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Signal Statistics, Yesterday and Today

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The subject of this Symposium is Signal Statistics or, more precisely, the study of statistical methods in problems of wave propagation. In sixteen papers, acknowledged authorities will tell you of the latest advances in their field. Six papers will present the mathematical background, the other ten cover ocean waves, radio waves, and light waves. These sixteen papers cover a great deal of ground. Quite certainly the next two days will offer us as many new ideas as it is possible to absorb in so short a time. Even so, such topics as the study of the statistics of languages and of music, of signals in biological systems, had to be excluded. In my address I will limit my remarks to the subject as it has been defined for this Symposium.

The research about which we shall talk is partly motivated by man's desire to understand natural phenomena, and partly by his wish to use this information for practical purposes. Some of the phenomena that have intrigued mankind for thousands of years: the twinkling of the stars, the blue color of the sky, the halos in the clouds, could only be fully understood by studies analogous to those referred to in this Symposium. On the applied research side we mention all the problems of fading in long-distance radio communications, the 12,000 miles of wide-band tropospheric scatter circuits installed, and the space communications satellites.

First of all I shall touch briefly on some aspects of the history of our subject, and you will see that serious studies started about a hundred years ago, after which I shall talk about the results in the field covered by two recent symposia.

Several papers at this Symposium will make reference to the Rayleigh distribution. Lord Rayleigh [1899–1920] derived this distribution in a paper in the Philosophical Magazine of 1880: "On the resultant of a large number of vibrations of the same pitch and of arbitrary phase." He first applied his results to the theory of sound. In 1888, [1888, 1889], he suggested that light might be represented by the superposition of a large number of independent frequency components with a Gaussian distribution. It was Kluyver [1906] who solved the problem of the drunk man's walk for steps of unequal length. (A man starts from a point 0 and walks a distance a_1 in a straight line; he then turns through any angle whatsoever and walks a distance a_2 in a second straight line. He repeats this process n times. It is required to find the probability that he is at a distance between r and $r+dr$ from his starting point.) Rayleigh [1919] later gave the full details of the analysis of the problem and then obtained the solution of the corresponding problem for flight in three dimensions.

The mathematical problems attached to the superposition of random vectors are not the only link between Lord Rayleigh and this Symposium. One of his first—now famous—papers that concerns us was written when he was 28 years old, and was entitled: "On the light of the sky, its polarization and colour", 1871. I quote from it: "It is now, I believe, generally admitted that the light which we receive from the clear sky is due in one way or another to small suspended particles which divert the light from its regular course. The experiments of Tyndall, 1852, seem quite decisive . . . . When light is scattered by particles which are very small compared with any of the wavelengths the ratio of the amplitudes of the vibrations of the scattered and incident light varies inversely as the square of the wavelengths and the intensity of the lights themselves as the inverse fourth power. The polarization is complete in a direction perpendicular to that of the incident beam." In 1873 Maxwell, who had read this paper, posed the following problem to Lord Rayleigh: "Suppose that there are N spheres of density ρ and diameter S in the unit of volume of the medium. Find the index of refraction of the compound medium and the coefficient of extinction of the light passing through it. The object is of course to obtain data about the size of molecules of air." Some aspects of this problem are still quite modern. It was not until 1899 that Rayleigh returned to the problem with a more detailed theory [1899]. Two other papers of Lord Rayleigh in our field are: "On the propagation of waves through a stratified medium", 1912, and "On the scattering of light by spherical shells and by complete spheres of periodic structure, when the refractivity is small" in 1918.

Almost all the contributions you will hear today and tomorrow have something to do with random processes. In a recent book MacDonald [1962] has outlined the history of one of the processes that drew early attention: the Brownian Movement. While examining small particles immersed in water, the biologist Robert Brown observed many of them evidently in motion. As a biologist Brown thought he had encountered the existence of an elementary form of life. In 1879 von Nägeli made an attempt to show that molecular bombardment was the cause of Brownian motion. The reasons why he rejected the idea make interesting reading. In 1905 Einstein [1905] published the first of a series of classical papers providing a clear theoretical analysis. In 1909 Einstein [1909] treated the subject of energy fluctuations in radiation. He applied the energy fluctuation formula to blackbody radiation in an enclosure at equilibrium temperature, and derived an equation that later was shown by Fürth [1928] to be directly derivable from the Bose-Einstein statistics of photons, irrespective of their spectral distribution. With the advent of lasers this equation has gained new importance.

That the twinkling of stars was of atmospheric origin was known to Newton, but a systematic study of its dependence on meteorological conditions was started by Dufour (Phil. Mag. [1860]). The atmospheric origin of the twinkling of radio-star was proved by correlation techniques.

The mathematical treatment of random processes owes much to Norbert Wiener, who in a famous Acta Mathematica paper entitled "On generalized harmonic analysis", 1930, laid down the foundations. The Wiener-Khinchine relation linking the energy spectrum and the autocorrelation function is of great importance in the development of the subject. Wiener then also derived and discussed the coherence matrix. The problem of coherence in optics was treated at about the same time by van Cittert [1934] and Zernike [1938] in Holland. Wiener and Kolmogoroff also developed new methods to predict the development of random processes in 1940–1942.

During and after World War II, signal statistics came in for so much interest that a full account of the process of growth would take too much time. Of the many books on statistical communication theory, I shall only mention those by Bendat [1958], Davenport and Root [1958], Helström [1960], Lawson and Uhlenbeck [1950], Lee [1960], Middleton [1960] (a 1200 page introduction!); Brekhovskih [1957], Chernov [1960], and Tatarsky (1960), published works on propagation in layered, random, or turbulent media; and Kotelnikov [1947], and Woodward [1955], on probabilistic detection.

Booker and Gordon [1950], Ratcliffe [1956], and others helped us to gain an insight into tropospheric and ionospheric diffraction and scattering problems. Scattering even developed into a new means of communication. In radio astronomy the detailed structure of our own galaxy was derived from the reception of the 1420 Mc/s hydrogen line. Hanbury Brown et al. [1952], discovered a new technique in his intensity interferometer, a development that came first in radio astronomy and then proved to be of great use in optical astronomy. In radar astronomy, echoes from the Moon, the Sun, and Venus have been studied in great detail, with spectacular results.

Let us now turn to the results of the Brussels Symposium on Information Theory 1962 and of the Plenary Assembly of the URSI in Tokyo, 1963. Again we have to make a choice; we do not hope to give you a complete survey, but only to stimulate an interest in the Transactions of the Symposium and the Monographs of the URSI. In a Commission 6 session—On stochastic aspects of radiation—Bremmer and Twersky were the invited speakers. Bremmer [1963], treated multiple scattering as a Markov process and derived a balance equation for the energy. This can be simplified to a Fokker-Planck equation, but a more rigorous treatment results from a Laplace transform of the balance equation. This may lead to greatly different results. Twersky [1963] considered a statistical ensemble of configurations of arbitrary scatterers. He begins with exact operational representations in the form of infinite series of orders of scattering, and in the form of integral relations (hierarchy equations). Consistent integral equation forms are derived by direct manipulation of the exact series. Explicit results for these functions and for the higher statistical moments are discussed for a simple class of scattering problems. The amount of computation involved looks formidable. Experimental confirmation has been found for some of the results. Blanc-Lapierre [1963] considered a single scattering model for propagation in random media. (Example: "reverberation effects" in underwater sound detection problems.) Karbowiak [1963] developed a waveguide mode approach for propagation in random media by introducing cross-coupling effects between the propagation modes of the system. This development was restricted to slowly varying time dependence and to enclosed structures, but it can be generalized. Simon [1963] treated a tropospheric scatter link 300 km in length. The frequency can be changed rapidly from 3300 to 3500 Mc/s. It is always possible to find a band of 2 Mc/s that is 5 dB above the average. Duration of measurement was 1 msec. Duration of of transmission was 100 msec. This may be more effective than diversity. Furutsu [1963a; 1963b] pointed out many analogies between quantum field theory and the statistical theory of waves. Basic equations in the statistical theory correspond closely to the commutation relations and the Heisenberg equation of motion in the field theory. Furutsu expects to publish an extension of his so far one-dimensional theory to three dimensions in the next few months. Skinner [1963] analysed the possibility of determining the correlation of a macroscopic density variation within a turbulent plasma by intensity and correlation measurements upon a beam of light. This analysis needs experimental verification.

In the Commission 6 session on time-varying channels, and in Brussels, Kailath [1962; 1963] discussed time-variant channels that can be described as linear time-variant filters. These filters can be modeled by a densely tapped delay line. The product of the maximum time and frequency spreads produced by the filter sets a limit to our ability to determine the instantaneous values unambiguously even in the absence of noise. A survey of suggested methods of measurement was given, but both this area and that of receiver evaluation need further study. According to Siforov [1963], the channel capacity in the case of multipath propagation depends on the difference between the total bandwidth of all randomly varying parameters and the bandwidth of the channel. If the latter is greater the information goes to infinity without noise. Siforov also calculated the quantity of information available from a moving source. Price [1963] described an adaptive receiver for a multichannel binary communication system. Each channel has a nondispersive, nonfading propagation path and additive white gaussian noise. The receiver measures path strength and phase by means of the information signals, and automatically carries out the necessary weighting and phase shifting. Error probabilities are calculated. The situation seems somewhat artificial, and should not be confused with multipath. Perry and Wozencraft [1962], and Wozencraft and Kennedy [1963] treated probabilistic sequential coding. In Brussels, Wozencraft concentrated on a description of the encoding and decoding processes and their implementation in a digital machine. In Tokyo he stressed the importance of signal design and received-signal processing, and discussed applications to a wide-band satellite channel, and to telephone channels with the required low error probability. Miss Mourier [1963] gave a test for the hypothesis that a random process is stationary, both for the case where there is only one realization and when there are more realizations of the process. Turin [1963] suggested that singular systems (no error) be studied further so as to provide a guide for improvement of real systems. The detailed structure of the optimum M-ary sequential receiver should be determined. The selection of the signal set which optimizes the transmitter-receiver combination as a whole, also needs more study, especially with random modulation delays.

In Commission 2 the interpretation of measurements of propagation and of meteorological evidence has led to representative models. Turbulent layers and laminar structures are often found next to each other at the same time. So the two models supplement each other, rather than being in opposition. In any model the parameters should be derived from the observations.

In Commission 3 ionospheric irregularities were discussed; new data were available from satellite observations above the F-layer. Instabilities in the ionospheric plasma result from incoming solar particles. These cause diffraction effects and may even act as lenses for passing waves. In Commission 4 a report has been prepared for C.C.I.R., which not only gives information on the world distribution of noise power, but also those details of the noise structure which are needed in assessing the required signal-to-noise ratio for a satisfactory radio service.

To sum up, I have tried to give you a survey of only those subjects in the two meetings mentioned that were nearest to signal statistics. There were of course many other valuable contributions, but we have no time to go into them here.

I hope you will conclude that the view I have given you is that of research in progress, of the efforts of scientists from many countries to advance our understanding a little further. Although spectacular results are not achieved every year, if you look over periods of ten or twenty years what a difference you see! Let this encourage those of us working in domains where the wrestling with intractable mathematical problems has not yet yielded the results that are well deserved.

In information theory a signal is defined as the physical embodiment of a message. I hope that all of you here, from your different disciplines will take away with you from this Symposium on Signal Statistics the message to continue your efforts, and to solve the next problem, because together we will assuredly make progress, even though we may never reach our goal and conclude our work. Thank you.

NOTE: Reprints of the documents URSI Tokyo 1963 were issued at the meeting in Tokyo. An URSI Monograph will be published, containing most papers and the discussions.

Eindhoven, November 26, 1963.

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