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Impedance of a Monopole Antenna With a Radial-Wire Ground System on an Imperfectly Conducting Half Space—Part III [1]

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The impedance of a vertical monopole placed over the surface of an imperfectly conducting earth and having a radial-wire ground system has been studied theoretically by J. R. Wait. An experimental investigation discussed in part I of this paper gave results in good agreement with Wait's theory. Part II presented calculations based on Wait's theory showing the behavior of antenna impedance as a function of antenna height, top loading, and number, size and length of radial wires. Part III concludes this study with a presentation of calculations showing the effect of the earth conductivity and permittivity on the antenna impedance.

1. Introduction

A vertical monopole antenna over an imperfectly conducting ground excites currents in the ground near the base of the antenna. The consequences of these currents are a substantial loss of power and a low antenna system efficiency. These undesirable effects can be diminished by lowering the effective surface impedance of the ground in the vicinity of the antenna. This in turn, can be accomplished by placing a metallic disk at the surface (or slightly below the surface) of the ground at the base of the antenna or by the similar but more practical method in which the disk is replaced by a system of radial wires. The effect of a radial-wire ground system on the antenna base impedance has been analyzed theoretically by Wait and Pope [1954, 1955], and experimentally by the authors. The experimental investigations produced results in reasonably good agreement with the theoretical work of Wait. This was reported in part I of this paper [Maley and King, 1962]. Although the experimental investigation was for a limited range of values of antenna system parameters, the success of Wait's formulas in predicting the effect of the radial-wire ground system upon the monopole base impedance for these special cases was considered adequate proof of the validity of the formulas.[2] It was then decided to make an extensive set of calculations from Wait's formulas for the range of values of the parameters which would most commonly occur in actual antenna systems.[3] Typical results of the first phase of these calculations were presented in the form of graphs in part II of this paper [Maley and King, 1964]. This was an analysis of the effects of monopole height, top loading, and number, length, and size of the radial wires on the antenna impedance. It was shown that, in general, there is little to be gained by using a number of radials, N, in excess of approximately 250 and that the normalized system radius, b/λ, rarely needs to be greater than 0.2 wavelength. The impedance. Z, is a rapidly varying function of b/λ for small values of b/λ. The real part of the antenna impedance is a monotonic increasing function of both the antenna height, h/λ, and the top-loading parameter, α; this is due to the increase of radiated power.

In this concluding part of the paper, an analysis is made of the effects of earth parameters. Calculated values of impedance are plotted as a function of the various ground system and antenna parameters. These calculations considerably extend those given by Wait and Pope [1955], who employed a combination of analytical and graphical procedures to evaluate the integrals.

2. Theory

The monopole base impedance, Z, as formulated by Wait and Pope [1954], and used throughout this paper, is given by [4] the approximate relation

$$Z \simeq Z^{\infty} + \frac{2\pi}{I_0^2}\int_a^b \eta_e(r)[H_\varphi^\infty(r,0)]^2 r dr + \frac{2\pi}{I_0^2}\int_b^\infty \eta_g [H_\varphi^\infty(r,0)]^2 r dr. \quad (1)$$

It should be stressed that this formula is only valid if b is somewhat greater than a "skin depth" in the ground.

[1] Part I appeared in J. Res. NBS **66D** (Radio Prop.) No. 2, 175–180 (Mar.–Apr. 1962); part II appeared in Radio Sci. J. Res. NBS/USNC-URSI **68D,** No. 2, 159–165 (Feb. 1964).

[2] For the configuration considered, the ground losses were primarily H-field losses [Wait, 1958]. The calculations in this paper consider only these losses.

[3] A more complete set of curves as well as a tabulation of data will appear in a scientific report to be published.

[4] The rationalized MKS System of units is used throughout, and the time dependence is $e^{i\omega t}$.

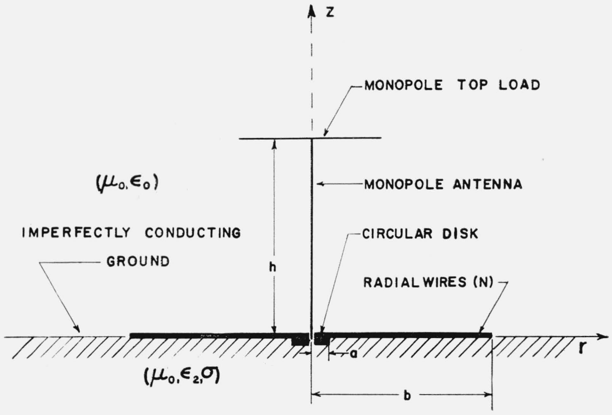

FIGURE 1. *Sketch of antenna system geometry.*

The geometry of the system is shown in figure 1, Z^∞ is the monopole base impedance which would exist if the ground were perfectly conducting. $H_\varphi^\infty(r, 0)$ is the tangential magnetic field for the same condition assuming an antenna base current of I_0; it is given by

$$H_\varphi^\infty(r, 0)=\frac{iI_0}{2\pi \sin \alpha}\left[\frac{e^{-i\beta\rho}}{r} \cos(\beta h-\alpha)-\frac{e^{-i\beta r}}{r}\cos\alpha + \frac{ihe^{-i\beta\rho}}{\rho r}\sin(\beta h-\alpha)\right], \quad (2)$$

h is the actual antenna height; $\beta=2\pi/\lambda$; λ is the free space wavelength and $\rho=(r^2+h^2)^{1/2}$. α is the top-loading parameter for the assumed sinusoidal antenna current distribution,

$$I(z)=I_0\frac{\sin(\alpha-\beta z)}{\sin\alpha}, \quad (3)$$

which is valid for this antenna. The virtual antenna height is $h+h'$; so $\alpha=\beta(h+h')$ and lies in the range $2\pi h/\lambda \leq \alpha \leq \pi/2$, the lower and upper limits corresponding to $h'=0$ and full top-loading respectively. $\eta_e(r)$ is the assumed effective surface impedance in the region $a<r<b$; it is given by [Wait, 1954]

$$\eta_e=\frac{\eta_g\eta_w}{\eta_g+\eta_w} \quad (4)$$

where

$$\eta_w=\frac{i2\pi\eta_0 r}{\lambda N}\ln\left[\frac{r}{Nc}\right] \quad (5)$$

and

$$\eta_g=\left[\frac{i\mu\omega}{\sigma+i\omega\epsilon_2}\right]^{1/2}, \quad (6)$$

η_0 is the intrinsic impedance of free space and c is the radius of the wire.

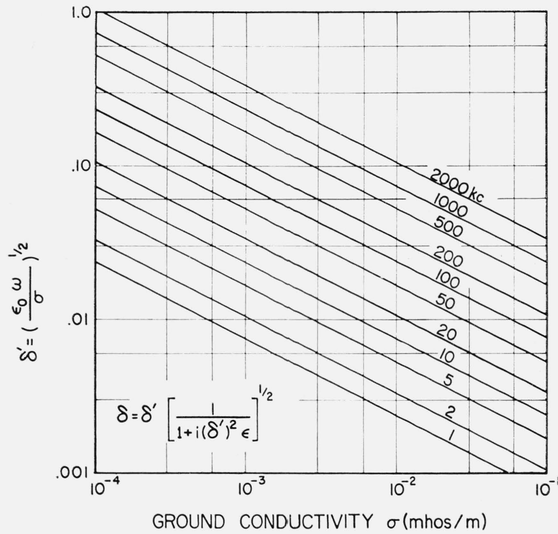

FIGURE 2. *δ' versus σ for various frequencies.*
(Use (9) to get δ.)

3. Calculations

Calculations were made from (1) for ΔZ, which is defined as $\Delta Z=Z-Z^\infty$. Thus, it is the amount by which the actual impedance differs from that for the ideal case of a perfectly conducting ground plane. The real and imaginary parts of ΔZ are ΔR and ΔX respectively.

The permittivity ϵ_2 and conductivity σ of the imperfectly conducting earth are accounted for in the parameter δ which can be defined in terms of η_g;

$$\eta_g=\left[\frac{i\mu\omega}{\sigma+i\omega\epsilon_2}\right]^{1/2}=\sqrt{i}\eta_0\left[\frac{(\delta')^2}{1+i(\delta')^2\epsilon}\right]^{1/2}=\sqrt{i}\eta_0\delta, \quad (7)$$

where $\epsilon=\epsilon_2/\epsilon_0$ is the dielectric constant and

$$\delta'=\left[\frac{\omega\epsilon_0}{\sigma}\right]^{1/2}. \quad (8)$$

Also

$$\delta=\delta'\left[\frac{1}{1+i(\delta')^2\epsilon}\right]^{1/2}=|\delta|e^{-i\psi}, \quad (9)$$

where

$$\psi=\frac{1}{2}\tan^{-1}[(\delta')^2\epsilon]. \quad (10)$$

If $(\delta')^2\epsilon<<1$, then $\psi\cong 0$ which corresponds to a highly conducting earth and negligible displacement currents, and if $(\delta')^2\epsilon>>1$, $\psi\simeq\pi/4$ which corresponds to large displacement currents and negligible conduction currents. Figure 2 [Wait, 1954 and 1955] may be used to determine δ' in (8) for known frequencies and conductivities. Equations (9) and (10) may then be used to calculate δ.

The ranges of δ and ψ were chosen to be $0.01\leq|\delta|\leq 0.3$ and $0\leq\psi\leq\pi/4$. Figures 3 through 7 show the effect of varying either $|\delta|$ or ψ for commonly used values of N and $C=c/\lambda$ of 100 and 10^{-6} respectively.

4. Conclusions

Figure 3 shows how the real and imaginary parts of $\Delta Z = \Delta R + i\Delta X$ vary with ground system radius b/λ using δ as a parameter with $\psi=0$. Note that for certain values of b/λ, $|\delta|$ has very little effect on ΔR, and for other values it has little effect on ΔX. A somewhat more meaningful presentation of the data is given in figures 4a, b, and c. These show that the variation of ΔR with δ is much more rapid for small b/λ and is proportional to $\sqrt{f/\sigma}$ except for very small values of δ. Again, one of the conclusions of part II is reached, namely that ΔZ is nearly independent of the earth parameters for b/λ greater than about 0.20.

A comparison of the two parts of figures 4a and b shows that if the top-loading is increased ΔR increases resulting in a greater power radiated by the antenna. Similarly, comparing figures 4a, b, and c for $\alpha=\pi/2$ shows that ΔR varies over an increasingly larger range as a function of δ with increasing antenna height. Part II shows that the variation of ΔR with h/λ is nearly proportional to $K(h/\lambda)^k$, where K and k are constant, especially for h/λ less than 0.10 and no top loading.

Figure 4c shows ΔR for $N=100$ and $N=\infty$ and the indicated system and ground parameters. Typically, the results are seen to be nearly the same for the two values of N if b/λ is less than 0.10, the difference being greatest for large σ (small δ). The reason for this is that the distance between adjacent radials at $r/\lambda > 0.10$ becomes so large that the radials are nearly ineffective. To make the radials effective at $r/\lambda > 0.10$ requires an increased number of radials.

Some of the data for ΔR in figures 4a, b, and c was plotted on figure 5. This shows that for a specified ΔR, b/λ is nearly proportional to $|\delta|$. It is evident that for small δ (large σ) a further increase of σ can have a considerable effect upon the value of b/λ necessary for a given ΔR, especially when ΔR itself is small. Thus, it is distinctly advantageous

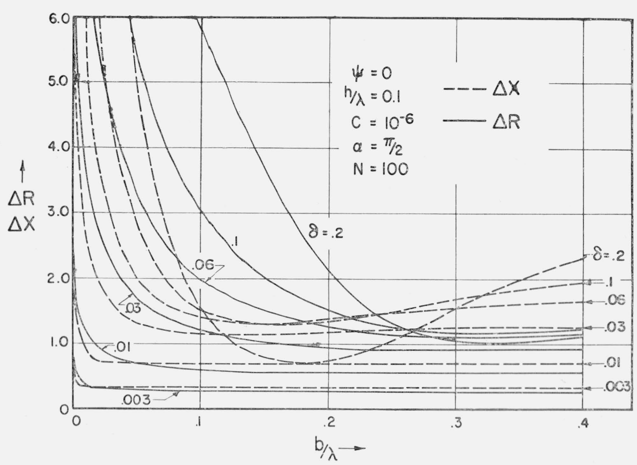

FIGURE 3. ΔR *and* ΔX *versus length of radial wires for selected values of* δ *and* $\psi=0$.

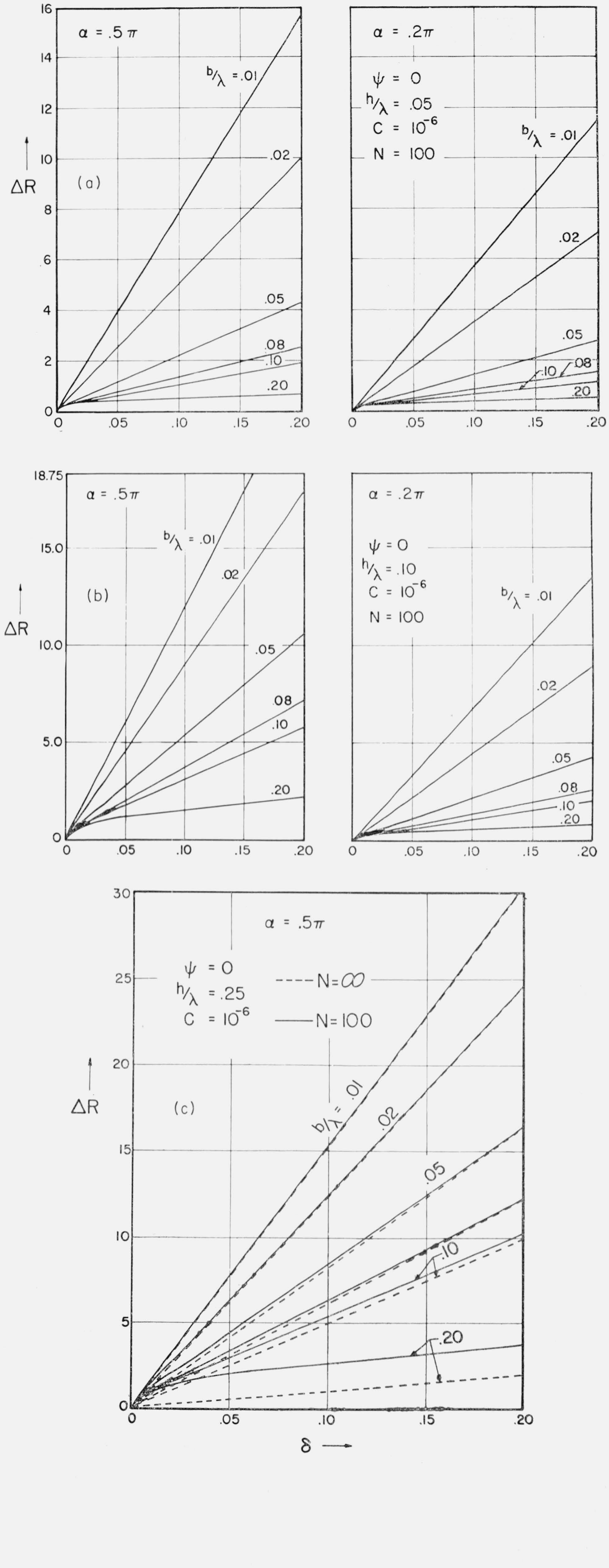

FIGURE 4. ΔR *versus* δ *for* $\psi=0$ *with* b/λ *and* α *as the parameters.*
(a) $h/\lambda=0.05$, (b) $h/\lambda=0.10$, (c) $h/\lambda=0.25$.

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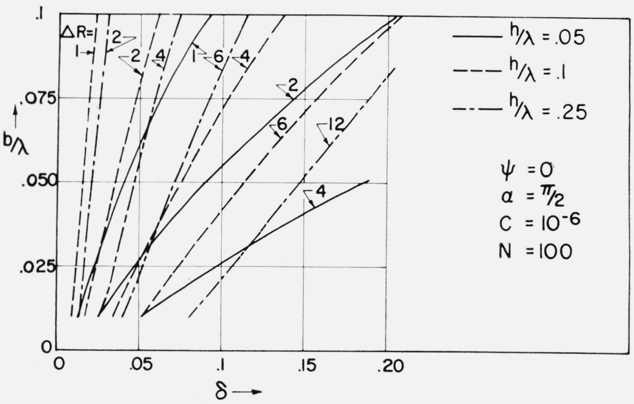

FIGURE 5. b/λ *versus* δ *for selected values of* ΔR *and three values of* h/λ.

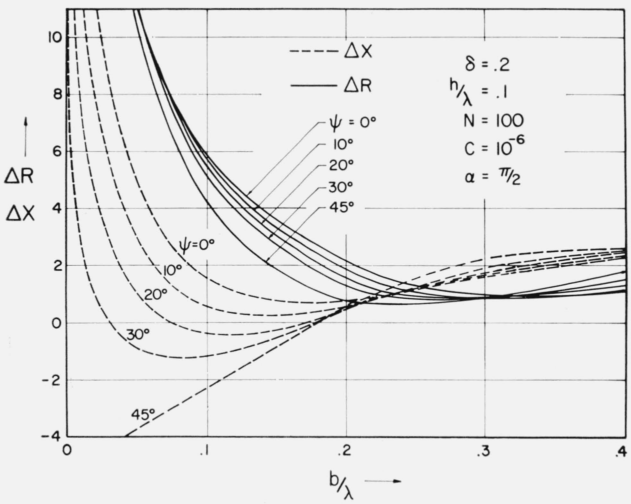

FIGURE 6. ΔR *and* ΔX *versus* b/λ *for selected values of* ψ.

$\psi=0$ corresponds to a highly conducting earth, and $\psi=\pi/4$ corresponds to a arge proportion of ground displacement currents to conduction currents.

to locate the antenna over a highly conducting ground because the length of the radials can then be made smaller. For example, for $h/\lambda=0.10$ and $\Delta R=2.0$ and the parameters specified for figure 5, b/λ can be decreased by a factor of 10 when σ is increased by a factor of approximately 13.

Figures 6 and 7 show the effect of ψ on the antenna impedance. The former shows that ΔX is generally affected much more than ΔR as ψ increases especially for small b/λ. Increasing ψ corresponds to decreasing conduction currents as compared to displacement currents. Note that ΔR is hardly affected by changes in ψ at certain values of b/λ and ΔX is hardly affected at other values of b/λ. The fact that ΔR is a slowly varying function of ψ is more clearly seen in figures 7a and b. Again, it is evident that ΔR is affected very little for large b/λ. Comparison of the two parts of figures 7a and b show that the antenna height has considerably more influence on ΔR than ψ does.

Although figures 6 and 7 may be useful for design purposes, they must be interpreted with caution since ψ cannot be varied without simultaneously varying

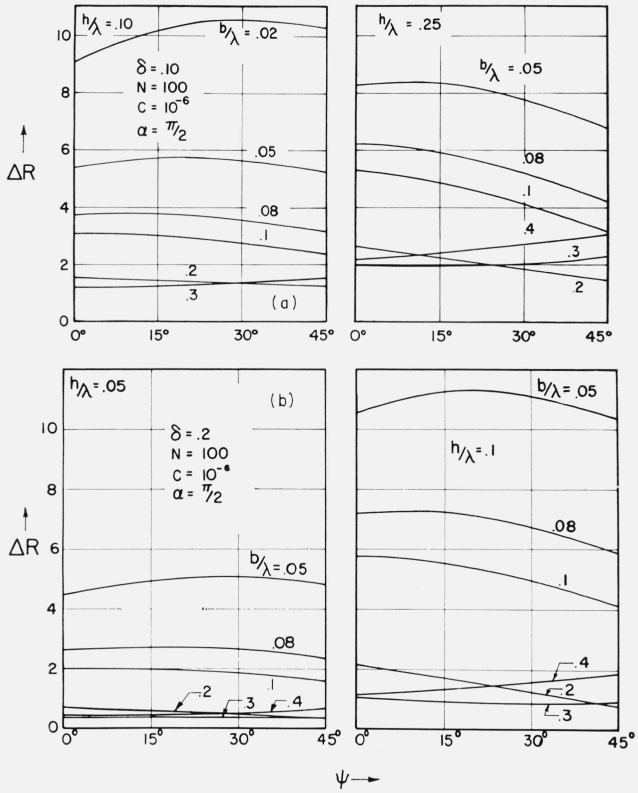

FIGURE 7. ΔR *versus* ψ *for selected values of* b/λ *and* h/λ

(a) $|\delta|=0.10$, (b) $|\delta|=0.20$.

$|\delta|$ due to the nature of (9). Cases where ψ is large are generally only of academic interest.

In summary, the following general conclusions can be made concerning antenna systems in which the ground power losses are predominately *H*-field losses:

1. It is usually not necessary to have a number of radials, N, in excess of approximately 250. However, when the length of the radials is large (which seldom happens) there is a definite advantage for using a large value of N.

2. Antenna system efficiency is increased by increasing the length of the radial wires, but the effect is small for increases beyond a certain limit which depends upon N, h, α and $|\delta|$. It is rarely worthwhile to make $b/\lambda>0.2$.

3. Changes in the size of the radial wires, within the range considered, have little effect on antenna impedance.

4. Increasing the antenna height and/or top-loading results in increased antenna resistance.

5. The incremental antenna resistance, ΔR, is approximately directly proportional to $\sqrt{f/\sigma}$. Location of the antenna over a highly conducting earth has a definite advantage over locating it over a poorly conducting earth.

6. If $|\delta|$ is kept constant and the angle ψ is allowed to change the antenna resistance is effected very little while its reactance may change considerably. Increasing ψ corresponds to increasing the proportion of displacement currents to conduction currents.

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5. References

Maley, S. W., and R. J. King (Mar.–Apr. 1962), Impedance of a monopole antenna with a radial-wire ground system on an imperfectly conducting half-space, part I, J. Res. NBS **66D** (Radio Prop.), No. 2, 175–180.

Maley, S. W., and R. J. King (Feb. 1964), Impedance of a monopole antenna with a radial-wire ground system on an imperfectly conducting half-space, part II, Radio Sci. J. Res. NBS/USNC–URSI **68D,** No. 2, 159–165.

Wait, J. R., and W. A. Pope (1954), The characteristics of a vertical antenna with a radial conductor ground system, Appl. Sci. Res. Sect. B **4,** 177–195. Note that in eq 5, $\sin(\beta h-\alpha)$ should be $\sin(\alpha-\beta h)$.

Wait, J. R., and W. A. Pope (May 1955), Input resistance of LF unipole aerials, Wireless Engineer **32,** 131–138.

Wait, J. R. (1958), On the calculations of transverse current loss in buried wire ground systems, Appl. Sci. Res. Sect. B **7,** 81–86.

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