RADIO SCIENCE Journal of Research NBS/USNC–URSI
Vol. 69D, No. 4, April 1965

Impedance of a Short Dipole in a Compressible Plasma [1]

K. G. Balmain

Department of Electrical Engineering, University of Illinois, Urbana, Ill.

(Received October 13, 1964; revised November 17, 1964)

The field of a cylindrical dipole antenna in a compressible, isotropic, lossy plasma is represented by the field of a cylindrical current sheet immersed in the plasma. The antenna is short compared to a free space wavelength and for this reason a triangular current distribution is assumed. A formula for the input impedance is derived and compared with existing formulas for spherical, cylindrical and planar geometries. In addition, the effect of contact between the plasma and a metal antenna is estimated in the low frequency limit for the case of an antenna biased to the point of ion sheath collapse.

1. Introduction

Electroacoustic oscillations in isotropic, uniform plasmas are characterized by the absence of an oscillating magnetic field. A propagating electroacoustic wave (plasma wave) is longitudinal and can exist at all frequencies above the plasma frequency if the electron gas is compressible (i.e., has a finite temperature). Electroacoustic effects have been studied extensively and a thorough discussion on the subject has been presented by Cohen [1961, 1962], whose work includes a calculation of the radiation resistance of a filamentary, sinusoidal current distribution; radiation resistance calculations have also been done by Chen [1963]. Hessel and Shmoys [1962] have calculated the field of an infinitesimal electric dipole in an infinite medium, and Hessel, Marcuvitz, and Shmoys [1962] have considered the problem of a magnetic current filament in free space over a compressible plasma half-space. Hall [1963] has obtained the impedance of a parallel plate capacitor and Fejer [1964] has carried out a similar computation for a single spherical electrode. Wait [1964] has studied the field of a slotted sphere and also [1965] the fields of finite and infinite cylindrical dipoles. In addition, Whale [1963], Mlodnosky and Garriott [1962], and Crawford and Mlodnosky [1964] have estimated the electroacoustic effect on impedance when an ion sheath is present.

The small-signal impedance of an antenna in a plasma is determined principally by three phenomena, electroacoustic oscillations, electromagnetic oscillations and contact (or ion sheath) effects. At present it cannot be said that any one of these is negligible. In fact, all three phenomena would be closely coupled in any practical situation. Nevertheless, in the following analysis of a dipole antenna, contact phenomena will be disregarded at first and the antenna fields will be approximated by the fields of a given current distribution in a uniform medium. Under such conditions the electroacoustic and electromagnetic oscillations make separate and distinct contributions to the input impedance.

[1] This research was supported by the National Aeronautics and Space Administration under contracts NsG–395 and NsG–511 and by the Air Force Cambridge Research Laboratories under contract AF 19(628)–3900. Some of the material in this paper was contained in a dissertation submitted in partial fulfillment of the requirements for the Ph.D. degree, University of Illinois, Urbana, Ill., 1963.

The approach outlined above requires a good estimate of the current distribution on the dipole. It is assumed that the dipole is short compared to a free space electromagnetic wavelength and that the electromagnetic oscillations alone determine the current distribution which, consequently, is approximately triangular (zero at the ends of the dipole and maximum at the center). Furthermore, although the antennas considered here are short and thin compared to an electromagnetic wavelength, they are generally very long and moderately thick compared to an electroacoustic wavelength. Therefore, in order to take the thickness into account, the antenna must be represented by a cylindrical sheet of current and not by a current filament.

2. Theoretical Development

In rationalized MKS units and for $e^{j\omega t}$ time variation, Maxwell's equations are

$$\nabla \times \overline{E} = -j\omega\mu_0 \overline{H} \tag{1}$$

$$\nabla \times \overline{H} = j\omega\epsilon_0 \overline{E} - Ne\overline{v} + \overline{J} \tag{2}$$

in which $\overline{J}$ is the source current, $\overline{v}$ the electron velocity (the ions are stationary), e the electron charge magnitude and N the average electron density in the plasma. Conservation of the number of electrons (continuity) is expressed by

$$N\nabla \cdot \overline{v} + j\omega n = 0 \tag{3}$$

in which n is the oscillating part of the electron density. The equation of motion is

$$j\omega N m\overline{v} = -Ne\overline{E} - \nabla p - Nm\nu\overline{v} \tag{4}$$

in which p is the scalar pressure, m the electron mass and ν the collision frequency. The adiabatic equation of state is

$$p = \gamma kTn \tag{5}$$

where k is Boltzmann's constant and T is the electron temperature. The factor γ is the ratio of specific heats and may be set equal to 3 for one-dimensional adiabatic compression [Spitzer, 1962]. Combining the equations of motion and state gives

$$j\omega UNm\overline{v} = -Ne\overline{E} - mV^2\nabla n \tag{6}$$

in which $U = 1 - jZ$, $Z = \dfrac{\nu}{\omega}$, and $V^2 = \dfrac{\gamma kT}{m}$. The quantities $\overline{E}$ and $\overline{v}$ may be eliminated from (2), (3), and (6) to yield the differential equation for n,

$$(\nabla^2 - \alpha^2)n = \frac{Ne}{j\omega\epsilon_0 mV^2}\nabla \cdot \overline{J} \tag{7}$$

in which $\alpha^2 = \dfrac{\omega^2(X-U)}{V^2}$, $X = \dfrac{\omega_N^2}{\omega^2}$ and $\omega_N^2 = \dfrac{Ne^2}{m\epsilon_0}$.

At this point it is convenient to separate $\overline{E}$ and $\overline{v}$ into electromagnetic and plasma (electroacoustic) components, designated by the subscripts e and p, respectively [Cohen, 1961]. Thus (6) can be written as two equations:

$$j\omega UNm\overline{v_e} = -Ne\overline{E_e} \tag{8}$$

$$j\omega UNm\overline{v_p} = -Ne\overline{E}_p - mV^2\nabla n\,. \tag{9}$$

The differential equations now may be separated into two groups. The electromagnetic group is

$$\nabla \times \overline{E}_e = -j\omega\mu_0 \overline{H} \tag{10}$$

$$\nabla \times \overline{H} = j\omega\epsilon_0 K_0 \overline{E}_e + \overline{J} \tag{11}$$

and the electroacoustic group consists of (7) and

$$\overline{v}_p = j\frac{V^2}{\omega U K_0 N}\nabla n \tag{12}$$

$$\overline{E}_p = \frac{mV^2(1-K_0)}{NeK_0}\nabla n \tag{13}$$

in which $K_0 = 1 - XU^{-1}$. Combining (7) and (13) gives the differential equation for the electroacoustic part of the electric field strength:

$$(\nabla^2 - \alpha^2)\overline{E}_p = \frac{1-K_0}{j\omega\epsilon_0 K_0}\nabla\nabla\cdot\overline{J}. \tag{14}$$

In order to evaluate the electroacoustic contribution to the impedance of a dipole it is necessary to select an appropriate current density function $\overline{J}$, find the component of $\overline{E}_p$ parallel to $\overline{J}$ by solving (14) and evaluate the impedance using the Poynting theorem. If $\overline{J}$ is taken to be in the z-direction and if E_z is the z component of $\overline{E}_p$, then (14) becomes

$$(\nabla^2 - \alpha^2)E_z = \frac{1-K_0}{j\omega\epsilon_0 K_0}\frac{\partial^2 J_z}{\partial z^2}. \tag{15}$$

For the case of unit input current, the current density is given by

$$J_z = \frac{\delta(r-\rho)}{2\pi\rho}J(z) \tag{16}$$

where ρ is the dipole radius, r is the cylindrical radial coordinate and $J(z)$ is the triangular function shown in figure 1.

Equation (15) will be solved using the transform pair

$$\tilde{f}(\gamma, k) = \int_{-\infty}^{\infty}\int_0^{\infty} f(r,z)e^{-jkz}J_0(\gamma r)r\,dr\,dz \tag{17}$$

$$f(r,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_0^{\infty}\tilde{f}(\gamma,k)e^{jkz}J_0(\gamma r)\gamma\,d\gamma\,dk. \tag{18}$$

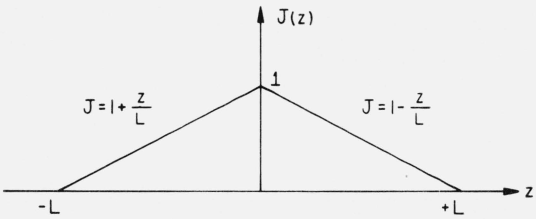

FIGURE 1. *The assumed longitudinal current distribution function.*

The transformed current density is

$$\tilde{J}_z = \frac{-1}{2\pi L k^2}\left(e^{-jkL} + e^{jkL} - 2\right) J_0(\gamma\rho) \tag{19}$$

in which $J_0(\gamma\rho)$ is the Bessel function of the first kind. The electric field can be expressed as

$$E_z = \frac{K_0 - 1}{j\omega\epsilon_0 K_0 L (2\pi)^2} \int_{-\infty}^{\infty}\int_0^{\infty} \frac{e^{-jkL} + e^{jkL} - 2}{k^2 + \gamma^2 + \alpha^2} e^{jkz} J_0(\gamma\rho) J_0(\gamma r)\, \gamma\, d\gamma\, dk. \tag{20}$$

In order to simplify the calculations, the impedance of a monopole of height L over a ground plane will be computed and the dipole impedance will be obtained by doubling the monopole impedance. Thus the field calculations may be limited to the range $0 \leq z \leq L$. Furthermore, for impedance calculations only the field at $r = \rho$ is needed. If, in addition, $\sqrt{\gamma^2 + \alpha^2}$ is taken to have a positive real part then integration of (20) with respect to k gives

$$E_z = \frac{K_0 - 1}{j\omega 4\pi\epsilon_0 K_0 L} \int_0^{\infty} \frac{e^{-(L-z)\sqrt{\gamma^2+\alpha^2}} + e^{-(L+z)\sqrt{\gamma^2+\alpha^2}} - 2e^{-z\sqrt{\gamma^2+\alpha^2}}}{\sqrt{\gamma^2 + \alpha^2}} J_0^2(\gamma\rho)\gamma\, d\gamma. \tag{21}$$

The use of the formula

$$J_0^2(\gamma\rho) = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} J_0(2\gamma\rho\cos\theta)\, d\theta \tag{22}$$

and the assumption that α has a positive real part permits integration with respect to γ (it is a form of "Sommerfeld's integral"):

$$E_z = \frac{K_0 - 1}{j\omega 4\pi\epsilon_0 K_0 L}\left[I(L) + I(-L) - 2I(0)\right] \tag{23}$$

where

$$I(L) = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{e^{-\alpha\sqrt{(z-L)^2 + (2\rho\cos\theta)^2}}}{\sqrt{(z-L)^2 + (2\rho\cos\theta)^2}}\, d\theta.$$

Integration with respect to θ will be delayed in order to simplify subsequent calculations.

For the case of unit input current the dipole impedance due to electroacoustic oscillations is found by integrating the product $(-J_z^* E_z)$ over the volume occupied by the current, a procedure which may be deduced from the Poynting theorem. Thus

$$Z_{in}^p = -2\int_0^L \left(1 - \frac{z}{L}\right) E_z\, dz. \tag{24}$$

Under the assumption that $L^2 \gg \rho^2$, integration with respect to z gives

$$Z_{in}^p = \frac{K_0 - 1}{j\omega\pi\epsilon_0 K_0 L}\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \left[K_0(2\alpha\rho\cos\theta) - 2E_1(\alpha L) + E_1(2\alpha L) + \frac{1}{2\alpha L}\left\{4e^{-\alpha L} - e^{-2\alpha L} - 3e^{-2\alpha\rho\cos\theta}\right\}\right] d\theta. \tag{25}$$

In this formula the modified Bessel function of the second kind is given by

$$K_0(\alpha x) = \int_x^{\infty} \frac{e^{-\alpha u}}{\sqrt{u^2 - x^2}}\, du \tag{26}$$

and the exponential integral is given by

$$E_1(\alpha L) = \int_L^\infty \frac{e^{-\alpha u}}{u}\, du. \tag{27}$$

Integration with respect to θ gives

$$Z_{in}^p = \frac{K_0 - 1}{j\omega\pi\epsilon_0 K_0 L}\Bigg[I_0(\alpha\rho)K_0(\alpha\rho) - 2E_1(\alpha L) + E_1(2\alpha L) + \frac{1}{2\alpha L}\Big\{4e^{-\alpha L} - e^{-2\alpha L} - 3I_0(2\alpha\rho) + 3L_0(2\alpha\rho)\Big\}\Bigg] \tag{28}$$

in which $I_0(x)$ is a modified Bessel function of the first kind and $L_0(x)$ is a modified Struve function. In most cases of interest αL is very large and $\alpha\rho$ is fairly small; under such conditions only the first term in (28) is significant.

A complete impedance formula can be obtained by combining the first term of (28) with the electromagnetic impedance contribution Z_{in}^e. For the case of a very short dipole, Z_{in}^e is predominantly reactive and the expression for it may be deduced readily from the free space expression [see for instance Schelkunoff and Friis, 1952]. Thus the complete formula is

$$Z_{in} = Z_{in}^e + Z_{in}^p = \frac{1}{j\omega\pi\epsilon_0 K_0 L}\left[ln\frac{L}{\rho} - 1 - (1-K_0)I_0(\alpha\rho)K_0(\alpha\rho)\right]. \tag{29}$$

This expression is convenient to use when $\omega < \omega_N$. However, when $\omega > \omega_N$, it is preferable to set $\alpha = j\beta$. Thus (29) becomes

$$Z_{in} = \frac{1}{j\omega\pi\epsilon_0 K_0 L}\left[ln\frac{L}{\rho} - 1 + (1-K_0)\frac{\pi}{2}\Big\{J_0(\beta\rho)N_0(\beta\rho) + jJ_0^2(\beta\rho)\Big\}\right]. \tag{30}$$

For a lossless plasma, Z_{in} has a positive real part which may be associated with the radiation of electroacoustic waves.

Similar formulas have been worked out for a lossless medium by Hall [1963] and Fejer [1964] who used boundary-value techniques to derive impedance formulas for a single sphere and a parallel-plate capacitor, respectively. The following is a summary of the various formulas; note that R is sphere radius and S and D are parallel plate area and spacing, respectively.

$$\text{Sphere:}\ Z_{in} = \frac{1}{j\omega 4\pi\epsilon_0(1-X)R}\left[1 - \frac{X}{1+\alpha R}\right] \tag{31}$$

$$\text{Plates:}\ Z_{in} = \frac{2D}{j\omega\epsilon_0(1-X)S}\left[1 - \frac{X\tanh\alpha D}{\alpha D}\right] \tag{32}$$

$$\text{Dipole:}\ Z_{in} = \frac{1}{j\omega\pi\epsilon_0(1-X)L}\left[ln\frac{L}{\rho} - 1 - XI_0(\alpha\rho)K_0(\alpha\rho)\right]. \tag{33}$$

Of the three expressions only (32) has no positive real part for $\omega > \omega_N$. The similarity of the formulas is most apparent if $\omega < \omega_N$ and if the temperature approaches zero ($\alpha \to \infty$):

$$\text{Sphere:}\ Z_{in} = \frac{1}{j\omega 4\pi\epsilon_0(1-X)R}\left[1 - \frac{X}{\alpha R}\right] \tag{34}$$

$$\text{Plates:}\ Z_{in} = \frac{2D}{j\omega\epsilon_0(1-X)S}\left[1 - \frac{X}{\alpha D}\right] \tag{35}$$

$$\text{Dipole:}\ Z_{in} = \frac{1}{j\omega\pi\epsilon_0(1-X)L}\left[ln\frac{L}{\rho} - 1 - \frac{X}{2\alpha\rho}\right]. \tag{36}$$

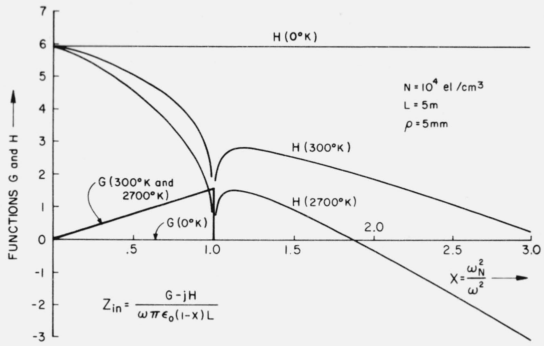

FIGURE 2. *Impedance calculations for* N$=10^4$ *el./cm.*^{3}

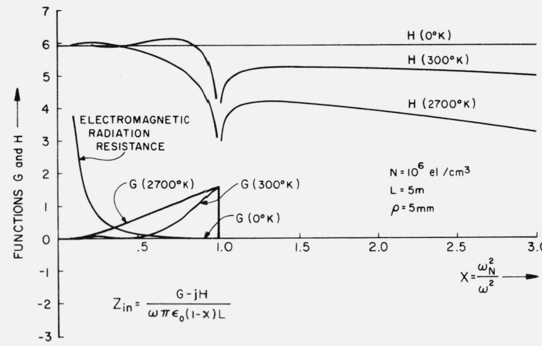

FIGURE 3. *Impedance calculations for* N$=10^6$ *el./cm.*^{3}

3. Discussion of Results

Equation (36) is helpful in estimating the magnitude of electroacoustic effects; for instance, if X is set equal to 2, a value of $(\alpha\rho)^{-1}$ of the order of unity clearly indicates an appreciable effect.

$$(\alpha\rho)^{-1}=\frac{V}{\omega\rho}=\frac{V\lambda}{2\pi c\rho} \qquad \text{at } X=2. \tag{37}$$

Representative parameters in the ionosphere are $T=300$ °K, $\lambda=300$ m and $\rho=1$ cm. Under these conditions, $(\alpha\rho)^{-1}=1.86$ and thus strong electroacoustic effects may be anticipated. In laboratory plasmas [see, for instance, Balmain, 1964] representative parameters are $T=300$ °K, $\lambda=30$ cm, and $\rho=1$ mm. Under these conditions, $(\alpha\rho)^{-1}=0.0186$ and thus electroacoustic effects are negligible according to the theory developed so far.

Computations were carried out for a dipole with dimensions $\rho=5$ mm. and $L=5$ m. and the results are shown in figures 2 and 3. In the computations $\nu=0$ and N and T are representative of the ionosphere. Figure 2 shows a low frequency resonance at $X\approx 2$ (all the curves would indicate a resonance if the X axis were extended far enough). Such a resonance has been noticed by Fejer [1963] and is contained in the formula derived by Hall [1963]. Inspection of (31) to (36) reveals that the resonant frequency depends on the size and shape of the antenna.

The radiation of electromagnetic waves has not been included in the discussion so far but its effect can be estimated by the use of the familiar formula for the radiation resistance of a short dipole [Jordan, 1950, for instance]. Upon insertion of the relative permittivity of a lossless plasma, the radiation resistance formula becomes

$$R=\frac{80\pi^2L^2\sqrt{1-X}}{\lambda^2} \tag{38}$$

in which λ is the free space wavelength. With suitable normalization (38) is plotted on figure 3 where it can be compared directly with the function G. The effect of electromagnetic radiation is appreciable only for small X in figure 3 and is entirely negligible in figure 2.

It is worthwhile to compare the electroacoustic radiation resistance as given by (30) with the resistances calculated by other authors for filamentary source currents. Cohen [1962], Chen [1963] and Wait [1965] all have carried out such calculations, and in all three cases the electroacoustic radiation resistance approaches a finite value as the electron temperature approaches zero. This behavior is entirely the result of having considered a source current of infinitesimal radius rather than one of finite radius. Equation (30) indicates that, for a dipole of finite radius, the entire electroacoustic part of the input impedance approaches zero as the electron temperature approaches zero. For finite temperature and vanishingly small radius, however, the electroacoustic radiation resistance in (30) is identical to that derived by Wait [1965] for a filamentary source current.

The computations shown in figures 2 and 3 indicate that the input reactance is very strongly affected by the compressibility of the plasma. This theoretical result is ample cause for a reexamination of the assumptions made at the beginning of the analysis. The assumption of a triangular current distribution is particularly suspect since it implies that the electroacoustic oscillations do not affect the current distribution. Without electroacoustic effects, the triangular current assumption is justified provided that the antenna is short compared to a wavelength under propagating conditions and short compared to the penetration length under cutoff conditions. However, the strong electroacoustic effect in the impedance suggests that there may be an equally strong electroacoustic effect in the current distribution. Such a conclusion has also been reached by Wait [1965] following his analysis of an infinitely long dipole.

Also open to question is the assumption that the antenna can be represented by a current sheet in a uniform medium. This is the approach used in the "induced EMF" method of impedance calculation, and such an approach to a certain extent neglects the physical presence of the antenna surface; consequently a high order of accuracy cannot be expected in the reactance calculations. Furthermore, the equation of continuity shows that the antenna model is one in which the electrons in the medium are not taken into the metal surface to become part of the antenna current. In this respect the model is similar to the one used by many authors who employ the rigid boundary condition [Fejer, 1964, and Hall, 1963 for instance]. It is evident that a better plasma-metal boundary condition must be found before full confidence can be placed in any impedance calculation.

4. Contact Effects

Any antenna immersed in a plasma is surrounded by an ion sheath and consequently the electron density is highly nonuniform within a few Debye lengths from the surface. If a uniform-medium impedance theory is to be compared with experiment, the sheath must be collapsed by giving the probe a positive d-c bias with respect to some large reference electrode. Although sheath collapse ideally results in a uniform electron density adjacent to the antenna surface, it also introduces an appreciable steady flow of electron current. If the plasma is compressible the application of an RF signal causes a density modulation and consequently an RF current in the stream of electrons flowing to the antenna surface.

The impedance contribution due to this effect may be computed very easily in the low frequency limit. When $\omega=0$ and $\nu=0$, (6) becomes

$$-Ne\bar{E}=mV^2\,\nabla\, n. \tag{39}$$

If $\bar{E}=-\nabla\psi$ where ψ is a scalar potential, integration of (39) between two plates of a capacitor gives

$$Ne(\psi_1-\psi_2)=mV^2(n_1-n_2). \tag{40}$$

For a symmetrical geometry it is possible to set $\psi_2=-\psi_1$ and $n_2=-n_1$ from which

$$Ne\psi_1=mV^2n_1. \tag{41}$$

The oscillating current to an electrode with area A may be expressed as

$$I=An_1e\sqrt{\frac{kT}{2\pi m}}. \tag{42}$$

Thus the input resistance for a cylindrical dipole is given by

$$R_{in}=\frac{2\psi_1}{I}=\frac{2\gamma\sqrt{2\pi mkT}}{ANe^2}=\frac{2\gamma}{\rho LNe^2}\sqrt{\frac{mkT}{2\pi}}, \tag{43}$$

R_{in} would constitute the total input impedance in the limit as frequency approaches zero and at higher frequencies one would expect to find a shunt resistance similar to R_{in}. The influence of the high frequency resistance is difficult to predict but, since $R_{in} \propto N^{-1}$, little contact effect would be expected at low electron densities (say 10^4 el/cm^3).

Under isothermal conditions γ is equal to unity and thus (43) reduces to the resistance discussed by Mlodnosky and Garriott [1962] whose formula is derived by taking the slope of the Langmuir probe voltage-current characteristic. Thus one might expect a transition from adiabatic to isothermal behavior as frequency is lowered; calculations by Pavkovich [1964] suggest that there is such a transition but that it is very gradual as frequency changes from ω_N to zero. It should be noted that the isothermal resistance has been used by Balmain [1964] to account for high apparent losses in laboratory impedance probe experiments. Evidently, contact effects are appreciable even in laboratory plasmas, and furthermore these effects are closely related to electroacoustic oscillations.

5. References

Balmain, K. G. (Sept. 1964), The impedance of a short dipole antenna in a magnetoplasma, IEEE Trans. Ant. Prop. **AP–12,** No. 5, 605–617.

Chen, Kun-Mu (July 1963), Interaction of a radiating source with a plasma—effect of an electroacoustic wave, paper delivered at the 1963 PTGAP International Symposium, Boulder, Colo., Report No. 4563-39–T, Radar Laboratory, University of Michigan.

Cohen, M. H. (Aug. 1, 1961), Radiation in plasmas, Phys. Rev. **123,** No. 3, pt. I, 711–721.

Cohen, M. H. (Apr. 15, 1962), Radiation in plasmas, Phys. Rev. **126,** No. 2, pt. II and III, 389–397, 398–404.

Crawford, F. W., and R. F. Mlodnosky (July 1, 1964), Langmuir probe response to periodic waveforms, J. Geophys. Res. **69,** No. 13, 2765–2773.

Fejer, J. A. (Nov. 1964), The interaction of an antenna with a hot plasma and the theory of resonance probes, Radio Sci., J. Res. NBS **68D,** No. 11, 1171–1176.

Hall, R. B. (Sept. 1963), Parallel plate plasma problem, Am. J. Phys. **31,** No. 9, 696–706.

Hessel, A., N. Marcuvitz, and J. Shmoys (Jan. 1962), Scattering and guided waves at an interface between air and a compressible plasma, IEEE Trans. Ant. Prop. **AP–10,** No. 1, 48–54.

Hessel, A. and J. Shmoys, (1962), Excitation of plasma waves by a dipole in a homogeneous, isotropic plasma, Electromagnetics and fluid dynamics of gaseous plasma, 173–183 (Polytechnic Press, Brooklyn, N.Y.).

Jordan, E. C. (1950), Electromagnetic waves and radiating systems, 311–312 (Prentice-Hall, Inc., Englewood Cliffs, N.J.).

Mlodnosky, R. F., and O. K. Garriott (July 1962), The VLF admittance of a dipole in the lower ionosphere, Proceedings of the International Conference on the Ionosphere, 484–491 (Institute of Physics and Physical Society, London).

Pavkovich, J. M. (Jan. 1964), Numerical calculations related to the RF properties of the plasma sheath, Report No. ARL 64017, Aerospace Research Laboratories, Office of Aerospace Research, U.S. Air Force.

Schelkunoff, S. A., and H. T. Friis (1952), Antennas; theory and practice, ch. 10 (John Wiley and Sons, Inc., New York, N.Y.).

Spitzer, L. (1962), Physics of fully ionized gases, p. 17 (Interscience Publishers Co., New York, N.Y.).

Wait, J. R. (1964), Theory of a slotted-sphere antenna immersed in a compressible plasma, parts I and II, Radio Sci, J. Res. NBS/USNC–URSI **68D,** No. 10, 1127–1136, 1137–1143.

Wait, J. R. (1965), On radiation of electromagnetic and electroacoustic waves in plasma, parts I and II, Appl. Sci. Res. Sec. B, **11** (in press).

Whale, H. A. (Jan. 15, 1963), The excitation of electroacoustic waves by antennas in the ionosphere, J. Geophys. Res. **68,** No. 2, 415–422.

(Paper 69B4–491)