

Some Techniques for Measuring Small Mutual Inductances

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A method of measuring small mutual inductances is presented. The smallest inductor measured was $0.1 \mu\text{H}$. The circuit is a transformer-ratio-arm bridge with multiple balances and is described in detail. Uncertainties are of the order of one part in 10^7 .

Several ideas for the design and construction of suitable mutual inductance standards are presented.

Key Words: Auxiliary generator, bridge, coaxial choke, mutual inductor, stray magnetic field, transformer.

1. Introduction

A recent theoretical study by Page [1]¹ suggests the desirability of making accurate measurements of mutual inductance at the $0.1 \mu\text{H}$ level and of stepping up to larger values. An experimental bridge has been constructed for the purpose of investigating some techniques which seemed appropriate in this range. Some of those techniques which appear to be particularly useful are described in this paper. The bridge is a transformer-ratio-arm bridge, having a ratio of 10 to 1, and has been designed to compare mutual inductors through five orders of magnitude, from $0.1 \mu\text{H}$ to 1 mH at a frequency of 1592 Hz. Sensitivity of a part in 10^7 is achieved at the $0.1 \mu\text{H}$ to $1 \mu\text{H}$ level provided the inductors carry 3A in their primary windings.

2. Bridge Circuit

A simplified version of the bridge is shown in figure 1. Mutual impedances Z_1 and Z_2 , which are both mainly inductive, are defined by the equations

$$Z_1 = j\omega M_1 + R_1$$

$$Z_2 = j\omega M_2 + R_2.$$

If both inductors have the same current, i , in their primary windings,² the comparison of Z_1 to Z_2 is achieved by measuring the ratio of the induced voltages e_1 and e_2 . Since $Z_1 = \frac{e_1}{i}$ and $Z_2 = \frac{e_2}{i}$, then $\frac{Z_1}{Z_2} = \frac{e_1}{e_2}$. The ratio of e_1/e_2 is compared with the 10 to 1 voltage ratio of an inductive voltage divider, which may be calibrated by a modification of the capacitance-ratio

method described by Cutkosky and Shields [2]. The bridge balance is accomplished by the adjustment of e_3 , which is an adjustable voltage source, small relative to e_1 and e_2 .

The detector consists of a tunable amplifier and a tuned high-impedance preamplifier, coupled to the bridge through an impedance matching transformer. The transformer is tuned with a variable capacitor to give maximum output voltage.

Accurate measurement of small inductances requires that all sources of stray magnetic field be well controlled. To that end, inductors and transformers are toroidal, bridge components are enclosed in copper eddy-current shields, components are interconnected by coaxial cables, and shield current of the coaxial leads is restricted to be nearly equal and opposite to current in their inner conductors. Figure 2 illustrates shielding and the use of the technique of threading a coaxial lead through a high permeability core [3] forming a coaxial choke to equalize current in the inner conductor and outer shield of the coaxial cable.

Following construction of the bridge, tests were made to establish the effectiveness of the design in keeping stray magnetic fields small. Such tests were made by using two coils of wire, one connected to a

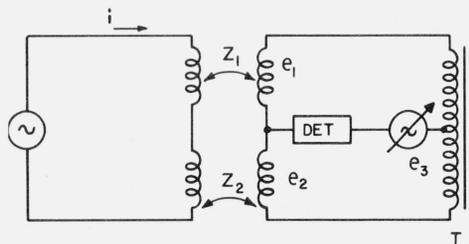


FIGURE 1. Simplified inductance bridge circuit.

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¹Figures in brackets indicate the literature references at the end of this paper.

²Ground capacitance current between the primary windings is negligible; it is discussed in section 4 of this paper.

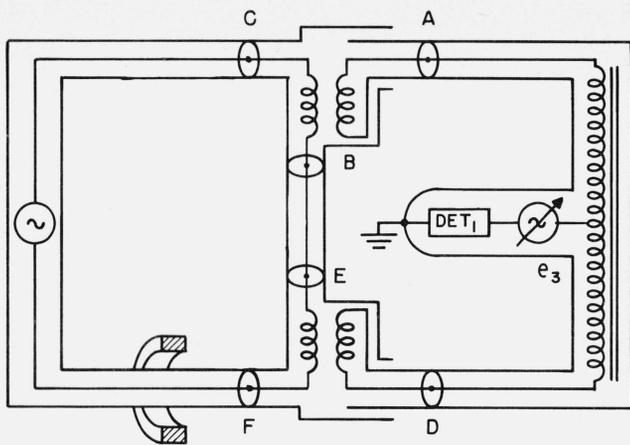


FIGURE 2. Inductance showing shielding, junction points (A, B, C, D, E, F) of the mutual inductor, and use of high-permeability core to equalize coaxial current.

detector and used to find sources of stray field, and the other connected to a generator and used to find places in the circuit that were sensitive to stray field. For example, the detector coil was moved around until a source of stray field was found, and the amount of detector off-balance was noted. The coil connected to the generator was driven so that its field gave about the same off-balance at a comparable distance from the detector coil. Then the driven coil was used as a probe to determine the effect of such a field on the main bridge balance when brought near various bridge components.

The largest sources of stray field were the coaxial leads in the loop that includes the inductor primary windings since the current is largest in that part of the circuit. It was determined that the coaxial choke, mentioned above, was not sufficient to equalize current in the inner conductor and outer shield of the coaxial cable. A greatly improved coaxial choke was realized by making it an active device using two cores in a manner described by Gibbins [4]. As it is used in this application, it may be called an "active coaxial choke." The principle is shown in figure 3. If $e_p = Z_p i$, $e_q = G e_p$, and $e_T = e_p + e_q$, it can be shown that $Z_T = Z_p(1 + G)$. Thus, the active choke reduces net current much more than a coaxial choke composed of a single magnetic core. The gain of the amplifier and the number of turns of cable through the device was adjusted to reduce net current by an amount which made the error negligible.

Small stray magnetic fields due to slight nonsymmetries in the coaxial cables remained. Using the probe technique discussed above, three modifications were made. First, the copper can serving as an eddy-current shield for the inductive divider was found to be unsatisfactory because the lid was electrically attached to the can at only one point. The shield was improved by bolting the lid to the can firmly with screws spaced about 2 in apart around its circum-

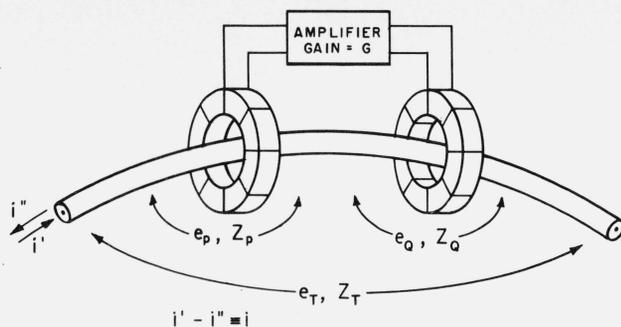


FIGURE 3. Active coaxial choke.

ference. Second, an uncertainty in the value of mutual inductance arises from the coupling between the leads connected to the inductor primary windings and the leads connected to the inductor secondary windings, if the leads move relative to each other. Subsequent inductors were designed to have the primary winding terminals on the opposite side of the shield can from the secondary terminals. Third, the smallest inductors (for which the ratio of mutual inductance of the leads to mutual inductance of the inductor is largest) were modified as follows: The eddy-current shield was extended over the secondary leads by passing these leads through a three-inch-long copper cylinder having $1/8$ -in wall thickness (about two skin depths at the frequency used).

The standards to be compared, as illustrated in figure 2, are defined as follows: Let i'_A be the current into the inner terminal of connector A, i''_A be the current out of the outer terminal, and e_A be the voltage from the inner to the outer terminal. Currents and voltages are similarly defined at connectors B, C, D, E, and F. Then the mutual impedances of interest are defined by $Z_{AB} = \frac{e_A}{i'_B}$ with the conditions $i'_B = i''_B$, $e_B = 0$, and $i'_A = i''_A = 0$; $Z_{DE} = \frac{e_D}{i'_E}$ with the conditions $i'_E = i''_E$, $e_E = 0$, and $i'_D = i''_D = 0$. The above conditions are similar to those used in the theoretical analysis by Cutkosky of four-terminal-pair networks [5].

3. Auxiliary Balances

The techniques used to assure the realization of the conditions defining the inductors involve the use of several auxiliary generators and detectors. Figure 4 shows the complete system. Each of the four auxiliary generators consists of a decade capacitance network in parallel with a decade conductance network, driven by a 40-turn winding on the magnetic core of the bridge supply transformer. Figure 4 shows two such 40-turn windings, the sign of the voltage of one being opposite to that of the other so that each capacitance decade or conductance decade can be switched

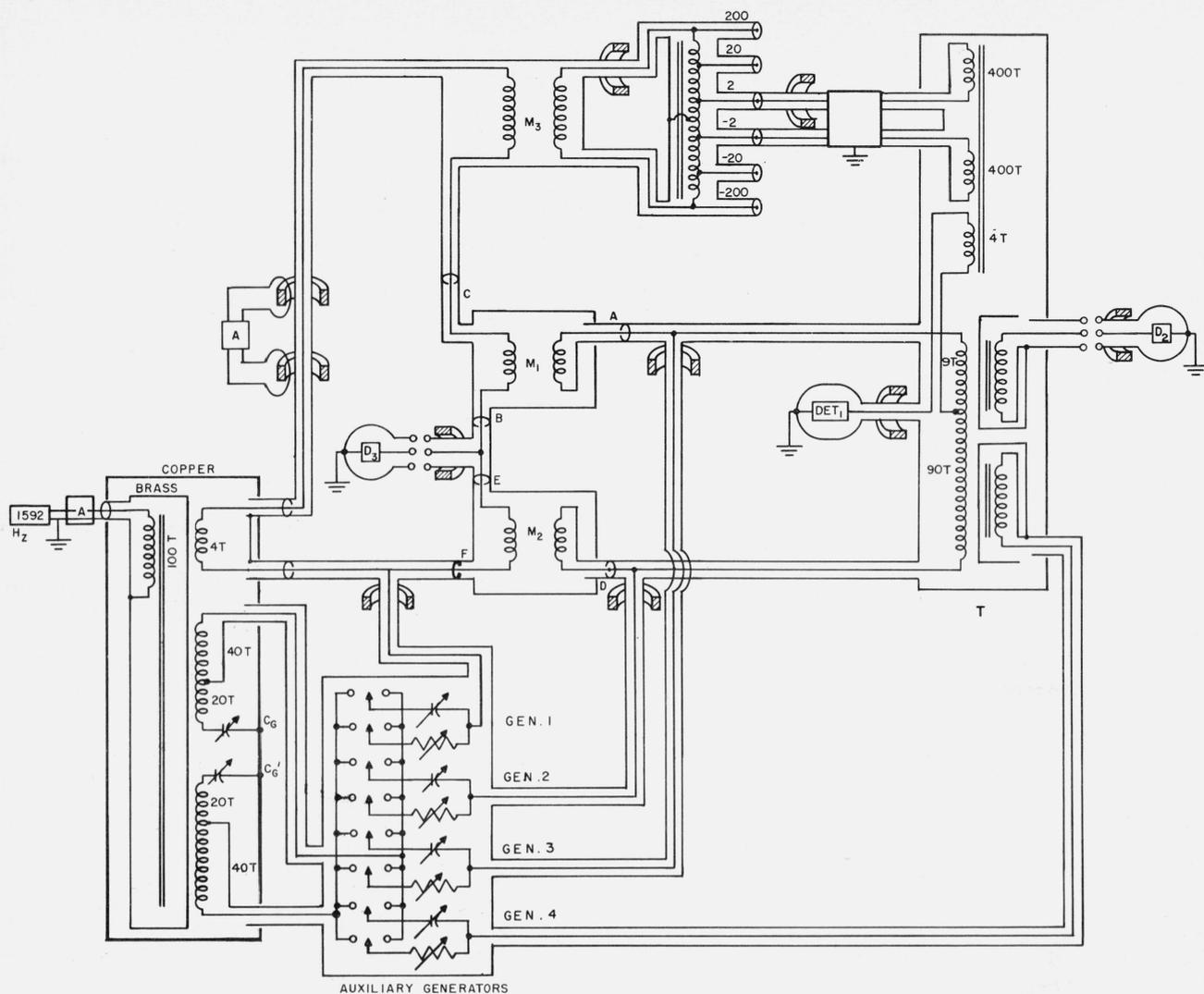


FIGURE 4. Complete system for measurement of small mutual inductances.

independently to reverse the sign of the voltage driving it. Stray capacitance from the 40-turn winding to the shield raises the potential of the shield and would result in current in the shield. This difficulty is overcome if an adjustable capacitor, C_G , is attached to the 20-turn winding, the voltage of which is opposite in sign to that of the 40-turn winding. A capacitance bridge is formed by placing a detector between the shield and the bottom of the 40-turn winding, and the bridge can be balanced by adjusting C_G . A similar bridge is formed with C'_G as one component. Permanent adjustments of C_G and C'_G were made following the completion of this component of the system.

The load on the secondary winding of each inductor is supplied by an auxiliary generator. The loading consists of capacitance in the cable connecting the

inductors to the inductive voltage divider, T , and capacitance in the inductive voltage divider itself. Each of the auxiliary generators is adjusted until unplugging the inner conductor of the coaxial connection at the inductor secondary shows no change in the null at detector No. 1. Such a condition can be expressed as $i'_A = 0$ and $i'_D = 0$. The impedances of the auxiliary generators are high relative to the inductor secondary windings so that convergence to balance is more quickly achieved. Without the auxiliary generators, the loading in the circuit is such that the voltage e_A at connector A of a $100 \mu\text{H}$ inductor was changed by approximately one part in 10^5 when the load was disconnected from the circuit by unplugging the lead to the secondary winding. Thus the auxiliary generators need to be adjusted to one part in 10^3 to obtain accuracy of one part in 10^8 .

At the 1 mH level, an error arises from a capacitance change at the secondary terminal when a coaxial cable is connected to the terminal. When such a connection is made, the inner connector of the coaxial cable slips over the inner member of the connector on the inductor, thus changing the capacitance from the inner connector of the inductor to its shield about 0.3 pF. The impedance of the secondary winding of the 1 mH inductor used in this work is about 20 Ω . A calculation shows that one could expect an error of about 6 in 10⁸ due to such a capacitance change. This error can be made reproducible by using the same coaxial cable each time the 1 mH inductor is measured.

The inductive voltage divider T is a two-stage device. Two stage devices have been analyzed by Gibbings [4] and by Cutkosky [6]. The auxiliary generator (gen. 4 in fig. 4) incorporated in the transformer supplies the excitation current in the ratio winding. With the bridge balanced, a detector winding on the second core assures that no excitation current is supplied by the inductor secondary windings. Thus this inductive voltage divider behaves like a device having infinite input impedance in that it draws no current. An additional feature of the two-stage design is a highly accurate 10 to 1 ratio [4].

The conditions $e_B=0$ and $e_E=0$ are met by adjustment of the auxiliary generator 1 in the part of the circuit containing the primary windings of the mutual inductors such that detector No. 3 indicates null. Shunt admittances in conjunction with the impedance of the coaxial cable from the detector junction to points B or E may cause an error. A calculation based on an assumed equivalent circuit and approximate values for the impedance and admittance indicates that such an error is in the order of 1 in 10⁸ or less for typical length of coaxial cable used at this point. An experimental verification of the above calculation was made by observing the difference in the main bridge balance caused by changing the length of coaxial cable between B and E.

Tests of interdependence of the detector balances were made by bringing the bridge to balance, and then changing one auxiliary generator voltage while observing changes in the null conditions at the detector junctions. Such tests and experience in bringing all the detectors to a simultaneous null indicate some dependence of one balance on another. However, the time involved in obtaining convergence of all balances is not unreasonably long.

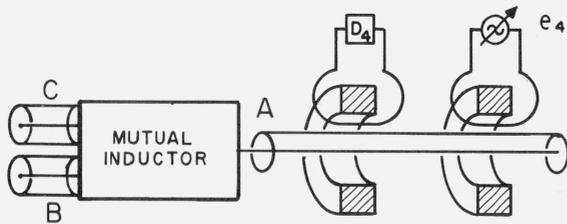


FIGURE 5. Method of assuring zero net current at inductor secondary terminal.

The condition $i'_A=i''_A$ can be assured by an auxiliary device illustrated in figure 5. One can adjust e_4 (to null D_4) to compensate for net current which can result from capacitance from the primary to the secondary winding of an inductor. However, it is possible to construct inductors in such a way that the change in the main bridge balance (detector 1) when e_4 is switched in and out is small enough so that the corresponding error is less than 1 part in 10⁸. The 1 mH mutual inductor was altered by inserting a permanent short between the low-voltage side of the primary winding to the low-voltage side of the secondary. Inductors smaller than 1 mH were not fixed with a short as described above since the error corresponding to the change in the main bridge balance when e_4 is switched in and out in their case was smaller than 1 in 10⁸ without the short.

4. Main Bridge Balance

The circuit used to obtain the small adjustable voltage, e_3 is shown in figure 6. That portion of figure 6 shown within dotted lines represents a component part of an "a-c direct-reading-ratio set" which has been described by Cutkosky [6]. This is an active circuit with high input impedance and low output impedance. Hence M_3 is not strongly loaded, and reasonably large loads can be driven by the circuit. The in-phase and quadrature components of e_3 are read directly from dials of two inductive voltage dividers, IVD#1 and #2 in figure 6. The mutual inductor M_3 has nearly the same current in its primary winding as the inductor M_1 . The voltage e_3 is related, through current i , to e_1 and e_2 . The relationship is controlled by the choice of the value of M_3 , which provides a scale factor. The value of M_3 has been calculated such that one step of the last dial is one part in 10⁸.

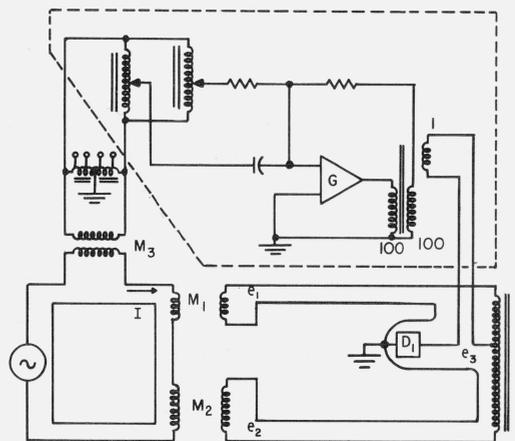


FIGURE 6. Basic bridge and, in dotted lines, circuit used to obtain balance.

The range of e_3 is ± 5 in 10^4 and the least count is 1 in 10^8 . The current in M_3 , therefore, must be no more than 2 in 10^5 different from that in M_1 for accuracy of 1 in 10^8 . A calculation shows that ground capacitance between M_1 and M_3 and in M_3 must not exceed 1000 pF for this accuracy. The inductor M_3 is constructed to have ground capacitance well below 1000 pF. Components of the apparatus shown within dotted lines in figure 6 were initially adjusted to an accuracy of a few parts in 10^5 . Past experience with the type components used in the apparatus indicates that at the time of these measurements the accuracy of the dial readings was better than 1 part in 10^3 of the dial reading. Thus if the inductors being compared (i.e., M_1 and M_2) are adjusted so that their values differ by less than 1 in 10^5 , the measurements can be relied upon to 1 in 10^8 .

5. Measurement Procedure

A single detector is used for all balances; it is plugged alternately into positions D_1 , D_2 , and D_3 (fig. 4). Balancing starts by bringing D_1 to null by the dials IVD#1 and IVD#2 (fig. 6). Next, leaving the detector at D_1 , the secondaries of the mutual inductors M_1 and M_2 are unplugged in turn. With the secondary M_1 unplugged, a null at D_1 is obtained by turning the dials of gen. 3 (fig. 4); similarly, the secondary of M_2 is unplugged, and the balance reached with gen. 2. The detector is then plugged into the D_2 position and a null is achieved by turning dials of gen. 4. Finally, the detector is plugged into the D_3 position and brought to null by gen. 1.

All balances described above are then repeated several times until convergence is reached.

6. Inductors

Some of the requirements of the inductors are that they be stable, astatic, have small phase angle, be close to nominal value, and be capable of carrying substantial current in their primary windings.

To make the inductors astatic, both the primary and secondary have toroidal symmetry about the same axis; the primary winding is outside the secondary winding and, therefore, has a larger cross section. The leads, as shown in figure 7, are brought out to the

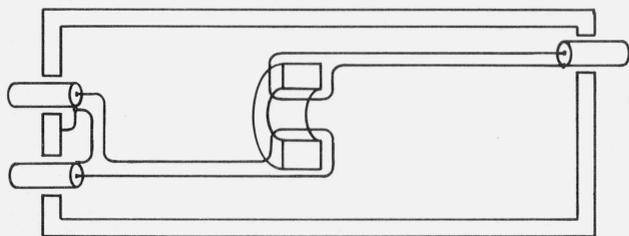


FIGURE 7. Lead geometry and shielding of mutual inductor.

coaxial connectors in such a way as to create a minimum loop. The inductor is mounted in a copper shield of $\frac{1}{8}$ -in thickness (about two skin depths at 1592 Hz). The coaxial connector of one of the primary leads is connected to the copper shield at one point inside the shield. The primary leads and the secondary leads are brought out on opposite sides of the toroid as shown in figure 7 to keep magnetic coupling to a minimum outside the toroid.

If two windings have toroidal symmetry about a common axis, and if one of these windings is totally within the other, the mutual inductor thus formed has been observed to have the following property: The phase angle of the inductor is practically independent of the thickness of the wire used for the outside winding; the phase angle depends on the thickness of the wire used on the inside winding, a larger phase angle being produced by thicker wire. The outside winding is, therefore, better suited for the primary, since the primary should have low resistance for minimum heat dissipation.

A few inductors have been constructed using toroidal forms of Bakelite; they were potted in epoxy resin and enclosed in $\frac{1}{8}$ -in thick copper shield cans. These inductors are useful for testing some of the features of the inductance bridge, but have temperature coefficients that are too large to make them useful as standards in this work.

A 0.1 μH inductor was made by painting silver windings onto a toroidal form of fused silica. The silica form, with the silver secondary winding, was baked and then a primary of copper wire was wound around the form but mechanically isolated from it. A critical problem is that of bringing the leads from the secondary out through the primary (see fig. 8). A cylindrical fused-silica rod was fused to the toroidal form as in figure 9 so that the leads could be brought out on a solid mount. An attempt to use silver painted leads was made but abandoned in favor of using #36 copper wire. These lead wires were tightly twisted

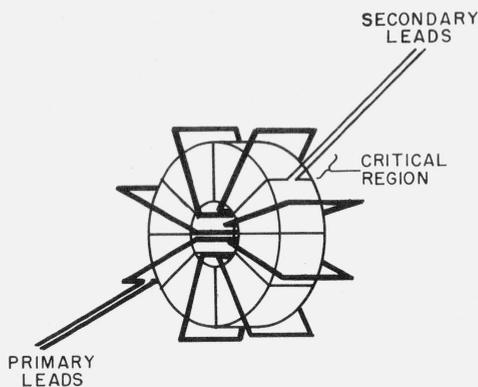


FIGURE 8. Winding geometry of mutual inductors having fused silica toroidal form.

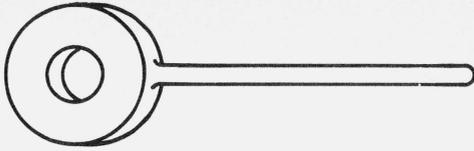


FIGURE 9. Fused silica toroidal form and lead mount.

and attached to the fused-silica rod with alkyd resin paint, and electrical contact to the secondary was made with silver paint. The gage of the wire was chosen to be as small as possible to keep the loop area small, but large enough such that lead resistance was below the resistance of the painted secondary.

A possible source of uncertainty due to relative motion between primary and secondary is the magnetic coupling from the "single turn" produced by the progression of the turns about the toroid. This is an effective loop around the toroid in which an emf can be induced by a component of magnetic field parallel to the axis of the toroid. To reduce this uncertainty all the skew of the turns was concentrated at the inner surface of the toroid as shown in (fig. 10). After a thin insulating coating of alkyd resin paint was applied, a return loop was painted on top of the skewed wires forming a loop in the opposite direction to the loop formed by the skew.

A nonuniformity of the secondary winding painted on the toroidal form could create a net loop which would cause the inductors in effect to be lopsided. Thus relative motion between the primary and secondary windings could change the inductance. A calculation for the $0.1 \mu\text{H}$ inductor indicates that such a net loop of 1 mm^2 would dictate that for stability of one part in 10^8 , the relative motion of primary to secondary must be 10^{-5} mm .

It appears that improved techniques in painting the secondary winding onto a fused-silica toroidal form would result in inductors of good stability.

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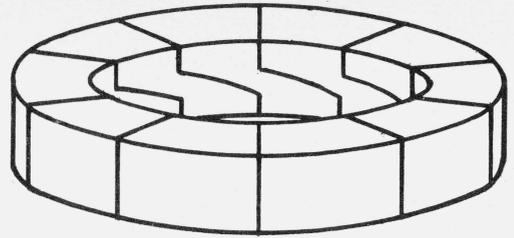


FIGURE 10. Illustration of painted secondary winding showing skew of turns concentrated in inner toroidal surface.

7. Conclusion

At the smallest level of inductance measured here, bridge sensitivity is one part in 10^7 . Sensitivity improves at larger values of inductance such that $100 \mu\text{H}$ can be compared with 1 mH with sensitivity of a few parts in 10^8 . It appears likely that the bridge techniques described here can be relied on to yield errors that are less than one part in 10^7 . An analysis of all errors in such a step-up from $0.1 \mu\text{H}$ to 1 mH must await the development of a more reliable set of mutual inductance standards.

The author thanks R. D. Cutkosky for suggesting the scheme of measurement used here.

8. References

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