divider

The 2 : 1 ratio divider is a center-tapped transformer that divides a voltage into two equal parts. It is convenient to define the complex ratio error, $\rho = \rho_r + j\rho_i$, as the error in the ratio of the output voltage to the input voltage. The output voltages e_1 and e_2 shown in figure 1 then become

$$e_1 = \frac{e_i}{2}(1 + \rho), \qquad e_2 = \frac{e_i}{2}(1 - \rho).$$

If $|\rho| \ll 1$, the ratio e_1/e_2 is approximately

$$\frac{e_1}{e_2} = 1 + 2\rho. \tag{1}$$

To get an expression for the ratio error, consider the divider as two impedances in series which differ by a small amount ΔZ as shown in figure 4. If no current is drawn from the center tap, the voltage ratio e_1/e_2 is

$$\frac{e_1}{e_2} = \frac{Z + \Delta Z}{Z} = 1 + \frac{\Delta Z}{Z}. \tag{2}$$

From (1) and (2) we get

$$\rho = \frac{\Delta Z}{2Z} = \frac{\Delta R + j\omega \Delta L}{R + j\omega L} \tag{3}$$

where $R + j\omega L$ is the open circuit input impedance, Z_{in}, of the transformer. The real and imaginary components of ρ obtained from (3) are

$$\rho_r = \frac{R\Delta R + \omega^2 L\Delta L}{R^2 + \omega^2 L^2} \tag{4}$$

$$\rho_i = \frac{\omega R\Delta L - \omega L\Delta R}{R^2 + \omega^2 L^2}. \tag{5}$$

If $Q^2 \gg 1$ as is often the case, where $Q \equiv \omega L/R$, (4) and (5) reduce to

$$\rho_r = \frac{\Delta L}{L} + \frac{\Delta R}{\omega L Q} \tag{6}$$

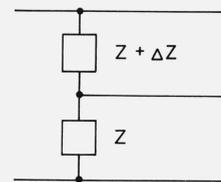$$\rho_i = -\frac{\Delta R}{\omega L} + \frac{\Delta L}{LQ}. \tag{7}$$

FIGURE 4. *Simplified circuit of a 2:1 ratio divider.*

These equations show the importance of winding the transformer so that it has a large inductance. The value of $\Delta L/L$ is made small by winding the transformer on a high permeability core to increase L, and by twisting the wires to minimize ΔL. A twisted cable is made by folding one wire in half and twisting the pair fairly tightly with a hand drill as shown in figure 5a. This cable is wound on the magnetic core and connected as shown in figure 5b. The wires are connected so that the current path from the upper tap to the center tap is the same as the current path from the center tap to the bottom tap. Thus, both windings link very nearly the same magnetic flux and have a small difference in inductance ΔL. Twisting the wires helps assure that both windings link the same flux.

Figure 6 shows the variation of ρ with frequency for a transformer having 42 turns of twisted No. 26 magnet wire wound on a ferrite torodial core. Note that ρ_r is essentially constant with frequency up to 2 MHz which is the upper frequency limit of our bridge. To see how high in frequency ρ_r will remain constant, we need to look at the equivalent circuit of the divider.

Figure 7 shows one of two exact equivalent circuits for an autotransformer [2] which is what the 2:1 ratio divider is. The Y_{oc} is the open-circuit admittance measured across the input terminals and Z_{sc} is the short-circuit "leakage" impedance measured across either output with the input terminals short-circuited. It can be seen from figure 7 that the voltage ratio V_1/V_2 at the output terminals will be equal to e_1/e_2 only if no current flows through Z_{sc}. If within the transformer housing the admittance from A to B differs from that from B to D by an amount ΔY_t, and if $|Z_{sc}\Delta Y_t| \ll 1$, then

$$\frac{V_1}{V_2} = \frac{e_1}{e_2} - 2Z_{sc}\Delta Y_t. \qquad (8)$$
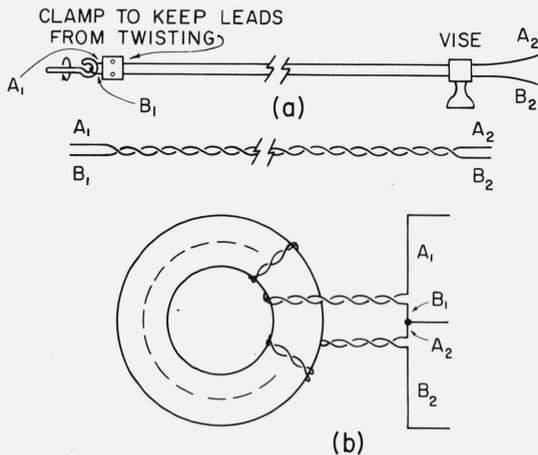
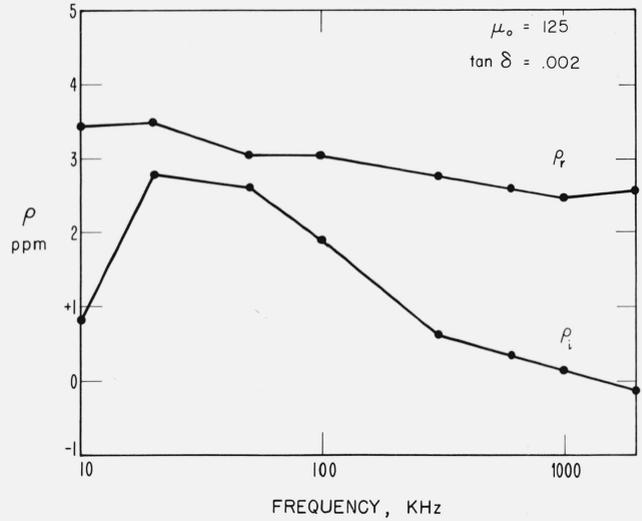If ΔY_t is mainly a difference in capacitance, ΔC_t,

FIGURE 6. *Error curve for one of the first experimental dividers showing how ρ_r and ρ_i are relatively constant with frequency.*

$$\frac{V_1}{V_2} = 1 + 2\rho + 2\omega^2 L_{sc}\Delta C_t - j2R_{sc}\omega\Delta C_t \qquad (9)$$

where L_{sc} is the leakage inductance defined by $Z_{sc} \equiv R_{sc} + j\omega L_{sc}$. A typical value of L_{sc}, such as for the transformer whose ratio error is shown in figure 6, is $L_{sc} = 0.1\ \mu H$. By using trimmer capacitors from A to B and B to D, ΔC_t can be made less than 0.01 pF giving, in ppm,

$$2\omega^2 L_{sc}\Delta C_t < 0.1\ f_{MHz}^2$$

and

$$2R_{sc}\omega\Delta C_t < 0.01\ f_{MHz}$$

where f_{MHz} is the frequency in MHz. Since this loading error is proportional to the frequency squared, it will be the predominant error term at frequencies much above 1 MHz. The transformer whose characteristic is shown in figure 6 has trimmer capacitors across the output leads to adjust ΔC_t to a minimum. Without adjusting ΔC_t to a minimum the ρ_r curve increases to a value of about 5 ppm at 2 MHz.

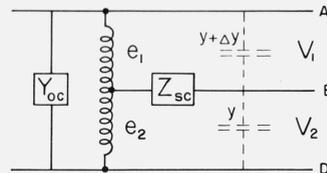FIGURE 5. *The wires are twisted, wound on the core and connected as shown here to make a 2:1 ratio divider.*

FIGURE 7. *An "exact" equivalent circuit for an autotransformer with lead admittances Y and Y + ΔY added.*

4. A Bridge to Measure ρ

A simplified diagram of the bridge used to measure ρ is shown in figure 8. The two arms of the transformer are balanced against two capacitors C', each having a nominal value of 1000 pF. The bridge is balanced with the capacitance C and the conductance G. The transformer is then removed from the bridge, rotated so that D is at the top and A is at the bottom, and then replaced on the bridge. The bridge is again balanced as before, changing C an amount ΔC and G an amount ΔG. Solving the bridge equations gives

$$\rho_r = \frac{\Delta C}{4C'} \tag{10}$$

$$\rho_i = \frac{-\Delta G}{4\omega C'} \tag{11}$$

where it has been assumed that $|\rho| \ll 1$, $C'' \ll C'$, and $G''^2 \ll \omega^2 C'^2$.

A more detailed diagram of the bridge is shown in figure 9a. The capacitors C and C' are three-terminal capacitors having admittances to ground shown as dotted lines. If these admittances are unequal they will load the transformer causing currents to flow through Z_{sc} and, thereby, change the apparent voltage
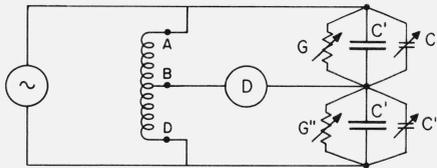
ratio of the transformer. To measure the true voltage ratio e_1/e_2 of the transformer, a second detector, D_2, is placed between the center tap of the transformer and ground in order to balance the admittances to ground with Y. With both detectors nulled for both positions of the transformer, the bridge equations yield (10) and (11) as before.

To measure the small changes in conductance, ΔG, the T-network shown in figure 9b was used for G. Changing C_s an amount ΔC_s gives a change in conductance

$$\Delta G = \omega^2 C_g R \Delta C_s \tag{12}$$

and a change in capacitance which is negligible compared to ΔC. Putting (12) into (11) gives

$$\rho_i = -\frac{R C_g}{4C'} \omega \Delta C_s. \tag{13}$$

The resolution for ρ_r and ρ_i is 0.025 ppm from 1 kHz to 1 MHz and 0.25 ppm to about 3 MHz. A photograph of the bridge is shown in figure 10. Bridges of similar design are described in references [3] and [4].

5. Optimum Transformer Design

Using the bridge shown in figure 10, a number of experiments were made to determine the optimum coil geometry, core material, type of wire, wire sizes, and amount of twist of the wire.

5.1. Coil Geometry

Theoretically the best coil geometry would be that which gives a maximum inductance for a minimum

FIGURE 8. *Simplified circuit of a bridge to measure the ratio error of a 2:1 ratio divider.*
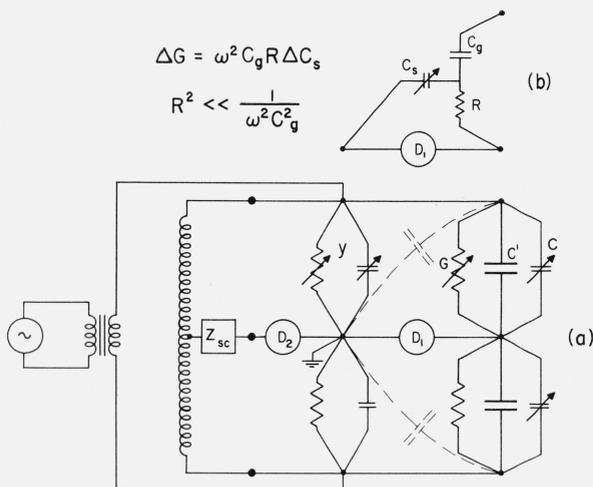
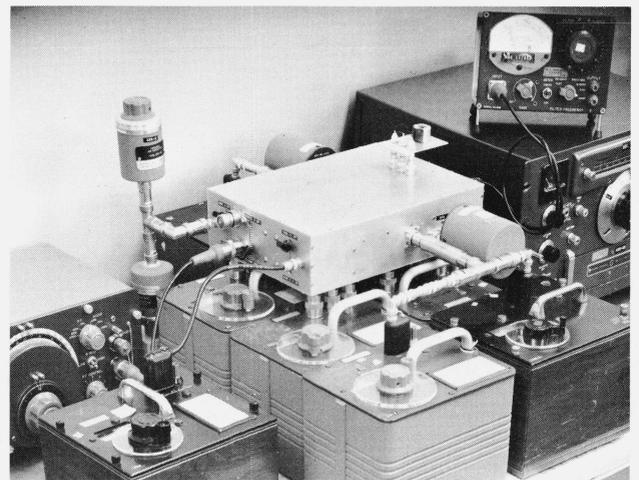FIGURE 9. *A more detailed drawing of the circuit shown in figure 8.*

FIGURE 10. *Photograph of bridge components, showing a divider, with its shield removed, connected at the top of the bridge.*

length of cable. Large inductance is needed to decrease ρ_r, and minimum cable length is needed to make L_{sc} small. The leakage inductance per unit length of cable is given approximately by

$$\frac{L_{sc}}{l} = 0.2\,\frac{P}{d}\,\tan^{-1}\frac{d}{P}\ \ \mu H/m \qquad (14)$$

where

$P = 2t + \delta$

t = wire insulation thickness

d = wire diameter

δ = skin depth $\left(\leqslant \dfrac{d}{2}\right)$

l = cable length.

If $\delta > d/2$, use $d/2$ in place of δ. Equation (14) is actually the inductance per unit length of two long parallel thin tapes of width d spaced a distance P apart. By defining P and d as stated under (14), the results from (14) agree with measured data to within 10 percent which is sufficiently accurate for most applications. Note that L_{sc} is independent of the core permeability. To decrease L_{sc} it is necessary to decrease the cable length. Changing the wire diameter or insulation thickness has only a secondary affect on L_{sc}. Measured values of L_{sc} and R_{sc} from 100 kHz to 2 MHz are shown in figure 11 for No. 24 wire. The leakage inductance decreases slightly due to skin effect. The resistance R_{sc} increases slightly faster than the square root of frequency primarily due to skin effect.

5.2. Core Material

Figure 12 shows how ρ_r is decreased by increasing the permeability μ. These four transformers were identical except for the type of core material used. One long twisted cable was made and cut in half. One half was used to wind No. 10 and the other half to wind No. 11. Transformers Nos. 12 and 13 were, likewise, made from one cable cut in half. The results show that ρ_r is inversely proportional to μ. The cores were all very low-loss ferrite toroids with loss tangents of about 0.002. Cores which are very lossy do not give the same results. The Q for transformers with lossy cores is so low that the second term on the right of (6) may become larger than the first. The second term is a function of the core loss as well as the permeability. The core material, therefore, should have a high μ but not be too lossy. The core material can be fairly lossy if ΔR is small enough to make the second term on the right of (6) negligible.

5.3. Type of Wire

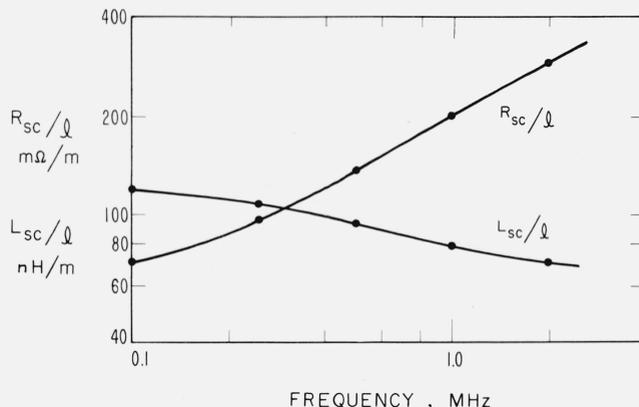A number of experiments with different types of wire led to the conclusion that wire which has a very

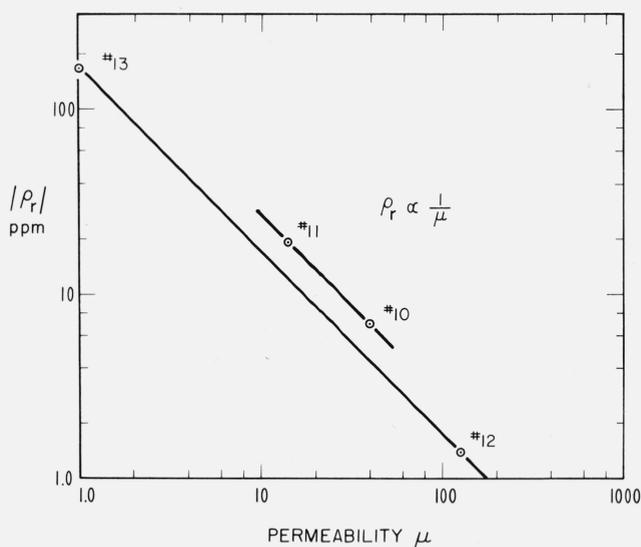FIGURE 11. *Variation of the leakage inductance and resistance with frequency.*

FIGURE 12. *Ratio error versus core permeability for four transformers made identical except for core permeability.*

uniform resistivity and diameter gives the lowest ΔR values. Figure 13 shows the errors for four transformers which are identical except for the type of wire used. These transformers have air cores to eliminate the effect of magnetic core properties. Another reason for using air core transformers is that the errors are larger and easier to measure. The shape of the single layer solenoid type winding is shown in figure 14. The different types of wire used were formvar coated magnet wire (C), TEFLON® coated magnet wire (A1 & A3), and copper thermocouple wire (T). Note that the thermocouple wire ρ_r curve is constant with frequency and quite small for an air core. This means that the second term in (6) is negligible even though the Q is fairly low. Using thermocouple wire we were consistently able to wind transformers with low errors which are constant with frequency.

The thermocouple wire transformers are true "inductive" voltage dividers. The other transformers are more nearly "resistive" dividers than "inductive" dividers. The inductance of each of these four transformers is about 15 μH. Since $\rho_r \doteq 10$ ppm for the thermocouple wire transformer, (6) gives $\Delta L \doteq 150$ pH. Typical values of ΔL are 100 to 500 pH. They are constant with frequency and independent of μ. Expressing L as $L = \mu N^2 L_a$, (6) becomes

$$\rho_r \doteq \frac{\Delta L}{\mu N^2 L_a}$$

where N = number of turns on the core and L_a is the geometric inductance of the core. Typical values of ΔR are 50 to 500 $\mu\Omega$ for thermocouple wire and 10 times that for the other wire we tried. These figures are all for No. 24 wire.

It should be noted that any wire, including thermocouple wire, generally gives larger errors if the wire is stretched. A number of transformers were made with wire that was stretched approximately 1 percent to remove slight kinks in the wire. The errors were significantly larger than for transformers made from the same spool of wire with the slight kinks left in the wire. It is best to purchase wire wound on large spools so that it is relatively free of kinks when removed from the spool.

5.4. Wire Size

The experiment described above was repeated with No. 20 wire and also with No. 30 wire. The larger wire transformers gave slightly lower ρ_i errors than the corresponding No. 24 wire transformers gave. There was no significant decrease in the ρ_r errors. The results are shown in figure 15. Transformers made with No. 30 wire had considerably higher errors than those made with No. 24 or No. 20 wire.

5.5. Cable Twist

To determine how much the wires should be twisted, a number of transformers were made identical except for the amount the wires were twisted. Figure 16a shows ρ_r and ρ_i plotted against the number of turns per unit length of the cable. The length, l_T of one turn is defined in figure 16b. These are all air core transformers wound with No. 24 thermocouple wire. The dots are single layer solenoid windings whereas the X's are multilayer coils wound on nylon bobbins. The inductance, L, of each transformer is approximately 15 μH. As can be seen from figure 16, any twist from four to ten turns/inch (160 to 400 turns/meter) gives about the same results. Below four turns/inch the error increases. At about 12 turns/inch (500 turns/meter) the wire breaks. In terms of the twist length, l_T, and wire diameter, d, satisfactory results were obtained in the range

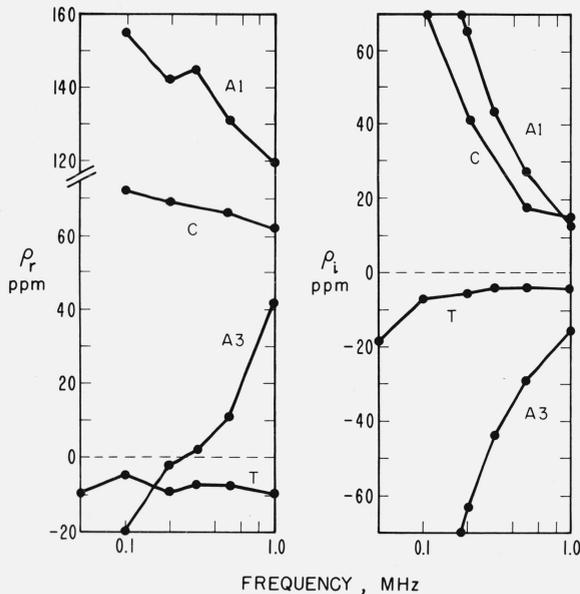$$0.08 < \frac{d}{l_T} < 0.2.$$

FIGURE 13. *Comparing results from four different types of No. 24 wire, where A1 and A3 are TEFLON ® coated magnet wire with single (0.0006 in) and triple (0.0015 in) insulation thickness; C is formvar coated magnet wire, and T is enamel coated copper thermocouple wire.*
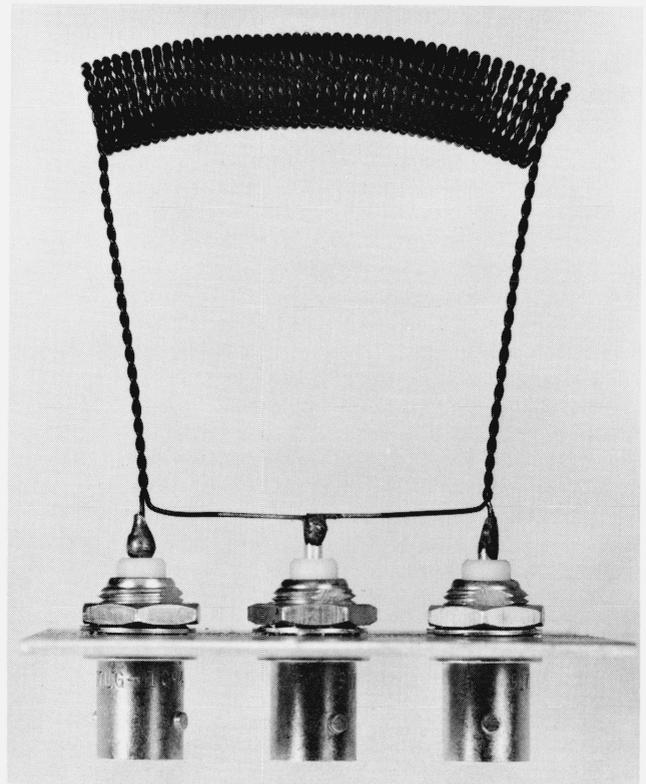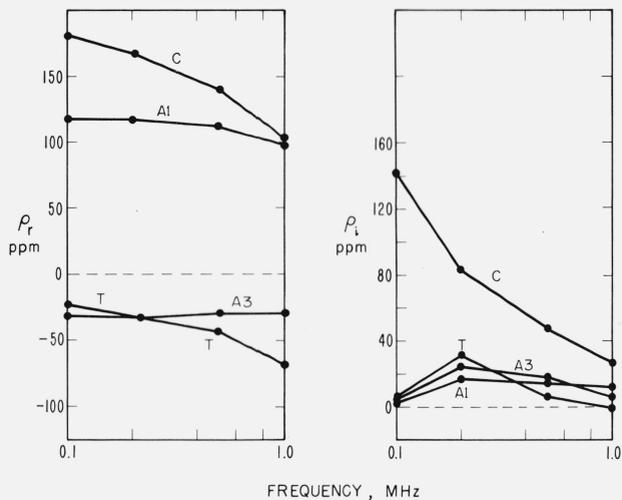

FIGURE 14. *An air core, solenoid type winding.*

FIGURE 15. *Comparing results from four different types of No. 20 wire where A1, A3, C, and T are as defined in figure 13.*

FIGURE 16. *Results of air core solenoid type (dots) and bobbin type (X's) transformers wound with cables twisted a different number of turns per inch.*

FIGURE 17. *Three different arrangements of the transformer leads.*

This equation should also be a reasonable guide for twisting wires other than size No. 24.

5.6. Lead Connections

One more important consideration in winding the transformers is how the leads are connected. The first transformers that we wound were connected as shown in figure 17a. The ratio errors were quite unstable and very sensitive to slight movement of the leads. To find a better way of connecting the leads, a number of air core transformers were made identical except for the lead connections. Figure 17 shows the three different ways the leads were connected. Of these three ways, the one shown in figure 17c was most stable and gave the smallest errors. The improvement in stability is probably due to the smaller mutual inductance and capacitance between leads. The lower error is probably a result of the ease with which the corresponding leads can be cut to the same length. It is important that in any arrangement of the leads that the leads on one side of the center tap be the same length as the corresponding leads on the other side. A difference in lead length of 0.001 in will give a ΔL of about 40 pH for No. 24 wire.

5.7. Cup-Core Design

One of the easiest ways of constructing a transformer with magnetic cores is to wind the cable on a bobbin and then clamp magnetic cup cores to the bobbin such as shown in figure 18. Figures 19 and 20 show the errors for three cup core transformers constructed like the one shown in figure 18. These transformers were made with all the optimum parameters that were found. The cables were made with No. 24 thermocouple wire twisted to six turns/inch (240 turns/meter). The cores
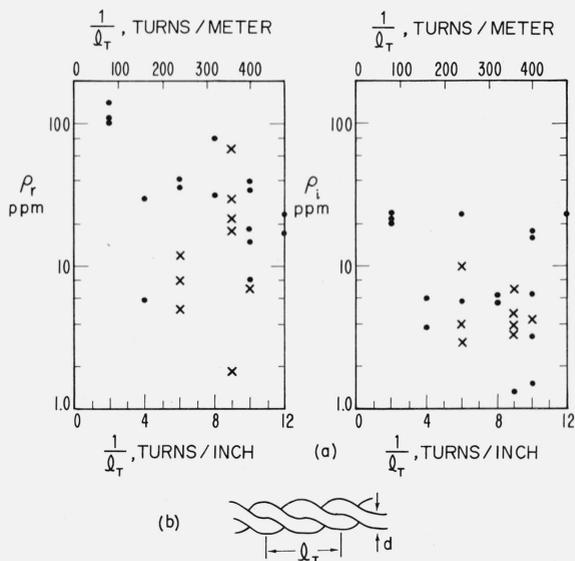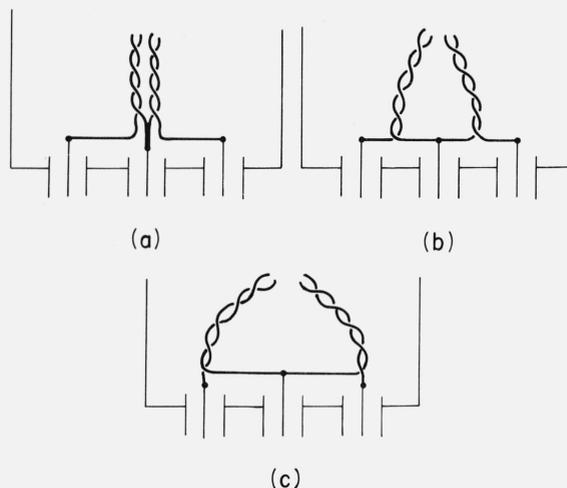
are high permeability, fairly low-loss ferrite cores. The increase in ρ_r and ρ_i at frequencies above 100 kHz is due to the capacitance between leads not being equal. If these capacitances were balanced as discussed in part 3, the errors would be less than 1 ppm to above 1 MHz.

With care, the error can be made less than 0.1 ppm. A fourth cup core transformer was adjusted to have a ρ_r error less than 0.1 ppm at 100 kHz by changing the length of one lead slightly. Then ρ_r was adjusted to less than 0.1 ppm at 1 MHz by adjusting small trimmer capacitors placed between the leads. As figure 21 shows, the resulting ρ_r error is less than 0.1 ppm from 30 kHz to above 1 MHz.
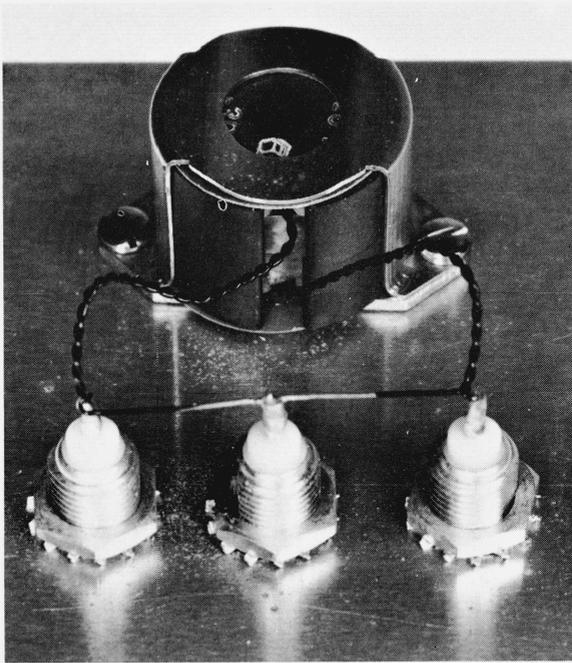
FIGURE 18. *A cup-core transformer having very low ratio errors like that shown in figures 19 and 20.*
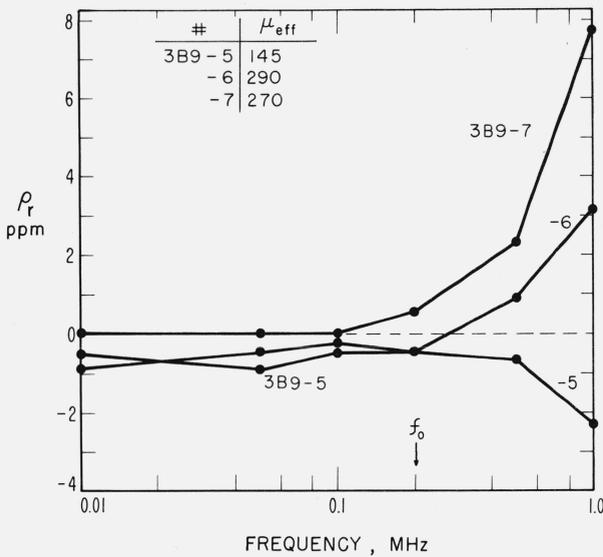The cover of the shield is not shown.

FIGURE 19. *Results from three cup-core transformers wound with No. 24 copper thermocouple wire twisted to six turns/inch.*

FIGURE 20. *Phase angle error for the transformers described in figure 19.*

FIGURE 21. *Ratio error of a cup core transformer which has been adjusted to have a minimum ρ_r by adjusting lead length and by balancing the capacitance between leads.*

FIGURE 22. *Ratio error of a transformer wound on a supermalloy 1 mil tape core having dimensions OD = 1.150, ID = 0.650 and H = 0.375 in.*

5.8. Tape-Core Transformers

Very broad band 2:1 ratio dividers can be made with high permeability magnetic tape cores. Figure 22 shows the ratio error of a divider wound on a supermalloy tape core having a tape thickness of 1 mil. The
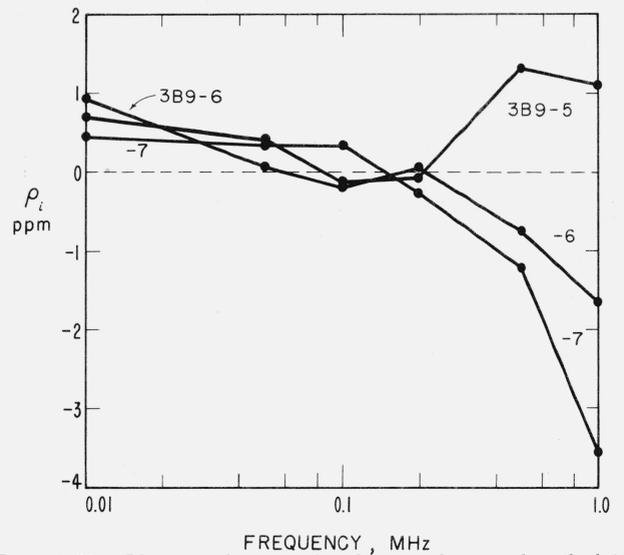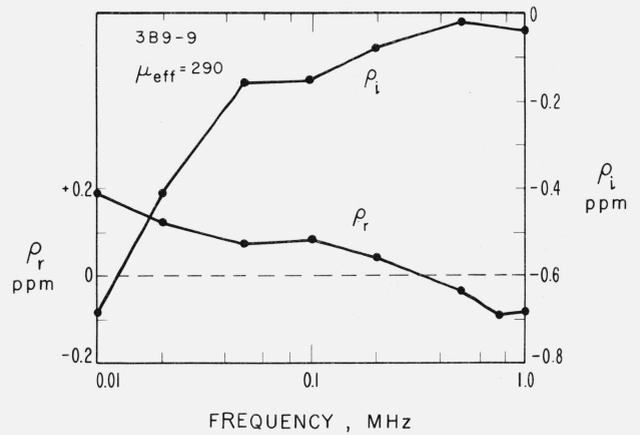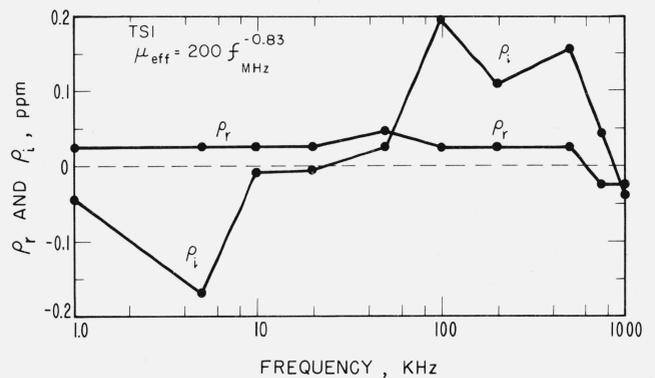
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core has 64 turns of No. 24 copper thermocouple wire twisted to about 8 turns/inch (320 turns/meter). The capacitance between leads has been adjusted to be equal. The ρ_r error is less than 5 parts in 10^8 from 1 kHz to 1 MHz.

6. Conclusion

The 2:1 ratio transformer is a simple, yet accurate, divider which is easy to construct. Using the optimum parameters discussed in part 5, it is easy to keep the ratio error less than 1 ppm from 1 kHz to 1 MHz. Air core transformers having no magnetic cores can be made with errors less than 10 ppm over a reduced frequency range. With extra care, ρ_r can be made less than 0.1 ppm to above 1 MHz. The ratio errors are quite insensitive to temperature, time, shock, and most any change in the environment since both windings of the transformer are influenced equally by any change in the environment.

Five cup-core transformers such as described in part 5.7 have been combined as shown in figure 2b to make an accurate 30 dB attenuator at 1 MHz. Details of this type of attenuator together with a complete error analysis will be given in a future publication.

7. References

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[4] Cutkosky, R. D., and J. Q. Shields (September 1960), The precision measurement of transformer ratios, IRE Trans. on Instrumentation **I–9,** No. 2, 243–250.