Dependence of Jupiter's Decimeter Radiation on the Electron Distribution in Its Van Allen Belts

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Numerical calculations are presented, which relate the intensity and polarization of Jupiter's decimeter radiation to the distribution of synchrotron-radiating electrons in its "Van Allen belts." The calculations are based on the simple model of a dipole magnetic field centered in the planet but inclined to the axis of rotation. Shadowing by the planet's disk is taken into account. An appropriate choice of the parameters of the model enables one to account for (1) the intensity, (2) the spectrum, (3) the beaming, and (4) the degree of polarization of Jupiter's decimeter radiation. However, the model *cannot* account for the observed asymmetries in the beaming and polarization.

1. Introduction

Nearly 4 years ago the author undertook a study of the theory of synchrotron radiation from stars with dipole magnetic fields (results reported in [Thorne, 1963]). As part of this study a model was constructed in which the beaming of the radiation was determined by the pitch-angle distribution and energy distribution of the radiating electrons. Only after this study was completed did the author become aware of the discovery by Morris and Berge [1962] of the obliquity of Jupiter's magnetic field and of the consequent ability of Earth-based observers to study the beaming of Jupiter's decimeter radiation. However, J. A. Roberts and M. M. Komesaroff realized immediately that the author's calculations for stars are also applicable to Jupiter; and they communicated with the author concerning the interpretation of the decimeter radiation in the light of those calculations. In order to make the calculations more applicable to Jupiter, the author modified the original stellar model to take into account shadowing by the planet's disk and the absence of electrons whose mirror points would be inside the planet. The results of calculations based on this modified model were sent to Roberts and Komesaroff, who used them in interpreting their recent observations of Jupiter's decimeter radiation [Roberts and Komesaroff, 1964, 1965].

The purpose of this paper is three fold: (1) To present the numerical results which were used by Roberts and Komesaroff in interpreting their observations; (2) to present results of additional numerical calculations based on the same model; (3) to use these new calculations to extend the interpretation begun by Roberts and Komesaroff.

In addition to the results reported here, theoretical calculations of synchrotron radiation from Jupiter have been made by Chang and Davis [1962]; Korchak [1963], and by Ortwein, Chang, and Davis [1965].

2. The Model

The model on which the calculations are based is as follows: (For additional details see [Thorne, 1963].) The magnetic field of Jupiter is idealized as a dipole field centered at the center of the planet. The radiating electrons are confined to a narrow dipolar shell of equatorial radius equal to 3 Jovian radii and of thickness ΔR. Shadowing by the planet is precisely confined to the region of the dipole shell hidden from the Earth by the planet's disk. At the equator the distribution of electron energies, E, and of pitch angles (angle between velocity vector and magnetic field), α_e, is taken as

$$N_v(E, \alpha_e) = \frac{A_0}{\Delta R} \frac{E^{-\gamma}}{(mc^2)^{1-\gamma}} \sum_q a_q \sin^q \alpha_e, \quad \text{if} \begin{cases} E_1 < E < E_2, \text{ and} \\ \alpha_{\text{crit}} < \alpha < (\pi - \alpha_{\text{crit}}); \end{cases} \tag{1}$$

$$N_v(E, \alpha_e) = 0, \text{ if} \begin{cases} E < E_1, \text{ or } E > E_2, \\ \text{or } \alpha_e < \alpha_{\text{crit}}, \text{ or } \alpha_e > (\pi - \alpha_{\text{crit}}); \end{cases} \tag{2}$$

where A_0, γ, E_1, E_2, a_q, and α_{crit} are constants. The constant α_{crit} is chosen so as to rule out electrons with mirror points inside the planet. It is that pitch angle which corresponds to electrons with mirror points at Jupiter's surface

$$\alpha_{\text{crit}} = \sin^{-1}(3^{-7/4}) = 8.4°$$

[cf Thorne, 1963 (12) and (1b)]. The constant A_0 fixes the total number of radiating electrons, and hence the intensity of the radiation. The constants E_1, E_2, and γ determine the electron energy distribution, and hence the spectrum of the radiation. The amplitude factors a_q fix the electron pitch angle distribution, and hence the beaming and polarization of the radiation.

This model differs from that of Thorne [1963] only in this, that shadowing by the planet's disk and the absence of electrons with mirror points inside the planet are taken into account. Comparison of numerical computations based on this model with computations based on the original model reveals that shadowing and high mirror-point emission have only minor effects on the integrated intensity and polarization of the radiation. For instance, the total effect on the intensity is $\lesssim 7$ percent for $\nu_{\min}<\nu<\nu_{\max}$ (cf (5)).[1]

3. Numerical Results

Roberts and Komesaroff [1965] have confirmed the finding of Gower [1963] that the spectrum of the decimeter radiation from Jupiter is flat

$$I_\nu \equiv dI/d\nu \propto \nu^0. \tag{4}$$

If the decimeter region lies in the frequency range $\nu_{\min}<\nu<\nu_{\max}$, where

$$\nu_{\min}=1.61\times 10^8 \times (B_{0_{\text{gauss}}})\times (E_{1\text{MeV}})^2 \text{ sec}^{-1},$$

$$\nu_{\max}=1.61\times 10^5 \times (B_{0_{\text{gauss}}})\times (E_{2\text{MeV}})^2 \text{ sec}^{-1}, \tag{5}$$

and B_0 is the equatorial field strength at 3 Jovian radii, *then* our model predicts [cf Thorne, 1963]

$$I_\nu \propto \nu^{-(\gamma-1)/2}, \tag{6}$$

and consequently, the energy exponent for the electrons of Jupiter's Van Allen belt must be $\gamma=1$. In table 1 we present data which enable one to calculate the intensity and polarization of the radiation as a function of observer's polar angle for $\gamma=1$ and $\nu_{\min}<\nu<\nu_{\max}$. In particular, we give the intensity, $I_{\nu q}$, and polarization, P_q, for the special distributions of pitch-angle,

$$a_{q'}=\delta_{qq'}=\begin{cases}0 \text{ if } q'\neq q\\ 1 \text{ if } q'=q.\end{cases} \tag{7}$$

For the more general distribution (1) the intensity and polarization are (superposition principle)

$$I_\nu=\sum_q a_q I_{\nu q},$$

$$P=(1/I_\nu)\left(\sum_q a_q P_q I_{\nu q}\right). \tag{8}$$

The data of table 1 are *not* the data used by Roberts and Komesaroff [1965]. Since data for $\gamma=1$ were not available at the time of their analysis, they used the results of computations for $\gamma=5/3$, which are presented in table 2.

We do not know *à priori* that the decimeter region lies in the frequency range $\nu_{\min}<\nu<_{\max}$. Consequently, it is of interest to examine the beaming and polarization for the regions $\nu<\nu_{\min}$ and $\nu>\nu_{\max}$. This is done in tables 3 and 4 for $\gamma=5/3$ (the analysis for other values of γ is much more difficult) and for $q=3.3$.

[1] In figures 6 and 7 of [Thorne, 1963] the author presented estimates of the effects of shadowing and of high-latitude emission. It was not stated there, but it should have been, that those estimates are merely upper bounds and that for $r_{e0}/R \lesssim 4$ they are considerably larger than the true effects of shadowing and high latitude emission. Hence, figures 6 and 7 are not useful in studying Jupiter's decimeter radiation, for which $r_{e0}/R \approx 3$.

TABLE 1. *Intensity and polarization as functions of* q *and* θ_0 *for* $\gamma=1.0$, $\nu_{min}<\nu<\nu_{max}$[a]

q	$\theta_0=90°$		$\theta_0=87°$		$\theta_0=83°$		$\theta_0=80°$		$\theta_0=77°$	
	$I_{\nu q}$	P_q	$I_{\nu q}$	P_q	$I_{\nu q}$	P_q	$I_{\nu q}$	P_q	$I_{\nu q}$	P_q
1	46.9	−0.063	47.1	−0.062	47.3	−0.059	47.4	−0.056	47.9	−0.047
2	27.4	+.103	27.4	+.103	27.4	+.104	27.4	+.103	27.4	+.104
3	19.8	.228	19.8	.228	19.7	.227	19.6	.225	19.4	.223
4	16.0	.310	15.9	.310	15.8	.309	15.7	.307	15.4	.304
5	13.7	.365	13.7	.365	13.5	.364	13.3	.361	13.0	.358
6	12.2	.404	12.1	.403	12.0	.402	11.7	.399	11.4	.397
8	10.2	.453	10.1	.452	9.91	.451	9.60	.448	9.22	.445
10	8.93	.482	8.87	.482	8.61	.480	8.25	.478	7.82	.475
13	7.69	.510	7.62	.509	7.31	.508	6.90	.506	6.42	.503
16	6.86	.527	6.77	.527	6.42	.525	5.97	.523	5.45	.520
20	6.07	.542	5.97	.541	5.58	.540	5.09	.538	4.53	.535
25	5.39	.553	5.28	.553	4.83	.552	4.30	.549	3.72	.547
30	4.89	.561	4.77	.561	4.28	.559	3.72	.557	3.12	.555
50	3.74	+.577	3.59	+.577	2.98	+.575	2.36	+.573	1.77	+.571

[a] The quantities q and γ are defined in (2) and (1); $\nu_{\min}$ and $\nu_{\max}$ are defined in (5); θ_0 is the angle between the magnetic dipole axis and the observer's line of sight ("observer's polar angle") and $I_{\nu q}$ and P_q are the intensity and polarization of the radiation for the special pitch-angle distribution (7). $I_{\nu q}$ is measured in units of

$$K'=\frac{A_0\sqrt{3}}{8\pi}(\mu_0 e^2 c)\left(\frac{eB_0}{2\pi m}\right)(3R)^2 \quad \text{(MKS)},$$

where R is the radius of Jupiter and B_0 is the equatorial field strength in the emission region (3 Jovian radii). This table was calculated from (26) and (27) of Thorne [1963] with obvious modifications for shadowing effects and for the absence of high mirror-point emission. The entries in the table are accurate to ± 1 percent.

TABLE 2. *Intensity and polarization as functions of* q *and* θ_0 *for* $\gamma=5/3$ *and* $\nu_{\min}<\nu<\nu_{\max}$[a]

q	$\theta_0=90°$		$\theta_0=77°$	
	$I_{\nu q}$	P_q	$I_{\nu q}$	P_q
2	16.4	+0.096	16.5	+0.131
3	10.8	.180	10.6	.174
3.3	9.86	.220	9.66	.213
4	8.33	.297	8.06	.288
5	6.99	.374	6.64	.365
6	6.12	.426	5.73	.417
8	5.06	.490	4.58	.481
10	4.41	.528	3.86	.519
13	3.78	.562	3.15	.554
16	3.35	.583	2.66	.574
20	2.96	.600	2.21	.592
25	2.62	.614	1.81	.606
30	2.38	.623	1.52	.616
50	1.82	+.641	0.856	+.634

[a]Data used by Roberts and Komesaroff [1965]. Symbols used here are described in footnote to table 1, except that $I_{\nu q}$ is measured in units of

$$K'=\frac{A_0\mu_0e^2c}{4\pi\sqrt{2}}\left(\frac{3}{2}\right)^{5/6}\left(\frac{eB_0}{2\pi m}\right)^{8/6}(3R)^2\nu^{-1/3}\ (\text{MKS}).$$

The entries in the table are accurate to ±1 percent.

TABLE 3. *Intensity and polarization as functions of* ν/ν_{min} *and* θ_0 *for* $\gamma=5/3$, $q=3.3$, *and* $\nu<\nu_{max}$ (*"Takeoff Region"*)[a]

$\nu/\nu_{\min}$	$\theta_0=90°$		$\theta_0=77°$	
	$I_{\nu q}$	P_q	$I_{\nu q}$	P_q
0.001	0.47	0.26	+0.46	+0.26
.003	.96	.26	.94	.26
.01	2.06	.268	2.02	.265
.03	3.85	.276	3.76	.273
.1	6.83	.289	6.67	.284
.3	9.02	.269	8.81	.262
1.0	9.71	.232	9.46	.222
	9.86	+.220	9.66	+.213

[a]For explanation of symbols see footnote to table 2. Accuracy: ±1 percent. Note that the entries in this table are related to those of table 4 by $I(\nu/\nu_{\min})_{\text{table }3}=[I(\nu/\nu_{\max}=0)-I(\nu/\nu_{\max}=0.001\ \nu/\nu_{\min})]_{\text{table }4}$.

TABLE 4. *Intensity and polarization as functions of* ν/ν_{max} *and* θ_0 *for* $\gamma=5/3$, q$=3.3$, *and* $\nu>\nu_{min}$ (*"Takeoff Region"*)[a]

$\nu/\nu_{\max}$	$\theta_0=90°$		$\theta_0=77°$	
	$I_{\nu q}$	P_q	$I_{\nu q}$	P_q
0.	9.86	+0.220	9.66	+0.213
1.	9.39	.218	9.20	.211
3.	8.90	.216	8.72	.209
10.	7.80	.208	7.64	.200
30.	6.01	.184	5.90	.175
100.	3.03	+.065	2.99	+.056
300.	0.835	−.304	0.852	−.293
1000.	.151	−.546	.206	−.196

[a]For explanation of symbols see footnote to table 2. Accuracy: ±1 percent.

4. Fitting of the Model to Jupiter

The simplest distributions of electron energies and equatorial pitch angles which seem capable of reproducing the decimeter radiation of Jupiter as measured by Roberts and Komesaroff [1965] are these:

$$N_v(E,\alpha_e)=\frac{2.2\times10^{-4}}{B_0\Delta R}\frac{\sin^3\alpha_e}{E}\frac{\text{cm}^{-3}}{\text{MeV}},\ \text{for 74 cm data;}\quad (9\text{a})$$

$$N_v(E,\alpha_e)=\frac{1.2\times10^{-4}}{B_0\Delta R}\frac{(\sin^2\alpha_e+2.0\sin^{40}\alpha_e)}{E},$$

$$\text{for 21 cm data;}\quad (9\text{b})$$

with $\nu_{\min}<\nu_{\text{decimeter}}<\nu_{\max}$—i.e., with

$$E_2^2B_0\gtrsim2\times10^4,\qquad E_1^2B_0\lesssim2.\qquad (10)$$

Here ΔR is the thickness of the radiating shell as measured in units of Jovian radii; B_0 is the equatorial magnetic field strength at 3 Jovian radii (the emission region) as measured in gauss; and the energies E, E_1, and E_2 are measured in MeV. These electron distributions are capable of reproducing (1) the intensity, 6.7×10^{-26} $\text{Wm}^{-2}(\text{c/s})^{-1}$, of the radiation observed at Earth; (2) the spectrum of the radiation, $I_\nu\propto\nu^0$; (3) the beaming of the radiation,

$$I_\nu\ (13°\ \text{magnetic latitude})/I_\nu\ (\text{magnetic equator})=\begin{cases}0.89 & \text{at 21 cm}\\ \sim1.0 & \text{at 74 cm;}\end{cases}$$

and (4) the degree of polarization of the radiation,

$$P\ (\text{magnetic equator})\approx0.22$$

$$P\ (13°\ \text{magnetic latitude})\approx\begin{cases}0.22\ \text{at 74 cm}\\ 0.18\ \text{at 21 cm.}\end{cases}\qquad (12)$$

However, so long as the magnetic field is assumed to be that of a centered dipole, no electron distribution can account for the asymmetries in the radiation observed by Roberts and Komesaroff.

Clearly, the electron distribution cannot be of the forms (9a) and (9b) simultaneously. Rather, as was first noted by Roberts and Komesaroff [1965], the true distribution may be a mixture of these, in which the lower energy electrons have pitch angles similar to (9a) while the higher energy electrons have pitch angles similar to (9b).[2]

The electron distributions (9) are not completely unique; there are other distributions which will also reproduce the observed features of the radiation. However, the following limits can be put on the distribution:

Pitch angles. Any one-component model which fits the 74 cm data must have $q\approx3$. For the 21 cm data, as was first noted by Roberts and Komesaroff [1965], one is forced to invoke *at least* two groups of

[2]J. A. Roberts reported at this conference that the apparent dependence of pitch angle on energy is in considerable doubt. Because his recent 48 cm data resemble closely the data at 21 cm, Roberts is inclined to disbelieve the low degree of equatorial beaming which seems to characterize his 74 cm data.

electrons—one group with pitch angles distributed nearly isotropically ($q \lesssim 2$) and the other with very flat pitches ($q \gtrsim 20$).[3]

Energy. The direction of polarization of Jupiter's decimeter radiation is observed to be orthogonal to the projection of the magnetic axis on the plane of the sky; but for $\nu \gtrsim 100\ \nu_{\max}$, our model demands (table 4; also Thorne [1963]) that the radiation be polarized parallel to the magnetic axis. Consequently, we can be certain that $\nu_{\text{decimeter}} \lesssim 100\ \nu_{\max}$, or, equivalently, that the high-energy cutoff in the electron energy distribution satisfies

$$[(E_2)_{\text{MeV}}]^2 \times (B_0)_{\text{gauss}} \gtrsim 200. \tag{13}$$

The more stringent bound, $\nu_{\text{decimeter}} < \nu_{\max}$ ($E_2^2 B_0 \gtrsim 2 \times 10^4$), seems quite likely, since the spectrum could be flat over a section of the region $\nu > \nu_{\max}$ only if $\gamma < 1$; and $\gamma < 1$ seems unlikely on physical grounds.

Although we are fairly certain that $\nu_{\text{decimeter}} < \nu_{\max}$, we cannot be sure that $\nu_{\min} < \nu_{\text{decimeter}}$ ($[E_{1\text{MeV}}]^2 B_{0_{\text{gauss}}} < 2$). Table 3 reveals that the beaming and polarization of the radiation in the region $\nu < \nu_{\min}$ is not too different from that for the region $\nu_{\min} < \nu < \nu_{\max}$. However, the spectrum is quite different in these regions. If $\nu_{\text{decimeter}} < \nu_{\min}$, then the energy exponent, γ, must exceed 1 in order for the decimeter spectrum to be flat. In fact, γ might be as large as 5 (the value for the Earth's Van Allen belt—[cf O'Brien et al., 1962]—if $\nu_{\min}$ were sufficiently large. Such a situation cannot be ruled out.

[3] In a private communication Roberts and Komesaroff note that the particular 2-component pitch angle distribution given in their paper [Roberts and Komesaroff, 1965] to fit the 21 cm data is not correct; but that the necessity for two components, one with $q \lesssim 2$ and the other with $q \gtrsim 20$, is unavoidable.

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Observations of Jupiter at 8.6 mm

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Attempts to obtain measurements of the brightness temperature of Jupiter at 8.6 mm were made on several occasions near the opposition in May, 1959. With the 10-ft reflector the expected antenna temperature was low, and it was necessary to average repeated drift scans to obtain a measurable deflection. Atmospheric fluctuations nullified the results on some occasions, but analytical criteria found effective in more recent work have enabled the measurements for three days to be evaluated with some confidence. These results were

1–2 May, 1959	308 ± 88 °K (p.e.)
6–7 May	291 ± 88 °K
8–9 June	260 ± 90 °K

These values, obtained with north-south polarization, exceed the expected temperature by roughly a factor of two, and seem to indicate an anomalous effect in this period. The stated uncertainties were derived from the random fluctuations in the drift scans, with a relatively small systematic error not included. However, it should be emphasized that some chance exists that these results are in large error. For example, on another day, June 10, data considered to be comparable in quality yielded a blurred unmeasurable result. Therefore, further observations at various periods would be useful.

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