

Splitting of Equivalent Points in Noncentrosymmetric Space Groups Into Subsets Under Homogeneous Stress

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Splitting of general positions in crystals into subsets of equivalent sites under homogeneous stress has previously been given for all centrosymmetric space groups; the tabulation is here completed for all space groups by listing the results for noncentrosymmetric space groups.

1. Introduction

The present paper presents results analogous to those previously submitted for centrosymmetric space groups [1]¹ and for point groups [2]; these previous papers should be consulted for detailed discussion.

It is assumed that the symmetry elements possessed by a homogeneously stressed crystal will be those common to the crystal and to the macroscopic state of stress. Application of stress either leaves a space group unaltered or lowers it to a subgroup. Such lowering can always be considered to take place in successive steps each of which leaves no group which is both a stress-induced subgroup of the initial group and a supergroup of the final group, and which is distinct from both. Each such step can be accomplished by a uniaxial stress; for the noncentrosymmetric space groups all but two of the symmetry reductions consisting of two or more successive steps can also be accomplished by uniaxial stress. These two require biaxial stress [2]. More general stress states are, however, consistent with many of the steps of symmetry lowering; we list the most general state of stress (in terms of the modified stress ellipsoid [1]) consistent with each step. The same stress is appropriate for all space-group-to-space-group transformations associated with a given point-group-to-point-group transformation. There are 25 of the latter which are minimum steps of symmetry lowering for noncentrosymmetric point groups so that the results for the noncentrosymmetric space groups are collected into 25 corresponding tables.

A set of points all of which are equivalent in the unstressed crystal frequently splits into two or more subsets under stress. For each space group all possibilities are taken into account by considering the behavior of the general position because the behavior of each special position can be derived by specializing the general position. This process of

specialization in space groups has been discussed and a technique for visualizing it in terms of stereograms of point groups has been described [2].

2. Results

2.1. Behavior of General Position

The splitting of the general position (set of equivalent sites having no symmetry) into subsets is listed in tables 1 through 25. Each table is headed by a point-group transformation which is a minimum step of symmetry lowering. Each of the space groups associated with the initial point group is listed in the table together with the coordinates of a set of sites making up a general position. The latter are collected into subsets; all of the sites in a subset remain equivalent after symmetry reduction to the final space group which is also listed. For some of the point-group reductions the final point group can occur in two or three non-equivalent orientations. These may correspond to different final space groups; in table 12, for example, one orientation corresponds to the caption at the top of the table and the other to the caption at the bottom. The stress is described by giving conditions on the axes X , Y , Z of the stress ellipsoid [1] to the crystal axes x , y , z ; the stress described is the most general (i.e., least restricted) consistent with the symmetry reduction. In many of the tables a single stress specification suffices for all space groups, but in some (table 10, for example) the stress must be specified for each space group because it is customary to choose the axes in different orientations with respect to the point group.

2.2. Stress Table

The most general stress consistent with each possible step of stress-induced symmetry lowering, minimum or compound, is listed in table 26 for all of the noncentrosymmetric point groups. The stress conditions for the minimum steps of stress-induced symmetry lowering are equivalent, though not always identical, to those given in tables 1–25.

¹Figures in brackets indicate the literature references on page 462.

TABLE 1. Reduction from $\bar{4}3m$ to $\bar{4}2m$

3. References

- [1] J. B. Wachtman, Jr., and H. S. Peiser, Splitting of a set of equivalent sites in centrosymmetric space groups into subsets under homogeneous stress, *J. Res. NBS* **69A** (*Phys. and Chem.*) No. 2, 193–207 (1965).
 - [2] H. S. Peiser and J. B. Wachtman, Jr., Reduction of crystallographic point groups to subgroups by homogeneous stress, *J. Res. NBS* **69A** (*Phys. and Chem.*) No. 4, 309–324 (1965).

TABLE 2. Reduction from $\bar{4}3m$ to $3m$

Space group of unstrained crystal, order 24 per lattice point		If stressed so that $X=Y; [111]\parallel Z$				Space group of strained crystal, order 6 per lattice point	
		Coordinates referred to axes of unstrained crystal					
No.	Symbol	1st Subset	2d Subset	3d Subset	4th Subset	No.	Symbol
215	$P\bar{4}3m$	(x, y, z)	(x, \bar{y}, \bar{z})	(\bar{x}, y, \bar{z})	(\bar{x}, \bar{y}, z)	160	$R3m$
		(y, z, x)	(\bar{y}, \bar{z}, x)	(y, \bar{z}, \bar{x})	(\bar{y}, z, \bar{x})		
		(z, x, y)	(\bar{z}, x, \bar{y})	(\bar{z}, \bar{x}, y)	(z, \bar{x}, \bar{y})		
		(y, x, z)	(\bar{y}, x, \bar{z})	(y, \bar{x}, \bar{z})	(\bar{y}, \bar{x}, z)		
		(z, y, x)	(\bar{z}, \bar{y}, x)	(\bar{z}, y, \bar{x})	(z, \bar{y}, \bar{x})		
		(x, z, y)	(x, \bar{z}, \bar{y})	(\bar{x}, \bar{z}, y)	(\bar{x}, z, \bar{y})		
216	$F\bar{4}3m$	(x, y, z)	(x, \bar{y}, \bar{z})	(\bar{x}, y, \bar{z})	(\bar{x}, \bar{y}, z)	160	$R3m$
		(y, z, x)	(\bar{y}, \bar{z}, x)	(y, \bar{z}, \bar{x})	(\bar{y}, z, \bar{x})		
		(z, x, y)	(\bar{z}, x, \bar{y})	(\bar{z}, \bar{x}, y)	(z, \bar{x}, \bar{y})		
		(y, x, z)	(\bar{y}, x, \bar{z})	(y, \bar{x}, \bar{z})	(\bar{y}, \bar{x}, z)		
		(z, y, x)	(\bar{z}, \bar{y}, x)	(\bar{z}, y, \bar{x})	(z, \bar{y}, \bar{x})		
		(x, z, y)	(x, \bar{z}, \bar{y})	(\bar{x}, \bar{z}, y)	(\bar{x}, z, \bar{y})		
217	$I\bar{4}3m$	(x, y, z)	(x, \bar{y}, \bar{z})	(\bar{x}, y, \bar{z})	(\bar{x}, \bar{y}, z)	160	$R3m$
		(y, z, x)	(\bar{y}, \bar{z}, x)	(y, \bar{z}, \bar{x})	(\bar{y}, z, \bar{x})		
		(z, x, y)	(\bar{z}, x, \bar{y})	(\bar{z}, \bar{x}, y)	(z, \bar{x}, \bar{y})		
		(y, x, z)	(\bar{y}, x, \bar{z})	(y, \bar{x}, \bar{z})	(\bar{y}, \bar{x}, z)		
		(z, y, x)	(\bar{z}, \bar{y}, x)	(\bar{z}, y, \bar{x})	(z, \bar{y}, \bar{x})		
		(x, z, y)	(x, \bar{z}, \bar{y})	(\bar{x}, \bar{z}, y)	(\bar{x}, z, \bar{y})		
218	$P\bar{4}3n$	(x, y, z)	(x, \bar{y}, \bar{z})	(\bar{x}, y, \bar{z})	(\bar{x}, \bar{y}, z)	161	$R3c$
		(y, z, x)	(\bar{y}, \bar{z}, x)	(y, \bar{z}, \bar{x})	(\bar{y}, z, \bar{x})		
		(z, x, y)	(\bar{z}, x, \bar{y})	(\bar{z}, \bar{x}, y)	(z, \bar{x}, \bar{y})		
		($\frac{1}{2}+y, \frac{1}{2}+x, \frac{1}{2}+z$)	($\frac{1}{2}-y, \frac{1}{2}+x, \frac{1}{2}-z$)	($\frac{1}{2}+y, \frac{1}{2}-x, \frac{1}{2}-z$)	($\frac{1}{2}-y, \frac{1}{2}-x, \frac{1}{2}+z$)		
		($\frac{1}{2}+z, \frac{1}{2}+y, \frac{1}{2}+x$)	($\frac{1}{2}-z, \frac{1}{2}-y, \frac{1}{2}+x$)	($\frac{1}{2}-z, \frac{1}{2}+y, \frac{1}{2}-x$)	($\frac{1}{2}+z, \frac{1}{2}-y, \frac{1}{2}-x$)		
		($\frac{1}{2}+x, \frac{1}{2}+z, \frac{1}{2}+y$)	($\frac{1}{2}-x, \frac{1}{2}-z, \frac{1}{2}-y$)	($\frac{1}{2}-x, \frac{1}{2}-z, \frac{1}{2}+y$)	($\frac{1}{2}-x, \frac{1}{2}+z, \frac{1}{2}-y$)		
219	$F\bar{4}3e$	(x, y, z)	(x, \bar{y}, \bar{z})	(\bar{x}, y, \bar{z})	(\bar{x}, \bar{y}, z)	161	$R3c$
		(y, z, x)	(\bar{y}, \bar{z}, x)	(y, \bar{z}, \bar{x})	(\bar{y}, z, \bar{x})		
		(z, x, y)	(\bar{z}, x, \bar{y})	(\bar{z}, \bar{x}, y)	(z, \bar{x}, \bar{y})		
		($y, x, \frac{1}{2}+z$)	($\bar{y}, x, \frac{1}{2}-z$)	($y, \bar{x}, \frac{1}{2}-z$)	($\bar{y}, \bar{x}, \frac{1}{2}+z$)		
		($z, y, \frac{1}{2}+x$)	($\bar{z}, \bar{y}, \frac{1}{2}+x$)	($\bar{z}, y, \frac{1}{2}-x$)	($z, \bar{y}, \frac{1}{2}-x$)		
		($x, z, \frac{1}{2}+y$)	($x, \bar{z}, \frac{1}{2}-y$)	($\bar{x}, \bar{z}, \frac{1}{2}+y$)	($\bar{x}, z, \frac{1}{2}-y$)		
220	$I\bar{4}3d$	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, \bar{z}$)	($\bar{x}, \frac{1}{2}+y, \frac{1}{2}-z$)	($\frac{1}{2}-x, \bar{y}, \frac{1}{2}+z$)	161	$R3c$
		(y, z, x)	($\frac{1}{2}-y, \bar{z}, \frac{1}{2}+x$)	($\frac{1}{2}+y, \frac{1}{2}-z, \bar{x}$)	($\bar{y}, \frac{1}{2}+z, \frac{1}{2}-x$)		
		(z, x, y)	($\bar{z}, \frac{1}{2}+x, \frac{1}{2}-y$)	($\frac{1}{2}-z, \bar{x}, \frac{1}{2}+y$)	($\frac{1}{2}+z, \frac{1}{2}-x, \bar{y}$)		
		($\frac{1}{4}+y, \frac{1}{4}+x, \frac{1}{4}+z$)	($\frac{3}{4}-y, \frac{3}{4}+x, \frac{1}{4}-z$)	($\frac{3}{4}+y, \frac{1}{4}-x, \frac{3}{4}-z$)	($\frac{1}{4}-y, \frac{3}{4}-x, \frac{3}{4}+z$)		
		($\frac{1}{4}+z, \frac{1}{4}+y, \frac{1}{4}+x$)	($\frac{1}{4}-z, \frac{3}{4}-y, \frac{3}{4}+x$)	($\frac{3}{4}-z, \frac{3}{4}+y, \frac{1}{4}-x$)	($\frac{3}{4}+z, \frac{1}{4}-y, \frac{3}{4}-x$)		
		($\frac{1}{4}+x, \frac{1}{4}+z, \frac{1}{4}+y$)	($\frac{3}{4}+x, \frac{1}{4}-z, \frac{3}{4}+y$)	($\frac{1}{4}-x, \frac{3}{4}-z, \frac{3}{4}+y$)	($\frac{3}{4}-x, \frac{3}{4}+z, \frac{1}{4}-y$)		

TABLE 3. Reduction from 432 to 422

TABLE 4. Reduction from 432 to 32

TABLE 5. Reduction from 23 to 222

Space group of unstrained crystal, order 12 per lattice point		If stressed so that $x, y, z \parallel X, Y, Z$, any permutation						Space group of strained crystal, order 4 per lattice point	
No.	Symbol	Coordinates referred to axes of unstrained crystal						No.	Symbol
		1st Subset		2d Subset		3d Subset			
195	P23	(x, y, z)		(y, z, x)		(z, x, y)		16	P222
		(x, \bar{y}, \bar{z})		(y, \bar{z}, \bar{x})		(z, \bar{x}, \bar{y})			
		(\bar{x}, y, \bar{z})		(\bar{y}, z, \bar{x})		(\bar{z}, x, \bar{y})			
		(\bar{x}, \bar{y}, z)		(\bar{y}, \bar{z}, x)		(\bar{z}, \bar{x}, y)			
196	F23	(x, y, z)		(y, z, x)		(z, x, y)		22	F222
		(x, \bar{y}, \bar{z})		(y, \bar{z}, \bar{x})		(z, \bar{x}, \bar{y})			
		(\bar{x}, y, \bar{z})		(\bar{y}, z, \bar{x})		(\bar{z}, x, \bar{y})			
		(\bar{x}, \bar{y}, z)		(\bar{y}, \bar{z}, x)		(\bar{z}, \bar{x}, y)			
197	I23	(x, y, z)		(y, z, x)		(z, x, y)		23	I222
		(x, \bar{y}, \bar{z})		(y, \bar{z}, \bar{x})		(z, \bar{x}, \bar{y})			
		(\bar{x}, y, \bar{z})		(\bar{y}, z, \bar{x})		(\bar{z}, x, \bar{y})			
		(\bar{x}, \bar{y}, z)		(\bar{y}, \bar{z}, x)		(\bar{z}, \bar{x}, y)			
198	P2 ₁ 3	(x, y, z)		(y, z, x)		(z, x, y)		19	P2 ₁ 2 ₁ 2 ₁
		($\frac{1}{2}x + r, \frac{1}{2}y - r, \bar{z}$)		($\frac{1}{2}y + r, \frac{1}{2}z - r, \bar{x}$)		($\frac{1}{2}z + r, \frac{1}{2}x - r, \bar{y}$)			
		($\bar{x}, \frac{1}{2}y + r, \frac{1}{2}z - r$)		($\bar{y}, \frac{1}{2}z + r, \frac{1}{2}x - r$)		($\bar{z}, \frac{1}{2}x + r, \frac{1}{2}y - r$)			
		($\frac{1}{2}x - r, \bar{y}, \frac{1}{2}z + r$)		($\frac{1}{2}y - r, \bar{z}, \frac{1}{2}x + r$)		($\frac{1}{2}z - r, \bar{x}, \frac{1}{2}y + r$)			
199	I2 ₁ 3	(x, y, z)		(y, z, x)		(z, x, y)		24	I2 ₁ 2 ₁ 2 ₁
		($\frac{1}{2}x + r, \frac{1}{2}y - r, \bar{z}$)		($\frac{1}{2}y + r, \frac{1}{2}z - r, \bar{x}$)		($\frac{1}{2}z + r, \frac{1}{2}x - r, \bar{y}$)			
		($\bar{x}, \frac{1}{2}y + r, \frac{1}{2}z - r$)		($\bar{y}, \frac{1}{2}z + r, \frac{1}{2}x - r$)		($\bar{z}, \frac{1}{2}x + r, \frac{1}{2}y - r$)			
		($\frac{1}{2}x - r, \bar{y}, \frac{1}{2}z + r$)		($\frac{1}{2}y - r, \bar{z}, \frac{1}{2}x + r$)		($\frac{1}{2}z - r, \bar{x}, \frac{1}{2}y + r$)			

TABLE 6. Reduction from 23 to 3

Space group of unstrained crystal, order 12 per lattice point		If stressed so that $X=Y=[111] \parallel Z$						Space group of strained crystal, order 3 per lattice point					
No.	Symbol	Coordinates referred to axes of unstrained crystal											
		1st Subset	2d Subset		3d Subset		4th Subset		No.	Symbol			
195	P23	(x, y, z)	(x	\bar{y} ,	\bar{z})	(\bar{x} ,	y ,	\bar{z})	(\bar{x} ,	\bar{y} ,	z)	146	R3
		(y, z, x)	(\bar{y} ,	\bar{z} ,	x)	(y ,	\bar{z} ,	\bar{x})	(\bar{y} ,	z ,	\bar{x})		
		(z, x, y)	(\bar{z}	x ,	\bar{y})	(\bar{z} ,	\bar{x} ,	y)	(z ,	\bar{x} ,	\bar{y})		
196	F23	(x, y, z)	(x	\bar{y} ,	\bar{z})	(\bar{x} ,	y ,	\bar{z})	(\bar{x} ,	\bar{y} ,	z)	146	R3
		(y, z, x)	(\bar{y} ,	\bar{z} ,	x)	(y ,	\bar{z} ,	\bar{x})	(\bar{y} ,	z ,	\bar{x})		
		(z, x, y)	(\bar{z}	x ,	\bar{y})	(\bar{z} ,	\bar{x} ,	y)	(z ,	\bar{x} ,	\bar{y})		
197	I23	(x, y, z)	(x	\bar{y} ,	\bar{z})	(\bar{x} ,	y ,	\bar{z})	(\bar{x} ,	\bar{y} ,	z)	146	R3
		(y, z, x)	(\bar{y} ,	\bar{z} ,	x)	(y ,	\bar{z} ,	\bar{x})	(\bar{y} ,	z ,	\bar{x})		
		(z, x, y)	(\bar{z}	x ,	\bar{y})	(\bar{z} ,	\bar{x} ,	y)	(z ,	\bar{x} ,	\bar{y})		
198	P2 ₁ 3	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y,$	\bar{z})	($\bar{x}, \frac{1}{2}+y, \frac{1}{2}-z$)	($\frac{1}{2}-x,$	$\bar{y}, \frac{1}{2}+z$)	146	R3				
		(y, z, x)	($\frac{1}{2}-y,$	$\bar{z}, \frac{1}{2}+x$)	($\frac{1}{2}+y, \frac{1}{2}-z,$	$\bar{x})$	($\bar{y}, \frac{1}{2}+z, \frac{1}{2}-x$)						
		(z, x, y)	($\bar{z}, \frac{1}{2}+x, \frac{1}{2}-y$)	($\frac{1}{2}-z,$	$\bar{x}, \frac{1}{2}+y$)	($\frac{1}{2}+z, \frac{1}{2}-x,$	$\bar{y})$						
199	I2 ₁ 3	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y,$	\bar{z})	($\bar{x}, \frac{1}{2}+y, \frac{1}{2}-z$)	($\frac{1}{2}-x,$	$\bar{y}, \frac{1}{2}+z$)	146	R3				
		(y, z, x)	($\frac{1}{2}-y,$	$\bar{z}, \frac{1}{2}+x$)	($\frac{1}{2}+y, \frac{1}{2}-z,$	$\bar{x})$	($\bar{y}, \frac{1}{2}+z, \frac{1}{2}-x$)						
		(z, x, y)	($\bar{z}, \frac{1}{2}+x, \frac{1}{2}-y$)	($\frac{1}{2}-z,$	$\bar{x}, \frac{1}{2}+y$)	($\frac{1}{2}+z, \frac{1}{2}-x,$	$\bar{y})$						

TABLE 7. Reduction from $\bar{6}m2$ to $mm2$

Space group of unstrained crystal, order 12 per lattice point		If stressed so that y and $z \parallel$ any two of X , Y , and Z			Space group of strained crystal, order 4 per lattice point	
		Coordinates referred to axes of unstrained crystal				
No.	Symbol	1st Subset	2d Subset	3d Subset	No.	Symbol
187	$P\bar{6}m2$	(x , y , z)	(\bar{y} , $x-y$, z)	($y-x$, \bar{x} , z)	38	$Amm2$
		(x , $x-y$, z)	(\bar{y} , \bar{x} , z)	($y-x$, y , z)		
		(x , y , \bar{z})	(\bar{y} , $x-y$, \bar{z})	($y-x$, \bar{x} , \bar{z})		
		(x , $x-y$, \bar{z})	(\bar{y} , \bar{x} , \bar{z})	($y-x$, y , \bar{z})		
188	$P\bar{6}c2$	(x , y , z)	(\bar{y} , $x-y$, z)	($y-x$, \bar{x} , z)	40	$Ama2$
		(x , $x-y$, $\frac{1}{2}+z$)	(\bar{y} , \bar{x} , $\frac{1}{2}+z$)	($y-x$, y , $\frac{1}{2}+z$)		
		(x , y , $\frac{1}{2}-z$)	(\bar{y} , $x-y$, $\frac{1}{2}-z$)	($y-x$, \bar{x} , $\frac{1}{2}-z$)		
		(x , $x-y$, \bar{z})	(\bar{y} , \bar{x} , \bar{z})	($y-x$, y , \bar{z})		
189	$P\bar{6}2m$	(x , y , z)	(\bar{y} , $x-y$, z)	($y-x$, \bar{x} , z)	38	$Amm2$
		(\bar{x} , $y-x$, z)	(y , x , z)	($x-y$, \bar{y} , z)		
		(x , y , \bar{z})	(\bar{y} , $x-y$, \bar{z})	($y-x$, \bar{x} , \bar{z})		
		(\bar{x} , $y-x$, \bar{z})	(y , x , \bar{z})	($x-y$, \bar{y} , \bar{z})		
190	$P\bar{6}2c$	(x , y , z)	(\bar{y} , $x-y$, z)	($y-x$, \bar{x} , z)	40	$Ama2$
		(\bar{x} , $y-x$, $\frac{1}{2}+z$)	(y , x , $\frac{1}{2}+z$)	($x-y$, \bar{y} , $\frac{1}{2}+z$)		
		(x , y , $\frac{1}{2}-z$)	(\bar{y} , $x-y$, $\frac{1}{2}-z$)	($y-x$, \bar{x} , $\frac{1}{2}-z$)		
		(\bar{x} , $y-x$, \bar{z})	(y , x , \bar{z})	($x-y$, \bar{y} , \bar{z})		

TABLE 8. Reduction from $6mm$ to $mm2$

Space group of unstrained crystal, order 12 per lattice point		If stressed so that y and $z \parallel$ any two of X , Y , and Z			Space group of strained crystal, order 4 per lattice point	
		Coordinates referred to axes of unstrained crystal				
No.	Symbol	1st Subset	2d Subset	3d Subset	No.	Symbol
183	$P6mm$	(x , y , z)	(\bar{y} , $x-y$, z)	($y-x$, \bar{x} , z)	35	$Cmm2$
		(x , $x-y$, z)	(\bar{y} , \bar{x} , z)	($y-x$, y , z)		
		(\bar{x} , \bar{y} , z)	(y , $y-x$, z)	($x-y$, x , z)		
		(\bar{x} , $y-x$, z)	(y , x , z)	($x-y$, \bar{y} , z)		
184	$P6cc$	(x , y , z)	(\bar{y} , $x-y$, z)	($y-x$, \bar{x} , z)	37	$Ccc2$
		(x , $x-y$, $\frac{1}{2}+z$)	(\bar{y} , \bar{x} , $\frac{1}{2}+z$)	($y-x$, y , $\frac{1}{2}+z$)		
		(\bar{x} , \bar{y} , z)	(y , $y-x$, z)	($x-y$, x , z)		
		(\bar{x} , $y-x$, $\frac{1}{2}+z$)	(y , x , $\frac{1}{2}+z$)	($x-y$, \bar{y} , $\frac{1}{2}+z$)		
185	$P6_{3}cm$	(x , y , z)	(\bar{y} , $x-y$, z)	($y-x$, \bar{x} , z)	36	$Cmc2_1$
		(x , $x-y$, $\frac{1}{2}+z$)	(\bar{y} , \bar{x} , $\frac{1}{2}+z$)	($y-x$, y , $\frac{1}{2}+z$)		
		(\bar{x} , \bar{y} , $\frac{1}{2}+z$)	(y , $y-x$, $\frac{1}{2}+z$)	($x-y$, x , $\frac{1}{2}+z$)		
		(\bar{x} , $y-x$, z)	(y , x , z)	($x-y$, \bar{y} , z)		
186	$P6_{3}mc$	(x , y , z)	(\bar{y} , $x-y$, z)	($y-x$, \bar{x} , z)	36	$Cmc2_1$
		(x , $x-y$, z)	(\bar{y} , \bar{x} , z)	($y-x$, y , z)		
		(\bar{x} , \bar{y} , $\frac{1}{2}+z$)	(y , $y-x$, $\frac{1}{2}+z$)	($x-y$, x , $\frac{1}{2}+z$)		
		(\bar{x} , $y-x$, $\frac{1}{2}+z$)	(y , x , $\frac{1}{2}+z$)	($x-y$, \bar{y} , $\frac{1}{2}+z$)		

TABLE 9. Reduction from 622 to 222

Space group of unstrained crystal, order 12 per lattice point		If stressed so that y and $z \parallel$ any two of X , Y , and Z			Space group of strained crystal, order 4 per lattice point	
		Coordinates referred to axes of unstrained crystal				
No.	Symbol	1st Subset	2d Subset	3d Subset	No.	Symbol
177	P622	(x, y, z)	($\bar{y}, x-y, z$)	($y-x, \bar{x}, z$)	21	C222
		($x, x-y, \bar{z}$)	($\bar{y}, \bar{x}, \bar{z}$)	($y-x, y, \bar{z}$)		
		(\bar{x}, \bar{y}, z)	($y, y-x, z$)	($x-y, x, z$)		
		($\bar{x}, y-x, \bar{z}$)	(y, x, \bar{z})	($x-y, \bar{y}, \bar{z}$)		
178	P6 ₁ 22	(x, y, z)	($\bar{y}, x-y, \frac{1}{3}z+z$)	($y-x, \bar{x}, \frac{2}{3}z+z$)	20	C222 ₁
		($x, x-y, \frac{5}{6}z-z$)	($\bar{y}, \bar{x}, \frac{5}{6}z-z$)	($y-x, y, \frac{1}{2}z-z$)		
		($\bar{x}, \bar{y}, \frac{1}{2}z+z$)	($y, y-x, \frac{5}{6}z+z$)	($x-y, x, \frac{1}{6}z+z$)		
		($\bar{x}, y-x, \frac{2}{3}z-z$)	($y, x, \frac{1}{3}z-z$)	($x-y, \bar{y}, \bar{z}$)		
179	P6 ₅ 22	(x, y, z)	($\bar{y}, x-y, \frac{2}{3}z+z$)	($y-x, \bar{x}, \frac{1}{3}z+z$)	20	C222 ₁
		($x, x-y, \frac{5}{6}z-z$)	($\bar{y}, \bar{x}, \frac{5}{6}z-z$)	($y-x, y, \frac{1}{2}z-z$)		
		($\bar{x}, \bar{y}, \frac{1}{2}z+z$)	($y, y-x, \frac{1}{6}z+z$)	($x-y, x, \frac{5}{6}z+z$)		
		($\bar{x}, y-x, \frac{1}{3}z-z$)	($y, x, \frac{2}{3}z-z$)	($x-y, \bar{y}, \bar{z}$)		
180	P6 ₂ 22	(x, y, z)	($\bar{y}, x-y, \frac{1}{3}z+z$)	($y-x, \bar{x}, \frac{2}{3}z+z$)	21	C222
		($x, x-y, \frac{1}{3}z-z$)	($\bar{y}, \bar{x}, \frac{2}{3}z-z$)	($y-x, y, \bar{z}$)		
		(\bar{x}, \bar{y}, z)	($y, y-x, \frac{1}{3}z+z$)	($x-y, x, \frac{1}{3}z+z$)		
		($\bar{x}, y-x, \frac{1}{3}z-z$)	($y, x, \frac{2}{3}z-z$)	($x-y, \bar{y}, \bar{z}$)		
181	P6 ₁ 22	(x, y, z)	($\bar{y}, x-y, \frac{1}{3}z+z$)	($y-x, \bar{x}, \frac{2}{3}z+z$)	21	C222
		($x, x-y, \frac{2}{3}z-z$)	($\bar{y}, \bar{x}, \frac{1}{3}z-z$)	($y-x, y, \bar{z}$)		
		(\bar{x}, \bar{y}, z)	($y, y-x, \frac{1}{3}z+z$)	($x-y, x, \frac{2}{3}z+z$)		
		($\bar{x}, y-x, \frac{2}{3}z-z$)	($y, x, \frac{1}{3}z-z$)	($x-y, \bar{y}, \bar{z}$)		
182	P6 ₅ 22	(x, y, z)	($\bar{y}, x-y, z$)	($y-x, \bar{x}, z$)	20	C222 ₁
		($x, x-y, \frac{1}{2}z-z$)	($\bar{y}, \bar{x}, \frac{1}{2}z-z$)	($y-x, y, \frac{1}{2}z-z$)		
		($\bar{x}, \bar{y}, \frac{1}{2}z+z$)	($y, y-x, \frac{1}{2}z+z$)	($x-y, x, \frac{1}{2}z+z$)		
		($\bar{x}, y-x, \bar{z}$)	(y, x, \bar{z})	($x-y, \bar{y}, \bar{z}$)		

TABLE 10. Reduction from $\bar{4}\bar{2}m$ to $mm\bar{2}$

Space group of unstrained crystal, order 8 per lattice point		If stressed so that	Coordinates referred to axes of unstrained crystal				Space group of strained crystal, order 4 per lattice point	
No.	Symbol		1st Subset		2d Subset		No.	Symbol
111	$P\bar{4}2m$	[110], [110], $z \parallel X, Y, Z$, any permutation.	(x, y, z)	(y, x, z)	(x, \bar{y}, \bar{z})	(y, \bar{x}, \bar{z})	35	Cmm2
			(\bar{x}, \bar{y}, z)	(\bar{y}, \bar{x}, z)	(\bar{x}, y, \bar{z})	(\bar{y}, x, \bar{z})		
112	$P\bar{4}2c$	Do.	(x, y, z)	($y, x, \frac{1}{2}+z$)	($x, \bar{y}, \frac{1}{2}-z$)	(y, \bar{x}, \bar{z})	37	Ccc2
			(\bar{x}, \bar{y}, z)	($\bar{y}, \bar{x}, \frac{1}{2}+z$)	($\bar{x}, y, \frac{1}{2}-z$)	(\bar{y}, x, \bar{z})		
113	$P\bar{4}21m$	Do.	(x, y, z)	($\frac{1}{2}+y, \frac{1}{2}+x, z$)	($\frac{1}{2}+x, \frac{1}{2}-y, \bar{z}$)	(y, \bar{x}, \bar{z})	35	Cmm2
			(\bar{x}, \bar{y}, z)	($\frac{1}{2}-y, \frac{1}{2}-x, z$)	($\frac{1}{2}-x, \frac{1}{2}+y, \bar{z}$)	(\bar{y}, x, \bar{z})		
114	$P42_1c$	Do.	(x, y, z)	($\frac{1}{2}+y, \frac{1}{2}+x, \frac{1}{2}+z$)	($\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}-z$)	(y, \bar{x}, \bar{z})	37	Ccc2
			(\bar{x}, \bar{y}, z)	($\frac{1}{2}-y, \frac{1}{2}-x, \frac{1}{2}+z$)	($\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}-z$)	(\bar{y}, x, \bar{z})		
115	$P\bar{4}m2$	$x, y, z \parallel X, Y, Z$, any permutation.	(x, y, z)	(x, \bar{y}, z)	(y, x, \bar{z})	(y, \bar{x}, \bar{z})	25	Pmm2
			(\bar{x}, \bar{y}, z)	(\bar{x}, y, z)	($\bar{y}, \bar{x}, \bar{z}$)	(\bar{y}, x, \bar{z})		
116	$P4c2$	Do.	(x, y, z)	($x, \bar{y}, \frac{1}{2}+z$)	($y, x, \frac{1}{2}-z$)	(y, \bar{x}, \bar{z})	27	Pcc2
			(\bar{x}, \bar{y}, z)	($\bar{x}, y, \frac{1}{2}+z$)	($\bar{y}, \bar{x}, \frac{1}{2}-z$)	(\bar{y}, x, \bar{z})		
117	$P\bar{4}b2$	Do.	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, z$)	($\frac{1}{2}+y, \frac{1}{2}+x, \bar{z}$)	(y, \bar{x}, \bar{z})	32	Pba2
			(\bar{x}, \bar{y}, z)	($\frac{1}{2}-x, \frac{1}{2}+y, z$)	($\frac{1}{2}-y, \frac{1}{2}-x, \bar{z}$)	(\bar{y}, x, \bar{z})		
118	$P\bar{4}n2$	Do.	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}+z$)	($\frac{1}{2}+y, \frac{1}{2}+x, \frac{1}{2}-z$)	(y, \bar{x}, \bar{z})	34	Pnn2
			(\bar{x}, \bar{y}, z)	($\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z$)	($\frac{1}{2}-y, \frac{1}{2}-x, \frac{1}{2}-z$)	(\bar{y}, x, \bar{z})		
119	$I\bar{1}m2$	Do.	(x, y, z)	(x, \bar{y}, z)	(y, x, \bar{z})	(y, \bar{x}, \bar{z})	44	Imm2
			(\bar{x}, \bar{y}, z)	(\bar{x}, y, z)	($\bar{y}, \bar{x}, \bar{z}$)	(\bar{y}, x, \bar{z})		
120	$I\bar{4}c2$	Do.	(x, y, z)	($x, \bar{y}, \frac{1}{2}+z$)	($y, x, \frac{1}{2}-z$)	(y, \bar{x}, \bar{z})	45	Iba2
			(\bar{x}, \bar{y}, z)	($\bar{x}, y, \frac{1}{2}+z$)	($\bar{y}, \bar{x}, \frac{1}{2}-z$)	(\bar{y}, x, \bar{z})		
121	$I\bar{4}2m$	[110], [110], $z \parallel X, Y, Z$, any permutation.	(x, y, z)	(y, x, z)	(x, \bar{y}, \bar{z})	(y, \bar{x}, \bar{z})	42	Fmm2
			(\bar{x}, \bar{y}, z)	(\bar{y}, \bar{x}, z)	(\bar{x}, y, \bar{z})	(\bar{y}, x, \bar{z})		
122	$I\bar{4}2d$	Do.	(x, y, z)	($y, \frac{1}{2}+x, \frac{1}{4}+z$)	($x, \frac{1}{2}-y, \frac{1}{4}-z$)	(y, \bar{x}, \bar{z})	43	Fdd2
			(\bar{x}, \bar{y}, z)	($\bar{y}, \frac{1}{2}-x, \frac{1}{4}+z$)	($\bar{x}, \frac{1}{2}+y, \frac{1}{4}-z$)	(\bar{y}, x, \bar{z})		

TABLE 11. Reduction from $\bar{4}2m$ to 222

Space group of unstrained crystal, order 8 per lattice point		If stressed so that	Coordinates referred to axes of unstrained crystal				Space group of strained crystal, order 4 per lattice point	
No.	Symbol		1st Subset		2d Subset		No.	Symbol
111	$P\bar{4}2m$	$x, y, z \parallel X, Y, Z$, any permutation.	(x, y, z)	(x, \bar{y}, \bar{z})	(y, x, z)	(y, \bar{x}, \bar{z})	16	P222
			(\bar{x}, \bar{y}, z)	(\bar{x}, y, \bar{z})	(\bar{y}, \bar{x}, z)	(\bar{y}, x, \bar{z})		
112	$P\bar{4}2c$	Do.	(x, y, z)	($x, \bar{y}, \frac{1}{2}-z$)	($y, x, \frac{1}{2}+z$)	(y, \bar{x}, \bar{z})	16	P222
			(\bar{x}, \bar{y}, z)	($\bar{x}, y, \frac{1}{2}-z$)	($\bar{y}, \bar{x}, \frac{1}{2}+z$)	(\bar{y}, x, \bar{z})		
113	$P\bar{4}2m$	Do.	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, \bar{z}$)	($\frac{1}{2}+y, \frac{1}{2}+x, z$)	(y, \bar{x}, \bar{z})	18	P ₂ 1 ₂ 1 ₂
			(\bar{x}, \bar{y}, z)	($\frac{1}{2}-x, \frac{1}{2}+y, \bar{z}$)	($\frac{1}{2}-y, \frac{1}{2}-x, z$)	(\bar{y}, x, \bar{z})		
114	$P\bar{4}2_{1}c$	Do.	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}-z$)	($\frac{1}{2}+y, \frac{1}{2}+x, \frac{1}{2}+z$)	(y, \bar{x}, \bar{z})	18	P ₂ 1 ₂ 1 ₂
			(\bar{x}, \bar{y}, z)	($\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}-z$)	($\frac{1}{2}-y, \frac{1}{2}-x, \frac{1}{2}+z$)	(\bar{y}, x, \bar{z})		
115	$P\bar{4}m2$	$[1\bar{1}0], [110], z \parallel X, Y, Z$, any permutation.	(x, y, z)	(y, x, \bar{z})	(x, \bar{y}, z)	(y, \bar{x}, \bar{z})	21	C222
			(\bar{x}, \bar{y}, z)	(\bar{y}, \bar{x}, z)	(\bar{x}, y, \bar{z})	(\bar{y}, x, \bar{z})		
116	$P\bar{4}c2$	Do.	(x, y, z)	($y, x, \frac{1}{2}-z$)	($x, \bar{y}, \frac{1}{2}+z$)	(y, \bar{x}, \bar{z})	21	C222
			(\bar{x}, \bar{y}, z)	($\bar{y}, \bar{x}, \frac{1}{2}-z$)	($\bar{x}, y, \frac{1}{2}+z$)	(\bar{y}, x, \bar{z})		
117	$P\bar{4}b2$	Do.	(x, y, z)	($\frac{1}{2}+y, \frac{1}{2}+x, \bar{z}$)	($\frac{1}{2}+x, \frac{1}{2}-y, z$)	(y, \bar{x}, \bar{z})	21	C222
			(\bar{x}, \bar{y}, z)	($\frac{1}{2}-y, \frac{1}{2}-x, \bar{z}$)	($\frac{1}{2}-x, \frac{1}{2}+y, z$)	(\bar{y}, x, \bar{z})		
118	$P\bar{4}n2$	Do.	(x, y, z)	($\frac{1}{2}+y, \frac{1}{2}+x, \frac{1}{2}-z$)	($\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}+z$)	(y, \bar{x}, \bar{z})	21	C222
			(\bar{x}, \bar{y}, z)	($\frac{1}{2}-y, \frac{1}{2}-x, \frac{1}{2}-z$)	($\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z$)	(\bar{y}, x, \bar{z})		
119	$I\bar{4}m2$	Do.	(x, y, z)	(y, x, \bar{z})	(x, \bar{y}, z)	(y, \bar{x}, \bar{z})	22	F222
			(\bar{x}, \bar{y}, z)	(\bar{y}, \bar{x}, z)	(\bar{x}, y, \bar{z})	(\bar{y}, x, \bar{z})		
120	$I\bar{4}e2$	Do.	(x, y, z)	($y, x, \frac{1}{2}-z$)	($x, \bar{y}, \frac{1}{2}+z$)	(y, \bar{x}, \bar{z})	22	F222
			(\bar{x}, \bar{y}, z)	($\bar{y}, \bar{x}, \frac{1}{2}-z$)	($\bar{x}, y, \frac{1}{2}+z$)	(\bar{y}, x, \bar{z})		
121	$I\bar{4}2m$	$x, y, z \parallel X, Y, Z$, any permutation.	(x, y, z)	(x, \bar{y}, \bar{z})	(y, x, z)	(y, \bar{x}, \bar{z})	23	I222
			(\bar{x}, \bar{y}, z)	(\bar{x}, y, \bar{z})	(\bar{y}, \bar{x}, z)	(\bar{y}, x, \bar{z})		
122	$I\bar{4}2d$	Do.	(x, y, z)	($x, \frac{1}{2}-y, \frac{1}{4}-z$)	($y, \frac{1}{2}+x, \frac{1}{4}+z$)	(y, \bar{x}, \bar{z})	24	I ₂ 1 ₂ 1 ₂
			(\bar{x}, \bar{y}, z)	($\bar{x}, \frac{1}{2}+y, \frac{1}{4}-z$)	($\bar{y}, \frac{1}{2}-x, \frac{1}{4}+z$)	(\bar{y}, x, \bar{z})		

TABLE 12. Reduction from $4mm$ to $mm2$

Space group of unstrained crystal, order 8 per lattice point		If stressed so that $x, y, z \parallel X, Y, Z$, any permutation				Space group of strained crystal, order 4 per lattice point	
		Coordinates referred to axes of unstrained crystal					
No.	Symbol	1st Subset		2d Subset		No.	Symbol
99	P4mm	(x, y, z)	(x, \bar{y}, z)	(y, x, z)	(y, \bar{x}, z)	25	Pmm2
		(\bar{x}, \bar{y}, z)	(\bar{x}, y, z)	(\bar{y}, \bar{x}, z)	(\bar{y}, x, z)		
100	P4bm	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, z$)	($\frac{1}{2}+y, \frac{1}{2}+x, z$)	(y, \bar{x}, z)	32	Pba2
		(\bar{x}, \bar{y}, z)	($\frac{1}{2}-x, \frac{1}{2}+y, z$)	($\frac{1}{2}-y, \frac{1}{2}-x, z$)	(\bar{y}, x, z)		
101	P4cm	(x, y, z)	($x, \bar{y}, \frac{1}{2}+z$)	(y, x, z)	($y, \bar{x}, \frac{1}{2}+z$)	27	Pcc2
		(\bar{x}, \bar{y}, z)	($\bar{x}, y, \frac{1}{2}+z$)	(\bar{y}, \bar{x}, z)	($\bar{y}, x, \frac{1}{2}+z$)		
102	P4 ₂ nm	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}+z$)	(y, x, z)	($\frac{1}{2}+y, \frac{1}{2}-x, \frac{1}{2}+z$)	34	Pnn2
		(\bar{x}, \bar{y}, z)	($\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z$)	(\bar{y}, \bar{x}, z)	($\frac{1}{2}-y, \frac{1}{2}+x, \frac{1}{2}+z$)		
103	P4cc	(x, y, z)	($x, \bar{y}, \frac{1}{2}+z$)	($y, x, \frac{1}{2}+z$)	(y, \bar{x}, z)	27	Pcc2
		(\bar{x}, \bar{y}, z)	($\bar{x}, y, \frac{1}{2}+z$)	($\bar{y}, \bar{x}, \frac{1}{2}+z$)	(\bar{y}, x, z)		
104	P4nc	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}+z$)	($\frac{1}{2}+y, \frac{1}{2}+x, \frac{1}{2}+z$)	(y, \bar{x}, z)	34	Pnn2
		(\bar{x}, \bar{y}, z)	($\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z$)	($\frac{1}{2}-y, \frac{1}{2}-x, \frac{1}{2}+z$)	(\bar{y}, x, z)		
105	P4 ₂ mc	(x, y, z)	(x, \bar{y}, z)	($y, x, \frac{1}{2}+z$)	($y, \bar{x}, \frac{1}{2}+z$)	25	Pmm2
		(\bar{x}, \bar{y}, z)	(\bar{x}, y, z)	($\bar{y}, \bar{x}, \frac{1}{2}+z$)	($\bar{y}, x, \frac{1}{2}+z$)		
106	P4 ₂ bc	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, z$)	($\frac{1}{2}+y, \frac{1}{2}+x, \frac{1}{2}+z$)	($y, \bar{x}, \frac{1}{2}+z$)	32	Pba2
		(\bar{x}, \bar{y}, z)	($\frac{1}{2}-x, \frac{1}{2}+y, z$)	($\frac{1}{2}-y, \frac{1}{2}-x, \frac{1}{2}+z$)	($\bar{y}, x, \frac{1}{2}+z$)		
107	I4mm	(x, y, z)	(x, \bar{y}, z)	(y, x, z)	(y, \bar{x}, z)	44	Imm2
		(\bar{x}, \bar{y}, z)	(\bar{x}, y, z)	(\bar{y}, \bar{x}, z)	(\bar{y}, x, z)		
108	I4em	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, z$)	($\frac{1}{2}+y, \frac{1}{2}+x, z$)	(y, \bar{x}, z)	45	Iba2
		(\bar{x}, \bar{y}, z)	($\frac{1}{2}-x, \frac{1}{2}+y, z$)	($\frac{1}{2}-y, \frac{1}{2}-x, z$)	(\bar{y}, x, z)		
109	I4 ₁ md	(x, y, z)	(x, \bar{y}, z)	($y, \frac{1}{2}+x, \frac{1}{4}+z$)	($y, \frac{1}{2}-x, \frac{1}{4}+z$)	44	Imm2
		(\bar{x}, \bar{y}, z)	(\bar{x}, y, z)	($\bar{y}, \frac{1}{2}-x, \frac{1}{4}+z$)	($\bar{y}, \frac{1}{2}+x, \frac{1}{4}+z$)		
110	I4 ₁ cd	(x, y, z)	($x, \bar{y}, \frac{1}{2}+z$)	($y, \frac{1}{2}+x, \frac{3}{4}+z$)	($y, \frac{1}{2}-x, \frac{3}{4}+z$)	45	Iba2
		(\bar{x}, \bar{y}, z)	($\bar{x}, y, \frac{1}{2}+z$)	($\bar{y}, \frac{1}{2}-x, \frac{3}{4}+z$)	($\bar{y}, \frac{1}{2}+x, \frac{3}{4}+z$)		
No.	Symbol	1st Subset	2d Subset	1st Subset	2d Subset	No.	Symbol
Space group of unstrained crystal, order 8 per lattice point		Coordinates referred to axes of unstrained crystal				Space group of strained crystal, order 4 per lattice point	
		If stressed so that $[1\bar{1}0], [110], z \parallel X, Y, Z$, any permutation					

TABLE 13. Reduction from 422 to 222

Space group of unstrained crystal, order 8 per lattice point		If stressed so that $x, y, z \parallel X, Y, Z$, any permutation					Space group of strained crystal, order 4 per lattice point			
		Coordinates referred to axes of unstrained crystal								
No.	Symbol	1st Subset		2d Subset		No.	Symbol			
89	P422	(x, y, z)	(x, \bar{y}, \bar{z})	(y, x, \bar{z})	(y, \bar{x}, z)	16	P222	21	C222	
		(\bar{x}, \bar{y}, z)	(\bar{x}, y, \bar{z})	($\bar{y}, \bar{x}, \bar{z}$)	(\bar{y}, x, z)					
90	P4 ₁ 2 ₂	(x, y, z)	($\frac{1}{2}x + y, \frac{1}{2}y - z, \bar{z}$)	(y, x, \bar{z})	($\frac{1}{2}y + y, \frac{1}{2}x - z, z$)	18	P2 ₁ 2 ₂	21	C222	
		(\bar{x}, \bar{y}, z)	($\frac{1}{2}x - y, \frac{1}{2}y + z, \bar{z}$)	($\bar{y}, \bar{x}, \bar{z}$)	($\frac{1}{2}y - y, \frac{1}{2}x + z, z$)					
91	P4 ₁ 2 ₂	(x, y, z)	($x, \bar{y}, \frac{1}{2}z - z$)	($y, x, \frac{3}{4}z - z$)	($y, \bar{x}, \frac{3}{4}z + z$)	17	P2 ₂ ₁	20	C222 ₁	
		($\bar{x}, \bar{y}, \frac{1}{2}z + z$)	(\bar{x}, y, \bar{z})	($\bar{y}, \bar{x}, \frac{1}{4}z - z$)	($\bar{y}, x, \frac{1}{4}z + z$)					
92	P4 ₁ 2 ₁ 2	(x, y, z)	($\frac{1}{2}x + y, \frac{1}{2}y - z, \frac{3}{4}z - z$)	(y, x, \bar{z})	($\frac{1}{2}y + y, \frac{1}{2}x - z, \frac{3}{4}z + z$)	19	P2 ₁ 2 ₁ 2 ₁	20	C222 ₁	
		($\bar{x}, \bar{y}, \frac{1}{2}z + z$)	($\frac{1}{2}x - y, \frac{1}{2}y + z, \frac{1}{4}z - z$)	($\bar{y}, \bar{x}, \frac{1}{2}z - z$)	($\frac{1}{2}y - y, \frac{1}{2}x + z, \frac{1}{4}z + z$)					
93	P4 ₂ 2 ₂	(x, y, z)	(x, \bar{y}, \bar{z})	($y, x, \frac{1}{2}z - z$)	($y, \bar{x}, \frac{1}{2}z + z$)	16	P222	21	C222	
		(\bar{x}, \bar{y}, z)	(\bar{x}, y, \bar{z})	($\bar{y}, \bar{x}, \frac{1}{2}z - z$)	($\bar{y}, x, \frac{1}{2}z + z$)					
94	P4 ₂ 1 ₂	(x, y, z)	($\frac{1}{2}x + y, \frac{1}{2}y - z, \frac{1}{2}z - z$)	(y, x, \bar{z})	($\frac{1}{2}y + y, \frac{1}{2}x - z, \frac{1}{2}z + z$)	18	P2 ₁ 2 ₂	21	C222	
		(\bar{x}, \bar{y}, z)	($\frac{1}{2}x - y, \frac{1}{2}y + z, \frac{1}{2}z - z$)	($\bar{y}, \bar{x}, \bar{z}$)	($\frac{1}{2}y - y, \frac{1}{2}x + z, \frac{1}{2}z + z$)					
95	P4 ₃ 2 ₂	(x, y, z)	($x, \bar{y}, \frac{1}{2}z - z$)	($y, x, \frac{1}{4}z - z$)	($y, \bar{x}, \frac{1}{4}z + z$)	17	P2 ₂ ₁	20	C222 ₁	
		($\bar{x}, \bar{y}, \frac{1}{2}z + z$)	(\bar{x}, y, \bar{z})	($\bar{y}, \bar{x}, \frac{3}{4}z - z$)	($\bar{y}, x, \frac{3}{4}z + z$)					
96	P4 ₃ 2 ₁ 2	(x, y, z)	($\frac{1}{2}x + y, \frac{1}{2}y - z, \frac{1}{4}z - z$)	(y, x, \bar{z})	($\frac{1}{2}y + y, \frac{1}{2}x - z, \frac{1}{4}z + z$)	19	P2 ₁ 2 ₁ 2 ₁	20	C222 ₁	
		($\bar{x}, \bar{y}, \frac{1}{2}z + z$)	($\frac{1}{2}x - y, \frac{1}{2}y + z, \frac{3}{4}z - z$)	($\bar{y}, \bar{x}, \frac{1}{2}z - z$)	($\frac{1}{2}y - y, \frac{1}{2}x + z, \frac{3}{4}z + z$)					
97	I422	(x, y, z)	(x, \bar{y}, \bar{z})	(y, x, \bar{z})	(y, \bar{x}, z)	23	I222	22	F222	
		(\bar{x}, \bar{y}, z)	(\bar{x}, y, \bar{z})	($\bar{y}, \bar{x}, \bar{z}$)	(\bar{y}, x, z)					
98	I4 ₁ 2 ₂	(x, y, z)	($x, \frac{1}{2}y - y, \frac{1}{4}z - z$)	(y, x, \bar{z})	($y, \frac{1}{2}x - x, \frac{1}{4}z + z$)	24	I2 ₁ 2 ₁ 2 ₁	22	F222	
		(\bar{x}, \bar{y}, z)	($\bar{x}, \frac{1}{2}y + y, \frac{1}{4}z - z$)	($\bar{y}, \bar{x}, \bar{z}$)	($\bar{y}, \frac{1}{2}x + x, \frac{1}{4}z + z$)					
No.	Symbol	1st Subset	2d Subset	1st Subset	2d Subset	-----	-----	-----	No.	Symbol
Space group of unstrained crystal, order 8 per lattice point		Coordinates referred to axes of unstrained crystal					Space group of strained crystal, order 4 per lattice point			
		If stressed so that $[1\bar{1}0], [110], z \parallel X, Y, Z$, any permutation								

TABLE 14. Reduction from 6 to 2

Space group of unstrained crystal, order 6 per lattice point		If stressed so that $z \parallel X, Y, \text{ or } Z$			Space group of strained crystal, order 2 per lattice point	
		Coordinates referred to axes of unstrained crystal				
No.	Symbol	1st Subset	2d Subset	3d Subset	No.	Symbol
168	P6	(x, y, z)	($\bar{y}, x - y, z$)	($y - x, \bar{x}, z$)	3	P2
		(\bar{x}, \bar{y}, z)	($y, y - x, z$)	($x - y, x, z$)		
169	P6 ₁	(x, y, z)	($\bar{y}, x - y, \frac{1}{3}z + z$)	($y - x, \bar{x}, \frac{2}{3}z + z$)	4	P2 ₁
		($\bar{x}, \bar{y}, \frac{1}{2}z + z$)	($y, y - x, \frac{5}{6}z + z$)	($x - y, x, \frac{1}{6}z + z$)		
170	P6 ₃	(x, y, z)	($\bar{y}, x - y, \frac{2}{3}z + z$)	($y - x, \bar{x}, \frac{1}{3}z + z$)	4	P2 ₁
		($\bar{x}, \bar{y}, \frac{1}{2}z + z$)	($y, y - x, \frac{1}{6}z + z$)	($x - y, x, \frac{5}{6}z + z$)		
171	P6 ₂	(x, y, z)	($\bar{y}, x - y, \frac{2}{3}z + z$)	($y - x, \bar{x}, \frac{1}{3}z + z$)	3	P2
		(\bar{x}, \bar{y}, z)	($y, y - x, \frac{2}{3}z + z$)	($x - y, x, \frac{1}{3}z + z$)		
172	P6 ₄	(x, y, z)	($\bar{y}, x - y, \frac{1}{3}z + z$)	($y - x, \bar{x}, \frac{2}{3}z + z$)	3	P2
		(\bar{x}, \bar{y}, z)	($y, y - x, \frac{1}{3}z + z$)	($x - y, x, \frac{2}{3}z + z$)		
173	P6 ₃	(x, y, z)	($\bar{y}, x - y, z$)	($y - x, \bar{x}, z$)	4	P2 ₁
		($\bar{x}, \bar{y}, \frac{1}{2}z + z$)	($y, y - x, \frac{1}{2}z + z$)	($x - y, x, \frac{1}{2}z + z$)		

TABLE 15. Reduction from $\bar{6}$ to m

Space group of unstrained crystal, order 6 per lattice point		If stressed so that $z \parallel X, Y, \text{ or } Z$			Space group of strained crystal, order 2 per lattice point	
		Coordinates referred to axes of unstrained crystal				
No.	Symbol	1st Subset	2d Subset	3d Subset	No.	Symbol
174	P $\bar{6}$	(x, y, z) (x, y, \bar{z})	($\bar{y}, x-y, z$) ($\bar{y}, x-y, \bar{z}$)	($y-x, \bar{x}, z$) ($y-x, \bar{x}, \bar{z}$)	6	Pm

TABLE 16. Reduction from $3m$ to m

Space group of unstrained crystal, order 6 per lattice point		If stressed so that	Coordinates referred to axes of unstrained crystal			Space group of strained crystal, order 2 per lattice point	
			1st Subset	2d Subset	3d Subset		
No.	Symbol					No.	Symbol
156	P3m1	[110] $\parallel X, Y, \text{ or } Z$	(x, y, z) (\bar{y}, \bar{x}, z)	($y-x, \bar{x}, z$) ($x, x-y, z$)	($\bar{y}, x-y, z$) ($y-x, y, z$)	8	Cm
			(x, y, z) (y, x, z)	($y-x, \bar{x}, z$) ($\bar{x}, y-x, z$)	($\bar{y}, x-y, z$) ($x-y, \bar{y}, z$)		
157	P31m	[1 $\bar{1}0$] $\parallel X, Y, \text{ or } Z$	(x, y, z) (y, x, z)	($y-x, \bar{x}, z$) ($\bar{x}, y-x, z$)	($\bar{y}, x-y, z$) ($x-y, \bar{y}, z$)	8	Cm
			(x, y, z) ($\bar{y}, \bar{x}, \frac{1}{2}+z$)	($y-x, \bar{x}, z$) ($x, x-y, \frac{1}{2}+z$)	($\bar{y}, x-y, z$) ($y-x, y, \frac{1}{2}+z$)		
158	P3c1	[110] $\parallel X, Y, \text{ or } Z$	(x, y, z) ($\bar{y}, \bar{x}, \frac{1}{2}+z$)	($y-x, \bar{x}, z$) ($x, x-y, \frac{1}{2}+z$)	($\bar{y}, x-y, z$) ($y-x, y, \frac{1}{2}+z$)	9	Cc
			(x, y, z) ($y, x, \frac{1}{2}+z$)	($y-x, \bar{x}, z$) ($\bar{x}, y-x, \frac{1}{2}+z$)	($\bar{y}, x-y, z$) ($x-y, \bar{y}, \frac{1}{2}+z$)		
159	P31c	[1 $\bar{1}0$] $\parallel X, Y, \text{ or } Z$	(x, y, z) ($y, x, \frac{1}{2}+z$)	($y-x, \bar{x}, z$) ($\bar{x}, y-x, \frac{1}{2}+z$)	($\bar{y}, x-y, z$) ($x-y, \bar{y}, \frac{1}{2}+z$)	9	Cc
			(x, y, z) (\bar{y}, \bar{x}, z)	($y-x, \bar{x}, z$) ($x, x-y, z$)	($\bar{y}, x-y, z$) ($y-x, y, z$)		
160	R3m hex. axes	[110] $\parallel X, Y, \text{ or } Z$	(x, y, z) (\bar{y}, \bar{x}, z)	($y-x, \bar{x}, z$) ($x, x-y, z$)	($\bar{y}, x-y, z$) ($y-x, y, z$)	8	Cm
			(x, y, z) ($\bar{y}, \bar{x}, \frac{1}{2}+z$)	($y-x, \bar{x}, z$) ($x, x-y, \frac{1}{2}+z$)	($\bar{y}, x-y, z$) ($y-x, y, \frac{1}{2}+z$)		
161	R3c hex. axes	[110] $\parallel X, Y, \text{ or } Z$	(x, y, z) ($\bar{y}, \bar{x}, \frac{1}{2}+z$)	($y-x, \bar{x}, z$) ($x, x-y, \frac{1}{2}+z$)	($\bar{y}, x-y, z$) ($y-x, y, \frac{1}{2}+z$)	9	Cc

TABLE 17. Reduction from 32 to 2

Space group of unstrained crystal, order 6 per lattice point		If stressed so that	Coordinates referred to axes of unstrained crystal			Space group of strained crystal, order 2 per lattice point	
			1st Subset	2d Subset	3d Subset		
No.	Symbol					No.	Symbol
149	P312	[1 $\bar{1}0$] $\parallel X, Y, \text{ or } Z$	(x, y, z) (\bar{y}, \bar{x}, z)	($\bar{y}, x-y, z$) ($y-x, y, z$)	($y-x, \bar{x}, z$) ($x, x-y, z$)	5	C2
			(x, y, z) (y, x, z)	($\bar{y}, x-y, z$) ($x-y, \bar{y}, z$)	($y-x, \bar{x}, z$) ($\bar{x}, y-x, z$)		
150	P321	[110] $\parallel X, Y, \text{ or } Z$	(x, y, z) (y, x, z)	($\bar{y}, x-y, z$) ($x-y, \bar{y}, z$)	($y-x, \bar{x}, z$) ($\bar{x}, y-x, z$)	5	C2
			(x, y, z) ($\bar{y}, \bar{x}, \frac{2}{3}-z$)	($\bar{y}, x-y, \frac{2}{3}+z$) ($y-x, y, \frac{2}{3}-z$)	($y-x, \bar{x}, \frac{2}{3}+z$) ($x, x-y, z$)		
151	P3 ₁ 21	[1 $\bar{1}0$] $\parallel X, Y, \text{ or } Z$	(x, y, z) ($\bar{y}, \bar{x}, \frac{2}{3}-z$)	($\bar{y}, x-y, \frac{2}{3}+z$) ($y-x, y, \frac{2}{3}-z$)	($y-x, \bar{x}, \frac{2}{3}+z$) ($x, x-y, z$)	5	C2
			(x, y, z) (y, x, z)	($\bar{y}, x-y, \frac{2}{3}+z$) ($x-y, \bar{y}, \frac{2}{3}-z$)	($y-x, \bar{x}, \frac{2}{3}+z$) ($\bar{x}, y-x, \frac{2}{3}-z$)		
152	P3 ₂ 12	[1 $\bar{1}0$] $\parallel X, Y, \text{ or } Z$	(x, y, z) ($\bar{y}, \bar{x}, \frac{2}{3}-z$)	($\bar{y}, x-y, \frac{2}{3}+z$) ($y-x, y, \frac{2}{3}-z$)	($y-x, \bar{x}, \frac{2}{3}+z$) ($x, x-y, z$)	5	C2
			(x, y, z) (y, x, z)	($\bar{y}, x-y, \frac{2}{3}+z$) ($x-y, \bar{y}, \frac{2}{3}-z$)	($y-x, \bar{x}, \frac{2}{3}+z$) ($\bar{x}, y-x, \frac{2}{3}-z$)		
153	P3 ₂ 12	[1 $\bar{1}0$] $\parallel X, Y, \text{ or } Z$	(x, y, z) ($\bar{y}, \bar{x}, \frac{2}{3}-z$)	($\bar{y}, x-y, \frac{2}{3}+z$) ($y-x, y, \frac{2}{3}-z$)	($y-x, \bar{x}, \frac{2}{3}+z$) ($x, x-y, z$)	5	C2
			(x, y, z) (y, x, z)	($\bar{y}, x-y, \frac{2}{3}+z$) ($x-y, \bar{y}, \frac{2}{3}-z$)	($y-x, \bar{x}, \frac{2}{3}+z$) ($\bar{x}, y-x, \frac{2}{3}-z$)		
154	P3 ₂ 21	[110] $\parallel X, Y, \text{ or } Z$	(x, y, z) (y, x, z)	($\bar{y}, x-y, \frac{2}{3}+z$) ($x-y, \bar{y}, \frac{2}{3}-z$)	($y-x, \bar{x}, \frac{1}{3}+z$) ($\bar{x}, y-x, \frac{2}{3}-z$)	5	C2
			(x, y, z) (\bar{y}, \bar{x}, z)	($\bar{y}, x-y, z$) ($x-y, \bar{y}, z$)	($y-x, \bar{x}, z$) ($\bar{x}, y-x, z$)		
155	R32 hex. axes	[110] $\parallel X, Y, \text{ or } Z$	(x, y, z) (y, x, z)	($\bar{y}, x-y, z$) ($x-y, \bar{y}, z$)	($y-x, \bar{x}, z$) ($\bar{x}, y-x, z$)	5	C2

TABLE 18. Reduction from 4 to 2

Space group of unstrained crystal, order 4 per lattice point		If stressed so that $z \parallel X, Y, \text{ or } Z$		Space group of strained crystal, order 2 per lattice point	
		Coordinates referred to axes of unstrained crystal			
No.	Symbol	1st Subset	2d Subset	No.	Symbol
75	P4	(x, y, z)	(\bar{y}, x, z)	3	P2
		(\bar{x}, \bar{y}, z)	(y, \bar{x}, z)		
76	P4 ₁	(x, y, z)	($\bar{y}, x, \frac{1}{4}+z$)	4	P2 ₁
		($\bar{x}, \bar{y}, \frac{1}{2}+z$)	($y, \bar{x}, \frac{3}{4}+z$)		
77	P4 ₂	(x, y, z)	($\bar{y}, x, \frac{1}{2}+z$)	3	P2
		(\bar{x}, \bar{y}, z)	($y, \bar{x}, \frac{1}{2}+z$)		
78	P4 ₃	(x, y, z)	($\bar{y}, x, \frac{3}{4}+z$)	4	P2 ₁
		($\bar{x}, \bar{y}, \frac{1}{2}+z$)	($y, \bar{x}, \frac{1}{4}+z$)		
79	I4	(x, y, z)	(\bar{y}, x, z)	5	C2
		(\bar{x}, \bar{y}, z)	(y, \bar{x}, z)		
80	I4 ₁	(x, y, z)	($\bar{y}, \frac{1}{2}+x, \frac{1}{4}+z$)	5	C2
		(\bar{x}, \bar{y}, z)	($y, \frac{1}{2}-x, \frac{1}{4}+z$)		

TABLE 19. Reduction from $\overline{4}$ to 2

Space group of unstrained crystal, order 4 per lattice point		If stressed so that $z \parallel X, Y, \text{ or } Z$		Space group of strained crystal, order 2 per lattice point	
		Coordinates referred to axes of unstrained crystal			
No.	Symbol	1st Subset	2d Subset	No.	Symbol
81	P $\bar{4}$	(x, y, z)	(\bar{y}, x, \bar{z})	3	P2
		(\bar{x}, \bar{y}, z)	(y, \bar{x}, \bar{z})		
82	I $\bar{4}$	(x, y, z)	(\bar{y}, x, \bar{z})	5	C2
		(\bar{x}, \bar{y}, z)	(y, \bar{x}, \bar{z})		

TABLE 20. Reduction from $mm\bar{2}$ to m

Space group of unstrained crystal, order 4 per lattice point		If stressed so that $x \parallel X, Y, \text{ or } Z$		Space group of strained crystal, order 2 per lattice point		If stressed so that $y \parallel X, Y, \text{ or } Z$		Space group of strained crystal, order 2 per lattice point	
		Coordinates referred to axes of unstrained crystal				Coordinates referred to axes of unstrained crystal			
No.	Symbol	1st Subset	2d Subset	No.	Symbol	1st Subset	2d Subset	No.	Symbol
25	Pmm2	(x, y, z)	(x, \bar{y}, z)	6	Pm	(x, y, z)	(\bar{x}, y, z)	6	Pm
		(\bar{x}, y, z)	(\bar{x}, \bar{y}, z)			(x, y, z)	(\bar{x}, \bar{y}, z)		
26	Pmc ₂ 1	(x, y, z)	($x, \bar{y}, \frac{1}{2}+z$)	6	Pm	(x, y, z)	(\bar{x}, y, z)	7	Pc
		(\bar{x}, y, z)	($\bar{x}, \bar{y}, \frac{1}{2}+z$)			($x, \bar{y}, \frac{1}{2}+z$)	($\bar{x}, \bar{y}, \frac{1}{2}+z$)		
27	Pcc2	(x, y, z)	($x, \bar{y}, \frac{1}{2}+z$)	7	Pc	(x, y, z)	($\bar{x}, y, \frac{1}{2}+z$)	7	Pc
		($\bar{x}, y, \frac{1}{2}+z$)	(\bar{x}, \bar{y}, z)			($x, \bar{y}, \frac{1}{2}+z$)	(\bar{x}, \bar{y}, z)		
28	Pma2	(x, y, z)	($\frac{1}{2}+x, \bar{y}, z$)	6	Pm	(x, y, z)	($\frac{1}{2}-x, y, z$)	7	Pc
		($\frac{1}{2}-x, y, z$)	(\bar{x}, \bar{y}, z)			($\frac{1}{2}+x, \bar{y}, z$)	(\bar{x}, \bar{y}, z)		
29	Pea ₂ 1	(x, y, z)	($\frac{1}{2}+x, \bar{y}, z$)	7	Pc	(x, y, z)	($\frac{1}{2}-x, y, \frac{1}{2}+z$)	7	Pc
		($\frac{1}{2}-x, y, \frac{1}{2}+z$)	($\bar{x}, \bar{y}, \frac{1}{2}+z$)			($\frac{1}{2}+x, \bar{y}, \frac{1}{2}+z$)	($\bar{x}, \bar{y}, \frac{1}{2}+z$)		
30	Pnc2	(x, y, z)	($x, \frac{1}{2}-y, \frac{1}{2}+z$)	7	Pc	(x, y, z)	($\bar{x}, \frac{1}{2}+y, \frac{1}{2}+z$)	7	Pc
		($\bar{x}, \frac{1}{2}+y, \frac{1}{2}+z$)	(\bar{x}, \bar{y}, z)			($x, \frac{1}{2}-y, \frac{1}{2}+z$)	(\bar{x}, \bar{y}, z)		
31	Pmn ₂ 1	(x, y, z)	($\frac{1}{2}+x, \bar{y}, \frac{1}{2}+z$)	6	Pm	(x, y, z)	($\frac{1}{2}+x, \bar{y}, \frac{1}{2}+z$)	7	Pc
		(\bar{x}, y, z)	($\frac{1}{2}-x, \bar{y}, \frac{1}{2}+z$)			(x, \bar{y}, z)	($\frac{1}{2}-x, \bar{y}, \frac{1}{2}+z$)		
32	Pba2	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, z$)	7	Pc	(x, y, z)	($\frac{1}{2}-x, \frac{1}{2}+y, z$)	7	Pc
		($\frac{1}{2}-x, \frac{1}{2}+y, z$)	(\bar{x}, \bar{y}, z)			($\frac{1}{2}+x, \frac{1}{2}-y, z$)	(\bar{x}, \bar{y}, z)		
33	Pna ₂ 1	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, z$)	7	Pc	(x, y, z)	($\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z$)	7	Pc
		($\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z$)	($\bar{x}, \bar{y}, \frac{1}{2}+z$)			($\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}+z$)	($\bar{x}, \bar{y}, \frac{1}{2}+z$)		
34	Pnn2	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}+z$)	7	Pc	(x, y, z)	($\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z$)	7	Pc
		($\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z$)	(\bar{x}, \bar{y}, z)			($\frac{1}{2}+x, \frac{1}{2}-y, z$)	(\bar{x}, \bar{y}, z)		
35	Cmm2	(x, y, z)	(x, \bar{y}, z)	8	Cm	(x, y, z)	(\bar{x}, y, z)	8	Cm
		(\bar{x}, y, z)	(\bar{x}, \bar{y}, z)			(x, \bar{y}, z)	(\bar{x}, \bar{y}, z)		
36	Cmc ₂ 1	(x, y, z)	($x, \bar{y}, \frac{1}{2}+z$)	8	Cm	(x, y, z)	($\bar{x}, y, \frac{1}{2}+z$)	9	Cc
		(\bar{x}, y, z)	($\bar{x}, \bar{y}, \frac{1}{2}+z$)			($x, \bar{y}, \frac{1}{2}+z$)	($\bar{x}, \bar{y}, \frac{1}{2}+z$)		
37	Ccc2	(x, y, z)	($x, \bar{y}, \frac{1}{2}+z$)	9	Cc	(x, y, z)	($\bar{x}, y, \frac{1}{2}+z$)	9	Cc
		($\bar{x}, y, \frac{1}{2}+z$)	(\bar{x}, \bar{y}, z)			($x, \bar{y}, \frac{1}{2}+z$)	(\bar{x}, \bar{y}, z)		
38	Amm2	(x, y, z)	(x, \bar{y}, z)	6	Pm	(x, y, z)	(\bar{x}, y, z)	8	Cm
		(\bar{x}, y, z)	(\bar{x}, \bar{y}, z)			(x, \bar{y}, z)	(\bar{x}, \bar{y}, z)		
39	Abm2	(x, y, z)	($x, \frac{1}{2}-y, z$)	7	Pe	(x, y, z)	($\bar{x}, \frac{1}{2}+y, z$)	8	Cm
		($\bar{x}, \frac{1}{2}+y, z$)	(\bar{x}, \bar{y}, z)			($x, \frac{1}{2}-y, z$)	(\bar{x}, \bar{y}, z)		
40	Ama2	(x, y, z)	($\frac{1}{2}+x, \bar{y}, z$)	6	Pm	(x, y, z)	($\frac{1}{2}-x, y, z$)	9	Cc
		($\frac{1}{2}-x, y, z$)	(\bar{x}, \bar{y}, z)			($\frac{1}{2}+x, \bar{y}, z$)	(\bar{x}, \bar{y}, z)		
41	Aba2	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, z$)	7	Pe	(x, y, z)	($\frac{1}{2}-x, \frac{1}{2}+y, z$)	9	Cc
		($\frac{1}{2}-x, \frac{1}{2}+y, z$)	(\bar{x}, \bar{y}, z)			($\frac{1}{2}+x, \frac{1}{2}-y, z$)	(\bar{x}, \bar{y}, z)		
42	Fmm2	(x, y, z)	(x, \bar{y}, z)	8	Cm	(x, y, z)	(\bar{x}, y, z)	8	Cm
		(\bar{x}, y, z)	(\bar{x}, \bar{y}, z)			(x, \bar{y}, z)	(\bar{x}, \bar{y}, z)		
43	Fdd2	(x, y, z)	($\frac{1}{4}+x, \frac{1}{4}-y, \frac{1}{4}+z$)	9	Ce	(x, y, z)	($\frac{1}{4}-x, \frac{1}{4}+y, \frac{1}{4}+z$)	9	Cc
		($\frac{1}{4}-x, \frac{1}{4}+y, \frac{1}{4}+z$)	(\bar{x}, \bar{y}, z)			($\frac{1}{4}+x, \frac{1}{4}-y, \frac{1}{4}+z$)	(\bar{x}, \bar{y}, z)		
44	Imm2	(x, y, z)	(x, \bar{y}, z)	8	Cm	(x, y, z)	(\bar{x}, y, z)	8	Cm
		(\bar{x}, y, z)	(\bar{x}, \bar{y}, z)			(x, \bar{y}, z)	(\bar{x}, \bar{y}, z)		
45	Iba2	(x, y, z)	($x, \bar{y}, \frac{1}{2}+z$)	9	Ce	(x, y, z)	($\bar{x}, y, \frac{1}{2}+z$)	9	Cc
		($\bar{x}, y, \frac{1}{2}+z$)	(\bar{x}, \bar{y}, z)			($x, \bar{y}, \frac{1}{2}+z$)	(\bar{x}, \bar{y}, z)		
46	Ima2	(x, y, z)	($\frac{1}{2}+x, \bar{y}, z$)	8	Cm	(x, y, z)	($\frac{1}{2}-x, y, z$)	9	Cc
		($\frac{1}{2}-x, y, z$)	(\bar{x}, \bar{y}, z)			($\frac{1}{2}+x, \bar{y}, z$)	(\bar{x}, \bar{y}, z)		

TABLE 21. Reduction from $mm\bar{2}$ to $\bar{2}$

Space group of unstrained crystal, order 4 per lattice point		If stressed so that $z \parallel X, Y$, or Z		Space group of strained crystal, order 2 per lattice point	
		Coordinates referred to axes of unstrained crystal			
No.	Symbol	1st Subset	2d Subset	No.	Symbol
25	Pmm2	(x, y, z)	(x, \bar{y}, z)	3	P2
		(\bar{x}, \bar{y}, z)	(\bar{x}, y, z)		
26	Pmc2 ₁	(x, y, z)	($x, \bar{y}, \frac{1}{2}+z$)	4	P2 ₁
		($\bar{x}, \bar{y}, \frac{1}{2}+z$)	(\bar{x}, y, z)		
27	Pcc2	(x, y, z)	($x, \bar{y}, \frac{1}{2}+z$)	3	P2
		(\bar{x}, \bar{y}, z)	($\bar{x}, y, \frac{1}{2}+z$)		
28	Pma2	(x, y, z)	($\frac{1}{2}+x, \bar{y}, z$)	3	P2
		(\bar{x}, \bar{y}, z)	($\frac{1}{2}-x, y, z$)		
29	Pca2 ₁	(x, y, z)	($\frac{1}{2}+x, \bar{y}, z$)	4	P2 ₁
		($\bar{x}, \bar{y}, \frac{1}{2}+z$)	($\frac{1}{2}-x, y, \frac{1}{2}+z$)		
30	Pnc2	(x, y, z)	($x, \frac{1}{2}-y, \frac{1}{2}+z$)	3	P2
		(\bar{x}, \bar{y}, z)	($\bar{x}, \frac{1}{2}+y, \frac{1}{2}+z$)		
31	Pmn2 ₁	(x, y, z)	($\frac{1}{2}+x, \bar{y}, \frac{1}{2}+z$)	4	P2 ₁
		($\frac{1}{2}-x, \bar{y}, \frac{1}{2}+z$)	(\bar{x}, y, z)		
32	Pba2	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, z$)	3	P2
		(\bar{x}, \bar{y}, z)	($\frac{1}{2}-x, \frac{1}{2}+y, z$)		
33	Pna2 ₁	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, z$)	4	P2 ₁
		($\bar{x}, \bar{y}, \frac{1}{2}+z$)	($\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z$)		
34	Pnn2	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}+z$)	3	P2
		(\bar{x}, \bar{y}, z)	($\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z$)		
35	Cmm2	(x, y, z)	(x, \bar{y}, z)	3	P2
		(\bar{x}, \bar{y}, z)	(\bar{x}, y, z)		
36	Cmc2 ₁	(x, y, z)	($x, \bar{y}, \frac{1}{2}+z$)	4	P2 ₁
		($\bar{x}, \bar{y}, \frac{1}{2}+z$)	(\bar{x}, y, z)		
37	Ccc2	(x, y, z)	($x, \bar{y}, \frac{1}{2}+z$)	3	P2
		(\bar{x}, \bar{y}, z)	($\bar{x}, y, \frac{1}{2}+z$)		
38	Amm2	(x, y, z)	(x, \bar{y}, z)	5	C2
		(\bar{x}, \bar{y}, z)	(\bar{x}, y, z)		
39	Abm2	(x, y, z)	($x, \frac{1}{2}-y, z$)	5	C2
		(\bar{x}, \bar{y}, z)	($\bar{x}, \frac{1}{2}+y, z$)		
40	Ama2	(x, y, z)	($\frac{1}{2}+x, \bar{y}, z$)	5	C2
		(\bar{x}, \bar{y}, z)	($\frac{1}{2}-x, y, z$)		
41	Aba2	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, z$)	5	C2
		(\bar{x}, \bar{y}, z)	($\frac{1}{2}-x, \frac{1}{2}+y, z$)		
42	Fmm2	(x, y, z)	(x, \bar{y}, z)	5	C2
		(\bar{x}, \bar{y}, z)	(\bar{x}, y, z)		
43	Fdd2	(x, y, z)	($\frac{1}{4}+x, \frac{1}{4}-y, \frac{1}{4}+z$)	5	C2
		(\bar{x}, \bar{y}, z)	($\frac{1}{4}-x, \frac{1}{4}+y, \frac{1}{4}+z$)		
44	Imm2	(x, y, z)	(x, \bar{y}, z)	5	C2
		(\bar{x}, \bar{y}, z)	(\bar{x}, y, z)		
45	Iba2	(x, y, z)	($x, \bar{y}, \frac{1}{2}+z$)	5	C2
		(\bar{x}, \bar{y}, z)	($\bar{x}, y, \frac{1}{2}+z$)		
46	Ima2	(x, y, z)	($\frac{1}{2}+x, \bar{y}, z$)	5	C2
		(\bar{x}, \bar{y}, z)	($\frac{1}{2}-x, y, z$)		

TABLE 22. Reaction from 222 to 2

Space group of unstrained crystal, order 4 per lattice point		If stressed so that $x \parallel X, Y, \text{ or } Z$		Space group of strained crystal, order 2 per lattice point		If stressed so that $y \parallel X, Y, \text{ or } Z$		Space group of strained crystal, order 2 per lattice point		If stressed so that $z \parallel X, Y, \text{ or } Z$		Space group of strained crystal, order 2 per lattice point	
		Coordinates referred to axes of unstrained crystal				Coordinates referred to axes of unstrained crystal				Coordinates referred to axes of unstrained crystal			
No.	Symbol	1st Subset	2d Subset	No.	Symbol	1st Subset	2d Subset	No.	Symbol	1st Subset	2d Subset	No.	Symbol
16	P222	(x, y, z)	(\bar{x}, y, \bar{z})	3	P2	(x, y, z)	(x, \bar{y}, \bar{z})	3	P2	(x, y, z)	(x, \bar{y}, \bar{z})	3	P2
		(x, \bar{y}, \bar{z})	(\bar{x}, \bar{y}, z)			(\bar{x}, y, \bar{z})	(\bar{x}, \bar{y}, z)			(\bar{x}, \bar{y}, z)	(\bar{x}, y, \bar{z})		
17	P222 ₁	(x, y, z)	($\bar{x}, y, \frac{1}{2}-z$)	3	P2	(x, y, z)	(x, \bar{y}, \bar{z})	3	P2	(x, y, z)	(x, \bar{y}, \bar{z})	4	P2 ₁
		(x, \bar{y}, \bar{z})	($\bar{x}, \bar{y}, \frac{1}{2}+z$)			($\bar{x}, y, \frac{1}{2}-z$)	($\bar{x}, \bar{y}, \frac{1}{2}+z$)			($\bar{x}, \bar{y}, \frac{1}{2}+z$)	($\bar{x}, y, \frac{1}{2}-z$)		
18	P2 ₁ 2 ₁ 2 ₂	(x, y, z)	($\frac{1}{2}-x, \frac{1}{2}+y, \bar{z}$)	4	P2 ₁	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, \bar{z}$)	4	P2 ₁	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, \bar{z}$)	3	P2
		($\frac{1}{2}+x, \frac{1}{2}-y, \bar{z}$)	(\bar{x}, \bar{y}, z)			($\frac{1}{2}-x, \frac{1}{2}+y, \bar{z}$)	(\bar{x}, \bar{y}, z)			(\bar{x}, \bar{y}, z)	($\bar{x}, y, \frac{1}{2}-z$)		
19	P2 ₁ 2 ₁ 2 ₁	(x, y, z)	($\bar{x}, \frac{1}{2}+y, \frac{1}{2}-z$)	4	P2 ₁	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, \bar{z}$)	4	P2 ₁	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, \bar{z}$)	4	P2 ₁
		($\frac{1}{2}+x, \frac{1}{2}-y, \bar{z}$)	($\frac{1}{2}-x, \bar{y}, \frac{1}{2}+z$)			($\bar{x}, \frac{1}{2}+y, \frac{1}{2}-z$)	($\bar{x}, \bar{y}, \frac{1}{2}+z$)			($\bar{x}, \bar{y}, \frac{1}{2}+z$)	($\bar{x}, y, \frac{1}{2}-z$)		
20	C222 ₁	(x, y, z)	($\bar{x}, y, \frac{1}{2}-z$)	5	C2	(x, y, z)	(x, \bar{y}, \bar{z})	5	C2	(x, y, z)	(x, \bar{y}, \bar{z})	4	P2 ₁
		(x, \bar{y}, \bar{z})	($\bar{x}, \bar{y}, \frac{1}{2}+z$)			($\bar{x}, y, \frac{1}{2}-z$)	($\bar{x}, \bar{y}, \frac{1}{2}+z$)			($\bar{x}, \bar{y}, \frac{1}{2}+z$)	($\bar{x}, y, \frac{1}{2}-z$)		
21	C222	(x, y, z)	(\bar{x}, y, \bar{z})	5	C2	(x, y, z)	(x, \bar{y}, \bar{z})	5	C2	(x, y, z)	(x, \bar{y}, \bar{z})	3	P2
		(x, \bar{y}, \bar{z})	(\bar{x}, \bar{y}, z)			(\bar{x}, y, \bar{z})	(\bar{x}, \bar{y}, z)			(\bar{x}, \bar{y}, z)	(\bar{x}, y, \bar{z})		
22	F222	(x, y, z)	(\bar{x}, y, \bar{z})	5	C2	(x, y, z)	(x, \bar{y}, \bar{z})	5	C2	(x, y, z)	(x, \bar{y}, \bar{z})	5	C2
		(x, \bar{y}, \bar{z})	(\bar{x}, \bar{y}, z)			(\bar{x}, y, \bar{z})	(\bar{x}, \bar{y}, z)			(\bar{x}, \bar{y}, z)	(\bar{x}, y, \bar{z})		
23	I222	(x, y, z)	(\bar{x}, y, \bar{z})	5	C2	(x, y, z)	(x, \bar{y}, \bar{z})	5	C2	(x, y, z)	(x, \bar{y}, \bar{z})	5	C2
		(x, \bar{y}, \bar{z})	(\bar{x}, \bar{y}, z)			(\bar{x}, y, \bar{z})	(\bar{x}, \bar{y}, z)			(\bar{x}, \bar{y}, z)	(\bar{x}, y, \bar{z})		
24	I2 ₁ 2 ₁ 2 ₁	(x, y, z)	($\bar{x}, \frac{1}{2}+y, \frac{1}{2}-z$)	5	C2	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, \bar{z}$)	5	C2	(x, y, z)	($\frac{1}{2}+x, \frac{1}{2}-y, \bar{z}$)	5	C2
		($\frac{1}{2}+x, \frac{1}{2}-y, \bar{z}$)	($\frac{1}{2}-x, \bar{y}, \frac{1}{2}+z$)			($\bar{x}, \frac{1}{2}+y, \frac{1}{2}+z$)	($\frac{1}{2}-x, \bar{y}, \frac{1}{2}+z$)			($\bar{x}, \bar{y}, \frac{1}{2}+z$)	($\bar{x}, \frac{1}{2}+y, \frac{1}{2}-z$)		

TABLE 23. Reduction from 3 to 1

Space group of unstrained crystal, order 3 per lattice point		No specialization of stress			Space group of strained crystal, order 1 per lattice point	
		Coordinates referred to axes of unstrained crystal				
No.	Symbol	1st Subset	2d Subset	3d Subset	No.	Symbol
143	P3	(x, y, z)	($\bar{y}, x-y, z$)	($y-x, \bar{x}, z$)	1	P1
144	P3 ₁	(x, y, z)	($\bar{y}, x-y, \frac{1}{3}+z$)	($y-x, \bar{x}, \frac{2}{3}+z$)	1	P1
145	P3 ₂	(x, y, z)	($\bar{y}, x-y, \frac{2}{3}+z$)	($y-x, \bar{x}, \frac{1}{3}+z$)	1	P1
146	R3 hex. axes	(x, y, z)	($\bar{y}, x-y, z$)	($y-x, \bar{x}, z$)	1	P1

TABLE 24. Reduction from m to 1

Space group of unstrained crystal, order 2 per lattice point		No specialization of stress			Space group of strained crystal, order 1 per lattice point	
		Coordinates referred to axes of unstrained crystal ($m \perp y$)				
No.	Symbol	1st Subset	2d Subset	No.	Symbol	
6	Pm	(x, y, z)	(x, \bar{y}, z)	1	P1	
7	Pc	(x, y, z)	($x, \bar{y}, \frac{1}{2}+z$)	1	P1	
8	Cm	(x, y, z)	(x, \bar{y}, z)	1	P1	
9	Cc	(x, y, z)	($x, \bar{y}, \frac{1}{2}+z$)	1	P1	

TABLE 25. Reduction from 2 to 1

Space group of unstrained crystal, order 2 per lattice point		No specialization of stress			Space group of strained crystal, order 1 per lattice point	
		Coordinates referred to axes of unstrained crystal ($2 \parallel y$)				
No.	Symbol	1st Subset	2d Subset	No.	Symbol	
3	P2	(x, y, z)	(\bar{x}, y, \bar{z})	1	P1	
4	P2 ₁	(x, y, z)	($\bar{x}, \frac{1}{2}+y, \bar{z}$)	1	P1	
5	C2	(x, y, z)	(\bar{x}, y, \bar{z})	1	P1	

TABLE 26. Most general stress consistent with a reduction of a noncentrosymmetric point group to any one of its stress-induced subgroups

Initial group	Subgroup										
	Minimum					Compound					
Self subgroup	42m	422	3m	32	3	mm2	222	m	2	1	
43m	X=Y=Z	X=Y; Z Z		X=Y; [111] Z		[1 $\bar{1}0$], [110], z X, Y, Z, any permutation	x, y, z X, Y, Z, any permutation	[110] X, Y, Z or x X, Y, Z	x X, Y, Z	Any	
432	X=Y=Z		X=Y; Z Z		X=Y; [111] Z		x, y, z or [1 $\bar{1}0$], [110], z X, Y, Z, any permutation	x or [110] X, Y, Z	Any		
23	X=Y=Z				X=Y; [111] Z		x, y, z X, Y, Z, any permutation	x X, Y, Z	Any		

TABLE—Continued

Initial group	Subgroup					
	Self subgroup	mm2	222	m	2	1
$\bar{6}m2$ (hex. axes)	$X=Y,$ $z \parallel Z$	$[1\bar{1}0], [110], z \parallel X, Y, Z, \text{any permutation}$		$z \parallel X, Y, \text{or } Z \text{ or}$ For $\bar{6}m2:$ $[110]; \text{for}$ $\bar{6}2m: [110]$ $\parallel X, Y, \text{or } Z$	For $\bar{6}m2:$ $[110]; \text{for}$ $\bar{6}2m: [110]$ $\parallel X, Y, \text{or } Z$	Any
6mm (hex. axes)	$X=Y,$ $z \parallel Z$	$z, y \parallel \text{any two of } X, Y, Z$		$[110] \text{ or } [\bar{1}\bar{1}0]$ $\parallel X, Y, \text{or } Z$	$z \parallel X, Y, \text{or } Z$	Any
622 (hex. axes)	$X=Y,$ $z \parallel Z$		$z, y \parallel \text{any two of } X, Y, Z$		$[110], [\bar{1}\bar{1}0], \text{or } z$ $\parallel X, Y, \text{or } Z$	Any
6 (hex. axes)	$X=Y,$ $z \parallel Z$				$X, z \parallel X, Y, \text{or } Z$	Any
$\bar{6}$ (hex. axes)	$X=Y,$ $z \parallel Z$			$z \parallel X, Y, \text{or } Z$		Any
$\bar{4}2m$	$X=Y,$ $z \parallel Z$	For $\bar{4}2m:$ $[110], [110],$ $z; \text{for } \bar{4}m2:$ $x, y, z \parallel X, Y, Z, \text{any permutation}$	For $\bar{4}2m:$ $x, y, z; \text{for}$ $\bar{4}m2: [110],$ $[110], z \parallel X, Y, Z, \text{any permutation}$	For $\bar{4}2m:$ $[110]; \text{for}$ $\bar{4}m2: x \parallel X, Y, \text{or } Z$	For $\bar{4}2m:$ $x; \text{for } \bar{4}m2:$ $[110] \parallel X, Y, \text{or } Z \text{ or } z \parallel X, Y, \text{or } Z$	Any
4mm	$X=Y,$ $z \parallel Z$	$x, y, z \text{ or } [\bar{1}\bar{1}0],$ $[\bar{1}\bar{1}0], z \parallel X, Y, Z, \text{any permutation}$		$x \text{ or } [110]$ $\parallel X, Y, \text{or } Z$	$z \parallel X, Y, \text{or } Z$	Any
422	$X=Y,$ $z \parallel Z$		$x, y, z \text{ or } [\bar{1}\bar{1}0],$ $[\bar{1}\bar{1}0], z \parallel X, Y, Z, \text{any permutation}$		$x, [110], \text{ or } z$ $\parallel X, Y, \text{or } Z$	Any
4	$X=Y,$ $z \parallel Z$				$X, z \parallel X, Y, \text{or } Z$	Any
$\bar{4}$	$X=Y,$ $z \parallel Z$				$X, z \parallel X, Y, \text{or } Z$	Any
3m (hex. axes)	$X=Y,$ $z \parallel Z$			For $3m1:$ $[110]; \text{for}$ $31m: [110]$ $\parallel X, Y, \text{or } Z$		Any
32 (hex. axes)	$X=Y,$ $z \parallel Z$				For $321:$ $[110]; \text{for}$ $312: [110]$ $\parallel X, Y, \text{or } Z$	Any
3 (hex. axes)	$X=Y,$ $z \parallel Z$					Any
mm2	$x, y, z \parallel X, Y, Z, \text{any perm.}$			$x \text{ or } y$ $\parallel X, Y, \text{or } Z$	$z \parallel X, Y, \text{or } Z$	Any
222	$x, y, z \parallel X, Y, Z, \text{any perm.}$				$x, y, \text{ or } z$ $\parallel X, Y, \text{or } Z$	Any
$m(\perp y)$	$y \parallel X, Y, \text{or } Z$					Any
$2(\parallel y)$	$y \parallel X, Y, \text{or } Z$					Any
1	Any					

Large square brackets indicate same subgroup in equivalent setting.

(Paper 69A5–368)