

Table of Dedekind Sums ¹

R. Dale Shipp ²

(June 17, 1965)

The Dedekind sums are of importance in the transformation formulae for the Dedekind modular form $\eta(\tau)$, and in discussing the characters of degree 1 of the modular group and its subgroups. These sums are rational numbers and a table of their exact values is given. In addition a comprehensive bibliography on these sums is included.

Key Words: Dedekind sums, tables, characters, modular form.

1. Introduction

The purpose of this paper is to present a short table of Dedekind sums $s(h, k)$. These sums arise in the theory of modular forms and in number theory as well as in various other areas of mathematics. We have included a fairly extensive bibliography.³

In order to define a Dedekind sum, we first introduce the following symbol:

$$\begin{aligned} ((x)) &= x - [x] - \frac{1}{2} && \text{for } x \text{ not an integer} \\ &= 0 && \text{for } x \text{ an integer} \end{aligned} \quad (1.1)$$

where $[x]$ denotes the greatest integer not exceeding x . Then a Dedekind sum is defined by

$$s(h, k) = \sum_{u=1}^k ((hu/k)) ((u/k)) \quad (1.2)$$

where h is any integer and k is any positive integer.

The paper of Rademacher and Whiteman [1941], contains a number of interesting theorems on Dedekind sums. A later paper by Rademacher [1956], extends some of these results. In section 2 we quote some of these theorems. The interested reader is referred to these papers for proofs.

2. Theorems on Dedekind Sums

In this paragraph we present, without proof, theorems on Dedekind sums which will be helpful in using and extending the table.

THEOREM 1. For integers h, q, k , with q and k positive $s(qh, qk) = s(h, k)$. [RW, p. 380].

This theorem allows us to consider only $(h, k) = 1$. In fact, Rademacher in [R] places this restriction on the definition of $s(h, k)$.

THEOREM 2. If $h_1 \equiv h_2 \pmod{k}$ then $s(h_1, k) = s(h_2, k)$. [R, p. 445].

THEOREM 3. $s(-h, k) = -s(h, k)$. [R, p. 445].

Theorems 1, 2, and 3 allow us to obtain $s(h, k)$ for

all h and for all k less than some fixed limit K from a table giving $s(h, k)$ only for $k = 2, 3, \dots, K$ and h each integer in the interval $[1, [k/2]]$ such that $(h, k) = 1$.

THEOREM 4. If $h_1 h_2 \equiv 1 \pmod{k}$ then $s(h_1, k) = s(h_2, k)$ [R, p. 445].

THEOREM 5. The denominator of $s(h, k)$ is at most $2k \cdot (3, k)$ where (n, m) denotes the greatest common divisor of n and m [R, p. 446].

The main theorem on Dedekind sums is the following reciprocity relation.

THEOREM 6. If $h > 0, k > 0, (h, k) = 1$ then

$$s(h, k) + s(k, h) = -\frac{1}{4} + (h/k + k/h + 1/hk)/12.$$

There are many proofs of this theorem (e.g. [RW, p. 381], [R, p. 445]). In computing our table we used the reciprocity relation for Dedekind sums. It allows the computation of $s(h_0, k_0)$ using a table of $s(h, k)$ for $k < k_0$ without directly computing the sum in the definition. This relation will also be quite useful to the reader who wishes values of $s(h, k)$ not in the table. For example, if $1 \leq h \leq 100$ then $s(h, k)$ can be obtained for any $k > 0$ by at most one application of the reciprocity formula.

3. Authentication and Description of Computing Procedures

The program was written in FORTRAN 60 and run on a CDC 1604 computer. After using a sieve type method to eliminate from consideration those h except for $1 \leq h \leq [k/2]$ and $(h, k) = 1$, $s(h, k)$ was computed (in rational form) by using the reciprocity relation. It was then reduced until the numerator and denominator were without common factor and entered into the table. This table remained in storage until computation was completed at which time it was punched on cards. The punched cards were then printed using an IBM 407 accounting machine.

This table was checked visually with another unpublished table (computed by Robert A. Tobey using (1.2) directly) and the two were found to be in complete agreement.

¹ This project was sponsored by Wisconsin Alumni Research Funds.

² Present address: University of Wisconsin, Madison, Wis.

³ The author wishes to acknowledge appreciation to Professor J. R. Smart for his assistance in the preparation of this paper.

k	h	$s(h, k)$	k	h	$s(h, k)$	k	h	$s(h, k)$	k	h	$s(h, k)$	k	h	$s(h, k)$	k	h	$s(h, k)$
2	1	0/1	23	6	17/46	32	13	19/64	40	9	23/80	46	13	-11/46	51	22	91/306
3	1	1/18	23	7	-1/46	32	15	-27/64	40	11	17/80	46	15	-45/46	51	23	-55/306
4	1	1/8	23	8	21/46	33	1	248/99	40	13	-13/16	46	17	3/46	51	25	-575/306
5	1	1/5	23	9	-7/46	33	2	112/99	40	17	-5/16	46	19	3/46	52	1	425/104
5	2	0/1	23	10	-1/46	33	4	32/99	40	19	-47/80	46	21	-5/46	52	3	119/104
6	1	5/18	23	11	-33/46	33	5	31/99	41	1	130/41	47	1	345/94	52	5	5/8
7	1	5/14	24	1	253/144	33	7	23/99	41	2	60/41	47	2	161/94	52	7	47/104
7	2	1/14	24	5	53/144	33	8	-32/99	41	3	39/41	47	3	105/94	52	9	41/104
7	3	-1/14	24	7	19/144	33	10	5/99	41	4	20/41	47	4	81/94	52	11	31/104
8	1	7/16	24	11	-37/144	33	13	-31/99	41	5	10/41	47	5	51/94	52	15	47/104
8	3	1/16	25	1	46/25	33	14	-23/99	41	6	25/41	47	6	65/94	52	17	-119/104
9	1	14/27	25	2	4/5	33	16	-112/99	41	7	25/41	47	7	37/94	52	19	31/104
9	2	4/27	25	3	2/5	34	1	44/17	41	8	-10/41	47	8	65/94	52	21	5/8
9	4	-4/27	25	4	4/25	34	3	11/17	41	9	0/1	47	9	5/94	52	23	-41/104
10	1	3/5	25	6	-4/25	34	5	9/17	41	10	-20/41	47	10	15/94	52	25	-87/104
10	3	0/1	25	7	0/1	34	7	9/17	41	11	9/41	47	11	-1/94	53	1	221/53
11	1	15/22	25	8	-2/5	34	9	4/17	41	12	3/41	47	12	81/94	53	2	104/53
11	2	5/22	25	9	4/25	34	11	-11/17	41	13	-11/41	47	13	7/94	53	3	68/53
11	3	3/22	25	11	-4/25	34	13	0/1	41	14	39/41	47	14	-15/94	53	4	39/53
11	4	3/22	25	12	-4/5	34	15	-4/17	41	15	9/41	47	15	-33/94	53	5	34/53
11	5	-5/22	26	1	25/13	35	1	187/70	41	16	-4/41	47	16	105/94	53	6	41/53
12	1	53/72	26	3	7/13	35	2	17/14	41	17	-3/41	47	17	1/94	53	7	17/53
12	5	-1/72	26	5	0/1	35	3	11/14	41	18	-4/41	47	18	-7/94	53	8	20/53
13	1	11/13	26	7	2/13	35	4	43/70	41	19	-11/41	47	19	51/94	53	9	41/53
13	2	4/13	26	9	7/13	35	6	37/70	41	20	-60/41	47	20	-37/94	53	10	11/53
13	3	1/13	26	11	-2/13	35	8	1/14	42	1	205/63	47	21	5/94	53	11	21/53
13	4	-1/13	27	1	325/162	35	9	43/70	42	5	29/63	47	22	-33/94	53	12	8/53
13	5	0/1	27	2	143/162	35	11	-13/70	42	11	20/63	47	23	-161/94	53	13	-39/53
13	6	-4/13	27	4	73/162	35	12	11/14	42	13	-2/63	48	1	1081/288	53	14	16/53
14	1	13/14	27	5	35/162	35	13	-1/14	42	17	29/63	48	5	161/288	53	15	-17/53
14	3	3/14	27	7	73/162	35	16	-13/70	42	19	-20/63	48	7	199/288	53	16	11/53
14	5	3/14	27	8	-1/162	35	17	-17/14	43	1	287/86	48	11	-1/288	53	17	-23/53
15	1	91/90	27	10	1/162	36	1	595/216	43	2	133/86	48	13	1/288	53	18	68/53
15	2	7/18	27	11	35/162	36	5	35/216	43	3	77/86	48	17	89/288	53	19	16/53
15	4	19/90	27	13	-143/162	36	7	-35/216	43	4	67/86	48	19	-161/288	53	20	20/53
16					117/56	36	11	-19/216	43	5	41/86	48	23	-217/288	53	21	-34/53
16					27/56	36	13	19/216	43	6	7/86	49	1	188/49	53	22	-8/53
16					13/56	36	17	-109/216	43	7	-7/86	49	2	88/49	53	23	0/1
16					-27/56	37	1	105/37	43	8	13/86	49	3	52/49	53	24	-21/53
16					-13/56	37	2	48/37	43	9	27/86	49	4	32/49	53	25	-23/53
17	1	20/17	28	13	-19/56	37	3	27/37	43	10	11/86	49	5	38/49	53	26	-104/53
17	2	8/17	29	1	63/29	37	4	15/37	43	11	67/86	49	6	8/49	54	1	689/162
17	3	5/17	29	2	28/29	37	5	14/37	43	12	5/86	49	8	-8/49	54	5	139/162
17	4	0/1	29	3	18/29	37	6	0/1	43	13	11/86	49	9	18/49	54	7	77/162
17	5	1/17	29	4	7/29	37	7	5/37	43	14	-77/86	49	10	38/49	54	11	139/162
17	6	5/17	29	5	13/29	37	8	8/37	43	15	35/86	49	11	18/49	54	13	-31/162
17	7	1/17	29	6	13/29	37	9	-15/37	43	16	-13/86	49	12	-32/49	54	17	-41/162
17	8	-8/17	29	7	-7/29	37	10	3/37	43	17	-41/86	49	13	8/49	54	19	41/162
18	1	34/27	29	8	4/29	37	11	-3/37	43	18	5/86	49	15	-8/49	54	23	-77/162
18	5	2/27	29	9	-3/29	37	12	-27/37	43	19	-27/86	49	16	-52/49	54	25	-31/162
18	7	-2/27	29	10	18/29	37	13	12/37	43	20	-35/86	49	17	24/49	55	1	477/110
19	1	51/38	29	11	4/29	37	14	8/37	43	21	-133/86	49	18	4/49	55	2	45/22
19	2	21/38	29	12	0/1	37	15	14/37	44	1	301/88	49	19	-4/49	55	3	27/22
19	3	9/38	29	13	-3/29	37	16	5/37	44	3	91/88	49	20	8/49	55	4	113/110
19	4	11/38	29	14	-28/29	37	17	-12/37	44	5	61/88	49	22	-8/49	55	6	27/110
19	5	11/38	30	1	203/90	37	18	-48/37	44	7	19/88	49	23	-24/49	55	7	17/22
19	6	-9/38	30	7	1/18	38	1	11/38	44	9	61/88	49	24	-88/49	55	8	17/22
19	7	3/38	30	11	13/90	38	3	33/38	44	13	5/88	50	1	98/25	55	9	-27/110
19	8	-3/38	30	13	1/18	38	5	15/38	44	15	-91/88	50	3	6/5	55	12	3/22
19	9	-21/38	31	1	145/62	38	7	11/38	44	17	5/88	50	7	0/1	55	13	1/22
20	1	57/40	31	2	65/62	38	9	-1/38	44	19	19/88	50	9	2/25	55	14	113/110
20	3	3/8	31	3	35/62	38	11	11/38	44	21	-59/88	50	11	-2/25	55	16	17/110
20	7	3/8	31	4	33/62	38	13	33/38	45	1	473/135	50	13	2/5	55	17	1/22
20	9	-7/40	31	5	5/62	38	15	-15/38	45	2	44/27	50	17	6/5	55	18	-27/22
21	1	95/63	31	6	-5/62	38	17	-1/38	45	4	77/135	50	19	2/25	55	19	63/110
21	2	40/63	31	7	13/62	39	1	703/234	45	7	10/27	50	21	-2/25	55	21	7/110
21	4	5/63	31	8	33/62	39	2	323/234	45	8	8/27	50	23	-2/5	55	23	3/22
21	5	-5/63	31	9	13/62	39	4	163/234	45	11	-77/135	51	1	1225/306	55	24	-17/110
21	8	4/63	31	10	-35/62	39	5	11/18	45	13	10/27	51	2	575/306	55	26	-63/110
21	10	-40/63	31	11	15/62	39	7	37/234	45	14	-23/135	51	4	17/18	55	27	-45/22
22	1	35/22	31	12	-1/62	39	8	11/18	45	16	23/135	51	5	125/306	56	1	495/112
22	3	7/22	31	13	-1/62	39	10	163/234	45	17	8/27	51	7	91/306	56	3	153/112
22	5	3/22	31	14	-15/62	39	11	-37/234	45	19	-13/135	51	8	71/306	56	5	55/112
22	7	-7/22	31	15	-65/62	39	14	53/234	45	22	-44/27	51	10	-125/306	56	9	15/112
22	9	3/22	32	1	155/64	39	16	19/234	46	1	165/46	51	11	89/306	56	11	-55/112
23	1	77/46	32	3	45/64	39	17	-19/234	46	3	45/46	51	13	17/18	56	13	23/112
23	2	33/46	32	5	19/64	39	19	-323/234	46	5	15/46	51	14	89/306	56	15	33/112
23	3	21/46	32	7	5/64	40	1	247/80	46	7	11/46	51	16	-35/306	56	17	-1/112
23	4	17/46	32	9	-5/64	40	3	13/16	46	9	-15/46	51	19	-71/306	56	19	153/112
23	5	7/46	32	11	45/64	40	7	5/16	46	11	-5/46	51	20	-55/306	56	23	1/112

k	h	$s(h, k)$	k	h	$s(h, k)$	k	h	$s(h, k)$	k	h	$s(h, k)$	k	h	$s(h, k)$	k	h	$s(h, k)$
56	25	15/112	61	10	-20/61	65	14	-24/65	69	11	31/207	73	7	51/73	76	27	35/152
56	27	-103/112	61	11	0/1	65	16	-64/65	69	13	59/207	73	8	6/73	76	29	21/152
57	1	770/171	61	12	-35/61	65	17	5/13	69	14	229/207	73	9	-6/73	76	31	35/152
57	2	364/171	61	13	5/61	65	18	0/1	69	16	59/207	73	10	27/73	76	33	-11/152
57	4	140/171	61	14	-5/61	65	19	9/65	69	17	-221/207	73	11	33/73	76	35	-85/152
57	5	121/171	61	15	-55/61	65	21	-3/5	69	19	32/207	73	12	-36/73	76	37	-203/152
57	7	14/171	61	16	15/61	65	22	21/13	69	20	49/207	73	13	17/73	77	1	475/77
57	8	-14/171	61	17	8/61	65	23	5/13	69	22	-58/207	73	14	-1/73	77	2	228/77
57	10	41/171	61	18	8/61	65	24	9/65	69	25	-31/207	73	15	41/73	77	3	150/77
57	11	-5/171	61	19	-15/61	65	27	-3/13	69	26	112/207	73	16	4/73	77	4	95/77
57	13	23/171	61	20	-85/61	65	28	6/13	69	28	68/207	73	17	5/73	77	5	80/77
57	14	-140/171	61	21	40/61	65	29	14/65	69	29	-32/207	73	18	-84/73	77	6	85/77
57	16	-4/171	61	22	9/61	65	31	-3/5	69	31	-49/207	73	19	24/73	77	8	48/77
57	17	-41/171	61	23	28/61	65	32	-32/13	69	32	-68/207	73	20	33/73	77	9	25/77
57	20	67/171	61	24	-16/61	65	1	520/99	69	34	-544/207	73	21	51/73	77	10	31/77
57	22	23/171	61	25	9/61	66	5	65/99	70	1	391/70	73	22	27/73	77	12	24/77
57	23	121/171	61	26	-34/61	66	7	61/99	70	3	23/14	73	23	-24/73	77	13	85/77
57	25	-4/171	61	27	-29/61	66	13	-65/99	70	9	39/70	73	24	-126/73	77	15	-15/77
57	26	-5/171	61	28	-16/61	66	17	56/99	70	11	11/70	73	25	60/73	77	16	25/77
57	28	-364/171	61	29	-40/61	66	19	61/99	70	13	3/14	73	26	1/73	77	17	-25/77
58	1	133/29	62	1	305/62	66	23	47/99	70	17	-5/14	73	27	0/1	77	18	4/77
58	3	38/29	62	2	95/62	66	25	16/99	70	19	-11/70	73	28	-17/73	77	19	-95/77
58	5	21/29	62	3	49/62	66	29	-16/99	70	23	-23/14	73	29	-71/73	77	20	36/77
58	7	11/29	62	5	53/62	66	31	-56/99	70	27	3/14	73	30	-5/73	77	23	-31/77
58	9	13/29	62	7	53/62	67	1	715/134	70	29	-1/70	73	31	-19/73	77	24	-25/77
58	11	2/29	62	9	53/62	67	2	341/134	70	31	-39/70	73	32	4/73	77	25	-59/77
58	13	13/29	62	11	23/62	67	3	209/134	70	33	-5/14	73	33	-19/73	77	26	150/77
58	15	14/29	62	13	15/62	67	4	171/134	71	1	805/142	73	34	-41/73	77	27	36/77
58	17	0/1	62	15	-17/62	67	5	117/134	71	2	385/142	73	35	-60/73	77	29	48/77
58	19	-38/29	62	17	23/62	67	6	55/134	71	3	253/142	73	36	-204/73	77	30	4/77
58	21	-2/29	62	19	-15/62	67	7	65/134	71	4	193/142	74	1	219/37	77	31	80/77
58	23	-21/29	62	21	95/62	67	8	53/134	71	5	105/142	74	3	69/37	77	32	-24/77
58	25	11/29	62	23	-1/62	67	9	71/134	71	6	145/142	74	5	44/37	77	34	-20/77
58	27	-14/29	62	25	49/62	67	10	43/134	71	7	35/142	74	7	21/37	77	36	-15/77
59	1	551/118	62	27	-1/62	67	11	-55/134	71	8	133/142	74	9	11/37	77	37	-59/77
59	2	261/118	62	29	-17/62	67	12	29/134	71	9	133/142	74	11	17/37	77	38	-228/77
59	3	171/118	63	1	1891/378	67	13	-15/134	71	10	-35/142	74	13	6/37	78	1	1463/234
59	4	131/118	63	2	899/378	67	14	51/134	71	11	75/142	74	15	44/37	78	5	19/18
59	5	111/118	63	4	451/378	67	15	71/134	71	12	145/142	74	17	-6/37	78	7	77/234
59	6	101/118	63	5	305/378	67	16	-5/134	71	13	75/142	74	19	24/37	78	11	-77/234
59	7	63/118	63	8	323/378	67	17	171/134	71	14	-105/142	74	21	-21/37	78	17	49/234
59	8	37/118	63	10	109/378	67	18	21/134	71	15	53/142	74	23	-4/37	78	19	-103/234
59	9	19/118	63	11	143/378	67	19	-65/134	71	16	33/142	74	25	69/37	78	23	49/234
59	10	101/118	63	13	181/378	67	20	-43/134	71	17	-25/142	74	27	17/37	78	25	-85/234
59	11	9/118	63	16	451/378	67	21	-5/134	71	18	193/142	74	29	-4/37	78	29	-23/234
59	12	111/118	63	17	53/378	67	22	-209/134	71	19	53/142	74	31	0/1	78	31	-19/18
59	13	-19/118	63	19	109/378	67	23	99/134	71	20	-15/142	74	33	11/37	78	35	-23/234
59	14	-11/118	63	20	-127/378	67	24	51/134	71	21	-1/142	74	35	-24/37	78	37	-103/234
59	15	131/118	63	22	127/378	67	25	-53/134	71	22	-13/142	75	1	2701/450	79	1	1001/158
59	16	-9/118	63	23	143/378	67	26	259/90	71	23	-97/142	75	2	260/450	79	2	481/158
59	17	63/118	63	25	-305/378	67	27	117/134	71	24	253/142	75	4	649/450	79	3	299/158
59	18	-3/118	63	26	53/378	67	28	29/134	71	25	25/142	75	7	13/18	79	4	241/158
59	19	-61/118	63	29	-181/378	67	29	11/134	71	26	23/142	75	8	43/90	79	5	201/158
59	20	171/118	63	31	-899/378	67	30	-11/134	71	27	1/142	75	11	251/450	79	6	91/158
59	21	111/118	64	1	651/128	67	31	-15/134	71	28	-49/142	75	13	29/90	79	7	99/158
59	22	-37/118	64	3	189/128	67	32	-99/134	71	29	13/142	75	14	-1/450	79	8	161/158
59	23	-3/118	64	5	131/128	67	33	-341/134	71	30	-23/142	75	16	1/450	79	9	101/158
59	24	29/118	64	7	-21/128	68	1	737/136	71	31	-33/142	75	17	7/90	79	10	161/158
59	25	-21/118	64	9	-21/128	68	3	-231/136	71	32	-15/142	75	19	649/450	79	11	21/158
59	26	-21/118	64	11	61/128	68	5	121/136	71	33	-49/142	75	22	-7/90	79	12	47/158
59	27	-29/118	64	13	131/128	68	7	87/136	71	34	-97/142	75	23	-29/90	79	13	-91/158
59	28	-61/118	64	15	-11/128	68	9	33/136	71	35	-385/142	75	26	251/450	79	14	71/158
59	29	-261/118	64	17	11/128	68	11	7/136	72	1	2485/432	75	28	-43/90	79	15	-1/158
60	1	1711/360	64	19	-3/128	68	13	1/8	72	5	413/432	75	29	-1/450	79	16	201/158
60	7	29/72	64	21	-189/128	68	15	-33/136	72	7	235/432	75	31	1/450	79	17	71/158
60	11	161/360	64	23	21/128	68	19	-1/136	72	11	35/432	75	32	-13/18	79	18	33/158
60	13	11/72	64	25	-21/128	68	21	1/8	72	13	-35/432	75	34	-251/450	79	19	-19/158
60	17	-29/72	64	27	-3/128	68	23	231/136	72	17	53/432	75	37	-259/90	79	20	241/158
60	19	-71/360	64	29	-61/128	68	25	1/136	72	19	163/432	76	1	925/152	79	21	1/158
60	23	-11/72	64	31	-139/128	68	27	-121/136	72	23	-181/432	76	3	275/152	79	22	33/158
60	29	-361/360	65	1	336/65	68	29	-87/136	72	25	181/432	76	5	125/152	79	23	13/158
61	1	295/61	65	2	32/13	68	31	7/136	72	29	413/432	76	7	155/152	79	24	-13/158
61	2	140/61	65	3	21/13	68	33	-159/136	72	31	235/432	76	9	93/152	79	25	-19/158
61	3	85/61	65	4	64/65	69	1	1139/207	72	35	-541/432	76	11	155/152	79	26	-299/158
61	4	55/61	65	6	61/65	69	2	544/207	73	1	426/73	76	13	85/152	79	27	143/158
61	5	35/61	65	7	6/13	69	4	221/207	73	2	204/73	76	15	-125/152	79	28	39/158
61	6	20/61	65	8	0/1	69	5	229/207	73	3	126/73	76	17	93/152	79	29	23/158
61	7	34/61	65	9	14/65	69	7	194/207	73	4	84/73	76	21	21/152	79	30	23/158
61	8	28/61	65	11	61/65	69	8	112/207	73	5	71/73	76	23	11/152	79	31	-39/158
61	9	29/61	65	12	3/13	69	10	194/207	73	6	36/73	76	25	-275/152	79	32	65/158

<i>k</i>	<i>h</i>	<i>s(h, k)</i>	<i>k</i>	<i>h</i>	<i>s(h, k)</i>	<i>k</i>	<i>h</i>	<i>s(h, k)</i>	<i>k</i>	<i>h</i>	<i>s(h, k)</i>	<i>k</i>	<i>h</i>	<i>s(h, k)</i>	<i>k</i>	<i>h</i>	<i>s(h, k)</i>
79	33	47/158	83	11	27/166	86	11	55/86	89	17	18/89	92	9	85/184	95	12	45/38
79	34	99/158	83	12	183/166	86	13	27/86	89	18	128/89	92	11	59/184	95	13	17/38
79	35	-101/158	83	13	49/166	86	15	39/86	89	19	-7/89	92	13	-91/184	95	14	103/190
79	36	-21/158	83	14	197/166	86	17	-85/86	89	20	28/89	92	15	-21/184	95	16	257/190
79	37	-65/158	83	15	-27/166	86	19	-35/86	89	21	18/89	92	17	29/184	95	17	11/38
79	38	-143/158	83	16	7/166	86	21	-45/86	89	22	-132/89	92	19	75/184	95	18	5/38
79	39	-481/158	83	17	107/166	86	23	39/86	89	23	49/89	92	21	13/184	95	21	-93/190
80	1	1027/160	83	18	13/166	86	25	19/86	89	24	19/89	92	25	-59/184	95	22	17/38
80	3	65/32	83	19	9/166	86	27	-15/86	89	25	-8/89	92	27	-29/184	95	23	-13/38
80	7	25/32	83	20	-43/166	86	29	189/86	89	26	19/89	92	29	-75/184	95	24	353/190
80	9	163/160	83	21	267/166	86	31	19/86	89	27	5/89	92	31	435/184	95	26	117/190
80	11	37/160	83	22	37/166	86	33	-27/86	89	28	-17/89	92	33	45/184	95	27	-31/38
80	13	-1/32	83	23	-13/166	86	35	15/86	89	29	-83/89	92	35	-13/184	95	28	11/38
80	17	7/32	83	24	53/166	86	37	61/86	89	30	203/89	92	37	237/184	95	29	-17/190
80	19	-27/160	83	25	75/166	86	39	-55/86	89	31	49/89	92	39	-45/184	95	31	-193/190
80	21	27/160	83	26	7/166	86	41	-45/86	89	32	8/89	92	41	85/184	95	32	93/38
80	23	25/32	83	27	-141/166	87	1	3655/522	89	33	5/89	92	43	-21/184	95	33	13/38
80	27	65/32	83	28	351/166	87	2	1763/522	89	34	0/1	92	45	-307/184	95	34	103/190
80	29	-37/160	83	29	43/166	87	4	883/522	89	35	-17/89	93	1	2093/279	95	36	17/190
80	31	-3/160	83	30	11/166	87	5	629/522	89	36	44/89	93	2	1012/279	95	37	5/38
80	33	7/32	83	31	-93/166	87	7	451/522	89	37	-35/89	93	4	437/279	95	39	13/190
80	37	-1/32	83	32	49/166	87	8	575/522	89	38	-79/89	93	5	364/279	95	41	47/190
80	39	-227/160	83	33	-189/166	87	10	253/522	89	39	12/89	93	7	221/279	95	42	-13/38
81	1	1580/243	83	34	37/166	87	11	575/522	89	40	-28/89	93	8	220/279	95	43	-13/38
81	2	760/243	83	35	9/166	87	13	127/522	89	41	-57/89	93	10	149/279	95	44	-47/190
81	4	320/243	83	36	11/166	87	14	35/522	89	42	-44/89	93	11	193/279	95	46	-193/190
81	5	220/243	83	37	63/166	87	16	163/522	89	43	-83/89	93	13	14/279	95	47	-141/38
81	7	158/243	83	38	-53/166	87	17	-5/18	89	44	-308/89	93	14	148/279	96	1	4465/576
81	8	40/243	83	39	-107/166	87	19	37/522	90	1	979/135	93	16	113/279	96	5	665/576
81	10	-40/243	83	40	-141/166	87	20	-127/522	90	7	32/27	93	17	193/279	96	7	559/576
81	11	58/243	83	41	-533/166	87	22	883/522	90	11	29/135	93	19	203/279	96	11	359/576
81	13	50/243	84	1	3403/504	87	23	125/522	90	13	32/27	93	20	148/279	96	13	217/576
81	14	112/243	84	5	683/504	87	25	451/522	90	17	4/27	93	22	-4/279	96	17	305/576
81	16	-220/243	84	11	269/504	87	26	-253/522	90	19	61/135	93	23	-437/279	96	19	-665/576
81	17	40/243	84	13	307/504	87	28	-233/522	90	23	22/27	93	25	23/279	96	23	-145/576
81	19	-40/243	84	17	683/504	87	31	-35/522	90	29	-79/135	93	26	-23/279	96	25	145/576
81	20	-320/243	84	19	109/504	87	32	-37/522	90	31	79/135	93	28	149/279	96	29	89/576
81	22	-58/243	84	23	269/504	87	34	-125/522	90	37	-4/27	93	29	-113/279	96	31	-305/576
81	23	-158/243	84	25	-53/504	87	35	629/522	90	41	29/135	93	32	202/279	96	35	359/576
81	25	50/243	84	29	323/504	87	37	-179/522	90	43	-22/27	93	34	68/279	96	37	217/576
81	26	-122/243	84	31	109/504	87	38	-163/522	91	1	1335/182	93	35	220/279	96	41	-559/576
81	28	122/243	84	37	-53/504	87	40	-179/522	91	2	645/182	93	37	-364/279	96	43	-89/576
81	29	112/243	84	41	-757/504	87	41	-5/18	91	3	405/182	93	38	4/279	96	47	-1009/576
81	31	-4/243	85	1	581/85	87	43	-1763/522	91	4	323/182	93	40	221/279	97	1	760/97
81	32	-104/243	85	2	56/17	88	1	1247/176	91	5	15/14	93	41	-68/279	97	2	368/97
81	34	-4/243	85	3	35/17	88	3	377/176	91	6	135/182	93	43	14/279	97	3	232/97
81	35	40/243	85	4	7/5	88	5	215/176	91	8	9/14	93	44	-203/279	97	4	160/97
81	37	-40/243	85	6	56/85	88	7	129/176	91	9	15/182	93	46	-1012/279	97	5	133/97
81	38	-104/243	85	7	7/17	88	9	127/176	91	10	-15/182	94	1	713/94	97	6	80/97
81	40	-760/243	85	8	12/17	88	13	71/176	91	11	57/182	94	3	217/94	97	7	123/97
82	1	270/41	85	9	59/85	88	15	113/176	91	12	69/182	94	5	143/94	97	8	32/97
82	3	81/41	85	11	46/85	88	17	-17/176	91	15	-135/182	94	7	89/94	97	9	78/97
82	5	46/41	85	12	-7/17	88	19	41/176	91	16	15/182	94	9	73/94	97	10	55/97
82	7	33/41	85	13	0/1	88	21	7/176	91	17	-15/182	94	11	23/94	97	11	70/97
82	9	0/1	85	14	-56/85	88	23	81/176	91	18	-15/14	94	13	27/94	97	12	-32/97
82	11	25/41	85	16	31/85	88	25	-129/176	91	19	83/182	94	15	7/94	97	13	67/97
82	13	15/41	85	18	4/17	88	27	-71/176	91	20	-5/182	94	17	-23/94	97	14	123/97
82	15	25/41	85	19	59/85	88	29	-377/176	91	22	-37/182	94	19	143/94	97	15	67/97
82	17	19/41	85	21	-7/5	88	31	17/176	91	23	323/182	94	21	73/94	97	16	-80/97
82	19	15/41	85	22	7/17	88	35	-215/176	91	24	83/182	94	23	-57/94	97	17	29/97
82	21	30/41	85	23	1/17	88	37	-41/176	91	25	43/182	94	25	-7/94	97	18	28/97
82	23	2/41	85	24	-16/85	88	39	-127/176	91	27	9/182	94	27	89/94	97	19	-35/97
82	25	2/41	85	26	-9/85	88	41	-113/176	91	29	-37/182	94	29	27/94	97	20	53/97
82	27	-81/41	85	27	-7/17	88	43	-279/176	91	30	-405/182	94	31	-217/94	97	21	21/97
82	29	19/41	85	28	-35/17	89	1	638/89	91	31	15/14	94	33	31/94	97	22	0/1
82	31	10/41	85	29	84/85	89	2	308/89	91	32	57/182	94	35	-17/94	97	23	-3/97
82	33	46/41	85	31	46/85	89	3	203/89	91	33	-57/182	94	37	-31/94	97	24	-160/97
82	35	-33/41	85	32	12/17	89	4	132/89	91	34	-9/14	94	39	-9/94	97	25	48/97
82	37	-10/41	85	33	-4/17	89	5	128/89	91	36	-93/182	94	41	-9/94	97	26	23/97
82	39	-30/41	85	36	-9/85	89	6	113/89	91	37	57/182	94	43	-17/94	97	27	28/97
83	1	1107/166	85	37	1/17	89	7	79/89	91	38	69/182	94	45	-57/94	97	28	39/97
83	2	533/166	85	38	0/1	89	8	22/89	91	40	-43/182	95	1	1457/190	97	29	-55/97
83	3	351/166	85	39	-16/85	89	9	98/89	91	41	-5/182	95	2	141/38	97	30	-1/97
83	4	267/166	85	41	-84/85	89	10	98/89	91	43	-93/182	95	3	93/38	97	31	-48/97
83	5	189/166	85	42	-56/17	89	11	-22/89	91	44	-15/14	95	4	353/190	97	32	-232/97
83	6	197/166	86	1	595/86	89	12	35/89	91	45	-645/182	95	6	257/190	97	33	112/97
83	7	183/166	86	3	189/86	89	13	57/89	92	1	1365/184	95	7	31/38	97	34	53/97
83	8	93/166	86	5	85/86	89	14	7/89	92	3	435/184	95	8	45/38	97	35	8/97
83	9	63/166	86	7	61/86	89	15	113/89	92	5	237/184	95	9	93/190	97	36	-8/97
83	10	75/166	86	9	35/86	89	16	12/89	92	7							

k	h	$s(h, k)$	k	h	$s(h, k)$	k	h	$s(h, k)$
97	38	-3/97	98	45	-16/49	99	46	-161/594
97	39	133/97	98	47	-44/49	99	47	-343/594
97	40	29/97	99	1	4753/594	99	49	-2303/594
97	41	-23/97	99	2	2303/594	100	1	1617/200
97	42	1/97	99	4	1153/594	100	3	99/40
97	43	-78/97	99	5	953/594	100	7	7/8
97	44	-70/97	99	7	343/594	100	9	33/200
97	45	-39/97	99	8	431/594	100	11	-33/200
97	46	-35/97	99	10	703/594	100	13	13/40
97	47	-112/97	99	13	235/594	100	17	29/40
97	48	-368/97	99	14	-343/594	100	19	-17/200
98	1	388/49	99	16	73/594	100	21	17/200
98	3	124/49	99	17	323/594	100	23	-13/40
98	5	68/49	99	19	-53/594	100	27	3/40
98	9	58/49	99	20	953/594	100	29	33/200
98	11	58/49	99	23	89/594	100	31	-33/200
98	13	4/49	99	25	1153/594	100	33	-99/40
98	15	-4/49	99	26	53/594	100	37	-3/40
98	17	12/49	99	28	-161/594	100	39	-17/200
98	19	-2/49	99	29	35/594	100	41	17/200
98	23	-12/49	99	31	73/594	100	43	7/8
98	25	44/49	99	32	-397/594	100	47	-29/40
98	27	4/49	99	34	397/594	100	49	-367/200
98	29	-4/49	99	35	323/594			
98	31	-2/49	99	37	-431/594			
98	33	124/49	99	38	-235/594			
98	37	16/49	99	40	343/594			
98	39	-68/49	99	41	35/594			
98	41	4/49	99	43	-89/594			
98	43	-4/49	99	46	-161/594			

4. Bibliography

T. M. Apostol, Generalized Dedekind sums and transformation formulae of certain Lambert series, *Duke Math. J.* **17**, 147–157 (1950); MR **11**, 641.

T. M. Apostol, Theorems on generalized Dedekind sums, *Pacific J. Math.* **2**, 1–9 (1952); MR **13**, 725.

L. Carlitz, Some theorems on generalized Dedekind sums, *Pacific J. Math.* **3**, 513–522 (1953); MR **15**, 12.

L. Carlitz, The reciprocity theorem for Dedekind sums, *Pacific J. Math.* **3**, 523–527 (1953); MR **15**, 12.

R. Dedekind, Erläuterungen zu den vorstehenden Fragmenten, *Riemann's Gesammelte Werke*, pp. 466–478; *Dedekind's Gesammelte Werke Bd 1*, 159–172 (1930).

R. Dedekind, Schreiben an Heir Borchardt über die Theorie der elliptischen Modulfunktionen, *J. Reine Angew. Math.* **83**, 265–292 (1877); also *Dedekind's Gesammelte Werke*, Bd **1**, pp. 174–201.

U. Dieter, Beziehungen zwischen Dedekindschen Summen, *Abh. Math. Sem. Univ. Hamburg.* **21**, 109–125 (1957); MR **19**, 395.

C. Meyer, Über einige Anwendungen Dedekindschen Summen, *J. Reine Angew. Math.* **198**, 143–203 (1957); MR **21**, #3396.

L. J. Mordell, Lattice points in a tetrahedron and generalized Dedekind sums, *J. Indian Math. Soc.* **15**, 41–46 (1951); MR **13**, 322.

L. J. Mordell, The reciprocity for Dedekind sums, *Amer. J. Math.* **73**, 593–598 (1951); MR **13**, 113.

Morris Newman, Construction and application of a class of modular functions. I, II, *Proc. London Math. Soc.* **7**, 334–350 (1957); MR **19**, 953; **9**, 373–387 (1959); MR **21**, #6354.

Hans Rademacher, Zur Theorie der Modulfunktionen, *J. Reine Angew. Math.* **167**, 312–336 (1931); Z **3**, 215.

Hans Rademacher, Bestimmung einer gewissen Einheitswurzel in der Theorie der Modulfunktionen, *J. London Math. Soc.* **7**, 14–19 (1932); Z **3**, 351.

Hans Rademacher, Ein arithmetische Summenformel, *Monatsh. Math. Phys.* **39**, 221–228 (1932).

Hans Rademacher, Über eine Reziprozitätsformel aus der Theorie der Modulfunktionen, *Mat. Fiz. Lap.* **40**, 24–34, Z **3**, 75; *FdM* **59**, 1060.

Hans Rademacher, On the partition function $p(n)$, *Proc. London Math. Soc. series 2*, **43**, 241–254 (1937); Z **16**, 246.

Hans Rademacher, The Ramanujan identities under modular substitutions, *Trans. Amer. Math. Soc.* **51**, 609–636 (1942); MR **3**, 271.

Hans Rademacher, Die Reziprozitätsformel für Dedekindsche Summen, *Acta Sci. Math. (Szeged)* **12B**, 57–60 (1950); MR **11**, 642.

Hans Rademacher, On Dedekind sums and lattice points in a tetrahedron, *Studies in mathematics and mechanics presented to Richard von Mises* (Academic Press, New York, 1954); pp. 49–53, MR **16**, 341.

Hans Rademacher, Generalization of the reciprocity formula for Dedekind sums, *Duke Math. J.* **21**, 1954, 391–397 (1954); MR **16**, 14.

Hans Rademacher, On the transformation of $\log \eta(\tau)$, *J. Indian Math. Soc.* **19**, 25–30 (1955); MR **17**, 15.

Hans Rademacher, Zur Theorie der Dedekindschen Summen, *Math. Z.* **63**, 445–463 (1956); MR **18**, 114.

Hans Rademacher, Lectures on analytic number theory, *Tata Institute of Fundamental Research, Bombay*, 1954–55.

Hans Rademacher and A. L. Whitman, Theorems on Dedekind sums, *Amer. J. Math.* **63**, 377–407 (1941); MR **2**, 249.

L. Redei, Elementarer Beweis und Verallgemeinerung einer Reziprozitätsformel von Dedekind, *Acta Sci. Math. (Szeged)* **12B**, 236–239 (1950); MR **11**, 641.

Hans Salie, Zum Wertevorrat der Dedekindscher Summen, *Math. Z.* **72**, 61–75 (1959); MR **21**, #5601.

Klaus Wohlfahrt, Über Dedekindsche Summen und Untergruppen der Modulgruppe, *Abh. Math. Sem. Univ. Hamburg*, **23**, 5–10 (1959); MR **21**, #1350.

(Paper 69B4–155)