Radar Scattering From Venus and Mercury at 12.5 cm[1]

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A theory of radar scattering from a rough planetary surface based strictly on the geometrical-optics approximation has been published by Muhleman [1964]. It was shown in that paper that the intensity of the scattered radiation back to the radar from an element of area on a sphere of radius R is (unit flux illumination)

$$\delta I_b = \frac{R^2}{4} p(\theta) \sin\theta \, d\theta \frac{d\nu}{2\pi} \tag{1}$$

where θ is the angle of incidence (and reflection for the radar backscatter case), and $p(\theta)/2\pi$ is a probability density. The geometrical-optics approximation assumes that the surface is made up of flat facets whose normal vectors are tilted from the mean-surface normal by the random angles θ with a uniform azimuthal distribution. Thus, $p(\theta)/2\pi$ is the probability density of these tilt angles.

The density function was normalized to unity over the angles in the referenced paper which yielded the correct angular spectrum of the scattered energy but not the correct *total* power return. The proper normalization is obtained by requiring that the total area of the facets when projected to the mean spherical surface by the insertion of $\cos\theta$ sum to the area of the sphere. Thus the normalized returned intensity is found to be

$$\delta I_b = \frac{R^2}{8\pi} \frac{p(\theta)\sin\theta d\theta \, d\nu}{\int_0^{2\pi} p(\theta)\cos\theta\sin\theta d\theta} \tag{2}$$

where $p(\theta)$, so normalized, is now identically the angular backscatter function. The total intensity back to the radar is then obtained by integrating over the hemisphere,

$$I_b = \frac{R^2}{4} \frac{\int_0^{\pi/2} p(\theta)\sin\theta d\theta}{\int_0^{\pi/2} p(\theta)\cos\theta\sin\theta d\theta}. \tag{3}$$

Now it is of major importance in radar astronomy to be able to interpret the total measured power in terms of the directivity of the surface (related to the backscatter law), and the reflectivity of the planet's surface materials (related to the electrical parameters of the surface). If the planet's pulse response or Doppler spectrum is known from measurements, then $p(\theta)$ is known and the separation of directivity g and reflectivity ρ may be carried out with (3). The directivity is defined as the ratio of the backscatter intensity to the *mean* intensity scattered into all directions. The mean intensity is clearly $\pi R^2/4\pi$; therefore, from (3),

$$g = \frac{\int_0^{\pi/2} p(\theta)\sin\theta d\theta}{\int_0^{\pi/2} p(\theta)\cos\theta\sin\theta d\theta}. \tag{4}$$

1. The Planetary Scattering Law

Muhleman [1964] has derived a theoretical scattering law based on experimentally distributed height and length variations of the scattering facets which fits the observed lunar radar scattering measurements over a wide range of wavelengths. This law contains a single parameter, α, which varies with wavelength. It is our purpose here to determine the applicability of this law to 12.5-cm radar measurements of Venus and Mercury, to determine the statistics of roughness for these planets, and to determine the 12.5-cm directivities. The *unnormalized* probability density so obtained is

$$S(\theta) = \frac{\cos\theta}{[\sin\theta + \alpha\cos\theta]^3}, \tag{5}$$

which yields a directivity of

$$g \simeq 1 + \alpha - 2\alpha^2; \qquad \alpha < 1.0.$$

It can be shown that α is the *one-dimensional* mean slope, i.e., the expectation value of $\tan\theta$ computed with (5). A physically more meaningful statistic is the two-dimensional mean slope, where the mean is taken over the spherical planetary surface and is defined by

$$E(\tan\theta) = \frac{\int_0^{\pi/2} \tan\theta \, S(\theta)\cos\theta\sin\theta d\theta}{\int_0^{\pi/2} S(\theta)\cos\theta\sin\theta d\theta}. \tag{6}$$

Using (5), we get

$$E(\tan\theta) \simeq \sqrt{\alpha} \qquad 0.09 \leq \alpha \leq 5. \tag{7}$$

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2. Applications to Venus and Mercury

A typical observed radar spectrum of Venus taken with the JPL–Goldstone radar system (operated at 12.5 cm) is shown in figure 1. Also shown by the solid line is the spectrum arising from the theoretical scattering law with $\alpha = 0.10$. The agreement is excellent. The deviation from the model at a frequency of 0.5 is due to the presence of a well-understood spectral feature [Carpenter, 1965]. This value of α yields $g = 1.08$, which means that Venus deviates very slightly from an isotropic scatterer at 12.5 cm (i.e., smooth sphere). From (7), $E(\tan\,\theta) = 0.316$.

The observed spectrum of Mercury measured with the Goldstone system in 1963 is shown in figure 2. The quality of the observations is considerably inferior to that of the Venus measurements. Consequently, two α parameters were used to bracket the measurements. Figure 3 shows the 1965 Goldstone measurements of Mercury, which are somewhat smoother, primarily due to the use of less resolution. The model with $\alpha = 0.2$ is in good agreement with the data. An $\alpha = 0.2$ yields $g = 1.16$ and $E(\tan\,\theta) = 0.45$. Thus Mercury appears considerably rougher than Venus at 12.5 cm and it can be shown that Mercury's roughness is very similar to that of the Moon.

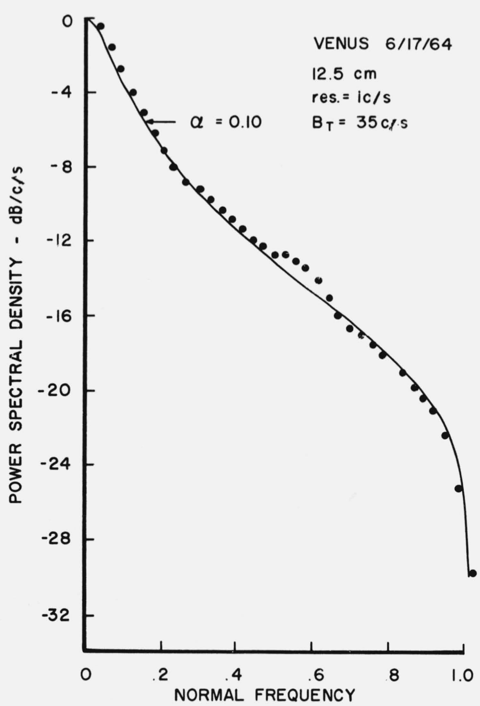

FIGURE 1. *Observed spectrum of Venus and the theoretical scattering model with $\alpha = 0.1$.*

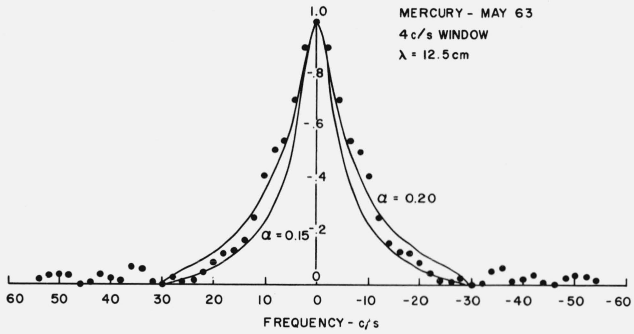

FIGURE 2. *Observed spectrum of Mercury and theoretical models with $\alpha = 0.15$ and 0.20.*

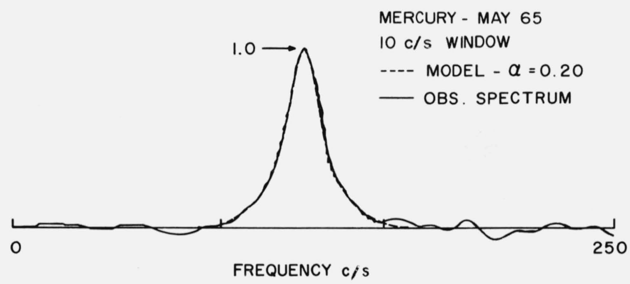

FIGURE 3. *Observed spectrum of Mercury and model assuming an 88 rotational period (1965).*

This interpretation of the Mercury spectra assumes a synchronous rotation rate for the planet which would cause a spectral width of about 70 c/s. The observed decay of the spectral wings supports this assumption. However, Pettengill [1965] reports a rotational period of 59 ± 5 days (as opposed to 88 days for synchronous rotation). A critical discussion of the rotational rate of Mercury is outside the scope of this paper, but we can say that the more rapid rate is not consistent with our observations.

3. References

Carpenter, R. (1965), 118th Meeting of the American Astronomical Society, Lexington, Kentucky.

Muhleman, D. O. (1964), Radar scattering from Venus and the Moon, Astron. J. **69**, No. 1, 34–41.

Pettengill, G. (1965), Symposium on Planetary Surfaces and Atmospheres, Dorado, Puerto Rico.

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