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Complete Scattering Parameters of Polydispersed Hydrometeors in the λ0.1 to λ10 cm Range

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The extinction, the albedo of single scattering, the differential scattering cross section, and the complete polarization properties, per unit volume of cloud and rain elements, irradiated by microwave radiation at various frequencies have been determined. Continuous drop-size distribution functions are introduced to represent real clouds and precipitation, and the absorption and scattering parameters are integrated with high accuracy. A Rayleigh approximation is found adequate for the cloud model, but the complete Mie expressions had to be used for precipitation-sized particles. Older estimates have been corroborated, but some new features have been brought out for the first time, such as the scattering intensity and polarization (including its ellipticity) as a function of scattering angle.

The quantitative results, presented graphically and in extensive tables included in the original study (available as RAND publication R–422–PR, 1963), will be useful in advancing cloud physics research by means of active and passive microwave techniques. The same results can be used in the theoretical interpretation of the observed continuous and discrete microwave emissions from certain planets.

1. Introduction

This new theoretical investigation was undertaken with the following objectives in mind: (a) to introduce a realistic size spectrum for precipitation drops, *including the smaller ones found near the base of the cloud*; (b) to evaluate as exactly as possible all the scattering parameters that are necessary inputs for the formulation of the equivalent radiative transfer problem; in addition to the scattering and absorption coefficients, these include the Stokes polarization parameters at all scattering angles, per unit volume of space with precipitation or cloud particles or both; and (c) to extend the range of previous estimates to shorter wavelengths down to 1 mm.

Such parameters are not readily available in the literature. Previous investigations have been generally limited to the determination of backscattering and attenuation cross sections for single particles as a function of the size parameter. We hope that the results presented here will lead to new research applications of passive and active microwave techniques in terrestrial cloud physics. These will also be useful in the formulation of a planetary problem that arises when microwave radiations of several frequencies are diffusely transmitted or reflected by an extensive atmosphere containing hydrometeors.

[1] This paper was presented at the World Conference on Radio Meteorology Sept. 14–18, 1964, National Bureau of Standards, Boulder, Colo.

Only a short summary of the results will be given here, since the full details describing the method and including numerical tables are available in the form of a RAND publication [Deirmendjian, 1963b] obtainable on request.

2. Models

The size distribution function chosen to represent cloud and precipitation particles (water or ice spheres) has the general form

$$n(r) = ar^{\alpha} \exp\{-br^{\gamma}\}, \tag{1}$$

where $n(r)$ is the number of particles per unit volume at the radius r, and a, α, b, and γ are positive constants that determine the total number of particles N, the shape and range of the distribution curve, and the radius r_c of maximum frequency.

The specific precipitation model used here is shown in figure 1, and it is sufficiently realistic to illustrate the main results. The parameters have been adjusted so that $N=10^3/\text{m}^3$, $r_c=0.05$ mm, and the liquid (rain) water content is $M=0.495$ g/m^3. Actual raindrop counts corresponding to two rainfall rates at the ground, as well as the so-called Marshall-Palmer [1948] distribution curve for a rainfall of 17 mm hr^{-1} are also inserted in figure 1 for comparison. It is evident that our model corresponds to greater numbers of drops with $r>1.5$ mm than is indicated by the *M–P* curve or by the actual counts. This disagree-

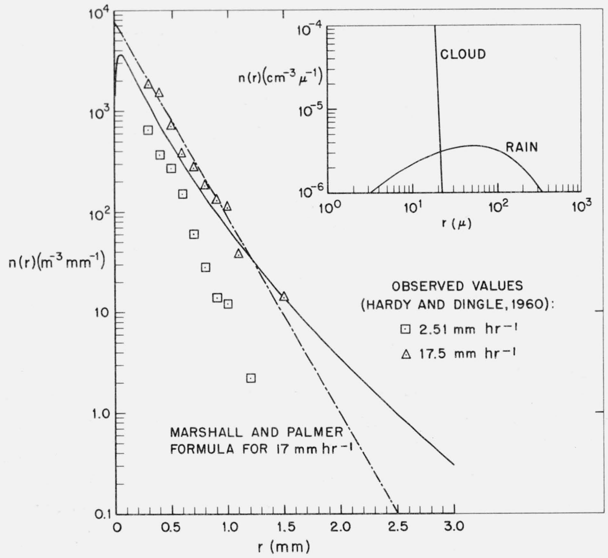

FIGURE 1. *The raindrop size distribution used (full line), some observed counts (squares and triangles), and the Marshall and Palmer curve (dash-dot).*

The inset shows the crossover point and local shape of the distribution curves used for the cloud and rain models.

ment is not serious, however, if one considers the facts that (a) no clear relation between the drop-size distribution *within or below a cloud base* and the rainfall rate and size distribution *at the ground* seems to have been established [Mason and Ramanadham, 1954]; and (b) as will be shown below, at least for scattering properties for subcentimeter radiation, the larger drops in the spectrum do not contribute significantly to their integral values. It should also be borne in mind that, although the empirically derived *M–P* exponential-type distribution reproduces *average* raindrop spectra at the ground fairly well, it cannot do so with respect to individual cases [Dingle and Hardy, 1962], and it shows a maximum number of raindrops at $r=0$.

For the cloud-droplet distribution we have adjusted the parameters in (1) so that $N=10^8\ m^{-3}$, $r_c=4\ \mu$ and the liquid (cloud) water content is $M=0.0626$ g/m³ This model is identical with the one used previously in a study of the infrared scattering properties of water clouds [Deirmendjian, 1963a]. The inset in figure 1 shows the crossover point between the rain- and cloud-drop distributions, respectively, to demonstrate how these theoretical models are able to differentiate (at least conceptually) between the two distributions.

3. Results

The total extinction and absorption coefficients and the differential-scattering cross sections per unit volume were integrated over the raindrop size distribution using the full Mie series in the expressions:

$$\beta_{\substack{ex\\ab}}(m,\lambda)=\pi\int_0^\infty x^2 f(x,\lambda)K_{\substack{ex\\ab}}(m,x)\,dx, \qquad (2)$$

$$P_j(m,\theta,\lambda)=\frac{4\pi}{\beta_{sc}}\int_0^\infty f(x,\lambda)i_j(m,x,\theta)\,dx,$$

$$j=1,2,3,4 \qquad (3)$$

where $x=2\pi r/\lambda$ is the Mie size parameter, $f(x,\lambda)$ is a function related to the size-distribution law, $K_{\substack{ex\\ab}}$ is the cross section for extinction or absorption, and the i_j are functions of the Mie amplitudes of the scattered wave for a sphere of index of refraction m and size x in the direction $\boldsymbol{\theta}$. For further details the reader is referred to the complete version [Deirmendjian, 1963b]. Here it is important to note that the integrals (2) and (3) converge quite rapidly after a certain characteristic size has been reached, thanks to the properties of the Mie functions, and the nature of the distribution function.

Figure 2 shows the manner of convergence of the product of the integrals

$$\beta_{sc}P(180°) \qquad (4)$$

as a function of the upper limit of integration, expressed in terms of the true raindrop radius, for three wavelengths. The product (4), where $\beta_{sc}=\beta_{ex}-\beta_{ab}$, is proportional to the back scattering (or radar) cross section. It is seen that for λ 2 mm radiation, the main contribution comes from drops with $r<1.5$ mm, whereas for λ 2 cm this limit is doubled, and for λ 5 cm it is quadrupled. Only in the latter case are the larger drops in the spectrum approaching the Rayleigh limit; but even here, their contribution to the overall radar cross section does not vary as r^6, as is usually assumed in relating radar echo intensities to rainfall rates. For the sake of consistency, in the results discussed below, we adopt the values of the integrals over the entire drop spectrum, with the understanding that these may be overestimated for wavelengths $\lambda>2$ cm *when we are considering raindrop spectra as observed at the ground.*

In the case of cloud-drop size distributions it can be shown that, in the integration of all the scattering parameters, the Rayleigh approximation is sufficient, and hence these parameters may be expressed essentially in terms of the liquid-water content regardless of the shape of the distribution function [Deirmendjian, 1963b, app. B].

The microwave extinction coefficients were evaluated by means of (2) at fixed wavelengths, not necessarily corresponding to those used in current radar meteorology, since the interest was mainly in examining the microwave continuum properties between λ 0.1 cm and λ 10.0 cm. The complex indices of refraction for liquid water and ice were computed in the usual way from the Debye formula. Figure 3 illustrates the extinction (or attenuation) curves as a function of λ for the various models. The values of the extinction coefficient β_{ex} are given in terms of a 1 km path through homogeneous cloud or rain; these may also be interpreted as a cross section per 10^5 cm³ of volume. Since the extinction is based on an exponential decay of directly transmitted energy, the values shown convert to decibels per kilometer by putting

$$\beta(\mathrm{km}^{-1}) = 0.23026\ \mathrm{dB}(\mathrm{km}^{-1}). \tag{5}$$

The curves marked "frozen rain" and "ice cloud" in figure 3 represent ice spheres of the same size, size distribution, and total number as the corresponding liquid-drop models; hence, strictly speaking, they do not represent natural hydrometeors. They are shown in order to illustrate the effect of a change of phase (or refractive index) only. To obtain the scattering coefficients β_{sc}, one must multiply the values shown by the albedo of single scattering per unit volume, that is, the ratio $A=\beta_{sc}/\beta_{ex}$ of the coefficients integrated over all sizes. Samples of these are given in table 1.

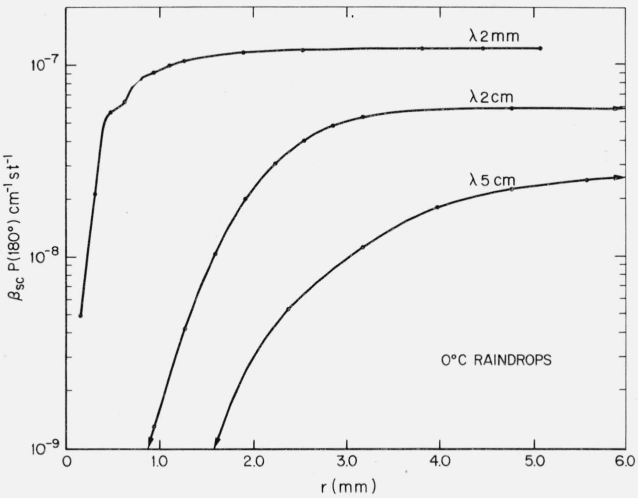

FIGURE 2. *Three examples of the integral representing the cumulative backscattering cross section of raindrops, plotted as a function of the maximum radius* r *in the distribution shown in figure 1.*

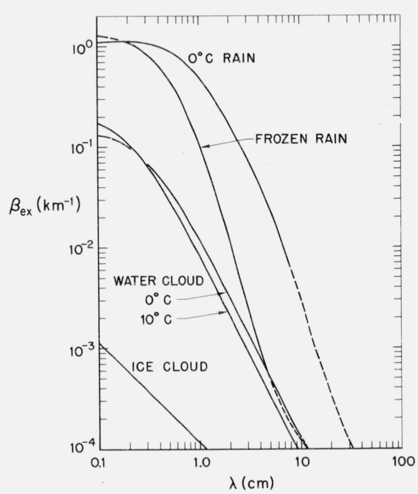

FIGURE 3. *The computed extinction curves for the cloud and precipitation models, and for liquid and ice spheres as indicated.*

TABLE 1. *Albedo* A *of single scattering for polydisperse precipitation*

λ	0 °C Liquid rain	10 °C Rain	0 °C Frozen rain
cm			
0.1	0.514	0.520	[a](0.978)
.3	.485	.516	[b](.987)
1.0	.438	.455	.984)
2.0	.294	[b](.310)	.953
5.0	.083	.086	.634

[a] Graphically extrapolated value.
[b] Graphically interpolated value.

The other parameters, integrated according to (3), represent the normalized intensity and polarization functions $P_j(\theta)$, $j=1, 2, 3, 4$ as a function of scattering angle θ ($\theta=180°$ for the radar echo case). The meaning of the first two functions is quite clear: their semisum gives the power singly scattered in the direction θ, per unit incident flux, unit volume, unit solid angle and unit scattering cross section at the given frequency. The remaining two parameters P_3 and P_4 are related to the state of polarization of the scattered power, including ellipticity and rotation of plane of polarization, given the state of polarization of the incident beam. Their exact definition including illustrative examples of their use will be found in the detailed version [Deirmendjian, 1963b, app. C].

All integrations were carried out numerically by means of an EDP computer program, which incorporates the Mie functions, the distribution function, and an increment in x sufficiently small to insure high accuracy in the final results.

A few graphical examples of these parameters will suffice to illustrate our results. Figures 4 and 5 are linear plots of all four normalized functions against the scattering angle θ at the wavelength of 2 cm. Figure 4 is for raindrops and figure 5 for ice spheres, both according to the distribution given by the solid curve of figure 1. The effect of change of phase (or index of refraction) is quite clear: for the liquid drops the forward and the backscattering intensities are about equal, whereas for ice spheres, the forward intensity up to about 30° is some three times the backward intensity between 130 and 180°. Using these parameters in combination with A and β_{ex}, it is seen that an unattenuated radar echo at this wavelength from an isolated liquid-rain element would be eight times more intense than one from a frozen-rain element. Although this result does not apply to ice crystals or snow flakes but only to solid ice spheres, it is compatible with the so-called bright band near the 0 °C isotherm.

Figures 6 and 7 show the equivalent situation for λ 2 mm radiation. Only the first two intensity elements, plotted logarithmically against θ, are shown. The forward scattering lobe is very pronounced in both the solid and liquid cases. As for a hypothetical radar echo at this wavelength, the intensity from ice spheres would be eight times that from liquid rain; that is, the opposite of the situation for λ 2 cm is observed.

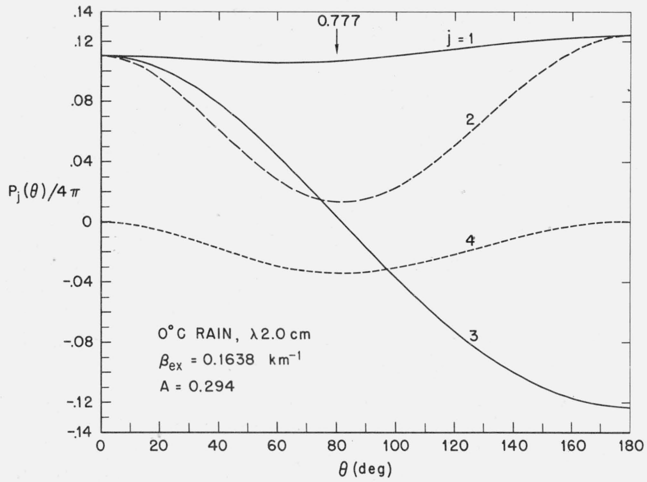

FIGURE 4. *The four normalized intensity functions* $P_j(\theta)/4\pi$ *for* λ *2.0 cm radiation scattered differentially by a volume of liquid raindrops.*

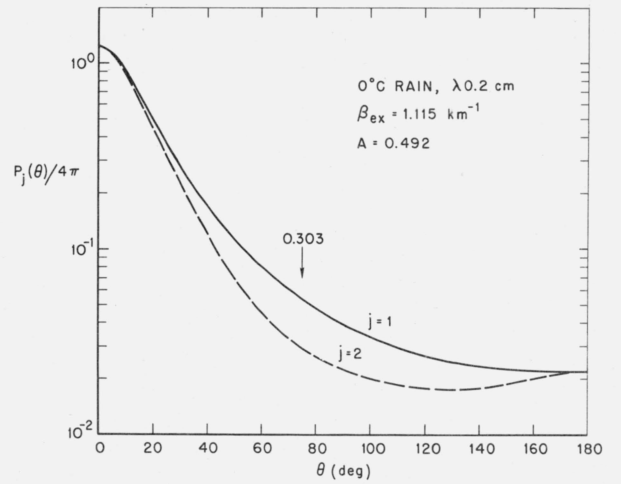

FIGURE 7. *The two normalized intensity elements* $P_1/4\pi$ *and* $P_2/4\pi$ *for* λ *0.2 cm radiation scattered by ice spheres.*

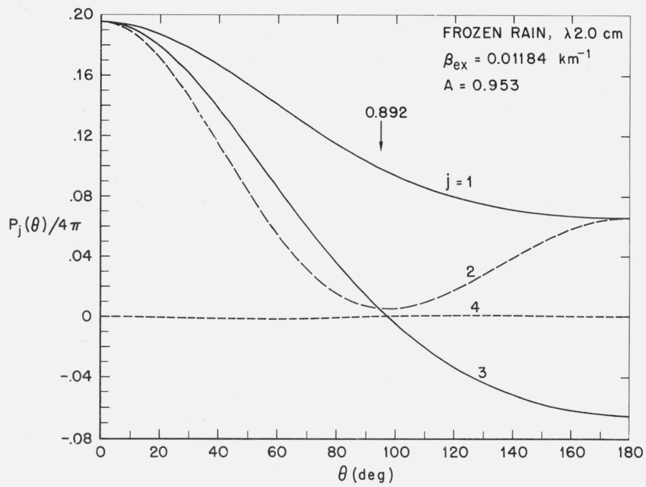

FIGURE 5. *The four normalized intensity functions* $P_j(\theta)/4\pi$ *for* λ *2.0 cm radiation scattered differentially by a volume of frozen raindrops (ice spheres).*

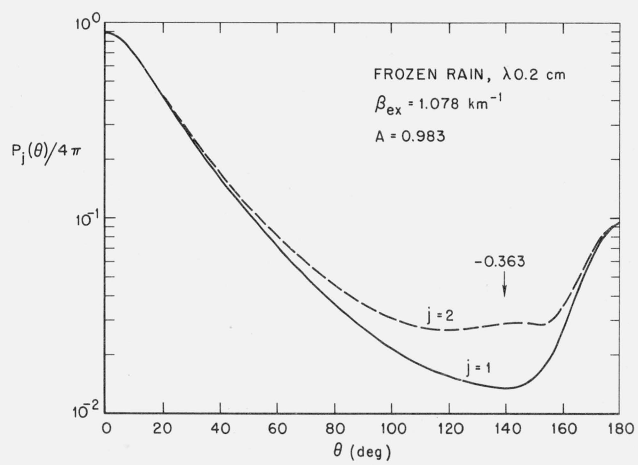

FIGURE 6. *The two normalized intensity elements* $P_1/4\pi$ *and* $P_2/4\pi$ *for* λ *0.2 cm radiation scattered by liquid raindrops.*

The scattering patterns at other intermediate wavelengths change gradually between the two shown above. The forward scattering is further enhanced at λ 1 mm, so that for rain, the ratio between the exact forward and backward directions is of the order of 300.

4. Discussion

In general our results as to the backscattering and attenuation properties of cloud and precipitation elements in the microwave region agree with previous theoretical estimates. However, in many cases they improve the accuracy, reveal new features and suggest new experimental techniques. Some of the main inferences may be listed as follows:

Liquid-water content of clouds. The attenuation is directly proportional to the water content and the radar echo to the square of the water content, *independently of the shape of the size distribution function.* This is a consequence of the validity of the Rayleigh approximation. Shorter wavelengths of course are to be preferred in cloud probing.

Liquid-water content of precipitation. This is not simply related to either the attenuation or the scattering power in any particular direction (except possibly near $\theta=45°$ provided the distribution function is known). Both of these parameters strongly depend on the shape of the size-distribution function and not merely on its integral with respect to the volume of the drops. This is a consequence of the inapplicability of the Rayleigh approximation in this case to the larger drops in the spectrum, even at say λ 8 cm. Our exact integration of the Mie expressions and comparison with the approximation clearly reveal this. It is not suprising, therefore, that the constants in the usual *empirical* relation between rainfall rate, water content and radar reflectivity vary widely depending on the investigator and local conditions. From the cloud-physics experimental point of view (and not that of operational weather radar) it appears that by means of a multichannel system bracketing the millimeter and centimeter region and capable of measuring attenuation, backscatter *and* scattering at some smaller angles, preferably less than 90°, it should be possible to fix the drop-size distribution with some degree of confidence. From the latter, the liquid-water content can then be estimated, if indeed this is a significant parameter. In our opinion, it is more important to investigate the *changes in the shape of the distribution function* within, near the base of, and below a precipitating cloud. Whether this can be accomplished more efficiently by actual sampling (without disturbing the medium) is another question.

Albedo of single scattering. This quantity, when properly integrated over a size distribution, is quite important in estimating the microwave emissivity of precipitating clouds. The integrated emissivity (or absorptivity) per unit volume of rainy air is given by the difference 1−A, and table 1 clearly shows that the radiation properties of precipitation at millimeter wavelengths is quite far from those of a perfect black or gray body. This is an important consideration in estimating the sky backgrounds or noise at these wavelengths in radio astronomy [Hogg and Semplak, 1961] as well as the terrestrial and atmospheric emission as viewed, say, from a satellite.

Polarization. The resultant polarizing properties of a raindrop distribution at various scattering angles in the microwave region have been computed for the first time. Except for their use in a complete solution of the planetary problem (see below), these quantities seem of academic interest. However, they may be useful in the proper choice of a microwave transmitting system for experimental or operational purposes.

Planetary problem. The kind of normalized scattering parameters evaluated in this work are essential in the interpretation of the microwave brightness spectra of planets covered with deep atmospheres. An eminent example is the microwave continuum of Venus. As indicated elsewhere [Deirmendjian, 1964], if the invisible planetary surface emits as a 600 °K blackbody (which seems to be the case), and if the visible atmosphere contains large particles (as is also most likely), we have a problem of *diffuse* transmission of microwave energy through an atmosphere with differential absorption and scattering properties, depending on the wavelength. If the large particles are further assumed to be H_2O condensation products, our results are precisely those needed to construct and investigate models with various amounts of cloud and precipitation-size particles, in order to deduce the surface pressure, structure and composition of a Venusian atmosphere that best fits the data.

5. References

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