RP338

THEORY OF DESIGN AND CALIBRATION OF VIBRATING-REED INDICATORS FOR RADIO RANGE BEACONS

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ABSTRACT

This paper gives a general treatment of the theory of design of vibrating-reed indicators, which was developed in connection with measurement and design work on the tuned-reed course indicator for the aircraft radio range beacon. The equations and conclusions may be readily adapted to apply to any similar vibrating system.

By assuming that a reed may be replaced by an equivalent particle, vibrating in the plane of the driving poles, the differential equation of motion is simplified greatly and becomes readily solvable for small vibrations. Equations are given for determining the constants of the equivalent particle from the dimensions and constants of the reed. The expression for the frequency of a loaded uniform reed, computed by a method equivalent to one given by Rayleigh, checks very closely with that obtained by Drysdale and Jolley for a similar reed. This theory for small vibrations is applicable when the amplitude is small enough so that its square may be neglected in comparison with the square of the air gap.

From an analysis of large vibrations of tuning forks by Mallett, the behavior of the reed at relatively large amplitudes of vibration is inferred, although an exact quantitative verification of the theory is difficult.

Design equations are given for uniform reeds and for the type used in the reed indicator. From the results of both theory and experiment, the effect of the various factors of design and operation upon the reed frequency is discussed, and the calibration procedure necessary to take account of these factors is outlined.

CONTENTS

I. INTRODUCTION

This theory of reed indicator design has been developed in connection with a program of measurement and design work on the tuned-reed visual course indicator for the aircraft radio range-beacon. The purpose of this work was to improve the sensitivity of the indicator, to standardize the design for production manufacture, and to develop apparatus and methods for laboratory and production calibration.

The theory of lateral vibrations of bars has been very completely developed by Lord Rayleigh.[1] However, his theory, while having

[1] Lord Rayleigh, Theory of Sound, 1, Ch. VIII, p. 255.

the advantage of great generality, is somewhat complicated for practical use, and is not applicable to damped, forced vibrations. Several investigators have developed simplifications of Rayleigh's theory, but these have not been found to be useful in connection with this problem. Mallett [2] has given a very thorough treatment of large vibrations of electrically driven tuning forks. With certain modifications, necessitated by differences in drives, his equations may be applied to vibrations of reeds and are used to explain some of the properties of reeds when vibrating at large amplitudes.

Rayleigh derived an equation for the frequency of free vibration of a uniform bar, but none of the investigations includes any equation for the frequency of a reed of nonuniform cross section, nor do they give any method for the calculation of the sensitivity of driven reeds. There is a definite need for these equations in indicator design, and they are developed in this work. The equation for frequency is obtained by Rayleigh's method, while an assumption that the reed is replaceable by an equivalent particle enables an expression for sensitivity to be derived.

The greatest problem encountered in production manufacture has been nonuniformity of indicators. This has necessitated the expenditure of an inordinately large amount of time in calibration, thus increasing costs of manufacture. In order to minimize this time, it is necessary to determine and eliminate the causes of nonuniformity. Also, careful and exact design results in a simplification of calibration procedure which is of great value to a manufacture.

It has also been necessary to develop apparatus and methods for calibration, which includes tuning the reed to the proper frequency and adjustment of its sensitivity and sharpness of resonance to the desired values.

II. THEORY FOR SMALL VIBRATIONS

A brief outline of the type of drive used in the reed indicator will give a clearer conception of what follows. Constructional details of the indicator are given in a publication by Dunmore,[3] and a diagrammatic sketch is shown in Figure 1. In this figure, *A* is the permanent magnet used to polarize the reed, *B* indicates the driving coils and pole pieces, and *C* is the reed. The magnetization of the permanent magnet is such that, with no current in the driving coils, both pole pieces have a polarity opposite to that of the reed and exert equal and opposite attractions upon the reed. The driving coils are connected in such a manner that when a direct current is passed through them, the ends of the coils nearest the reed have opposite magnetic polarities. Consequently, when a direct current flows in the coils, the attraction of one pole for the reed is increased while that of the other pole is decreased, and a force thus acts on the reed tending to displace it from its neutral position. If an alternating current flows in the coils, the attractions of the poles vary periodically with the current and the force on the reed varies correspondingly.

It is obvious that, as soon as the reed moves from the neutral position, a second force due to the permanent magnet comes into play,

[2] E. Mallett, Resonance Curves of Tuning Forks, Physical Society Proc., **39**, p. 334; 1927.
[3] Design of Tuned-Reed Course Indicators for Aircraft Radio-Beacon, B. S. Jour. Research, **1** (RP28); November, 1928.

inasmuch as the reed is then nearer to one pole than to the other. This force also tends to pull the reed away from its neutral position.

Figure 2 shows a top view of the reed, with the damping vane attached at the narrow end.

For small vibrations, the square of the deflection from the neutral position may be neglected in comparison with the square of the gap.

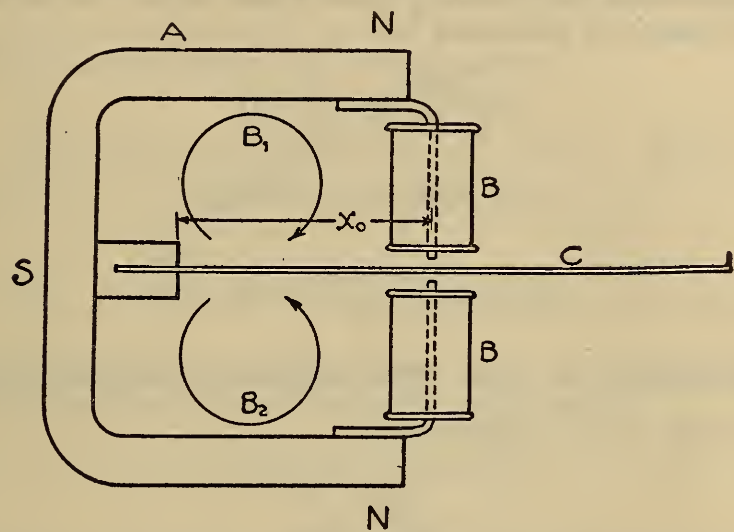

FIGURE 1.—*Schematic diagram of drive used in reed indicator (side view)*

If it is assumed that the entire reed may be replaced by an equivalent particle between the driving poles; that is, a particle of which the displacement at each instant is the same as that of a point on the reed between the driving poles, and which has the same kinetic energy, the same potential energy, and thus suffers the same loss of energy per unit of time as the reed, then the equation of motion may readily be written and solved. This assumption is the same as saying that the phase is the same at every point on the reed. There is little but

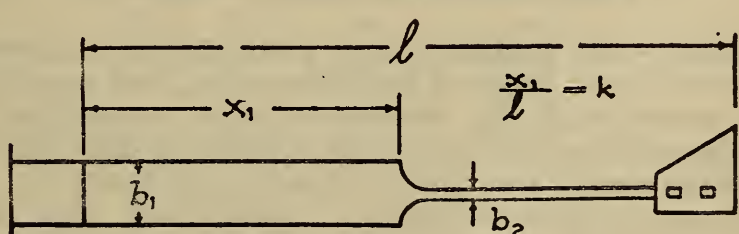

FIGURE 2.—*Top view of reed used in reed indicator*

pragmatic justification for this assumption, although its success in numerous similar cases warrants its adoption here. The final justification, of course, lies in the experimental verification of equations dependent upon the assumption. If the mass of this hypothetical particle is m, the damping force per unit velocity is r, and the restoring force per unit of displacement is s, the equation of motion is

$$my'' + (r + A_1)y' + (s - A_2)y = BI \cos \omega t \tag{1}$$

where y is the displacement of that portion of the reed immediately between the magnet poles, B is the force per unit current, I is the maximum instantaneous value, and ω the angular velocity of the current in the driving coils, and the primes over the y's indicate differentiation with respect to the time t. A_1 is a constant representing the increase in damping caused by magnetic and mechanical hysteresis, and A_2 is a constant denoting the decrease in stiffness caused by the permanent magnetic field. The quantities, $r+A_1$ and $s-A_2$, are replaced by r' and s' in the following. The solution of (1) for a steady state of vibration is

$$y=\frac{BI}{Z}\sin(\omega t+\theta) \tag{2}$$

where

$$\left.\begin{aligned} Z&=\sqrt{\omega^2 r'^2+(s'-\omega^2 m)^2} \\ \theta&=\tan^{-1}\frac{s'-\omega^2 m}{\omega r'}=\tan^{-1}\frac{X}{r'} \end{aligned}\right\} \tag{3}$$

and

X being written for $\frac{s'}{\omega}-\omega m$. The amplitude of this vibration is a maximum when Z is a minimum or

$$\omega=\sqrt{\frac{s'}{m}-\frac{r'^2}{2m^2}} \tag{4}$$

This equation shows that the reed vibrates with maximum amplitude when the frequency of the driving current is somewhat lower than the frequency of resonance of the damped reed outside a magnetic field because the quantity s' is less than the normal stiffness of the reed, and the quantity r' is greater than the normal damping force. The amount of this lowering of the frequency of resonance by the magnetic positional force is dependent upon the constants of the magnetic circuit and the position of the driving coils.

1. EQUIVALENT-PARTICLE CONSTANTS

There remains the problem of determining the constants of the equivalent particle in terms of the dimensions and constants of the reed. For convenience the determination of the constants will be based on the average values of the kinetic energy, the potential energy, and the rate of dissipation of energy of the reed. The same final results are obtained as would be obtained by using the instantaneous values. If $Y=f(x)$ is the maximum value of the displacement at any point x on the reed (x being measured from the base of the reed), the r. m. s. values of displacement, velocity, and acceleration at any point are $\frac{1}{\sqrt{2}}Y$, $\frac{1}{\sqrt{2}}\omega Y$, and $\frac{1}{2}\omega^2 Y$, respectively. The average kinetic energy of a small portion of the reed dx is

$$dT=\frac{1}{4}\omega^2 Y^2 A\rho dx \tag{5}$$

A being the cross-sectional area of the reed and ρ its density. The kinetic energy of a load of mass M on the reed is $\frac{1}{4}\omega^2 M Y_M^2$, Y_M being the value of the displacement at the load. The total average kinetic energy is, then

$$T=\frac{\omega^2}{4}\int_0^l A\rho Y^2dx+\frac{1}{4}\omega^2\Sigma MY^2_M \tag{6}$$

If this is divided by one-half the square of the r. m. s. velocity at the driving point, the result will be the mass of the equivalent particle having the same kinetic energy as the reed and the same velocity as a point on the reed between the driving poles

$$\mathrm{m}=\frac{\frac{\omega^2}{4}\int_0^l A\rho Y^2\,dx+\frac{\omega^2}{4}\Sigma MY^2{}_M}{\frac{\omega^2}{4}Y^2{}_{x_0}}=\frac{\rho\int_0^l AY^2dx+\Sigma MY^2{}_M}{Y_{x_0}} \tag{7}$$

Y_{x_0} is the value of Y at the driving point, and l is the length of the reed.

Similarly, the total energy dissipated per unit of time divided by the square of the velocity at the driving point gives

$$r=\frac{\int_0^l k_1Y^2dx+k_2\,A_vY^2{}_v}{Y^2{}_{x_0}} \tag{8}$$

and the potential energy of flexure divided by the square of the displacement at the driving point gives

$$s=\frac{E}{Y^2{}_{x_0}}\int_0^l I\left(\frac{d^2Y}{dx^2}\right)^2 dx \tag{9}$$

In these expressions k_1 and k_2 are constants depending upon the resistance of the air to the motions of the reed and the damping vane respectively, A_v the area of the damping vane, Y_v the value of Y at the center of area of the damping vane, Y_{x_0} the value of Y at the driving point, E Young's modulus for the material of the reed, and I the moment of inertia of the reed cross section about its neutral axis. This use of the letter I will be readily distinguishable from its use to designate the current in the driving coils of the reed, as these two quantities appear in entirely different expressions in the following work, and the context will indicate which quantity is meant. The letters A in equation (7) and I in equation (9) are kept under the integral signs because the reed may not have a uniform cross section. Of course, if the reed were not of the same material throughout, ρ and E would also have to be placed within the integrals. This would be a rare case, however.

The above equations define the motion of the reed, provided that the square of the amplitude in the air gap is negligible in comparison with the square of the gap itself.

III. EFFECT OF LARGE VIBRATIONS

When the vibration becomes so large that the square of the amplitude becomes comparable in magnitude with the square of the gap, these equations no longer hold. The frequency of resonance becomes a function of the amplitude, and the resonance curve becomes unsymmetrical, tending to drop off more sharply on the low frequency side than on the high frequency side.

Mallett's treatment [4] of large vibrations of tuning forks may be applied to the reed indicator and admirably explains these effects. When properly transformed to conform to the conditions existing in the reed indicator and the notation used here, Mallett's solution becomes

$$\left.\begin{aligned}\left(\frac{s'-\omega^2 m}{\omega r'}\right) Y - CY^3 &= DI\cos\theta\\ Y + GY^3 &= DI\sin\theta\end{aligned}\right\} \tag{10}$$

Here, ω, I, m, r', and s' are the same as before, except that s' contains a small term depending upon the current in addition to the terms

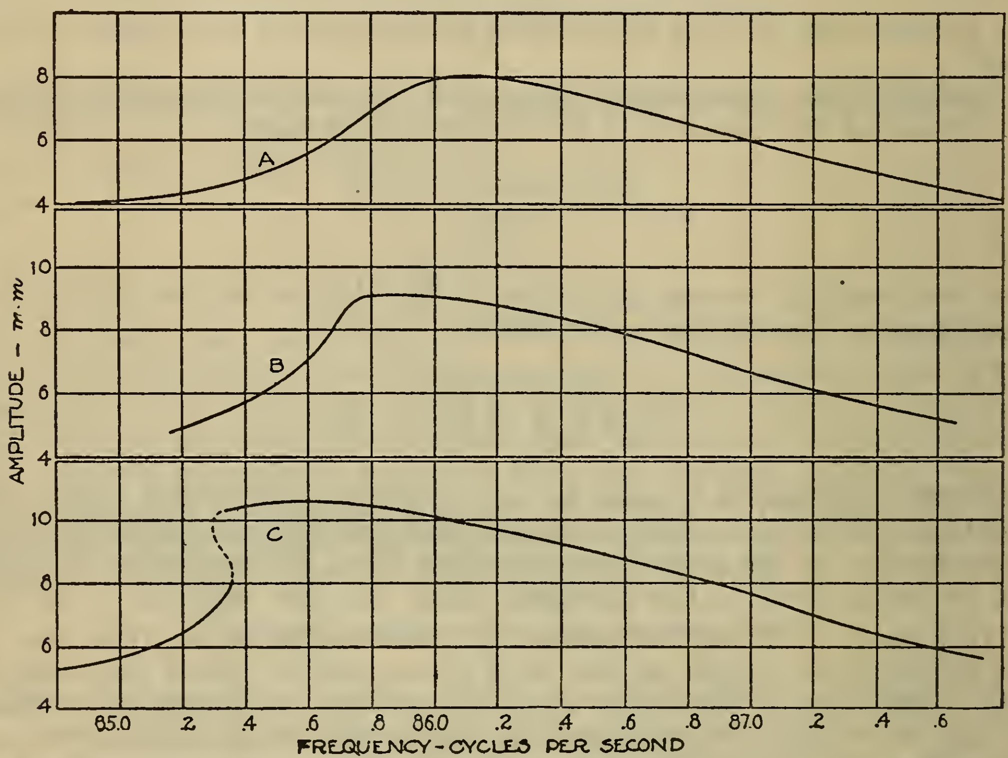

FIGURE 3.—*Resonance curves taken with constant current, showing increasing distortion as amplitude is increased*

A, 0.70 ma.; *B*, 0.83 ma.; *C*, 0.98 ma.

previously mentioned; Y is the amplitude of vibration of the equivalent particle; C, D, and G are constants depending upon the characteristics of the permanent magnet and coils and upon the dimensions of the magnetic circuits; and θ is the phase angle of the current, referred to the phase of the vibration as standard.

Resonance curves plotted from graphical solutions of these equations show an increasing dissymmetry as the amplitude is increased. The equations also show that the damping increases with amplitude. The fact that the current amplitude enters into the quantity s' also indicates that the frequency of resonance varies slightly with the driving current at constant amplitude of vibration, the frequency

[4] See footnote 2, p. 196.

decreasing as the current increases. Also, the damping should change slightly with current at constant amplitude, but this effect is too small to be detectable.

Experimental resonance curves show all of the above characteristics, the upper portions of curves taken at constant current being displaced toward the low-frequency side and becoming broader as the maximum amplitude is increased (fig. 3), while the upper portions of curves taken with variable current, the amplitude being held constant as the frequency is varied, are displaced toward the high-frequency side. (Fig. 4.) A sufficiently exact evaluation of the constants entering into equation (10) to permit a thorough quantitative check of these results has not yet been made.

It has been found desirable to sacrifice a certain amount of sensitivity in the indicator to eliminate the effects of large vibrations, since any change of the frequency of resonance with amplitude of either or

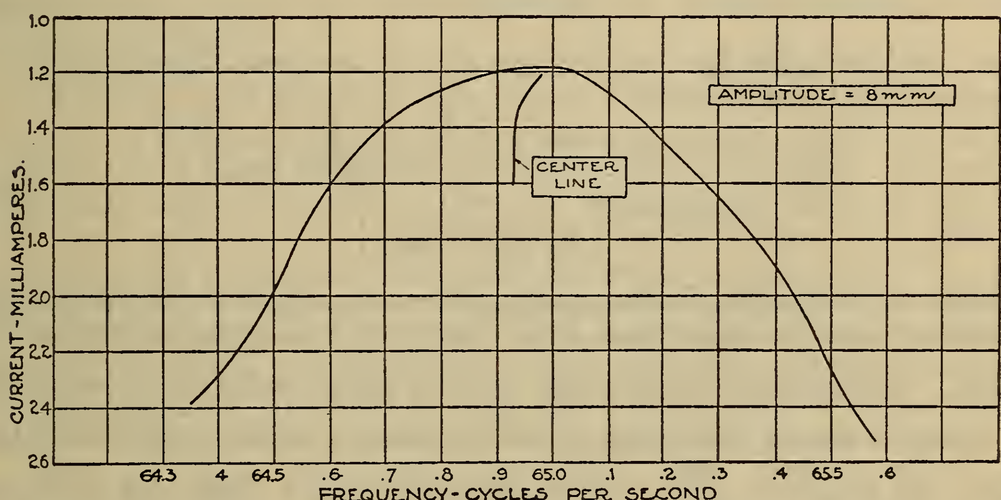

Figure 4.—*Resonance curve at constant amplitude, showing increase of frequency as current is decreased*

both reeds in an indicator (inasmuch as such a change is nearly always different for the two reeds), will almost certainly result in a shift of the course as the operating amplitude of the reeds varies. Such variation of operating amplitude is inevitable, since it is virtually impossible to maintain the output of an airplane radio receiving set constant under operating conditions. Therefore, the gaps between the reed and pole pieces are made so large that distortion of the resonance curve at normal operating amplitude is negligible. For the purposes of design, then, the vibration may be considered small, although an accurate calibration procedure must still take account of the large vibration effects.

IV. APPLICATION OF THEORY TO REED DESIGN

In order that equations (7), (8), and (9) may be used for the design of reeds, the form of the function Y, which defines the curve of deflections assumed by the reed during vibration, must be determined. If the cross section is uniform (provided the mass of the load is small in comparison with the mass of the reed, as is usually the case), Lord Rayleigh has shown theoretically that the curve assumed by a vibrating bar is very nearly the same as that which it would assume statically if deflected by a force acting at a distance from the base of three-fourths the total length. Later, however, Garrett[5] showed by experimental methods that the curve assumed during vibration is more accurately approximated if the force is considered to be applied four-fifths of the length of the reed from the base. By use of these data, the curve assumed by a uniform reed vibrating freely is found to be

$$Y=\frac{Px^2}{30EI}(12l-5x)\left(0<x<\frac{4}{5}l\right) \tag{11}$$

$$Y=\frac{8Pl^2}{375EI}(15x-4l)\left(\frac{4}{5}l<x<l\right) \tag{12}$$

where P is the force deflecting the reed statically. By substituting these expressions for Y in the formulas for m and s and carrying out the integrations with x_0 $(<\frac{4}{5}l)$ equal to the value of x at the driving poles, and with the load at the free end of the reed, these constants are found to be as follows:

$$\left.\begin{aligned} m&=(12.40A\rho l+49.5M)\left[\frac{l^3}{x_o^2\,(12l-5x_o)}\right]^2 \\ s&=\frac{153.6\;EI}{l^3}\left[\frac{l^3}{x_o^2\,(12l-5x_o)}\right]^2 \end{aligned}\right\} \tag{13}$$

Now, even though the damping of the reed is greatly increased by means of the air vane, the total damping still remains negligibly small. For example, in the case of a 65-cycle reed with $\frac{r'}{2m}$ equal to 3, which is about the largest value encountered in actual indicator reeds, the value of $\frac{r'^2}{4m^2}$ is approximately 5×10^{-5} times the value of $\frac{s'}{m}$, so that the effect of the damping upon the frequency of free vibration is less than 1 part in 20,000. Accordingly, the frequency of free vibration of a reed outside a magnetic field may be expressed

$$f=\frac{1}{2\pi}\sqrt{\frac{s}{m}} \tag{14}$$

Substituting for s and m the values given above,

$$f=\frac{0.1617\;cV}{(1+2R)\,l^2} \tag{15}$$

in which c is the thickness of the reed, $V=\sqrt{\frac{E}{\rho}}$ is the velocity of sound in the material of the reed, and R is the ratio of mass of load to mass of reed (this is assumed small). For a similar reed, Drysdale and Jolley[6] give

$$f=\frac{0.1637\;c\,V}{(1+2.05R)\,l^2} \tag{16}$$

[5] C. A. B. Garrett, On the Lateral Vibrations of Bars, Phil. Mag., 8 p. 581; 1904.
[6] Electrical Measuring Instruments, 2, p. 261. (A misprint has been corrected.)

The agreement between these two formulas affords excellent confirmation of the correctness of the assumptions made regarding the shape of the curve of deflections, since in the derivation of their formula Drysdale and Jolley used entirely different assumptions and a method different from that used here.

It should be pointed out, however, that the validity of equation (15) does not confirm the equivalent-particle assumption. The expression for the frequency may be obtained by equating the average kinetic and potential energies of the reed. Since the equivalent particle is assumed to have the same kinetic and potential energies as the reed, equating the average kinetic and potential energies of the particle necessarily gives the same frequency, as may be seen by substituting in equation (14) the values of s and m from equations (9) and (7). The method here given of computing the frequency, then, involves the determination of the form of the curve of deflections, $Y=f(x)$, and it is then equivalent to the approximate method of Lord Rayleigh.[7]

Frequencies computed by means of these formulas will be somewhat higher than those actually given by the reeds in the indicator because of the lowering due to the magnetic action. If all the factors entering into these frequency lowering effects are known accurately, they may be included in the formula, but their presence complicates the result considerably. Furthermore, these factors are very difficult to determine before the reed itself is made, so that, generally, it is far simpler to use the above formula and design the reed, considering R to be zero, to have a frequency sufficiently higher than the desired frequency so that, with the reed in the indicator, a certain amount of loading must still be added to bring the frequency down to the desired value. This permits adjustment of the load for accurate tuning.

Since Drysdale and Jolley have shown[8] that their equation (16) for the frequency of a reed checks very closely with experimental results, either equation (15) or (16) may be used to determine the velocity of sound in the alloy used for reeds in the indicator. This alloy, known as "Allegheny electric metal," is used because of its high magnetic permeability and low thermoelastic coefficient. The value of Young's modulus of this material, which is necessary for the calculation of the velocity of sound in it, is not available at present. From observations on a reed made from sheet stock 0.020 inch thick cold-rolled to a final thickness of 0.015 inch, the value of V was found to be 4×10^5 centimeters per second. This can be considered as an approximation only, since experience shows that V varies appreciably with the thickness to which the 0.020 inch stock is cold-rolled, the thinner material having somewhat lower values of V.

To obtain an expression for the sensitivity of a reed, equation (2) is used. This equation gives the value of the displacement of the reed at the driving poles. It is necessary to multiply this displacement by the ratio of the amplitude at the tip of the reed to the amplitude in the gap in order to determine the deflection of the free end of the reed. Thus, for the tip of the reed

$$y_l = \frac{Y_l}{Y_{x_0}} \cdot \frac{BI}{Z} \sin(\omega t + \theta) \tag{17}$$

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[7] See footnote 1, p. 195.

[8] See footnote 6, p. 202.

For values of ω near resonance, Z may be considered as approximately equal to $\omega r'$. Consequently

$$y_l = \frac{Y_l}{Y_{x_0}} \cdot \frac{BI}{\omega r'} \sin(\omega t + \theta) \tag{18}$$

or

$$Y_l = \frac{BI}{\omega r'} \cdot \frac{Y_l}{Y_{x_0}} \tag{19}$$

Now, r' is different for reeds of different frequencies, but since it is necessary that all reeds in the indicator have resonance curves of the same shape, the sharpness of resonance S_R must be constant for all reeds. Sharpness of resonance is here defined by the equation

$$S_R = \frac{f_R}{f_1 - f_2} \tag{20}$$

where f_1 and f_2 are the frequencies at which the amplitude on the resonance curve is 0.707 of the maximum amplitude, and f_R is the frequency at maximum amplitude. It may readily be shown that, defined in this manner

$$S_R = \frac{\omega_0 m}{r'} \tag{21}$$

where ω_0 is the angular velocity at resonance. On substitution of the value of r' obtained from equation (21), equation (19) becomes

$$Y_l = \frac{BIS_R}{\omega_0^2 m} \cdot \frac{Y_l}{Y_{x_0}} \tag{22}$$

or, since $\omega_0^2 m = s'$

$$Y_l = \frac{BIS_R}{s'} \cdot \frac{Y_l}{Y_{x_0}} \tag{23}$$

To a first approximation, s may be used in place of s', since A_2 is rarely greater than 5 per cent of s.

Now in the case of a forced vibration it is possible that even a uniform reed does not conform to the curve of deflections already given, unless the driving force is applied at a point four-fifths of the length from the base, which is not usually the case. Furthermore, reeds having other than uniform cross sections may be desirable in many instances, as in the reed indicator. In general, therefore, the curve of deflections must be determined experimentally and the corresponding equation developed. If it is possible to approximate to the curve of deflections by assuming the reed to be deflected by a single force at some point, the task of finding the equation of this curve may be simplified greatly. It should be pointed out here, however, that this method is useless in the case of a nonuniform reed unless the dimension (or dimensions) of the reed which varies with the distance from the base is expressible as a function of x which can be integrated. If this is not the case, it is necessary to fit a power series or other function of x to the observed curve. The equations for m, r, s, frequency, and sensitivity, will then have different forms from those given above. However, it may be shown, by suitable transformations of variables and functions within the integrals, that the occurrence, in the equations for m, r, s, frequency, and sensitivity, of all quantities independent of x is not affected by the form of the function representing the curve of deflections. Thus, these equations may be of some value even though the exact form of the function representing the curve of deflections is not known. For a reed of the type that is used in the reed indicator (fig. 2), m and s may be written

$$m=\frac{\rho lc[f_1(b_1,\ b_2,\ k)+f_2(b_1,\ b_2,\ k)R]}{F_h^2} \tag{24}$$

$$s=\frac{c^3Ef_3(b_1,\ b_2,\ k)}{N_o l^3 F_h^2} \tag{25}$$

Here, b_1 and b_2 are the widths of the wide and narrow portions of the reed, k is the ratio of the length of the wide portion to the total length, and N_o is a numerical constant. F_h is a function derived from the equation of the curve of deflections and is equal to the value of F,

$$F=\frac{Y}{Y_l}=F\left(\frac{x}{l}\right) \tag{26}$$

for $x=x_o$, that is

$$F_h=\frac{Yx_o}{Y_l}=F\left(\frac{x_o}{l}\right)=F(h) \tag{27}$$

From these values for m and s

$$f=\frac{N_1cV}{l^2}\sqrt{\frac{f_3\ (b_1,\ b_2,\ k)}{f_1\ (b_1,\ b_2,\ k)+f_2\ (b_1,\ b_2,\ k)\ R}} \tag{28}$$

$$Y=\frac{B'I\,S_R}{E}\,\frac{l^3\,F_h}{c^3\,f_3\ (b_1,\ b_2,\ k)} \tag{29}$$

where N_1 is a numerical constant and B' is the constant B of equation (4) multiplied by the numerical factors arising from the substitutions and transformations made to obtain equation (29). In these equations, the manner of occurrence of the quantities, c, l, ρ, E, and V is independent of the form of the function representing the curve of deflections. Therefore, if the values of b_1, b_2, and k are, or can be, fixed, and an approximate value for F_h obtained, either experimentally or mathematically, the equations have a certain field of usefulness as design equations. Thus, for the type of reed in use in the radiobeacon course indicator, F_h is found to be very nearly proportional to h^2, and the following relations are found useful, in connection with experimental data:

$$f\propto\frac{c}{l^2} \tag{30}$$

$$Y\propto\frac{l^3h^2}{c^3} \tag{31}$$

An exact equation for f may be derived by fitting to the experimentally observed curve of deflections an empirical equation and using this equation to determine the values of N_1 and the functions of b_1, b_2, and k. This has been done in the present investigation as follows: The curve of deflections of the indicator reed was determined by measuring the double amplitude of the vibrating reed at a number of points along its length, by means of a traveling micrometer microscope. The distance from the base of the reed to the measuring point was determined in each case. The values for the double amplitude were then corrected for the thickness of the reed, divided by two, and plotted against the corresponding distances from the base of the reed. To fit this observed curve, it was found necessary to use three separate equations—the first, containing terms in x and x^2, applied from the base to the end of the wide portion; the second, also quadratic in x, applied from the end of the wide portion to a point three-fourths of the length from the base; and the third, which was linear, applied to the remaining portion of the reed. By calculation from these equations, taking $b_1 = 0.794$ cm, $b_2 = 0.127$ cm., and $k = 0.465$ (which are the standard values for all indicator reeds), the values of f_1, f_2, and f_3 were found to be as follows:

$$f_1\ (b_1,\ b_2,\ k) = 0.037.$$
$$f_2\ (b_1,\ b_2,\ k) = \ .172.$$
$$f_3\ (b_1,\ b_2,\ k) = \ .179.$$

Also

$$N_1 = \frac{1}{2\pi}$$

Upon the introduction of these values, equation (28) becomes

$$f = \frac{0.35\ c\ V}{l^2\sqrt{1+4.65R}} \tag{32}$$

or, if R is small

$$f = \frac{0.35\ c\ V}{l^2(1+2.33R)} \tag{33}$$

This equation and the relations (30) and (31) have been found to check very well with experimental data taken on a large number of indicators. Figure 5 shows the current required to drive a reed at 8 mm. total tip amplitude plotted against h^2. Equation (32), of course, does not include the effect of magnetic action upon the reed frequency. As a consequence, frequencies calculated by this equation are consistently higher, by approximately 2 per cent, than the frequencies of the reeds in an indicator. In designing reeds, approximately five cycles per second allowance is made for the effect of magnetic action and load; that is, the dimensions for a 65-cycle reed are calculated from equation (32) by using $f = 70$ cycles per second. In special cases, when an unusually large load must be attached to the reed, greater allowances must be made.

An interesting application of these results occurs in the 3-reed or 12-course indicator. Because of space limitations in this type of indicator, it is not possible to set the driving coils at the proper positions to give equal reed sensitivities for equal gaps. In this case, the distance from the driving pole to the free end of the reed is considered constant and the reed sensitivity varied by moving the base of the reed, thus varying the length and driving distance (x_0) simultaneously.

As the length of the reed is varied, the thickness must also be changed to keep the ratio $\frac{c}{l^2}$, and hence the frequency, constant. If d_1, d_2, and d_3 represent the distances from driving poles to reed tips, and l_1, l_2, and l_3 the lengths of the 65, 86⅔, and 108⅓ cycle reeds, respectively, the following equations are obtained from the given conditions:

$$\frac{\left(1-\frac{d_1}{l_1}\right)^2}{27l_1^3}=\frac{\left(1-\frac{d_2}{l_2}\right)^2}{64l_2^3}=\frac{\left(1-\frac{d_3}{l_3}\right)^2}{125l_3^3} \tag{34}$$

Each of these fractions is proportional to the sensitivity of the corresponding reed, so that a graph of the value of each fraction against

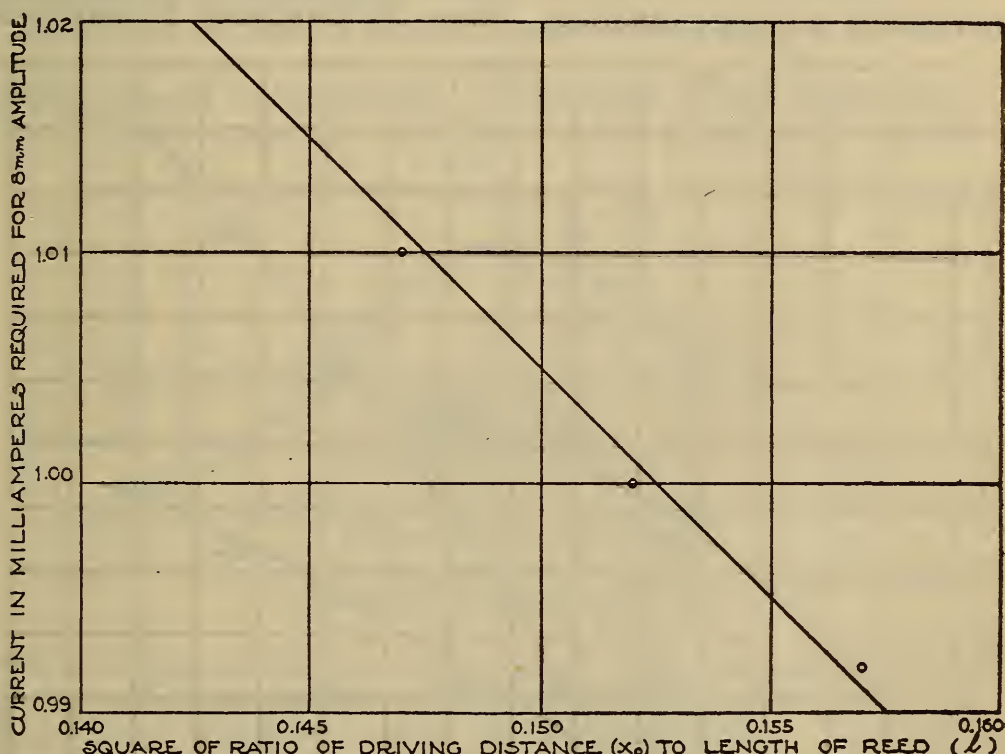

FIGURE 5.—*Relation between current necessary to drive reed at 8 mm tip double amplitude and square of ratio of driving distance (x_o) to length of reed (l)*

the corresponding length will show how the sensitivity of that reed changes as the length is varied under the prescribed conditions. Figure 6 shows the values of all three of the fractions plotted against a common scale of lengths for $d_1=d_2=4.70$ cm, and $d_3=3.64$ cm. The intersections of any horizontal line with these curves give the lengths required for equal sensitivities. For actual design, the horizontal line is taken as high as possible to give maximum sensitivity. After the lengths are determined from these curves, the corresponding thicknesses can readily be calculated.

The damping of the reed is not so easily obtained. The air-damping vane used on the reed introduces many factors the magnitudes of which have been difficult to control. Consequently, at the present time, no thorough experimental verification of theoretical expressions for damping is available, and the design of the air dampers continues to be mainly empirical. Equation (8) indicates that the value of r', for a reed of fixed dimensions driven at a fixed point, should be a linear function of A_v, the area of the damping vane. The data available at present, however, when plotted against the area of the damping vane, give points that are too scattered definitely to determine the shape of the required curve. Further experimental work, perhaps, accompanied by a more detailed theoretical analysis, will be necessary for a final determination of equations for the design of damping vanes.

V. CALIBRATION OF REED INDICATORS

The tuning of the reed is of the utmost importance, as the operating requirements are very exacting. Since the frequency of maximum

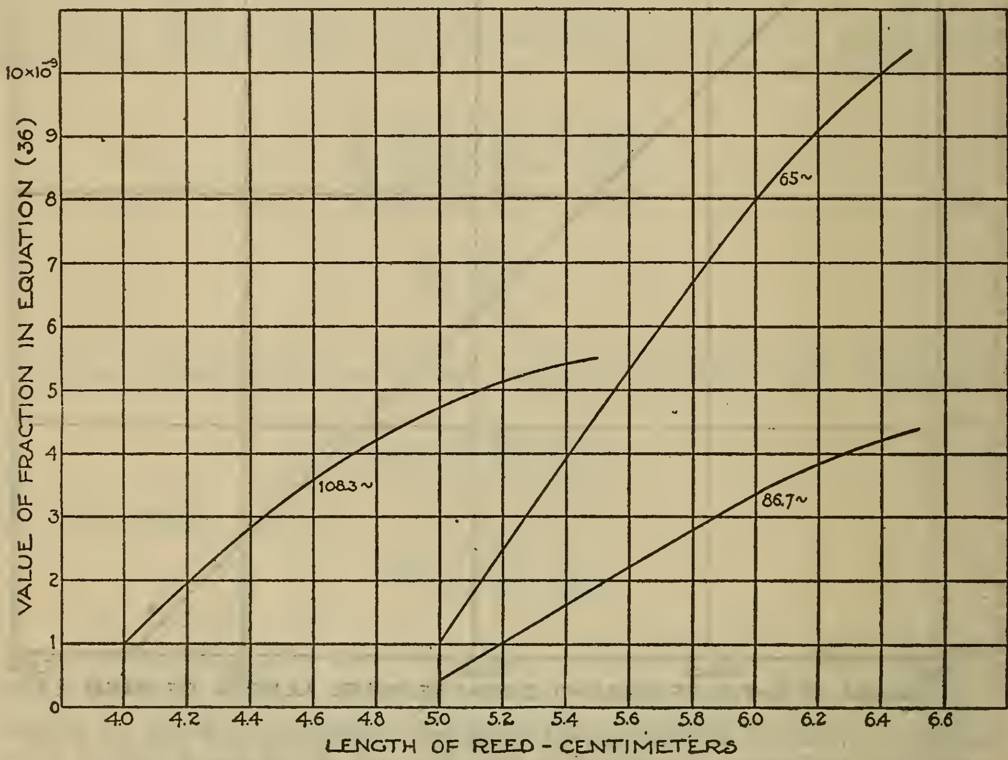

FIGURE 6.—*Curves for determining length of reeds in 12-course indicator*

response and the damping change with amplitude, a standard calibrating amplitude must be chosen and all measurements made at this amplitude. A very convenient method for determining the frequency and sharpness of resonance at a given amplitude Y consists in adjusting the driving source so that, as the frequency is varied, the maximum amplitude obtainable is equal to $\sqrt{2}Y$. If, then, the frequency alone is varied so that the reed vibrates with amplitude Y (which will occur at two frequencies), and the frequencies giving this amplitude measured, the frequency and sharpness of resonance may readily be computed. The measured frequencies are denoted by f_1 and f_2, and S_R is found by means of equation (20) while the frequency of resonance, f_R is given by

$$f_R = \frac{f_1 + f_2}{2} \tag{35}$$

As noted above, resonance curves taken with constant current show a marked dissymmetry, the upper portions being displaced toward the low-frequency side. If the voltage across the indicator instead of the current through it is held constant, this dissymmetry is even more pronounced. Curves taken at constant amplitude, with variable current, on the other hand, show only a slight dissymmetry, and here the displacement of the upper portions is toward the high-frequency side. The frequency and sharpness of resonance for the same reed will have different values when determined from these different resonance curves. Consequently, it is necessary to select as a standard, one of the three possible methods of tuning a reed—at constant voltage, at constant current, or at constant amplitude. The proper choice obviously will depend upon the conditions under which the indicator is to be used.

Before this analysis is taken up, however, the cause of the difference between resonance curves at constant current and those at constant voltage should be considered. If the reed is held stationary in its neutral position and the impedance of the driving coils measured at

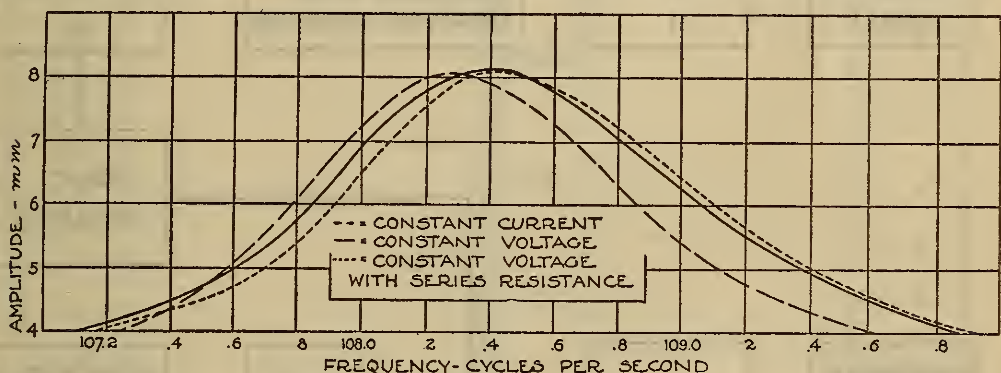

FIGURE 7.—*Resonance curves showing effect of impedance variations upon frequency and sharpness of resonance*

various frequencies in the neighborhood of the frequency of resonance of the reed, this impedance will be found to be exactly the same as that of any impedance having the same constants. This is termed the damped impedance. If, now, the reed is allowed to vibrate, and the impedance again measured, at frequencies near the frequency of resonance of the reed the impedance will have values considerably different from the damped impedance. At frequencies slightly below resonance, the impedance is greater than the damped impedance, while at frequencies above resonance the impedance is less than the damped impedance. Consequently, when the voltage is held constant for measurement of a resonance curve, the current through the driving coils will vary with the frequency, being lower for frequencies lower than resonance than it is for frequencies higher than resonance. This explains the greater dissymmetry evidenced by curves taken at constant voltage. These impedance variations must be considered in tuning the reed if the indicator is to be operated under conditions such that its impedance variations will cause variations of the current through it. (See fig. 7.)

Now, the reed indicator was developed for use in the output of a radio set as a visual course indicator for the radio range beacon. Furthermore, pilots are instructed to use it at a fixed amplitude

(8 mm). The volume control on the radio set may be either manual or automatic. If manual, the indicator may be considered to be operating in a series circuit comprising the indicator, a resistance equal to the output impedance of the set used, and a source of constant voltage but slightly varying frequency (because of slight variations in the modulating frequencies of the beacon transmitter). The conditions are the same if an automatic volume control operated by the received carrier wave is used. However, if an automatic volume control operated by the voltage across the indicator such as that recently developed by the National Bureau of Standards, is used, the fluctuations of the voltage across the indicator caused by variations in the modulation frequencies will be corrected by the volume control; and the voltage directly across the reed indicator, rather than that of the source in the series circuit mentioned above, will be maintained constant. Generally, the reed will be operated on a

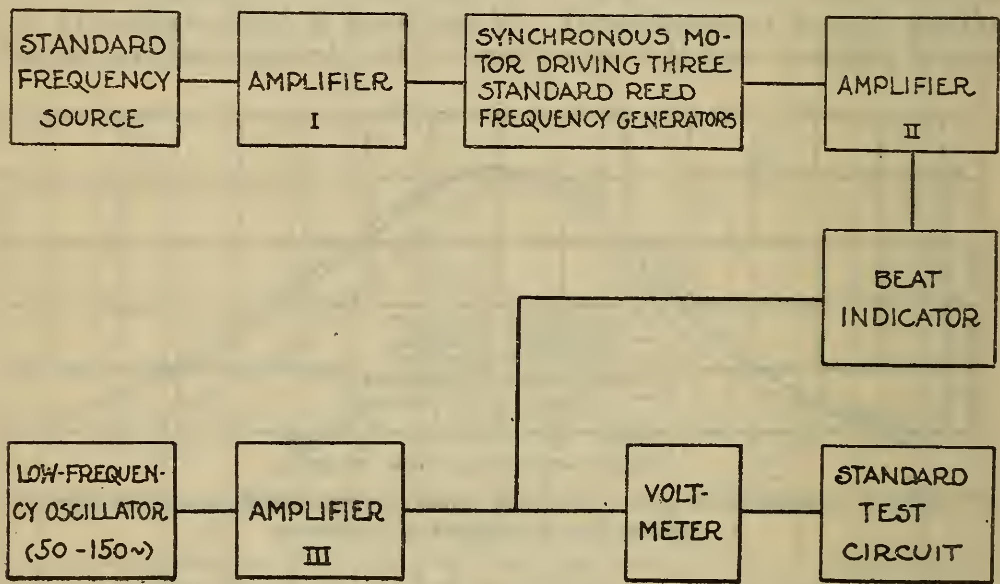

FIGURE 8.—*Schematic diagram of reed calibrating equipment*

resonance curve with a maximum amplitude of 8 mm. (This includes the width of the tab on the front of the reed, the actual amplitude of vibration being only 6 mm).

For use with a radio set having manual or carrier-operated automatic volume control, the indicator should be connected in series with a resistance equal to the output impedance of the radio set and a variable voltage of controllable frequency inserted in series with this circuit. This voltage is then adjusted so that the maximum amplitude obtainable, as the frequency is varied, is 8 mm, and, for determination of f_R and S, the frequencies f_1 and f_2 giving 6.2 mm amplitude are measured, the voltage in the circuit being held constant. This value of 6.2 mm is obtained by taking 70.7 per cent of the actual amplitude of vibration, 6 mm, and adding the 2 mm width of the tab on the tip of the reed. The sensitivities of all reeds in the indicator are adjusted so that their amplitudes are the same for a given voltage impressed in the circuit described above.

For use with a radio set equipped with an automatic volume control operated by the indicator voltage, sensitivity adjustments are made as above, and tuning adjustments are made at the same amplitude. The voltage, during tuning adjustments, is maintained constant directly across the indicator terminals.

When tuned by either of these methods and operated under the corresponding conditions, the reeds show a minimum amount of variation due to the unavoidable causes pointed out in the above analysis of the operation of the reed.

In practical calibration work, the reed is driven by a vacuum-tube oscillator and when this oscillator is adjusted to one of the "test

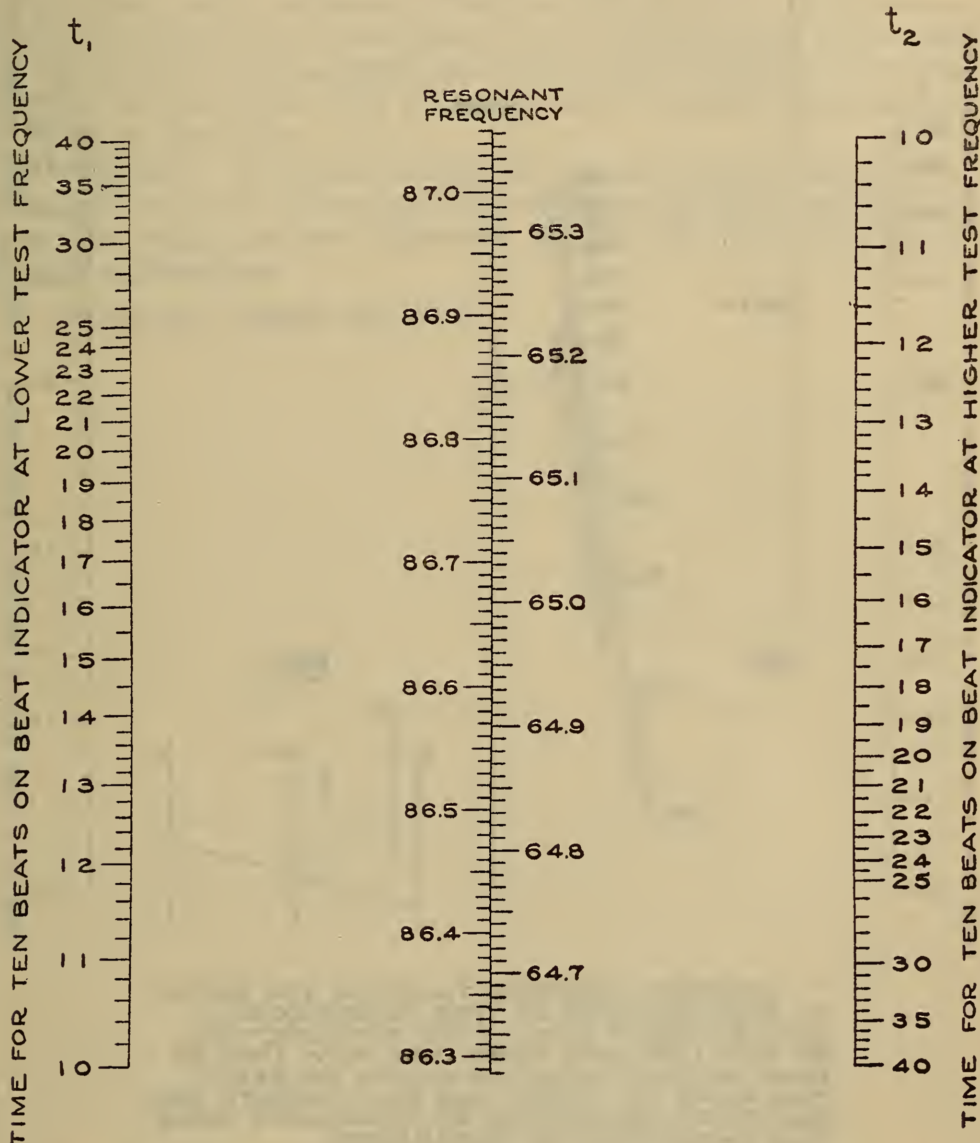

FIGURE 9.—*Chart for determining frequencies of resonance of reeds*

frequencies" (f_1 and f_2 of equation (20)) its frequency is determined by measuring with a stop watch the time required for 10 beats between the driving oscillator and a standard frequency source. The standard frequency source used at the National Bureau of Standards consists of three alternators, giving 65.000, 86.667, and 108.333 cycles per second, all mounted on the same shaft and driven by a 1,000-cycle synchronous motor. The current to drive this motor is obtained from the primary-standard frequency equipment. Beats between the standard frequency and the driving oscillator are indicated by a vacuum-tube beat frequency indicator. A schematic diagram of the complete calibrating equipment is shown in Figure 8.

To eliminate the arithmetical work necessary to calculate the frequency and sharpness of resonance from the measured times for 10 beats at the test frequencies, two nomographic or alignment charts

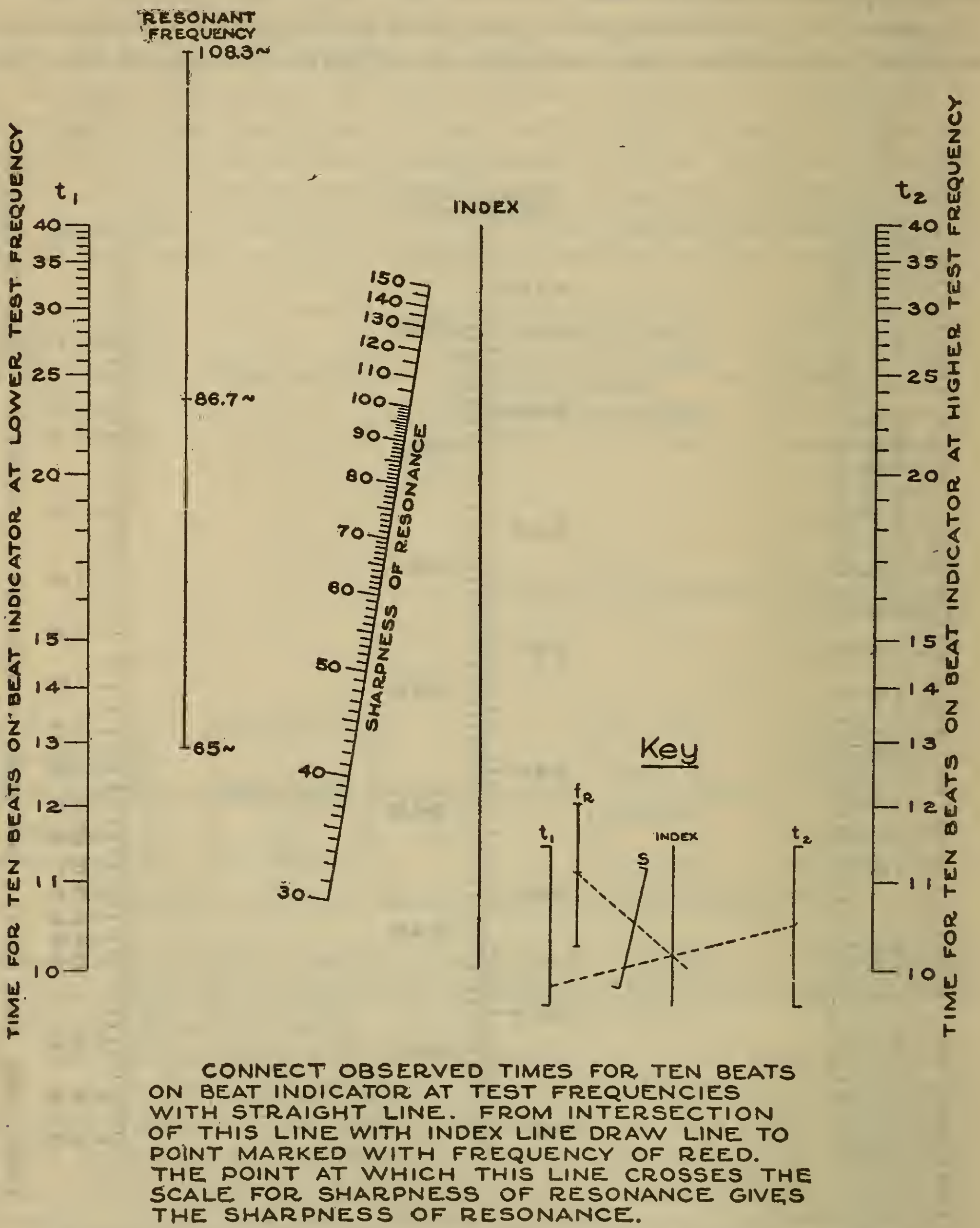

FIGURE 10.—*Chart for determining sharpness of resonance of reeds*

have been designed to facilitate the work. These charts are shown in Figures 9 and 10, the former being used to obtain the frequency and the latter for sharpness of resonance. In the first figure, the two measured times are located on the scales indicated for them, and the straight line connecting these two points crosses the central line at a point which determines the frequency of resonance. The second figure is self-explanatory.

VI. CONCLUSION

By means of Rayleigh's approximate method, equations are obtained for the frequency of free vibration of a uniform reed and for a particular type of nonuniform reed.

To obtain an expression for the sensitivity of a driven reed, the entire reed is assumed to be replacable by an equivalent particle located at the driving point. The equation thereby derived for the sensitivity of the reed checks very closely with results observed with the type of reed used in the reed indicator.

The theory also points out factors affecting the performance of the reeds which must be included in any consideration of procedures of calibration and adjustment, and it enables a method of calibration to be developed which will give the most desirable performance of the reeds in operation.

In conclusion, the writer wishes to express appreciation to F. W. Dunmore for much information regarding practical considerations in connection with the reed indicator, and to H. Diamond for many valuable suggestions.

WASHINGTON, March 25, 1931.