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Effects of Induced Earth Currents on Low-Frequency Electromagnetic Oscillations[1]

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The information about the effects of induced earth currents that can be obtained from analyses of observations and from calculations based on electromagnetic induction theory is examined. The nature of these effects is discussed and estimates of their magnitude for fields of varying extent and frequency are obtained. The influence of underlying geology and the effects of the oceans are considered. The type of mathematical problem that needs to be solved to get more detailed information is described.

1. Introduction

It is well known that geomagnetic micropulsations of all types arise from sources external to the earth's surface. The primary oscillating electromagnetic fields of these external sources give rise, however, to induced electric currents within the earth, which considerably modify the observed surface fields. In seeking to discover the precise nature of the external sources, one must try to assess the effects of the induced earth currents and remove them from the surface observations. In this paper we consider what is known about these effects, and what can be inferred from analyses of the observations together with calculations based on electromagnetic induction theory. It is well known that the induced currents tend to reduce the amplitude of the Z-component and increase that of the H-component. An accurate estimate of these changes is needed, for example, when estimating the height and/or horizontal position of certain ionospheric sources, such as jet currents. There are also other effects produced by the induced currents such as changes of phase and of polarization. Our point of view in the present discussion is that the induced currents are something of a hindrance to our investigation of the primary sources of the phenomena, and we study them merely in order to get rid of their effects. There is of course another point of view with regard to these earth currents, namely, that their study may give some information about the distribution of electrical conductivity within the earth. Two methods have been developed for obtaining such information, (i) the magnetic potential method based on analyses of magnetic variations over the earth's surface or over some suitable part of it, and (ii) the magnetotelluric method based on the relations at different frequencies of mutually orthogonal horizontal components of the electric and magnetic variations at a given station. A valuable and extensive discussion of the theory of the magnetotelluric method has been given by Wait [1962]. In the present paper, however, we shall be concerned only with the effects of induced earth currents on the magnetic field.

2. Order of Magnitude of the Conductivities Involved

The conductivity of dry surface rocks is frequently of order $\sigma = 10^{-4}$mho/m(10^{-15}emu) or less, that of moist earth and sedimentary rocks $10^{-1} - 10^{-2}$mho/m and of sea water, 4 mho/m. Thus the conductivity of layers near the surface may vary by several orders of magnitude.

The estimates of the conductivity at greater depths have been obtained mainly from analyses of the slower magnetic variations, combined with theoretical studies of electromagnetic induction. A summary of results of these studies is given in figure 1, taken from a recent paper by Eckhardt, Larner, and Madden [1963]. It will be seen that the mean conductivity near the surface is estimated at between 10^{-3} and 10^{-2} mho/m and rises sharply in the region of about 600 km depth to over 1 mho/m. The Cantwell-McDonald curve shows a rather rapid rise of σ at a few tens of kilometers depth to a value greater than 5.10^{-2} mho/m. This part of the profile was deduced by Cantwell from magnetotelluric measurements. It should be stated that the magnetic variations results obtained by Lahiri and Price [1939] certainly did not preclude a high conductivity at say 60 to 70 km depth, but they indicated that the conductivity must in that case decrease to appreciably less than 10^{-2} mho/m beyond this depth and then rise again in the region of about 500 to 600 km depth.

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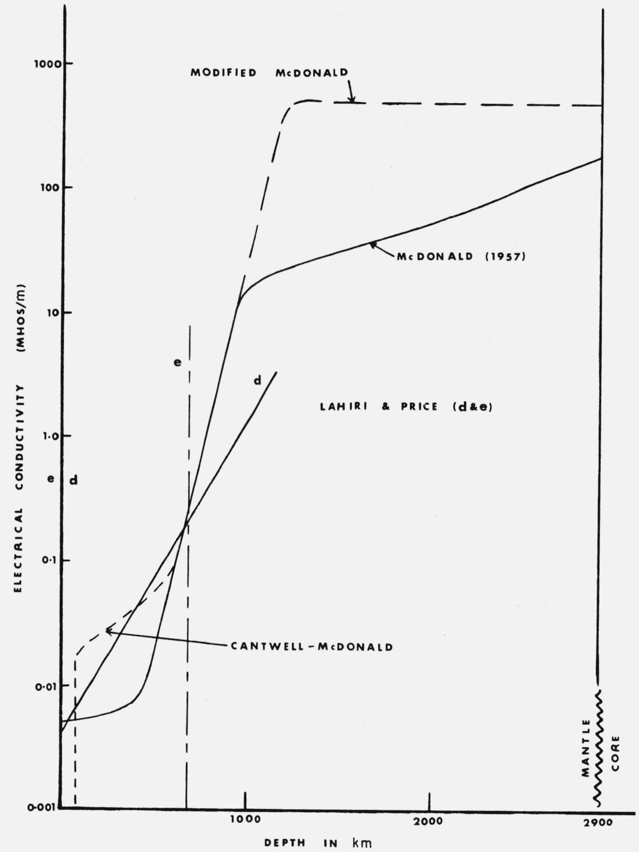

FIGURE 1. *Theoretical electrical conductivity profiles for the earth's mantle* (after Eckhardt, Larner, and Madden).

3. Calculation of the Induced Currents

In the present discussion we consider only the range of frequencies from 1 to 0.001 c/s, i.e., periods T from 1 to 1000 sec. For these values of T (or any greater values) and for linear dimensions of the conductor not greater than those of the earth, it is permissible to ignore displacement currents and use the diffusion equation for the electromagnetic field. For values of T appreciably less than 1 sec, this is not permissible, but the problems which then arise are of a different kind, and in these problems it is often sufficient to treat the earth as a perfect conductor.

We shall thus consider only the induction of currents by varying *magnetic* fields. Reference has sometimes been made to the possibility of electrostatic induction of currents by varying atmospheric electric fields, but apart from thunderstorm effects, any earth currents likely to be produced in this way would be some orders of magnitude smaller than those produced by varying magnetic fields, and will therefore be ignored.

In view of the considerable range of conductivities of the crustal layers of the earth, it is clear that those induced currents that are near the surface will vary greatly in strength from one place to another. Hence if the frequency of the primary field is high enough to confine the induced currents, because of the "skin effect" to the crustal layers, the effects of these currents on the observed micropulsation fields will differ greatly from one place to another. It is therefore necessary to examine the depth of penetration of the induced currents. Before doing this we may note that the mathematical problems which arise in the study of earth currents are of two kinds. In the first we are concerned with inducing fields of global dimensions and with averaged or smoothed conductivities of the earth as a whole; these we may call *global problems.* In the second kind the inducing field may or may not be of global dimensions but we are now concerned with strictly local values of the conductivity, we may call these *local problems.* The problems arising in the interpretation of surface observations of rapid fluctuations such as micropulsations are often of the second kind, but there are exceptions. For example, studies of the overall effect of the oceans lead to problems of a more global character. On the other hand, the investigation of coastal effects is a local problem. Local problems are concerned with the great differences of conductivity of different geological strata. In these local problems we can often ignore the sphericity of the earth and treat it as a semi-infinite body having a nonuniform distribution of conductivity, our attention being confined to some limited region. On the other hand, the inducing field will often be of large dimensions and therefore effectively uniform over the region being studied. This has sometimes been taken to imply that the actual finite dimensions and distribution of the inducing field can be ignored in studying the induced currents, but this is not always the case. A knowledge of the inducing field in the immediate neighborhood of some particular station and of the surrounding ground conductivity is not usually sufficient to determine the strength of the induced currents in that neighborhood. This will be affected by the nature and distribution of the entire inducing field and by the average properties of the conductor over a region of corresponding dimensions. It is also true, of course, that the entire induced current system will contribute to the magnetic field at a particular station, but this fact is not so important because usually the largest contribution comes from currents in the immediate neighborhood. The really important point is that, not only the local properties, but the properties of the conductor as a whole—its geometry and the distribution of its conductivity—determine the *strength* of the currents in any neighborhood being studied.

Thus the problem of calculating the contribution of the induced field to the observed total field will frequently reduce to finding out how, in a particular neighborhood, some extended system of induced currents is redistributed by local differences of conductivity. The extended current system will usually need to be determined or estimated from results obtained in some global type of problem, where the earth is treated as a sphere with a suitable smoothed distribution of bulk conductivity. Many problems of this type are discussed in the literature. When the extended average current system has been found in this way, the determination of the local redistribution of these currents is not purely a problem in the disturbance of steady current flow, because the currents are alternating and a skin effect will usually be in evidence.

4. Separation of the External and Internal Sources by Analysis of the Surface Field

It would not be necessary, of course, to actually calculate the induced currents for the purpose of removing their effects from the observed surface field if one could separate the external and internal (induced current) sources at any instant by analyzing directly the observed surface distribution of the field. Theoretically, it would be possible to do this by using well-known results in potential theory, but in practice we do not have a sufficiently detailed knowledge of the surface distribution to do it satisfactorily. Hence it is necessary to supplement any studies of the surface field by deductions drawn from suitably illustrative calculations of the induced currents.

One of the difficulties inherent in the analysis of the surface distribution of the field of fairly rapid geomagnetic oscillations is that there are two patterns present in this distribution. One is *usually* a small scale pattern determined by the distribution of conductivity in the surface layers. This conductivity may change greatly in a distance of a few tens of kilometers. The other is generally a large scale pattern determined by the inducing field, which is often of global dimensions. If one seeks to determine the general distribution and overall dimensions of the field from observations at stations over a sufficiently extended area, it is obviously necessary to have regard to the fact that the field at any one station may be considerably affected by local geological (possibly subterranean) conditions. Due allowance must be made for this by estimating the effects from local surveys, or otherwise. The local irregularities can of course be reduced to a minimum by siting the stations in an area as uniform as possible.

A program of *simultaneous* observations of various types of micropulsations over an extended area would certainly provide information of the greatest value about the fields involved. It is understood that a program of this kind is being organized in the United States. One of the most elementary pieces of information we require in order to assess the importance of induction effects is a knowledge of the overall extent of the inducing fields, because this is an important factor in determining the intensity and depth of penetration of the induced currents.

Other measurements of great value in the study of induction effects are those that have been made on ice islands in the Arctic Ocean, such as those reported by Hessler [1962 see also Swift and Hessler, 1964], Heirtzler [1963], and Zhigalov [1960]. The ocean affords a horizontally uniform environment, with the conductivity of the sea water known and that of the seabed probably very much smaller. Also an ice island, unlike an ordinary oceanic island, does not seriously disturb the induced currents in the immediate vicinity. Hence the conditions are most favorable for interpreting the observations and studying the induction effects. Some of the results obtained by Zhigalov, from measurements on a drifting ice station "North Pole 6" are shown in figure 2, which illustrates the attenuation of the vertical component by electric currents induced in the sea. The ratios of the amplitudes of the vertical variations r_Z to those of the horizontal variations r_H are compared with the depth of the sea at a number of positions of the drifting station. It will be seen that the ratio r_Z/r_H decreases considerably as the depth of sea increases, and the curve of r_Z/r_H is very similar to the ocean depth profile. Also the ratio is considerably smaller for the variations of shorter period. These results are in accordance with the effects expected from currents induced in the sea.

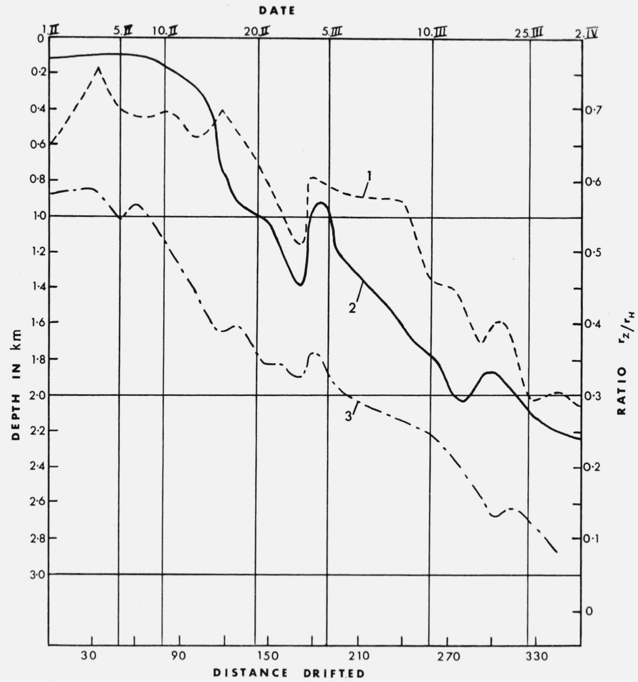

FIGURE 2. *Curves of the ratio r_z/r_H compared with the ocean profile for the ice-island station North Pole 6, February-March 1958 (after Zhigalov).*

Lat 79°30′ – 80°44′ N, long. 153°57′ – 150°17′ E.
1. Ocean depth in meter,
2. ratio r_Z/r_H, mean for all hours of day,
3. ratio r_Z/r_H, for variations of period $T \leq 10$ min.

5. Depth of Penetration of Induced Currents and Magnetic Field

The well-known formula for the skin effect for alternating currents in a conductor is derived from the solution of the diffusion equation

$$\frac{\partial^2 I}{\partial z^2} = \sigma \mu i \omega I \qquad (1)$$

for the current density I in a semi-infinite uniform conductor occupying the half-space $z<0$ say. The period is $2\pi/\omega$ in seconds, and σ, μ, are the conductivity and magnetic permeability, respectively, in MKS units. The permeability μ may be taken as that of free space (μ_0) in all the cases that need be considered. The expression for the current density at depth z is then

$$I=I_0 \exp\{-z(\mu_0\sigma i\omega)^{1/2}\} \tag{2}$$

so that the amplitude contains the attenuation factor $\exp\left\{-z\left(\frac{\mu_0\sigma\omega}{2}\right)^{1/2}\right\}$. The skin depth is correspondingly taken as

$$d=\left(\frac{\mu_0\sigma\omega}{2}\right)^{-1/2} \tag{3}$$

Typical values of d in kilometers, for various conductivities and periods are shown in table 1.

TABLE 1. *Values of the skin depth d in kilometers corresponding to various values of the conductivity and period* $T=2\pi/\omega$

Period	T	1	10	100	1000
Conductivity in mho/m	10^{-4}	50	159	500	1,590
	10^{-2}	5	15.9	50	159
	4	0.25	0.8	2.8	8.0

This table suggests that the depth of penetration of currents in continental areas is quite considerable even for the short periods considered. But it must be remembered that the formula used to obtain these results is based on a number of simplifying assumptions that may not be applicable to the geophysical problem under consideration. The sphericity of the earth is ignored and the conductor is assumed to extend downwards to infinity. Also the current flow is assumed to be uniform over any horizontal plane. It is probable that the assumption that the sphericity of the earth can be ignored does not lead to any serious error in most cases, but the other two assumptions can both conceivably do so, and it is necessary to examine them further.

When the actual distribution and extent of the inducing field is taken into account, the induced current distribution is no longer uniform over each horizontal plane, and the depth of penetration of the currents is reduced. An estimate of the effect produced by the finite scale of the inducing field can be obtained by considering the field whose potential, just above the surface of the conductor, is

$$\Omega=-Ae^{\nu z+i\omega t}P(x,y) \tag{4}$$

where P satisfies the equation

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}+\nu^2 P=0 \tag{5}$$

because Ω satisfies Laplace's equation. For example, if the field is independent of y, P will be a trigonometric function, say $\sin \nu x$, so that the field is periodic in the x-direction with wavelength $2\pi/\nu$. In the slightly more general case corresponding to (5) we may regard $2\pi/\nu$ as a length representing the scale of the field pattern.

It can be shown [Price, 1950, 1962] that the density at depth z of the resulting induced currents in the conductor, assumed to extend downwards to infinity, is given by

$$\mathbf{I}=-\frac{2\sigma i\omega\mu_0}{\theta+\nu}Ae^{-\theta z+i\omega t}\left(\frac{\partial P}{\partial y},-\frac{\partial P}{\partial x},0\right) \tag{6}$$

where

$$\theta^2=\mu_0 i\sigma\omega+\nu^2. \tag{7}$$

The attenuation factor is therefore

$$\exp\left\{-z\left[\left(\frac{1}{4}\mu_0^2\sigma^2\omega^2+\frac{1}{4}\nu^4\right)^{1/2}+\frac{1}{2}\nu^2\right]^{1/2}\right\} \tag{8}$$

which shows that the attenuation is greater than when the field is uniform, corresponding to $\nu=0$. The increase in the attenuation will be significant if ν is not too small compared with $\sqrt{(\mu_0\sigma\omega)}$.

Now $2\pi/\nu$ is not likely to be much less than 4×10^5m, i.e., about four times the height of the E layer where some of the current sources may be situated, and it cannot be greater than 4×10^7m, the circumference of the earth. Of course fields of global dimensions can only be properly discussed by treating the earth as a sphere, but since the depth of penetration is usually a small fraction of the earth's radius, it is possible to treat the earth's surface as a plane for getting an approximate idea of the magnitude of the effect we are considering. From the above we see that the values of ν for natural fields are likely to range from 1.6×10^{-7} for global fields to 1.6×10^{-5} for local fields.

We consider first the case where the conductivity is low and the period fairly long so that we have the maximum penetration of the field. We take $\sigma=10^{-4}$ mho/m and $T=100$ sec. We then have $\sqrt{(\mu_0\sigma\omega)}\sim 2.8\times10^{-6}$, which is certainly within the likely range of values of ν. We can infer that, in this case, the scale of the inducing field affects the depth of penetration appreciably.

If we take a higher conductivity say $\sigma=10^{-1}$ mho/m—which is probably greater than that of most continental strata but still considerably less than that of sea water—and a shorter period, say $T=10$ sec, the value of $\sqrt{(\mu_0\sigma\omega)}$ becomes 2.8×10^{-4}. Hence, in this case, the depth of penetration is not likely to be appreciably affected by the scale of any natural inducing field.

In the case of sea water $\sigma \sim 4$mho/m, so that $\sqrt{(\mu_0 \sigma \omega)} \sim 5.7 \times 10^{-3}(T)^{-1/2}$. Hence, *if one could treat the sea as of infinite depth*, the depth of penetration of the induced currents would apparently not be sensibly affected by the scale of the inducing field, unless the period of the oscillations was at least a few hours. But actually the finite depth of the sea has a very important effect, and we find that the scale of the inducing field becomes important when the depths of the real oceans are considered.

6. Depth of Penetration of Currents and Field in the Oceans

If we consider an ocean of depth D m and assume that the conductivity of the seabed is negligible compared with that of the sea water itself, the intensity of the induced currents at depth z is found to be [Price 1962]

$$\frac{I(z)}{I(0)} = \frac{e^{-\theta z}\{1+\beta e^{-2\theta(D-z)}\}}{1+\beta e^{-2\theta D}} \tag{9}$$

where

$$\beta = \frac{\theta - \nu}{\theta + \nu} \tag{10}$$

and θ is defined by (7).

The vertical component Z of the field attenuates in exactly the same way as the current density I, i.e., $Z(z)/Z(0)$ is given by the same formula (9). The horizontal component H attenuates more rapidly, however, and is given by

$$\frac{H(z)}{H(0)} = \frac{e^{-\theta z}\{1-\beta e^{-2\theta(D-z)}\}}{1-\beta e^{-2\theta D}} \tag{11}$$

These results were obtained from the direct solution of the diffusion equation with appropriate boundary conditions at $z=0$ and $z=D$. It will be seen that if the expressions (9) and (11) are expanded by the binomial theorem, they can also be regarded as representing the effect at depth z of the incident (evanescent) wave $e^{-\theta z}$ together with the effects of the sequence of waves reflected, with the appropriate transformations, from the lower and upper surfaces. The important difference between the formulas (9) and (11) should be noted. This disappears when $D \rightarrow \infty$, i.e., all the field components attenuate in the same way in a conductor of infinite depth, but when the depth is finite H attenuates more rapidly than I or Z.

The formulas will apply equally well, of course, to any highly conducting stratum over a poorly conducting basement. More elaborate formulas applicable to a series of layers of different conductivities have been obtained for purposes of electromagnetic deep sounding, but these will not be considered here.

To illustrate the effect of the finite value of D on the depth of penetration of the horizontal component of the field, the values of the amplitude ratio mod $\{H(z)/H(0)\}$ at various depths z in an ocean of depth 1 km are compared with the values at the same depths in an ocean of infinite depth. The period T is taken as 100 sec and the value of ν as 10^{-6}. The results are shown in table 2.

TABLE 2. *The reduction in amplitude of* H *at various depths* z *in (i) an ocean of infinite depth, (ii) an ocean of depth 1 km, (iii) an ocean of depth* z

$T=100, \nu=10^{-6}$

z (m) =	10	10^2	5×10^2	9×10^2	10^3	2×10^3	5×10^3
$D=\infty$	1.0	0.96	0.82	0.70	0.67	0.45	0.14
$D=10^3$	0.99	.90	.50	.80	.003	....	....
$D=z$	.30	.03	.006	.004	.003	.002	.001

In this table the first row of values gives the ratios of the amplitude of H at depth z to that at depth zero in an ocean of infinite depth, the second row the values in an ocean of 1 km depth. The third row gives the values at the bottom of the sea when the sea has depth z.

The remarkable differences in these values should be noted. It is clear that the currents are much more concentrated near the surface when D is small, and the field is therefore much more attenuated than the usual skin formula indicates.

It is also interesting to find that when D is small compared with the skin depth, the dimensions of the inducing field $2\pi/\nu$ become important even when σ is large. We find, in fact, that when

$$D \ll |\theta|^{-1} \ll 2\pi/\nu \tag{12}$$

the above formula (11) for $H(z)/H(0)$, when $z=D$, reduces to

$$\frac{H(D)}{H(0)} = \frac{1}{1+\mu_0 i\sigma\omega D/\nu} \tag{13}$$

which is the well-known formula for shielding by a thin conducting plate. Thus doubling the scale $\frac{2\pi}{\nu}$ of the inducing field would reduce the amplitude ratio for H, i.e., increase the attenuation, by the same amount as doubling the frequency (ω) or the conductivity σ.

The above results all relate to the horizontal component H. When the vertical component is considered, we find a very different result. Thus for the same values of T, ν, as above, the reduction of the amplitude of Z at various depths z are shown in table 3.

TABLE 3. *The reduction in amplitude of* Z *at various depths* z *in (i) an ocean of infinite depth, (ii) an ocean of depth 1 km, (iii) an ocean of depth* z

$T=100, \nu=10^{-6}$

z (m)	10	10^2	5×10^2	9×10^2	10^3	2×10^3	5×10^3
$D=\infty$	1.00	0.96	0.82	0.70	0.67	0.45	0.14
$D=10^3$	1.00	.99	.96	.91	.89	....	....
$D=z$	1.00	.98	.93	.90	.89	.82	.28

This table shows that even at the bottom of an ocean of depth 2,000 m there is only slight attenuation of the amplitude of the vertical component. It must however be remembered that the vertical component is usually already greatly reduced at the surface by the induced currents whereas the horizontal component is increased there.

7. Effect of Induced Currents on Surface Values of H and Z

The induced currents will tend to reduce the amplitude of the Z-component and increase that of the H-component. For very high frequencies—specifically those for which $(\mu_0\omega\sigma)^{-1}$ can be taken as zero in the calculations—the Z-component will be practically extinguished, while, on a flat surfaced conductor, H will be doubled. When the curvature of the earth has to be taken into account, e.g., when fields and conductivity distribution are of global dimensions, H would be increased in the ratio $(2n+1)/(n+1)$, where n is the order of the spherical harmonic involved.

To obtain numerical estimates of the effects for other frequencies, we first make calculations for a single layer conductor, since, as we have seen, this seems an appropriate model for discussing the effects of the oceans.

Denoting the contributions to the surface field (H, Z) of the inducing and induced fields by (H_0, Z_0) and (H_1, Z_1) respectively, we find

$$\frac{H_1}{H_0} = -\frac{Z_1}{Z_0} = \frac{\beta(1-e^{-2\theta D})}{1-\beta^2 e^{-2\theta D}}. \tag{14}$$

This formula has been used to estimate the enhancement of the H variations and the reduction of the Z variations on the surface of an ocean of depth D for various values of D. The scale length $2\pi/\nu$ of the inducing field has been taken as 10^6m (i.e., 1000 km), the conductivity σ as 4 mho/m and the period T as 1000 sec. The results will be unaltered if σ and T are both changed in the same ratio, e.g., if we take $\sigma = 3$ mho/m which is likely to be a more representative value for the Arctic Ocean, then the results will apply for $T = 750$ sec. The results are shown in the first two rows of table 4. They show that, for the variations considered, the Z component would be practically extinguished over an ocean of depth 5 km and the H component would be nearly doubled.

The differences of phase between the inducing and induced field, determined from (14) are shown in the third row of the table. It will be seen that the phase differences are quite small.

To obtain the relationship between the total Z and H variations at the surface we easily find from (14) that

$$\frac{Z}{H} = \frac{Z_0+Z_1}{H_0+H_1} = \frac{Z_0}{H_0}\frac{\nu}{\theta}\frac{1+\beta e^{-2\theta D}}{1-\beta e^{-2\theta D}}. \tag{15}$$

TABLE 4. *The effects of induced currents on* H *and* Z *Variations at the surface of an ocean of depth* D

	$D(m)$	5×10^2	10^3	2×10^3	3×10^3	4×10^3	5×10^3
(1)	$\lvert H_1/H_0\rvert$	1.56	1.71	1.83	1.88	1.91	1.93
(2)	$\lvert Z_1/Z_0\rvert$	0.44	0.29	0.17	0.12	0.09	0.07
(3)	$\arg(H_1/H_0)$	.0°	.0°	.1°	.6°	.9°	1.1°
(4)	r_Z/r_H (calculated)	.29	.17	.09	.06	.05	0.04
(5)	r_Z/r_H (observed)	.53	.39	.15			
(6)	r_Z/r_H (calculated)	.37	.19	.10	.06		
(7)	r_Z/r_H (calculated)	.53	.28	.14	.09		

The values in row (4) are calculated from (15) with $2\pi/\nu = 10^6$, those in row (6) from (16) with $2\pi/\nu = 10^6$ and those in row (7) from (16) with $2\pi/\nu = 7\times10^5$. For all cases $\sigma = 4$ mho/m with $T = 10^4$ sec or, alternatively, $\sigma = 3$ mho/m with $T = 750$ sec.

The amplitude ratio $|Z_0/H_0|$ for the external source fields will of course depend on the position of the station where the particular variations are observed. If, for simplicity, we assume that the *average* amplitude ratio for these variations is unity, the average amplitude ratio, say r_Z/r_H, for the total variations may then be calculated from (15). The calculated average values of r_Z/r_H are shown in the fourth row of table 4, and the values of these ratios estimated from Zhigalov's observations (fig. 2) are shown in the fifth row. It will be seen that the calculated values are less than those obtained from the observations. This could of course be due to several different factors. For example, our assumption that for the inducing field $(r_Z/r_H)_0 \sim 1$, may not be true for the variations observed by Zhigalov. Again we have taken L to be 1000 km, and the average scale factor of the real inducing fields may be greater or less than this. Finally, the fact that the ocean has a finite area has been ignored, and there is probably an important coast effect extending over the shallower parts near the edge.

A rough idea of the nature of the dependence of r_Z/r_H on the parameters ν, ω, σ, and D can be obtained by treating the ocean as a thin conducting plate of integrated conductivity σD. This gives

$$\frac{Z}{H} = \frac{Z_0}{H_0}\frac{1}{1+\mu_0 i\sigma\omega D/\nu} \tag{16}$$

which is, in fact, the approximate form of (15) when

$$\nu D << 1 \qquad \text{and } |\theta D| << 1. \tag{17}$$

This approximate formula has been used to calculate r_Z/r_H for $L = 1000$ km and $L = 700$ km, and the results are shown in rows 6 and 7 of table 4. It will be seen that the latter value of L gives a good fit to the values deduced by Zhigalov from the observed amplitude of the variations of period $\leqslant 10$ min. Though the formula is not very accurate for the values of $|\theta D|$ involved, we may reasonably conclude that Zhigalov's results can be adequately explained by induction of currents in the sea. Further, since the induction effects appear to be quite sensitive to the scale length L (or $2\pi/\nu$) of the inducing field, it may be possible, from studies of similar and more extensive ice island observations, to obtain useful estimates of L as well as of the other parameters involved in these induction effects. In more elaborate studies, it would, of course, be desirable to take into account the possible effects of the conductivity and topography of the ocean bottom.

8. Induction Effects Over Land Areas

The general agreement between the calculated and observed induction effects over the oceans, where conditions are fairly uniform, gives promise that, by using other and usually rather more elaborate mathematical models, some idea of the nature and magnitude of induction effects over land areas may be obtained. A simple model may first be used to get a general idea of the *magnitude* of the effects to be expected. More elaborate models must then be used to discuss the distribution of these effects and their variation with frequency and other parameters. We consider first a uniform conductor occupying the half-space $z>0$, and take various values of the conductivity σ that would represent possible *average* or effective values of σ throughout a depth somewhat greater than the skin depth for any particular frequency $\omega/2\pi$ of the oscillations. For this model we find

$$\frac{H_1}{H_0}=-\frac{Z_1}{Z_0}=\beta=\frac{\sqrt{(\mu_0 i\sigma\omega+\nu^2)}-\nu}{\sqrt{(\mu_0 i\sigma\omega+\nu^2)}+\nu}. \tag{18}$$

For the induction effects to be important, $\mu_0\sigma\omega$ must not be small compared with ν^2, but if it is large compared with ν^2, then the effects are simply the extinquishing of Z and the approximate doubling of H. More interesting changes of the induction effects with frequency of oscillations will occur when $\mu_0\sigma\omega$ is comparable in magnitude with ν^2.

TABLE 5. *Values of mod β and arg β for different values of the ratio $\alpha=\mu_0\sigma\omega/\nu^2$*

$\alpha=$	0.01	0.25	1.0	4.0	25	100
mod $\beta=$	0.0025	0.065	0.22	0.48	0.75	0.87
arg $\beta=$	90°	77°	66°	39°	16°	8°

Table 5 gives the values of mod β and arg β for values of the ratio $\alpha=\mu_0\sigma\omega/\nu^2$. This table shows that the induced currents just begin to be effective when α reaches 0.25, H then being increased by a factor 1.065 and Z decreased by a factor 0.935. The effects may be regarded as becoming really significant when α reaches 1, since H is then increased by 20 percent and Z decreased by that amount, so that the corresponding ratio of the amplitudes of H and Z is then two-thirds that for the inducing field alone. When α reaches 100, H is increased by a factor 1.87 and Z decreased by a factor 0.13. Hence, for $\alpha \geqslant 100$, the induction effects are predominant and Z is practically extinquished compared with H.

To determine the possible ranges of the parameters σ, T, ν which correspond to the above values of α we may note that

$$\sigma \sim \alpha\nu^2 T/2\pi\mu_0. \tag{19}$$

Now we have already seen that ν for the inducing field is probably within the range 1.6×10^{-7} to 1.6×10^{-5} so that ν^2 ranges from 2.5×10^{-14} for global fields to 2.5×10^{-10} for local fields. At the lower limit of ν (global fields) with $\alpha=1$ (just significant induction effects) we have $\sigma\sim3\times10^{-9}$ T. This shows that induction effects will be significant for these worldwide fields for all variations of period $T<10^3$ sec, provided the effective bulk conductivity is greater than 3×10^{-6} mho/m (3×10^{-17} emu) and this is probably true in almost all cases. It must however be pointed out that this result refers only to fields of the widest possible extent, i.e., those for which $2\pi/\nu=4\times10^7$ m, and the coefficient α—the "induction response" coefficient—varies as ν^{-2}. If we take $\nu\sim1.6\times10^{-6}$, corresponding to fields extending to about 4,000 km, we find from (19) that induction effects for variations of period 10^3 sec are significant if $\sigma>3\times10^{-4}$ mho/m. This value is probably near the upper limit of the effective bulk conductivity.

The induction effects will be *predominant* ($\alpha=100$) in fields of maximum global extent for all variations with $T<10^3$ sec, if $\sigma>3\times10^{-4}$ mho/m. For fields extending to about 4000 km, and $T=10^3$ sec, σ would have to be greater than 3×10^{-2} mho/m, which seems unlikely in most continental areas. Assuming σ is of order 3×10^{-4} mho/m, induction effects will predominate for these fields when $T<10$ sec.

For strictly local fields, corresponding to $\nu\sim1.6\times10^{-5}$, $\sigma\sim3\times10^{-5}$ α T. Hence for $T=10^3$ sec, σ would have to be as high as 3×10^{-2} mho/m for there to be significant induction effects, and as high as 3 mho/m for these effects to be predominant. For $T=1$ sec, the induction effects are likely to be significant even for these local fields, but will not be predominant unless the effective σ is greater than 3×10^{-3} mho/m.

We conclude that for $T\sim1$ sec, the induction effects are always important and will be predominant except for inducing fields of very limited extent. When $T\sim10^3$ sec, the importance of the induction effects in continental areas will depend very much on the dimensions of the inducing field.

9. Local Effects Due to Nonuniform Conductivity

The discussion in sections 5–8 has been based on the assumption that the earth can be treated as a conductor with horizontal layers of uniform conductivity. In many land areas, however, there are considerable variations of σ in the horizontal directions, and in some cases actual discontinuities, depending on the geology of the area. This nonuniform distribution of conductivity will affect the distribution of the induced currents. Hence the induction effects, particularly for short period oscillations, will vary considerably over such an area. The effect of a nonuniform distribution of conductivity is also shown by the increase of r_Z/r_H at land stations near the coastline of a deep ocean [Schmucker, 1964]. This increase is caused by the induced currents in the relatively highly conducting sea water, which tend to concentrate near the boundary of the ocean because of self induction.

The nature of the mathematical problems that arise in considering these local features has already been referred to in section 3. They are not usually direct induction problems of the type considered in the previous sections. What is now usually required is a method for calculating the disturbance produced by a local nonuniform distribution of conductivity in some average system—possibly quite extensive—of induced currents. In this sense it is a disturbed skin effect problem. Very few complete solutions of this type of problem, applicable to oscillations of period 1000 sec or less, have as yet been published, though a number of people are working in this field, and one may hope that eventually a library of solutions of relevant mathematical problems will be available for helping to interpret the observations.

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