It is worth mentioning the abnormal reflection associated with the crater Tycho [Pettengill and Henry, 1962], where the reflection was about five times greater than its surroundings at 440 Mc/s. Figure 6 indicates that this corresponds to a thickness of the lower layer of 1 cm, if our interpretation is correct.

We can remark that, dealing with the nature of compact materials, a dielectric constant of 6 and 7 is in good agreement with that of tektites [Olte and Siegel, 1960].

3. Conclusions

We have shown that the observed variation of the effective scattering cross section of the Moon suggest an increase of the reflectivity with wavelength, although we cannot disregard the possibility of a constant reflective coefficient of a homogeneous surface structure.

If this variation is due to a superficial layer, where the density increases with depth, the following conclusions can be stated:

The thickness of the layer is of the order of a few tens of centimeters, at least for a good part of the slopes that are responsible for the quasi-specular component of the received echos.

The dielectric constant of the material that the layer is formed of is higher than 5 at the compact state.

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(Paper 69D12–622)

Moon Distance Measurement by Laser[1]

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1. Introduction

It seems hardly necessary to recall the main characteristics of the ruby laser, the first to have functioned and still the most widely used today. These characteristics are essentially:

(a) A very monochromatic light,

(b) a very narrow beam,

(c) lastly, and in particular with the so-called *Q*-switched lasers, an emission effected by very short pulses of which the peak power attains considerable values.

The small aperture of the radiated beam, in conjunction with the very high peak power of the emission, means that the laser represents a source of considerable brightness, reaching, for example, 10^{10} W/cm^2/sterad.

A light source presenting such properties could certainly not fail to attract the attention of all those particularly interested in the location and ranging of distant objects.

The first realizations in this field were infantry and tank telemeters, destined to measure distances not exceeding a few kilometers. Then, as laser power increased, the measurement of far greater distances came to be attempted, for example, that of a satellite, taken successfully a short time ago in the United States [Plotkin, 1963] and in France [Bivas, 1965]. It was therefore tempting to try to measure, using a laser as light source, the Earth-Moon distance, already measured by radar [Yaplee et al., 1963].

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This determination would present two advantages:

At a first stage, the direct calculation of the lunar radius directed towards the Earth (since we know from astronomical calculations the distance from the Moon's center of gravity to the Earth). We would thus hope to calculate this radius to 0.1 percent.

Next, if the signal:noise ratio at reception proves sufficient, the complete determination of the form of the selenoid.

First of all we are going to establish, in specifying the physical orders of magnitude of this project, that only a Q-switched laser would in fact enable us to contemplate this measurement [Smullin and Fiocco, 1962; Le Boiteux, 1965]. After that, we shall try to determine first the performance of the emitting optical system coupled with the laser and then that of the optical system and of the receiving apparatus. Finally we shall conclude with a brief evocation of some of the problems arising in laser utilization, and we shall propose a way of exploiting the expected results.

2. Determination of the Experiment

2.1. Emission

a. Choice of Light Source

The proposed experiment is interesting insofar as it will permit an accuracy comparable with or superior to that of existing radar measurements. This supposes that one may determine the moment of the echo return to less than 10^{-6} sec. Since the rise time of the detectors used must be inferior to this figure, the best source will be that allowing the receiving apparatus sufficient energy for detection in the shortest possible time (and in any case less than 10^{-6} sec).

Let us note that the power received depends both on the emitted power and on the "spread" of the echo resulting, in particular, from the curvature and relief of the Moon. In order, therefore, to reduce the influence of this second factor, it will be necessary either that the zone lighted on the Moon be no more than a few kilometers in diameter, or that, whatever the zone lit, only a very small zone be observed at reception. This would amount to the same thing. We shall see later that the second possibility, involving a considerable loss of light, is to be set aside.

The reduction in diameter of the lighted zone generally demands an optical system comprising two lenses, such as that represented in figure 1. The aperture of this system will depend on the light source chosen.

Let D be the diameter of the largest lens and d that of the light source (or the part of this source used). The dimensions required for the light patch on the Moon will determine the angular aperture Θ of the transmitted beam and consequently the aperture θ of the light from the source which may be utilized.

In the most favorable instance, supposing the lighted zone to have a diameter of 40 km (corresponding to a 60-m distance difference between center and edge of the lighted spot) we have

$$\Theta = \frac{40}{400{,}000} = 10^{-4} \text{ rad.} \tag{1}$$

With $D = 1$ m 90, we get $\theta . d = 1.8\ 10^{-2}$cm. Thus from a source of 1 cm², we would use only the light contained in a 1° cone.

(a) Let us now suppose that we take an electric flash as our source; the fraction of energy in the discharge which can in fact be utilized is very small; in this way a very powerful flash (100 MW for example) would provide only a few effective kilowatts.

(b) The free-running laser constitutes another possible source. With this we may obtain several hundred joules of light energy over a period of 1 or 2 msec. This energy, emitted in an angle of a fraction of a degree, lends itself to the required concentration and we shall see that the energy received would be relatively high. However, because of the length of emission, we end up with a signal:noise ratio which leaves us no grounds for hoping to identify the beginning of the echo received, nor consequently to measure the distance accurately.

(c) We are left with the Q-switched laser, possibly followed by one or more amplifying stages. The energy emitted (between one and several joules) can be amplified by induced emission, and, after this amplification, we may count on obtaining between 5 and 50 J in about 100 nsec. Under these conditions, the angular aperture of the beam transmitted is about 3 to 10 times the maximum angle imposed by diffraction, that is, $1.22\lambda/D$. This corresponds to a 4-km diam spot

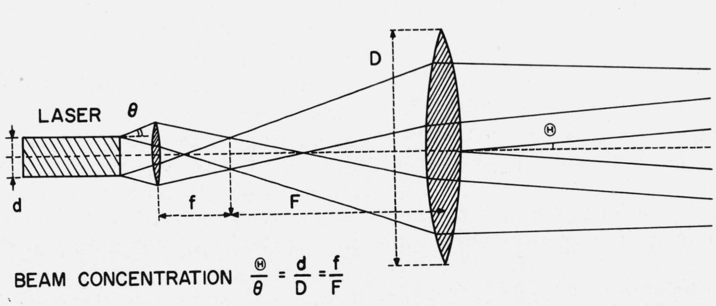

FIGURE 1. *The laser source and optics.*

on the Moon, if we use for the emission a 105-cm otpics, such as the Pic du Midi observatory telescope.

In conclusion, it would seem that of all available light sources, the Q-switched laser will permit us to receive the maximum energy in return, within the time set by the required accuracy of measurement, that is, 10^{-6} to 10^{-7} sec.

b. Choice of Transmitting Optics

The energy picked up being very small, we have seen that in order to avoid spreading the return echo, we will have to restrict the diameter of the spot lighted on the moon and consequently the angular aperture of the transmitted beam. This constitutes a first reason for choosing an optical system of large diameter.

There is another reason which prevents us from falling below a certain diameter for the transmitting optics, when they are constituted by a telescope, for astronomical use: such telescopes, intended to receive on any wavelength, are usually aluminium or silver coated. Such a film absorbs from 5 to 20 percent of the incident light and consequently part of the laser energy. With the energy we have in mind, the reflecting layer is likely to be overheated if the mirror has too small a diameter (see below).

In conclusion, the high degree of geometric concentration prescribed for the radiated beam, in order to achieve the accuracy required and to obtain a sufficient signal:noise ratio, together with the necessity of limiting the heating of the mirror, imposes an optics diameter of over 50 cm (for 50 J of energy transmitted).

There is another supporting reason. We will see further on that a number of successive shots will be necessary in order to ensure detection, and so the telescope (if an instrument of this type is chosen) will have to track the Moon. The laser ought then to be fastened onto the telescope,[2] which will be unable to follow adequately unless it is built strongly enough to take the extra weight of the laser (nearly 100 kg).

We shall assume, subsequently, that the optical system used is the 105-cm telescope of the Pic du Midi observatory (of which the mobile part weighs about 1500 kg).

2.2. Reception

a. Generalities

The receiving equipment should have the following characteristics: a short rise time (under 10^{-6} sec), sufficient sensitivity, and an intrinsic noise level as low as possible. The second and third points prescribe a photomultiplier detection. This photomultiplier, placed in the focus of a collecting mirror of largest possible diameter, will receive both the laser signal thrown back by the lunar ground and various parasitic lights, such as scattered moonlight, sky background, etc. Finally, it will be necessary to take into account the dark current issuing from the photomultiplier cathode and the noise generated by the electronic circuits following the detector.

In order to endow the receiving equipment with the highest possible efficiency, we are going to try to evaluate in turn the useful signal received and the signals arising from different noise sources.

b. Evaluation of Energy Received

Suppose that we receive, on the photocathode, a whole image of the circle on the Moon lit by the laser. Let us provisionally adopt a 105-cm optics diameter, for receiving as for transmitting, and let us assume that there is a filter in front of the photomultiplier. We shall see later why this filter is necessary.

If t_e is the transmission of the emitting optics,
t_t is the transmission of the telescope
t_a is the transmission of the atmosphere
t_f is the transmission of the filter;

and if a is the albedo of the Moon, the energy received relative to the energy emitted will be

$$W_r/W_e = \frac{1}{\pi}\, a\, t_e\, t_f\, t_t^2\, t_a^2\, \Omega, \qquad (2)$$

if we accept Lambert's law to have been proved (brightness of the lighted zone independent of the angle of emission) and if Ω is the angle under which the receiver is viewed from the diffusing surface.

With $t_e = 0.90$ (4 bloomed air-glass surfaces)
$t_t = 0.90$ (1 metallic reflection)
$t_f = 0.31 = 0.60$ (interference filter) X 0.85 (mirror) X 0.60 (blazed grating)
$t_a = 0.85$
$a = 0.10$[3]

and

$W_e = 5$ J, we find
$W_r \simeq 1.5\ 0.10^{-12}$ erg, or about 0.5 photons per pulse.

Using a Lyot filter instead of the grating should yield about the same received energy.

c. Choice of Detector

It appears straightway that the signal received will be very faint, and this leads us to choose a photomultiplier having a cathode of very high quantum yield. Then again, and for the same reason, it is important that the dark current be reduced as far as possible.

[2] In fact, the insufficient resistance of mirrors to the laser pulse prevents the utilization of image relays using a number of small-sized mirrors.

[3] This value would correspond to a relatively bright zone, the average albedo of the Moon being slightly inferior.

Typical values at 0.7 μ would be

Quantum yield	Cathode dark current*	Number of electrons per second
2.5 10^{-2}**	10^{-14}ca 25 °C to 10^{-18}ca-100 °C	10^4 to 1

*Amperes; when T is reduced by 10 °C the current is divided by 6 to 8, that is, by 10^6 for a 100 °C cooling—if the nature of the cathode permits such treatment.
**Trialkali cathode.

Under the best conditions, such photomultipliers show gains of the order of 10^7, so that for each cathodic photoelectron we obtain packets of 10^7 electrons on the anode.

d. Choice of Wavelength

With regard to the choice of photomultiplier, let us note that the quantum yields of the best 1.06-μ cathodes are no more than a few percent of the values indicated at 0.7 μ for trialkali cathodes. On the other hand, neodymium lasers, which emit at 1.06 μ, furnish hardly any more power than ruby lasers (0.7 μ). Lastly, when one passes from 0.7 μ to 1 μ, the optical noise rather tends to increase.

All these reasons, to which may be added the poor transparency of the atmosphere at 1.06 μ, due to a water-vapor absorption band, lead us to choose the ruby laser 0.7-μ wavelength to carry out our experiment.

e. Noise Evaluation

Having evaluated the level of the signal, we must now evaluate that of the noise. This noise has two origins: optic and electronic.

Optical Noise. Optical noise is constituted by the parasitic light which reaches the receiver and which arises from three sources:

Light from the lunar surface illuminated by the Earth, equivalent to "earth-light" for the Moon; this light appears directly in the field of observation of the telescope.

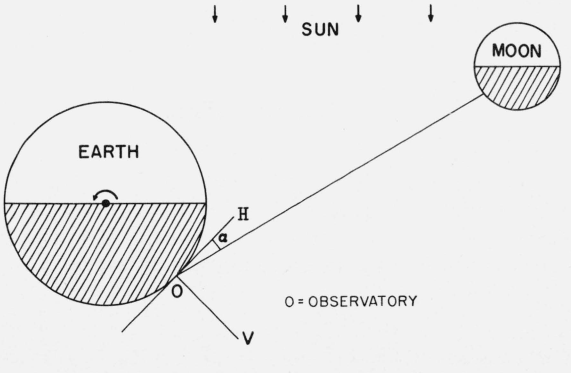

FIGURE 2. *Favorable Earth-Moon configuration.*

"Moonlight" scattered onto the receiver by the atmosphere and the inner surfaces of the telescope (even though the latter will not observe the lighted zone on the Moon).

Lastly, "sky background" light (stars, etc. . .), diffused onto the detector in the same way.

In order to reduce these parasitic lights, we are going to take three simultaneous courses:

(a) To choose the most favorable Moon phases. The region to be explored should be in shadow, the earthlight should not be too bright, and the crescent should not be too bright either (because of the light diffused inside the apparatus). We shall scarcely observe beyond 60° and it is desirable to take measurements on sizeable portions of the diameters passing through the center of the lunar disk. We therefore prefer to operate in the few days preceding the first quarter and in those following the last quarter (see fig. 2).

(b) To observe only the zone lighted by the laser. For the same energy emitted by the laser, the smaller the zone observed, the lower the flux of lunar origin picked up in the same solid angle. Here again we see the advantage of a laser whose energy is concentrated in the smallest possible area.

(c) To take advantage of the monochromaticity of the laser signal in order to let pass only a narrow spectral band of lunar light and scattered light. The smallness of the image to be dealt with makes it possible to use either a filter or a dispersing system.

When the above conditions are fulfilled, we find [Danjon and Rougier, 1936] on the detector cathode, the following noise flux per square meter of the collecting surface and for a zone of 2000 km² observed through a 2.5 Å filter:

4000 photons coming from the earth-light,
100 photons coming from the night sky,

about 20,000 photons coming from the scattered moonlight (this value depends on the instrument and on the atmospheric conditions).

The conclusion to be drawn from these evaluations is that the moonlight scattered by the atmosphere constitutes the principal noise source, with a flux of 20.10^3 photons/sec for an observed diameter of about 50 km, (and a 2.5-Å band). In order to ensure detection, this noise flux should be kept at a level notably lower than the signal level. We may achieve this result by adjustment of the observation period, the field of view of the receiver, or the spectral bandwidth of the detector.

It is not possible, for theoretical or technological reasons, to choose arbitrarily the values of the three above quantities. Let us examine each of them:

(a) Observation period. The probability of the noise fluctuations' reaching a level comparable with that of the expected signal increases with time. It is in our interest, for that reason, to reduce as much as possible the length of the observation interval. We are, however, limited in this direction by the prevalent uncertainty concerning the Earth-Moon distance, known by astronomical methods to about ± 1 km. Corresponding to such uncertainty, there is a minimum observation time of 12 μsec. As a precaution we shall observe during a 100-μsec "window." The noise is then reduced to approximately 2 photons on the cathode, still for an observed zone of 2000 km^2, and a band of 2.5 Å.

(b) Receiver field of view. Generally speaking, the noise lessens with the area observed, as indicated in figure 3. (The noise level is calculated for 2 filter bandwidths.) It would in fact be best to observe only the lighted surface. We must note, however, that the angular diameter of this zone is of the order of 2 sec. Under these conditions, we run the risk of not obtaining the required tracking accuracy on the part of the receiver, and it seems more reasonable to reckon on an observed diameter of 20 km.

(c) Filter bandwidth. The filters ought to permit us to isolate as narrow a spectral band as possible, even for light rays relatively inclined on the axis. Under these conditions the only possibilities are:

Lyot or Solc filters, having a bandwidth which can be reduced to 0.75 Å and an angular aperture of the order of f/10. These filters are very compact and fairly light.

Grating monochromators, which can be utilized because of the smallness of the image to be filtered. We could obtain in this way, for a 0.3-mm image, a resolving power of 1.5 Å. This filtering system, though less costly than the previous one, requires rather critical adjustments.

Evaluation of the Electronic Noise. Let us now recall that electronic noise is added to the optical noise. We shall likewise evaluate this electronic noise. It has three origins, the photocathode dark current, the noise generated inside the photomultiplier load, and the noise generated inside the amplifier following the photomultiplier.

(a) We saw at the beginning how the choice of a trialkali cathode photomultiplier, possibly cooled, would enable us to bring the dark current down to very low values, ranging from a few tens to a few hundred electrons per second. The noise flux is then of the order of 10^{-2} electrons during the 100-μsec "window" chosen.

(b) If we give a value of around 5000 Ω to the load resistance, the corresponding noise will range around 20 μV at a normal temperature.

(c) Lastly, the amplifier noise equivalent input will reach a similar value.

The photomultiplier gain being of the order of 10^7, we shall obtain, for each photoelectron emitted from the cathode, this same number of electrons on the anode. Hence the voltage of the pulse will range about 20 to 50 mV at the amplifier input (taking into account the photomultiplier gain fluctuations), if we suppose that the photomultiplier is loaded with a 40 $\mu\mu$F capacitance.

Consequently we may set the detection threshold at 1 mV; under such conditions, the probability of getting a false alarm arising from electronic noise voltage fluctuations is practically nil, and the only

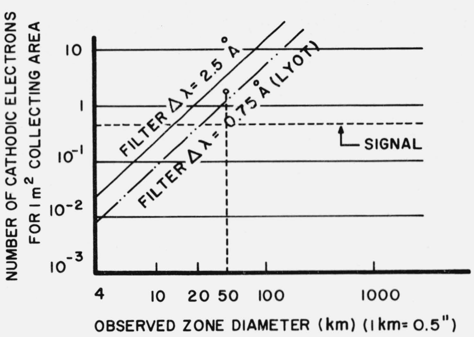

FIGURE 3. *Optical noise as a function of area observed.*

pulses detected will be those due to thermal electrons emitted by the cathode. These pulses are in fact indistinguishable from those due to the emission of photoelectrons.

We have seen that the optical noise (calculated, let us recall, in a 100-μsec "window", and with a bandwidth of 2.5 Å) is reduced to about 0.5 photons per square meter of collecting surface for an observed zone 20 km in diameter. To this noise coming from parasitic lights may be added the thermal electrons emitted by the cathode detector, at a rate of 0.01 for 100 μsec. (This would correspond to an incident light flux of 0.4 photon.)

f. Receiving Optics

Now it remains to decide upon an optical system allowing the best possible detection. For this, the collecting optics, in association with the receiving apparatus, will have to fulfill three distinct conditions, to collect a sufficient quantity of laser light to ensure detection, to be close enough to the transmitter to allow sufficiently accurate synchronization between them, and, at the same time, to permit a signal: noise ratio making it possible to identify the useful signal. We are going to examine these three imperatives in turn:

(a) Light collection. The preceding evaluations led to a 0.5-photon return flux per square meter of the collector and for a pulse emitted of 5 J. The adoption of a collector with a diameter of 3.5 m would, under the same conditions, allow the reception of 5 to 6 photons, bringing the detection probability up to 15 percent. An even bigger collector would clearly be advantageous, but a 3.5-m diam represents more or less the workable limit, if we take into account the geometrical conditions determined further on.

(b) The next problem arising is that of measuring the time interval between emission and reception. The precision required (of the order of 0.5 μsec or less) makes this measurement difficult at intervals exceeding some tens of kilometers. In fact, we must exclude a prior synchronization by radio waves reflected in the ionosphere, because of the fluctuation in altitude of the ionized layers. There remain the VLF waves propagated along the ground, and the radio or light waves propagated in direct view.

(c) Lastly, the optical system should allow us to obtain a sufficient S:N ratio. Let us recall that the principal noise sources consist of the earthlight observed directly by the receiver, and the solar light diffused by the Moon and scattered into the atmosphere and the receiving instrument. Now, the light picked up by the optical system from these two sources is in direct ratio to the solid angle observed by the instrument. As it is also proportional to the collecting surface, we conclude that *the signal:noise ratio depends only on the solid angle observed.*

We must then consider the various combinations rendered possible by a given received signal and signal:noise ratio. As a function of the area of the collecting optics (that is, as a function of the average number of photoelectrons liberated by the return echo) we are going to look for the minimum solid angle to be isolated at reception in order to permit detection. But before that we shall describe the detection method used.

First of all, it is evident that, as the received signal comprises only a few photons, the detection probability will be very slight. Again, the S:N ratio is itself very low. Consequently it will be impossible to content oneself with a detecting apparatus simply consisting, as in a radar, of an oscilloscope triggered (after a suitable delay) by the departure of the light pulse. It will therefore be necessary to have recourse to another method, which we shall call detection by "signal sampling and storage."

g. Detection by Signal Sampling and Storage

The principle of this method is as follows: The return signal is sliced into time periods of a given length, and the signals received in the different periods are stored, during several successive firings, in digital memories or channels.

Let us suppose that we examine the contents of these channels after N shots. The channel corresponding to the instant of the echo return will have received, on an average, a few more pulses than the others, which only received noise. In that case we imagine it to be possible to designate with certainty the channel having received the echo. The distance we are looking for will correspond to this channel, and the narrower the channel, the more accurate will be our measurement.

We are limited by the uncertainty of the instant of the echo's arrival, which is of the order of 0.1 μsec (curvature of the Moon for a frontal spot 10 km in diameter) + 0.2 μsec (emission time X 2), or about 0.3 μsec.

More precisely, we call X_B the average number of noise electrons emitted by the cathode during the total observation time T, X_S the average number of signal photoelectrons emitted at each laser echo return, c the number of channels contained in the total measurement interval T, and finally, P_0, P_1 . . . P_k . . . the probabilities of obtaining, after a single shot, 0, 1, 2 . . . k . . . cathodic electrons. (Naturally, we have $P_0+P_1+P_2+\ldots P_k+\ldots=1$.)

The results of successive shots being considered to be independent, the different probabilities resulting from a series of N experiments will be equal to the terms of development of

$$(P_0+P_1+\ldots+P_k+\ldots)^N = \sum P_0^{\alpha 0}\, P_1^{\alpha 1}\, P_2^{\alpha 2} \ldots P_n^{\alpha n}. \qquad (3)$$

In particular, the probability of obtaining k electrons will be equal to the sum of those of the above terms for which

$$\alpha_1+2\alpha_2+\ldots+N\alpha_N=k \qquad (4)$$

Let P_N^k be this probability. The $P_0, P_1, \ldots P_k \ldots$ representing a Poisson distribution, we know the distribution of P_N^k to be likewise a Poisson distribution, of which the mean value is NX_s; and consequently

$$P_N^k=\frac{(NX_s)^k}{k!}\, e^{-NX_s}, \qquad (5)$$

or again

$$P_N^k=\frac{N^k}{k!}\,(X_s)^k\, e^{-NX_s}. \qquad (6)$$

h. Calculation of Signal: Noise Ratio

The measurement interval T having been divided into c channels, the probabilities of obtaining at each firing a "signal" electron in the channel concerned,[4] and a noise electron in any channel, are respectively

$$X_s \text{ and } \frac{X_b}{c}.$$

If we fire N times, and if we put down $NX_s=\mu$, there will be a 50-percent probability of obtaining μ or more electrons in the signal channel.

Let us fix the detection threshold at this level, that is, μ electrons per channel.

The probability of obtaining exactly μ noise electrons in a given channel is equal to

$$P_N^\mu=\frac{N^\mu}{\mu!}\left(\frac{X_B}{c}\right)^\mu e^{-N\frac{X_B}{c}}, \text{ or again} \qquad (7)$$

$$P_N^\mu=\frac{\mu^\mu}{\mu!}\left(\frac{X_B}{cX_s}\right)^\mu \cdot e^{\frac{-\mu XB}{cX_s}} \qquad (8)$$

and the probability of obtaining μ electrons or more in this same channel is equal to

$$\sum_\mu^\infty P_N=\lambda\, P_N^\mu \text{ where } \lambda<\frac{1}{1-\dfrac{X_B}{cX_s}}. \qquad (9)$$

[4] That is, if we suppose that the "signal" electrons always go into the same channel $\left(\text{fluctuations} < \frac{T}{c}\right)$ or at worst into two adjacent channels. We should know the moment of departure of the pulse and the delay accurately enough to justify this supposition.

If we now look for the probability of false detection, we shall find it to be equal (since we assume these probabilities to be slight) to the probability of obtaining μ electrons or more in a particular channel, multiplied by the number of channels. Thus,

$$P_\beta = C \cdot \lambda \cdot \frac{\mu^\mu}{\mu!} \left(\frac{X_B}{cX_s}\right) \mu e^{-\frac{\mu X_B}{cX_s}} \qquad (10)$$

In working out this formula, we are going to evaluate the possible S:N ratios in terms of the number of channels c and of the number [5] of firings,

$$N = \frac{\mu}{X_s}. \qquad (11)$$

In practice, the duration of the channels is set by a compromise between the desired accuracy and the complexity of the electronic system; given the total measurement interval T, the number c is determined. We shall consider the following cases: $c=50$, 100, 200, 10^3 and 10^4. Lastly, we shall grant that the noise is acceptable if P_β is equal to a tenth of the detection probability; that is,

$$P_\beta = 0.05. \qquad (12)$$

We then find $\frac{X_B}{X_s}$ in terms of μ and of c:

These results can be said to confirm the intuition that detection is always possible, so long as X_B does not become of the same order as X_s. Moreover they will permit us to determine the geometric parameters of the collecting optics.

Let us note that if we wish to maintain an angular accuracy of the order of a second, it will be difficult to spread the series of successive experiments over more than 5 min. Since technological reasons limit the functioning rate of the laser itself to one shot every few seconds, N must remain inferior to about 50.

TABLE 1

For $\mu=$	1	2	3	4	5
and $c=50$	0.05	1.2	3	5.5	8
and $c=100$	0.05	1.5	5	8.8	14
and $c=200$	0.05	2.2	7.5	15	22
and $c=10^3$	0.05	5	22	50	75

Then again, we know X_s, which depends on the optics diameter (generally $X_s \ll 1$) and can deduce from it:

$$\mu = NX_S.$$

As the number of channels c is given, we can finally determine the maximum noise level $= X_B$ permitting the maintenance of P_β below 0.05. We have performed this calculation for $c=50$ and $c=100$, and the results are indicated in table 2. For convience, and because (with a given optics diameter) the noise depends solely on the solid angle observed, we have expressed the noise levels X_B in terms of angular resolving powers, indicated at the bottom of the table.

[5] Let us note that the number of shots intervenes only by its relation to $\frac{1}{X_s}$, that is, by μ.

TABLE 2

Collecting optics diameter (m)		1.20	3	4.5
$X_S = nq$		0.02	0.10	0.20
Noise: Signal ratio allowing detection for 50 channels and N firings	$N=50$	0.05	8	
	$N=20$		1.2	5.5
	$N=10$		0.05	1.2
Noise: Signal ratio allowing detection for 100 channels and N firings	$N=50$	0.05	1.3	
	$N=20$		1.5	9
	$N=10$		0.05	1.5
Angular resolving power corresponding to the higher and lower noise levels in each column		2″	40″ 2″	35″ 10″

The very real advantage of a large diameter optical system can be seen straightway, even if it collects a considerable amount of noise.

We could thus contemplate using plastic mirrors, or solar furnace mirrors, for reception (on condition that their angular aperture can be brought below 0.1, without which it would be impossible to use narrow-band filters).

i. Conclusion: Choice of Receiving Optics

(a) The synchronization condition examined in the first place prevents us from separating transmitter and receiver by more than about 50 km.

(b) The desire to reduce both atmospheric absorption and scattered light leads us to choose high altitudes for the installation of the transmitter and above all, the receiver.

(c) The latter's resolving power, that is to say, the allowed noise level, varies with the received signal level, that is, the optics diameter.

Another result of the increase in this diameter and the correlative reduction in the resolving power needed is that the axes of the emitting and receiving optics no longer have to be so strictly parallel; for example, for a 3-m 50 diam, a disalinement of the order of 5 sec of arc would be allowed.

3. Expected Results

Now that the principal data of the experiment have been specified, we are going to ask what the results are likely to be, and above all with what accuracy we may hope to measure the distance.

3.1. Accuracy of Distance Measurement

This accuracy is limited by several factors:

(a) The duration of the pulse emitted by the laser, that is, about 50 nsec. This time interval corresponds to an uncertainty of 15 m in the distance.

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(b) The spread of the return pulse arising from the curvature of the Moon. For a 4-km diam spot, this spread represents 1 m at the frontal point (but 4 km at a latitude of 45°).

(c) The spread due to the relief of the Moon. Astronomical and radar measurements show, towards the frontal point, altitude gradients of up to 500 m for a 1° displacement on the lunar ground, that is, about 31 km. Thus, for a 4-km spot, we may in certain cases expect a further spread of nearly 100 m.

(d) To the above causes of uncertainty, inherent in the radar method, we must add the detection error, due to the fact that we shall not know at what instant the echo enters the channel where it will be detected. For this same reason we must try to reduce the duration of the successive channels. With 0.5-μsec channels, which seem feasible, accuracy would be of about ± 40 m (not counting lunar relief).

In conclusion, the possible precision will thus range from about 80 m to over 150 m, according to the lunar relief, the point sighted on the Moon, and the width of the channels used for detection.

It has been supposed that the speed of light propagation was known exactly; in actual fact, we must add to the preceding errors an uncertainty of the order of ± 1000 m, coming from *c*.

Since the radial speed[6] of the Moon may attain nearly 100 m/sec, we will have to know in advance to within 30 msec the instant of laser impact. Moreover, an adjustable retardation will have to be introduced in the release of the multichannel system, precisely in order to balance continually the Earth-Moon distance variation. This is possible, for—unlike the distance itself—the law governing the variation of this distance is known exactly.

The speed of displacement of the observatory, carried along by earth rotation, can attain (in projection on the Earth-Moon direction) nearly 300 m/sec. We must therefore also take this distance variation into account.

Lastly, atmospheric fluctuations introduce yet another error in the measurement taken. We know experimentally that these fluctuations are expressed by angular variations of the order of 1 to 2 sec. Given this figure, we can deduce an order of magnitude for the variations of the optical Earth-Moon path, since the same atmospheric fluctuations give rise to both phenomena. Thus we find that, on condition that the Moon's height exceeds 10° at the time of experiment, these distance fluctuations remain of less than a meter, and consequently negligible.

3.2. Accuracy of Angle Measurement

Once the distance of a point on the lunar ground is determined, a second problem arises: how to locate this point with precision, that is to say, be able to determine, for example, its latitude and longitude so that it may be found again when the Moon is presented differently to the instrument. Several factors contribute to the difficulty of such a location. In particular the resolving power of the telescope itself is limited to about 500 m, lunar relief does not always offer landmarks characteristic enough to allow precise reference, and finally, atmospheric refraction fluctuations can attain 1 or 2 arc seconds.

In any case, the two first points mentioned carry location errors of the order of a kilometer, so that, particularly in the neighborhood of the frontal point, the accuracy in measurement of the distance of a surface element will be far superior to that of the location of this element. Insofar as the uncertainties due to atmospheric fluctuations are concerned, it is possible to rid ourselves of them to a certain extent if we send enough pulses. We can then bring down the location error to below a certain limit which may be set arbitrarily.

To these various difficulties, common to all lunar observations, are added those resulting from the use of a laser and of a determined instrument. These are

(a) The errors connected with the telescope: mechanical vibrations, backlash, bendings, etc. . . . These errors, to which we must add tracking errors when the telescope is in movement, are known, as a rule, and remain inferior to 0.2 sec, that is, very slight. Generally speaking, however, they risk being accentuated by the installation of relatively heavy elements, such as the laser head, on the telescope. At all events, these errors can probably only be accurately estimated experimentally, by loading the telescope.

(b) Errors made in the orientation of the laser and of the transmitting equipment; in fact, it will be necessary to position the optical axis of the telescope with an accuracy of the order of 1 to 2 sec. For this we have two distinct methods available, a direct method, by receiving on another telescope the transmitted beam, and an indirect method, utilizing auxiliary optics inserted between the laser and the Cassegrain mirror of the telescope.

In conclusion, in order to reduce the influence of alinement errors, we shall have to increase the area observed, to the detriment of the S:N ratio. It seems a reasonable compromise to observe 2000 km^2, allowing a disalinement of $\pm 10''$.

To reduce the influence of atmospheric fluctuations, we shall have to send a sufficient number of laser pulses to the same point. This supposes a laser able to function a considerable number of times.

Finally, because of the lack of lunar landmarks, it will be necessary to take distance measurements in directions, if not well defined in relation to the Moon, at least known with exactitude in relation to each other.

Thus we shall be able to measure in turn the distance to the frontal point and the distances to points of which we know the exact angular divergence (seen from the Earth) with respect to the frontal point and each other. We shall explore in this way successive circles centered on the frontal point. By using a wedge plate to deviate the beam we can ensure, with-

[6] The corresponding Doppler laser frequency shift is negligible, that is, within, the filter band pass.

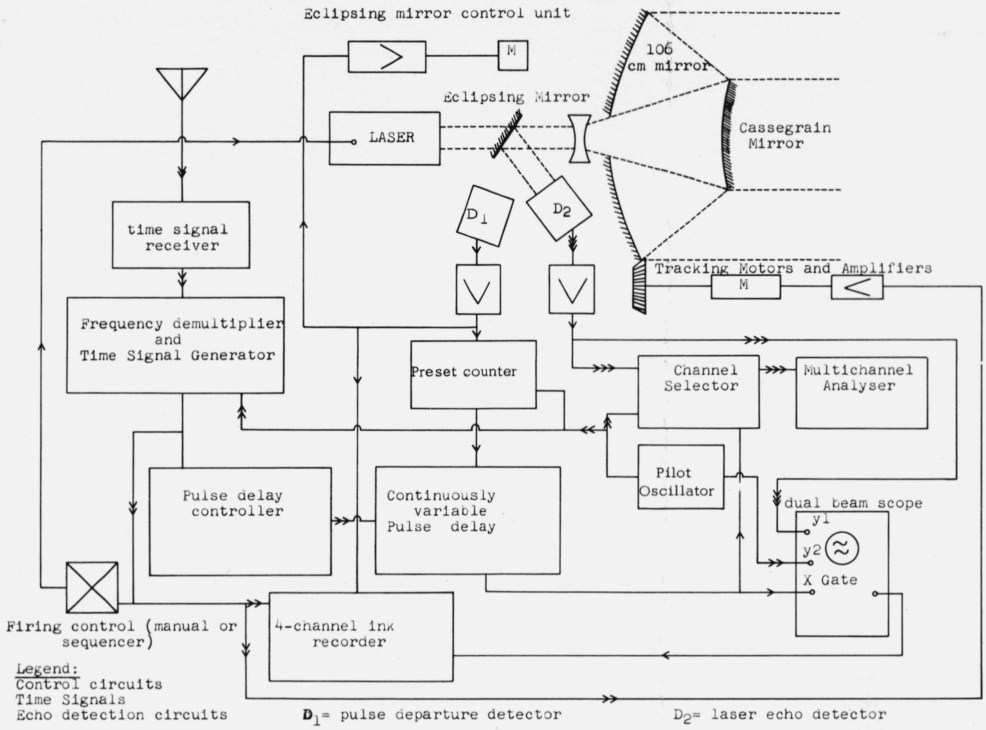

FIGURE 4. *The proposed experiment in block diagram form.*

out modifying any adjustment, that the successive circles are indeed centered onto the frontal point.

These series of shots will then result in a polar coordinate display—the pole being the observatory—of the Moon, in the neighborhood of its frontal point. It will then be a question of making the display thus obtained coincide as well as possible with existing maps, for example, by applying the methods advocated by several authors [Eckhardt and Hunt, 1960] at the Leningrad Congress.

4. Conclusion: Draft of the Experiment

We shall end this account by indicating (fig. 4) a summary draft of the proposed installation, and by briefly mentioning some of the problems posed in the utilization of a very powerful light source. These are the ionization of air and optical materials, the heating of the telescope transmitter mirrors, the radiation pressure on the mirrors, and lastly the influence of the pumping discharge on the receiving electronics and detector.

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