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Functions for Thermal Stress Calculation Near a Transient Heat Source on a Flat Surface

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When an initially unstressed elastic solid at uniform temperature is subject to a transient, locally two-dimensional heat flux on a flat surface, the two-dimensional total stress field near the wall is locally determined for short times and may be constructed from the functions described in this report; Fortran programs are available for their computation. In particular, maximum total stress and total stress fields for initially large heat fluxes are readily obtained for estimations of yield probability.

Key Words: Elastic stress, heat transfer, plane strain, thermal stress, yield stress.

1. Introduction

When heat is suddenly applied to or withdrawn from the surface of an elastic solid, large superficial stresses are set up which can cause surface damage when local elastic limits are exceeded. This occurs, for example, when hot or cold material is suddenly brought into contact with the solid surface. If the surface is reasonably flat and the initial heat flux distribution rapidly varying in one direction only (for example, near the perimeter of contact of the hot material mentioned above), the initial transient stresses near the wall are obtainable from plane strain theory.

In this report, plane strain theory is used to analyze the total stress field near the surface of a semi-infinite elastic solid during a short time interval during which a large two-dimensional heat flow occurs; formulae are presented for numerical evaluation of the stress for a class of heat flux pulses which may be approximated by a simple physically relevant form. Such a stress field is useful for estimating the probability and location of fracture from either maximum stress-yield strength data or a more sophisticated probability theory.

We shall consider then a semi-infinite elastic solid occupying the region $y \geqslant 0$ which at time $t=0$ has vanishing stress components, $\sigma_{ij}=0$, and uniform temperature $T=0$. Cartesian coordinates x and z are taken to lie in the surface and a given *outward* heat flux $q(x, t)$ independent of z is stipulated, so that the temperature $T(x, y, t)$, the stresses $\sigma_{ij}(x, y, t)$, the strains $\epsilon_{ij}(x, y, t)$, and the displacements $u_i(x, y, t)$, $(i=1, 2)$, do not depend on z. We assume also (plane strain) $u_3(x, y, t)=0$.

We shall neglect the heat generated in the solid by the strains, and assume that the temperature is governed by the heat equation

$$T_{,t}=D(T_{,xx}+T_{,yy}), \tag{1}$$

where D is the coefficient of thermal diffusion. We shall also neglect the inertial effects in the solid, assuming that the stress is quasi-stationary near the surface. Then

$$\sigma_{ij,j}=0, \qquad (i, j=1, 2, 3), \tag{2}$$

where the stress-stain relation [1][3] is given by

$$\sigma_{ij}=\lambda\delta_{ij}\epsilon_{kk}+2\mu\epsilon_{ij}-mT\delta_{ij} \tag{3}$$

the strains are given in terms of the displacements by:

$$\epsilon_{ij}=\tfrac{1}{2}(u_{i,j}+u_{j,i}), \qquad (i, j=1, 2, 3). \tag{4}$$

Here λ and μ are the Lame constants, α is the coefficient of linear thermal expansion, and $m=\alpha(3\lambda+2\mu)$. The Lame constants λ and μ are given in terms of the Young's modulus E and Poisson's ratio ν as follows [1]:

$$\lambda=\frac{\nu E}{(1+\nu)(1-2\nu)}, \qquad \mu=\frac{E}{2(1+\nu)}.$$

The validity of the assumptions made in the above equations will depend on the specific heat flux involved, $q(x, t)$, and the region being studied, and should be checked a posteriori.

We shall assume further that a single scalar, the total stress (the first stress invariant),

$$\sigma \equiv \sigma_{ii}=\sigma_{11}+\sigma_{22}+\sigma_{33}, \tag{5}$$

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[3] Figures in brackets indicate the literature references on page 270.

suffices for yield analysis. Yield criteria may be based on extreme values of σ or on the more sophisticated probability theories using the stress distribution [2, 3].

Boundary conditions on the stress system are that surface forces vanish on $y=0$:

$$\sigma_{12}=\sigma_{22}=0, \qquad \text{on } y=0. \tag{6}$$

It is well known [4] that when two bodies at different temperatures are put in contact at $t=0$, heat is transferred at a rate $0(t^{-1/2})$ for sufficiently short times. We shall assume the heat flux time history in a time interval of interest, $0 \leqslant t \leqslant t_F$, is approximated as follows:

$$q(x, t)=\sum_{n=0}^{3} A_n(x)t^{n-1/2}, \qquad (0 \leqslant t \leqslant t_F). \tag{7}$$

In particular, we shall find the total stress fields $\sigma_{n\delta}(x, y, t)$ associated with the line source heat fluxes:

$$q_{n\delta}(x, t)=\delta(x)t^{n-1/2}, \qquad (n=0, 1, 2, 3). \tag{8}$$

Then the stress $\sigma(x, y, t)$ for the general heat flux of (7) is given by

$$\sigma(x, y, t)=\sum_{n=0}^{3}\int_{-\infty}^{\infty} dsA_n(s)\sigma_{n\delta}(x-s, y, t). \tag{9}$$

Our principal task is to construct the functions $\sigma_{n\delta}(x, y, t)$.

2. Analytic Evaluation of the Stress $\sigma_{n\delta}(x, y, t)$

For the case of plane strain, the strains and stresses are given in terms of the displacements as follows:

$$\begin{cases} \epsilon_{11}=u_{1,x} \\ \epsilon_{12}=\epsilon_{21}=\frac{1}{2}(u_{1,y}+u_{2,x}) \\ \epsilon_{22}=y_{2,y} \\ \epsilon_{i3}=\epsilon_{3i}=0 \qquad (i=1, 2, 3) \\ \epsilon_{ii}=u_{1,x}+u_{2,y} \end{cases} \tag{10}$$

$$\begin{cases} \sigma_{11}=\lambda\epsilon_{ii}+2\mu u_{1,x}-mT \\ \sigma_{12}=\sigma_{21}=\mu(u_{1,y}+u_{2,x}) \\ \sigma_{22}=\lambda\epsilon_{ii}+2\mu u_{2,y}-mT \\ \sigma_{33}=\lambda\epsilon_{ii}-mT \\ \sigma_{23}=\sigma_{32}=\sigma_{13}=\sigma_{31}=0 \\ \sigma=\sigma_{ii}=(3\lambda+2\mu)\epsilon_{ii}-3mT. \end{cases} \tag{11}$$

The equations (2) become:

$$\begin{aligned} &(\lambda+2\mu)u_{1,xx}+\lambda u_{2,xy}-mT,_x+\mu(u_{1,yy}+u_{2,xy})=0 \\ &(\lambda+2\mu)u_{2,yy}+\lambda u_{1,xy}-mT,_y+\mu(u_{1,xy}+u_{2,xx})=0. \end{aligned} \tag{12}$$

For a function $f(x, y, t)$, satisfying appropriate conditions, we introduce the Laplace transform $\mathscr{L}$ in t and the Fourier transform $\mathscr{F}$ in x:

$$\begin{aligned} F(y; k, s) &\equiv \mathscr{F}\mathscr{L}f(x, y, t) \\ &=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dxe^{-ikx}\int_0^{\infty} dte^{-st}f(x, y, t) \end{aligned} \tag{13}$$

with the inverse:

$$\begin{aligned} f(x, y, t) &=\mathscr{F}^{-1}\mathscr{L}^{-1}F(y; k, s) \\ &=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dke^{ikx} \\ &\quad\cdot\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dse^{st}F(y; k, s). \end{aligned} \tag{14}$$

The transforms of (u_1, u_2, T, σ) are denoted, respectively, by (U, V, Θ, Σ).

We obtain the system of ordinary differential equations:

$$\begin{cases} (s+k^2D)\Theta-D\Theta,_{yy}=0 \\ -(\lambda+2\mu)k^2U+\mu U,_{yy}+(\lambda+\mu)ikV,_y-ikm\Theta=0, \\ -\mu k^2V+(\lambda+2\mu)V,_{yy}+(\lambda+\mu)ikU,_y-m\Theta,_y=0. \end{cases} \tag{15}$$

Boundary conditions are boundedness of the transforms as $y\to\infty$, and the following conditions at $y=0$ follow from (6):

$$U,_y+ikV=0, \qquad ik\lambda U+(\lambda+2\mu)V,_y-m\Theta=0, \tag{16.1}$$

and by (8) at $y=0$:

$$K\Theta,_y=\frac{1}{\sqrt{2\pi}}\frac{\Gamma(n+\frac{1}{2})}{s^{n+1/2}}, \tag{16.2}$$

where K is the coefficient of heat conduction. The system of (15) has solutions of the form

$$\begin{pmatrix}\Theta\\ U\\ V\end{pmatrix}=\begin{pmatrix}\Theta_1\\ U_1\\ V_1\end{pmatrix}e^{-\nu_1 y}+\begin{pmatrix}0\\ U_2+yU_3\\ V_2+yV_3\end{pmatrix}e^{-kye}, \tag{17}$$

where:

$$\nu_1=\left(k^2+\frac{s}{D}\right)^{1/2},$$
$$\epsilon=\operatorname{sgn} k, \tag{18}$$

and ν_1 has nonnegative real part. Substitution of (17) into (15) and use of the conditions (16.1) gives the following:

$$\begin{cases} U=[q_{11}e^{-\nu_1 y}+(q_{12}+yq_{13})e^{-1ky}]\Theta_1 \\ V=[q_{21}e^{-\nu_1 y}+(q_{22}+yq_{23})e^{-1ky}]\Theta_1, \end{cases} \tag{19}$$

where:

$$q_{11}=\frac{ikmD}{(\lambda+2\mu)s},$$

$$q_{21}=-\frac{\nu_1 mD}{(\lambda+2\mu)s},$$

$$q_{12}=\frac{mD}{2iks(\lambda+\mu)(\lambda+2\mu)}\{2\epsilon(\lambda+2\mu)k\nu_1-2\mu k^2\},$$

$$q_{13}=\frac{imD}{2\mu s(\lambda+2\mu)}\{2\mu k\nu_1-2\mu k^2\epsilon\}, \tag{20}$$

$$q_{22}=\frac{mD}{2k\mu s(\lambda+\mu)(\lambda+2\mu)}\{-2\mu^2 k\nu_1+2\mu(\lambda+2\mu)k^2\epsilon\},$$

$$q_{23}=i\epsilon q_{13}.$$

The stress transform Σ is readily found to contain no term linear in y:

$$\Sigma=\left[\alpha_1 e^{-\nu_1 y}+\left(\alpha_2\frac{\nu_1|k|}{s}+\alpha_3\frac{k^2}{s}\right)e^{-|k|y}\right]\Theta_1 \tag{21}$$

where the α_i do not depend on (k, s, y):

$$\alpha_1=-\frac{4\mu m}{\lambda+2\mu}$$

$$\alpha_2=-\alpha_3=\frac{2\mu mD(3\lambda+2\mu)}{(\lambda+\mu)(\lambda+2\mu)}. \tag{22}$$

We notice that the first term on the right-hand side of (21) gives a term directly proportional to the temperature (17). By (16.2) and (17),

$$\Theta_1=-\frac{1}{K\sqrt{2\pi}}\frac{\Gamma(n+\frac{1}{2})}{\nu_1 s^{n+1/2}}. \tag{23}$$

Inverse transforms of the terms in (21) may be obtained by manipulation of tabulated transforms. It is convenient to introduce the variables:

$$r=\sqrt{x^2+y^2},\qquad (\xi,\eta,\rho)=\frac{(x,y,r)}{\sqrt{4Dt}},$$
$$\varphi=\tan^{-1}\frac{\xi}{\eta},\qquad z=\rho^2 e^{-2i\varphi},\qquad |\arg z|\leqslant\pi. \tag{24}$$

Then for the first of the terms in (21), we note [5a]:

$$\mathscr{L}^{-1}\mathscr{F}^{-1}(e^{-\nu_1 y}/\nu_1 s^{n+1/2})$$
$$=\frac{1}{\sqrt{2\pi}}t^{n-1/2}e^{-|z|}\psi\left(\frac{1}{2}+n,\ 1;\ |z|\right), \tag{25}$$

where we have made use of the relation between the Whittaker function $W_{-n,0}(z)$ and the confluent hypergeometric [6a] function $\psi(\frac{1}{2}+n,\ 1;\ z)$.

$$\mathscr{L}^{-1}\mathscr{F}^{-1}\frac{|k|e^{-|k|y}}{s^{n+3/2}}=\frac{1}{\sqrt{2\pi D}}\frac{t^{n-1/2}}{\Gamma\left(n+\frac{3}{2}\right)}\frac{\cos 2\varphi}{|z|} \tag{26}$$

For the third term of (21) we note that

$$\mathscr{L}^{-1}\mathscr{F}^{-1}\frac{k^2e^{-|k|y}}{\nu_1 s^{n+3/2}}=\sqrt{\frac{2}{\pi}}\frac{\partial^2}{\partial y^2}I_n(x,y,t), \tag{27}$$

where:

$$I_n(x,y,t)\equiv\mathscr{L}^{-1}\frac{1}{s^{n+3/2}}J(x,y,s),$$

$$J(x,y,s)\equiv\int_0^\infty dk\,\frac{\cos kx\cdot e^{-ky}}{\sqrt{k^2+\frac{s}{D}}}. \tag{28}$$

Then in terms of Struve's function $\mathbf{H}_0(z)$ and Bessel's function [5c] of the second kind $Y_0(z)$,

$$J(x,y,s)=\frac{\pi}{4}\left[\mathbf{H}_0\left(re^{-i\varphi}\sqrt{\frac{s}{D}}\right)-Y_0\left(re^{-i\varphi}\sqrt{\frac{s}{D}}\right)\right]$$
$$+\frac{\pi}{4}\left[\mathbf{H}_0\left(re^{i\varphi}\sqrt{\frac{s}{D}}\right)-Y_0\left(re^{i\varphi}\sqrt{\frac{s}{D}}\right)\right]. \tag{29}$$

Then [5d, 6b],

$$I_{-1}(x,y,t)=\frac{1}{2\sqrt{\pi t}}\operatorname{Real}\left\{e^{z/2}K_0\left(\frac{z}{2}\right)\right\}$$
$$=\frac{1}{2\sqrt{t}}\operatorname{Real}\left\{\psi\left(\frac{1}{2},1;z\right)\right\}, \tag{30}$$

where K_0 is the modified Bessel function of the second

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kind. It follows that [5e]:

$$I_n(x, y, t) = \left(\int_0^t dt \cdot \right)^{n+1} I_{-1}(x, y, t)$$

$$= \frac{1}{2}(-)^{n+1} \left(\frac{r^2}{4D}\right)^{n+1/2} \text{Real} \left\{ e^{-i\varphi(2n+1)} \cdot \left(\int_\infty^z \frac{dz}{z^2} \right)^{n+1} z^{1/2} \psi\left(\frac{1}{2}, 1; z\right) \right\} \quad (31)$$

where $(\int dt \cdot)^{n+1}$ is the $(n+1)$-th iterated integral. Each integral is independent of the argument of the lower limit for z if $|\arg z| < 3\pi/2$. It is easy to establish by recursion the following formula [6e]:

$$\left(\int_{-\infty}^z \frac{dz}{z^2} \cdot \right)^{n+1} z^{1/2} \psi\left(\frac{1}{2}, 1; z\right) = \frac{C_n^*}{z^{n+1/2}} \psi\left(-n-\frac{1}{2}, 1; z\right) - \sum_{K=0}^{n} \frac{(-)^K}{z^K} \frac{C_{n-K}^*}{K!}, \quad C_n^* \equiv \left[\frac{\pi}{\Gamma\left(n+\frac{3}{2}\right)} \right]^2$$

The finite sum removes from the first term of the right-hand side those terms of the asymptotic series which would give a nonvanishing result as $z \to \infty$ [6d].

We introduce the following real-valued functions:

$$G_{1n}(\xi, \eta) = \text{Real} \left\{ \psi\left(\frac{1}{2} - n, 1; z\right) \right\},$$

$$G_{2n}(\xi, \eta) = \text{Real} \left\{ (4n^2-1) z \psi\left(\frac{3}{2} - n, 3; z\right) + (2n+1)\psi\left(\frac{1}{2} - n, 2; z\right) \right\}. \quad (33)$$

Finally then we are able to write the total stress as follows:

$$\sigma_{n\delta}(x, y, t) = \frac{m}{K} t^{n-1/2} \{ \Lambda_1 f_n(\xi, \eta) + \Lambda_2 h_n(\xi, \eta) \} \quad (34)$$

where the functions (f_n, h_n) are even in ξ and do not depend on the material parameters:

$$f_n(\xi, \eta) = -\frac{1}{\pi} \frac{2}{2n+1} \frac{\cos 2\varphi}{\rho^2} + \frac{(-)^{n+1}}{(2n+1)\Gamma\left(n+\frac{3}{2}\right)} \cdot \left\{ G_{2n}(\xi, \eta) - \sum_{K=0}^{n} \frac{(-)^K}{K!} \left[\frac{\Gamma\left(n+\frac{3}{2}\right)}{\Gamma\left(n-K+\frac{3}{2}\right)} \right]^2 \cdot (2n+1-2K)(2n-2K)\rho^{2n-1-2K} \cos \varphi \, (2n-1-2k) \right\}, \quad (35.1)$$

$$h_n(\xi, \eta) = \frac{\Gamma(n+\frac{1}{2})}{\pi} e^{-\rho^2} \psi(n+\tfrac{1}{2}, 1; \rho^2). \quad (35.2)$$

The constants Λ_i in (34) are the following:

$$\Lambda_1 = \frac{\mu(3\lambda+2\mu)}{2(\lambda+\mu)(\lambda+2\mu)}, \qquad \Lambda_2 = \frac{2\mu}{(\lambda+2\mu)}. \quad (36)$$

We note the useful relation:

$$T_{n\delta}(x, y, t) = -\frac{1}{2K} t^{n-1/2} h_n(\xi, \eta), \quad (37)$$

giving the temperature field in the solid due to the line heat source of (8).

3. Numerical Evaluation of (f_n, h_n)

Expressions are given in appendix A for the interim functions $G_{1n}(\xi, \eta)$, $G_{2n}(\xi, \eta)$, both convergent and asymptotic series. Using double precision computation (25 digit mantissa) for the convergent series, single precision computation (10 digit mantissa, exponent less than 308) for the asymptotic series, agreement between the two was to six significant figures at $\rho^2 = 20$ for several values of φ on $0 \leq \varphi \leq \frac{\pi}{2}$, and thus $\rho = \sqrt{20}$ was taken as the crossover point.

Expressions for $h_n(\xi, \eta)$ are also given in appendix A in terms of the functions $G_{1n}(0, \rho)$ using a recursion relation.

As $\rho \to 0$, the G_{2n} have integrable singularities:

$$f_n \to \frac{1}{\pi} \left\{ \ln \rho^2 + \left(\psi\left(\frac{1}{2} - n\right) - \psi(1) - \psi(2) + 2 \right) + 0(\rho) \right\},$$

$$h_n \to -\frac{1}{\pi} \left\{ \ln \rho^2 + \left(\psi\left(\frac{1}{2} + n\right) - 2\psi(1) \right) + 0(\rho) \right\}, \quad (38)$$

where $0(\rho)$ vanishes as $\rho \to 0$, and $\psi(x)$ is the Digamma function [7]. We may then integrate $\sigma_{n\delta}$ over $y = 0$ as required in the convolution (9).

For large ρ, the simple forms,

$$f_n \sim -\frac{1}{\pi} \frac{2}{2n+1} \frac{\cos 2\varphi}{\rho^2} + \frac{\Gamma\left(n+\frac{1}{2}\right)}{\pi\Gamma(n+2)} \frac{\cos 3\varphi}{\rho^3} + 0(\rho^{-5}),$$

$$h_n \sim 0, \quad (39)$$

give better than one percent accuracy for $\rho \geq 10$. These formulae show that

$$\sigma_{n\delta} \underset{\rho}{\sim} \left[\frac{m\Lambda_2}{K} \frac{\Gamma\left(n+\frac{1}{2}\right)\sqrt{4D}}{\pi\Gamma(n+1)} \right] \cdot \frac{t^n y}{x^2+y^2}, \tag{40}$$

so that for short times ($r/\sqrt{4Dt} \to \infty$), the stress field of a line source dies off $0(1/x^2)$ for $|x| \gg y$ (although only $0(1/y)$ for $y \gg |x|$). This integrable decay rate in x makes the convolution (9) useful since the large stresses expected near the surface at early times are locally induced, and the plane stress theory is applicable in more general situations.

A subroutine for the calculation of $\{G_{1n}(\xi, \eta), G_{2n}(\xi, \eta)\}$, and a program employing this subroutine for the calculation of $\{f_n(\xi, \eta), h_n(\xi, \eta)\}$ for a given value of η on the set of values,

$$\xi = 0.0(0.1)10.0, \tag{41}$$

is discussed in appendix B. The calculated values are stored on tape in consecutive values of ξ in the following order for each ξ:

$$\{f_0(\xi, \eta), \ldots, f_3(\xi, \eta), h_0(\xi, \eta), \ldots, h_3(\xi, \eta)\}. \tag{42}$$

The values are sufficiently slowly varying that Simpson's rule integration in (9) is normally adequate. Calculation of these functions on the set (41) for a given value of η takes on the average about two minutes, and was done for the set of integer values

$$\eta = 0.0(1.0)10.0. \tag{43}$$

These data are now on magnetic tape, and are available on punched cards. (The singular values ($\xi=0$, $\eta=0$) have been replaced by the values at ($\xi=0.1$, $\eta=0$) as being adequate for integration.)

4. Step-Input Stress Response

A special case of heat flux of particular interest for stress evaluation near the perimeter of an (almost) uniform source is obtained from the general source of (7) by setting

$$A^n(x) = A_n h(x), \tag{44}$$

where the A_n are constants and

$$h(x) = \begin{cases} 0 & (x<0) \\ 1 & (x \geq 0). \end{cases} \tag{45}$$

Then the resulting stress σ_h is found by (9) to be

$$\sigma_h(x, y, t) = \frac{m\sqrt{4D}}{K} \sum_{n=0}^{3} A_n t^n \{\Lambda_1 F_n(\xi, \eta) + \Lambda_3 H_n(\xi, \eta)\}, \tag{46}$$

where:

$$\begin{pmatrix} F_n \\ H_n \end{pmatrix} \equiv \int_{-\infty}^{\xi} ds \begin{pmatrix} f_n(|s|, \eta) \\ h_n(|s|, \eta) \end{pmatrix}, \tag{47}$$

$$(\xi, \eta) = \frac{(x, y)}{\sqrt{4Dt}}.$$

When the entire path of integration (ξ_1, ξ_2) lies outside the circle of radius $\rho=10$, the asymptotic forms (39) are used to evaluate:

$$\phi_n(\xi_2, \xi_1; \eta) = \int_{\xi_1}^{\xi_2} ds\, f_n(|s|, \eta), \tag{48}$$

each term of (39) having a simple closed-form integral. Then for $0 \leq \eta \leq 10$, we have

$\underline{\xi \leq -10.:}$

$$\begin{pmatrix} F_n(\xi, \eta) \\ H_n(\xi, \eta) \end{pmatrix} = \begin{pmatrix} \phi_n(\xi, -\infty; \eta) \\ 0 \end{pmatrix}; \tag{49}$$

$\underline{-10. \leq \xi \leq 10.:}$

$$\begin{pmatrix} F_n(\xi, \eta) \\ H_n(\xi, \eta) \end{pmatrix} = \begin{pmatrix} F_n(-10, \eta) \\ 0 \end{pmatrix} + \int_{-10}^{\xi} ds \begin{pmatrix} f_n(|s|, \eta) \\ h_n(|s|, \eta) \end{pmatrix}; \tag{50}$$

$\underline{\xi \geq 10.:}$

$$\begin{pmatrix} F_n(\xi, \eta) \\ H_n(\xi, \eta) \end{pmatrix} = \begin{pmatrix} F_n(10, \eta) \\ H_n(10, \eta) \end{pmatrix} + \begin{pmatrix} \phi_n(\xi, 10; \eta) \\ 0 \end{pmatrix}. \tag{51}$$

for $\eta > 10$, (49) is valid for all ξ.

Appendix C describes a program for determining $\{F_n, H_n\}$ for given η, (one of the set (43) of course) on the set

$$\xi = 10.(0.2)20., \tag{52}$$

using Simpson's rule for the evaluation of the integrals in (50). These results are on magnetic tape and are available on punched cards. Calculations took about one-half minute for each value of η.

When $A_0=1$, $A_1=A_2=A_3=0$, the total stress field is self-similar in time, dependent ξ and η only, and rises monotonically (dominated by $G_n(\xi, \eta)$) in ξ to a maximum value at $\xi=\infty$ independent of η. This value is attained at ($\eta=0$, $\xi=10$). to the accuracy of the calculation, and may be determined from the values

$$\begin{pmatrix} F_0(20, 0) \\ H_0(20, 0) \end{pmatrix} = \begin{pmatrix} -0.0637 \\ 1.7724 \end{pmatrix}. \tag{53}$$

For large values of ξ, the stress field σ_h given by (46) is uniform in x for any coefficient set $\{A_n\}$, and agree with the results for heat flux $q(t)$ uniform in x described in appendix D.

5. Conclusions

Formulae are presented for the estimation of total stress fields near the surface of a solid during a short time interval during which a large heat transfer occurs. Such a stress field is useful for estimating the probability and location of fracture, either from maximum stress-yield strength data or from a more sophisticated probability theory. The stress fields obtained may be expected to be valid in a region adjacent to a point on the solid surface of dimension $l \approx \sqrt{4Dt}$ and for the time interval $0 \leqslant t \leqslant t_F$, the period of validity of the heat flux approximation (7), so long as the following orders hold:

$$\sqrt{4Dt} \ll \{L, R_m, R_q\}, \tag{54}$$

where L is a length scale for the solid, and R_m and R_q are, respectively, the mean radius of curvature of the solid surface and the radius of curvature of lines of constant outward heat flux at the point in question.

It would be desirable to have a "maximum principle" available, stating, for example, that if $q(x, t)=0$ for $t > t_F$, then the maximum value of $\sigma(x, y, t)$ over (x, y) for any $t > t_F$ does not exceed the maximum value of $\sigma(x, y, t_F)$ over (x, y). The part of the stress associated with $h_n(\xi, \eta)$ shares with the temperature field given by (37) such a maximum principle, but no more general result appears available.

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6. References

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7. Appendix A. The Functions $\{G_{1n}(\xi, \eta), G_{2n}(\xi, \eta), h_n(\xi, \eta)\}$

The following expansions for G_{1n}, G_{2n} are readily obtained from the definitions of (33) and the properties [6c] of the confluent hypergeometric functions $\psi(a, b; z)$:

$$G_{1n}(\xi, \eta)=\frac{-1}{\Gamma\left(\frac{1}{2}-n\right)}\left\{\sum_{r=0}^{\infty}\frac{1}{(r!)^2}\frac{\Gamma\left(\frac{1}{2}-n+r\right)}{\Gamma\left(\frac{1}{2}-n\right)}\cdot\rho^{2r}[(\ln\rho^2)\cos 2r\varphi-2\varphi\sin 2r\varphi]\right.$$
$$+\sum_{r=0}^{\infty}\frac{\Gamma\left(\frac{1}{2}-n-r\right)}{\Gamma\left(\frac{1}{2}-n\right)}\frac{1}{(r!)^2}\left[\psi\left(\frac{1}{2}-n+r\right)-2\psi(1+r)\right]\left.\cdot\rho^{2r}\cos 2r\varphi\right\}, \tag{A1}$$

$$G_{1n}(\xi, \eta)\underset{\rho}{\sim}\sum_{m=0}^{\infty}(-)^m\frac{\rho^{2n-1-2m}}{m!}\cos\varphi(2n-1-2m)\cdot\left[\frac{\Gamma\left(m+\frac{1}{2}-n\right)}{\Gamma\left(\frac{1}{2}-n\right)}\right]^2,\ \left(|\varphi|\leqslant\frac{\pi}{2}\right)\cdot \tag{A2}$$

$$G_{2n}(\xi, \eta)=\frac{(-)^{n+1}}{\pi}\Gamma\left(n+\frac{3}{2}\right)\left\{(2n+1)\ln\rho^2\right.$$
$$+(1+2n)\left(\psi\left(\frac{1}{2}-n\right)-\psi(1)-\psi(2)+2\right)+\frac{2\cos 2\varphi}{\rho^2}$$
$$+(1+2n)\sum_{l=1}^{\infty}\frac{(1+2l)}{l!}\frac{\Gamma\left(\frac{1}{2}-n+l\right)}{\Gamma\left(\frac{1}{2}-n\right)\Gamma(l+2)}\rho^{2l}$$
$$\cdot[(\ln\rho^2)\cos 2l\varphi-2\varphi\sin 2l\varphi]$$
$$+(1+2n)\sum_{l=1}^{\infty}\frac{\Gamma\left(\frac{1}{2}-n+l\right)}{l!\Gamma(l+2)\Gamma\left(\frac{1}{2}-n\right)}\cdot\left[2+(2l+1)\right.$$
$$\left.\left.\cdot\left(\psi\left(\frac{1}{2}-n+l\right)-\psi(1+l)-\psi(2+l)\right)\right]\rho^{2l}\cos 2l\varphi\right\}, \tag{A3}$$

$$G_{2n}(\xi, \eta) \underset{\rho}{\sim} \sum_{m=0}^{\infty} \frac{(-)^m}{m!} \left[\frac{\Gamma\left(-\frac{1}{2}-n+m\right)}{\Gamma\left(-\frac{1}{2}-n\right)} \right]^2$$

$$(2n+1-2m)(2n-2m) \cdot \rho^{2n-1-2m} \cos \varphi(2n-1-2m),$$

$$\left(|\varphi| \leq \frac{\pi}{2}\right). \qquad \text{(A4)}$$

Let

$$h_n^*(\xi, \eta) \equiv \frac{\Gamma\left(n+\frac{1}{2}\right)}{\pi} e^{-\rho^2} G_{1n}(0, \rho),$$

$$(n=1, 2). \qquad \text{(A5)}$$

Then $h_n(\xi, \eta)$ are found from the following expressions:

$$h_0 = h_0^*,$$

$$h_1 = 2\rho^2 h_o^* - 4h_1^*,$$

$$h_2 = \frac{1}{3}[4\rho^2(2+\rho^2)-1]h_0^* - \frac{8}{3}(2+\rho^2)h_1^*, \qquad \text{(A6)}$$

$$h_3 = \frac{3}{10}\left\{\left[\frac{4}{9}(4+\rho^2)(4\rho^2(2+\rho^2)-1)-4\rho^2\right]h_0^* - 8\left[\frac{4}{9}(4+\rho^2)(2+\rho^2)-1\right]h_1^*.\right.$$

8. Appendix B. Description and Use of Fortran Programs for Calculation of $\{f_n, h_n\}$

From the expansions for G_{1n} and G_{2n}, given in appendix A, it can be seen that there are multiplicative factors in the sums which are dependent only on n and the index of summation. These factors were therefore computed using the program GCOEFFS and were punched on cards in a format suitable for use directly as part of an object deck in another program. It was decided that the maximum number of terms necessary for required accuracy is 20 for the asymptotic expansions and 100 for the convergent series.

Computation of the functions $\{f_n(\xi, \eta), h_n(\xi, \eta)\}$ is done in program FORMFG using subroutines GCOEF1, GCOEF2, FSGSHS, and GSUMS written in CDC 3600 Fortran. The main program FORMFG provides the input values ξ and η and writes the computed functions on a tape. Subroutine FSGSHS performs the actual computation of the functions (f_n, h_n) for each value of (ξ, η). Subroutine GSUMS computes the functions $\{G_{1n}(\xi, \eta), G_{2n}(\xi, \eta)\}$ for each (ξ, η) using the coefficients of G_{1n} and G_{2n} as computed in the program GCOEFFS. Subroutines GCOEF1 and GCOEF2 set up tables of these coefficients for use by GSUMS.

The first input variable, ISKIP, is used to position the output tape, giving the number of files to be skipped before any writing is done on that tape. Since the functions $\{f_n, h_n\}$ are computed for $\xi = -10.(0.1)10.$ for each η, it is necessary to read in only the variable η(DETA). The program will read in an η, do the computations and return to read the next η, continuing until no cards remain. Note that η is read in as a double precision variable.

Program listings for the CDC 3600 may be obtained from the authors.

9. Appendix C. Description and Use of Fortran Programs for Calculation of $\{F_n, H_n\}$

The program INTEGFGH computes the functions $\{F_n, H_n\}$ using the formulas (49), (50), (51). The functions $\{f_n, h_n\}$ are read from the tape generated by program FORMFG. Results are written on an outtape in the same order as are the $\{f_n, h_n\}$.

Input to this program is in two parts: the tape generated by FORMFG, and cards. The first card input variable is KSKIP, which positions the input tape by skipping KSKIP files. This option makes it possible to select the value of η for which the computations are to be performed. The upper limit of ξ(XIF) and an approximation to $-\infty$(AA) are also card input parameters. After each complete integration the input tape is rewound and the next value of KSKIP is read in, followed by the upper limit of ξ and the approximation to $-\infty$. The program continues until no cards remain to be read.

Program listings for the CDC 3600 may be obtained from the authors.

Appendix D. Heat Flux Uniform in x

When the heat flux of the wall in independent of x, the stress field will be also. Simple results may be obtained for this case, and these results have been used to corroborate the numerical results described in the text.

Putting $k=0$ in (21) gives the x-independent case for the Laplace transforms

$$\sum (y, s) = \alpha_1 e^{-y\sqrt{\frac{s}{D}}} \Theta_1(s). \qquad \text{(D1)}$$

If the outward heat flux $q(t)$ has Laplace transform

$Q(s)$, we have

$$Q(s) = -K\sqrt{\frac{s}{D}}\,\Theta_1(s),$$

so

$$\sum(y, s) = \frac{\alpha_1 e}{-K\sqrt{\frac{s}{D}}}\,Q(s).$$

Using the convolution theorem for Laplace transforms, we may obtain the following formula:

$$\sigma(y, t) = -2\sqrt{\frac{t}{\pi}}\,A\int_0^1 dp\,q(t(1-p^2))\,e^{-\frac{n^2}{p^2}}, \quad \text{(D2)}$$

where

$$A = \frac{\sqrt{D}}{K}\,\alpha_1. \quad \text{(D3)}$$

A useful relation can be derived for the wall stress $\sigma(0, t)$ in terms of the wall temperature $T(0, t)$ only, independent of the heat flux history. Setting $y=0$ in (D1) leads to the relation

$$\sigma(0, t) = \alpha_1 T(0, t). \quad \text{(D4)}$$

The relations (D2) and (D4) are preserved by numerical tests of (46).

(Paper 70B4–188)

Publications of the National Bureau of Standards *

Selected Abstracts

Inequalities for the permanental minors of non-negative matrices, R. A. Brualdi and M. Newman, *Can. J. Math.* **18,** *608–615 (1966).*

It is shown that if A is an $n \times n$ non-negative substochastic matrix then the rth permanental compound of A is also substochastic. If in addition A is doubly stochastic, then it is shown that the sum of all principal permanental minors of order r does not exceed $\binom{n-1}{r}+\binom{n-1}{r-1}$ per (A), with equality if and only if A is the identity matrix.

Real two-dimensional representations of the free product of two finite cyclic groups, J. Lehner and M. Newman, *Proc. Camb. Phil. Soc.* **62,** *135–141 (1966).*

All real discrete representations of the free product of two finite cyclic groups by a group of linear fractional transformations are determined.

Congruence subgroups of positive genus of the modular group, M. Newman and M. Knopp, *Ill. J. Math.* **9,** *No. 4, 577–583 (Nov. 1965).*

Let $\Gamma(n)$ be the principal congruence subgroup of Γ of level n, Γ_n the subgroup of Γ generated by $\Gamma(n)$ and $S=\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$, and $\Gamma_0(n)$ the subgroup of Γ consisting of all elements $\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ of Γ such that $c \equiv 0 \pmod{n}$. Then it is proved that if G is a subgroup of Γ containing Γ_n, then either $G=\Gamma$ or $G \subset \Gamma_0(d)$, $d \mid n$, $d>1$. This is used to prove that a free congruence subgroup of Γ of level n (i.e., a free subgroup of Γ containing $\Gamma(n)$) is of positive genus, provided that $(n, 2.3.5.7.13)=1$.

Scattering properties of concentric soot-water spheres for visible and infrared light, R. W. Fenn and H. Oser, *J. Appl. Opt.* **4,** *1504–1509 (Nov. 1965).*

Based on the theory by Aden and Kerker, computations of the scattering and absorption properties for concentric spherical water-and-particles have been performed for visible and infrared wave lengths. Computations were performed for size parameter values up to 250. Results indicate that, for compound particles with a nucleus smaller than about one-tenth of the total diameter of the particle, the optical properties are almost completely determined by the outer shell. Some results of the computations are presented in graphical form for the scattering and absorption efficience factors and the angular scattering functions.

The frequency dependence of the complex dielectric constant is not taken into account in the actual computations: absorption, scattering and total extinction are given for constant index of refraction. The theory does allow to take this dependence into account, however, if one wishes to do so.

Other NBS Publications

J. Res. NBS 70A (Phys. and Chem.), No. 4 (July–Aug. 1966), $1.00.

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Phase equilibria as related to crystal structure in the system niobium pentoxide-tungsten trioxide. R. S. Roth and J. L. Waring.
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