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Comments on H. Volland's "Remarks on Austin's Formula"

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In a recent paper (published in Nachr. Techn. Zeitschr.) Volland proposes an analytical model of the earth-ionosphere waveguide which is claimed to be in basic agreement with the predictions of the semiempirical transmission formula of L. W. Austin for frequencies less than 300 kc/s. In this note, the flat-earth assumptions made by Volland are questioned and any agreement of his result with Austin's formula is attributed to a fortuitous cancellation of errors.

It is rather interesting to note that H. Volland [1964] has recently published a paper under the title "Bemerkungen zur Austin'schen Formel" (Remarks on Austin's formula). This basic radio transmission formula was proposed by L. W. Austin and published in the Bulletin of the Bureau of Standards over 50 years ago [Austin, 1915]. Volland proposes an analytical model of the earth-ionosphere waveguide which is claimed to be in basic agreement with the predictions of the semi-empirical Austin formula for frequencies less than 300 kc/s. At the same time, Volland states that a flat-earth assumption is adequate and he asserts that the curvature of the earth leads only to second-order corrections and, thus, may be neglected. Since this latter statement is in violent contradiction to most of the recent theoretical work on VLF and LF mode theory of propagation, some comments on Volland's paper are called for.

To properly orient the reader, a brief survey of Volland's formulation is given. His model consists of a flat homogeneous earth with a dielectric constant ϵ_e and conductivity σ_e. The ionosphere is represented by a reflecting level at a height h above the earth's surface. His formula, in cylindrical coordinates (ρ, ϕ, z), for the vector potential of a vertical dipole source of moment Il is given by

$$\Pi_e = -\frac{Il}{4hf}\sum_{n=0}^{\infty} G_n H_0^{(2)}(kS_n\rho)\,[2\cos kC_n(z-z_0)$$

$$+R_e \exp(-ikC_n(z+z_0)) + R_e^{-1}\exp(+ikC_n(z+z_0))\,], \quad (1)$$

where

f = operating frequency

$k = 2\pi f/c$

$\left.\begin{matrix} z \\ z_0 \end{matrix}\right\} = \text{height of} \begin{cases} \text{receiving antenna} \\ \text{transmitting antenna} \end{cases}$

$\left.\begin{matrix} R_e \\ R_i \end{matrix}\right\} = \text{reflection coefficient of} \begin{cases} \text{earth} \\ \text{ionosphere} \end{cases}$

$S_n = (1 - C_n^2)^{1/2}$

$$G_n = \frac{1}{\left\{1 + \frac{i}{2khR_eR_i}\frac{\partial R_eR_i}{\partial C}\right\}_{C=C_n}}. \quad (2)$$

The "eigenvalue" C_n of the nth mode is to be determined from

$$R_e(C_n)R_i(C_n)\exp(-i2khC_n) = \exp(-i2\pi n) \quad (3)$$

$$(n = 0, 1, 2, \ldots);$$

C_n can be interpreted as the cosine of the (complex) angle of incidence θ_n of the nth mode in the waveguide.

Now, it may be remarked that (1) is an exact representation of the mode sum for a flat-earth model. An identical result has been given by Wait [1960, 1962] and it also agrees with the special case [$\sigma_e = \infty$ or $R_e = 1$] derived by Al'pert [1955] and Budden [1962]. However, it might be mentioned, in passing, that even for a flat earth, (1) is not complete as, in addition, there is a contribution from the "branch-line integrals." Fortunately, for the earth-ionosphere waveguide, these are negligible, as pointed out in the quoted references.

A more serious objection to (1) has to do with the influence of earth curvature. Volland attempts to deal with this question by multiplying each term in the mode sum by a factor B_n which is defined by

$$B_n = \left(\frac{\theta}{\sin\theta}\right)^{1/2}\exp\left(-\frac{ikhS_n\rho}{2a}\right), \quad (4)$$

where $\theta = \rho/a$ and a is the radius of the earth. Clearly, the logic behind this step is that the factor $(\theta/\sin\theta)^{1/2}$ accounts for the horizontal convergence, while the exponential factor simply transfers the phase reference from a point midway in the waveguide to the bottom surface (i.e., the ground). Volland further approximates by setting $S_n=1$ in the exponential factor.

It is the contention of this writer that Volland's curvature corrections are inadequate for the upper part of the VLF band and certainly for the whole LF band. The reasons for invoking higher order curvature corrections were pointed out by Wait and Spies [1960]. They showed that (1) is only an adequate approximation to the spherical-earth mode equation when the following conditions are simultaneously satisfied:

$$|C_n^2| \gg (h/a) \text{ and } \mathrm{Re}\, C_n(ka/2)^{1/3} > 2 \text{ or } 3.$$

To indicate how seriously violated these conditions may be, we note that under typical daytime conditions Re C_n may actually be zero at a frequency near 14 kc/s when $n=1$. This corresponds to the phase velocity of the dominant mode coinciding with c, the velocity of light. In this case, for frequencies less than 14 kc/s, the velocity would be greater than c whereas, for frequencies greater than 14 kc/s, the velocity is less than c. Such a behavior has been observed experimentally and such results are discussed in an available book [Wait, 1962]. It is quite clear that any flat-earth model will be inadequate in this situation, even when semiempirical curvature corrections are made. Furthermore, the attenuation rates of the curved-earth model may differ by as much as a factor of three over those computed for the flat-earth model for frequencies of the order of 30 kc/s [Wait and Spies, 1960].

The reader might now wonder how Volland's formulas could predict field strengths of the correct order when the computed attenuation rates of the modes are grossly in error. The answer appears to be related to his "excitation factors" D_n which, again, are computed on the basis of a flat earth. These are defined by

$$D_n = \frac{G_n S_n^{3/2}}{4}(2 + R_e + 1/R_e),$$

where G_n is given by (2). Now, near grazing incidence (i.e., $C_n \to 0$), this formula would indicate that D_n tends to vanish since R_e tends to -1 for a finitely conducting ground. This is certainly in accord with Volland's calculation for the LF region of the spectrum. As a result of this behavior, the vanishing of D_n tends to compensate partially for his low attenuation rates of the modes. Unfortunately, however, the above formula for D_n is not applicable for a curved earth when C_n is small. It is only when the condition Re $C_n(ka/2)^{1/3} > 2$ or 3 is satisfied that the above simple formula for D_n may be used. As it turns out, the correct form of the excitation factor for a spherical earth does not vanish as $C_n \to 0$ [e.g., see Wait, 1962 and Watt and Croghan, 1964].

Apart from the quantitative disagreement between Volland's calculated results in the LF band and the corresponding results based on a curved-earth model, some important differences exist in the qualitative description of the propagation phenomena. For example, Volland claims that the ground conductivity plays the dominant role in the propagation to large distances in the LF band. (In fact, his formulas would show enormous fields for an all sea water path.) On the other hand, the correct spherical-earth formulas for the low-order modes in the LF band would indicate that earth curvature reduces the excitation factor, primarily because of diffractive effects. This is a consequence of the "earth- detached" character of the low-order modes in the curved waveguide.

In solving the flat-earth mode equation given here by (1), Volland makes various kinds of approximations which depend on the frequency range considered. For example, at ELF, R_e and R_i are regarded to be near $+1$; at VLF, $R_e \sim +1$ and $R_i \sim -1$; at LF, R_e and $R_i \sim -1$. In this way, explicit expressions for the propagation constants are obtained. In the ELF range, these are equivalent to the results of Schumann [1952], while in the VLF band, the results are a refinement of the ones given by Wait [1957]. The latter results are based essentially on a flat-earth model although, for frequencies below 10 kc/s, the error introduced by neglecting curvature is not great [e.g., see Johler and Berry, 1962, 1964]. However, in the LF band, the idea that the solution may be obtained by perturbing about $R_e \sim -1$ would seem to be in serious conflict with the full theory for the curved waveguide [i.e., Wait, 1962]. Consequently, this limiting condition is highly artificial and for this reason alone many of Volland's numerical results could be questioned.

A comment on Volland's remarks on the influence of the earth's magnetic field is also called for. He points out correctly that linearly polarized LF waves reflected from the ionosphere at highly oblique incidence produce only small cross-polarized components. He then argues that the earth's magnetic field should have only a small effect. To show that this argument is fallacious, we may simply note that for a purely transverse magnetic field, there is no cross polarization, yet the reflection coefficient for vertical polarization is actually nonreciprocal as first pointed out by Crombie [1958]. As indicated, both by Crombie [1960] and by Wait and Spies [1960], this leads to attenuation rates for east-to-west propagation which may be as much as a factor of three greater than propagation from west to east at 18 kc/s, for example. Clearly, this nonreciprocal effect should be considered in any prediction formulas for VLF or LF propagation.

It is a pity that Volland would publish these invalid prediction formulas with continued references to Dr. L. W. Austin, who was a pioneer in the experimental study of LF radio propagation. Austin's later papers [e.g., 1926, 1931a, 1931b], not mentioned by Volland, are also well worth reading by present-day investigators.

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