Momentum Transfer Collisions in Oxygen for Thermal Electrons

Michael H. Mentzoni

Contribution From the Applied Research Laboratory, Sylvania Electronic Systems, a Division of Sylvania Electric Products, Inc., Waltham, Mass. 02154

The electron collision frequency for momentum transfer, ν_m, has been determined by microwave methods in an oxygen plasma for temperatures between 300 and 900 °K. It was found that the measured average electron-neutral collision frequency was consistent with ν_{em} being proportional to the square of the electron velocity when a Maxwellian velocity distribution is assumed for the electrons; e.g., $\nu_{em}(u) = 4.4 \times 10^{-8} \times N(O_2)\ x\ u$ (sec^{-1}), where u is electron energy in electron volts and $N(O_2)$ is the number density of oxygen molecules. The results also indicated that electron-ion collisions are important at the lower temperatures with the values of ν_{ei} in agreement with the ones recently reported by Chen.

1. Introduction

The electron collision frequency for momentum transfer, ν_m, and its energy dependence in atmospheric gases is of special technical interest.

In atmospheric environment the total electron collision frequency for momentum transfer is determined by adding the collision frequencies for the various gas constituents in proportion to their number densities. Thus by laboratory determinations of ν_m as a function of electron energy for gases such as N_2, O_2, NO, N, O, one is able to compute the composite value of ν_m. This value will vary with altitude according to the density profiles of the species and the electron temperature, the latter covering the temperature range of the present investigation between altitudes from about 110 to 150 km [Nawrocki and Papa, 1961].

The collisional absorption and its effects on earth-space radio propagation has been recently reviewed by Lawrence et al. [1964] who, in this respect, points out the importance of D layer, the physical processes of which have been surveyed by Reid [1964]. Calculations of EM-wave propagation based upon the Appleton-Hartree magneto-ionic theory incorporate an effective velocity independent collision frequency thus simplifying the analysis. It has been shown by Phelps [1960], that such a simplification may lead to significant errors.

During reentry, signal absorption due to the ionized shock layer and wake constitutes a serious problem the solution of which necessitates a knowledge of ν_m in shock heated air [Bachynski et al., 1960].

Temperatures in a wake are in general less than 3,000 °K for reentry speeds and the ν_m measured here would pertain to the far wake which is of importance for radar detectability

A number of laboratory studies have been reported in recent years pertaining to nitrogen. For oxygen and dry air the available information is still rather limited [Nielsen and Bradbury, 1937; Healey and Reed, 1941; Van Lint et al., 1959; Phelps, 1960; Shkarofsky et al., 1961; Carruthers, 1962; Gilardini, 1963]. For thermal and near thermal energies, the dependence of ν_m on the electron energy u has been difficult to determine experimentally. This is so because the generation of monoenergetic electron beams, used so successfully in scattering experiments above 1 eV becomes increasingly difficult at lower energies. However, Pack and Phelps [1959] have improved the design of the mobility tube allowing measurements at thermal energies. It seems that the reanalysis by Shkarofsky et al. [1961] of results from the early drift tube experiments of Nielsen and Bradbury [1937] with oxygen pertaining to u higher than the gas temperature, T_{gas}, is consistent with a collision frequency given by the formula

$$\nu_{em} N(O_2)^{-1} = 4 \times 10^{-9} u^{1/2} + 5 \times 10^{-8} u \text{ cm}^3 \text{ sec}^{-1}$$

where $N(O_2)$ is the molecular number denisty of oxygen and ν_{em} refers to elastic collisions between electrons and molecules. Measurements pertaining to $T_{gas} = 300$ °K, assuming thermalized electrons, give results that are in agreement with the $\nu_{em} N(O_2)^{-1} = 7 \times 10^{-8} u$ cm^3 sec^{-1} (Phelps, 1960); it seems

desirable to perform *direct* measurements at some other gas temperatures within the suggested range of applicability of this equation and thus obtain a more complete verification.

Finally, electron-ion collisions, ν_{ei}, which are important at high electron-ion densities and lower temperatures are considered and their collision cross section evaluated. For a review of ν_{em} and ν_{ei} pertaining to the atmosphere the reader is referred to publications by Nicolet [1959], Nawrocki and Papa [1961], and Dalgarno [1961].

2. Experimental Method and Procedure

The experimental arrangement has been described previously [Mentzoni, 1963; Mentzoni and Row, 1963] and only the salient points will be discussed here.

A short duration, pulsating d-c voltage, periodically created an oxygen plasma inside a cylindrical quartz tube which also functioned as a waveguide operating in the "quasi" TE_{11} mode due to a gold coating on the outer surface. A low power X-band microwave signal propagating along the 80 cm long plasma column acted as the probing wave. An interferometer technique together with accurate measurements of plasma insertion losses permitted determination of the ratio.

$$\frac{\Delta A}{\Delta \phi} = \frac{L\Delta\alpha}{L\Delta\beta} = \int_0^R \sigma_r E^2 r dr \Big/ \int_0^R \sigma_i E^2 r dr, \qquad (1)$$

here ΔA, $\Delta \phi$, $\Delta \alpha$, and $\Delta \beta$ represents changes in the total attenuation (nepers), phase shifts (radians), attenuation and phase constant respectively due to the plasma, L is length of the interaction region, E is electric field intensity, R is inner radius of the quartz cylinder and σ_r and σ_i are the real and imaginary components of the rf conductivity

$$\sigma = \sigma_r + j\sigma_i = -\frac{4\pi N_e^2}{3m} \int_0^\infty \frac{v^3}{\nu_m + j\omega} \frac{\partial f_0}{\partial v}\, dv. \qquad (2)$$

Here N_e, e, m, and v are electron number density, charge, mass, and velocity, respectively, and f_0 is the spherically symmetrical part of the electron velocity distribution function normalized such that

$$\int_0^\infty 4\pi v^2 f_0 dv = 1$$

$$\left(f_0 = \left(\frac{m}{2\pi k T_e}\right)^{3/2} e^{-\frac{mv^2}{2kT_e}} \text{ in the Maxwellian case} \right).$$

Equation (1) is valid provided the first order perturbation analysis is sufficiently accurate [Golant and Zhilinskii, 1960] which means that

$$\overline{N}_e = \int_0^R N_e r dr / \pi R^2 \ll n_{cr}(1 + (\nu_m/\omega)^2)^{1/2}/A_F$$

with $\omega/2\pi = 8.97 \times 10^3 n_{cr}^{1/2}$ and,

$$A_F = \frac{\int_0^R N_e E^2 r dr}{\int_0^R E^2 r dr} \Bigg/ \frac{2\pi \int_0^R N_e r dr}{\pi R^2},$$

A_F goes from unity to 1.3 for the TE_{11} mode as N_e goes from a uniform to a $J_0(2.405r/R)$ distribution. Actually the quartz walls cause a further (and in this case insignificant) perturbation of the TE_{11} mode as shown by Unger [1957].

Even though the probing field is nonuniform, it is selected to be very weak, satisfying the criterion

$$E^2 \ll E_p^2 = \frac{2G\langle u \rangle m}{e^2} (\omega^2 + \nu_m^2)$$

with $\langle u \rangle$ and G the average electron energy and energy loss factor (G-factor), respectively, so that one has that the average electron energy is much larger than the energy perturbation caused by the probing field, i.e.,

$$\langle u \rangle \gg \frac{e^2 E^2}{2m G(\omega^2 + \nu_m^2)}$$

and thus can be regarded as spatially uniform. When a larger field intensity is used, such as in the cross modulation experiment, the distribution function can still be regarded as independent of position provided the thermal conductivity of the electrons is very large.

In the present arrangement the gas is heated to any desired ambient temperature up to about 1000 °K by surrounding the waveguide-discharge configuration by an electric oven. A microwave radiometer utilizing the noise sampling technique is used to monitor the electron radiation temperature, one can thus find the time after which isothermal conditions prevail during the afterglow [Mentzoni, 1964] which ranged typically from about 30 to 50 μsec. In comparison, the total measurable time decay for electrons and ions, characterized by $N_e > 5 \times 10^8$ cm^{-3}, lasted always more than 1 msec.

In general one has $\nu_m = \nu_{em} + \nu_{ei} + \nu_{ee}$, where the new right-side terms indicate the frequencies for electron-ion and electron-electron collisions. In the following ν_{ee} will not be considered. Whereas ν_{em} is independent of spatial coordinates, this is, in general, not so with ν_{ei} which depends on the ion density, N_i according to the relation [Shkarofsky, 1961].

$$\nu_{ei} = C_1 \frac{N_i}{v^3} \ln [C_2 T_e^{3/2} N_e^{-1/2}]$$

where C_1 and C_2 are constants.

Ignoring the logarithmic term one can write, since $N_i = N_e$,

$$\nu_{ei} = \nu_{ei}^0(v) F(r)$$

where $F(r)$ describes the electron density profile defined by $N_e(r)=N_0F(r)$, with N_0 being the electron density at the center.

When isothermal conditions prevail ($T_e=T_{gas}$) the electrons obey a Maxwellian distribution so that $\langle u \rangle = \frac{3}{2}kT_{gas}$, this is also a reasonable assumption when T_e is slightly in excess of T_{gas} due to an rf heating pulse (cross modulation). In this case and for $\nu_m \ll \omega$ (low pressure approximation) one has from eqs (1) and (2)

$$\frac{\Delta A}{\Delta \phi} = -\frac{4\pi}{3\omega}\left[\int_0^\infty \nu_{em}v^3\frac{\partial f_0}{\partial v}dv\right.$$

$$\left. +\frac{\int_0^R E(r)^2 F(r)^2 r dr}{\int_0^R E(r)^2 F(r) r dr}\int_0^\infty \nu_{ei}^0 v^3 \frac{\partial f_0}{\partial v}dv\right]. \quad (3)$$

It is seen that for a nearly uniform electron distribution, $F(r)\approx 1$ one has

$$\frac{\Delta A}{\Delta \phi} = -\frac{4\pi}{3\omega}\int_0^\infty \nu_m v^3 \frac{\partial f_0}{\partial v}dv. \quad (3a)$$

In general ν_m is a function of electron energy $u(=\frac{1}{2}mv^2)$ and it can be shown [Phelps et al., 1951] that one can deduce from the measured average quantities the energy dependence of ν_{em} under certain conditions. In this study the condition $\nu_m \ll \omega^2$ is satisfied and the right side of (3) can be written, assuming $\nu_{em} \gg \nu_{ei}$,

$-\langle \nu_m \rangle / \omega$ with $-\langle \nu_m \rangle = \frac{1}{3}\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{m}{kT_e}\right)^{5/2}$

$$\int_0^\infty \nu_m(v)v^4 \exp\left(\frac{-mv^2}{2kT_e}\right)dv. \quad (4)$$

Based on previously reported results it is reasonable to assume a power law $\nu_{em}=NCv^h$ and one obtains from (4)

$$\langle \nu_{em}\rangle = \frac{4}{3}\pi^{-1/2}\left(\frac{m}{2kT}\right)^{-\frac{h}{2}}NC\,\Gamma\left(\frac{5+h}{2}\right) \quad (4a)$$

where C and h are constants which may be found empirically from a best fit procedure, and N is gas density in molecules per cubic centimeter. If indeed there exists such a simple power law for ν_{em} and measurements are performed with the same N, but various temperatures, $T_e=T_{gas}$, one obtains h from the simple relation

$$\frac{\langle \nu_{em}\rangle_{Tek}}{\langle \nu_{em}\rangle_{Tel}}=\frac{(\Delta A/\Delta \phi)_{Tek}}{(\Delta A/\Delta \phi)_{Tel}}=\left(\frac{T_{ek}}{T_{el}}\right)^{\frac{h}{2}}. \quad (5)$$

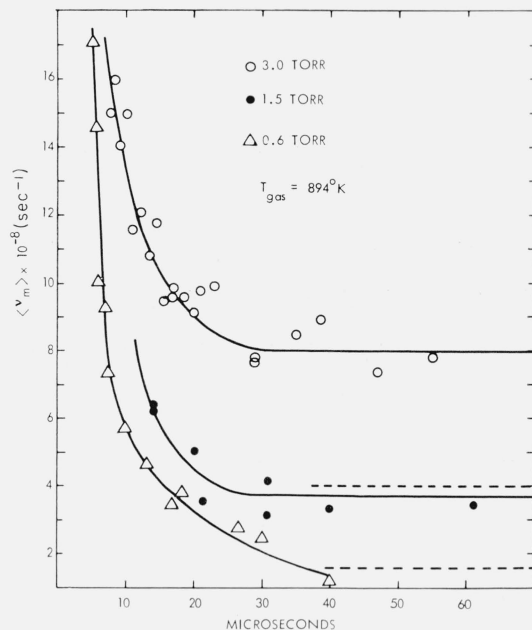If h can be determined in this manner, C can be determined from (3) and (4).

FIGURE 1.—*Typical time variations of $\langle \nu_m \rangle$ as computed from plasma insertion-loss measurements during the afterglow.*

The scatter of the experimental points represents differences between several runs of measurements. The broken lines indicate where the steady-state value of $\langle \nu_m \rangle$ should have been, based on the one found at $p=3.0$ torr.

3. Results and Discussion

In the preceding section it was shown how it is possible to obtain $\nu_{em}(u)$ for the low pressure approximation under the assumption that ν_{em} depends on v according to a simple power law. Figure 1 shows a plot of $(\Delta A/\Delta \phi)\omega$ versus time at $T_{gas}=894$ °K. It is seen that isothermal conditions are not present during the very early afterglow, thus perhaps invalidating (4). This is so because the assumption f_0 being Maxwellian may not be true even though the inequality $\nu_m \ll \omega$ ($=2\pi\ 10^{10}$ cycles sec^{-1}) is well satisfied. At later times constancy of $(\Delta A/\Delta \phi)\omega$ is obtained indicating $T_e=T_{gas}$ in agreement with electron radiation temperature measurements. The results for this case are given in table 1 and it is seen that $h=2$ yields good agreement with experimental results. The relationship thus found reads $\nu_{em}(u)=4.4\times 10^{-8}\times u\ N(O_2)$ sec^{-1} with u in electron volts or $\langle \nu_{em}\rangle=3.0\times 10^5\ pT_e$ sec^{-1}, with p (torr) referred to 273 °K. At temperatures below 569 °K steady state values of $\langle \nu_m \rangle$ as found for the higher temperatures were not as conclusive pointing to the possible noticeable contribution from $\langle \nu_{ei}\rangle$.

The spatial distribution of N_i and thereby ν_{ei} must in this connection be considered since (3) is the basis for the interpretation of the measurements. As has been pointed out [Mentzoni, 1965] the plasma seems to be axially and tangentially uniform based on luminosity measurements. The radial variation of $N_e(r)$ depends on the initial distribution and the rate of ambipolar diffusion during

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the afterglow. At the pressures pertaining to the present experiment, it is reasonable to assume a radially uniform initial distribution. The ratio $\int_0^R E(r)^2 F(r)^2 \, r \, dr / \int_0^R E(r)^2 F(r) \, r \, dr$ appearing in (3) is the spatial average $\overline{F(r)}$ of the electron-ion profile weighted by the quantity $E(r)^2 F(r)$ with $E(r)$ varying approximately as $J_0\left(k\dfrac{r}{R}\right)$, thus, $\overline{F(r)}$ is most sensitive to $\overline{F(r)}$ in the center region.

TABLE 1.—*Ratios of $<\nu_{em}>$ for various temperatures under isothermal conditions compared with the case of $\nu_{em} = NC$ $v^h (h=2)$ and $<\nu_{em}>$ being averaged over a Maxwellian velocity distribution*

T_{ek}	T_{el}	$<\nu_{em}>_{Tek}/<\nu_{em}>_{Tel}$	$(T_{ek}/T_{el})^{h/2}$ $h=2$
894	734	1.27	1.218
734	569	1.29	1.290
894	569	1.64	1.571
569	450	1.4	1.264

TABLE 2. *Electron-ion density profiles for various times during the afterglow, T=300 °K, p=0.3 torr and D_ap=110 cm² sec⁻¹ torr* [1]

Time	$N_e(t,r)/N_o = F(t,r)$				
	$r/R=0$	$r/R=0.2$	$r/R=0.4$	$r/R=0.6$	$r/R=0.8$
μsec					
10	0.980	0.980	0.980	0.980	0.961
20	.980	.980	.980	.979	.880
30	.980	.980	.980	.973	.796
40	.980	.980	.980	.959	.725
50	.980	.980	.978	.959	.667
60	.980	.980	.975	.915	.618

[1] D_a is ambipolar diffusion coefficient pertaining to the ions O_2^+.

TABLE 3. *Some composite results of $<\nu_m> = <\nu_{em}> + <\nu_{ei}>$ for $T_{gas} = T_e = 300$ °K*

$10^{-10} \times N_e$	p	$A = 10^{-8} \times <\nu_{em}>^a$	$B = 10^{-8} \times <\nu_{ei}>^b$	$A+B$	$10^{-8} \times <\nu_m>^c$
cm⁻³	Torr	sec⁻¹	sec⁻¹	sec⁻¹	sec⁻¹
3.325	3.0	2.70	1.53	4.23	4.35±.85
6.650	3.0	2.70	2.89	5.59	6.07±1.17
9.975	3.0	2.70	4.20	6.90	8.5
6.650	1.5	1.35	2.89	4.25	3.65±.05
9.975	1.5	1.35	4.20	5.55	5.0

[a] Based on present results, $<\nu_{em}> = 3.0 \times p \times T_e \times 10^5$ (sec⁻¹).

[b] Based on Chen's experimental results, $<\nu_{ei}> = 3.6 \dfrac{N_i}{T_e^{3/2}} \ln \left(\dfrac{2.0 \times 10^4 T_e^{3/2}}{N_e^{1/2}} \right)$ (sec⁻¹).

[c] Present direct measurements.

Considering diffusion losses under isothermal conditions only [Mentzoni, 1964] $F(r)$ can be computed as illustrated in table 2. At room temperature, $T_{gas} = 300$ °K, some of the results obtained are illustrated in table 3. Due to the lack of isothermal conditions in the time range of the high electron-ion

densities at the lowest pressures the experimental points tend to lie below those to be expected from theory and previous experiments (see fig. 2) assuming $T_e = T_{gas} = 300$ °K which in general follow the law

$$<\nu_{ei}> = C_1 N_i T_e^{-3/2} \ln \left(C_2 T_e^{3/2} N_e^{-1/2} \right) \qquad (6)$$

where C_1, C_2 are constants. Several theoretical and two experimental evaluations of C_1 and C_2 have been made previously yielding values of C_1 which varies between the limits 2.25 and 4.84×10³, and C_2 between 3.32×10³ and 20×10³.

It should be noted that Ginzburg and Gurevich [1960] and Nicolet [1959] quote expressions for $<\nu_{ei}>$ which are somewhat modified in form as compared to (6) being $<\nu_{ei}> \approx 5.5 N_i T_e^{-3/2}$ ln $(2 \times 10^2 T_e/N_e^{1/3})$ and $<\nu_{ei}> = [59.1 + 4.18 \log_{10} (T_e^3/N_e)] N_i/T_e^{3/2} 10^{-6}$, respectively, in the latter equation the electron density is given per meter.[3] At the pressures $p=3.0$ mm Hg and $p=1.5$ mm Hg it is seen from table 2 that the agreement with Chen's [1964] results is fair considering the experimental error of the measurements.

From figure 2 it is seen that the experimental points for the two different pressures converge for the lower electron densities and electron temperature indicating that $<\nu_{ei}>$ is dominating. At the higher densities on the other hand, the increasing divergence of the experimental points for the two pressures is to be expected in the case $<\nu_{em}>$ is dominating. The solid curve in figure 2a indicate that even under nonisothermal conditions the present result for $<\nu_{em}>$ coupled with $<\nu_{ei}>$ as found by Chen is reasonably consistent with experimental observation of $<\nu_m>$ and electron radiation temperature measurements. As the electron energy increases the

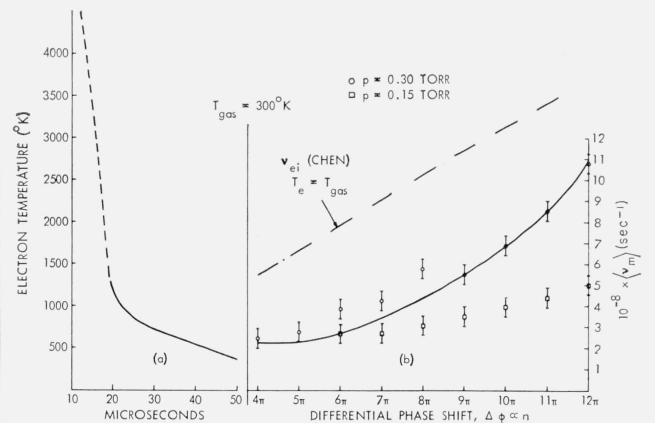FIGURE 2.—(b) $<\nu_m>$ versus the differential phase-shift $\Delta\phi$ for $T_{gas} = 300$ °K and pressures 0.30 and 0.15 torr.

The measured values are indicated by points, the dash-dotted curve represents $<\nu_{ei}>$ under isothermal conditions according to Chen, and the solid curve is computed from $<\nu_m> = <\nu_{em}> + <\nu_i> = 3.0 \, p \, T_e \times 10^5 + 3.6$

$$\times \frac{N_i}{T_e^{3/2}} \ln \left(\frac{2.0 \times 10^4 T_e^{3/2}}{N_e^{1/2}} \right)$$

with $p=0.30$ torr (referred to 273 °K) and T_e following the time decay curve shown in (a). The solid portion of this curve (time ≥ 20 μsec) has been measured.

inelastic electron collisions involving rotational and vibrational excitation of the oxygen molecules will also become increasingly important, however, their contribution to $<\nu_m>$ is expected to be small at the temperatures considered here.

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