RADIO SCIENCE Journal of Research NBS/USNC-URSI
Vol. 69D, No. 6, June 1965

Propagation of Waves Across a Magnetoplasma-Vacuum Boundary [1]

Robert L. Gallawa

Central Radio Propagation Laboratory, National Bureau of Standards, Boulder, Colo.

(Received January 13, 1965; revised January 23, 1965)

The coupling of electroacoustic and electromagnetic energy at a magnetoplasma-vacuum boundary is studied via the plane wave solution of Maxwell's equations. The effect of the static magnetic field is illustrated graphically. It is found that the coupling depends significantly on the angles, increased coupling being in favor of the grazing angles of transmission. The electron density of the plasma is seen to be an important parameter in most cases.

1. Introduction

In recent years there has been a considerable amount of energy expended in an effort to understand more completely the coupling mechanism which governs the conversion of electromagnetic energy at a plasma density discontinuity. In this general problem is included the interaction of electromagnetic energy with the plasma and a completely general account of the mechanism is very difficult. In order to reduce the problem to a tractable form, simplifying (nonphysical) assumptions are made which necessarily restrict the domain of application. Thus, for example, the linearized form of the hydrodynamic equations are used [Oster, 1960]; this restricts the amplitudes to small (perturbation) values.

The problem of energy conversion due to a boundary has received some attention by considering the reflection from and transmission into an isotropic plasma half-space [Hessel et al, 1962; Kritz and Mintzer, 1960; and Wait, 1964a], and more recently by Wait [1964b] for a magnetoplasma half-space. In these cases the problem has been one of characterizing the wave in a plasma half-space due to a wave incident from a vacuum. In recent years there has been an effort to understand the reciprocal problem, that of determining the consequences of a wave incident from the plasma onto a plasma-vacuum boundary [Field, 1956; Johler, 1964].

2. Formulation of the Equations

In this study a plane interface is postulated which separates a half-space of plasma and a half-space of vacuum. The plasma is taken to be a one-component, uniform, electron fluid; i.e., heavy ionic motion is neglected. However, collisions between electrons and neutral particles are accounted for by a constant collision frequency, ν. In addition, the finite compressibility is considered. The plasma is anisotropic by virtue of a steady magnetic field which permeates the region. Effort will be directed towards a description of the conversion factor which characterizes the (transverse) electromagnetic field existing in the vacuum due to a longitudinal (electroacoustic) wave from the plasma onto the interface. The wave magnitudes are assumed to be very small,

[1] This work was supported by Rome Air Development Center under Contract No. AF30(602)–2488.

permitting the use of linearized hydrodynamic equations [Oster, 1960] which are given by

$$mn_0\left(\nu+\frac{\partial}{\partial t}\right)\overline{V}=n_0e(\overline{E}+\overline{V}\times\overline{B})-\nabla p, \tag{1}$$

$$u^2mn_0\nabla\cdot\overline{V}=-\frac{\partial p}{\partial t}, \tag{2}$$

$$\nabla\times\overline{E}=-\mu_0\frac{\partial\overline{H}}{\partial t}, \tag{3}$$

$$\nabla\times\overline{H}=\epsilon_0\frac{\partial\overline{E}}{\partial t}+n_0\epsilon\overline{V}, \tag{4}$$

where e and m are the charge and mass of the electron, respectively; μ_0 and ϵ_0 are the permeability and dielectric constant of free space, respectively; u is the speed of sound in the electron gas, $u^2\cong\frac{KT}{m}$, $K=$Boltzmann's constant, $T=$absolute temperature; p is the deviation of the electron pressure from the mean and $\overline{V}$ is the mean electron velocity; n_0 is the constant electron number density. $\overline{E}$ and $\overline{H}$ are, as usual, the electric and magnetic fields. $\overline{B}$ is the steady magnetic induction. Without loss of generality, a (surpressed) time factor exp $(i\omega t)$ is assumed.

3. Homogeneous, Infinite Plasma

In order to understand the propagation mechanism, it is appropriate to seek a plane-wave solution which is proportional to exp $(-ik\overline{n}\cdot\overline{r})$, where $\overline{n}$ is the unit vector in the direction of propagation and r is the vector to the variable point. Under these conditions,

$$\nabla=-ik\overline{n}. \tag{5}$$

Unit vectors $\overline{a}$ and $\overline{t}$ are also introduced such that $\overline{n}$, $\overline{a}$, and $\overline{t}$ form a right-handed triad and the basis of a rectangular coordinate system. Attention will be restricted to the case where $\overline{B}=\overline{a}B_0$. Then for the assumed time variation

$$\begin{aligned}&\overline{n}E_n\left[-k^2+k^2\frac{u^2}{v^2}\right]+[\overline{n}E_n+\overline{a}E_a+\overline{t}E_t][k^2-\kappa^2]\\&+\overline{t}E_n\left[\frac{i\omega\omega_c}{v^2}+\frac{\omega_ck^2}{\nu+i\omega}\right]-\overline{n}E_t\left[\frac{i\omega\omega_c}{v^2}+\frac{\omega_ck^2}{\nu+i\omega}\right]-\overline{t}E_n\frac{\omega_ck^2}{\nu+i\omega}=0,\end{aligned} \tag{6}$$

or

$$E_n\left[k^2\frac{u^2}{v^2}-\kappa^2\right]-E_t\left[\frac{i\omega\omega_c}{v^2}+\frac{\omega_ck^2}{\nu+i\omega}\right]=0, \tag{7a}$$

$$E_a[k^2-\kappa^2]=0, \tag{7b}$$

$$E_n\left[\frac{-i\omega\omega_c}{v^2}\right]-E_t[k^2-\kappa^2]=0, \tag{7c}$$

where

$$\kappa^2=\omega^2\mu_0\epsilon_0-i\frac{\omega\mu_0\epsilon_0\omega_p^2}{\nu+i\omega}, \tag{8}$$

$$v^2=\frac{\nu+i\omega}{i\omega\mu_0\epsilon_0}, \tag{9}$$

$$\omega_p^2=\frac{n_0e^2}{m\epsilon_0}, \tag{10}$$

$$\omega_c=-eB_0/m, \tag{11}$$

and

$$\overline{E}=\overline{n}E_n+\overline{a}E_a+\overline{t}E_t. \tag{12}$$

It is clear from (7) that E_a is not coupled to the remaining components and has a propagation constant

$$k^2=\kappa^2.$$

The field components E_n and E_t are coupled, however, by virtue of the static magnetic field. The modes which propagate are then quasi-longitudinal and quasi-transverse. This description indicates the fact that neither a purely longitudinal wave nor a purely transverse wave can be supported in this medium. If the static magnetic field vanishes, it is readily seen that the medium can support three waves, each propagating independently. The wave which is polarized in the direction of propagation (the longitudinal wave) has propagation constant

$$k^2=\kappa^2\frac{v^2}{u^2}.$$

The transverse waves, under these conditions, each has propagation constant

$$k^2=\kappa^2.$$

It is interesting to note that there is no thermal correction for the wave number, k, for the mode which is polarized in the direction of the magnetostatic field. However, there is a thermal correction for the wave numbers associated with the other two waves by virtue of the term u^2/v^2 which appears in (7a). If the magnetostatic field vanishes, the thermal correction for the component E_t vanishes and the index of refraction for the transverse waves is the same as it would be for a cold (incompressible) plasma.

4. Case of Zero Static Magnetic Field

In order to develop concepts which will be useful in later work, consideration is given to the case of vanishing static magnetic field. In that case, (1) is modified to the extent that $\overline{B}$ is set equal to zero. Following earlier workers [Hessel and Shmoys, 1961; Hessel, Marcuvitz, and Shmoys, 1962; Chen, 1964; Field, 1956; Yildiz and Silberg, 1964], it is then convenient to split $\overline{E}$ into longitudinal and transverse components. Let

$$\overline{E}=\overline{E}_L+\overline{E}_T,$$

where

$$\nabla\times\overline{E}_L=0, \tag{13a}$$

$$\nabla\cdot\overline{E}_T=0. \tag{13b}$$

Then by (2), (4), and (13b),

$$\nabla \cdot \overline{E}_L = \frac{e}{u^2 \epsilon_0 m} p. \tag{14a}$$

In addition,

$$\nabla \times \overline{E}_T = -i\omega\mu_0 \overline{H}. \tag{14b}$$

From these equations, it is evident that $\overline{E}_L$ is in the direction of propagation (and hence is longitudinal) while $\overline{E}_T$ is entirely transverse to the direction of propagation. Equations (13) and (14) completely specify $\overline{E}_L$ and $\overline{E}_T$. The equation satisfied by $\overline{E}$ is found in the usual manner by taking the curl of (3):

$$-\nabla^2 \overline{E}_T - k_0^2 \overline{E}_T - k_0^2 \overline{E}_L + i\omega\mu_0 n_0 e \overline{V} = 0, \tag{15}$$

where $k_0^2 = \omega^2 \mu_0 \epsilon_0$. The velocity $\overline{V}$ may also be broken into longitudinal and transverse components:

$$\overline{V} = \overline{V}_L + \overline{V}_T.$$

Then from (1)

$$mn_0(\nu + i\omega)\overline{V}_L + mn_0(\nu + i\omega)\overline{V}_T = n_0 e \overline{E}_L + n_0 e \overline{E}_T - \nabla p. \tag{16}$$

It is convenient and congruous to let

$$mn_0(\nu + i\omega)\overline{V}_L = n_0 e \overline{E}_L - \nabla p, \tag{17a}$$

$$mn_0(\nu + i\omega)\overline{V}_T = n_0 e \overline{E}_T. \tag{17b}$$

Then (15) becomes

$$[-\nabla^2 - \kappa^2]\, \overline{E}_T = \left[k_0^2 - \frac{\omega_p^2}{v^2}\right] \overline{E}_L + \frac{i\omega\mu_0 e}{m(\nu + i\omega)} \nabla p. \tag{18}$$

Taking the curl of (18) yields

$$[-\nabla^2 - \kappa^2]\, \nabla \times \overline{E}_T = 0.$$

Since the curl of $\overline{E}_T$ is nonzero unless $\overline{E}_T$ is zero, this demands that

$$\nabla^2 \overline{E}_T + \frac{\omega^2}{c^2}\left[1 - \frac{i\omega_p^2}{\omega(\nu + i\omega)}\right] \overline{E}_T = 0. \tag{19}$$

Taking the divergence of (18) yields

$$\nabla^2 p + \frac{\omega^2}{u^2}\left[1 - \frac{\omega_p^2}{\omega^2} - i\frac{\nu}{\omega}\right] p = 0. \tag{20}$$

Equations (19) and (20) agree with the equations of Wait [1964a]. They show explicitly the propagation constants associated with the transverse electromagnetic wave and the pressure wave.

Furthermore, the pressure wave has an associated longitudinal electromagnetic wave by virtue of (14a). The two types of waves travel independently and with propagation constants

$$k_T^2 = \frac{\omega^2}{c^2}\left[1 - i\,\frac{\omega_p^2}{\omega(\nu + i\omega)}\right] = \kappa^2, \tag{21a}$$

$$k_L^2 = \frac{\omega^2}{u^2}\left[1 - \frac{\omega_p^2}{\omega^2} - i\,\frac{\nu}{\omega}\right] = \kappa^2\,\frac{c^2}{u^2}\,(1 - i\,\frac{\nu}{\omega}) = \kappa^2\,\frac{v^2}{u^2}. \tag{21b}$$

It is interesting to note that in the absence of collisions, the ratio of the two propagation constants is just the ratio c/u.

5. Finite Static Magnetic Field

The set of equations (7a) and (7b) leads to solutions for k by equating the determinant of the coefficients to zero. This leads to

$$k^2 = \frac{\kappa^2 + \kappa^2\,\dfrac{u^2}{v^2} - \dfrac{i\omega\omega_c^2}{v^2(\nu + i\omega)}}{2\,\dfrac{u^2}{v^2}}$$

$$\pm \frac{\sqrt{\left(-\kappa^2 - \kappa^2\,\dfrac{u^2}{v^2} + i\,\dfrac{\omega\omega_c^2}{v^2(\nu + i\omega)}\right)^2 - 4\,\dfrac{u^2}{v^2}\left(\kappa^4 - \dfrac{\omega^2\omega_c^2}{v^4}\right)}}{2\,\dfrac{u^2}{v^2}}. \tag{22}$$

It can readily be shown that in the limit of zero static magnetic field, the upper sign leads to

$$k^2 = \kappa^2\,\frac{v^2}{u^2}, \tag{23a}$$

and the lower sign yields

$$k^2 = \kappa^2. \tag{23b}$$

Comparison of (23) with (21) indicates that the upper sign should be associated with the quasi-longitudinal wave (k_L) and the lower sign with the quasi-transverse wave (k_T).

6. Coupling at a Plasma-Vacuum Boundary

The mechanism of coupling at a boundary can be studied by examining the mechanism of reflection and transmission at a plasma-vacuum boundary. Attention is restricted to the case of coupling from an electroacoustic wave. In order to do this, a quasi-longitudinal plane wave is assumed to be incident on the boundary from the plasma (see fig. 1). Such a wave could originate in the solar corona as discussed by Field [1956].

The magnetostatic field is taken to be in the y-direction, as discussed.[2] It is supposed that the incident wave is given by

$$\bar{E}_{Li} = E_{0L}\,[\bar{a}_x \cos\theta + \bar{a}_z \sin\theta + R(-\bar{a}_x \sin\theta + \bar{a}_z \cos\theta)] \cdot \exp\,[-ik_L(x\cos\theta + z\sin\theta)], \tag{24}$$

[2]The treatment here is abbreviated since a closely related problem has already been considered by Wait [1964b].

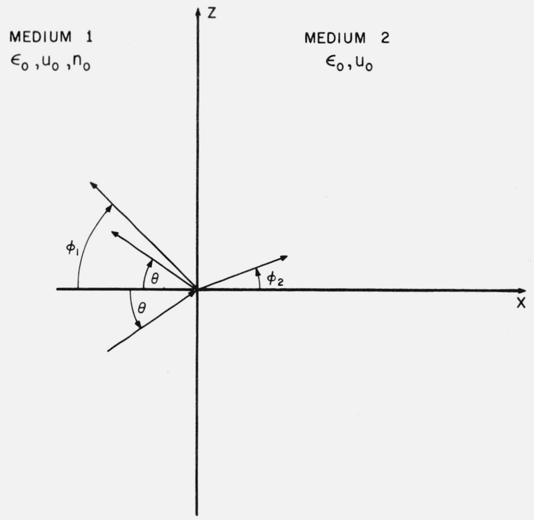

FIGURE 1. *Coordinate system showing the two media and illustrating the angles of incidence, reflection, and transmission.*

where

$$R = \frac{-i\omega\omega_c}{v^2(k_L^2 - \kappa^2)}. \tag{25}$$

The reflected longitudinal wave is

$$\overline{E}_{Lr} = E_{Lr}[-\overline{a}_x \cos\theta + \overline{a}_z \sin\theta + R(-\overline{a}_x \sin\theta - \overline{a}_z \cos\theta)] \cdot \exp[-ik_L(-x\cos\theta + z\sin\theta)]. \tag{26}$$

The reflected transverse wave can be of two types:

$$\overline{E}_{Tr}^{(1)} = E'_{Tr}[-\overline{a}_x \sin\varphi_1 - \overline{a}_z \cos\varphi_1 + T(-\overline{a}_x \cos\varphi_1 + \overline{a}_z \sin\varphi_1)] \cdot \exp[-ik_{T1}(-x\cos\varphi_1 + z\sin\varphi_1)]. \tag{27}$$

$$\overline{E}_{Tr}^{(2)} = \overline{a}_y E''_{Tr} \exp[-ik_{T1}(-x\cos\varphi_1 + z\sin\varphi_1)], \tag{28}$$

where

$$T = \frac{(k_{T1}^2 - \kappa^2)v^2}{-i\omega\omega_c}. \tag{29}$$

The transmitted wave can likewise be of two types:

$$\overline{E}_{TT}^{(1)} = E'_{TT}[\overline{a}_x \sin\varphi_2 - \overline{a}_z \cos\varphi_2] \exp[-ik_{T2}(x\cos\varphi_2 + z\sin\varphi_2)], \tag{30}$$

$$\overline{E}_{TT}^{(2)} = \overline{a}_y E''_{TT} \exp[-ik_{T2}(x\cos\varphi_2 + z\sin\varphi_2)]. \tag{31}$$

The notation used on the angles is as follows: θ is used in association with the longitudinal waves and φ is used in specifying the transverse waves (see fig. 1). The subscript 1 or 2 is used to designate plasma or vacuum, respectively.

In order to determine the magnitudes of the unknown amplitudes, the boundary conditions are applied at the interface. Because the interface is assumed to be rigid, the boundary condition on the electron velocity is that the normal component is continuous (zero) at the interface. This is the condition used by earlier workers in the field [Yildiz and Silberg, 1964; Wait, 1964a, b; Johler, 1964; Field, 1956; Hessel, Marcuvitz, and Shmoys, 1962]. In addition, the usual conditions of continuous tangential electric and magnetic fields are used.

In applying the boundary conditions to determine the unknown constants, it is immediately found that

$$E''_{TT} = E''_{Tr} = 0,$$

which is to be expected from the discussion of equations (7). The equations governing the remaining terms can be written in the compact form

$$\begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \dfrac{E_{Lr}}{E_{0L}} \\ \dfrac{E'_{Tr}}{E_{0L}} \\ \dfrac{E'_{TT}}{E_{0L}} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \tag{32}$$

where

$$a_{11} = \cos\theta + R\sin\theta - \eta_L^2 R\sin\theta,$$

$$a_{12} = \sin\varphi_1 + T\cos\varphi_1 - \eta_{T1}^2\sin\varphi_1,$$

$$a_{21} = \sin\theta - R\cos\theta,$$

$$a_{22} = -\cos\varphi_1 + T\sin\varphi_1,$$

$$a_{23} = \cos\varphi_2,$$

$$a_{31} = -\eta_L R,$$

$$a_{32} = -\eta_{T1},$$

$$a_{33} = -\eta_{T2},$$

$$b_1 = -R\sin\theta + \cos\theta + \eta_L^2 R\sin\theta,$$

$$b_2 = -\sin\theta - R\cos\theta,$$

$$b_3 = -\eta_L R.$$

In these equations, the indexes of refraction, η_L and $\eta_{T\,1,\,2}$, have been introduced.

$$k_L = \frac{\omega}{c}\,\eta_L,$$

$$k_T = \frac{\omega}{c}\,\eta_T.$$

The solution of (32) leads readily to the expression

$$\frac{E'_{TT}}{E_{0L}} = \frac{1}{\Delta}\{2\cos\theta\sin\theta\eta_{T1}(-1-R^2) + 2\eta_L^2\eta_{T1}R^2\sin\theta\cos\theta$$

$$+2\eta_L R^2\cos\theta[\sin\varphi_1 + T\cos\varphi_1 - \eta_{T1}^2\sin\varphi_1] + 2\eta_L R\cos\theta[-\cos\varphi_1 + T\sin\varphi_1]\}, \qquad (33a)$$

where

$$\Delta = [\cos\theta + R\sin\theta - \eta_L^2 R\sin\theta]\,[-\eta_{T2}(-\cos\varphi_1 + T\sin\varphi_1) + \eta_{T1}\cos\varphi_2]$$

$$+[\sin\varphi_1 + T\cos\varphi_1 - \eta_{T1}^2\sin\varphi_1]\,[-\eta_L R\cos\varphi_2 + \eta_{T2}(\sin\theta - R\cos\theta)]. \qquad (33b)$$

In the case of zero static magnetic field, this reduces to

$$\frac{E'_{TT}}{E_{0L}} = \frac{-2\eta_{T1}\cos\theta\sin\theta}{\eta_{T1}[\cos\theta\cos\varphi_2 - \eta_{T1}\sin\theta\sin\varphi_1] + \sin\theta\sin\varphi_1 + \cos\theta\cos\varphi_1}. \qquad (34)$$

In (38), η_{T2} has been set equal to unity.

The conversion factors given by (33) and (34) represent the desired solution to the original problem. It is seen that conversion depends significantly on the angles. In the case of vanishing electron gyrofrequency, there is no radiation into the vacuum for normal incidence. This is not so for finite gyrofrequency, there being radiation into the plasma by virtue of the transverse wave which is generated in the plasma.

An interesting aspect of the above development involves the direction of propagation. Everything so far developed holds equally well for complex angles. In fact, if the angle φ_2 is to be real, the angle of incidence, θ, must be complex. This fact comes out of Snell's law,

$$k_L\sin\theta = k_{T1}\sin\varphi_1 = k_{T2}\sin\varphi_2.$$

Since k_{T2} is real and k_L is complex ($\nu \neq 0$), θ must be complex if φ_2 is to be real. This result is not surprising and in fact may be expected from earlier work. If θ is real, φ_2 is complex. The decision as to how to choose the angles cannot be answered here. Instead consideration should be given to each of the two cases; i.e., θ real (φ_2 complex) or θ complex (φ_2 real). Indeed, both cases can be expected to exist [Caron and Stewart, 1964] and, in fact, both angles may be complex. This is implicit in earlier published work [Wait, 1964b].

If consideration is given to the case where φ_2 is complex, then primary concern should be directed toward the factor which determines the attenuation in the x-direction. That factor will be dependent upon the incident angle as well as the propagation constants. The determining factor is the imaginary part of $\cos\varphi_2$ and the propagation constant in vacuum. By Snell's law,

$$k_{T2}\cos\varphi_2 = \pm k_{T2}\sqrt{1 - \frac{k_L^2}{k_{T2}^2}\sin^2\theta} \qquad (35)$$

the choice of signs being made such that the field decays properly at large distances from the interface. In the present case, the lower sign is taken.

It is interesting to note that in the absence of a static magnetic field and collisions, the term given by (35) is either real or imaginary, depending on the ratio of the plasma frequency to the radian frequency. In that case,

$$\exp(-ixk_{T2}\cos\varphi_2)=\exp\left(-x\frac{\omega}{c}\sqrt{\frac{c^2}{u^2}\left[1-\frac{\omega_p^2}{\omega^2}\right]\sin^2\theta-1}\right). \tag{36}$$

If $\omega_p<\omega$, then except for $\left(\frac{\omega_p}{\omega}\right)^2$ near unity and very small incidence angles, θ, the wave will decay exponentially with x because (c/u) is usually much greater than unity.

There is an additional factor which lends insight to the nature of the conversion. This is the concept of power conversion. It is anticipated that this concept will be examined in a future publication.

7. Discussion

The variation of the electric field conversion factor, which describes the conversion of longitudinal (electroacoustic) field to electromagnetic field, is given in figures 2 through 6. The curves are parametric in electron density and are given as functions of frequency and (real) angle φ_2. The value of $\frac{u}{c}$ is taken to be 10^{-3} throughout (T is of the order of 10^4 deg Kelvin). The ordinate is a dimensionless conversion factor and can be related to perturbation pressure by a constant. Thus, if the incident longitudinal wave has a magnitude of 1 V/m, the associated pressure perturbation is $4.52\times10^{-7}\ |k_L|\ \mathrm{N/m^2}$. The magnitude of k_L varies from about 10^{-3} at the lower frequencies to about 10^{-6} at the higher frequencies for electron densities of about 10 per cm³. At

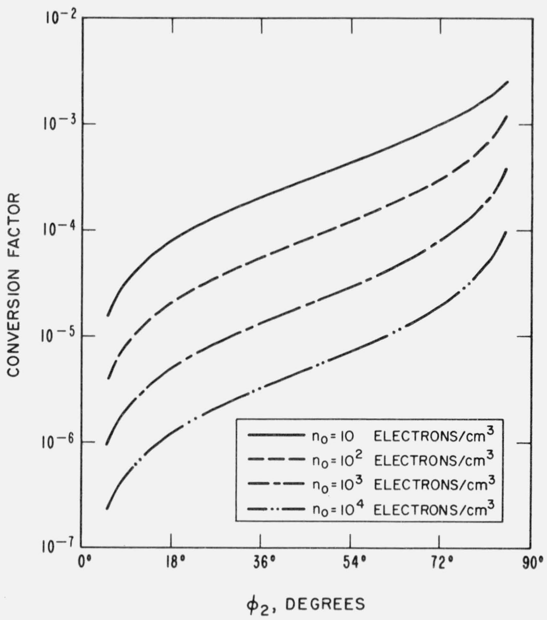

FIGURE 2. *Dimensionless conversion factor showing magnitude of electromagnetic field transmitted into vacuum for incident electroacoustic wave of unity magnitude.*
$B_0=0$; frequency $=1$ kc/s; collision frequency $=10^5$ sec^{-1}.

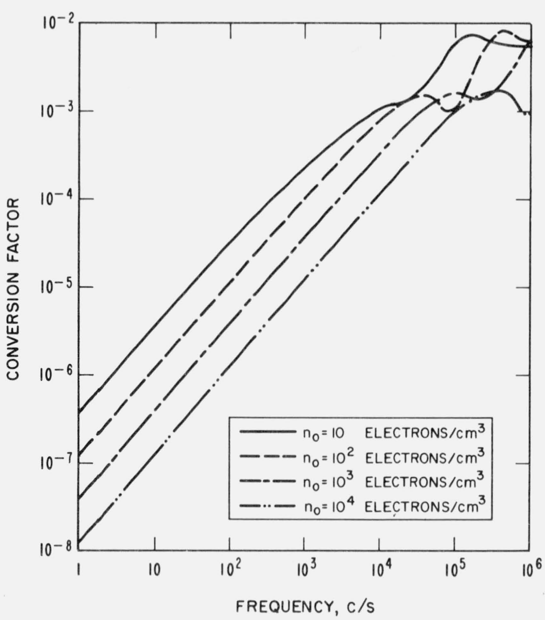

FIGURE 3. *Dimensionless conversion factor showing magnitude of electromagnetic field transmitted into vacuum for incident electroacoustic wave of unity magnitude.*
$B_0=0$; $\varphi_2=80°$; collision frequency $=10^5$ sec^{-1}.

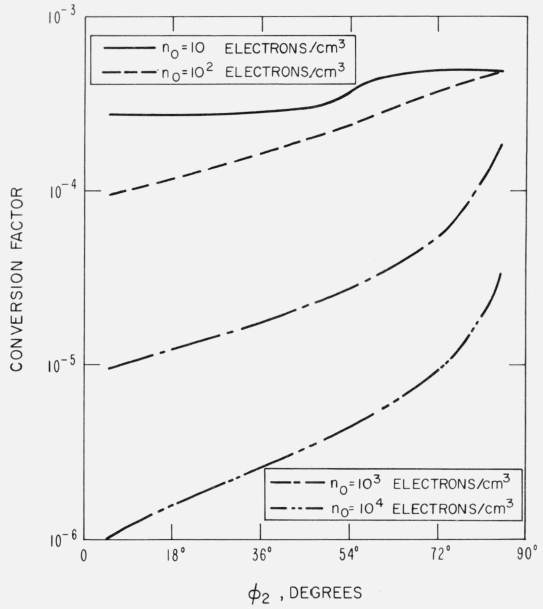

FIGURE 4. *Dimensionless conversion factor showing magnitude of electromagnetic field transmitted into vacuum for incident electroacoustic wave of unity magnitude.* $B_0 = 0.5$ Gauss; frequency = 1 kc/s; collision frequency = 10^5 sec^{-1}.

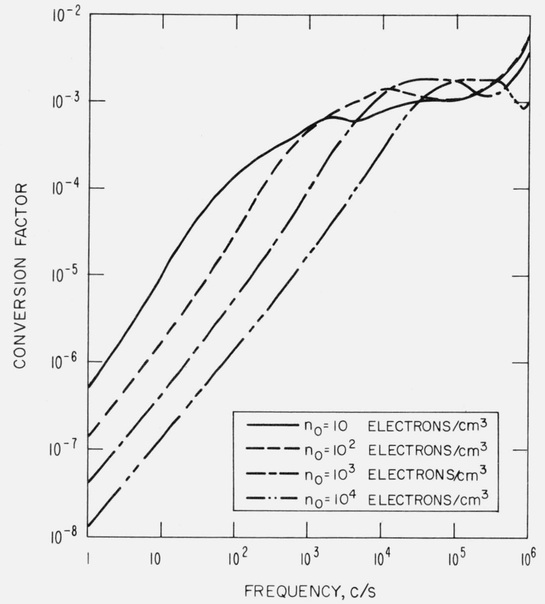

FIGURE 5. *Dimensionless conversion factor showing magnitude of electromagnetic field transmitted into vacuum for incident electroacoustic wave of unity magnitude.* $B_0 = 0.5$ Gauss; $\varphi_2 = 80°$; collision frequency = 10^5 sec^{-1}.

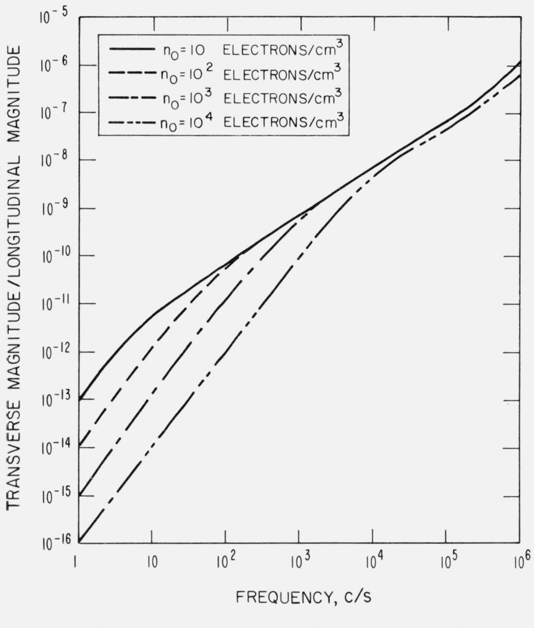

FIGURE 6. *Longitudinal-transverse coupling factor as a function of frequency for various electron densities;* $B_0 = 0.5$ *Gauss;* $\varphi_2 = 80°$; *collision frequency* = 10^5 *sec*$^{-1}$.

1 kc/s, $|k_L|$ has a value of about 10^{-4}. The results are given for the case of zero magnetostatic field (figs. 2–3) and also (figs. 4–5) for the case of a magnetostatic field having a magnitude of 0.5×10^{-4} Wb/m^2 (0.5 Gauss). The latter curves should be supplemented with the curve which shows the magnitude of R, which is the magnitude of the transverse component of the quasi-longitudinal wave present in the plasma, for unity longitudinal component, by virtue of the static magnetic field, figure 6. Thus, some of the electromagnetic energy transmitted into the vacuum is generated by the transverse component incident on the boundary.

The dips which can be seen in the field conversion factors occur approximately at the plasma frequency in each case without the magnetostatic field. The introduction of the magnetic field causes this dip to shift slightly toward a lower frequency, especially for the lower electron densities.

A problem complementary to that treated in section 3 concerns propagation in the direction of the static magnetic field. In that case, equations analogous to (7) can be found which show that there is no thermal correction to the index of refraction for the transverse components of the wave. It is then the longitudinal component which may propagate independently, the transverse components being coupled through the static magnetic field.

8. Conclusions

The E field conversion from an electroacoustic to an electromagnetic wave has been shown to be generally an increasing function of frequency and angle of transmission, up to one Mc/s. There is a slight dip at approximately the plasma frequency and this dip moves down the frequency scale when the magnetic field is introduced. The magnetic field tends to enhance the coupling at the small angles of transmission but detracts at the grazing angles.

I thank my colleague, Leslie A. Berry, and Dr. E. Bahar of the University of Colorado for their constructive criticism and valuable suggestions.

9. References

Caron, P. R., and G. E. Stewart (1964), Radiation from a line source in a ground plane covered by a warm plasma slab, Aerospace Corporation, General Research Report No. ATN–64(9230)–3.

Chen, Kun-Mu (1964), Interaction of a radiating source with a plasma, Proc. IEE (London) **111,** No. 10, 1668–1678.

Field, G. B. (1956), Radiation by plasma oscillations, Astrophys. J. **124,** No. 3, 555–570.

Hessel, A., N. Marcuvitz, and J. Shmoys (1962), Scattering and guided waves at an interface between air and a compressible plasma, IEEE Trans. Ant. Prop. **AP–10,** No. 1, 48–54.

Hessel, A., and J. Shmoys (1961), Excitation of plasma waves by a dipole in a homogeneous isotropic plasma, Polytechnic Institute of Brooklyn, Proc. of the Symposium on Electromagnetics and Fluid Dynamics of Gaseous Plasma, Polytechnic Press, (Interscience Publishers, New York, N.Y.).

Johler, J. R. (1964), Audio and sub-audio frequency EM waves propagated from spherical plasma sources excited by acoustic perturbations (private communication).

Kritz, A. H., and D. Mintzer (1960), Propagation of plane waves across a density discontinuity, Phys. Rev. **117,** No. 2, 382–386.

Oster, L. (1960), Linearized theory of plasma oscillations, Rev. Mod. Phys. **32,** No. 1, 141–168.

Wait, J. R. (1964a), Radiation from sources immersed in a compressible plasma media, Can. J. Phys. **42,** 1760–1780.

Wait, J. R. (1964b), On the theory of reflection of electromagnetic waves from the interface between a compressible magneto-plasma and a dielectric, Radio Sci. J. Res. NBS **68D,** No. 11, 1187–1191.

Yildiz, A., and P. A. Silberg (1964), Natural oscillations of a bounded plasma, Phys. of Fluids **1,** No. 1, 96–102.

10. Additional Related References

Ginzburg, V. L. (1964), Propagation of electromagnetic waves in plasma (Pergamon Press, Publishers, New York).

Seshadri, S. R. (1964), Wave propagation in a compressible ionosphere, Pts. I and II, Radio Sci. J. Res. NBS **68D,** No. 12, 1285–1307.

Yildiz, A. (1963), Scattering of plane plasma waves from a plasma sphere, Il Nuovo Cimento, Serie **X,** 30, 1182–1207.

(Paper 69D6–513)