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Self Distortion of Radio Signals in the *D* Region

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A procedure for calculating the self distortion of amplitude modulated radio waves in the *D* region is described. The technique includes calculation of the time dependence of the electron energy as well as the variation of the collision frequency with energy. Experimental values for collision frequency and average fractional energy loss per collision are used. Calculations are performed for a number of examples using two model electron densities. Neither dispersion nor the earth's magnetic field are included in the calculations.

1. Introduction

A number of authors have considered the problem of self distortion of radio beams in the ionosphere. Bailey [1935, 1937] and Bailey and Martyn [1935] first predicted the phenomenon, and Hibberd [1955, 1957] has investigated the possibility of use of the phenomenon for ionospheric experiments. Ginzberg and Gurevich [1960] have analyzed the problem briefly from the engineering point of view and predict substantial demodulation of signals from very powerful transmitters. The general problem in plasmas has been considered by Sodha and Palumbo [1962]. None of the treatments have been completely satisfactory and in fact accurate quantitative results in a form usable for engineering design are unavailable. It is the purpose of this paper to partially fill the need for quantitative data, to investigate to a limited extent the accuracy required of the input parameters to the theory, and to outline the method by which the computational techniques presented here may be elaborated if more extensive results are necessary.

The calculation which it is desired to make here requires, in principle, a combination of the standard propagation equations as treated, for example by Sen and Wyller [1960], with the Boltzmann equation to calculate the energy of the electrons. The resulting equations can be solved only after making severe assumptions.

It is intended here to adopt a philosophy quite separate from one which results in a closed solution and to treat the problem numerically.

Effects caused by dispersion will not be discussed except for a simple calculation of the reflection height. In the *D* region of the atmosphere, dispersive effects are small compared to those which are the principal purpose of this paper. If for some particular problem it should prove necessary to include deviative effects in the computations, this can readily be done to an accuracy comparable to that which is obtained by these calculations. Primary attention, then, will be paid to the nonlinear effects arising from the increase of electron energy (and consequent change in collision frequency) due to the radio frequency field itself.

The basic problem solved is as follows: A plane wave of power $P_0 W/\text{cm}^2$ and frequency f is assumed to be incident at angle ψ to the normal upon the *D* region. The assumed model electron density profile is treated numerically, allowing flexibility in the chosen model. The beam is assumed to be spreading as the inverse square of the path length traversed. The power, including nonlinear effects, is calculated step by step throughout the *D* region, taking into account after each step the absorbed power and the spreading of the beam. The beam is also assumed to be perfectly reflected at some height h_m, a datum which is calculated from the plasma frequency at that height. Effects due to the fact that two rays, (one reflected, one primary) can traverse the same volume of space are not considered.

It is assumed that the power is modulated at a frequency β so that the power as a function of time is given by

$$P_0(t)=P_Z(1-a\cos 2\pi\text{ft})^2. \tag{1}$$

The modulation depth, M, is given by

$$M=\frac{E_{\max}-E_{\min}}{E_{\max}}=\frac{P^{1/2}{}_{\max}-P^{1/2}{}_{\min}}{P^{1/2}{}_{\max}}. \tag{2}$$

Before nonlinear absorption has occurred, then, $M=\frac{2a}{1+a}$. The presence of the earth's magnetic field will be ignored even though for frequencies near the gyrofrequency its influence may become very

strong in the calculation of both the absorption and the dispersion of the extraordinary ray. The power absorbed per unit volume (or unit length if we are considering unit area) can be calculated from

$$\frac{dP}{ds}=R(\mathbf{E}\cdot\underset{\approx}{\sigma}\cdot\mathbf{E}), \tag{3}$$

where $\mathbf{E}$ is the electric field and σ is the conductivity tensor of which R indicates the real part. If we ignore the magnetic field, $\underset{\approx}{\sigma}$ reduces to a scalar, σ, and

$$\frac{dP}{ds}=R(\sigma E^2). \tag{4}$$

The difficulty of applying (3) or (4) lies in the fact that σ is a function of the electric field and, indeed, to a small extent a function of the frequency of the field. [Carleton and Megill, 1962; Phelps, 1963.]

Before making reasonable approximations to allow the solution of (4), it might be advisable to discuss the manner in which, in principle, one can most accurately calculate σ. In the steady state one can calculate real part of σ from Allis [1965] or Fehsenfeld, Megill, and Droppleman [1965]

$$R(\sigma)=-\frac{ne^2}{m}\int_0^\infty \frac{\nu(u)}{\nu(u)^2+\omega^2}\frac{\partial f(u)}{\partial u}u^{3/2}du \tag{5}$$

where $\nu(u)$ is the collision frequency for monoenergetic electrons at energy u and $\omega=2\pi f$. The function $f(u)$ describes the distribution in energy of the electrons. The distribution function is so normalized that

$$\int_0^\infty f(u)u^{1/2}du=1. \tag{6}$$

Note that for the case in which $\nu\gg\omega$ the conductivity is proportional to the average of $\frac{1}{\nu}$ over $\frac{\partial f}{\partial u}u^{3/2}$ while when $\nu\ll\omega$ the conductivity is proportional to a similar average of ν. Fehsenfeld, Megill, and Droppleman [1965] have calculated these two averages for nitrogen using a Maxwell distribution function and compared them to the average collision frequency

$$\nu_{\text{avg}}=\int \nu f u^{1/2}du. \tag{7}$$

They find that over reasonable ranges of energy these three quantities are equal to within 50 percent in all cases (and usually much closer), a fact which will be used later.

The calculations of $f(u)$ from the Boltzman equation for D region conditions in the steady state has been described by Carleton and Megill [1962], but in many cases of interest in this problem a time dependent solution is required. Such solutions are not at present available.

Most of the effects which have just been discussed involve corrections to a more simplified theory of only a few percent. For the purposes of these calculations it is possible to obtain approximate results, accurate to within our knowledge of the D region, using experimentally determined average parameters which include the above effects. It should be possible, by properly designed experiments, to obtain experimentally determined quantities which accurately include the proper averages over the more fundamental atomic parameters. These parameters do not seem to be immediately available in the literature, largely because of the fact that the experiments usually have as an aim the derivation of the atomic collisional cross sections, not conductivities.

2. Computational Procedure

The very detailed calculations as discussed above will, then, not be attempted, and a more simplified analysis will be undertaken with the realization that the results may well be in error by 20 to 50 percent because of the assumptions made. The sensitivity of the results to reasonable changes in the input data will be investigated to determine what parameters are needed to higher accuracy.

Consider electrons in a gas with temperature T. In the absence of an electric field, the average energy of the electrons will be $u_0=3/2\ kT$. In the presence of an electric field, the electrons with an average energy u will gain *an average* amount of energy $\epsilon(u)$ per collision from the electric field and will lose an amount $g(u)\ (u-u_0)$ per collision. The quantity $g(u)$ is the average fractional energy (in excess of thermal) lost per collision. The quantities $\epsilon(u)$ and $g(u)$ are designated as functions of the average energy u of the electron for generality. The gain term $\epsilon(u)$ can be shown to be [Megill and Carleton, 1962], in the absence of a magnetic field,

$$\epsilon(u)=\frac{m}{2}\left(\frac{eE}{m}\right)^2\frac{1}{\omega^2+\nu(u)^2}. \tag{8}$$

The quantity $g(u)$ is an experimentally determinable quantity and may be, as stated before, slightly dependent upon the ratio ν/ω because of the effect on the electron energy distribution function. Any such effect is ignored in these computations. In this analysis, $g(u)$ is maintained in tabular form so that the effect of varying $g(u)$ may be determined. The average energy of the electrons may be calculated from

$$\frac{du}{dt}=\nu[\epsilon(u)-g(u)\ (u-u_0)]. \tag{9}$$

From the investigation of (9), one can determine two facts. The first is that in the steady state the average energy of the electrons, U_s is

$$u_s=\frac{\epsilon(u_s)}{g(u_s)}+u_0=\frac{m}{2g(u_s)}\left(\frac{eE}{m}\right)^2\frac{1}{\omega^2+\nu(u_s)^2}+u_0. \tag{10}$$

The second fact is that in changing the energy a small amount, $\triangle u$, from u_b to $u_b+\triangle u$ the energy varies in time so that

$$u(t)=u_b+\triangle u[1-\exp(-t/\tau(u_b))] \qquad (11)$$

where $\tau(u_b)=\dfrac{1}{g(u_b)\nu(u_b)}$. In this case, $\triangle u$ must be small enough that $g(u)$ and $\nu(u)$ are essentially constant. For a particular height, the electron energy at each moment of time during the passage of modulated wave may be determined by first determining the electron energy u_1 at t_1 with density P_1. The steady state energy due to a power density P_2 is then calculated from (10). If the interval $\triangle u$ is small, the electron energy at time t_2 may then be determined using (11). Since the analysis is being done completely numerically, the calculations are done in sufficient detail that the energy intervals are small enough for the conditions for use of (11) to be met. The above analysis lacks only a starting electron energy. An appropriate energy is easily obtained by using an iterative procedure to calculate the average electron energy u_0 at the beginning of a modulation cycle. It is, therefore, possible to calculate, for each height h, the electron energy as a function of time throughout a period of the modulation frequency. If one knows, in addition, the collision frequency as a function of energy, the absorbed power at each instant per unit path length may be calculated from the equation,

$$\frac{dP(t)}{ds}=R[\sigma(t)E(t)^2]=-\frac{ne^2}{m}\frac{\nu(t)}{\nu(t)^2+\omega^2}E(t)^2$$

$$=-\frac{ne^2}{m}\frac{\nu(t)}{\nu(t)^2+\omega^2}\eta P(t), \qquad (12)$$

where η is the impedance of the medium.

Note that σE^2 is exactly equal to n (the electron density) times the average energy gain per collision times the total collision frequency. This is just the energy flowing from the electric field to the electron gas in unit time. By considering the absorption at each portion of time during a period of the modulation frequency, for each of many regions of the atmosphere thin enough that their power density is substantially constant throughout, it is possible to calculate the absorbed power at every step in the path of the radiation. It is this technique which has been applied in the calculations to be described here.

3. Electron Density Model

The state of knowledge about the nighttime D region is such that it is not easy to choose a model density distribution with any confidence as to its reliability. Two arbitrary profiles have been chosen which are shown in figure 1. These profiles more or less bracket a set of profiles which have been replotted from a figure given by Mechtly [1962]. The calculations, then, show results for a range of electron densities which should include most nighttime conditions met outside of the auroral zones.

4. Collision Frequency

The collision frequency is, as described earlier, not necessarily a unique function of the electron energy for all excitation frequencies or in time varying conditions. Data supplied by Phelps [1964] have been used as shown in figure 2. This figure shows the collision frequency for air (normalized to one molecule per cm^3) which has been used in this study. In addition, the values of $g(u)$ which may be deduced from the data supplied by Phelps are shown. These data correspond to a gas temperature of 232 °K. The curve shown has been used for all gas temperatures which are encountered in the D region. The increase (above the gas temperature) in average energy

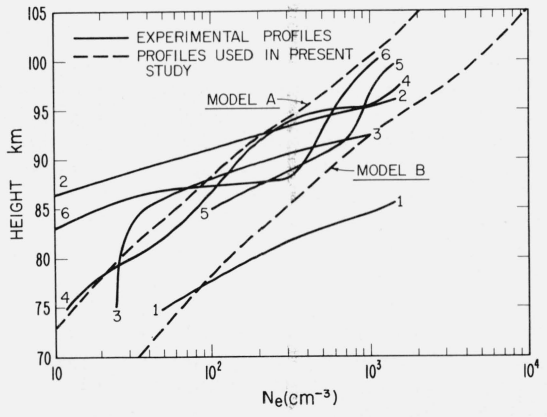


FIGURE 1. *This figure shows a collection of nighttime* D *region electron density profiles as given by Mechtly [1962].*

We show for comparison two models, "A" for a relatively low density case and "B" for a high density case. The curves as shown are due to several authors and are for various times. The references and times are (1) Mechtly [1960] 0100 hours, (2) Mechtly [1962] 0200 hours, (3) Ferraro [1959] (4) Smith [1962] 2200 hours, (5) Parkinson [1955] 2400 hours and (6) Smith [1962] 0400 hours.

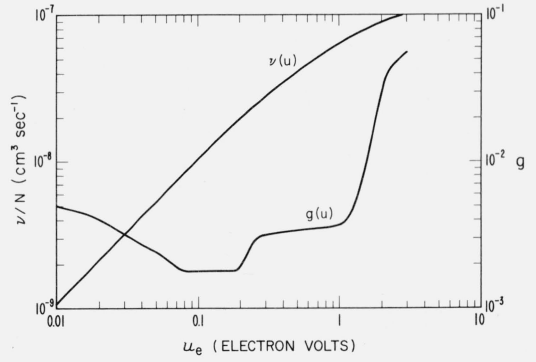


FIGURE 2. *This figure gives the average collision frequency normalized to a neutral particle density of one per* cm^3 *as a function of average electron energy and the average energy loss per collision as a function of electron energy.*

These data are due to Phelps [1964] and are deduced from data obtained in drift tube experiments.

of the electrons to be expected is calculated and referred to the curve of figure 2. Calculations for $g(u)$ as shown in figure 2 and for $g=6\times10^{-3}$ have been made in order to investigate the sensitivity of the theory to this parameter (see figs. 8a and 8b).

5. Results

Two sets of calculations, using the techniques just described, have been made. These two sets correspond to *D* region electron density profiles A and B of figure 1. Profile A involves a low electron density in comparison to the experimental results plotted, while Profile B represents a rather high electron density. Each set of data involves three carrier frequencies of 0.5, 1.0, and 1.5 Mc/s with a modulating frequency of 1000 c/s and three modulating frequencies of 300, 1000, and 3000 c/s per second with a carrier frequency of 1.0 Mc/s. These sets of data are designed to yield information as to the effect of various operating conditions which can be of use in engineering decisions.

In figure 3 we show graphically the result of a set of calculations with a carrier frequency of 1.0 Mc/s and a modulation frequency of 300 c/s at an effective radiated power of 10^8 W and *D*-region model A. The solid curve shows the envelope of the carrier beam before distortion occurs. The dashed curve shows the envelope after distortion occurs. The ordinate is power in arbitrary units. The two curves are normalized at their minima. In this figure one can clearly see an effective "phase shift" of the modulating frequency as well as a decrease in the modulation depth. In these calculations the square root of the ordinate of the dashed curve (to obtain the voltage versus time rather than power versus time) is further analyzed to determine the content of second harmonic, the phase shift of the first harmonic and the percent demodulation.

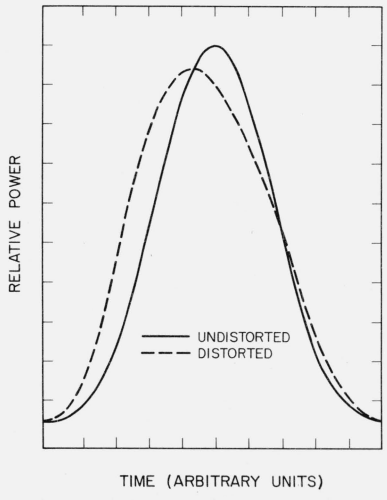


FIGURE 3. *A plot of the modulating wave form before and after nonlinear absorption.*

The solid curve gives the undisturbed wave form, the curve the distorted wave form. The ordinate is power in arbitrary units and the abscissa is time in arbitrary units. The two curves are normalized at the *minima* of the curves. The details are given in the text.

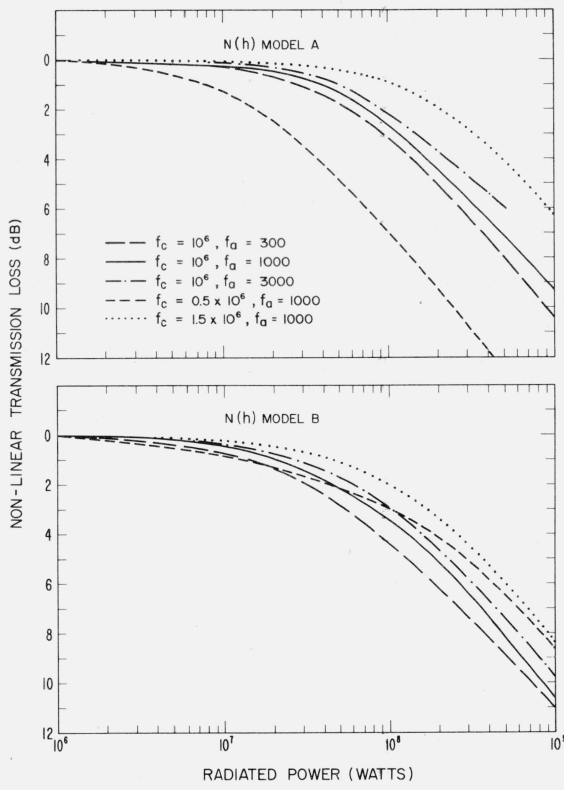


FIGURE 4. *Nonlinear transmission loss as a function of radiated power for various operating conditions.*

The data are calculated for two model atmospheres. All absorption is normalized to that calculated for a low energy wave. The abscissa is power from an isotropic radiator in watts, f_c is the frequency of the carrier wave, f_a is the frequency of the modulating signal.

In figure 4 we show the nonlinear transmission loss in decibels of the radio beams for the various cases. The ordinate is determined by normalizing the absorption to that calculated without taking nonlinear effects into account. In practice, this value is obtained by making the calculations under conditions in which the power is so low that no nonlinear effects are observed. The abscissa is the radiated power in watts, assuming an isotropic radiator. As an example of the use of this plot, suppose that we have an antenna with a gain of 10 over an isotropic radiator and a radiated power of 1 MW. This will be an effective radiated power of 10^7 W. In the case of model profile "A" and a 0.5 Mc/s transmitted frequency the nonlinear absorption is about 1.5 dB; i.e., the received power level will be 1.5 dB below the power which would be calculated on the basis of the absorption of a low power beam. If, on the other hand, one were to increase the power density by a factor of 10 either by increasing the antenna gain or by increasing the transmitted power, the received signal would be 7 dB below that calculated from the low power absorption.

In investigating the data as displayed in figure 4, most of the trends are more or less as one would intuitively expect as the carrier frequency or the electron density profile is changed. There is one case, however, which requires some special explana-

tion. In this case the lowest carrier frequency 0.5 Mc/s demonstrates *less* nonlinear absorption for electron density profile "B" at high powers than does the 1.0 Mc/s carrier. This does not occur in the case of profile "A" so that some explanation is required. First, recall that all these curves are normalized to the absorption at low power. The effects shown are the nonlinear ones due to the heating of the electrons by the beam itself, and the consequent change in the electron collision frequency. In the case in question, much of the absorption takes place at relatively low altitudes where the collision frequency is larger than the angular frequency of the radio wave. In this case, then, absorption as given in figure 3 varies as $1/\nu$ so that the effect of the electron heating is to reduce the absorption. In fact, in a constant density case as might occur in laboratory plasmas, this effect can cause an increase in transmission to occur. In the present problem the effects at high altitudes still overweigh those at lower altitudes so that the net effect is toward some increase in absorption with power level, for this frequency.

In order to calculate demodulation effects, the initial modulation was assumed to be 80 percent in all cases presented here. The percent of demodulation ($100 \times (M_{in} - M_{out}/M_{in}$, where M_{in} and M_{out} represent the modulation before and after absorption) as shown in figure 5 can be either positive or negative, depending upon the frequencies involved and where the principal absorption takes place. It seems nearly impossible to make general statements concerning this effect except to point out that the range can run from as much as 16 percent demodulation; i.e., the out coming wave is about 67 percent modulated rather than the original 80 percent, to a negative demodulation of about 8 percent. Again it may be said that these effects are more extreme in a constant density situation than in the atmosphere because there are competing effects entering at different altitudes.

Since the modulating wave is distorted, one parameter by which it can be characterized is the phase shift of the first harmonic. The phase shift data as calculated are presented in figure 6. A second parameter of some interest is the amount of power deposited in the second harmonic of the modulating frequency. These results are shown in figure 7. The experience gained in calculating these results leads one to think that these two parameters might be good ones to measure in laboratory studies of nonlinear absorption. It is not apparent, however, that unique answers could be obtained from measurement of such parameters in atmospheric experiments without the availability of additional evidence from other sources.

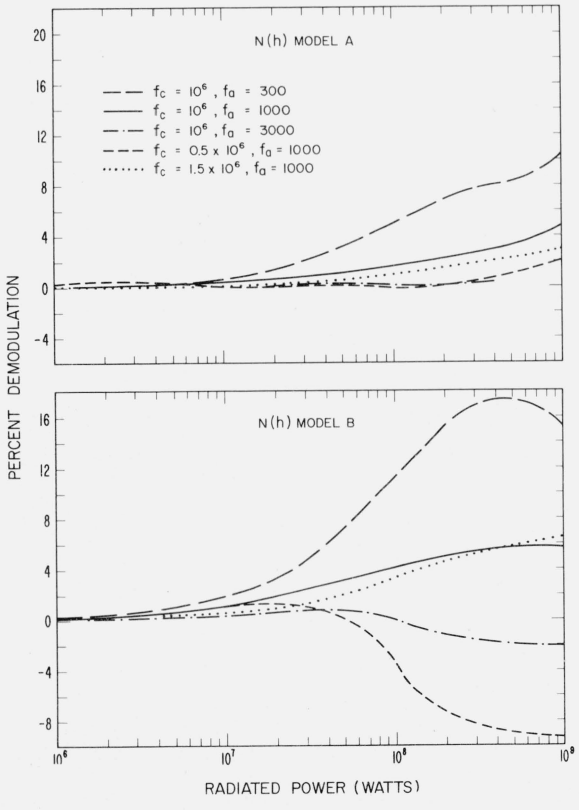


FIGURE 5. *A plot of percent demodulation versus isotropic radiated power for various operating conditions and two models of electron density.*

An initial modulation of 80 percent was assumed in all cases. The abscissa, f_c, and f_a are as defined in the caption for figure 4.

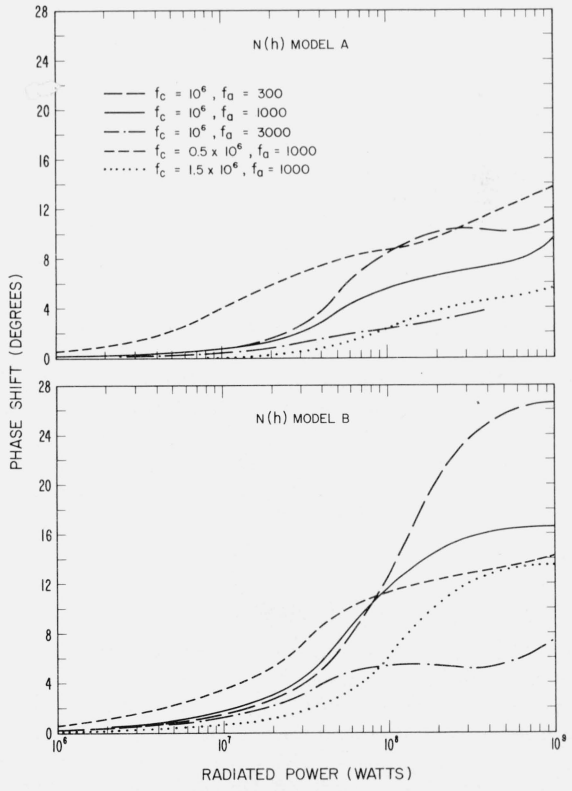


FIGURE 6. *Phase shift of the first harmonic versus isotropic radiated power.*

The abscissa, f_c, and f_a are as defined in the caption for figure 4.

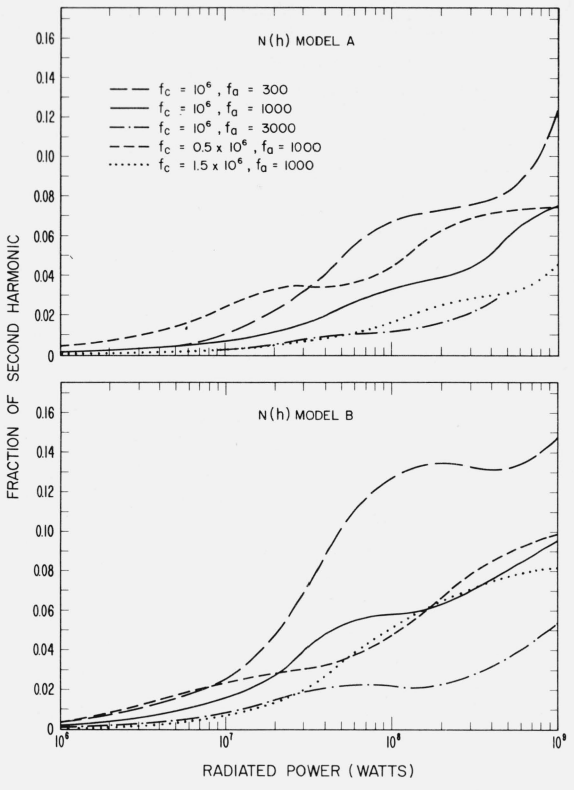
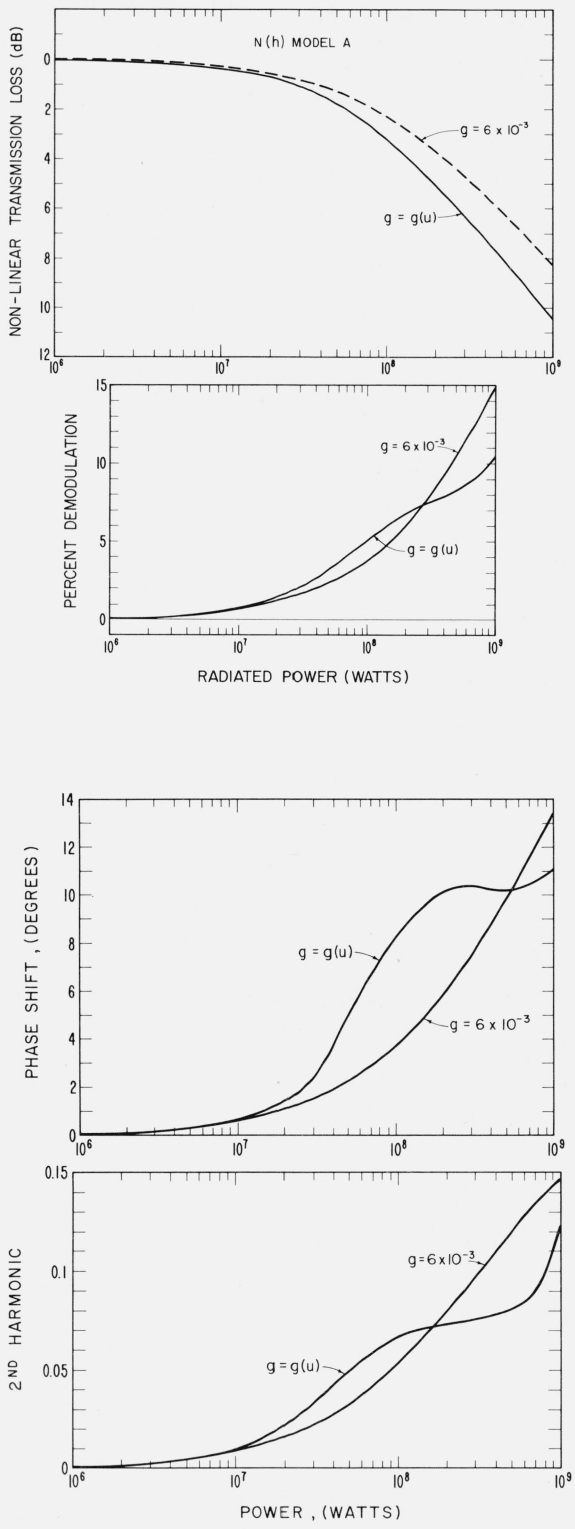


FIGURE 7. *Fractional amount of power contained in the second harmonic versus isotropic radiated power.*
The abscissa, f_c, and f_a are as defined in the caption for figure 4.

In all the data presented so far, the dependence of the fractional energy loss per collision parameter $g(u)$ on energy, as shown in figure 8, was used. The low energy dependence of this parameter is not well established, with the principal uncertainty being in the knowledge of energy losses in molecular oxygen. In order to demonstrate the sensitivity of the effects to changes in this parameter, a set of calculations have been carried out with $g = 6 \times 10^{-3}$ for all electron energies. The carrier frequency used in the calculations was 1.0 Mc/s and the modulation frequency 300 c/s. As can be seen, the differences would not be very significant for most engineering calculations. If one were interested in interpreting an experiment, however, these variations could, in principle, yield information as to the energy loss processes in the gas in question.

6. Conclusions

A set of calculations has been made which should be of help in making estimates of the value of increasing the power of already powerful radio stations. While a number of simplifications were made in the process of making the computations, it is expected that they do not invalidate the results of the calculations except, possibly, for carrier frequencies near the gyrofrequency in which case nonlinear effects may be expected to be much more severe.



FIGURES 8a and 8b. *Nonlinear effects for two types of dependence of g the average fractional energy loss per collision as a function of radiated power as discussed in the text and the previous figures.*
The abscissa, f_c, and f_a are as defined in the caption for figure 4.

The detailed evaluation of the effects depend, of course, upon the type of signal being transmitted. The acceptable limits to the distortion can only be set after detailed consideration of the transmission requirements.

These results are not accurate enough to allow one to interpret propagation experiments, but modifications which will increase the accuracy are expected to be in terms of added complexity, not of fundamental concepts.

The author recognizes the contributions of Mrs. Bernice Bender in programming the problem and running the initial calculations and of T. M. Georges in the extension of the work to more than one electron density profile. Helpful discussions have been had with W. K. Klemperer, F. C. Fehsenfeld, E. K. Smith, and T. M. Georges. It is also desired to thank A. V. Phelps of the Westinghouse Laboratories for allowing the use of his data in advance of publication. It should also be pointed out that the approximations made in the use of the data are the responsibility of the author and not of A. V. Phelps.

(Paper 69D3–472)

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