Temperature Coefficent of RF Permeability Measurement Using an Impedance Bridge as an Equality Indicating Device

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Measurements are made on toroidal coils, without and with low-loss powdered iron samples inside, from approximately 23 to 50 °C and 300 kHz to 1.5 MHz. A high-precision Maxwell impedance bridge is used as an equality indicating device. Impedance changes of the toroidal coils are compensated by adjusting parallel capacitance and resistance standards which are external to the bridge. Data on several materials show that temperature coefficients of permeability of approximately $5 \times 10^{-6}/°C$ may be evaluated.

Key Words: Equality indicating device, magnetic materials, Maxwell bridge, powdered iron materials, rf permeability, temperature coefficient, toroidal coils.

Definitions of Symbols

$(TC)_{\mu'}$ = Temperature coefficient of permeability.

$(TC)_{\mu''}$ = Temperature coefficient of the imaginary component of permeability.

μ_T', μ_T'' = Real and imaginary components of initial permeability at temperature T_2.

μ', μ'' = Real and imaginary components of initial permeability at temperature T_1.

$\Delta T = T_2 - T_1$.

ω = Angular frequency.

L_a = Equivalent air inductance of sample.

C_p, R_p = Parallel capacitance, parallel resistance to impedance Z of toroidal coil.

C_s, R_s = Parallel capacity, parallel resistance to impedance Z of toroidal coil with a sample inside.

C_e, R_e = Parallel capacity, parallel resistance to impedance Z of toroidal coil *without* a sample inside.

L_s, R_s = Toroidal coil inductance, resistance with a sample inside.

L_e, R_e = Toroidal coil inductance, resistance *without* a sample inside.

N = Number of turns of toroidal coil.

L_{space} = Equivalent inductance of space of toroidal form.

α = Coefficient of linear expansion.

1. Introduction

Magnetic materials with low-temperature coefficients of permeability are useful in stable inductors.

This manuscript describes (1) a method for measuring low-temperature coefficient properties of low-loss materials, and (2) measurement results on toroidally shaped powdered iron samples.

The temperature coefficients of complex permeability, $(TC)_{\mu'}$ and $(TC)_{\mu''}$, are conventionally derived from the expressions

$$\mu_T' = \mu'[1 + (TC)_{\mu'}\Delta T], \tag{1a}$$

$$\mu_T'' = \mu''[1 + (TC)_{\mu''}\Delta T], \tag{1b}$$

from which

$$(TC)_{\mu'} = \frac{\mu_T' - \mu'}{\mu'\Delta T} = \frac{(\Delta L)_{\mu'}}{\mu' L_a \Delta T}, \tag{2a}$$

$$(TC)_{\mu''} = \frac{\mu_T'' - \mu''}{\mu''\Delta T} = \frac{\Delta R}{R\Delta T}, \tag{2b}$$

where $(\Delta L)_{\mu'} = (\mu_T' - \mu')L_a$. For some powdered iron materials a fractional change of permeability of the order of 10^{-6} per degree centigrade may be observed.

The accuracy of determining $(TC)_{\mu'}$ is largely dependent upon the accuracy in the determination of $(\Delta L)_{\mu'}$. Errors arise in the determination of $(\Delta L)_{\mu'}$ because (1) it is extremely small, especially for rod-shaped samples; (2) it may be masked by systematic inductance changes resulting from the movement or expansion of the wire winding, from the interaction of electric flux with the sample, etc; and (3) it may vary with the thermal cycling procedure because of magnetic aftereffect phenomena. Errors in $(TC)_{\mu''}$ are not discussed in this manuscript because of the difficulty in measuring the losses of low-loss materials. The problems related to the evaluation of $(TC)_{\mu'}$ are dis-

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cussed. Dunton [1] [1] gives a good discussion of specific measurement problems, some of which are related to other techniques.

Before describing the method, which is the subject of this manuscript, it is desirable to discuss briefly other methods [1–5]. A resonance method often utilized to determine inductance change involves the shift in frequency of an LC oscillator in which the inductor generally contains a rod of material surrounded by a coil subject to a temperature change. Temperature coefficients are computed after making corrections for the demagnetizing field at the ends of the sample and the inductance change of the coil alone. In another method the capacitance of a Q meter is adjusted to measure the inductance change of a toroidally shaped coil with a sample inside. A Maxwell bridge may also be used to measure directly the inductance change of toroidal coils. There are no demagnetizing field corrections. The change of inductance is larger than that for rods. Because it is difficult to determine, the coil inductance is often included in the measured temperature coefficient in the last two techniques. Dunton considers these two methods, although they are less sensitive than the LC oscillator technique, to yield better estimates of temperature coefficient and to be more suitable for the measurement of toroids. A method previously developed at the National Bureau of Standards makes use of the rf permeameter [2] [4, 6] in which a toroidal sample is measured in a small copper oven. The body and primary of the permeameter are held at room temperature by means of a water bath. The permeameter and a parallel capacitance standard are attached to an admittance bridge. Input impedance changes of the permeameter due to sample changes are compensated through adjustments of the external capacitor and the bridge conductance. Although convenient for measuring relatively high temperature coefficients, as found in some ferrite materials, this method does not yield the precision necessary for low temperature coefficient measurements.

In evaluating these methods, a measurement procedure similar to that using the rf permeameter was devised. It encompasses the convenience of using a bridge and a parallel capacitance and yields high precision. Subjected to a temperature change, the inductance changes of toroidal coils, without and with a low-loss powdered iron sample inside, are measured using a high precision Maxwell impedance bridge as an equality indicating device. The inductance changes are compensated by adjusting parallel capacitance, C_p, and parallel resistance, R_p, standards attached externally to avoid bridge errors. The circuit is shown in figure 1. The temperature and frequency ranges are approximately 23 to 50 °C and 300 kHz to 1.5 MHz, respectively. It appears that these ranges may be extended somewhat.

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[1] Figures in brackets indicate the literature references at the end of this paper.

[2] The rf permeameter is a shorted section of coaxial line surrounding a primary winding which transforms the low coaxial impedance up to values which may be measured with commercially available instruments.
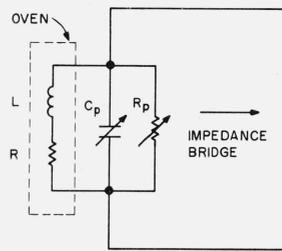
FIGURE 1. *Circuit used to measure temperature coefficient of permeability.*

2. Description of Method

Descriptions of the measurement procedure, of the equations pertinent to the method, and of the coils, bridge, standards and connectors, thermal system, and typical data follow.

2.1. Measurement Procedure

Two coils, one without and one with a sample inside, are carefully mounted in the oven. One of the coils is connected with the parallel components, C_p and R_p, to the bridge. (See fig. 1.) Although the two coils go through the thermal cycling procedure, only one is measured at a time; therefore, reference is made to the one coil being measured. This coil is thermally cycled one or more times, in 40 min intervals from an ambient temperature of $\sim$ 23 to 50 °C and return, and then given 40 min to reach thermal equilibrium. The bridge is balanced, using its controls and the adjustable parallel capacitance and resistance standards, state 1. Additional time may be required to make certain that the frequency, temperature and bridge are adequately stable. During the rest of the procedure the capacitance and resistance standards are adjusted to compensate for any impedance change of the coil. The coil is heated to 50 °C in approximately 20 min and given approximately 40 additional minutes to reach thermal equilibrium, and the bridge is rebalanced, state 2. The coil is then cooled to ambient temperature and given approximately 40 min to reach thermal equilibrium at ambience, and the bridge is rebalanced, state 3. It requires at least 7 hr to complete a set of data on the two coils. Because of the near linearity of the temperature coefficient of permeability and the small changes observed, measurements are taken at only states 1, 2, and 3.

2.2. Equations

Each coil to be measured has an impedance of the form, $Z = R + j\omega L$, where L and R change with temperature. These changes are compensated by adjusting C_p and R_p to maintain the same impedance across the Maxwell bridge terminals.

To maintain the simplicity and accuracy of the measurements, it is necessary that

$$\omega^2 L^2 \gg R^2, \tag{3}$$

and that

$$L_{T2}^2 \approx L_{T1}^2 \tag{4}$$

and

$$[L_s - (\mu' - 1)N^2 L_a]^2 \approx L_e^2, \tag{5}$$

to within approximately 1 percent.

The impedance connected across the bridge terminals is expressed as at temperature T_1

$$R_{in} + j\omega L_{in} = \cfrac{1}{\cfrac{1}{R_{T1} + j\omega L_{T1}} + \cfrac{1}{R_{pT1}} + j\omega C_{pT1}}, \tag{6}$$

and, at temperature T_2,

$$R_{in} + j\omega L_{in} = \cfrac{1}{\cfrac{1}{R_{T2} + j\omega L_{T2}} + \cfrac{1}{R_{pT2}} + j\omega C_{pT2}}. \tag{7}$$

From these expressions, it can be shown that to a close approximation

$$\Delta L = L_{T2} - L_{T1} \approx (C_{pT1} - C_{pT2})\omega^2 L_{T1}^2 = \Delta C \omega^2 L_{T1}^2, \tag{8}$$

and

$$\Delta R = R_{T2} - R_{T1} \approx \left(\frac{1}{R_{pT1}} - \frac{1}{R_{pT2}}\right)\omega^2 L_{T1}^2 = \Delta G \omega^2 L_{T1}^2. \tag{9}$$

Considering (2) and the impedance changes of the coils, the temperature coefficients of the real and imaginary components of permeability can be expressed as

$$(TC)_{\mu'} = \frac{\Delta\mu'}{\mu'\Delta T} = \frac{\Delta L_s - \Delta L_e}{\mu' N^2 L_a \Delta T}, \tag{10}$$

and

$$(TC)_{\mu''} = \frac{\Delta R}{R\Delta T} = \frac{\Delta R_s - \Delta R_e}{R\Delta T}, \tag{11}$$

making use of the relation $(\mu' - 1) = (L_s - L_e)/N^2 L_a$ given by Danielson and Harrington [7]. In their formula, errors in permeability measurements subtract out. To obtain higher accuracy in permeability, they recommend that an air gap be introduced between the sample and the coil winding to reduce the interaction with electric flux. They used an air gap of 0.050 in. We used one greater than 0.12 in.

Equations (10) and (11) are more useful if expressed in terms of measured parameters. It can be shown that in (10) and (11)

$$\mu' N^2 L a = (L_s - L_e)\mu'/(\mu' - 1) \tag{12}$$

and

$$R = R_s - R_e. \tag{13}$$

Therefore, substituting (8), (9), (12), and (13) into (10) and (11), we get

$$(TC)_{\mu'} = \frac{(\mu' - 1)\omega^2(\Delta C_s L_s^2 - \Delta C_e L_e^2)}{(L_s - L_e)\mu'\Delta T} \tag{14}$$

and

$$(TC)_{\mu''} = \frac{\omega^2(\Delta G_s L_s^2 - \Delta G_e L_e^2)}{(R_s - R_e)\Delta T}. \tag{15}$$

When samples have very low losses, $(TC)_{\mu''}$ cannot be determined accurately; therefore, such data are not presented in this paper.

Adequate resolution is obtained for the changes in the parallel capacitance standard and the coil inductance in the following way. An expression for L_a in terms of $N^2 L_{\text{space}}$ is determined for the lowest possible value of the temperature coefficient, $(TC)_{\mu'+\text{coil}}$, to be evaluated assuming $\Delta L_e = 0$.

From (8) and (10) we get

$$(TC)_{\mu'+\text{coil}} = \frac{\omega^2 \Delta C_s L_s^2}{\mu' N^2 L_a \Delta T}. \tag{16}$$

Neglecting leakage and violations of an assumption of a uniform current sheet, which subtract out in the expression of permeability, the coil inductance is

$$L_s \approx N^2(\mu' - 1)L_a + N^2 L_{\text{space}}. \tag{17}$$

Therefore,

$$(TC)_{\mu'+\text{coil}} \approx \frac{\omega^2 \Delta C_s[N^2(\mu' - 1)L_a + N^2 L_{\text{space}}]^2}{\mu' N^2 L_a \Delta T}, \tag{18}$$

and

$$\frac{\partial(TC)_{\mu'+\text{coil}}}{\partial L_a} = \frac{\omega^2 \Delta C_s N^2}{\mu'\Delta T}\left[(\mu' - 1)^2 - \frac{L_{\text{space}}^2}{L_a^2}\right] = 0, \tag{19}$$

for a maximum or minimum value of $(TC)_{\mu'+\text{coil}}$.

From (19) for a minimum, the inductance is

$$L_a \approx \frac{L_{\text{space}}}{(\mu' - 1)}. \tag{20}$$

Consequently, the minimum or lowest temperature coefficient is

$$_{\min}(TC)_{\mu'+\text{coil}} \approx \frac{4\omega^2 \Delta C_s N^2 L_{\text{space}}(\mu' - 1)}{\mu'\Delta T} \tag{21}$$

and

$$N^2 L_{\text{space}} \approx \frac{(TC)_{\mu'+\text{coil}}\mu'\Delta T}{4\omega^2\Delta C_s(\mu'-1)}. \qquad (22)$$

To find $N^2 L_{\text{space}}$, estimates are made for all the parameters on the right of (22) as follows:

$$_{\min}(TC)_{\mu'+\text{coil}} = \text{lowest value to be detected}$$
$$\mu' \approx (\mu'-1)$$
$$\Delta T = \text{range to be covered}$$
$$\omega = \text{upper limit of the frequency range}$$
$$\Delta C_s = \text{lowest value yielding the desired resolution in } _{\min}(TC)_{\mu'+\text{coil}}.$$

The following effects need to be considered in relation to the equations given: (1) the thermal expansion of the sample material and (2) the change of the self-capacitance of the coil. Considering the first, it can be shown, for $\mu'_T \approx \mu'$, that (14) is

$$(TC)_{\mu'} \approx \alpha + \frac{\mu'_T - \mu'}{\mu'\Delta T}. \qquad (23)$$

Apparently the error in the true temperature coefficient due to an expansion may be large. Equation (14) is used to avoid measuring the thermal expansion. However, the temperature coefficients given by (14) or the right side of (23) are, in either case, linear and useful properties. Referring to the second problem, the computed change of self-capacitance from the expansion of the lead wires is negligible. Computed values of temperature coefficient without self-capacitance corrections do not vary appreciably with frequency regardless of the sample considered. However, the temperature coefficient of the coil inductance for a given solid wire decreases with frequency. The frequency dependence of the skin depth may affect the temperature dependence of the coil inductance.

2.3. Coils

For highest accuracy, two coils, one without and one with the sample inside, should be wound identically with wire of very uniform physical and electrical properties. Excluding any change from a sample, identical windings should result in similar impedance changes with temperature. Ideally a coil, after thermal cycling, should return to its original impedance at the initial temperature. Fused silica forms with temperature coefficient of linear expansion $\approx 5 \times 10^{-7}/$°C are used. These forms have grooves and spacers with holes precisely determining the path of the winding and lead wire. (See fig. 2.) Each form is a toroidally shaped cup having a wall thickness of 0.1 in. and one open end for inserting a sample before applying a winding. The cup is designed to allow air circulation. Each wire across either face of the cup makes an 18° angle with the radius except on the tenth and final turn where the leads are separated 0.1 in. The wires along the sides of the form lay perpendicular to the faces. Even though some undesir-
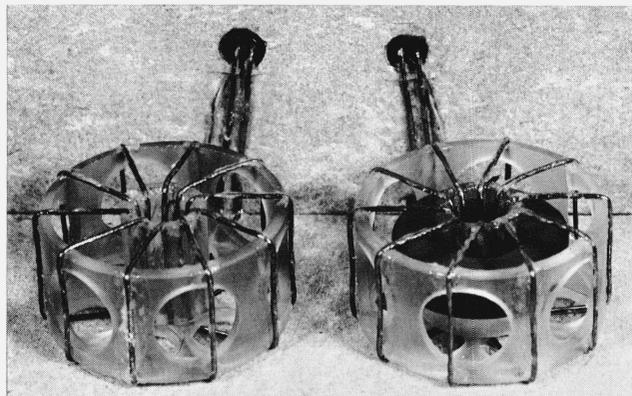
FIGURE 2. *Toroidal coils without and with a sample as they appear in the oven.*
The extra pairs of leads go to thermoelements used to measure temperature.

able stresses may thereby be introduced in the wire, the windings are glued in position with an electrical insulating cement. A toroid of the material to be measured does not touch the winding and is symmetrically spaced within it.

The type and condition of the wire itself are important considerations. Litz and solid copper wires with insulation were tested. A given piece of litz wire (90 strand No. 24) formed into a winding and subjected to a given temperature change at a given frequency gave erratic values of $(\Delta L/\Delta T)$; each new wire also resulted in a different value. Perhaps a different kind of litz wire would give better results. However, No. 20 solid copper wire proved satisfactory. It was thermally cycled from ~ 23 to 50 °C before each set of measurements to relieve stresses, which are mostly introduced when making the winding. High temperature stress relieving is not possible. The cycling also relieves a material somewhat from magnetic aftereffect.[3] For several pairs of coils, using No. 20 solid copper wire with insulation, the precision in temperature coefficient of permeability measurements was approximately 5×10^{-6}. The temperature coefficient of inductance of the coils without a sample is about $80 \times 10^{-6}/$°C.

2.4. Bridge

A high-frequency Maxwell bridge is used which may have an uncertainty in the magnitude of the impedance of the order of 0.1 percent. The frequency range is of the order of 1 kHz to 2 MHz. The minimum inductance, which can be measured directly, is 2×10^{-9} H; and the minimum resistance is 2×10^{-4} Ω. By using the bridge as an equality indicating device in the manner already discussed, changes of inductance two orders of magnitude smaller than this minimum may be determined. Errors from bridge instability are negligible.

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[3] Interestingly, measurements show that any apparent aftereffect phenomena are greater in the materials with bakelite binder than those with Teflon. The former materials need cycling several times while the latter generally once.

FIGURE 3. *Equipment used in making temperature coefficient of permeability measurements.*

On the left, coil leads are shown from the oven to the bridge. On the right and center, standards are shown attached to the bridge (cylindrical objects).

2.5. Standards and Connectors

High precision adjustable NBS standards of capacitance and resistance fitted with NBS connectors are utilized as parallel components compensating for impedance changes in the coil forms being evaluated. (See fig. 3.) These components have well-defined immittances at their NBS terminals. A cube connector (fig. 3) with six NBS terminals is used to make the component and bridge connections of the circuit shown in figure 1. The uncertainty in temperature coefficient of permeability due to the uncertainties in these standards is negligible when compared to the precision of the measurement.

The ranges of the standards and their calibration frequencies are

Range of standards	Calibration frequencies
Capacitance, 50–1100 pF	300 kHz to 1.5 MHz
Resistance, 600–3000 Ω	300 kHz
500–610 Ω	600 kHz and 1 MHz
1000–1110 Ω	1 MHz and 1.5 MHz

2.6. Thermal System

The uncertainty in the oven temperature is ± 0.1 °C. The circulating air is warmed by heating coils and cooled by injecting CO_2. The heater, injector, and circulator are turned off during the short periods of actual data taking. Copper-constantan thermocouples monitor the temperature of the coils and samples.

2.7. Data

The temperature coefficient of permeability data shown in tables 1, 2, 3, and 4 have an estimated uncer-

tainty of approximately 5×10^{-6}/°C. Accuracy is affected by any nonuniformity in the winding. (See tables.) Material 1 data for three frequencies and coil pairs 1 and 2 show excellent agreement. Coil pairs 3 and 4 do likewise, but differ somewhat from coil pairs 1 and 2, presumably because of inadvertent overheating of the sample. Measurements not shown on two different size samples of material 1, using another size form, yielded $(TC)_{\mu'}$ of 73×10^{-6}/°C and 74×10^{-6}/°C at 1 MHz, which indicates a uniform $(TC)_{\mu'}$. The fourth set of material 1 data also indicates no apparent change in properties after the toroidial ID is increased from 0.7 in. to 0.9 in. The purpose in reducing the volume of the sample was to ascertain whether the data were being affected by the proximity of the toroid to the center of the fused silica form. There appears to be no apparent change in $(TC)_{\mu'}$ with the frequency for each material, implying that the self-capacitance change of the coils with temperature is negligible. Data obtained using litz wire, not reported for reasons already discussed, show an effective $(TC)_{\mu'}$ unchanged to 1.5 MHz, where the greatest correction, if any, from the change of self-capacitance would occur. When coils with more uniform properties can be made, the accuracy in temperature coefficient of permeability measurements should improve. Tables 1, 2, 3, and 4 temperature coefficient of permeability, $(TC)_{\mu'}$, for the temperature interval of approximately 23 to 50 °C, low-loss, powdered iron toroids were measured.

TABLE 1. *Powdered iron material 1, Teflon binder, $\mu' = 6.32$*

Sample dimensions: O. D. 1.402 in., I. D. 0.701 in., and H 0.333 in. $(TC)_{\mu'}$

Frequency	Coil pair 1 [a]	Coil pair 2 [a]	Coil pair 3 [b]	Coil pair 4 [c]
300 kHz	$77 \cdot 10^{-6}$/°C	$74 \cdot 10^{-6}$/°C	$69 \cdot 10^{-6}$/°C	$69 \cdot 10^{-6}$/°C
600 kHz	78	74	72	71
1 MHz	79	78	75	74

[a] Obtained without grooves and lead spacers on the form to position precisely the path of the windings and leads.
[b] Sample inadvertently heated at 75 °C for 1 hr between pair 2 and pair 3.
[c] Sample 1 with I. D. increased from 0.7 in. to 0.9 in.; measured after coil pair 3.

TABLE 2. *Powdered iron material 2, carbonyl SF, $\mu' = 8.15$*

Sample dimensions: O. D. 1.300 in., I. D. 0.701 in., and H 0.300 in.

Frequency	$(TC)_{\mu'}$
300 kHz	$36 \cdot 10^{-6}$/°C
600 kHz	41
1 MHz	40

TABLE 3. *Powdered iron material 3, Teflon binder, $\mu' = 8.52$*

Sample dimensions: O. D. 1.316 in., I. D. 0.714 in., and H 0.301 in.

Frequency	$(TC)_{\mu'}$
300 kHz	$177 \cdot 10^{-6}$/°C
600 kHz	177
1 MHz	182

TABLE 4. *Powdered iron material 4, carbonyl E, $\mu' = 9.30$*

Sample dimensions: O. D. 1.250 in., I. D. 0.701 in., and H 0.275 in.

Frequency	$(TC)\mu'$
300 kHz	$67 \cdot 10^{-6}/°C$
600 kHz	72
1 MHz	69

Pertinent Information for Data in Tables 1, 2, 3, and 4

Fused silica form dimensions: O. D. 2.000 in., I. D. 0.480 in., H 1.000 in., and walls 0.100 in. $L_{space} = 7.04 \cdot 10^{-9}$ H.

Winding materials and turns: No. 20 solid copper wire with insulation coating; ten turns.

Data Used: Mean value of inductance changes between states 2 and 1 and between states 2 and 3 without sample and with sample coils. See section 2.1 for a description of the states. Using heating and cooling data, the difference in values of temperature coefficient of permeability, $(TC)_{\mu'}$, is of the order of $1 \times 10^{-6}/°C$.

Formula used: Equation (14) of text.

Estimated uncertainty: $\approx 5 \times 10^{-6}/°C$.

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