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Some Numerical Results Based on the Theory of Radio Wave Propagation Over Inhomogeneous Earth

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Several numerical examples are presented to show the predominant features of radio wave propagation over an inhomogeneous earth. These are based on the theories derived previously [Furutsu, 1957a, 1957b, 1959, 1963] in which the height and also the electrical properties of the earth's surface were assumed to change discontinuously several times along the wave path; thus, the terrain represented could include ridges, cliffs, bluffs at a coastline, etc. The theory is briefly reviewed, and numerical results are presented for the spherical earth approximation and are compared to those for the flat earth approximation. For a perfectly conducting flat earth, there are well-known formulas available in terms of the Fresnel integral, and the spherical earth results are compared to those obtained using these formulas to show the agreement at short distances. A few interesting phenomena are also illustrated, such as the obstacle gain due to a ridge on a lossy ground and the variation of field strength caused by a change of receiver (or transmitter) height when the wave is propagated over a mixed path. Finally, sets of graphs are also included to aid in evaluating the effects of a ridge or a cliff on a homogeneous earth; they can be used when the propagation distances are sufficiently large on each side of the ridge or cliff.

1. Introduction

The purpose of this paper is to determine the field strength when a radio wave is propagated over a terrain varying both in height and in electrical properties.

The problems of mixed paths over a smooth earth have been investigated by many authors [Bremmer, 1954; Clemmow, 1950; 1953; Feinberg, 1944; 1946; 1959; Godzinski, 1958; Wait, 1956; 1957; 1961], and one of the authors of this paper [Furutsu, 1955a; 1955b; 1955c]. Some of their results were given in terms of a convolution integral which is usually evaluated by a numerical method. Mixed paths mean that the wave path is over ground consisting of several sections of different electrical properties, but in these papers, no change of height of terrain had been taken into account.

However, it had been the main purpose of some papers by Furutsu [1957a; 1957b; 1959; 1963] to determine the effects on the groundwave propagation over an inhomogeneous earth [1] caused by significant changes in the height of terrain, such as ridges, cliffs, etc., and also how these effects could be treated. The model of the terrain, the height of a spherical earth surface and its electrical properties were assumed to change discontinuously along the wave path several times. This is illustrated in figure 1.[2] The formula of field strength for this model was obtained in the form of a multiple residue series. This formula reduces to that for mixed paths in the special case of a *smooth* earth and further to the ordinary Van der Pol-Bremmer series in the case of a homogeneous earth.

The basic terrain used in this paper is illustrated in figure 2. This figure shows the terrain consisting of two sections of different earth radii, a_2 and a_4, and different propagation constants,

[1] See, however, papers by Hufford [1952] and also by Wait [1963].

[2] In this figure, a_n is the radius of the nth section, r_n is its length, x_1 and x_{n+1} indicate the transmitter and receiver locations.

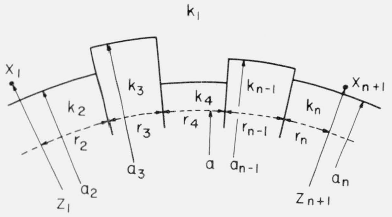


FIGURE 1. *The form of the terrain used in the theory of inhomogeneous earth.*

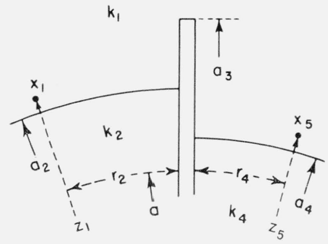


FIGURE 2. *The form of the terrain consisting of two sections for (3).*

k_2 and k_4, respectively, with a ridge of radial distance a_3 (measured from the earth's center to the top of the ridge) at the boundary of the two sections. The transmitter and the receiver are on each side of the ridge at the points x_1 and x_5, whose radial distances are z_1 and z_5, respectively. Their distances from the ridge are r_2 and r_4, being measured along a mean earth surface of radius a.

The attenuation coefficient A is defined by (A.2.1),[3] i.e., if E is the field strength to be obtained,

$$E=2AE_0, \qquad E_0=\frac{k_1^2}{4\pi(r_2+r_4)}\,e^{-ik_1(r_2+r_4)}, \qquad k_1=2\pi/\lambda,$$

where λ is the wavelength in free space and E_0 may be regarded as the field strength in free space. The attenuation coefficient A thus defined is conveniently given in terms of the numerical distances c_2 and c_4 defined by

$$c_2=(r_2/a)(k_1a/2)^{1/3}, \qquad c_4=(r_4/a)(k_1a/2)^{1/3}, \tag{1}$$

and also the numerical heights y_1, y_2, y_3, y_4, and y_5 defined by

$$\begin{aligned} &y_1=k_1(z_1-a)(2/k_1a)^{1/3}, \qquad y_2=k_1(a_2-a)(2/k_1a)^{1/3}, \qquad y_3=k_1(a_3-a)(2/k_1a)^{1/3}, \\ &y_4=k_1(a_4-a)(2/k_1a)^{1/3}, \qquad y_5=k_1(z_5-a)(2/k_1a)^{1/3}. \end{aligned} \tag{2}$$

Thus, using (A.3.1), A can be given in the form

$$A=\sum_{t_4,t_2}\{(c_4+c_2)/c_4\}^{1/2}A(y_{54},c_4)_{t_4}T^{(3)}(c_2)_{t_4,\,t_2}f_{t_2}(y_{12}), \tag{3a}$$

with the conditions

$$\begin{aligned} &(z_1-a_2)/r_2<<1, \qquad (a_3-a_2)/r_2<<1, \qquad (z_5-a_4)/r_4<<1, \qquad (a_3-a_4)/r_4<<1, \\ &k_1r_2>>1, \qquad k_1r_4>>1, \end{aligned} \tag{3b}$$

where

$$y_{ij}=y_i-y_j=-y_{ji}, \tag{4}$$

$$A(y_{54},c_4)_{t_4}\equiv(\pi c_4)^{1/2}(t_4-q_4^2)^{-1}f_{t_4}(y_{54})\exp\left[-i\{c_4(y_4+t_4)+\pi/4\}\right], \tag{5}$$

$$T^{(3)}(c_2)_{t_4,\,t_2}=\{q_4f'_{t_4}(y_{34})f_{t_2}(y_{32})-q_2f_{t_4}(y_{34})f'_{t_2}(y_{32})\}\{y_4-y_2+t_4-t_2\}^{-1}(t_2-q_2^2)^{-1}\exp\left[-ic_2(y_2+t_2)\right]. \tag{6}$$

Here, the set of values $t_m(m=2,\,4)$ stands for the roots of the equation

$$W'(t_m)-q_mW(t_m)=0; \tag{7}$$

$W'(t)$ is the first derivative of the function $W(t)$ defined by

$$W(-t)=(\pi t/3)^{1/2}e^{-i2\pi/3}H_{1/3}^{(2)}(\tfrac{2}{3}t^{3/2}), \tag{8}$$

[3] Equation numbers preceded by an A refer to equations in Part I of Furutsu [1963].

and

$$iq_m=(k_1 a/2)^{1/3}\times\begin{cases} k_1\sqrt{k_m^2-k_1^2}/k_m^2, & \text{Vertical Polarization} \\ \sqrt{k_m^2-k_1^2}/k_1, & \text{Horizontal Polarization.}\end{cases} \tag{9}$$

The function $f_{t_m}(y)$ is the ordinary height-gain function defined by

$$f_{t_m}(y)=W(t_m-y)/W(t_m), \tag{10}$$

and the function $f'_{t_m}(y)$ is defined by

$$q_m f'_{t_m}(y)=-(\partial/\partial y)f_{t_m}(y)=W'(t_m-y)/W(t_m). \tag{11}$$

Thus, it can be seen from (7) that

$$f'_{t_m}(0)=f'_{t_m}(0)=1. \tag{12}$$

In the special case where $a_3=a_2(a_2\geq a_4)$ or $a_3=a_4(a_2\leq a_4)$, the terrain would represent a cliff, and then

$$T^{(3)}(c_2)_{t_4,t_2}=T(c_2)_{t_4,t_2}\equiv\{y_4-y_2+t_4-t_2\}^{-1}(t_2-q_2^2)^{-1}$$

$$\times\exp[-ic_2(y_2+t_2)]\times\begin{cases} q_4 f'_{t_4}(y_{24})-q_2 f_{t_4}(y_{24}), & y_2\geq y_4 \\ q_4 f_{t_2}(y_{42})-q_2 f'_{t_2}(y_{42}), & y_2\leq y_4. \end{cases} \tag{13}$$

In the case of $a_2=a_4$ and $q_2=q_4$, the terrain would represent a ridge on a homogeneous ground. In this case, both the numerator and the denominator of

$$T^{(3)}(c_2)_{t_4,t_2}$$

vanish for $t_4=t_2$; then

$$T^{(3)}(c_2)_{t_2,t_2}=[\{1-y_{32}(t_2-q_2^2)^{-1}\}f_{t_2}^2(y_{32})$$
$$+q_2^2(t_2-q_2^2)^{-1}\{f_{t_2}^2(y_{32})-f'^2_{t_2}(y_{32})\}]\exp[-ic_2(y_2+t_2)]. \tag{14}$$

Further, in the case of smooth earth where $y_2=y_3=y_4=0$, (13) gives, on using (12) and (14):

$$T^{(3)}(c_2)_{t_4,t_2}=(q_4-q_2)(t_4-t_2)^{-1}(t_2-q_2^2)^{-1}e^{-ic_2t_2} \qquad (q_4\neq q_2)$$

$$=\begin{cases} e^{-ic_2t_2}, & t_4=t_2 \\ 0, & t_4\neq t_2, \end{cases} \qquad (q_4=q_2) \tag{15}$$

and thus, formula (3) becomes:

$$A=\sum_{t_2,t_4}\{\pi(c_2+c_4)\}^{1/2}(q_4-q_2)(t_4-t_2)^{-1}(t_2-q_2^2)^{-1}(t_4-q_4^2)^{-1}$$
$$\times f_{t_2}(y_1)f_{t_4}(y_5)\exp[-i\{c_2t_2+c_4t_4+\pi/4\}], \qquad (q_4\neq q_2) \tag{16}$$

$$=\sum_{t_2}\{\pi(c_2+c_4)\}^{1/2}(t_2-q_2^2)^{-1}f_{t_2}(y_1)f_{t_2}(y_5)\exp[-i\{(c_2+c_4)t_2+\pi/4\}], \qquad (q_4=q_2). \tag{17}$$

Equation (16) is a special case of the established formula for mixed paths over a smooth spherical earth consisting of several different sections [Furutsu, 1955c], and (17) is the ordinary formula for a homogeneous earth.

In the Van der Pol-Bremmer series for a homogeneous, smooth earth, the convergence becomes poor when the propagation distance is sufficiently short, and likewise, the convergence of the series (3) becomes very poor when the propagation distance on one or both sides of the ridge is very short. However, in the latter case, the effect of the earth's curvature is so small that the flat earth approximation can be used; the result in the flat earth approximation then provides the asymptotic form of the series (3) as $a \to \infty$.

In order to present the flat earth formula corresponding to (3), it is convenient to introduce the parameters defined by:

$$
\begin{aligned}
&d_2 = iq_2^2 c_2 = -i(k_1/k_2')^2 k_1 r_2/2,\\
&d_4 = iq_4^2 c_4 = -i(k_1/k_4')^2 k_1 r_4/2,\\
&f_1 = \tfrac{1}{2} c_2^{-1/2} y_{12} e^{i\pi/4} = (z_1 - a_2)(k_1/2r_2)^{1/2} e^{i\pi/4},\\
&f_5 = \tfrac{1}{2} c_4^{-1/2} y_{54} e^{i\pi/4} = (z_5 - a_4)(k_1/2r_4)^{1/2} e^{i\pi/4},\\
&f_2 = \tfrac{1}{2} c_2^{-1/2} y_{32} e^{i\pi/4} = (a_3 - a_2)(k_1/2r_2)^{1/2} e^{i\pi/4},\\
&f_4 = \tfrac{1}{2} c_4^{-1/2} y_{34} e^{i\pi/4} = (a_3 - a_4)(k_1/2r_4)^{1/2} e^{i\pi/4},\\
&n_2 = r_2/(r_2 + r_4),\\
&n_4 = r_4/(r_2 + r_4),
\end{aligned}
\tag{18}
$$

with

$$
k_m' = \begin{cases} k_m^2/\sqrt{k_m^2 - k_1^2}, & \text{Ver. Pol.} \\ k_1^2/\sqrt{k_m^2 - k_1^2}, & \text{Horiz. Pol.} \end{cases} \qquad (m=2,\ 4) \tag{19}
$$

all of which are independent of the earth radius a. It may also be noted that d_2 and d_4 are the Sommerfeld numerical distances on the respective sides of the ridge.

In the special case where both the transmitter and the receiver are on the ground, i.e., when $z_1 = a_2$, $z_5 = a_4$ or $f_1 = f_5 = 0$, the attenuation coefficient A is given, with the aid of (A.4.2) to (A.4.4), by

$$
\begin{aligned}
A = F(d_4, f_4 | d_2, f_2) \equiv e^{-(f_4^2 + f_2^2)} \Big[&\mathcal{E}(\sqrt{n_2} f_4 + \sqrt{n_4} f_2) - i\sqrt{\pi}(\sqrt{d_4/n_4} + \sqrt{d_2/n_2})^{-1} \\
&\times \big[\sqrt{d_4 d_2 / n_4 n_2}\, \mathcal{E}(f_4 + i\sqrt{d_4})\, \mathcal{E}(f_2 + i\sqrt{d_2}) + (d_4/n_4)\{\mathcal{E}(\sigma_4, (\sqrt{n_2} f_4 + \sqrt{n_4} f_2)/\sigma_4) \\
&- \mathcal{E}(\rho_4, (f_4 + i\sqrt{d_4})/\rho_4)\} + (d_2/n_2)\{\mathcal{E}(\sigma_2, (\sqrt{n_2} f_4 + \sqrt{n_4} f_2)/\sigma_2) - \mathcal{E}(\rho_2, (f_2 + i\sqrt{d_2})/\rho_2)\}\big]\Big].
\end{aligned}
\tag{20}
$$

Here the vertical line in the arguments of the function F defined above is used to distinguish between the variables on each side of the boundary, and

$$
\begin{aligned}
&\sigma_4 = \sqrt{n_4} f_4 - \sqrt{n_2} f_2 + i\sqrt{d_4/n_4}, \qquad \rho_4 = i\sqrt{d_4 n_2/n_4} - f_2,\\
&\sigma_2 = \sqrt{n_2} f_2 - \sqrt{n_4} f_4 + i\sqrt{d_2/n_2}, \qquad \rho_2 = i\sqrt{d_2 n_4/n_2} - f_4,
\end{aligned}
\tag{21}
$$

and

$$\mathcal{E}(z)=\frac{2}{\sqrt{\pi}}e^{z^2}\int_z^\infty e^{-x^2}dx,$$

$$\mathcal{E}(z, n/z)=\left(\frac{2}{\sqrt{\pi}}\right)^2 e^{z^2+n^2}\int_z^\infty dxe^{-x^2}\int_{(n/z)x}^\infty e^{-y^2}dy. \tag{22}$$

The analytic expansions for $\mathcal{E}(z)$ and $\mathcal{E}(z,n/z)$are treated in the appendix of Furutsu [1963].

In the general case of $z_1\neq a_2$ and $z_5\neq a_4$,

$$A=F(d_4, f_4+f_5|d_2, f_2+f_1)+\tfrac{1}{2}\{F(d_4, f_4+f_5|0, f_2-f_1)-F(d_4, f_4+f_5|0, f_2+f_1)$$
$$+F(0, f_4-f_5|d_2, f_2+f_1)-F(0, f_4+f_5|d_2, f_2+f_1)\}+\tfrac{1}{4}\{F(0, f_4+f_5|0, f_2+f_1)+F(0, f_4-f_5|0, f_2-f_1)$$
$$-F(0, f_4+f_5|0, f_2-f_1)-F(0, f_4-f_5|0, f_2+f_1)\}. \tag{23}$$

When the earth's surface is perfectly conducting and the wave is vertically polarized, (18) and (19) yield $d_2=d_4=0$. Thus, (20) becomes simply

$$A=e^{-(f_4^2+f_2^2)}\mathcal{E}(\sqrt{n_2}f_4+\sqrt{n_4}f_2),$$

$$\sqrt{n_2}f_4+\sqrt{n_4}f_2=\left\{\frac{a_3-a_4}{r_4}+\frac{a_3-a_2}{r_2}\right\}\sqrt{\frac{k_1r_4r_2}{2(r_4+r_2)}}\,e^{i\pi/4},(z_1=a_2,\ z_5=a_4). \tag{24}$$

As is seen from the definition of the function $\mathcal{E}(z)$ in (22), (24) is given by the Fresnel integral and, as expected, is exactly the same as the result for ordinary knife-edge diffraction.

On the other hand, when the earth's surface is smooth and both the transmitter and receiver are on the ground so that

$$f_1=f_2=f_3=f_4=f_5=0,$$

formula (20) becomes

$$A=1-i\sqrt{\pi}(\sqrt{d_4/n_4}+\sqrt{d_2/n_2})^{-1}[\sqrt{d_4d_2/n_4n_2}\,\mathcal{E}(i\sqrt{d_4})\,\mathcal{E}(i\sqrt{d_2})$$
$$+(d_4/n_4)\{\mathcal{E}(i\sqrt{d_4/n_4})-\mathcal{E}(i\sqrt{d_4n_2/n_4},\sqrt{n_4/n_2})\}$$
$$+(d_2/n_2)\{\mathcal{E}(i\sqrt{d_2/n_2})-\mathcal{E}(i\sqrt{d_2n_4/n_2},\sqrt{n_2/n_4})\}]. \tag{25}$$

This equation is essentially equivalent to the formula first obtained by Clemmow [1953] and also the same as by Furutsu [1955a]. Further, if the earth is homogeneous so that $d_2/n_2=d_4/n_4$, (25) becomes

$$A=1-i(\pi d_2/n_2)^{1/2}\mathcal{E}(i\sqrt{d_2/n_2})=1-i2(d_2/n_2)^{1/2}e^{-d_2/n_2}\int_{i\sqrt{d_2/n_2}}^\infty e^{-x^2}dx, \tag{26}$$

which is the formula commonly used for a homogeneous flat earth. In deriving this formula, the following relation is used:

$$\mathcal{E}(a, b/a)+\mathcal{E}(b, a/b)=\mathcal{E}(a)\,\mathcal{E}(b). \tag{27}$$

In formula (3), it frequently happens that the propagation distance on one side of the ridge, say r_2, is small enough so that $c_2<<1$, while the propagation distance on the other side is large enough so that $c_4\geq 1$. Under these conditions the flat earth approximation may be used on the c_2 side, while keeping all spherical properties as they are on the c_4 side; i.e., when

$$c_2<<1,\qquad |c_2(y_{42}+t_4)|<<1,\qquad c_2y_{12}<<1,\qquad c_2y_{32}<<1, \tag{28}$$

it has been shown [Furutsu, 1957b] that

$$\sum_{t_2} T^{(3)}(c_2)_{t_4,\,t_2} f_{t_2}(y_{12}) \simeq B_{t_4}(y_{34}, d_2, f_2, f_1) \equiv [\{(q_4/q_2) f'_{t_4}(y_{34}) - f_{t_4}(y_{34})\}\sqrt{d_2}\,J(\sqrt{d_2}, f_2+f_1)$$
$$+\tfrac{1}{2}(q_4/q_2) f'_{t_4}(y_{34})\sqrt{d_2}\{J(0, f_2-f_1) - J(0, f_2+f_1)\} + \tfrac{1}{2} f_{t_4}(y_{34})\{J(f_2-f_1) + J(f_2+f_1)\}]e^{-i\{(a_2/a)-1\}k_1 r_2}, \tag{29}$$

where f_1, f_2, and d_2 are the same as defined in (18) and

$$J(x, f) = e^{-f^2}\{\mathcal{E}(f) - \mathcal{E}(f+ix)\}/x,$$
$$J(f) = e^{-f^2}\mathcal{E}(f). \tag{30}$$

Furthermore, when $y_{34} << 1$,

$$f_{t_4}(y_{34}) \simeq 1 - y_{34}q_4 = 1 + i(k_1/k'_4)k_1(a_3 - a_4), \qquad y_{34} << 1$$

and

$$(q_4/q_2) f'_{t_4}(y_{34}) \simeq (q_4/q_2) - y_{34}t_4/q_2 = (k'_2/k'_4) - ik'_2(a_3 - a_4)(2/k_1 a)^{2/3} t_4, \qquad y_{34} << 1. \tag{31}$$

Also, when $y_{34}=0$, the right-hand side of (29), $B_{t_4}(0, d_2, f_2, f_1)$, turns out to be independent of t_4 and thus applying formula (29) to (3) on the condition (28), we get

$$A \simeq \sum_{t_4} A(y_{54}, c_4)_{t_4} B_{t_4}(0, d_2, f_2, f_1) = A|_{c_2=0} B_{t_4}(0, d_2, f_2, f_1), \quad A|_{c_2=0} = \sum_{t_4} A(y_{54}, c_4)_{t_4}. \tag{32}$$

The latter series is the ordinary Van der Pol-Bremmer residue series for a homogeneous earth.

In this paper, the above formulas are used to find the attenuation for several typical cases of inhomogeneous earth. Emphasis is first placed on the comparison of the numerical values in the spherical earth approximation with those in the flat earth approximation, since they are expected to asymptotically approach each other at short distances. Although there is almost no other literature available for wave propagation over a terrain of *finite* conductivity, there is the well-known formula given by the Fresnel integral for a ridge on a *perfectly* conducting *plane*, which therefore gives an interesting means of checking the corresponding spherical earth values. Secondly, a few interesting physical phenomena (such as the "obstacle gain" and a complicated variation of the field strength versus the height over mixed paths, etc.) are discussed with numerical illustrations. Finally, a set of graphs is presented for use in calculating the effects of a ridge and a cliff on the groundwave propagation; this is an extension of the work shown in figures 8 and 9 in [Furutsu, 1963]. They can be used in the same way as the ordinary height gain function for the transmitter and/or the receiver, when the propagation distances on both sides of the ridge are large enough so that

$$c_2 - y_{32}^{1/2} - y_{12}^{1/2} >> 1, \qquad c_4 - y_{34}^{1/2} - y_{54}^{1/2} >> 1. \tag{33}$$

However, the above condition may be too strict for practical use, as will be seen by a few examples shown in the appendix.

In the last section, the more general case is treated in which the inhomogeneous earth consists of several different sections with a ridge at each section boundary. This is illustrated in figure 12 and the formula for the attenuation coefficient is given, which was derived in earlier work [Furutsu, 1957b]; this formula corresponds to (3) for two sections with a ridge between them.

2. A Ridge on a Homogeneous Earth

The basic formula for the attenuation coefficient is given by (3) in the case of a spherical earth and by (20) or (23) in the case of a flat earth; for a ridge on a homogeneous earth, the

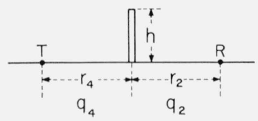


FIGURE 3. *A ridge on a homogeneous earth (flat or spherical) for figures 4 and 5.*

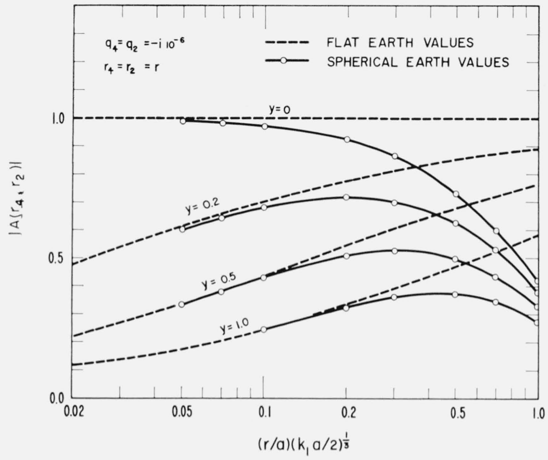


FIGURE 4. *Comparison of the spherical and the flat earth values of attenuation due to diffraction by a ridge on a perfectly conducting earth (vertical polarization).*

Attenuation versus the numerical distance $(r/a)(k_1a/2)^{1/3}$ when $r_2=r_4=r$, for several values of the numerical ridge height $y=k_1h(2/k_1a)^{1/3}$.

attenuation is found when $y_{32}=y_{34}=y\equiv k_1h(2/k_1a)^{1/3}$ and $q_2=q_4$ or $k_2=k_4$. For the numerical illustration the transmitter and receiver were placed on the ground at equal distances from the ridges, as shown in figure 3, and thus $r_2=r_4=r$ or $c_2=c_4=c\equiv(r/a)(k_1a/2)^{1/3}$ and $z_1=a_2=a_4=z_5$ or $y_{12}=y_{54}=0$.

2.1. Case of a Perfectly Conducting Earth

When the earth is perfectly conducting and the wave is vertically polarized, $q_2=q_4=0$ according to (9). Furthermore, if the earth is assumed to be flat, the attenuation coefficient A is given by the Fresnel integral of (24). In figure 4, the absolute value of A is plotted versus

$$c=(r/a)(k_1a/2)^{1/3}$$

for several values of y and is displayed by the broken lines.[4] The corresponding values in the case of a spherical earth, shown in the same figure by solid lines, were obtained by evaluating formula (3) for $q_2=q_4=-i10^{-6}$ (which is effectively zero). As expected, the figure clearly shows that the spherical earth values asymptotically approach the corresponding flat earth values when $c<<1$. It may also be noted that the attenuation increases with the height of the ridge in the range of variables presented.

[4] Although both the variables y and c are dependent on the earth's radius a, the attenuation coefficient A is independent of a, as is seen from relation (18).

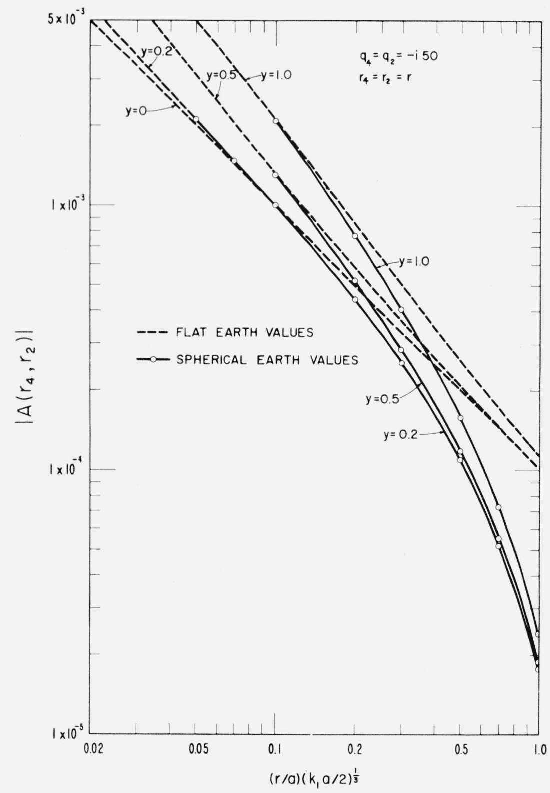


FIGURE 5. *Comparison of the spherical and the flat earth values of attenuation due to diffraction by a ridge on an earth with a finite propagation constant.*

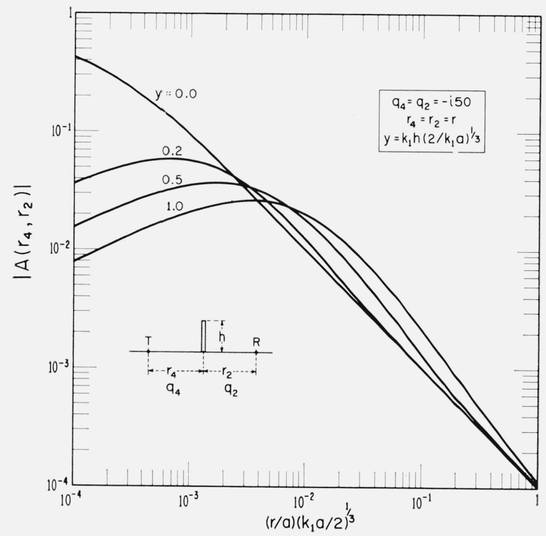


FIGURE 6. *Mountain gain and diffraction loss by a ridge on a lossy surface.*

2.2. Case of a Finite Conducting Earth

The result of the comparison using the same data except for $q_2 = q_4 = -i50$ is presented in figure 5. As in figure 4, the spherical earth values approach the flat earth values when $c << 1$. However, there is a remarkable difference between the characteristics of the two figures; i.e., in figure 5, the higher the ridge, the *smaller* the attenuation. This implies that the propagation loss along the lossy smooth surface is much greater than the diffraction loss by a ridge on the surface. Thus, we have the so-called obstacle gain. However, since the diffraction loss increases with a decrease of distance from the ridge, the obstacle gain is apparent only in a limited range. This situation is illustrated in figure 6, which is a continuation of the flat earth values in figure 5.

On the other hand, when the propagation distances on both sides of the ridge are large enough to satisfy the conditions given in (33), the convergence of the series in formula (3) is fast enough so that only the first term of the series is necessary to give a sufficiently accurate value. This is reflected in figure 5 by the fact that the three solid lines for the spherical earth values are almost parallel to each other for the large distances occurring when $c_2 = c_4 \sim 1$. Therefore, in this case, the effect of the ridge (of height h), located on a homogeneous spherical earth of earth constant q_2, can be obtained by multiplying the attenuation coefficient for the homogeneous earth by the ridge gain factor $T_R(\rho)$ given by:

$$T_R(\rho) = \{1 - y(t_2^0 - q_2^2)^{-1}\} f_{t_2^0}^2(y) + q_2^2 (t_2^0 - q_2^2)^{-1} \{ f_{t_2^0}^2(y) - f_{t_2^0}'^2(y) \}, \qquad y = k_1 h (2/k_1 a)^{1/3}. \quad (34)^{5}$$

[5] This equation is the same as (A.3.3) except that the second term is omitted in the latter. However, all the values in fig. 5 [Furutsu, 1959] and fig. 8 [Furutsu, 1963], were computed using the correct equation (compare with fig. 13 in this paper).

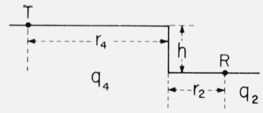


FIGURE 7. *A cliff or a bluff on an earth (flat or spherical) for figures 8, 9, and 10.*

This is the term in square brackets on the right side of (14) where t_2^0 stands for the first value of t_2 for the homogeneous earth specified by q_2; for later convenience, ρ is defined as

$$\rho=2^{-1/3}y=k_1h(k_1a)^{-1/3}, \qquad h=\text{height of ridge}, \tag{35}$$

and q_2 can be expressed as

$$q_2=2^{-1/3}K^{-1}e^{-i(b/2+\pi/4)}, \tag{36}$$

where K and b are the parameters introduced by Norton [1941].

A set of graphs is presented in figure 13 for $|T_R(\rho)|$ versus ρ for various values of K and b.

3. A Cliff or a Bluff at a Coastline

When the obstacle is a cliff or a bluff (of height h), the basic formula in this instance is again given by (3) for the spherical earth and by (20) or (23) for the flat earth with $y_{34}=0$ and $y_{32}=k_1h(2/k_1a)^{1/3}$. Further, if c_4 is chosen sufficiently large to satisfy the condition (28) for t_4, formula (29) is available for the range of $c_2<<1$. The latter is the case of figures 8b, 9, and 10 to be explained in the following, where the absolute values of the ratio $A/(A|_{c_2=0})$ are plotted versus c_2 for several values of the parameters.

For the numerical illustration, the antennas were again placed on the ground ($z_1=a_2$, $z_5=a_4=a_3$, or $y_{12}=y_{54}=0$), as shown in figure 7.

3.1. A Cliff

In this case, $k_2=k_4$ or $q_2=q_4$ and, when the earth's surface is perfectly conducting and the wave is vertically polarized, $q_2=q_4=0$. Then the attenuation coefficient is again given by the Fresnel integral of (24) with $f_4=0$ for the flat earth approximation. These values are shown in figure 8a and 8b by the broken lines, and the corresponding spherical earth values are shown by the solid lines (as in section 2, $q_2=q_4$ was again chosen to have a very small value $-i10^{-6}$, which is effectively zero). Figure 8a shows the absolute values of A versus $(r/a)(k_1a/2)^{1/3}$, when $r_2=r_4=r$, while figure 8b shows the relative values of $|A|$ versus the numerical distance $c_2=(r_2/a)(k_1a/2)^{1/3}$, when $c_4=10$; the latter values of $|A|$ were computed using formula (32).

A similar comparison is presented in figure 9 for the same data except for $q_2=q_4=-i50$, where the flat earth values were computed using the function $B_{t_4}(y_{34}, d_2, f_2, f_1)$ defined in (29) with $y_{34}=f_1=0$ and $q_2=q_4$. It may be noticed here that, when $z_1=a_2$ or $f_1=0$, B_{t_4} is independent of the value of $q_2=q_4$ on account of the identity (12). Thus, the flat earth values are the same as in figure 8b.

As in the case of a ridge on a homogeneous earth, when the propagation distances on both sides of the cliff are large enough to satisfy the conditions in (33) for $y_{34}=0$, the effect of a cliff (of height h) can be obtained by multiplying the attenuation coefficient by the cliff gain factor $T_C(\rho)$. On referring to (13) and figure 7, the factor $T_C(\rho)$ is found to be

$$T_C(\rho)=q_2(t_2^0-q_2^2)^{-1}y^{-1}\{f_{t_2^0}(y)-f'_{t_2^0}(y)\}, \tag{37}$$

where $\rho=2^{-1/3}y=k_1h(k_1a)^{-1/3}$.

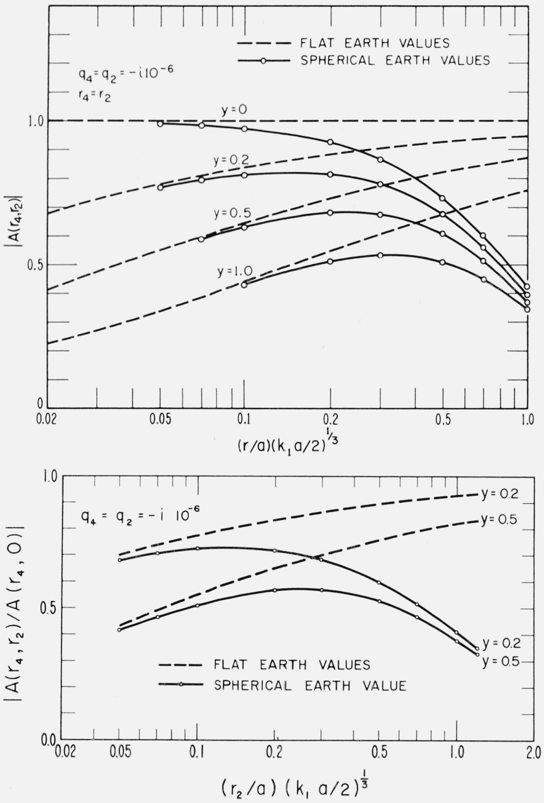


FIGURE 8. *Comparison of the spherical and the flat earth values of attenuation due to diffraction by a cliff on a perfectly conducting earth (vertical polarization).*

(a) Attenuation coefficient versus the numerical distance $(r/a)(k_1a/2)^{1/3}$ when $r_2=r_4=r$.
(b) Relative values of the attenuation coefficient versus $c_2=(r_2/a)(k_1a/2)^{1/3}$ when $c_4=10$.

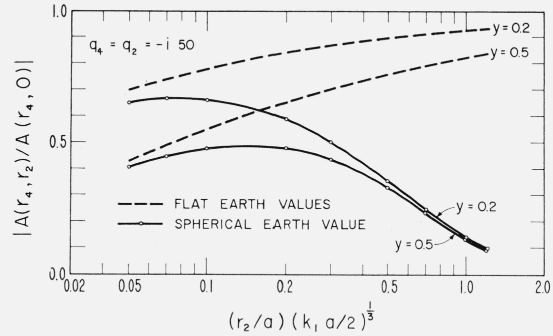


FIGURE 9. *Comparison of the spherical and the flat earth values of attenuation due to diffraction by a cliff on an earth with a finite propagation constant.*

Relative values of attenuation versus the numerical distance $(r_2/a)(k_1a/2)^{1/3}$ when $c_4=10$, for several values of the numerical cliff height $y=k_1h(2/k_1a)^{1/3}$.

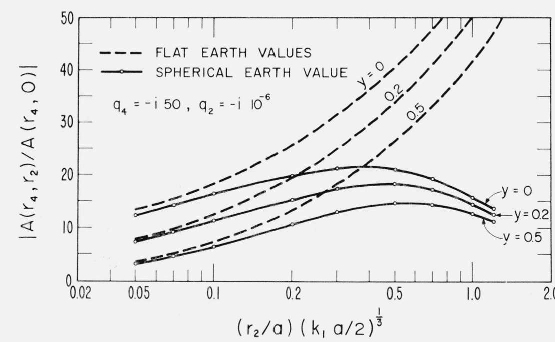


FIGURE 10. *Comparison of the spherical and the flat earth values of attenuation due to diffraction by a bluff on a coastline.*

Upon reference to (31), we see that $T_C(\rho)$ tends to 1 as $\rho \rightarrow 0$.

A set of graphs (fig. 14) is included for $|T_C(\rho)|$ versus ρ for various values of K and b as defined by (36). These figures together with figure 13 for $|T_R(\rho)|$ may be used to determine the gain (or loss) by a cliff or a ridge on a homogeneous earth; they are used in the same way as the ordinary height gain function to evaluate the gain caused by the variation in height of the transmitter and/or the receiver.

3.2. A Bluff at Coastline

This is the case where $|q_2| << |q_4|$ for vertical polarization or $|q_2| >> |q_4|$ for horizontal polarization (see fig. 7). For the numerical illustration, q_2 and q_4 were chosen to be $q_4 = -i50$ and $q_2 = -i10^{-6}$ and the values obtained are shown in figure 10. The flat earth values were computed using formula (32) with

$$y_{32} = y = k_1 h (2/k_1 a)^{1/3},$$

h being the height of the bluff. Figure 10 again shows that the flat earth values give the asymptotic values for $c_2 << 1$. Also shown here is the interesting phenomenon of increasing field strength with the increase in distances from the bluff on the q_2 side (sea).

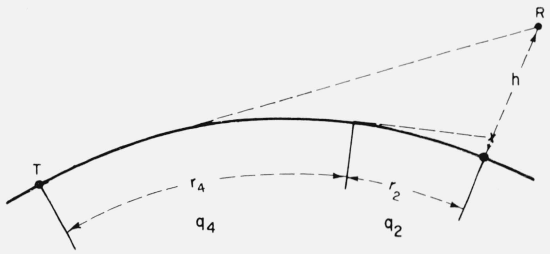


FIGURE 11a. *The form of terrain used in conjunction with figure 11b.*

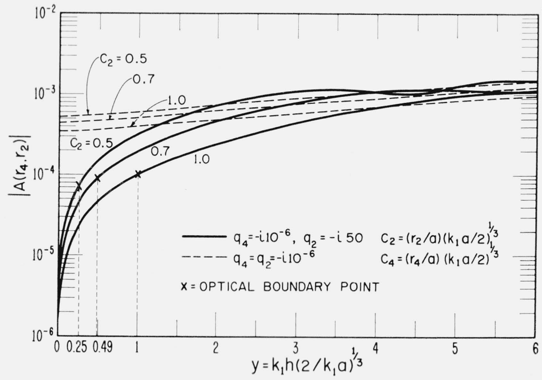


FIGURE 11b. *Field distribution over mixed paths consisting of two different sections.*

Attenuation versus the numerical height $k_1h(2/k_1a)^{1/3}$ of the receiver for several values of the numerical distance c_2.

4. Field Distribution as a Function of Height Over Mixed Paths

The field distribution as a function of the height over a homogeneous earth has been numerically treated by many people. However, there have been few attempts made to find the field in the case of mixed paths. A sketch of mixed paths is shown in figure 11a. In this sketch, the transmitter and the receiver are located over sections of different electrical properties and one of them, say the receiver, is allowed to vary in height.

In figure 11b, the field strength is plotted versus [6]

$$k_1h(2/k_1a)^{\frac{1}{3}}$$

for $c_4=10$ and some values of

$$c_2=(r_2/a)(k_1a/2)^{\frac{1}{3}}$$

both for this case (solid lines) and for a homogeneous earth (dotted lines) specified by q_4. When the receiver is above the optical boundary point (noted in both figs. 11a and 11b by a cross), the field strength becomes more and more dependent on the earth's electrical properties on the transmitter (q_4) side; this is indicated by the asymptotic behavior of the curves in figure 11b. It may be remarked that, although the field strength approaches the values for homogeneous earth at large heights, the points of approach are much higher than the optical boundary points; furthermore, the field strength oscillates with the height at larger heights. This oscillation may be reasonably interpreted as the interference between the principal wave and the wave induced at the boundary of discontinuity between the two sections of the inhomogeneous earth.

5. Formula for the More General Case

Formula (3) gives the attenuation when the wave is propagated over an inhomogeneous earth consisting of two different sections. This formula can be extended for the more general case, illustrated in figure 12. The terrain in this figure consists of several sections (each of which is a homogeneous surface having different heights and electrical properties), with a ridge at each section boundary [Furutsu, 1957b]. The transmitter and the receiver are located at the points x_1 and x_{n+1}, and the attenuation coefficient A is then given by:

$$A=\sum\nolimits_{t_n, t_{n-2}, \ldots, t_4, t_2}(c/c_n)^{1/2}A(y_{n+1,n}, c_n)_{t_n}T^{(n-1)}(c_{n-2})_{t_n, t_{n-2}}$$
$$\times T^{(n-3)}(c_{n-4})_{t_{n-2}, t_{n-4}} \ldots T^{(5)}(c_4)_{t_6, t_4}T^{(3)}(c_2)_{t_4, t_2}f_{t_2}(y_{12}) \quad (38)$$

[6] h denotes the height of the receiver.

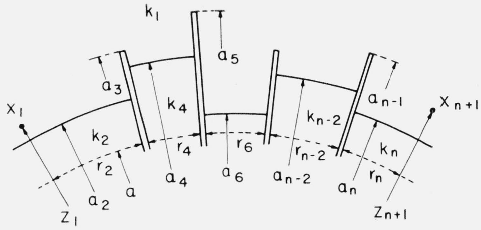


FIGURE 12. *The general form of the terrain as used for (38).*

with the conditions similar to (3b). Here, the functions

$$A(y_{n+1,\,n},\, c_n)_{t_n} \text{ and } T^{(m-1)}(c_{m-2})_{t_m,\, t_{m-2}}$$

($m=n$, $n-1$, . . ., 4) and $f_{t_2}(y_{12})$ are the same as those defined in (5), (6), and (10), respectively, and c is the total numerical distance.

The above formula, (38), agrees with (3) in the special case of $n=4$. Also, when $a_n=a_{n-1}=a_{n-2}=\ .\ .\ .\ =a_3=a_2=a$, formula (38) reduces to the established formula for mixed paths on a smooth earth [Furutsu, 1955c]. Further, the following lemmas hold:

$$\lim_{c_n\to+0}\sum_{t_n} T^{(m)}(c_n)_{t_l,\,t_n} T^{(p)}(c_q)_{t_n,\,t_q} = \begin{cases} T^{(m)}(c_q)_{t_l,\,t_q}, & a_m \geqq a_p \\ T^{(p)}(c_q)_{t_l,\,t_q}, & a_m \leqq a_p, \end{cases} \tag{39}$$

$$\lim_{c_q\to+0}\sum_{t_q} T^{(p)}(c_q)_{t_n,\,t_q} f_{t_q}(y_{sq}) = f_{t_n}(y_{sn}), \qquad z_s \geqq a_p. \tag{40}$$

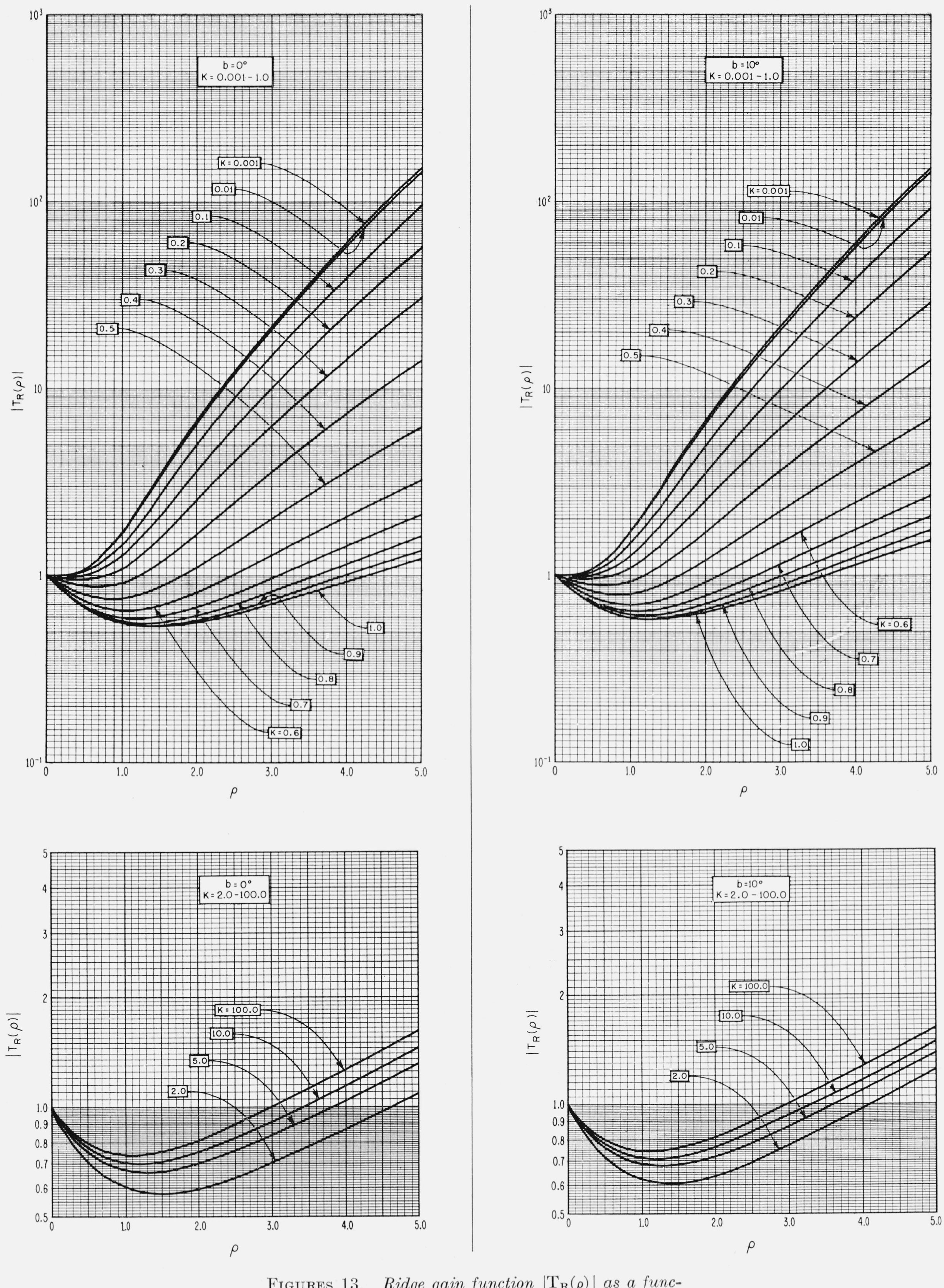
These are required from the self-consistency of the formula when the length of any section involved in the inhomogeneous earth approaches zero, or when the receiver (or the transmitter) approaches the boundary of the section.

6. Summary and Discussion

Some numerical results, based on theory developed in previous paper [Furutsu, 1957a, 1957b, 1959, 1963], are presented for several typical cases of inhomogeneous earth. The results are found for both the spherical earth approximation and the flat earth approximation, and, as expected, the spherical earth values approach the flat earth values at short distances. In the special case of a ridge on a perfectly conducting flat earth, the numerical results are also favorably compared with those obtained using the Fresnel integral.

Many problems which seem to be independent (e.g., mixed paths on a smooth earth, terrains including ridges, cliffs, and bluffs at a coastline, etc.), can be treated by special applications of the same formula. For the first illustration, the effect of a ridge on a lossy homogeneous earth is presented, which shows the obstacle gain at large distances, and the diffraction loss at small distances. Using the same formula in the second illustration, the field strength is found for a varying receiver (or transmitter) height when the wave is propagated over a mixed path. These values are presented along with the values for a homogeneous earth, and the comparison is quite interesting, especially from the optical viewpoint; they approach the optical values at much higher points than expected and also show an interference with another wave which is supposedly induced at the boundary of discontinuity of the mixed path.

The effect of locating a ridge or a cliff on a homogeneous earth is also found by the same basic formula, and sets of graphs (figs. 13 and 14) are presented showing this effect when the



FIGURES 13. *Ridge gain function $|T_R(\rho)|$ as a function of the numerical height ρ.*

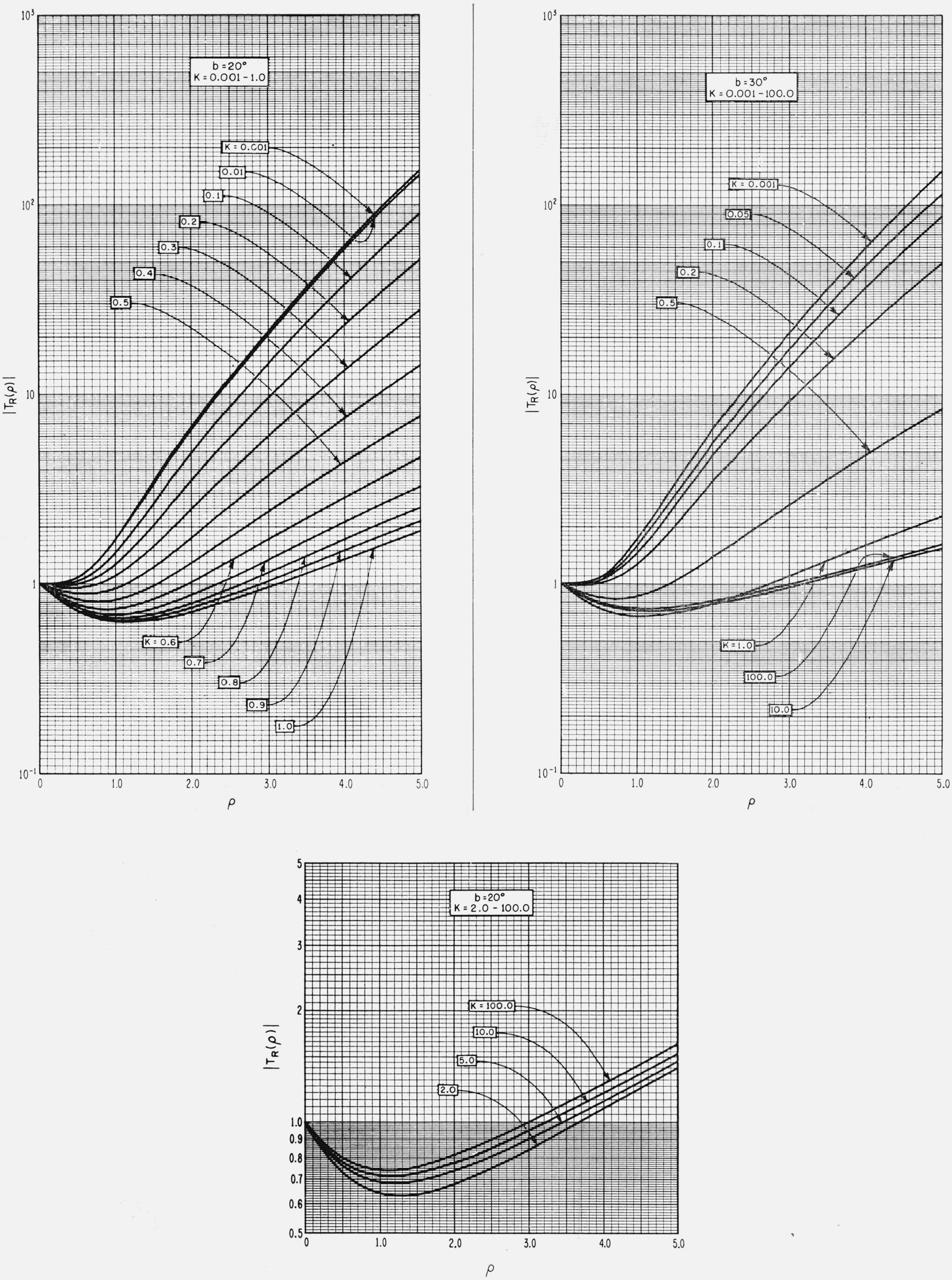


FIGURE 13.—Continued. *Ridge gain function* $|T_R(\rho)|$ *as a function of the numerical height* ρ.

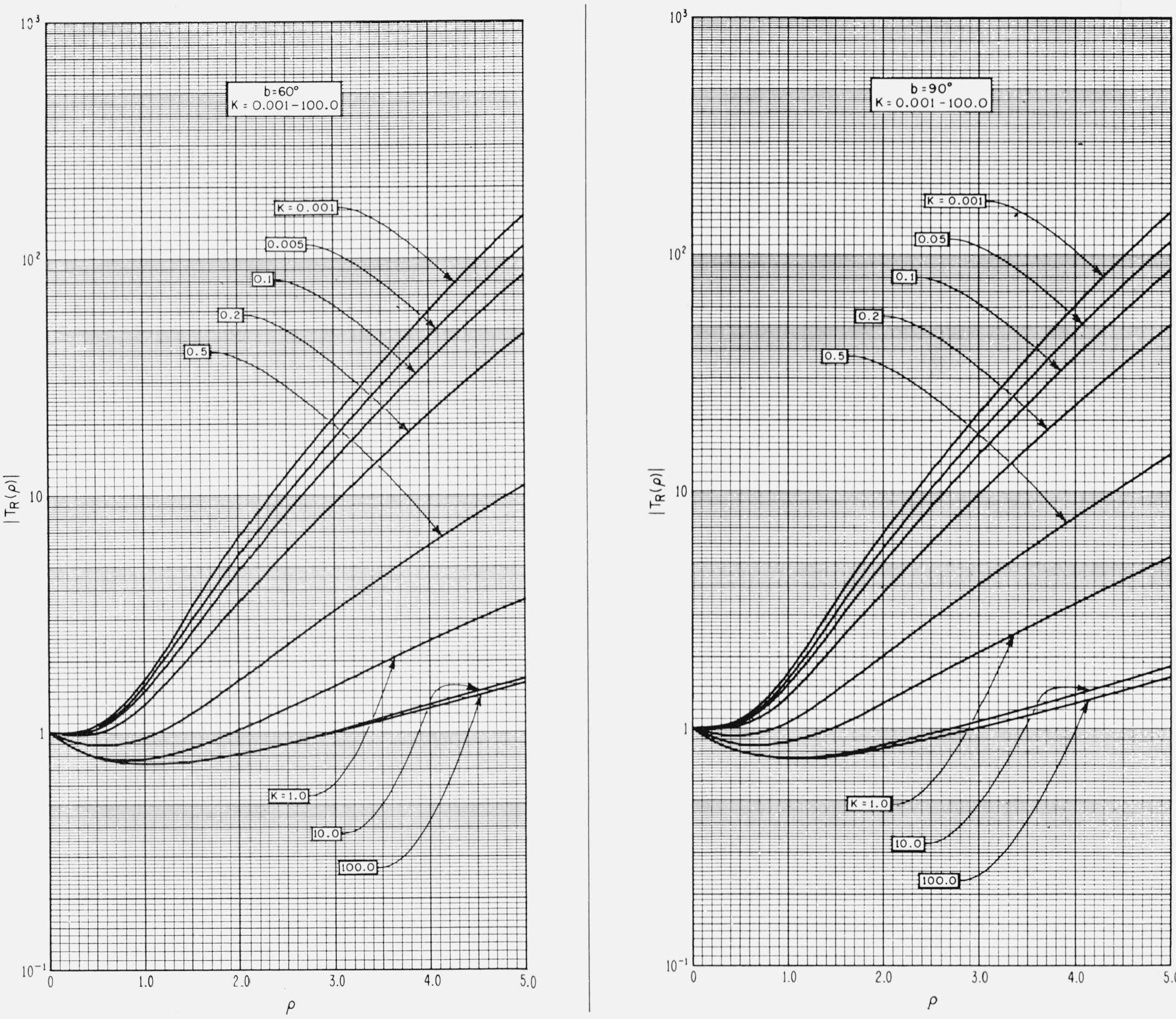


Figure 13.—Continued. *Ridge gain function* $|T_R(\rho)|$ *as a function of the numerical height* ρ.

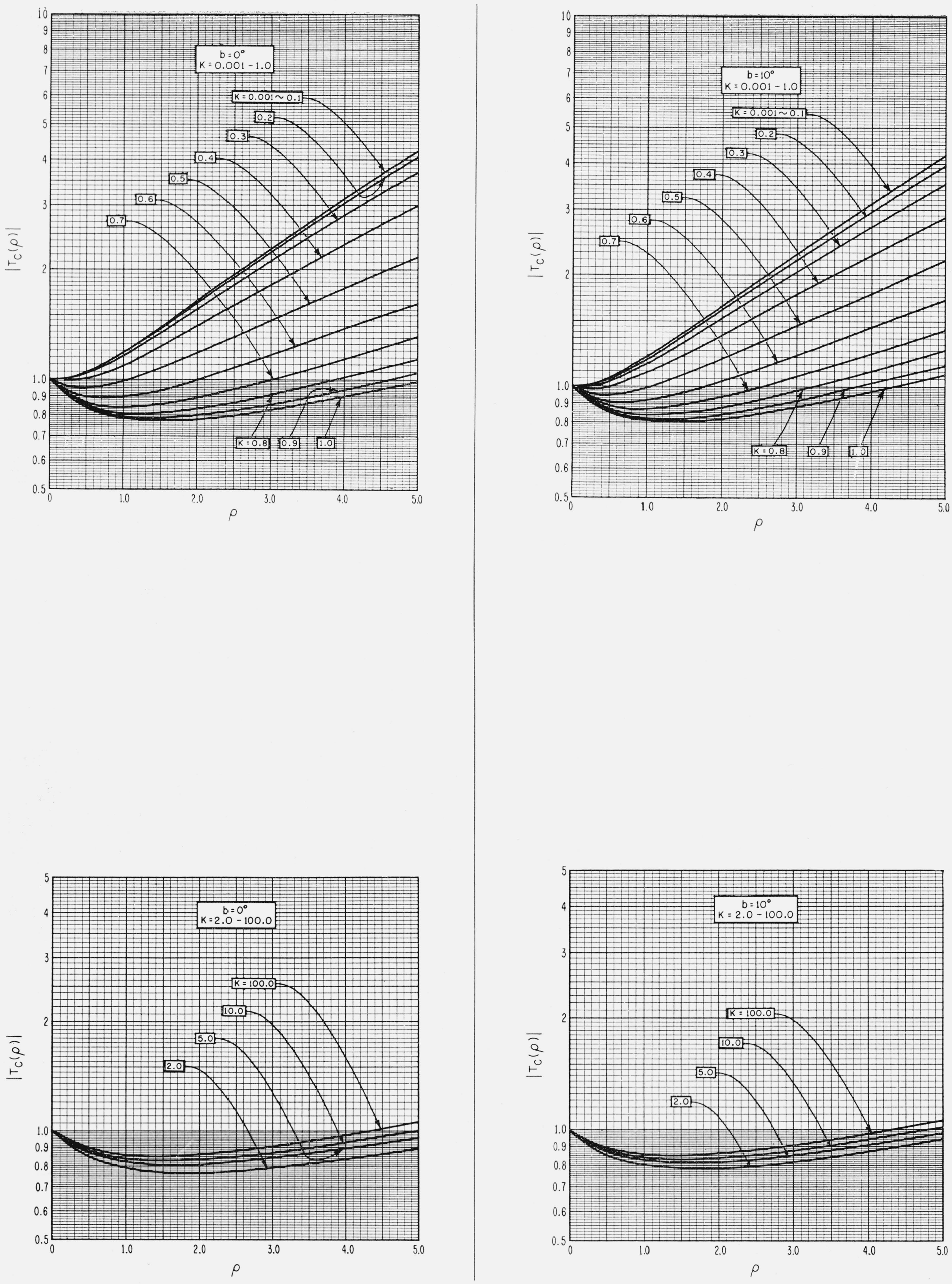


FIGURE 14. *Cliff gain function* $|T_C(\rho)|$ *as a function of the numerical height* ρ.

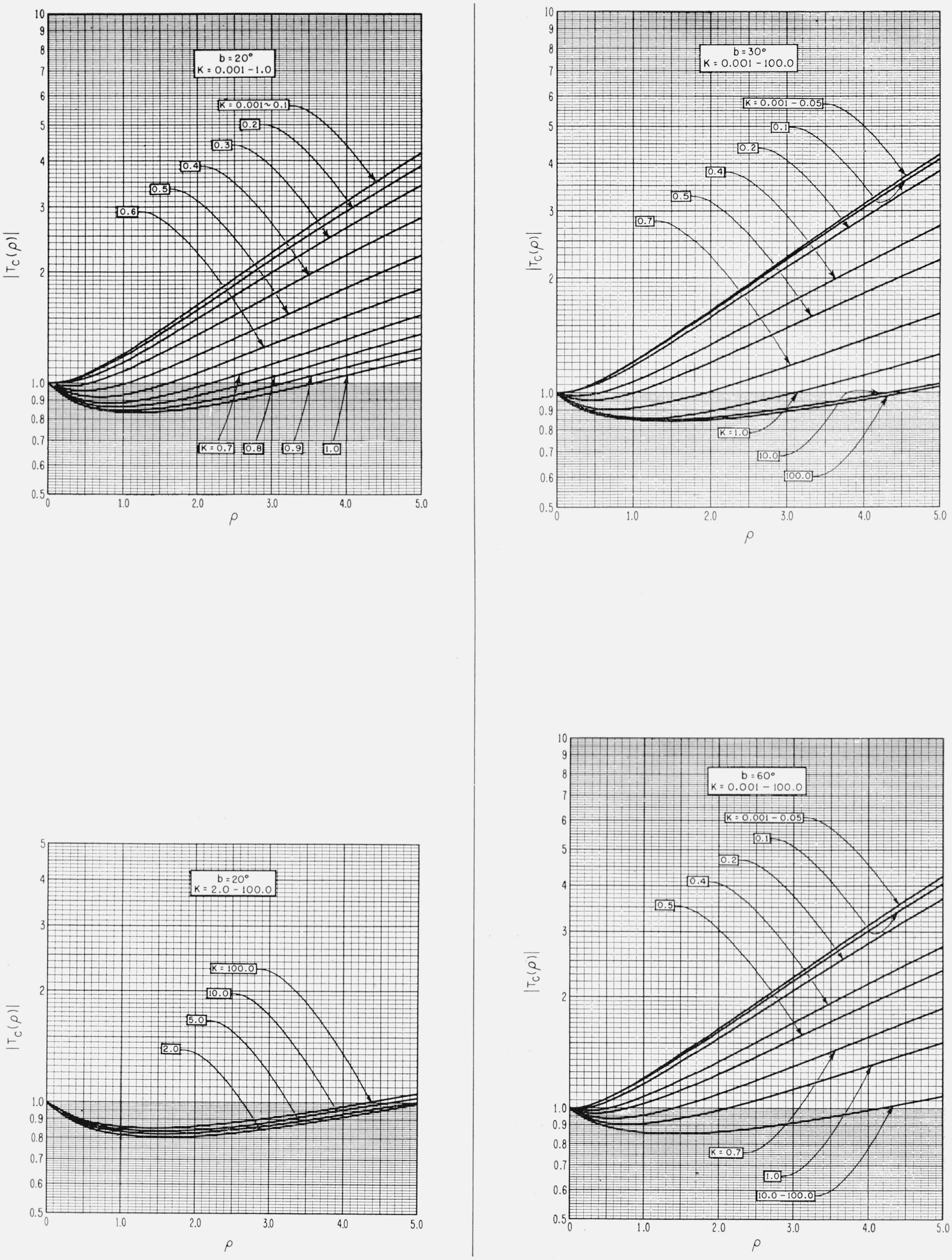


FIGURE 14.—Continued. *Cliff gain function* $|T_C(\rho)|$ *as a function of the numerical height* ρ.

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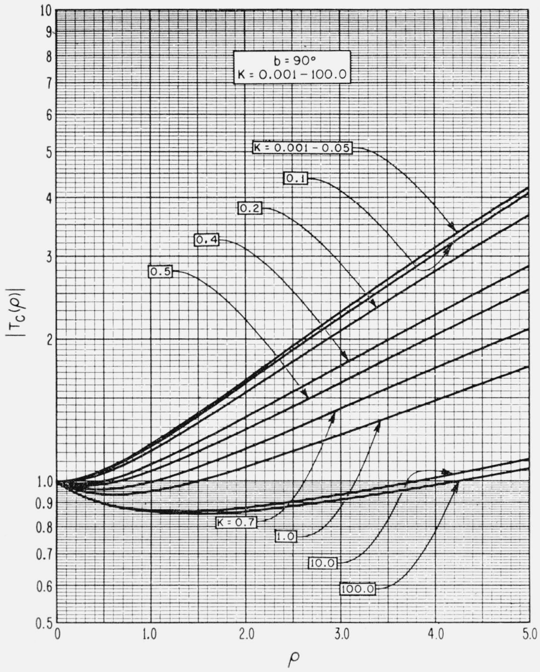

Figure 14.—Continued. *Cliff gain function* $|\mathrm{Tc}(\rho)|$ *as a function of the numerical height* ρ.

distance on both sides of the obstacle is large. As the graphs indicate, the field strength increases continuously with height (denoted by ρ) when the earth is a poor conductor ($K<<1$), but decreases for small ridge (or cliff) heights as the earth becomes a better conductor. It may also be noted that for very large heights the gain caused by a ridge is much larger than that for a cliff of the same height. These sets of graphs may be used in the same way as the ordinary height gain function has been used for evaluating the effect of antenna height of the receiver and/or the transmitter.

A subsequent paper will be published containing sets of graphs to aid in finding the field strength for various values of earth constants, ridge or cliff heights, propagation distances, etc.

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7. Appendix

In this appendix a few examples are presented showing the use of the set of graphs (fig. 13) for the ridge gain function.

The basic variables are the numerical distance c, the numerical height $y=2^{1/3}\rho$, and the earth constants K and b defined in (1), (2), (35), and (36), respectively. In terms of more practical variables, they are given as follows:

$$c=2.188\times f^{1/3}a^{-2/3}r,$$

$$y=9.579\times 10^{-3}\times f^{2/3}a^{-1/3}h,$$

$$\rho=2^{-1/3}y=7.603\times 10^{-3}\times f^{2/3}a^{-1/3}h,$$

$$K=0.3627\times(fa)^{-1/3}\times\begin{cases}(\epsilon^2+s^2)^{1/2}\{(\epsilon-1)^2+s^2\}^{-1/4} & \text{Ver. Pol.}\\ \{(\epsilon-1)^2+s^2\}^{-1/4} & \text{Hor. Pol.,}\end{cases}$$

$$b=\begin{cases}2\tan^{-1}(\epsilon/s)-\tan^{-1}\{(\epsilon-1)/s\} & \text{Ver. Pol.}\\ 180^\circ-\tan^{-1}\{(\epsilon-1)/s\} & \text{Hor. Pol.,}\end{cases} \tag{A.1}$$

with

$$s=18\sigma/f.$$

Here f is the frequency in megahertz, a is the effective radius of the earth in kilometers, r is the propagation distance in kilometers, h is the height in meters, ϵ is the relative dielectric constant of the earth (referred to a vacuum), and σ is the conductivity of the earth in millimhos/meter.

In the following, referring to figure 2, a ridge (of height 100, 200, or 300 m) is located on a homogeneous earth of effective radius 8500 km having the constants $\epsilon=10$ and $\sigma=0.1$ (10^{-15} emu) and a vertically polarized radio wave of frequency 300 MHz is propagated from the transmitter to the receiver, both having the height 10 m, over the distances $r_2=r_4=100$ km. Then, by using (A.1), the following values are obtained for the necessary parameters:

$$\begin{aligned}&c_2=c_4=3.52,\\ &K=0.00885,\\ &b=89.99^\circ,\\ &y_{12}=y_{54}=0.210,\\ &y_{32}=y_{34}=2.10\quad(100),\quad 4.21\quad(200),\quad 6.31\quad(300),\\ &\rho=2^{-1/3}y_{32}=1.67\quad(100),\quad 3.34\quad(200),\quad 5.00\quad(300).\end{aligned} \tag{A.2}$$

Here the values in parentheses are the corresponding ridge heights in meters.

The exact values of attenuation for the data in (A.2) are found, by using formula (3), to be

$$|A|=6.20\times10^{-7}\quad(100),\qquad 4.27\times10^{-6}\quad(200),\qquad 1.91\times10^{-5}\quad(300), \tag{A.3}$$

where the values in parentheses are again the corresponding ridge heights in meters.

On the other hand, when there is no ridge, the corresponding attenuations can be obtained by using the Bremmer series for a homogeneous earth, and the method for obtaining these homogeneous earth values has been given [Norton, 1941]. The homogeneous earth value, $|A_0|$, is then found to be

$$|A_0|=1.49\times10^{-7}. \tag{A.4}$$

Thus, the ridge gains $|A/A_0|$ are

$$|A/A_0|=4.17\quad(100),\qquad 28.7\quad(200),\qquad 128.\quad(300). \tag{A.5}$$

Here, if B is defined by

$$B\equiv c_2-y_{12}^{1/2}-y_{32}^{1/2}, \tag{A.6}$$

giving the left side of condition (33), it has the following values for the data in (A.2):

$$B=1.61\quad(100),\qquad 1.01\quad(200),\qquad 0.55\quad(300). \tag{A.7}$$

Since $b\doteq90^\circ$ according to (A.2), the ridge gains may be obtained from figure 13.6, and the values in (A.5) are found to be the same as those from the graph within the range of graphical error. Indeed, the exact values are

$$4.22\quad(100),\qquad 30.2\quad(200),\qquad 147\quad(300). \tag{A.8}$$

Here it is noticed that, as is seen from the values of B in (A.7), the condition given by (33) is not satisfied for the above data used. However the values in (A.8) may be sufficiently accurate for practical purposes.

It will be worth while to check a case where B is negative; e.g., when $r_2=r_4=50$ km or $c_2=c_4=1.76$ and the other parameters are the same as in (A.2), the exact value of the ridge gain for $h=100$ m is found to be

$$|A/A_0|=3.52 \qquad (100) \tag{A.9}$$

with

$$B=-0.5.$$

Here, the corresponding ridge gain from figure 13.6 is about 4.4, and therefore the error caused by the use of the graph turns out to be about 25 percent or 2 dB.

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