

Additional Remarks on a Theorem of M. Riesz¹

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Let V be a real four-dimensional vector space, whose underlying geometry is the metric defined by the matrix

$$K = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

The following theorem is proved.

THEOREM: *If A and B are real, skew-symmetric, orthogonal (with respect to K) linear transformations on V , then $[A, B] = AB - BA$ is a multiple of a real, skew-symmetric, orthogonal (with respect to K), linear transformation on V .*

The theorem is proved by use of the first and second regular representations of the real quaternions. Methods are given for constructing all 4×4 matrices which are skew-symmetric and orthogonal with respect to K , and all 4×4 matrices which are skew-symmetric (in the Euclidean sense) and orthogonal with respect to K .

Key Words: Commutator, matrix, orthogonal, quaternions, skew-symmetric, regular representation.

1. Introduction

M. Riesz, in a series of lectures [4]² dealing with Clifford algebras delivered at the University of Maryland (September, 1957, to February, 1958) proved the following theorem.

THEOREM: *If A and B are real, skew-symmetric, orthogonal linear transformations on E_4 (a real four-dimensional vector space on which there has been imposed a Euclidean metric), then $[A, B] = AB - BA$ is a multiple of a real, skew-symmetric, orthogonal transformation on E_4 .*

Subsequently, M. Pearl [3] proved this theorem using only the properties of the matrices associated with these transformations and the first and second regular representations of the real quaternions. Furthermore, Pearl proved that if the underlying geometry is changed from the Euclidean metric to the Lorentzian metric then the theorem is satisfied vacuously.

The purpose of this paper is to prove the theorem in the remaining four-dimensional case. That is, when the underlying geometry of the vector space is the metric defined by the matrix

$$K = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

The main tools used are the first and second regular representations of the real quaternions. Methods are given for constructing all 4×4 matrices which are skew-symmetric and orthogonal with respect to K , and for constructing all 4×4 matrices which are skew-symmetric (in the Euclidean sense) and orthogonal with respect to K .

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²Figures in brackets indicate the literature references at the end of this paper.

2. Regular Representations

Consider any algebra \mathfrak{A} over the field \mathfrak{F} . If \mathfrak{A} has a basis consisting of e_1, e_2, \dots, e_n then

$$e_i e_j = \sum_{k=1}^n c_{ijk} e_k, \quad c_{ijk} \in \mathfrak{F} \quad (1)$$

Using the method and notation of Pearl [3], if we define the $n \times n$ matrices $R_i = (c_{isr})$ and $S_i = (c_{ris})$, where r denotes the row index and s the column index, by associativity in \mathfrak{A} we have

$$R_i R_j = \sum_{k=1}^n c_{ijk} R_k \quad (2)$$

$$S_i S_j = \sum_{k=1}^n c_{ijk} S_k \quad (3)$$

and
$$R'_i S_j = S_j R'_i \quad (4)$$

where R'_i is the transpose of R_i .

If for any element a of \mathfrak{A} ,

$$a = \alpha_1 e_1 + \alpha_2 e_2 + \dots + \alpha_n e_n, \quad \alpha_i \in \mathfrak{F}$$

we define

$$R(a) = \alpha_1 R_1 + \alpha_2 R_2 + \dots + \alpha_n R_n, \\ S(a) = \alpha_1 S_1 + \alpha_2 S_2 + \dots + \alpha_n S_n \quad (5)$$

then the mappings $a \rightarrow R(a)$ and $a \rightarrow S(a)$ of \mathfrak{A} into the algebras of the matrices, \mathfrak{R} and \mathfrak{S} , are homomorphisms called the first and second regular representations of \mathfrak{A} . Furthermore, if \mathfrak{A} has an identity element, then these mappings are isomorphisms.

In the case in which \mathfrak{A} is the algebra of the real quaternions, \mathfrak{F} is the real field and \mathfrak{A} has basis elements e_0, e_1, e_2, e_3 where e_0 is the identity element. It follows that $R_0 = S_0 = I$, the identity matrix, and

$$R_1 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad R_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \\ R_3 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad S_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

$$S_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \quad (6)$$

Since $R_1, R_2, R_3, S_1, S_2, S_3$ clearly span the six dimensional vector space \mathfrak{D} of 4×4 skew-symmetric matrices, it follows that they are linearly independent and form a basis for \mathfrak{D} .

Thus, every 4×4 skew-symmetric matrix Q has a unique representation as $R(a) + S(b)$ where a and b are pure quaternions. (A pure quaternion is a quaternion whose e_0 -coefficient is zero.)

3. Skew-Symmetric Matrices

Now let us consider some properties of skew-symmetric matrices, in particular, the matrices $R(a)$ and $S(b)$.

Since each of the R 's and S 's is skew-symmetric, (4) becomes $R(a)S(b) = S(b)R(a)$. Now consider the mapping σ of the real vector $\mathbf{a} = (\alpha_1, \alpha_2, \alpha_3)$ into the pure quaternion $a = \sigma(\mathbf{a})$. Using this, Pearl [3] proves the following:

LEMMA 1: If a and b are orthogonal, pure quaternions

$(\mathbf{a} \cdot \mathbf{b} = 0)$, then $R(a)R(b) = -R(b)R(a)$ and $S(a)S(b) = -S(b)S(a)$.

For the pure quaternion $a = \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3$ we define the norm, $N(a)$, to be $\alpha_1^2 + \alpha_2^2 + \alpha_3^2$. Clearly for any real number α , $N(\alpha a) = \alpha^2 N(a)$ and furthermore $a^2 = -N(a)e_0$. Thus

$$R(a)^2 = S(a)^2 = -N(a)I. \quad (7)$$

For the skew-symmetric matrix $Q = R(a) + S(b)$ we define the conjugate, \bar{Q} , to be $R(a) - S(b)$, and the norm, $N(Q)$, to be $N(b) - N(a)$. Thus,

$$Q\bar{Q} = \bar{Q}Q = R(a)^2 - S(b)^2 = N(Q)I. \quad (8)$$

Furthermore, if \mathbf{c} is any vector orthogonal to \mathbf{b} , then by lemma 1

$$S(c)QS(c)^{-1} = S(c)[R(a) + S(b)]S(c)^{-1} = R(a) - S(b) = \bar{Q} \quad (9)$$

Thus, \bar{Q} and Q are similar and hence taking determinants of both sides of (8) we have

$$\text{Det } Q \text{ Det } \bar{Q} = (\text{Det } Q)^2 = N(Q)^4$$

and since the determinant of a real skew-symmetric matrix is nonnegative, we have

$$\text{LEMMA 2: Det } Q = \text{Det } \bar{Q} = N(Q)^2.$$

4. *K*-Skew-Symmetric, *K*-Orthogonal Matrices

Let the metric on the real four-dimensional vector space V be defined by the matrix

$$K = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

We defined a matrix A to be orthogonal with respect to K (K -orthogonal) if $A'KA = K$, and we define a matrix B to be skew-symmetric with respect to K (K -skew-symmetric) if $x'BKx = 0$ for every vector x in V . Thus B is K -skew-symmetric if and only if BK is skew-symmetric in the Euclidean sense.

Consider now a matrix A which is both K -skew-symmetric and K -orthogonal. Since A is K -skew-symmetric there exists a skew-symmetric matrix Q such that $AK = Q$ or $A = QK$. Thus we have

$$A' = KQ' = -KAK. \quad (10)$$

Furthermore, since A is K -orthogonal, by definition $A'KA = K$ and hence by (10)

$$A^2 = -I.$$

$$\text{Thus } A^{-1} = -A. \quad (11)$$

$$\text{But also } A^{-1} = (QK)^{-1} = KQ^{-1}. \quad (12)$$

If we define the norm of A , $N(A)$, to be the norm of Q for $AK = Q$ we have

LEMMA 3: *If the matrix A is K -skew-symmetric and K -orthogonal then $N(A) = \pm 1$.*

PROOF: $A'KA = K$ and $AK = Q$. Hence $Q'KQ = K$, and taking determinants of the terms $(\det Q)^2 = 1$. Thus $\det Q = \pm 1$. Since Q is skew-symmetric $\det Q = 1$. By lemma 2, $\det Q = N(Q)^2$. Thus $N(A) = N(Q) = \pm 1$.

LEMMA 4: $KR_1K = \pm R_1$, $KS_1K = \pm S_1$, + if $i = 1$, - if $i = 2, 3$. Now we may prove the following

THEOREM 1: *The matrix A is K -skew-symmetric and K -orthogonal if and only if one of the following conditions is satisfied:*

1. $N(A) = 1$ and A has the form $\alpha_2 R_2 K + \alpha_3 R_3 K + \beta_1 S_1 K$.

2. $N(A) = -1$ and A has the form $\alpha_1 R_1 K + \beta_2 S_2 K + \beta_3 S_3 K$.

PROOF: Let A be K -skew-symmetric and K -orthogonal. Then by (11) and (12), for $A = QK$ we have

$$A^{-1} = -QK = KQ^{-1}.$$

$$\text{Thus by (8)} \quad -KQK = \frac{1}{N(Q)} \bar{Q}. \quad (13)$$

Let $Q = R(a) + S(b)$ where $R(a) = \alpha_1 R_1 + \alpha_2 R_2 + \alpha_3 R_3$, and $S(b) = \beta_1 S_1 + \beta_2 S_2 + \beta_3 S_3$. Then we have two cases.

Case 1. $N(A) = N(Q) = +1$.

Substituting in (13) for Q and \bar{Q} and applying lemma 4 we obtain

$$-2\alpha_1 R_1 + 2\beta_2 S_2 + 2\beta_3 S_3 = 0.$$

Since R_1, S_2 , and S_3 are linearly independent, $\alpha_1 = \beta_2 = \beta_3 = 0$ and hence if $N(A) = 1$, $A = \alpha_2 R_2 K + \alpha_3 R_3 K + \beta_1 S_1 K$.

Case 2. $N(A) = N(Q) = -1$.

Again, substituting in (13) for Q and \bar{Q} and applying lemma 4, we obtain $\alpha_2 = \alpha_3 = \beta_1 = 0$ and hence if $N(A) = -1$, $A = \alpha_1 R_1 K + \beta_2 S_2 K + \beta_3 S_3 K$.

Conversely, if $N(A) = 1$ and $A = \alpha_2 R_2 K + \alpha_3 R_3 K + \beta_1 S_1 K$, AK is skew-symmetric, and hence A is K -skew-symmetric. Furthermore,

$$\begin{aligned} A'KA &= (-KQ)(KQK) = +KQ\bar{Q} \text{ by lemma 4} \\ &= K[N(Q)I] = K. \end{aligned}$$

Thus A is K -orthogonal. Similarly if $N(A) = -1$ and $A = \alpha_1 R_1 K + \beta_2 S_2 K + \beta_3 S_3 K$, A is K -skew-symmetric and K -orthogonal.

Using (6) and lemma 4 we may construct the following table of commutators.

[,]	R_1K	R_2K	R_3K	S_1K	S_2K	S_3K
R_1K	0	0	0	0	$2S_3K$	$2S_2K$
R_2K	0	0	$2S_1K$	$-2R_3K$	0	0
R_3K	0	$-2S_1K$	0	$-2R_2K$	0	0
S_1K	0	$2R_3K$	$2R_2K$	0	0	0
S_2K	$-2S_3K$	0	0	0	0	$2R_1K$
S_3K	$-2S_2K$	0	0	0	$-2R_1K$	0

We can now prove the analog to Riesz' Theorem. Let A and B be two K -skew-symmetric, K -orthogonal matrices. Then we have,

Case 1. $N(A) = N(B) = 1$. Using the linearity of commutators and the results of the table,

$$\begin{aligned}
[A, B] &= [\alpha_2 R_2 K + \alpha_3 R_3 K + \beta_1 S_1 K, \gamma_2 R_2 K \\
&\quad + \gamma_3 R_3 K + \delta_1 S_1 K] \\
&= 2(\beta_1 \gamma_3 - \alpha_3 \delta_1) R_2 K + 2(\beta_1 \gamma_2 - \alpha_2 \delta_1) R_3 K \\
&\quad + 2(\alpha_2 \gamma_3 - \alpha_2 \gamma_2) S_1 K
\end{aligned}$$

which is a multiple of a K -skew-symmetric, K -orthogonal matrix with norm 1, or zero if $A = \pm B$.

Case 2. $N(A) = N(B) = -1$.

$$\begin{aligned}
[A, B] &= [\alpha_1 R_1 K + \beta_2 S_2 K + \beta_3 S_3 K, \gamma_1 R_1 K + \delta_2 S_2 K \\
&\quad + \delta_3 S_3 K] = 2(\beta_2 \delta_3 - \beta_3 \delta_2) R_1 K \\
&\quad + 2(\alpha_1 \delta_3 - \beta_3 \gamma_1) S_2 K + 2(\alpha_1 \delta_2 - \beta_2 \gamma_1) S_3 K
\end{aligned}$$

which is a multiple of a K -skew-symmetric, K -orthogonal matrix with norm -1 , or zero if $A = \pm B$.

Case 3. $N(A) = -N(B) = -1$.

$$\begin{aligned}
[A, B] &= [\alpha_2 R_2 K + \alpha_3 R_3 K + \beta_1 S_1 K, \gamma_1 R_1 K \\
&\quad + \delta_2 S_2 K + \delta_3 S_3 K] \\
&= 0.
\end{aligned}$$

Thus the theorem is satisfied in all possible cases.

COROLLARY: Let A and B be K -skew-symmetric, K -orthogonal matrices such that $A \neq \pm B$. Then A and B commute if and only if $N(A)N(B) = -1$.

5. Skew-Symmetric, K -Orthogonal Matrices

Since we have a method for constructing all K -skew-symmetric, K -orthogonal matrices, and Pearl [3] gives a method for constructing all skew-symmetric, orthogonal matrices, the question arises, what is the form of a skew-symmetric, K -orthogonal matrix?

THEOREM 2: The matrix M is skew-symmetric (in the Euclidean sense) and K -orthogonal if and only if $M = AK$ where A is a K -skew-symmetric, K -orthogonal matrix. (Thus by Theorem 1, M is either of the form

$$\alpha_2 R_2 + \alpha_3 R_3 + \beta_1 S_1, \beta_1^2 - \alpha_2^2 - \alpha_3^2 = 1, \text{ or}$$

$$\alpha_1 R_1 + \beta_2 S_2 + \beta_3 S_3, \beta_2^2 + \beta_3^2 - \alpha_1^2 = -1.)$$

PROOF: If M is skew-symmetric and K -orthogonal, then M' and $-M$ and $M'KM = K$. Hence

$$(KM)^2 = (MK)^2 = -I$$

and thus

$$M^{-1} = -KMK$$

But by (8), $M^{-1} = \frac{1}{N(M)} \bar{M}$. Thus we have

$$-KMK = \frac{1}{N(M)} \bar{M}.$$

This is the same condition as (13). Thus $M = AK$ where A is a K -skew-symmetric, K -orthogonal matrix.

Conversely, if $M = AK$, where A is K -skew-symmetric and K -orthogonal, by definition M is skew-symmetric and since $A'KA = K$ we have $M'KM = K$.

6. Conclusion

This paper, together with Pearl's [3], completes the proof of Riesz' Theorem on all real, four-dimensional vector spaces, regardless of the metric imposed on the space.

Unfortunately, the methods of this paper are rather restricted, and it is not expected that they may be readily applied in the consideration of spaces of higher dimension.

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