THE RELATIVE SPECTRAL ENERGY DISTRIBUTION AND CORRELATED COLOR TEMPERATURE OF THE N. P. L. WHITE-LIGHT STANDARD

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ABSTRACT

The spectral characteristics of the two Guild-Young filters have been deduced from their specifications. The first of these filters, in conjunction with a source having the relative spectral energy distribution given by Planck's formula for a temperature of 2,360° K., is used to calibrate a gas-filled lamp. This lamp is used in turn with the second filter to produce the N. P. L. standard white light.

Our computations indicate that the chromaticity of the first source-and-filter combination approximates closely that of a Planckian radiator at 2,931° K. Using as a source the relative spectral energy distribution corresponding to 2,931° K. in combination with the second filter we have obtained the spectral energy distribution and the chromaticity of the N. P. L. white-light standard. This latter turns out to be approximately that of a Planckian radiator at 4,831° K. Comparison is made with the Davis-Gibson source-filter combination for obtaining a color temperature of 4,800° K.

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I. INTRODUCTION

About five years ago Guild [1] described two filters devised by his colleague, Dr. A. F. A. Young, to be used in establishing a working standard of "white light" for colorimetric purposes. The first of these filters was designed to convert the light from a lamp operating at a color temperature of 2,360° K. to a chromaticity approximating that of 3,000° K., so that a second lamp might be calibrated at this latter chromaticity by the 2,360° K. source-and-filter combination using standard photometric apparatus. The second filter was to be used with this second lamp, calibrated as described, to produce the N. P. L. white light, having a chromaticity approximating that of 5,000° K. It was emphasized in Guild's paper that the values given were "provisional" only, and that full details as to the energy distribution would be forthcoming later.

Recently Wright [2] has used these filters in setting up a stimulus which he used in a redetermination of the "trichromatic coefficients" for 10 observers. In the appendix to Wright's paper the following statement appears: "During the discussion of the paper it was pointed out by Mr. Guild that the standard white specified by the National

[1] J. Guild, Trans. Opt. Soc. vol. 27; pp. 122-124; 1925-26.
[2] W. D. Wright, A Redetermination of the Trichromatic Coefficients of the Spectral Colors, Trans. Opt. Soc., vol. 30, pp. 141-164; 1928-29.

Physical Laboratory did not, as stated, provide a source with a color temperature of 5,000° K., but one with a color temperature more nearly 4,800° K. To avoid confusion, the complete specification is given in this appendix." There follows the specification [3] of the two Guild-Young filters, one used to calibrate a gas-filled lamp, which is in turn used with the second filter to produce the standard white. In this connection it is remarked, "In this way the gas-filled lamp is rated to a definite color temperature, although the actual value of this temperature does not appear in the specification. * * * The color temperature actually obtained in this way is at present believed to be very close to 2,900° K. The energy distribution (of the standard white) departs appreciably from that of a black body but the integral color matches that of a black body at a temperature of approximately 4,800° K."

Apparently the spectral energy distribution of the N. P. L. standard white was not published until recently, although the desirability of such publication is quite evident.[4] Because of this lack of spectrophotometric data, Dr. D. B. Judd, of the colorimetry section, requested the authors, in 1930, to determine the spectral energy distribution of the N. P. L. standard white light, as he desired to make an analytical study of Wright's [5] new data.

II. METHOD USED TO DERIVE THE VALUES

We have derived the spectral energy distribution of the N. P. L. standard white from the specification of the two liquid filters. Each filter consists of two compartments; the absorbing materials in one compartment are copper sulphate and cobalt sulphate, for which standard data were already available;[6] that in the other is ammoniacal copper sulphate. The spectral transmittancies [7] of the specified solutions of ammoniacal copper sulphate were determined by methods previously described by the authors.[8] They are given in Table 1. From these component data the spectral transmissions of the filters were computed; these values for the white-light filter are given in Table 1.

The relative spectral distribution of energy was then computed for the lamp-rating filter combined with a Planckian radiator at 2,360° K., as specified by Guild. From this distribution the trilinear coordinates were computed on the basis of mean sun [9] at the center of the Maxwell triangle. As these r, g, and b values did not agree

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[3] As noted in B. S. Misc. Pub. No. 114, p. 9, footnote 4, the specification of the white-light filter as given by Wright is slightly different from that originally given by Guild. In the present computations the revised specification was used. This revised specification is also given in a recent paper by Guild, reference in footnote 16.

[4] See discussion of Wright's paper (Trans. Opt. Soc., vol. 30, p. 161, 1928–29) by Prof. L. C. Martin.

[5] See footnote 2.

[6] Raymond Davis and K. S. Gibson, Filters for the Reproduction of Sunlight and Daylight and for the Determination of Color Temperature, Misc. Pub. 114, Tables 5 and 6, p. 42. The values for the 2-compartment cell filled with water, Table 7, p. 43, were likewise assumed to be valid for the present computations.

[7] For the spectrophotometric terminology used here, see B. S. Misc. Pub. No. 114, pp. 38–39.

[8] However, the measurements on which the present data are based are far less extensive than t hose carried out in the determination of the characteristics of the pyridine-mannite solution of copper sulphate specified in Table 4, p. 41, of B. S. Misc. Pub. No. 114. No photoelectric or thermoelectric measurements were made. Only one solution of each kind was examined, visual measurements being made on two thicknesses of each of the two solutions at each of 18 wave lengths distributed between 404.7 and 710 mμ. The temperature of the solutions was 25° C. The authors are indebted to Mabel E. Brown for these measurements. In reducing the data the experimental values thus obtained were compared with data previously obtained on solutions of different ammonia content. Such comparison assisted considerably in the adoption of values at every 10 mμ below 500 mμ, but contributed much less at wave lengths above 500 mμ.

[9] See B. S. Misc. Pub. No. 114, Table 1, fig. 12.

exactly with those for the Planckian radiator[10] at any temperature, it was necessary to deduce the nearest color temperature resulting from this combination. This was done by Davis's method for determining "correlated color temperature," described in a recent publication[11] and previously used by the authors in connection with certain of their filters.[12]

After the correlated color temperature had thus been obtained with the 2,360° K. source and the lamp-rating filter, similar computations were made for a Planckian radiator at this temperature in combination with the N. P. L. white-light filter.

TABLE 1.—*Spectrophotometric data for the N. P. L. standard white as determined by the present authors*

Wave length in mμ	Spectral transmittancy of n grams of $CuSO_4.5H_2O$ and 100 n cc 28 per cent ammonia per 1,000 cc of aqueous solution.		Spectral transmission of N. P. L. white-light filter	Spectral energy distribution of N. P. L. standard white	Wave length in mμ	Spectral transmittancy of n grams of $CuSO_4.5H_2O$ and 100 n cc 28 per cent ammonia per 1,000 cc of aqueous solution		Spectral transmission of N. P. L. white-light filter	Spectral energy distribution of N. P. L. standard white
	n=1.41	n=2.30				n=1.41	n=2.30		
380	0.991	0.955	0.822	33.83	560	0.574	0.407	0.300	111.84
390	.993	.968	.832	41.88	570	.542	.368	.285	113.34
400	.993	.975	.830	50.33	580	.515	.336	.265	112.12
410	.993	.979	.823	59.49	590	.494	.310	.243	108.78
420	.993	.982	.811	69.00	600	.479	.290	.222	104.82
430	.991	.979	.780	77.42	610	.470	.275	.204	101.63
440	.986	.973	.730	83.63	620	.467	.267	.191	99.60
450	.979	.962	.664	87.10	630	.469	.265	.180	98.19
460	.966	.944	.599	89.10	640	.475	.269	.171	97.43
470	.946	.916	.547	91.74	650	.485	.277	.163	96.81
480	.923	.879	.502	94.08	660	.499	.288	.156	96.43
490	.891	.832	.456	94.86	670	.514	.302	.1487	94.97
500	.849	.774	.402	92.36	680	.531	.317	.1412	93.24
510	.802	.713	.356	89.93	690	.548	.333	.1323	90.20
520	.752	.649	.330	90.73	700	.566	.349	.1239	86.93
530	.703	.583	.320	95.50	710	.585	.366	.1160	83.69
540	.655	.518	.316	102.06	720	.603	.384	.1078	79.77
550	.612	.458	.311	108.09					

III. RESULTS OBTAINED

The spectral distribution of energy of the N. P. L. standard white as thus derived is given in Table 1. The colorimetric data for the two lamp-and-filter combinations—viz, the temperature of the Planckian radiator taken as the source in each case, and the trilinear coordinates, the correlated color temperature, the color departure,[11] and the conjunctive wave length [11] for the combinations—are given in Table 2. The color departure and the conjunctive wave length together indicate the nature of the difference between the colors represented by the source-and-filter combinations and the Planckian radiator at the respective correlated temperatures. The unit used in specifying the color departure is equal to about one-fifth of a satron.[13]

[10] Values of r, g, and b for the Planckian radiator over the temperature range from 1,600° to 20,000° K. are given in Table 11 of B. S. Misc. Pub. No. 114. They are based on computations with mean sun at the center of the Maxwell triangle.

[11] Raymond Davis, A Correlated Color Temperature for Illuminants, B. S. Jour. Research, vol. 7, p. 659; 1931.

[12] B. S. Misc. Pub. No. 114, Tables 13 to 15.

[13] See B. S. Misc. Pub. No. 114, p. 121, for definition of satron.

Recently Guild has published the relative spectral energy distribution of the N. P. L. standard white.[14] The values disagree somewhat from ours. The two sets of relative energy values, along with those for a Planckian radiator at 4,800° K., are shown in Figure 1. The colorimetric characteristics corresponding to the distribution given by Guild are presented in Table 2. It may be noted that the correlated color temperature obtained from Guild's values is 4,816° K. as compared with 4,831° K. obtained from our data. Judd's analysis indicates [15] that the difference in energy distribution and color obtained in these two determinations has no important bearing on the significance of Wright's data. Part of the discrepancy lies, of course, in the source used with the filter. We used the relative energy distribution of a Planckian radiator at 2,931° K., whereas Guild used an energy distribution corresponding to 2,900° K.

TABLE 2.—*Colorimetric characteristics of the Guild-Young lamp-and-filter combinations*

Designation of filter in combination with specified light source	Source of energy data	Source (Planckian radiator) at—	Trilinear coordinates			Correlated color temperature	Color departure $\frac{s}{\delta}$	Conjunctive wave length λ_j
			r	g	b			
								mμ
Lamp-rating filter	Authors	2,360° K	0.4640_6	0.3618_5	0.1740_8	2,931° K	0.9	576
White-light filter	do	2,931°	$.3442_7$	$.3301_2$	$.3256_1$	4,831°	2.2	568
Do	Guild	(2,900°)	$.3463_1$	$.3323_6$	$.3213_3$	4,816°	3.9	568

Guild has published [16] values of spectral "transmission coefficient" for the white-light filter over a wave-length range from 400 to 780 mμ. His values, however, appear to be what we would define as transmittance, or transmission corrected for reflection losses. His values were multiplied by 0.9 (in order to introduce approximately these reflection losses) and then compared with ours, and it was found that from 400 to 690 mμ, inclusive, the algebraic sum of the discrepancies is practically zero; and, except at 690 mμ, the maximum deviation from the mean is not greater than ± 1.5 per cent of the mean value. The discrepancy becomes considerable at 720 mμ, but this has little effect on the colorimetric use of the filter.

IV. COMPARISON WITH DAVIS-GIBSON SOURCE-FILTER COMBINATION

It was of interest to the authors to compare the energy distribution obtained for the N. P. L. standard white with that resulting from one of their filters—that designed to convert the chromaticity of a Planckian radiator at 2,848° K. to that for 4,800° K. The energy distribution for this particular filter had not previously been computed, although it could be closely estimated from the distributions

[14] J. Guild, The Colorimetric Properties of the Spectrum, Phil. Trans. Roy. Soc. London, vol. 230, p. 179, June, 1931.

[15] D. B. Judd, Comparison of Wright's data on Equivalent Color Stimuli with the O. S. A. data, J. Opt. Soc. Am., 21; 1931.

[16] J. Guild, On the Fixed Points o a Colorimetric System (advanced proof), Trans. Opt. Soc., 32, p. 21, 1930–31. In this paper are also give Guild's revised specifications for the white-light filter.

given in Charts 22 to 25 of B. S. Misc. Pub. No. 114. The concentrations of the components of this filter are, however, given in Table 12 (p. 61) of that publication.

The energy distribution derived from these concentrations (with source at 2,848° K., as with the other filters of this series) is shown in Figure 1, and also in the accompanying chart (fig. 2), which gives the chemical and spectrophotometric specification for this filter. The trilinear coordinates derived from this energy distribution do not differ from those for a Planckian radiator at 4,800° K. by more than 0.0001. This is an excellent check on the reliability of the concentrations of the components given at the various intermediate temperatures in

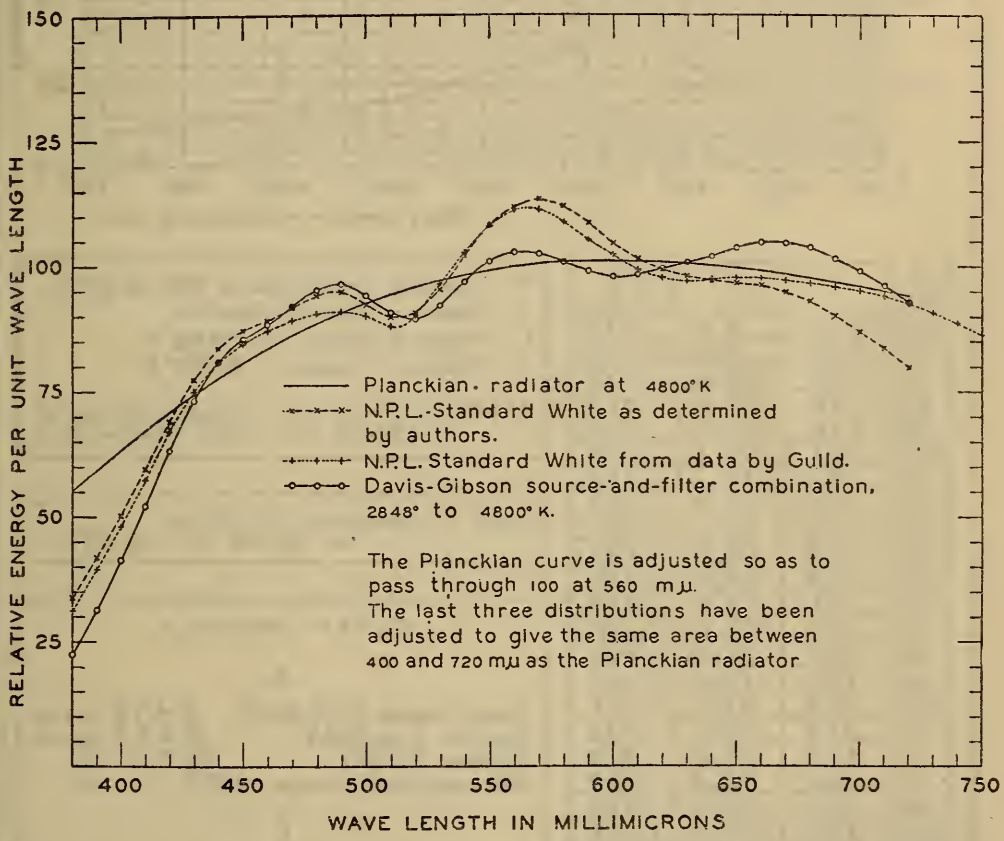
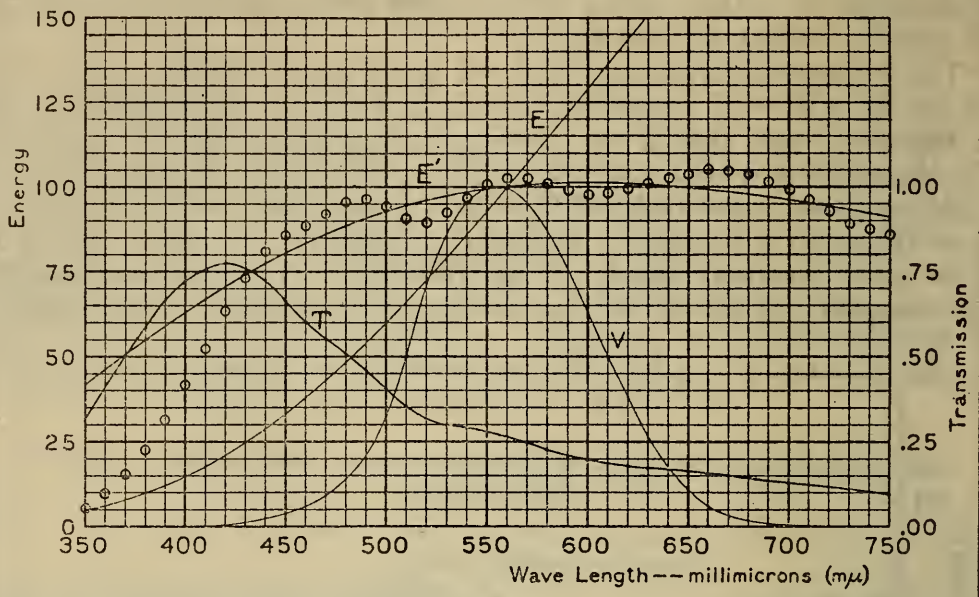


FIGURE 1.—*Relative spectral energy distribution of N. P. L. standard white compared with that for a Planckian radiator at 4,800° K. and a Davis-Gibson 2,848°-K.-to-4,800° K. source-and-filter combination.*

Table 12 referred to above. The value of the light transmission, 0.249, given on the chart, is taken from that table and not derived from independent computation.

Both of the relative spectral energy distributions produced by the respective source-filter combinations shown in Figure 1 deviate notably from that of the 4,800° K. curve. The deviations are so distributed in the case of the authors' filter, however, as to result in an accurate chromaticity match with the 4,800° K. curve according to the O. S. A. "excitation" curves. With the N. P. L. standard, on the other hand, the excess energy in the yellow-green results in failure to match the chromaticity of the Planckian radiator at any temperature, the nature and magnitude of the departure being shown in Table 2.

2848°K TO 4800°K



λ (mμ)	T	E''*	E''/E'
350	0.309	5.7	.134
60	.403	9.7	.207
70	.498	15.2	.297
80	.588	22.4	.406
90	.666	31.3	.528
400	.721	41.3	.652
10	.757	52.1	.776
20	.774	63.2	.893
30	.761	73.1	.984
40	.723	80.8	1.041
450	.663	85.4	1.059
60	.601	88.3	1.059
70	.551	92.0	1.068
80	.507	95.2	1.076
90	.460	96.5	1.064
500	.404	94.2	1.017
10	.353	90.7	.962
20	.317	89.5	.934
30	.300	92.2	.949
40	.290	96.9	.986
550	.279	101.0	1.018
60	.264	102.8	1.028
70	.246	102.6	1.020
80	.227	101.0	1.000
90	.209	99.2	.980
600	.195	98.0	.968
10	.186	98.5	.974
20	.178	99.7	.987
30	.172	101.0	1.003
40	.166	102.2	1.019
650	.162	103.9	1.041
60	.157	105.0	1.058
70	.151	104.9	1.064
80	.1440	103.9	1.063
90	.1360	101.6	1.048
700	.1284	99.1	1.031
10	.1210	96.2	1.011
20	.1134	92.9	.986

Bureau of Standards. Davis-Gibson Filters

T -- Spectral Transmission of Filter at 25°C
V -- Relative Visibility Function
E -- Relative Energy of 2848°K
E' -- Relative Energy of 4800°K

E'' (=T x E)* -- o o o o -- Relative Energy of 2848°K and Filter Combination

Light Transmission of Filter for 2848°K = 0.249 **

FILTER FORMULA

A

Copper Sulphate ($CuSO_4 \cdot 5H_2O$)	2.452	grams
Mannite ($C_6H_8(OH)_6$)	2.452	grams
Pyridine (C_5H_5N)	30.0	cc
Water (distilled) to make	1000.	cc

B

Cobalt Ammonium Sulphate ($CoSO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O$)	21.710	grams
Copper Sulphate ($CuSO_4 \cdot 5H_2O$)	16.110	grams
Sulphuric Acid (sp. gr. 1.835)	10.0	cc
Water (distilled) to make	1000.	cc

These data are for a one centimeter layer each of solutions A and B in a double cell with three plates of borosilicate crown glass (refractive index, D line, = 1.51), each 2.5 mm thick.

* Adjusted to make sum of E'' - E' from 400 to 720 mμ equal practically to zero

** Factor to be used to multiply the candle-power of the light-source to obtain the candle-power of the source-and-filter combination

FIGURE 2.—*Chart describing Davis-Gibson filter, 2,848° K. to 4,800° K.*

This filter is selected from those of Series II, B. S. Misc. Pub. No. 114. (See Table 12 of that publication.)

The superiority of the N. P. L. standard in the violet results from the higher temperature of the source used, as well as from the higher transmittancy of the filter in this region. This advantage of the higher temperature of source is, however, counteracted somewhat by the consequent shorter life of the lamp.

WASHINGTON, August 12, 1931.