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An Anisotropic Electron Velocity Distribution for the Cyclotron Absorption of Whistlers and VLF Emissions[1]

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In 1962, Scarf proposed cyclotron absorption as the physical mechanism explaining the high-frequency cutoff of nose whistlers. Using the Scarf proposal, two anisotropic electron velocity distributions for the magnetosphere are assumed and the complex refractive index is evaluated for each. The transverse (with respect to the ambient earth's field) velocity distribution in each case is assumed Maxwellian. The first longitudinal distribution of velocities considered is Maxwellian to several mean square velocities and then is proportional to v^{-1}. The cyclotron damping term is then evaluated; however, upon investigation, the rate of change of whistler damping with frequency is found to be insufficiently rapid to agree with the observed whistler cutoff. The second velocity distribution considered is a double-humped Maxwellian, i.e., a thermal electron distribution, and a resonant electron stream. This distribution allows for cyclotron absorption and, at the same time, is consistent with the whistler dispersion and attenuation. Arising from the analysis of the double-humped Maxwellian distribution is a transverse instability developed by Bell and Buneman which leads to VLF emissions. The relative importance of cyclotron damping vis-a-vis VLF emissions is examined and a qualitative explanation of some whistler-induced emissions is suggested.

1. Introduction

Whistlers are naturally occurring electromagnetic signals in the audio frequency range that result from the dispersion of lightning energy which has traveled through the outer ionosphere along the lines of force of the earth's magnetic field. The term *nose whistler* designates a particular class of whistlers that exhibits simultaneous rising and falling tones starting at a particular frequency called the *nose frequency*. The dispersion of the energy from the lightning source results from the action of free electrons in the presence of a circularly polarized electromagnetic wave (whose sense of rotation is the same as that of the free electrons in the ambient magnetic field).

A multipath whistler is a group of whistler traces originating from a single atmospheric that excites different magnetosphere paths. A typical frequency-time characteristic of a multipath whistler is illustrated in figure 1 of an accompanying article appearing in this journal [Guthart, 1965]. The spectrogram shows the whistler amplitude, measured by the display intensity, as a function of time (abscissa) and frequency (ordinate). The vertical lines in the spectrogram indicate impulsive atmospherics. The horizontal lines indicate constant-frequency signals, usually power-line harmonics. The three strongest whistler traces of Guthart's [1965] figure 1 are identified at the bottom of the spectrogram by the letters A, B, and C.

Each of the three marked traces is in reality a group of several whistler-components, a fact most easily seen in Trace B. The nose frequency of the leading whistler of each of the three traces is indicated by the frequency indicated to the left of the spectrogram. The whistler traces are derived from a single atmospheric source 1.78 sec prior to the arrival of the nose frequency of Trace A. This source is not shown in the figure, but its character is very much like many of the vertical lines seen in Guthart's [1965] figure 1.

The nose-whistler upper cutoff frequency is rarely as much as 60 percent greater than the nose frequency. The upper cutoff frequency is most often extremely sharp (see annotated whistler of fig. 5b in Guthart [1965]). The damping at cutoff is of the order of 20 dB for frequency changes of

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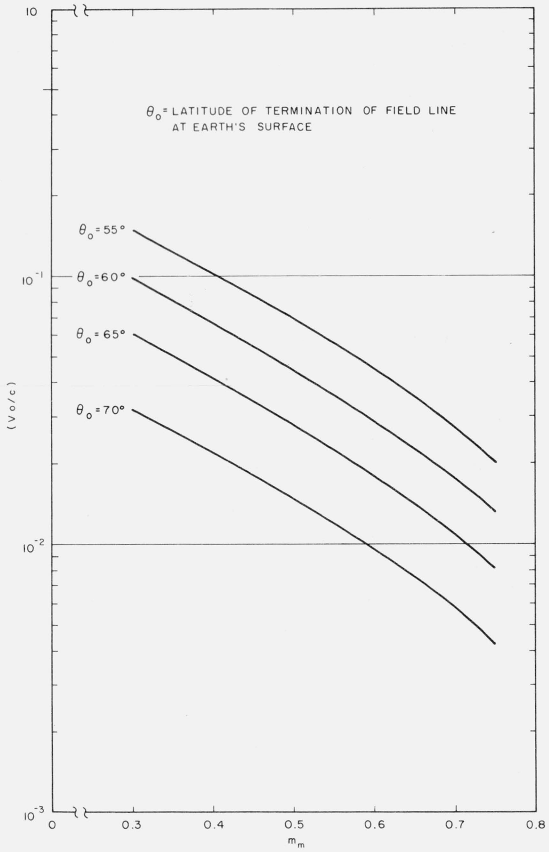

FIGURE 1. *Normalized stream velocity* (V_0/c) *required for cyclotron resonance as a function of the maximum normalized frequency*, m_m.

less than 2 percent. The damping rate is estimated by reducing the intensity of the whistler spectrogram, using a precision attenuator, until it is no longer discernible from the background. In many instances, VLF emissions are triggered at the upper end of nose whistlers; in a few cases, the upper cutoff is defined by a more gradual rate of change of attenuation of whistler energy with frequency. It will be shown that the absorption and emission of VLF energy are manifestations of the same phenomenon. The more gradual rate of attenuation around cutoff of some whistlers can then be interpreted as the near cancellation of the decay and growth processes.

The first proposed explanation for the relatively low upper cutoff frequency of nose whistlers (cold-plasma theory predicts that the upper cutoff frequency for whistler propagation is the minimum gyrofrequency along the path) was made by Smith [1960]. Smith elegantly derived an explanation of the nose whistler observations, based upon trapping of the whistler energy in ducts of enhanced ionization. Smith notes that the magnetosphere has a very low collision frequency; hence, the particles are constrained by the magnetic field to move along the field lines, giving long life to density irregularities perpendicular to the field. He then assumes that the duct properties change very slowly in the direction of the field lines, so that he can represent the propagation path by a slab model in which the field is uniform and in which the electron density has a gradient only in a direction transverse to the field. For density gradients that are small in the space of a wavelength in the medium, ray theory is then applied.

Using the ray-tracing approach, Smith found, for example, that for a density enhancement of 5 percent, the whistler energy was trapped in the duct when the frequency was less than one-half the nose frequency, and for wave normals within 45° of the magnetic field. Furthermore the effective group velocity of the trapped energy is most always within 1 percent of the value obtained assuming purely longitudinal propagation (i.e., the wave normal and magnetic field are alined).

For a uniform duct, the upper cutoff frequency for trapped energy remains at 50 percent of the gyrofrequency. For a duct whose properties change slowly in the direction of the field lines, the cutoff can exceed 50 percent of the minimum gyrofrequency and still be trapped in the duct. Smith's theory for nose whistler propagation is impressively supported by experimental evidence. Only the whistler properties at the upper cutoff frequency fail to satisfy the duct hypothesis. They fail to satisfy the duct hypothesis in two respects. First, the upper cutoff frequency of 50 percent predicted by the uniform duct theory is frequently exceeded. Second, the experimentally observed whistlers exhibit a sharp cutoff frequency. It appears that both these inconsistencies of the duct theory may possibly be made less extreme by the calculation of a complete ray tracing in nonuniform ducts. Such a calculation would restrict the duct shape as well as the angular distribution of whistler wave normals. At the present time, there is no experimental evidence to support such restriction. As a result, a body of work has evolved which accepts the general results of the duct hypothesis, but looks for an alternative explanation of the high-frequency cutoff.

As an alternative explanation of the high-frequency cutoff, Scarf [1962] and Tidman and Jaggi [1962] proposed the cyclotron absorption mechanism to account for the observed high-frequency cutoff of whistlers. Cyclotron absorption is a mechanism to account for the absorption of energy from a wave by particles in a collision-free plasma, and corresponds to Landau damping, with the principal difference being that the resonant particles absorb energy from the wave in a plane perpendicular to the drift motion of the particles along the magnetic field. The absorption of energy from the wave is large when a large number of particles have streaming velocities such that the Doppler-shifted frequency of the wave as "seen" by the particles equals the particle gyrofrequency or its harmonics in magnitude and sense. In this article we shall consider only the gyrofrequency as the harmonics are cut off for a wave propagating along the magnetic field in the whistler mode. The resonant condition for the electron gyrofrequency is mathematically expressed by the condition

$$\omega - kv - \Omega = 0$$

where ω is the whistler radian frequency, k and v are respectively, the whistler wave number and electron velocity along the magnetic field, and Ω is 2π times the electron gyrofrequency. Because whistler frequencies always lie below the electron gyrofrequency, it is necessary for the streaming velocity of nonrelativistic particles to be in opposite sense to the wave velocity.

The first analysis accounting for the upper cutoff frequency of nose whistlers in terms of cyclotron absorption assumed an isotropic electron velocity distribution function [Liemohn and Scarf, 1962]; however these distributions do not agree with the whistler observables in two respects [Guthart, 1963; Liemohn and Scarf, 1964]. First, these distributions do not account for the occurrence of nose whistlers at high magnetic latitudes; i.e., for these distributions, nose whistlers traversing high magnetic latitude should not be observable. Second, the magnetosphere temperature required by these distributions is too high.

If cyclotron absorption is to explain the upper cutoff frequency of nose whistlers, alternate electron velocity distributions are required. In this article, a pair of anisotropic electron velocity distributions will be considered and tested for agreement with the whistler observations. Specifically, a satisfactory magnetosphere electron velocity distribution must satisfy the following:

(1) It must be stable. The general character of the upper cutoff frequency of nose whistlers is highly repeatable, hence the velocity distribution function responsible for the phenomenon must be ever present.

(2) It must provide for a rapid attenuation at the whistler upper cutoff frequency. The rate is of the order of 20 dB for a frequency change of 2 percent.

(3) It must provide for no measurable increase in dispersion beyond the zero-temperature estimate. A measurable increase in dispersion is approximately 2 percent.

2. Theory

The refractive index, n_p, of the whistler mode in a thermal magnetoplasma can be shown [Stix, 1962] to be given by

$$n_p^2 = 1 + \frac{\pi e^2}{m\epsilon_0\omega^2}\int_{-\infty}^{\infty} dv_z \int_0^{\infty} dv_\perp\, v_\perp^2 \frac{\left[kv_\perp \frac{\partial f_0}{\partial v_z} + (\omega - kv_z)\frac{\partial f_0}{\partial v_\perp}\right]}{(\omega - kv_z - \Omega)}, \tag{1}$$

where $\vec{v}$ = electron velocity
e = electron charge
m = electron mass
ω = radian frequency of the wave
ϵ_0 = free space permittivity
k = wave number for the longitudinally propagating transverse wave
$\Omega = 2\pi$ (electron gyrofrequency)
$f_0 = f_0\left(\frac{v_\perp^2}{2}, v_z\right)$ = electron distribution function

and the subscripts x, y, z define a rectangular coordinate system with the ambient earth's field, B_0, alined along the Z-axis. The subscript $\perp$ refers to the perpendicular direction with respect to the static magnetic field. Consider the transverse velocity distribution to be Maxwellian, so that

$$f_0\left(\frac{v_\perp^2}{2}, v_z\right) = \frac{Ng(v_z)}{\pi a^2} \exp - \frac{v_\perp^2}{a^2},$$

and

$$g(v_z) = z\text{-directed distribution of electron velocity}$$

such that

$$\int_{-\infty}^{\infty} dv_z g(v_z) \sim 1,$$

$$a^2 = 2k_B T/m,$$

T = transverse electron temperature,

N = electron density,

k_B = Boltzmann's constant.

Substituting into (1) for f_0, the refractive index becomes

$$n_p^2 = 1 - \frac{i\omega_p^2 a^2}{2k\omega^2} < \Theta >, \tag{2}$$

where

$$< \Theta > = ik\int_{-\infty}^{\infty} dv_z \frac{\Theta(v_z)}{\omega - kv_z - \Omega},$$

and

$$\Theta = -\frac{2g}{a^2} + \frac{k}{\omega}\left(\frac{dg}{dv_z} + \frac{2v_z g}{a^2}\right).$$

We shall now consider two longitudinal electron velocity distributions.

2.1. Cyclotron Absorption by Resonant Electrons With a v_z^{-1} Distribution

As a first trial function consider a distribution related to one recently proposed by Liemohn and Scarf [1964]. They have suggested a distribution for the resonant electrons (the longitudinal electron energy for the resonant electrons is typically between 0.2 and 2.0 keV) of $g_r(v_z) \propto v_z^{-1}$ so that $[(1/N_0)\ (dN/dE_z)] \propto E_z^{-1}$ when dN/dE_z is the rate of change of electron density with energy. The electron energy, E_z, is equal to $[(1/2)\ mv_z^2]$. The resonant electrons are those electrons participating in the cyclotron absorption, namely, those electrons whose drift velocity, V_0, is approximately given by

$$V_0 = \frac{\omega - \Omega}{k}.$$

For simplicity we shall assume that the longitudinal distribution, $g_r(v_z)$, has a v_z^{-1} beyond the energy range proposed by Liemohn and Scarf. Although this assumption is obviously not physically realistic, the distribution of the nonresonant electrons does not significantly affect the cyclotron absorption mechanism. The following calculation, which attempts to test the validity of the proposal for cyclotron absorption of whistler energy by resonant particles having a v_z^{-1} distribution, is thus unperturbed by the assumption on the distribution of nonresonant particles. For the least energetic electrons, we shall assume a Maxwellian distribution. Therefore, let

$$g(v_z) = \begin{cases} \dfrac{\exp - (v_z^2/a)^2}{\pi^{1/2} a} & |v_z| \leqslant sa \text{ where } s \text{ is a positive integer} \\[2ex] \dfrac{\exp(-s^2)}{\pi^{1/2} a} \dfrac{sa}{|v_z|} & c > |v_z| \geqslant sa \\[2ex] 0 & |v_z| \geqslant c \end{cases} \tag{3}$$

so that

$$\int_{-\infty}^{\infty} g(v_z) dv_z \sim 1 + \frac{2s \exp - s^2}{\pi^{1/2}} \quad \overline{v_z^2} = \int_{-\infty}^{\infty} v_z^2 g(v_z) dv_z \sim a^2 + \frac{s \exp(-s^2)}{\pi^{1/2}} (c^2 - s^2 a^2).$$

Let us further assume that s is sufficiently great so that $\overline{v_z^2} \sim a^2$, and that the apparent magnetosphere electron temperature is approximately the Maxwellian value $\lesssim 10^4$ °K. The refractive index for this electron density distribution can be determined upon substitution of (3) into (2) and integrating. The integration of (2) is straightforward when it is remembered that the integral

$$\left[\int_{-c}^{-sa} \frac{dv}{\alpha a - v_z} \right]$$

has a pole at $v_z = -|\alpha| a$ and is equal to

$$\int_{-c}^{-sa} \frac{dv}{\alpha a - v_z} = -\pi i + \ln \left| \frac{c + \alpha a}{\alpha a + sa} \right|.$$

The quantity α is defined by

$$\alpha = \frac{\omega - \Omega}{ka},$$

and is less than zero for propagating whistlers. The refractive index squared for the distribution of (3) is thus given by

$$n_p^2 = 1 - \frac{(\omega_p/\omega)^2}{(1-y)}\left[1 + \frac{s\exp(-s^2)}{\pi^{1/2}}\left\{2\left(1+\frac{1-y}{2\alpha^2}\right)\ln\frac{c}{sa} - \left(y + \frac{1-y}{2\alpha^2}\right)\left[-\pi i + \ln\frac{c^2-\alpha^2 a^2}{\alpha^2 a^2 - s^2 a^2}\right]\right\}\right], \quad (4)$$

where $y = \Omega/\omega$, and $[(1-y)/\alpha]$ has been substituted for (ka/ω).

Let

$$n_p = n_{pr} + i n_{pi},$$

and

$$n_0^2 = 1 + \frac{(\omega_p/\omega)^2}{y-1}.$$

When $n_{pi} << n_{pr} \sim n_0$,

$$k_i = \frac{\omega}{c} n_{pi} \sim \frac{\pi^{1/2}\omega_p \Omega s \exp(-s^2)}{2c\omega^{1/2}(\Omega-\omega)^{1/2}}. \quad (5)$$

The total attenuation, L, experienced by an electromagnetic wave propagating in the whistler mode through the magnetosphere is given by

$$L = 8.68 \int_S k_i(\theta) ds \text{ decibels.} \quad (6)$$

Since the total attenuation is linearly related to k_i, the rate of change of attenuation with wave frequency is linearly related to the rate of change of $k_i(0°)$ with wave frequency. As noted previously, the rate of change of damping at cutoff for nose whistlers is most often extremely sharp. The damping at cutoff is of the order of 20 dB for frequency changes about the cutoff frequency of 2 percent. The rate of change of k_i for the v_z^{-1} distribution is

$$\frac{dk_i}{d\omega} = \frac{\pi^{1/2}\omega_p\Omega s\exp(-s^2)}{2c\omega^{1/2}(\Omega-\omega)^{1/2}}\left[\frac{2\omega-\Omega}{2\omega(\Omega-\omega)}\right],$$

$$\frac{dk_i}{d\omega} = k_i\left[\frac{1-\dfrac{\Omega}{2\omega}}{(\Omega-\omega)}\right].$$

A representative set of whistler parameters are [Liemohn and Scarf, 1962]: $\Omega_1 = 70.9(10^3)$ rad/sec, $\theta_0 = 61°$, $\omega = 39.2(10^3)$ rad/sec, $k_i(0°) = 1.5(10^{-6})\text{m}^{-1}$. For these parameters, the change in attenuation is less than 0.6 dB for a 1.5-percent change in wave frequency. As a consequence, this distribution does not satisfy a whistler observable (the rate change of attenuation about the cutoff frequency is too small); hence, it is rejected as a likely magnetosphere distribution.

2.2. Cyclotron Absorption by a Double-Humped Maxwellian Distribution; Calculation of Refractive Index

As a second trial function, consider a double-humped Maxwellian distribution, specified by

$$g(v) = \frac{1}{\pi^{1/2}a}\exp\left(-\frac{v^2}{a^2}\right) + \frac{\chi}{\pi^{1/2}a_1}\exp\left[-\frac{(v-V_0)^2}{a_1^2}\right], \quad (7)$$

where

$$\chi = \frac{N_1}{N_0},$$

$$\int_{-\infty}^{\infty} g(v)dv = 1 + \chi \sim 1 \quad \chi << 1,$$

and

$$\int_{-\infty}^{\infty} v^2 g(v)dv = a^2 + \chi(a_1^2 + V_0^2).$$

For simplicity, we shall further assume

$$\int_{-\infty}^{\infty} v^2 g(v)dv \sim a^2,$$

although recent data reported by Serbu [1964] indicate that this is not always true.

The velocity distribution of electrons has two components. The first is an ambient isotropic electron distribution defined by a temperature $T = (ma^2/2k_B)$. The second is an electron stream of mean longitudinal velocity, V_0, having a longitudinal temperature T_1.

As before,

$$n_p^2 = 1 - \frac{i\omega_p^2 a^2}{2k\omega^2} < \Theta > \cdot \tag{2}$$

After Stix [1962], the refractive index for the $g(v)$ specified by (7) can be determined. As in the previous example, caution must be exercised when integrating (2). The square of the refractive index becomes

$$n_p^2 = 1 - \frac{\omega_p^2}{\omega(\omega - \Omega)} - \frac{\omega_p^2 \chi(\omega - kV_0)}{\omega^2(\omega - kV_0 - \Omega)} + i\frac{\omega_p^2 \chi \pi^{1/2}}{n_p(a_1/c)\omega^3}\left[(\omega - kV_0 - \Omega)\frac{T_\perp}{T_1} + \Omega)\right] \exp(-\alpha_1^2), \tag{8}$$

where $\alpha_1 = \dfrac{\omega - kV_0 - \Omega}{ka_1} < -1$.

To obtain this solution, the integral

$$-\frac{2ki}{a^2}\int_{-\infty}^{\infty} \frac{dv_z\left[\frac{1}{\pi^{1/2}a}\exp - \frac{v_z^2}{a^2} + \frac{\chi}{\pi^{1/2}a_1}\exp - \frac{(v_z - V_0)^2}{a_1^2}\right]}{(\omega - kv_z - \Omega)} \sim -\frac{2}{a^3}F_0 - \frac{2\chi}{a^2 a_1}F_1,$$

where

$$F_0 = \frac{ik}{\pi^{1/2}}\int_{-\infty}^{\infty} \frac{dv_z \exp - \frac{v_z^2}{a^2}}{\omega - \Omega - kv_z} \sim \frac{i}{\alpha},$$

and

$$F_1 = \frac{ik}{\pi^{1/2}}\int_{-\infty}^{\infty} \frac{dv_z \exp - \left[\frac{(v_z - V_0)^2}{a_1^2}\right]}{\omega - kv_z - \Omega} \sim \frac{i}{\alpha_1} + \pi^{1/2}\exp - \alpha_1^2.$$

The quantities α and α_1 have been previously defined. It should be noted that (8) is also the solution for the refractive index squared for the following electron distribution function:

$$f_0\left(\frac{v_\perp^2}{2}, v_z\right)=\frac{N}{\pi^{3/2}a^3}\exp\frac{(v_\perp^2+v_z^2)}{a^2}+\frac{\chi N}{\pi^{3/2}a_\perp^2 a_1}\exp-\left(\frac{v_\perp^2}{a_\perp^2}\right)\exp-\frac{(v_z-V_0)^2}{a_1^2}. \tag{9}$$

The same degree of approximation is involved in the solution of (1) for the distribution of (9) as has been made previously. For the distribution of (9), the subscripts $\perp$ and 1 refer to stream parameters only. As a consequence we shall hereafter consider (8) as the solution for the distribution given by (9), and therefore consider the transverse and longitudinal stream temperatures as quantities independent of the ambient plasma temperature. For $n_p=n_{pr}+in_{pi}$ and if $n_p^2>>1$, then,

$$n_0^2=\frac{\omega_p^2}{\omega(\Omega-\omega)}, \tag{10a}$$

$$n_{pr}\sim\left[1+\frac{\chi(\omega-kV_0)(\Omega-\omega)}{2\omega(\Omega-\omega+kV_0)}\right]n_{p0}, \tag{10b}$$

$$n_{pi}\sim\frac{\chi\pi^{1/2}\left[(\omega-kV_0-\Omega)\frac{T_\perp}{T_1}+\Omega\right](\Omega-\omega)}{2(a_1/c)\omega^2}\exp-\left(\frac{\omega-kV_0-\Omega}{\omega n_{p0}(a_1/c)}\right)^2. \tag{11}$$

Equation (10) specifies the real part of the refractive index as a function of the plasma and wave parameters. It is seen that the effect of the stream is to perturb the isotropic ambient plasma solution.

Guthart [1965] has examined the dispersion of 23 nose whistlers and found the whistler dispersion to be satisfied by the zero-temperature dispersion formulae; i.e., the measured whistler dispersion exhibits no temperature effects. (The accuracy of the dispersion measurement, which determines the degree to which temperature effects can be observed, is 2 percent.) If the double-humped Maxwellian distribution is an allowed one, then the proposed stream parameters should generate no observable increase in dispersion.

Equation (11) specifies the imaginary part of the refractive index as a function of the wave and plasma parameters. It is seen that the imaginary part of the refractive index is primarily a function of the wave and stream parameters; i.e., this term becomes significant only when the stream velocity becomes equal to the Doppler-shifted wave velocity. The expression for the imaginary part of the refractive index is an even more rapid function of magnetic latitude and frequency than the isotropic Maxwellian solution considered previously. As a consequence, each frequency above the cutoff frequency is absorbed at a magnetic latitude for which the Doppler-shifted stream velocity is resonant with the wave velocity; hence,

$$L(f)\propto k_i[\theta(f)] \qquad f_{H1}\geq f\geq f_m.$$

An interesting result of the determination of n_{pi} is gleaned upon investigation of the term within the brackets of (11):

$$(\omega-kV_0-\Omega)\frac{T_\perp}{T_1}+\Omega.$$

For a propagating magnetosphere whistler, $(\omega-kV_0-\Omega)<0$, so that if

$$\frac{T_\perp}{T_1}>\frac{\Omega}{|(\omega-kV_0-\Omega)|}>1, \tag{12}$$

then the imaginary part of the refractive index is negative, indicating wave growth representative of a transverse instability. The general requirement for transverse instability (that $T_\perp/T_1 > 1$) was discussed by Stix [1962] and by Sudan [1963]. The physical mechanism for this growth was first suggested by Brice [1963], and subsequently proved by Bell and Buneman [1964] as a mechanism for VLF emissions.

Bell and Buneman considered an ambient isotropic plasma permeated by a tenuous stream having a mean square transverse velocity spread, $\overline{v_\perp^2}$, and a delta-function longitudinal distribution of stream velocity, $\delta(v_z - V_0)$. Bell and Buneman determined the conditions for wave growth of such a distribution, and then estimated the cyclotron absorption. Equation (11) more exactly specifies the relative importance of growth to cyclotron absorption. To obtain significant wave growth, (12) requires the transverse temperature, $T_\perp$, to be much greater than the longitudinal temperature; the exponential factor becomes sufficiently small only when $(\omega - kV_0 - \Omega) \rightarrow 0$.

We shall postpone further discussion of the growing-wave solution of (11) and calculate some stream parameters satisfying the whistler observables as well as establishing the stability of such a stream.

3. Representative Stream Parameters

To determine an allowed set of plasma parameters, we shall assume a value for the resonant particle flux, and then solve (10) and (11) for the set of plasma parameters so that (1) is consistent with the whistler observables and the assumed particle flux, and (2) has no longitudinal instabilities.

The flux of resonant particles participating in cyclotron damping is given by

$$\overline{nv_r} = \int_{V_0 - \Delta v}^{V_0 + \Delta v} v g(v) dv,$$

where Δv is a measure of the velocity spread of the resonant particles. Jackson [1960] estimates this velocity to be

$$\Delta v = \left(\frac{2eE}{mk}\right)^{1/2},$$

i.e., those particles whose relative velocities (with respect to the Doppler-shifted wave velocity) are insufficient to escape the potential well of the wave contribute to the damping.

For the double-humped Maxwellian distribution, the flux about the resonant particle velocity is

$$j_r = \overline{nv_r} = \frac{Na_\perp}{\pi^{1/2}} \int_{\alpha_0 - \frac{\Delta v}{a_1}}^{\alpha_0 + \frac{\Delta v}{a_1}} \left(\frac{v}{a_\perp}\right) \exp\left(-\frac{v^2}{a_\perp^2}\right) d\left(\frac{v}{a_\perp}\right) + \frac{\chi N a_1}{\pi^{1/2}} \int d\left(\frac{v}{a_1}\right)\left(\frac{v}{a_1}\right) \exp$$

$$-\left(\frac{v - V_0}{a_1}\right)^2 \sim 0 + \frac{\chi N a_1}{\pi^{1/2}} \int_{\alpha_1 - \frac{\Delta v}{a_1}}^{\alpha_1 + \frac{\Delta v}{a_1}} \left(y + \frac{V_0}{a_1}\right) \exp(-y^2) dy,$$

$$\overline{nv_r} = \frac{\chi N a_1}{\pi^{1/2}} \exp - \left[\alpha^2 + \left(\frac{\Delta v}{a_1}\right)^2\right] \sinh \frac{2\alpha_1 \Delta v}{a_1} + \sqrt{2}\chi N V_0 \left\{\Phi\left[\sqrt{2}\left(\alpha + \frac{\Delta v}{a}\right)\right] - \Phi\left[\sqrt{2}\left(\alpha - \frac{\Delta v}{a}\right)\right]\right\}, \quad (13)$$

where

$$\Phi(z) = \frac{1}{\sqrt{2\pi^1}} \int_{-\infty}^{z} \exp(-z^2/2) dz.$$

A second constraint on the stream parameters is that the velocity distribution be stable to longitudinal waves, i.e., the distribution should not give rise to longitudinal plasma oscillations. Bell and Buneman [1964] estimate the relative significance for the growth of longitudinal waves vis-a-vis the Landau damping of these waves. For a stable distribution,

$$\frac{V_0}{a_1} 0.69\chi^{1/3} \lesssim 1. \tag{14}$$

The foregoing stability criterion may be unduly severe. The cyclotron absorption mechanism as well as the Bell-Buneman transverse instability are dependent on the steepness of the distribution function for velocity, v being greater than the magnitude of the stream velocity, $|V_0|$. The longitudinal instability depends on the parameters of the distribution function wherein $v < |V_0|$, i.e., in that part of the distribution function where $dg(v)/dv > 0$. As a consequence, a distribution can by hypothesized that has a non-Maxwellian character for $v < |V_0|$ and a Maxwellian form for $v > |V_0|$. Such a distribution would retain all the cyclotron absorption and transverse instability properties embodied in (11) and, at the same time, relax the constraint of (14).

The final constraint on the allowed distribution is imposed by the experimental observation that the whistler dispersion is in good agreement (within 2%) with the cold-plasma approximation; i.e.,

$$\frac{D_1}{D_0} = \frac{\int_s (n_{gr} - n_{g0}) ds}{\int_s n_{g0} ds} \lessapprox 0.02.$$

For the double-humped Maxwellian distribution in the vicinity of resonance,

$$(n_{gr} - n_{g0}) = \frac{d[\omega(n_{pr} - n_0)]}{d\omega} \sim \frac{\chi \omega_p (\omega - kV_0)(\Omega - \omega)^{1/2}}{2\omega^{1/2}(\Omega - \omega + kV_0)}, \tag{15a}$$

$$n_{g0} = \frac{\omega_p \Omega}{2\omega^{1/2}(\Omega - \omega)^{3/2}}. \tag{15b}$$

Assume that the resonant particle flux, $\overline{nv_r}$, is 10^7 (electrons cm^{-2} sec^{-1}), the whistler field strength $E = 10^{-3}$ v/m [Carpenter, private communication], and that the whistler observables are those of the College, Alaska, whistler [Liemohn and Scarf, 1962]:

$$N(0°) = 250 \text{ cc}^{-1}$$

$$\Omega_1 = 7.1\ (10^4) \text{ rad/sec}$$

$$\omega_m = 3.9\ (10^4) \text{ rad/sec}$$

$$n_{p0} \sim 26$$

$$k_i \sim 1.5\ (10^{-6}) \text{ N}/m.$$

Using (11), (14), and (15), a set of allowable stream parameters can be derived (which is not unique) satisfying these constraints:

$$\chi = \frac{\text{stream density}}{\text{ambient density}} = 10^{-3}$$

$$a_1 = 4(10^7) \text{ cm/sec}$$

$$V_0 = 8.4(10^8) \text{ cm/sec.}$$

A stream with these parameters moving in the specified plasma accounts for the upper cutoff of the College, Alaska, whistler without increasing the whistler dispersion at cutoff.

The double-humped Maxwellian distribution of the longitudinal velocity of electrons is seen to be an allowed distribution from the standpoint of whistler observables and longitudinal stability. Each "hump" of the distribution offers a separate and distinct contribution to the observed whistler characteristics. The ambient, isotropic, electrons account for the observed dispersion characteristics of the whistler. The hypothesized electron stream, with its steep gradient in electron density for a velocity that is greater than the mean stream velocity, $|V_0|$, accounts for the whistler cutoff properties. As previously noted, the cyclotron absorption mechanism requires a steep density gradient for $v > |V_0|$, but imposes no restriction on the electron density gradient in velocity for $v < |V_0|$; this mitigates the difficulties associated with longitudinal instabilities.

The required stream speed for cyclotron resonance at the magnetic equator is

$$|V_0| = \frac{\Omega_1 - \omega_m}{k} = \frac{m_m^{-1/2}(1-m_m)^{3/2} f_{H1}^{1/2} c}{9K_1^{1/2}}, \tag{16}$$

where, as previously,

$m_m = \omega_m/\Omega_1 =$ maximum normalized cutoff frequency,

$f_{H1} =$ minimum gyrofrequency along the whistler path,

$c =$ velocity of light in free space, and

$K_1 = N/f_H =$ constant, relating electron density and gyrofrequency for the gyrofrequency model of electron density in the magnetosphere.

The stream will cut off all frequencies above the maximum observable whistler frequency, ω_m. This can be seen when it is remembered that the magnetic field strength varies along a magnetic field line. The minimum field strength (or electron gyrofrequency) is at the magnetic equator. As a consequence, a stream that is in resonance with a given frequency at the magnetic equator will resonate with any higher frequency at some nonequatorial location.

Equation (16) is plotted in figure 1 for $K_1 = 2(10^4)$. It is seen from (16) that if the maximum normalized frequency and the electron density constant K_1 are invariant, the resonant stream velocity varies as the 1/2 power of the minimum gyrofrequency along the whistler path. For the College, Alaska, nose whistler, the stream velocity is seen to be $V_0 = 8.4(10^9)$ cm/sec, corresponding to an electron energy of approximately 250 eV.

Regarding the upper cutoff frequency of "Eckersley" whistlers, it should be observed that the upper cutoff frequency of many of these whistlers is determined by (1) the frequency content of the atmospheric source, and (2) the propagation path absorption of the whistler energy. It has been shown [Arnold and Pierce, 1964; Dennis and Pierce, 1964] that VLF atmospherics have an approximately inverse-square frequency dependence in the high-frequency limit. In addition, the absorption experienced by the whistler energy in traversing the D layer is approximately proportional to the square root of frequency. Both facts limit the high-frequency content of the "Eckersley" whistlers, particularly at low and median latitudes, so that the rate of change of attenuation in the vicinity of cutoff is more gradual than is usually observed in nose whistlers. The source spectrum and propagation path absorption are not factors in multipath nose whistler cutoff for the low-latitude traces demonstrate the existence of the high-frequency components.

The variation of stream velocity with radial distance from the earth can be inferred from the examination of multipath nose whistlers. A representative multipath whistler group samples the magnetosphere through one earth radius as measured by the nose frequencies of the whistler traces. Accepting the hypothesis for cyclotron absorption by a double-humped Maxwellian distribution, we can solve (16) for stream velocity for each of the traces of the whistler group, and so determine the variation of stream velocity with radial distance. A preliminary analysis of a pair of whistler groups leads to a stream velocity, V_0, distribution such that

$$V_0 \propto f_{H1}.$$

The existence of highly energetic electron streams is well established [O'Brien et al., 1962]; however, recent experimental evidence indicates the existence of streaming particles having energies greater than 100 eV. Serbu [1964] has reported satellite measurements out to 15 earth radii derived from a retarding potential analyzer that measures the energy distribution of ions and electrons up to 100 eV. The instrument operates in such a manner as to measure the flux into the analyzer of ions or electrons whose energies are greater than a given retarding potential. Serbu's measurements indicate the existence of electron streams having energies greater than 100 eV. His measurements further indicate the absence of particles in the 20 to 100 eV range. Serbu further notes that the electrons exhibited thermal energies for geocentric distances less than 4.5 earth radii, and that the average electron energy then increases gradually to values above 100 eV at 8 earth radii. This would suggest that our assumption of $\int_{-\infty}^{\infty} v^2 g(v) dv \sim a^2$ is not appropriate. In any case, the fact does not modify the proposed analysis, which can be adapted to account for any observable energy.

In addition to Serbu's measurements, Sharp, et al., [1964] have measured the energy influx of particles (at 300 km) precipitating on the auroral zones by particle detectors aboard orbiting satellites. The integrated energies of precipitating electrons of energy greater than 80 eV were sampled. Electron streams as soft as a few hundred eV were observed.

Throughout this analysis we have assumed that the whistler wave normal is parallel to the magnetic field. When the wave normal is not alined with the field, the cyclotron resonance with a stream of velocity V_0 occurs when

$$|\omega - k_z V_0 - \Omega| \to 0,$$

where k_z is the component of the wave number along the magnetic field. As a consequence, nonducted whistlers would experience cyclotron absorption. In fact, Bell [1964] has recently extended his proposal for VLF emissions to account for nonzero wave-normal angles so that the cyclotron absorption and emission mechanisms are only quantitatively complicated by the introduction of nonzero wave normals.

4. Whistler-Triggered VLF Emissions

Many of the nose whistler sonograms studied to date give evidence of an emission triggered at the end of the whistler as, for example, in Guthart's [1965] figure 1. We can obtain a qualitative interpretation of this phenomenon by considering the previous results. It has been noted that

(1) The mechanism of cyclotron absorption couples energy to the orbiting electrons. This energy is coupled into the plane perpendicular to the electron's drift motion along the magnetic field; i.e., the transverse electron velocity is increased.

(2) The transverse electron stream temperature, $T_\perp$, must exceed the longitudingal stream temperature, T_1, by the factor $(\Omega/|\omega - kV_0 - \Omega|)$ for the Bell-Buneman transverse instability mechanism to dominate the cyclotron damping.

Consider a magnetosphere having the electron stream superimposed on the ambient, thermal plasma. In the undisturbed state, the stream parameters are such that

$$\frac{T_\perp}{T_1} < \frac{\Omega}{(\omega - kV_0 - \Omega)}.$$

A whistler traversing this medium suffers cyclotron damping, surrendering energy to the normal component of electron motion until

$$\frac{T_\perp}{T_1} > \frac{\Omega}{(\omega - kV_0 - \Omega)},$$

thereby triggering an emission. The electrons now return the energy previously absorbed by cyclotron damping, leaving the medium in the steady-state condition that existed prior to the whistler traversal.

5. Conclusions

An anisotropic particle distribution is suggested to explain the high-frequency cutoff of nose whistlers. The distribution consists of an ambient electron stream drifting along the magnetic field superimposed on an isotropic, thermal plasma. This distribution is consistent with the whistler observables while at the same time accounting for some VLF emissions. The suggested distribution is further consistent with recent magnetosphere flux measurements.

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